Lattice calculation of nucleon EDM

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for RBC/UKQCD collaboration

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Outline

- Introduction
  - Motivation of lattice calculation of EDM
- Strategy and method in lattice QCD
  - Lattice QCD
  - Extraction of EDM from correlation function
- Recent update (preliminary)
- Summary and future work

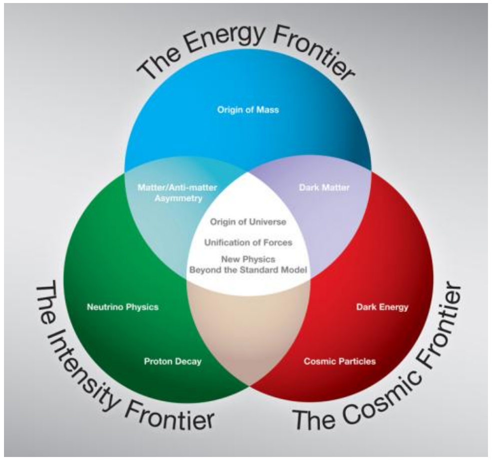
Neutron EDM

- CPV in QCD and the new physics (NP)
- Since 1960's, sensitivity of experiment has been developed.
  - Current nEDM upper limit is $|d_N^{\rm exp}| < 2.9 \times 10^{-26}\,{\rm e \cdot cm}$
- Sensitive observable to NP
  - Naturally QCD has CPV from θ term, but it seems to be unnaturally small. (strong CP problem)
  - Direct search of CPV from NP

    BSM (SUSY, etc) says the discover is coming soon…
  - Intensity frontier physics

    Alternative direction from high energy collision.

    Precision of the SM calculation is necessary.

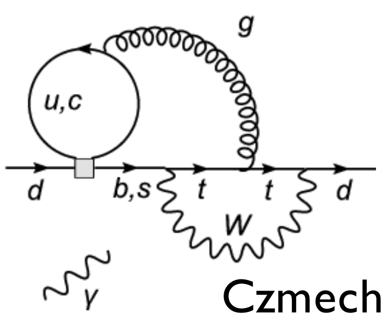


http://www.fnal.gov/pub/science/frontiers/

Nucleon EDM in EW

- Contribution to EDM in weak interaction is very small
  - No CP phase in 1-loop ($|V_{dq}|^2$) and 2-loop diagram (cancelation)
  - Three-loop order(short) and pion loop correction (long):

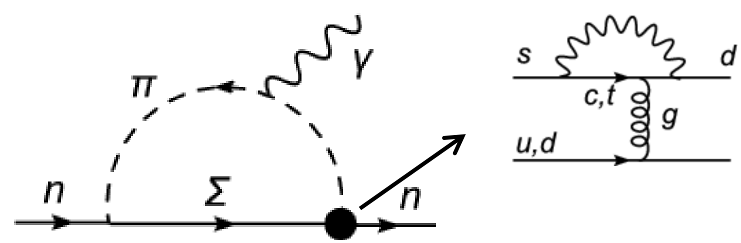
Short distance



Czmechi, Krause (1997)

$$d_N^{\mathrm{KM\,short}} \sim \mathcal{O}(\alpha_s G_F^2) \sim -10^{-34}\ \mathrm{e \cdot cm}$$

Long distance



Khriplovich, Zhitnitsky (1982)

$$d_N^{\mathrm{KM\,long}} \sim 10^{-30} – 10^{-32}\ \mathrm{e \cdot cm}$$

$$\Rightarrow d_N^{\mathrm{KM}} = d_N^{\mathrm{KM\,short}} + d_N^{\mathrm{KM\,long}} \simeq 10^{-30} – 10^{-32}\ \mathrm{e \cdot cm}$$

which is the 6-order magnitude below the experimental upper limit.
(to confirm, non-pertubative estimate is also needed)

1. Introduction

Nucleon EDM in QCD

- θ term in QCD Lagrangian

$$\mathcal{L} = \bar{q}_L M q_R + \bar{q}_R M^\dagger q_L + \theta/(64\pi^2) G\widetilde{G}$$

$$\Rightarrow \mathcal{L}_\theta = \bar{\theta}\frac{1}{64\pi^2}G\widetilde{G}, \quad \bar{\theta} = \theta + \arg\det M, \quad \arg\det M \sim \eta\frac{m_u m_d m_s}{m_u m_d + m_d m_s + m_s m_u}$$

  - Renormalizable and CPV.
  - $d_N/\theta \sim 10^{-16}$ e・cm (quark model, current algebra, etc)

  θ and arg det M is *unnaturally* canceled. Crewther, et al. (1979), Ellis, Gaillard (1979)

- Possible solution

  1. Massless quark (m_u = 0) Blum, et al. (2010)

  from lattice QCD+QED, m_u = 2.24(35) MeV, m_d = 4.65(35) MeV.

  It is hard to explain $\bar{\theta} = 0$.

  2. *Axion model (assumption of PQ symm.),* invisible axion model

  3. *Spontaneous CP breaking.* θ is calculable in loop order.

1. Introduction

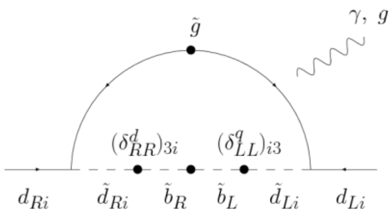
Nucleon EDM in BSM

- Higher dimension operators of CPV

$$H_{CP} = \sum_k C_k(\mu)\, \mathcal{O}_k \qquad \begin{aligned} \mathcal{O}_{\mathrm{qEDM}} &= d_q \bar{q}(\sigma \cdot F)\gamma_5 q \\ \mathcal{O}_{\mathrm{cEDM}} &= d_q^c \bar{q}(\sigma \cdot G)\gamma_5 q \\ \mathcal{O}_{\mathrm{Weinberg}} &= d^G G G \tilde{G} \end{aligned}$$

: Quark-photon (5-dim)
: Quark-gluon (5-dim)
: Pure gluonic (6-dim)

- In effective Hamiltonian, the new CPV term appears.
- In BSM, q(c)EDM corresponds to CP phase of heavy particle.
- d_q d_q^c are determined by BSM



Hisano, et al. (2009)

To obtain EDM, we need to estimate QCD effect in nucleon.

Nucleon EDM in BSM

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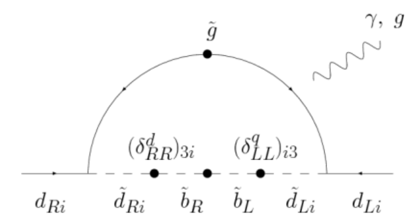
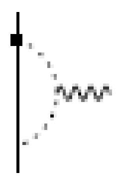


Hisano, et al. (2009)

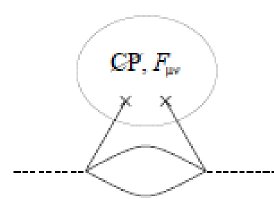
To obtain EDM, we need to estimate QCD effect in nucleon.

Using baryon CHPT or QCD sum rule, there are several evaluations,



…

Mereghetti, Vries, Hockings, Maekawa, Kolck, Timmermans, ...



…

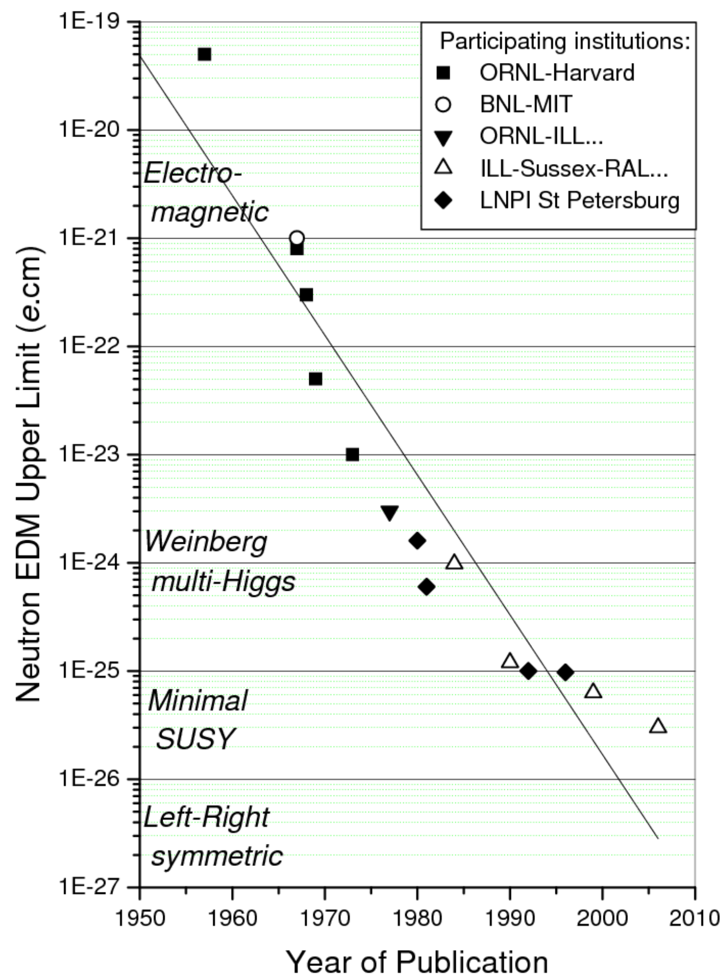
Pospelov, Ritz, Hisano, Shimizu, Nagata, Lee, Yang, …

$$\begin{aligned} d_N &= d_N^{\rm QCD}\bar{\theta} + d_N(d_q, d_q^c) + d_N(d^G) \\ &\sim 10^{-17}[\mathrm{e}\cdot\mathrm{cm}]\bar{\theta} + (1.4\text{–}0.47)d_d - (0.12\text{–}0.35)d_u + O(10^{-2})d_q^c \\ &\sim O(10^{-25}\text{–}10^{-27})\ \mathrm{e}\cdot\mathrm{cm} \end{aligned}$$

