Hadronic contributions to the muon g-2

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(HIRSCHEGG 2014)

Overview

- Introduction
- Hadronic Vacuum Polarisation
- Hadronic Light-by-Light Scattering
- Conclusions

Overview

- **Introduction**
- Hadronic Vacuum Polarisation
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Anomalous magnetic moment of the muon

$$\vec{\mu} = g_l \frac{e\hbar}{2m_l c} \vec{S}$$

g_l: Gyromagnetic ratio for spin $\frac{1}{2}$ particle

Dirac: $g = 2$

QFT: $g > 2$ (screening: Effective n-body interaction)

$$a_l = \frac{1}{2}(g-2)$$

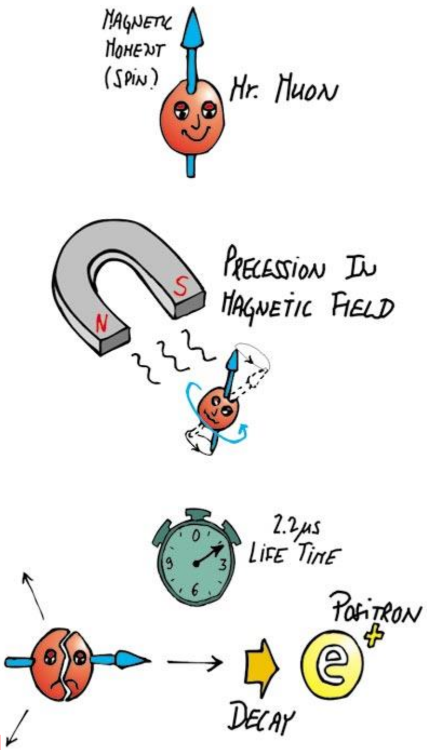
- Quantum corrections due to heavy particles

$$\frac{\delta a_l}{a_l} \propto \frac{m_l^2}{M^2}$$

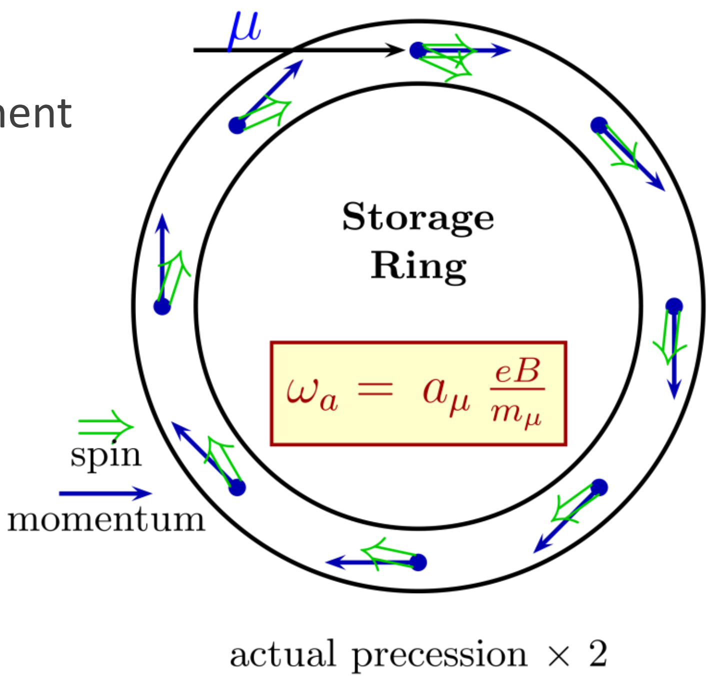
m_l^2/m_e^2 : **Muon** more sensitive by $\sim 200^2$

Experimental Measurement: *muon*

- Larmor Procession
  - *B* field exerts torque on the magnetic moment



[MaNEP]



[Jegerlehner & Nyffeler, 2009]

$$a_\mu = 116\,592\,089.0\;(63.0) \times 10^{-11}$$

[G. W. Benett et al (E821 at BNL), PRD 73, 072003 (2006)]

Muon Theory Budget

QCD dominates theory error

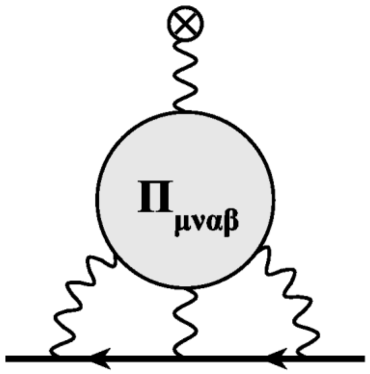
New physics = physics beyond QED + Weak

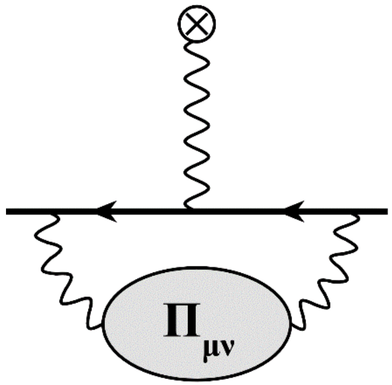
- QED/Weak perturbative
- QCD non-perturbative