Hisano, Shimizu (04), Ellis, Lee, Pilaftsis (08), Hisano, Lee, Nagata, Shimizu (12)

Constraint on nEDM

- EDM experiment
  (p,d)EDM experiment @ BNL,
  nEDM experiment @ ORNL, ILL, FRM-2,
  FNAL, PSI/KEK/TRIUMF, …
  Charged particle (d,p)EDM @ COSY
  Lepton EDM @ J-PARC, FNAL
  aiming for a sensitivity to 10^{-29} e・cm !
- Current estimate of QCD effect is based on quark model or BChPT, and it includes sort of model dependence.
- Non-perturbative contribution of θ term, qEDM, cEDM, etc is needed.
- πNN coupling from lattice QCD is also useful for study of atomic EDM (schiff moment)



Harris, 0709.3100

What lattice QCD can do for nEDM

- In principle
  - Direct estimate of hadronic contribution to neutron and proton EDM for θ term, higher dim. CPV operators
  - Matrix elements (or condensate) including higher dimension operators

    $\rightarrow$ for QCD sum rule, ChPT, …

Bhattacharya et al, Lattice 2012

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  - Statistical noise

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- In practice there are some difficulties
  - Statistical noise

    gauge background (topological charge, sea quark) and *disconnected diagram* (flavor singlet contraction) are intrinsically noisy.
  - Systematic study

    Volume effect may be significant. (e.g. BChPT discussion)

    O'Connell, Savage, PLB633, 319(2006), Guo, Meissner, 1210.5887

    Chiral behavior is also important check, $d_N \sim O(m)$.

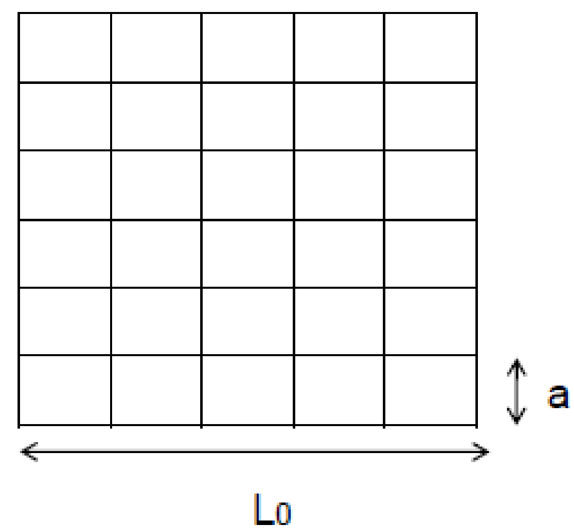
Strategy and method in lattice QCD

Lattice QCD

In lattice regularization, the path integral is computed by Monte-Carlo integral:

$$\langle O \rangle = Z^{-1} \int D\Psi\, O(\Psi) e^{-S(\Psi)} \simeq \sum_{\Psi} O(\Psi) P(\Psi)$$

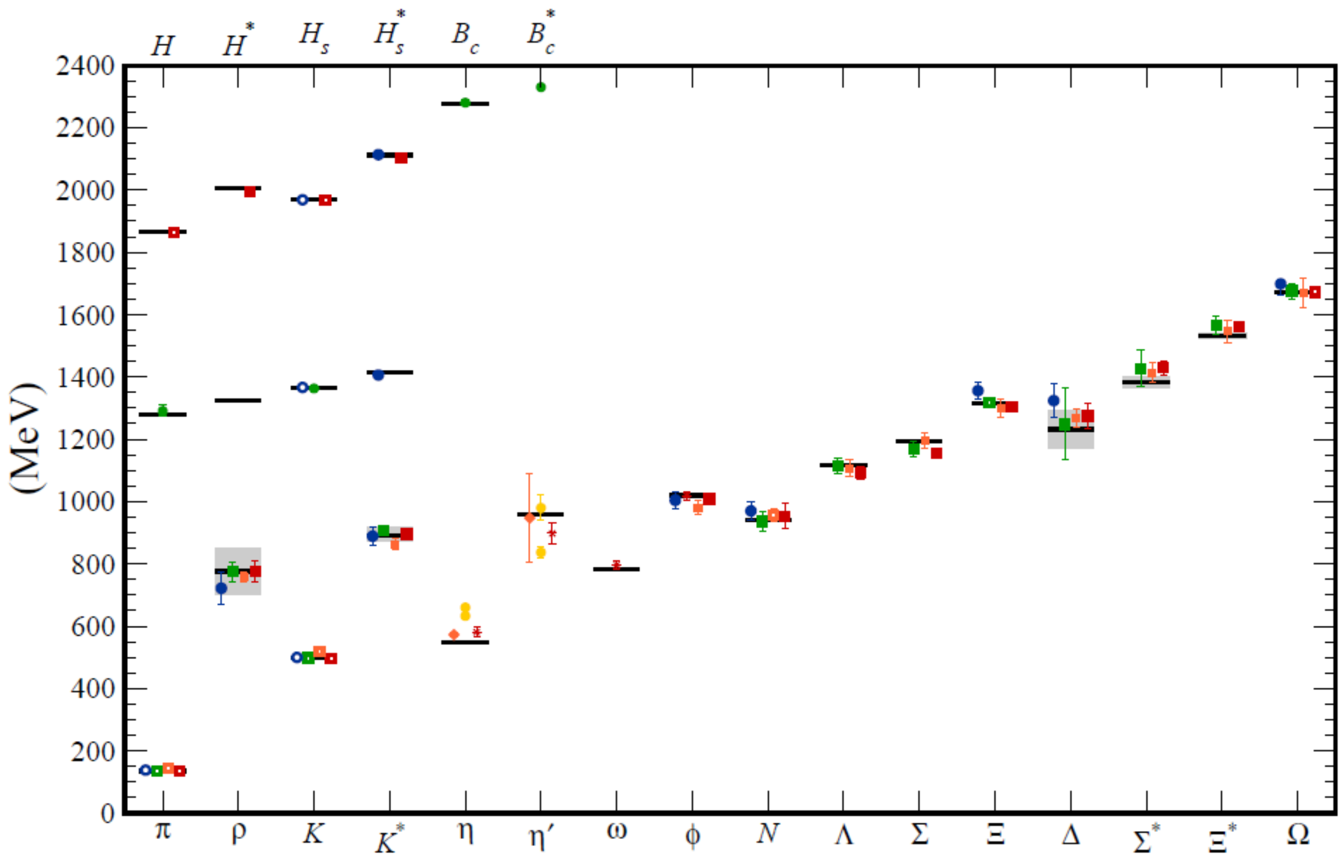
- Exact QCD calculation (enough large number of sampling N)
- Gauge invariant
- Translational invariant
- Ultraviolet cut-off a (lattice spacing)
  Infrared cut-off $V=L_0^D$ (lattice volume)
- Continuum limit, and infinite volume are important.
- The development of machine (BG, GPGPU, …)
  and algorithm, which make much progress.



Hadron spectrum in lattice QCD

- Good agreement with various lattice action and fermion with experimental results !

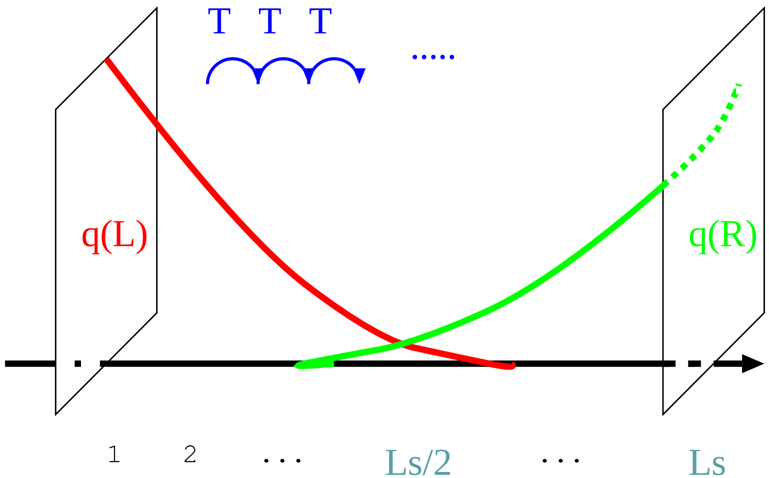
Kronfeld, 1209.3468



Choice of lattice fermion

- There are several kinds of fermion definition on the lattice.
- Require "realistic" fermion for the precise calculation
  - which has good approximated chiral symmetry on the lattice.
  - Good suppression of O(a) effect
  - Domain-wall fermion is appropriate selection.
- Domain-Wall fermion (DWF)

[Blum Soni, (97), CP-PACS(99), RBC(00), RBC/UKQCD. (05 --)]



- L, R fermion are localized on boundaries.
  Exact chiral symmetry is realized if $L_s \to \infty$.
- In finite L_s
  Violation of chiral symm. is suppressed as
  $am_{res} \sim \exp(-L_s) \ll 1$.
- Because of additional dimension, computational cost is much higher than Wilson fermin or staggered fermion.

Lattice methods of nEDM

- Spectrum method
  1. S.Aoki and A. Gocksch, Phys. Rev. Lett. 63, 1125 (1989).
  2. S.Aoki, A. Gocksch, A. V. Manohar, S. R. Sharpe, Phys. Rev. Lett. 65, 1092 (1990), in which they discussed about the possible lattice artifact in ref.1 results
  3. ES, et al., for CP-PACS collaboration, Phys. Rev. D75, 034507 (2007)
  4. ES, S. Aoki, Y. Kuramashi, Phys. Rev. D78, 014503 (2008)
- Form factor
  1. ES, et al., for CP-PACS collaboration, Phys. Rev. D72, 014504 (2005).
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EDM Form factor

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$$\begin{aligned}\langle n(P_1)|J_\mu^{\mathrm{EM}}|n(P_2)\rangle_\theta \;&=\; \bar{u}_N^\theta\Big[\underbrace{\frac{F_3^\theta(q^2)}{2m_N}\gamma_5\sigma_{\mu\nu}q_\nu}_{\text{P,T-odd}} + \underbrace{F_A(q^2)(iq^2\gamma_\mu\gamma_5 - 2m_N q_\mu\gamma_5)}_{\text{P-odd}} \\ &+\; \underbrace{F_1(q^2)\gamma_\mu + \frac{F_2(q^2)}{2m_N}\sigma_{\mu\nu}q_\nu}_{\text{P,T-even}}\Big]u_N^\theta\end{aligned}$$

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$$\sum_s u_N^\theta(s)\bar{u}_N^\theta(s) = \frac{ip\cdot\gamma + m_N e^{i\alpha_N^\theta\gamma_5}}{2E_N}$$

CPV phase α_N in nucleon propagator

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$$\langle\theta|\eta_N J_\mu^{\mathrm{EM}}\bar{\eta}_N|\theta\rangle = \langle 0|\eta_N J_\mu^{\mathrm{EM}}\bar{\eta}_N|0\rangle + i\theta\langle 0|\eta_N J_\mu^{\mathrm{EM}} Q\bar{\eta}_N|0\rangle$$

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$$\langle 0|\eta_N(t_1)J_\mu^{\rm EM}(t)Q\bar{\eta}_N(t_0)|0\rangle$$

Computation

$$= \frac{\alpha_N}{2}\gamma_5\left[F_1\gamma_\mu + F_2\frac{q_\nu\sigma_{\mu\nu}}{2m_N}\right]\frac{ip\cdot\gamma + m_N}{2E_N} + \frac{1+\gamma_4}{2}\left[F_1\gamma_\mu + F_2\frac{q_\nu\sigma_{\mu\nu}}{2m_N}\right]\frac{\alpha_N}{2}\gamma_5$$

Subtraction

$$+\frac{1+\gamma_4}{2}\left[F_3\frac{q_\nu\gamma_5\sigma_{\mu\nu}}{2m_N} + F_A(iq^2\gamma_\mu\gamma_5 - 2m_N q_\mu\gamma_5)\right]\frac{ip\cdot\gamma + m_N}{2E_N}$$

Extraction

- Subtraction of CP-odd phase, α_N, in nucleon propagator and CP-even part $F_{1,2}$

EDM Form factor

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CPV phase α_N in nucleon propagator

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Computation

$$= \frac{\alpha_N}{2}\gamma_5\left[F_1\gamma_\mu + F_2\frac{q_\nu\sigma_{\mu\nu}}{2m_N}\right]\frac{ip\cdot\gamma + m_N}{2E_N} + \frac{1+\gamma_4}{2}\left[F_1\gamma_\mu + F_2\frac{q_\nu\sigma_{\mu\nu}}{2m_N}\right]\frac{\alpha_N}{2}\gamma_5$$

Subtraction

$$+\frac{1+\gamma_4}{2}\left[F_3\frac{q_\nu\gamma_5\sigma_{\mu\nu}}{2m_N} + F_A(iq^2\gamma_\mu\gamma_5 - 2m_N q_\mu\gamma_5)\right]\frac{ip\cdot\gamma + m_N}{2E_N}$$

Extraction

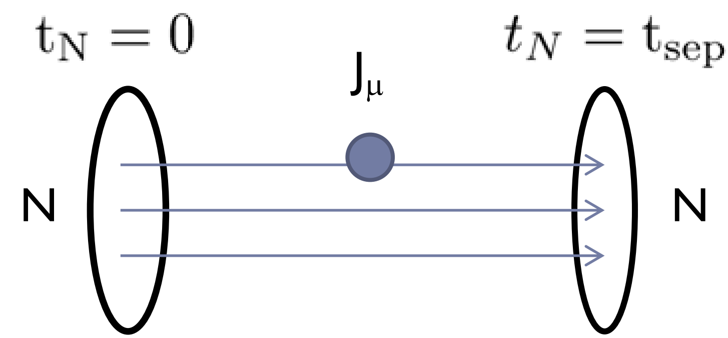
- Subtraction of CP-odd phase, α_N, in nucleon propagator and CP-even part $F_{1,2}$
- EDM is given in zero momentum transfer limit $d_N = \lim_{Q^2\to 0} F_3(Q^2)/2m_N$

3-pt function

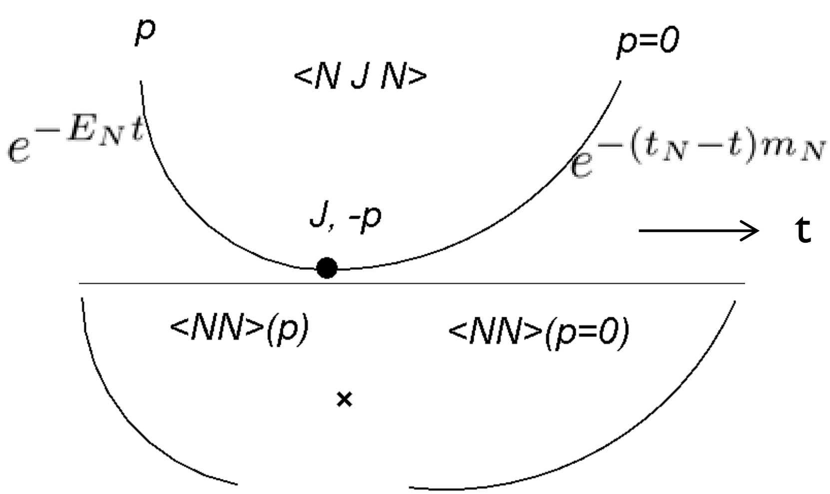
- (Nucleon)-(EM current)-(Nucleon)
- Location of operators

  $t = 0$ and $t = t_N$: nucleon op.

  EM current inserts between nucleon ops.

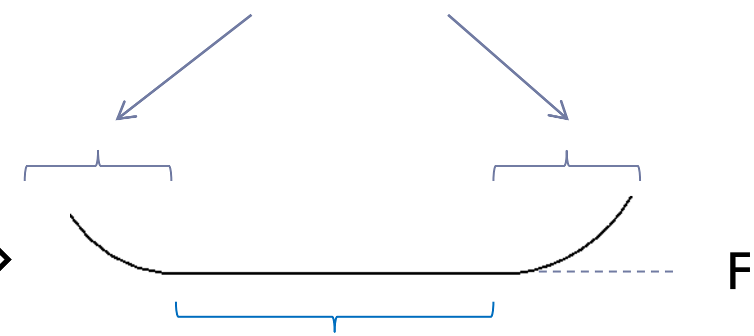


- Comparison of different t_{sep} is good check of excited state contamination.
- Ratio of 3-pt and 2-pt



$\Rightarrow$

Excited state contamination



The signal appears as plateau.

Recent update

Recent work on EDM from lattice

- θ term
  - New developed algorithm, called as **AMA method**, which is error reduction techniques without additional cost.