Contribution	$a_\mu[\times 10^{-11}]$	%
QED	116 584 718.1(0.2)	99.994
Weak	153.2 (1.8)	0.00013
QCD LOHVP	6 949.1 (58.2)	0.006
QCD HOHVP	-98.4 (1.0)	0.00008
QCD HLBL	105 (26)	0.00009
SM	116 591 827.0 (64)	100
Experiment	116 592 089.0 (63)	-
Difference	**262 (89)**	**-**

QCD Corrections

- Hadronic Vacuum Polarisation
  - *Related to data (e+ e- annihilation)*
  - *Early lattice QCD calculations*
  - *Model calculations*

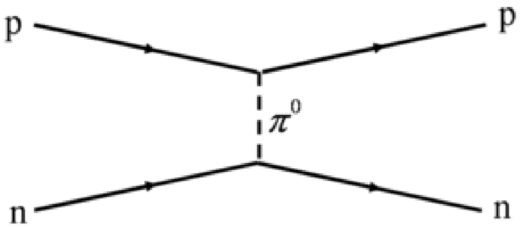


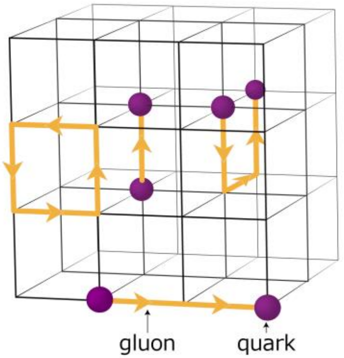


- Hadronic light-by-light scattering
  - *No independent experimental input*
  - *Very difficult lattice QCD calculation*
  - *Model calculations: systematics?*

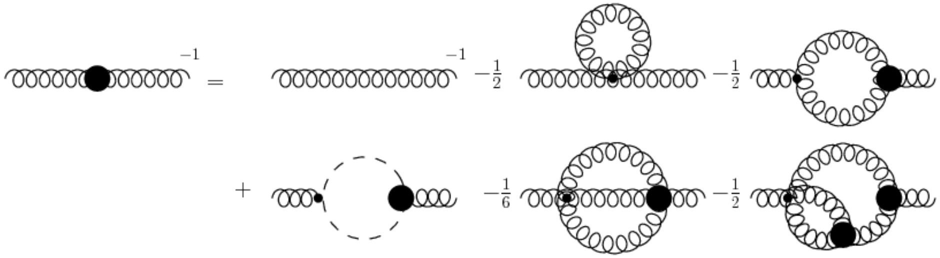
Non-perturbative tools

- Effective Field Theories
  - $1/N_c$ counting: ENJL, LMD, CQM
  - Chiral counting: ChPT
- Functional Methods
  - Lattice QCD
  - FRG/DSE/BSE