    Blum, Izubuchi, ES, Phys.Rev. D88, 094503 (2013), ES (lattice 2012)

  - Extremely high statistics for form factors in DWF
  - Ingredients to extract EDM form factor: ES, Blum, Izubuchi, Lattice 2013
    - EM form factor
    - topological charge distribution
    - CPV phase of nucleon wave function, α_N
    - 3pt function in CP-odd sector

Parameters

- DWF
  - $24^3 \times 64$ lattice, a^{-1} = 1.73 GeV (~3 fm^3 lattice)
  - L_s = 16 and am_{res} = 0.003
  - m = 0.005, 0.01 corresponding to m_π = 0.33, 0.42 GeV
  - Two temporal separation of N sink and source in 3 pt. function
    t_{sep} = 12 (t_{source} = 0, t_{sink} = 12), t_{sep} = 8 (t_{source} = 0, t_{sink} = 8)
  - # configs = 751 (m=0.005), 700 (m=0.01) [t_{sep} = 12]
    # configs = 180 (m=0.005), 132 (m=0.01) [t_{sep} = 8]

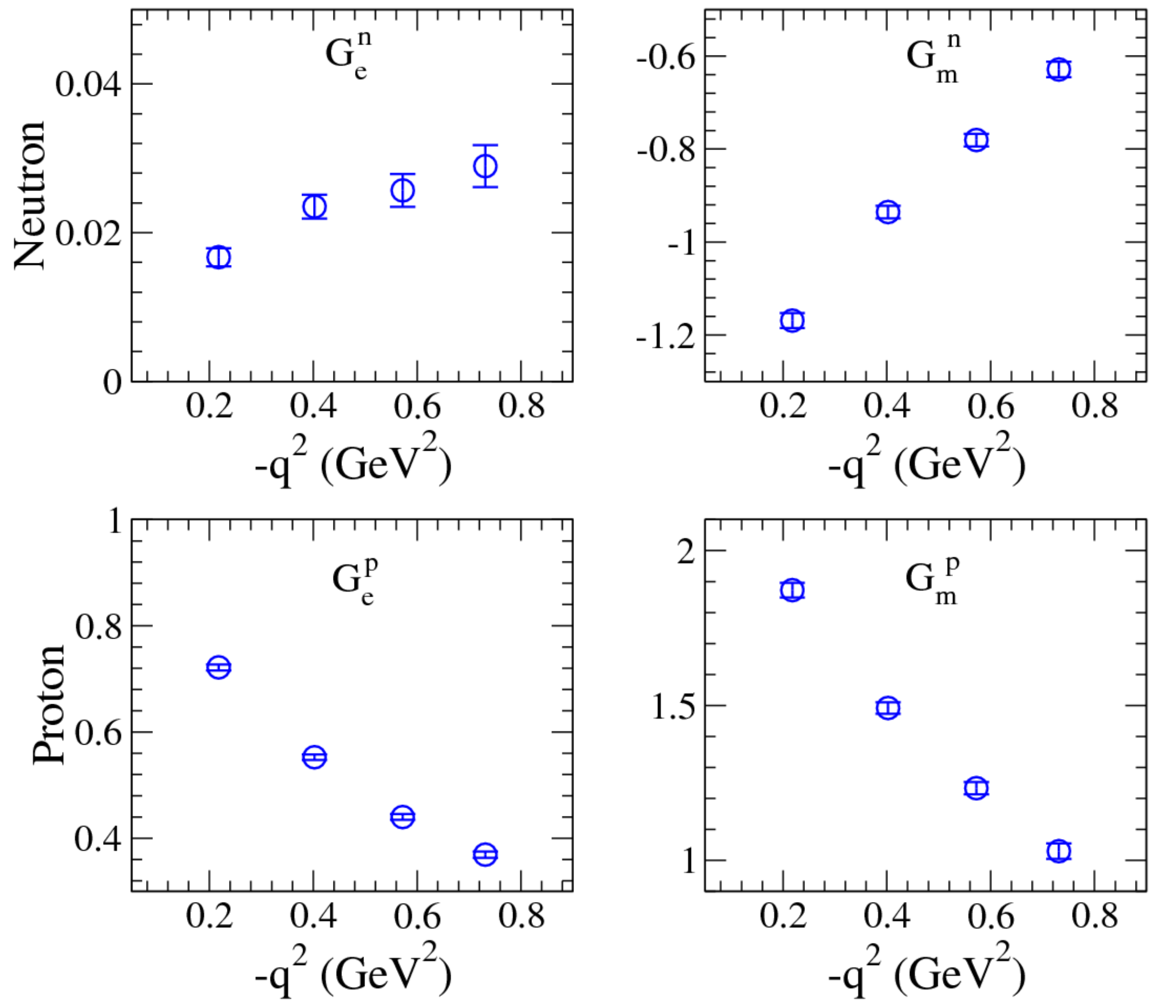
    Comparison to check the higher excited state contamination
- AMA
  - # of low-mode : N_λ = 400 (m=0.005), 180 (m=0.01)
  - Stopping condition, $|r| < 0.003$
  - N_G = 32 (2 separation for spatial, 4 separation for temporal direction of source localtion) → effectively O(10^4) statistics

3. Recent update (preliminary)

EM form factor

- By using AMA algorithm, statistical error of these observables achieve below 5% level.

- Compared with previous works (RBC PhysRevD79(2009)), computational time can be reduced by factor 5 and more.
⇒ higher precision



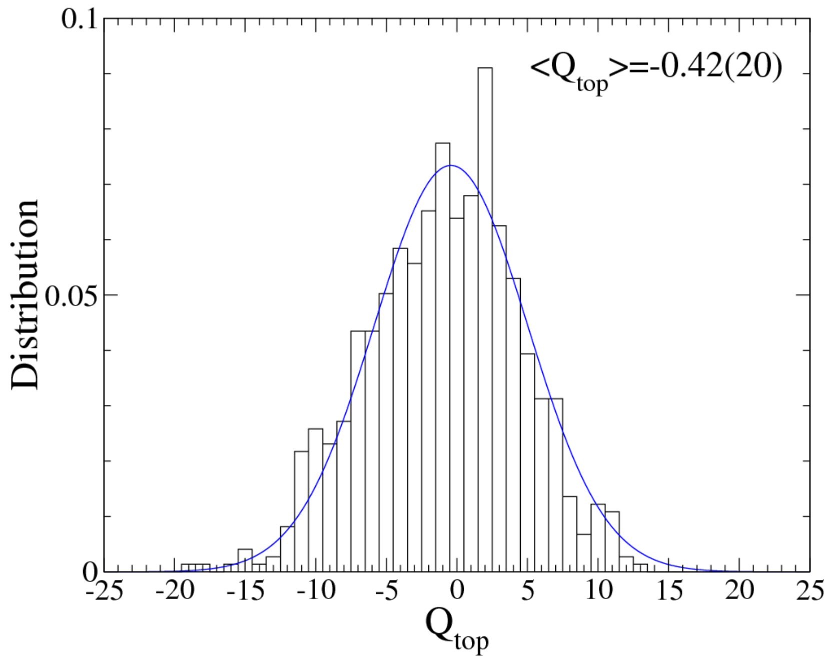
Topological charge distribution

- Topological susceptibility

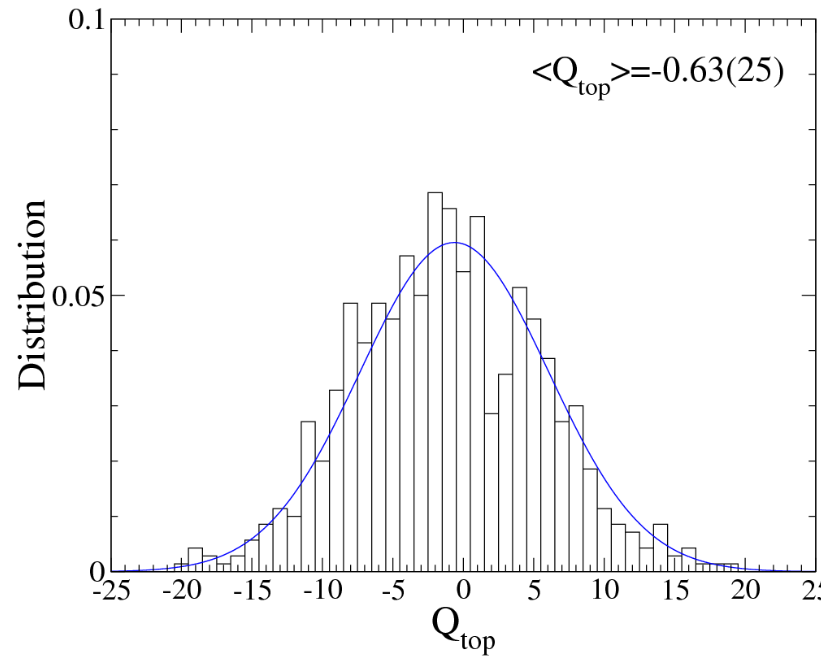
$$\left.\begin{aligned}\langle Q^2\rangle/V &= 3.0(1)\times 10^{-4}\,\mathrm{GeV}^4\;(m=0.005)\\ &= 4.6(2)\times 10^{-4}\,\mathrm{GeV}^4\;(m=0.01)\end{aligned}\right\}$$

Suppression by quark mass as expected in ChPT

m=0.005



m=0.01



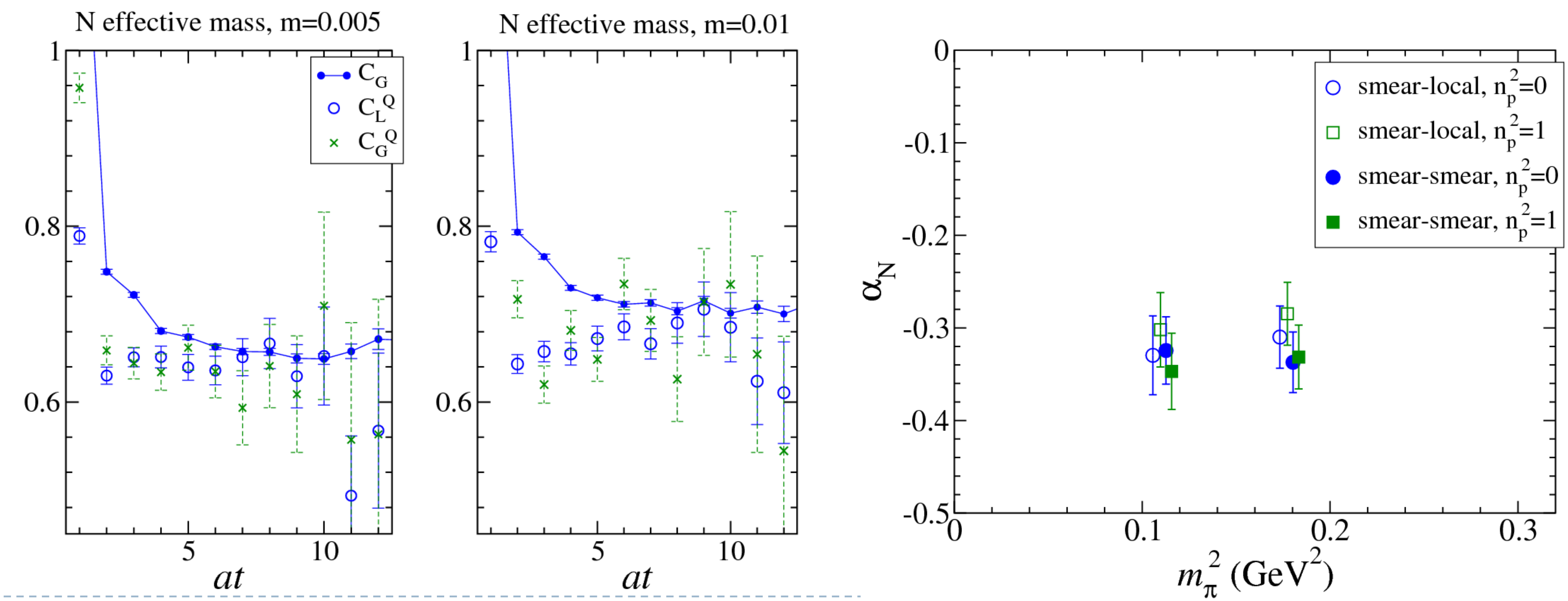
3. Recent update (preliminary)

α_N : CP-odd phase of wave function

- Projection with γ_5 for 2 pt in θ term, perform global fitting

$$\mathrm{tr}\left[\gamma_5\langle N(t)\bar{N}(0)Q\rangle\right] = Z_N\frac{2m_N}{E_N}\alpha_N e^{-E_N t} + O(e^{-E_{N^*}})$$

- By using AMA, this factor is determined within 15 % error.
- It does not depend on smearing function and momentum (mass dependence is not so clear)

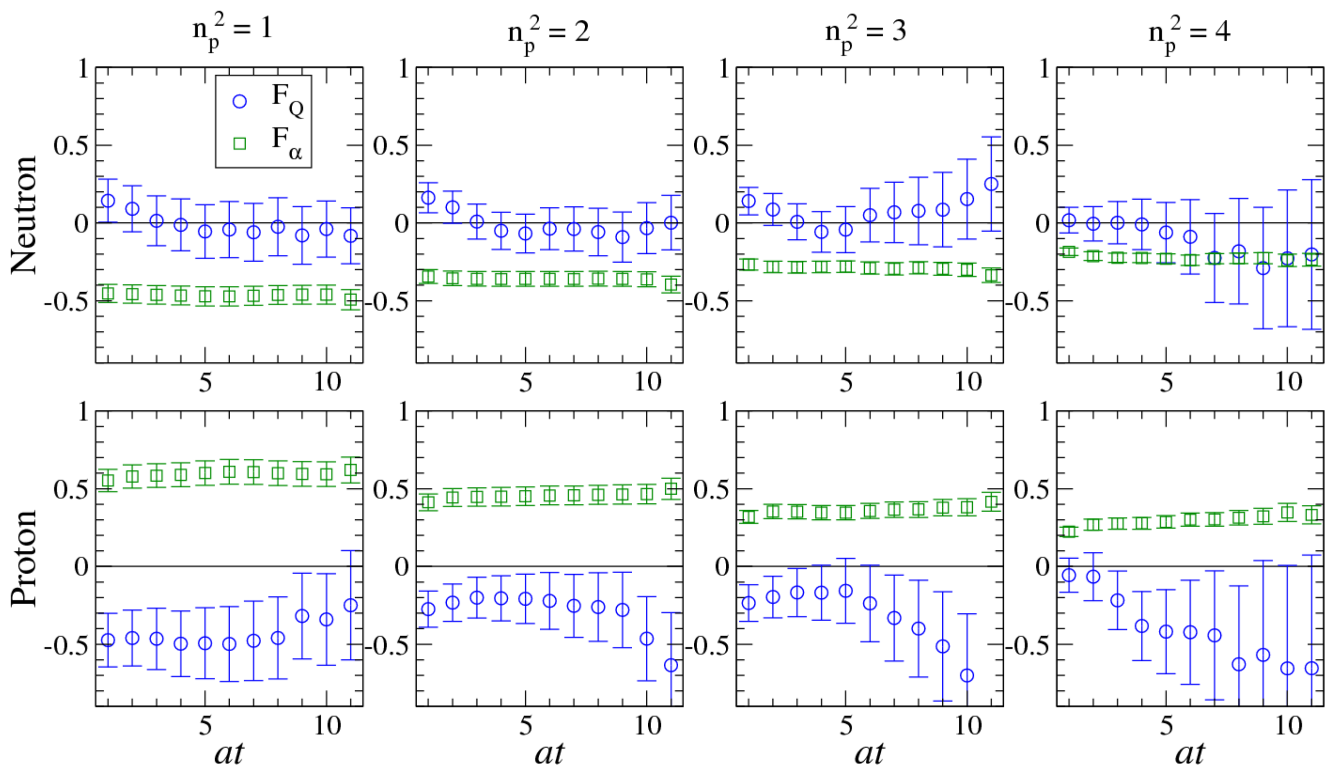


Subtraction term and 3pt function

- Splitting EDM form factor into two parts:

$$F_3 = F_Q + F_\alpha,\ F_Q = C(m_N)\langle N J_t^{\mathrm{EM}} \bar{N} Q\rangle,\ F_\alpha = F(\alpha_N, F_{1,2})$$

- F_α is good precision, and fluctuation of F_Q is large.