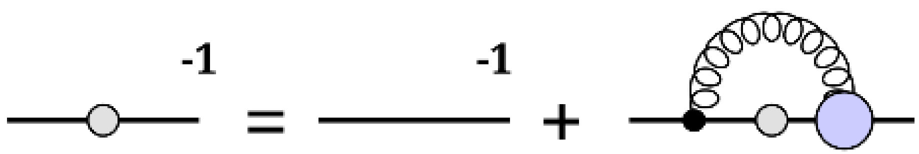


Can apply to both LO and NLO Hadronic contributions

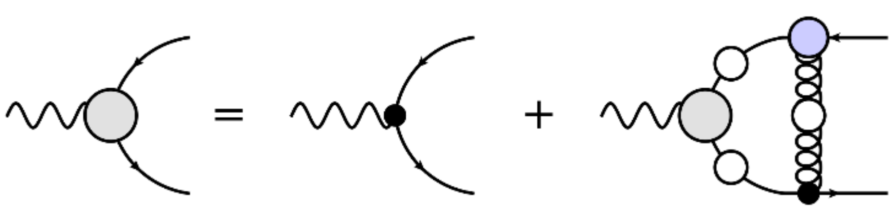


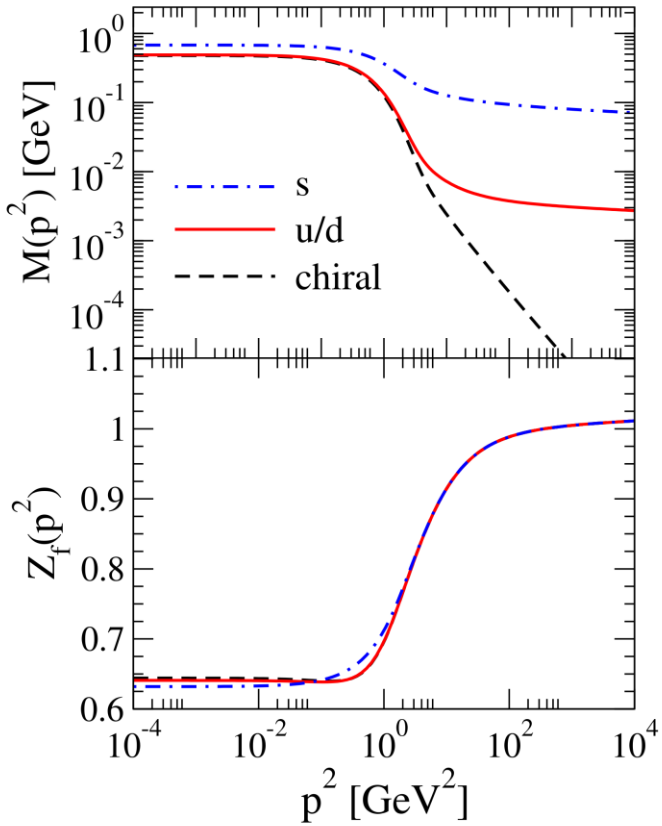
Dyson-Schwinger Equations

❖Quark propagator DSE



❖Quark-Photon Vertex DSE





Integral Equations for Green's Functions of QFT

Overview

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- Hadronic Light-by-Light Scattering
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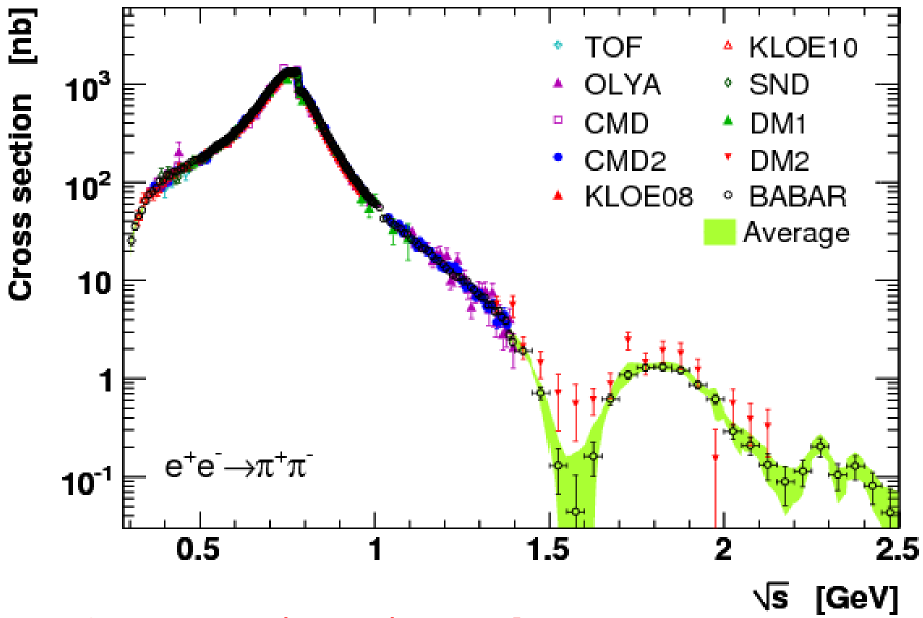
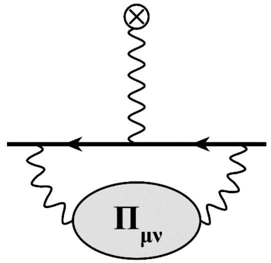
Hadronic Vacuum Polarisation

$\Pi_{\mu\nu}$: satisfies dispersion relation

$$a_\mu^{LO,HVP} = \frac{1}{4\pi^3}\int_{m_\pi^2}^{\infty} ds\ \sigma_{had}^0(s)K(s)$$



[Davier et al, EPJ C71 (2011) 1515]



- Direct scan
  - SND/BES II/CMD
- Radiative Return
  - KLOE/BELLE/BaBar

$e^+e^- \to 2\pi$

Provides $\sim 70\%$

Hadronic Vacuum Polarisation

- Data Extraction

LO: $(6951 \pm 45) \times 10^{-11}$
HO: $(-98 \pm 10) \times 10^{-11}$

[Hagiwari et al, JPG 38 (2011) 085003]

LO: $(6923.0 \pm 42) \times 10^{-11}$
HO: $(-97.9 \pm 9) \times 10^{-11}$

[Davier et al, EPJ C71 (2011) 1515]

- HLS eff. Interaction / global fit

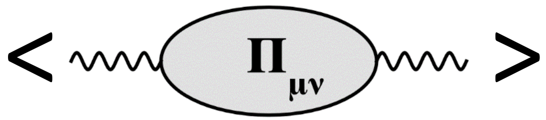
LO: $\left(6818 + \begin{bmatrix} +12.6 \\ -5.9 \end{bmatrix}_{\phi} \pm 45\right) \times 10^{-11}$

[Benayoun et al, EPJ C73 (2013) 2453]

- Limited by availability / quality of data
  - Systematics under control? Reasonable errors?