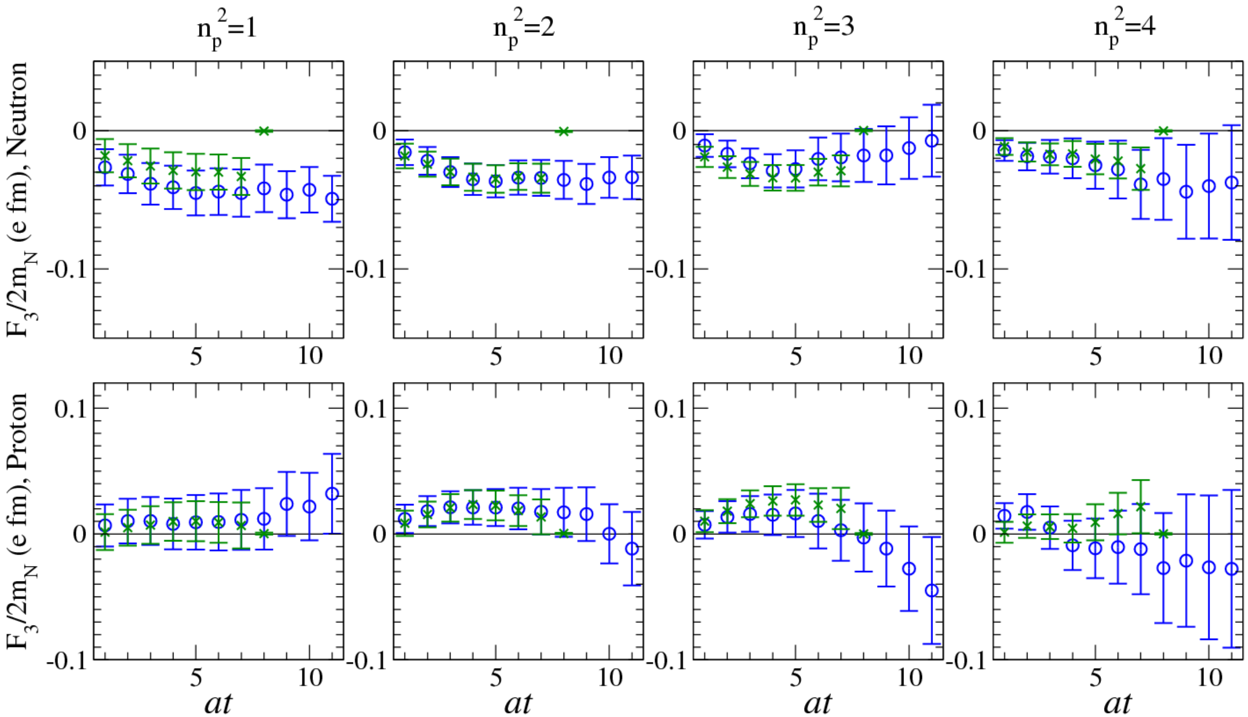
Comparison with different t_{sep}

- The sink and source separation in 3pt function enables us to control the statistical noise and excited state contamination

- Comparison

  t_{sep} = 12 (blue), [N_{conf} = 751]
  t_{sep} = 8 (green) [N_{conf} = 180]

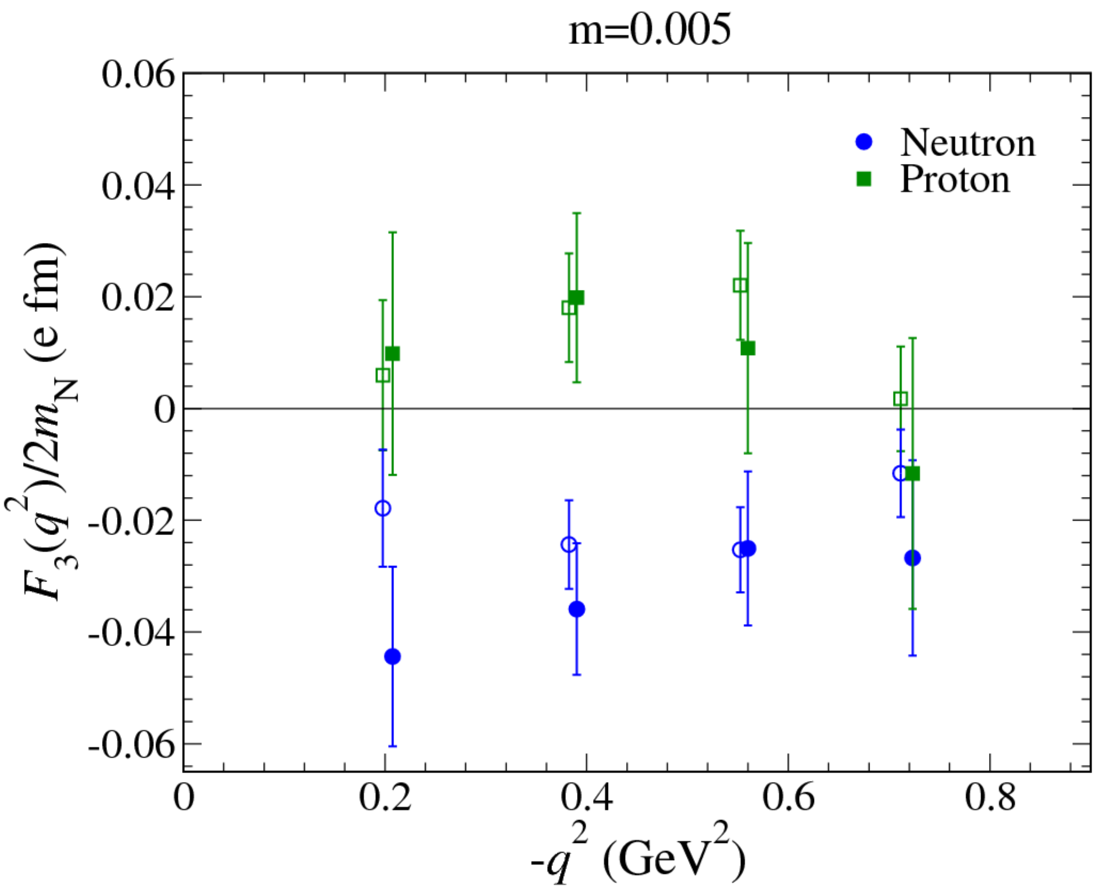
  - Good consistency between them.
  - Precision in t_{sep}=8 is much better.



3. Recent update (preliminary)

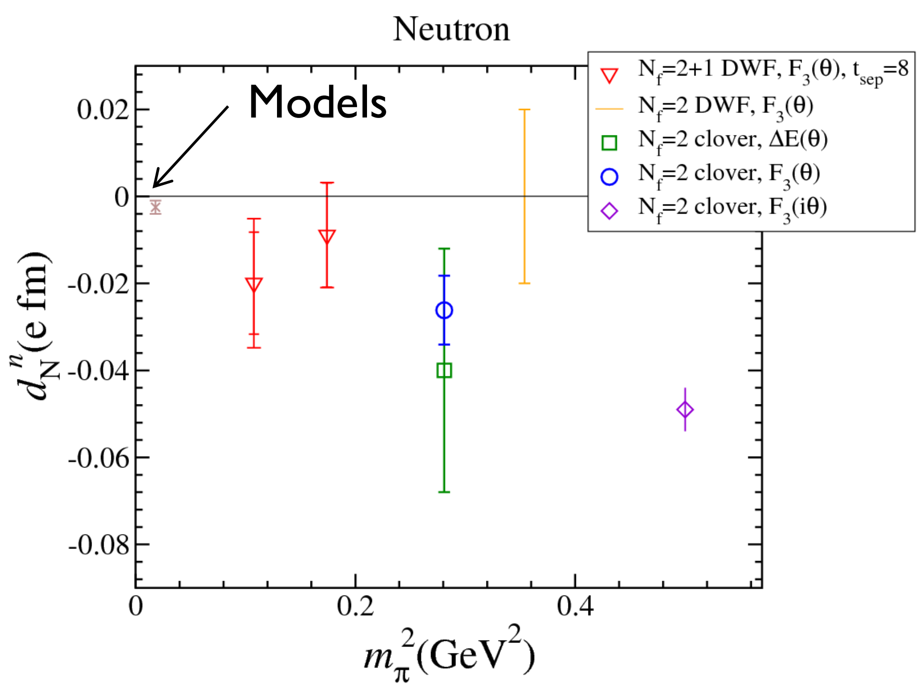
q^2 dependence

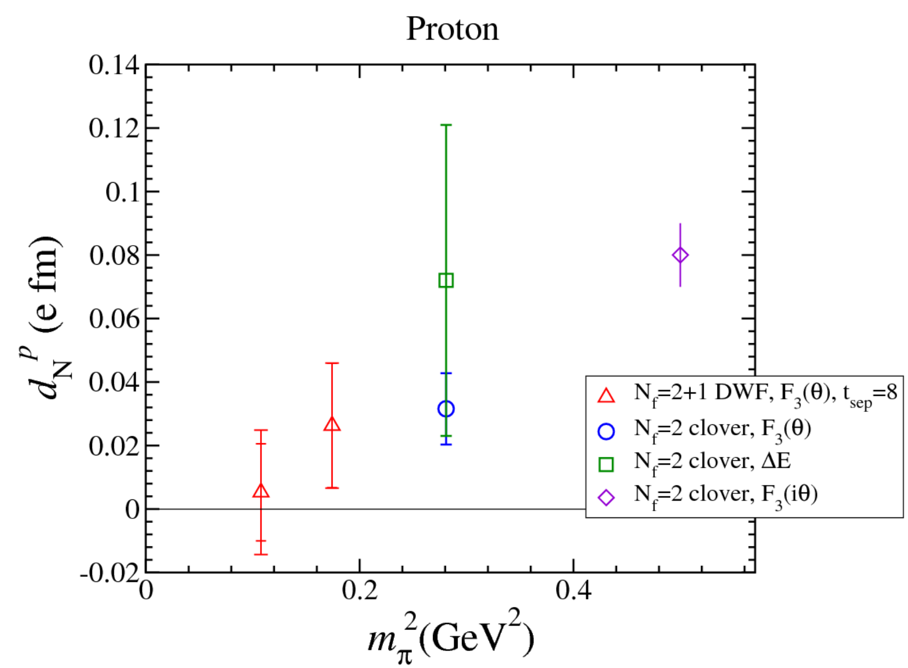
- Fitting data of EDM form factor at each momentum
- Open(t_{sep}=8) [N_{conf} = 751], filled (t_{sep}=12) [N_{conf} = 180]
- Fitting function
  3 point linear :
  $-q^2 < 0.55$ GeV^2
  2 point linear:
  $-q^2 < 0.4$ GeV^2
- Estimate of systematic error in extrapolation
- Fitting with BChPT is interesting extension.
  ⇒ estimate of LECs corresponding to πNN coupling



Mass dependence

- Comparison
  - Statistical error is still dominant rather than systematic one.
  - Central value is 10 times larger than models.
  - M_π^2 dependence is not clear, however its sign is consistent with magnetic moment, $d_N \sim \mu_m m_\pi^2 \Delta m$ Abada et al., PLB256(1991), Aoki, Hatsuda PRD45(1992)





3. Recent update (preliminary)

Statistical error

- Comparison between AMA error reduction and number of configurations.
- Number of configurations : reduce stat. error and relating to Q distribution

  AMA error reduction : reduce stat. error
- Error rate

  = Error(full)/Error(N)
- AMA works well
- Reduction rate when

  increase of confgs.

  is slightly better.

w/o correlation

↓

36%

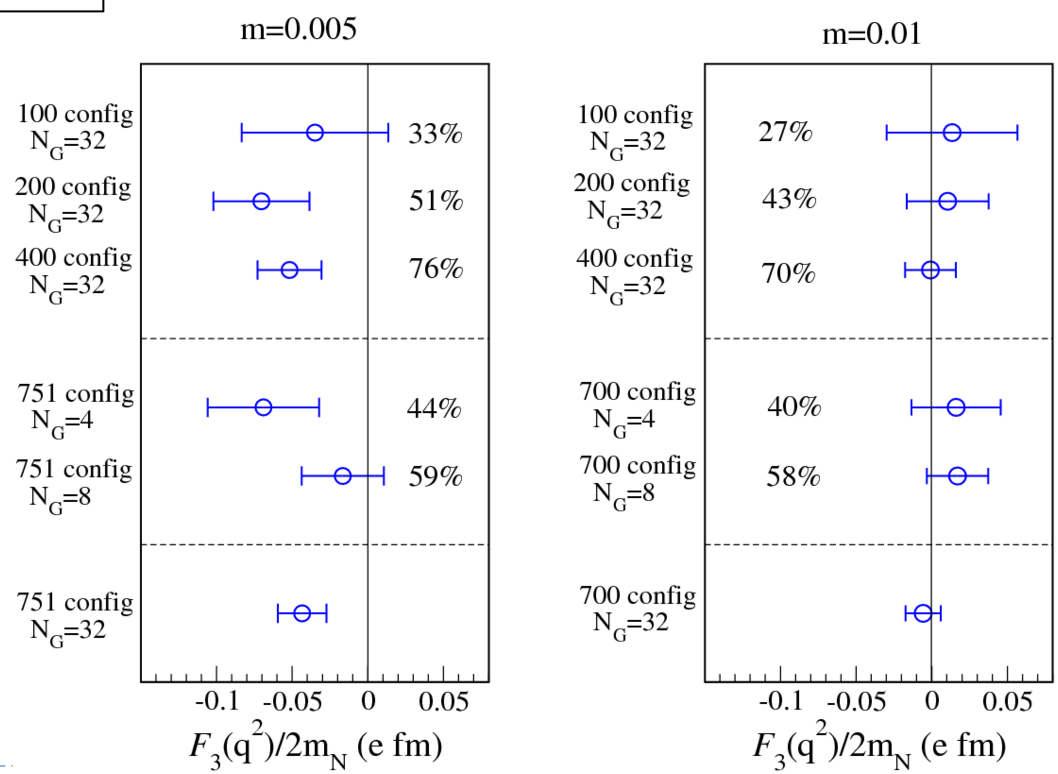
52%

73%

35%

50%

Full statistics →



Summary and future plan

- Nucleon EDM in $N_f = 2+1$ DWF in θ vacuum
  - Signal of EDM within 40% statistical error using AMA techniques.
  - 3-pt function is still noisy.
  - Short t_{sep} allows us to reduce the statistical error without large excited state contamination effect.

Summary and future plan

- Nucleon EDM in $N_f = 2+1$ DWF in θ vacuum
  - Signal of EDM within 40% statistical error using AMA techniques.
  - 3-pt function is still noisy.
  - Short t_{sep} allows us to reduce the statistical error without large excited state contamination effect.
- (Near) physical point of DWF configurations
  - Ensembles near physical points and large volume are available.
  - AMA with Möbius-DWF approximation is helpful. Hantao, Lattice2013
  - Remove chiral extrapolation → less than 10% precision

Lattice size	Physical size	a	L_s	Gauge action	Pion mass
$32^3 \times 64$	4.6 fm^3	0.135 fm	32	DSDR	171 -- 241 MeV
$48^3 \times 96$	5.5 fm^3	0.115 fm	16	Iwasaki	135 MeV

Thank you for your attention !

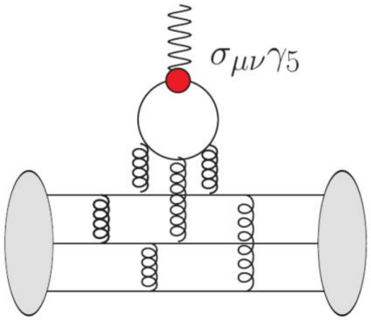
Backup

q(c)EDM term into QCD action

Bhattacharya et al, Lattice 2012

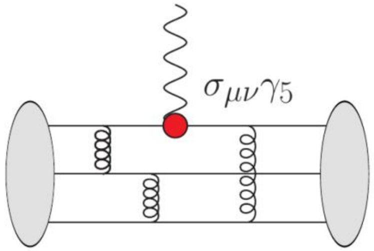
- Plan to do extension toward BSM action
  - Matrix element including BSM operator, quark EDM and chromo EDM (PQ symmetry is assumed)
  - The q(c)EDM term is CP-violating tensor charge of nucleon, connected diagram should be leading contribution→ statistical signal will be clear.
  - External E filed method may be easy way.
  - qEDM
    - Tensor charge matrix element + matrix element with qEDM:

$$\partial_{A_\mu}\langle N|d_q(\bar{q}\gamma_5\sigma\cdot Fq)|N\rangle_E$$
$$=\langle N|d_q(\bar{q}\gamma_5 q^\nu\sigma_{\mu\nu}q)|N\rangle+\langle N|J_\mu d_q(\bar{q}\gamma_5\sigma\cdot Fq)|N\rangle$$



  - Quark chromo EDM
    - Matrix element with chromo EDM term:

$$\partial_{A_\mu}\langle N|d_{cq}(\bar{q}\gamma_5\sigma\cdot Gq)|N\rangle_E=\langle N|J_\mu d_{cq}(\bar{q}\gamma_5\sigma\cdot Gq)|N\rangle$$



Examples of CAA

- Lowmode averaging (LMA)

  Guisti et al.(04), Neff et al.(01), DeGrand et al. (04)

  - Using lowlying eigenmode of Dirac operator to approximate propagator:

$$\mathcal{O}^{(\mathrm{appx})} = \sum_{\lambda}^{N_\lambda} \mathcal{O}_\lambda^{\mathrm{low}}$$

  where N_λ is number of lowmode computed by Lanczos.

  Except for computational cost of eigenmode, Cost(LMA) $\simeq$ 0, but approximation is only lowmode part (long distance contribution).

- All-mode averaging (AMA)
  - Using sloppy CG (loose stopping condition),

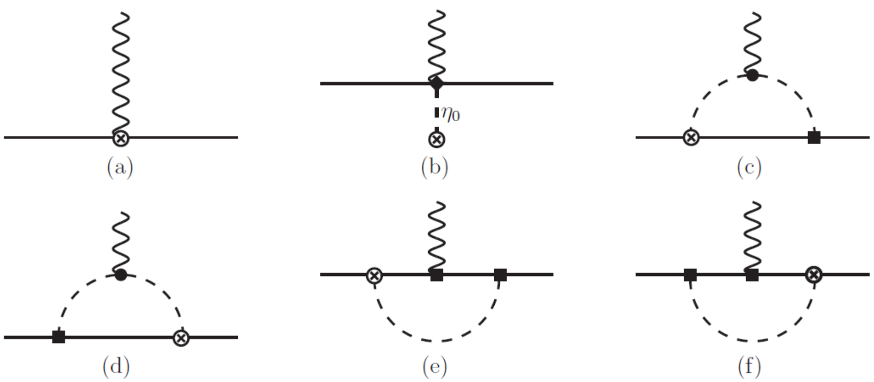
$$\mathcal{O}^{(\mathrm{appx})} = \mathcal{O}^{\mathrm{sloppy}}$$

  If stopping cond. is 0.003, Cost(AMA) $\simeq$ Cost(CG)/50(without deflation).

  Approximation becomes better than LMA for other than lowmode dominanted observables (nucleon, finite momentum hadron, ...).

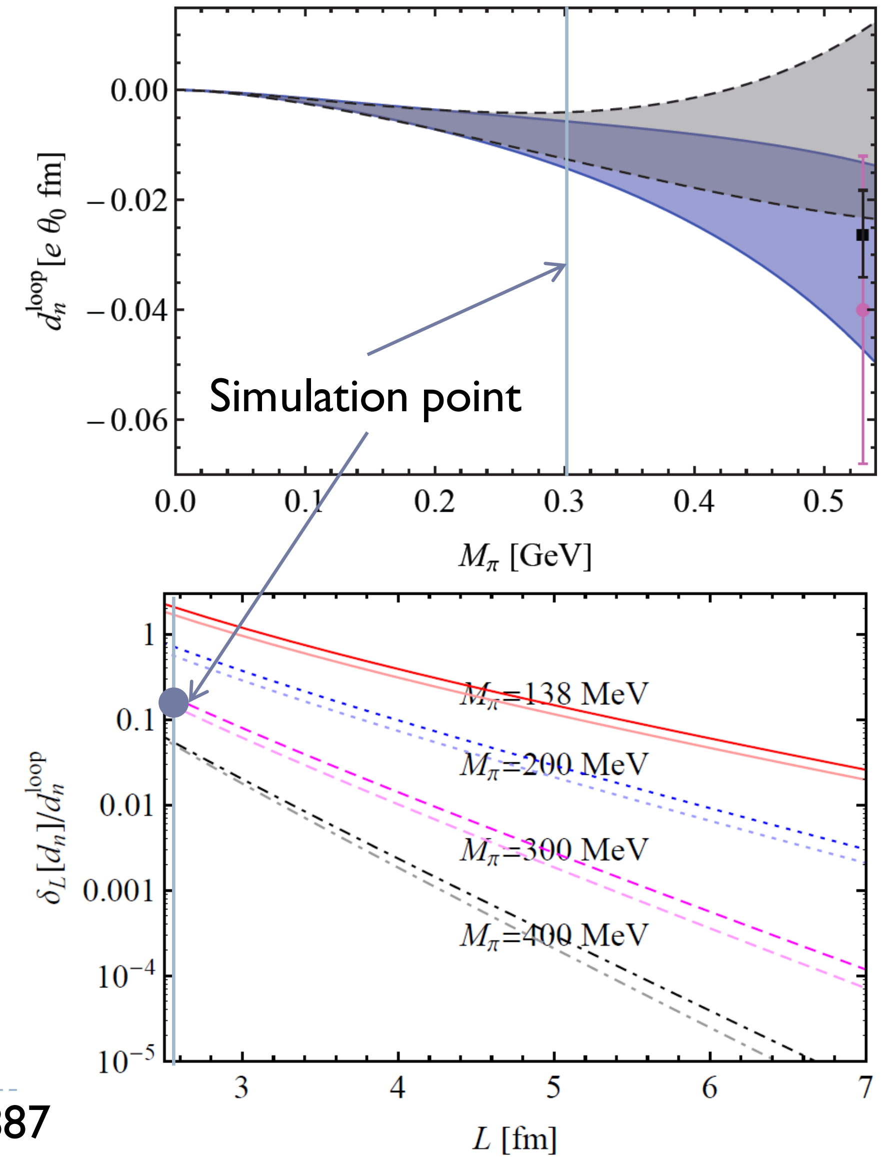
Volume effect ?

- BChPT analysis



$\otimes$ CP violating coupling

In LO, NLO BChPT analysis, there may be more than 20% finite size effect.



Guo, Meissner, 1210.5887

3. Recent update (preliminary)

Error reduction techniques

Blum, Izubuchi, ES, 1208.4349,
ES (lattice 2012),
Chulwoo, plenary on Fri

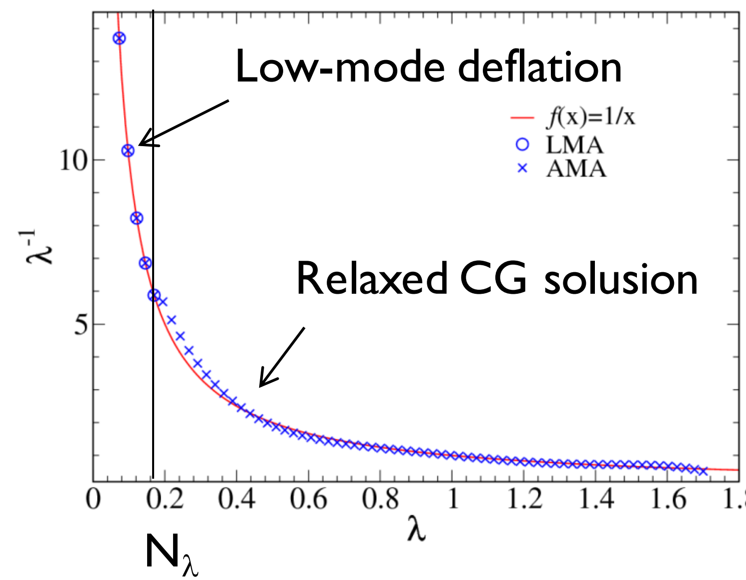
- Covariant approximation averaging (CAA)
  - For original correlator O, (unbiased) improved estimator is defined as

$$\mathcal{O}^{(\mathrm{imp})} = \mathcal{O}^{(\mathrm{rest})} + \frac{1}{N_G}\sum_{g\in G}\mathcal{O}^{(\mathrm{appx}),g}, \quad \mathcal{O}^{(\mathrm{rest})} = \mathcal{O} - \mathcal{O}^{(\mathrm{appx})}$$

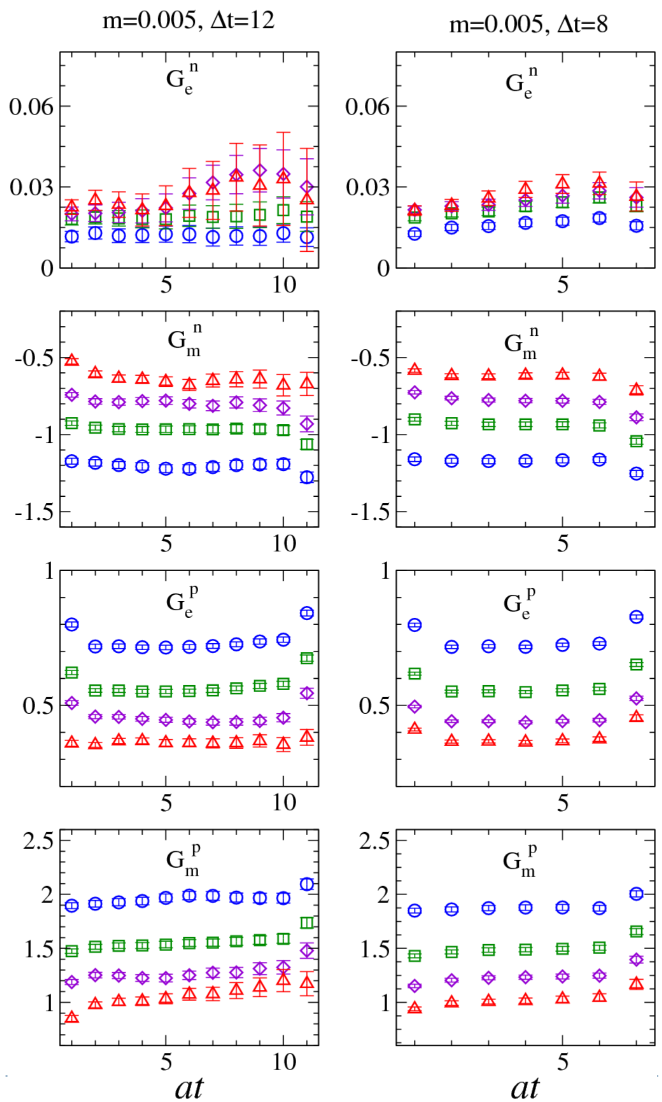
  - $\langle O \rangle = \langle O^{(\mathrm{imp})} \rangle$ if approximation has covariance under lattice symmetry g
  - Improved error $\mathrm{err}^{\mathrm{imp}} \simeq \mathrm{err}/\sqrt{N_G}$
  - Computational cost of $O^{(\mathrm{imp})}$ is cheap.
- All-mode-averaging (AMA)
  - Relaxed CG solution for approximation

$$\mathcal{O}^{(\mathrm{appx})} = \mathcal{O}[S_l],\ S_l = \sum_{\lambda=1}^{N_\lambda} v_\lambda v_\lambda^\dagger \frac{1}{\lambda} + P_n(\lambda)|_{|\lambda|>N_\lambda}$$

  - $P_n(\lambda)$ is polynomial approximation of $1/\lambda$
    - Low mode part : # of eigen mode
    - Mid-high mode : degree of poly.



EM form factor



3. Recent update (preliminary)

Comparison with μ = t, z

- EDM form factor is given from two directions of EM current
- Two signals are consistent, and data in t direction is much stable.