Hadronic Vacuum Polarisation: Theory

- Photon polarisation tensor / two-current correlator



$$\Pi_{\mu\nu}(q) = \int d^4x\, e^{i\,q\cdot x} \langle j_\mu(x) j_\nu(0) \rangle_{1PI,hadr}$$

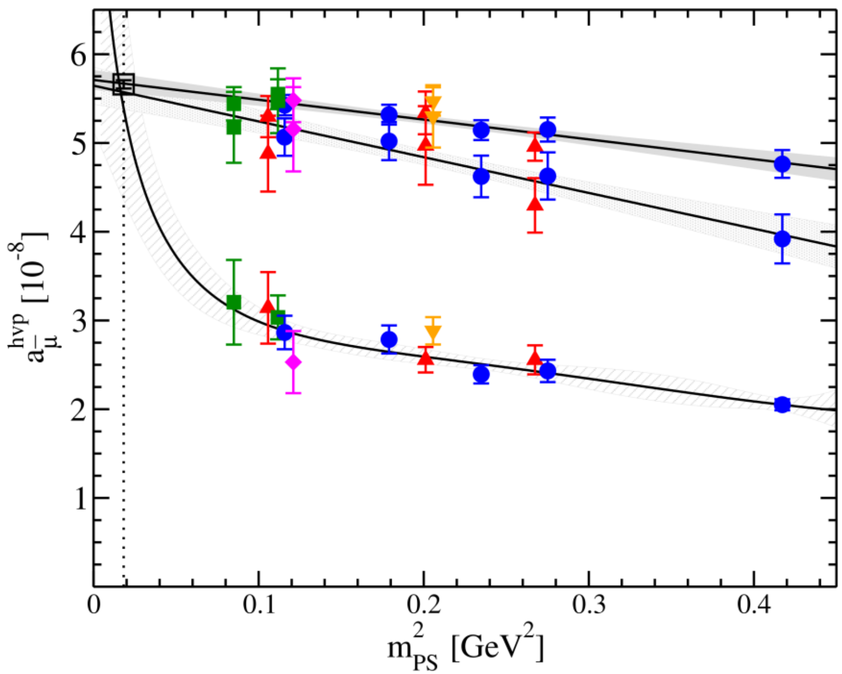
- Quark current:

$$j_\mu = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s + \cdots$$

- WI implies transversality

$$\Pi_{\mu\nu}(q) = \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) q^2 \Pi(q^2)$$

Hadronic Vacuum Polarisation: Lattice



- $N_f = 2$
  - "experiment"

    $a_\mu = 5660(\ 50) \times 10^{-11}$

  - Linear fit

    $a_\mu = 2950(450) \times 10^{-11}$

  - Vector meson fit

    $a_\mu = 5660(110) \times 10^{-11}$

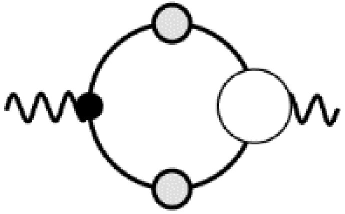
[Jansen et al, 2011 (ETMC)]

- $N_f = 2 + 1$ $a_\mu = 6410(33)(32) \times 10^{-11}$ [Boyle et al, 2011 (RBC-UKQCD)]

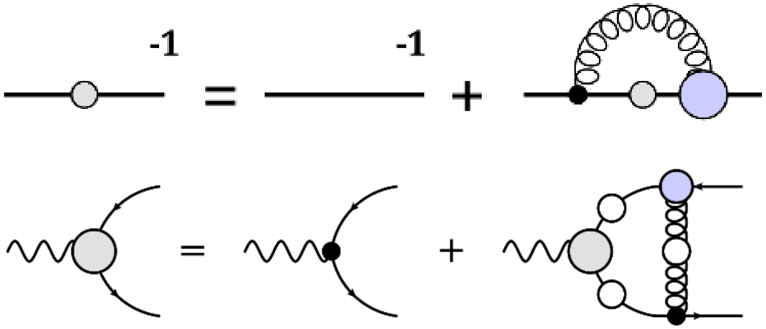
Hadronic Vacuum Polarisation: DSE

- Exact equation

Corresponds to



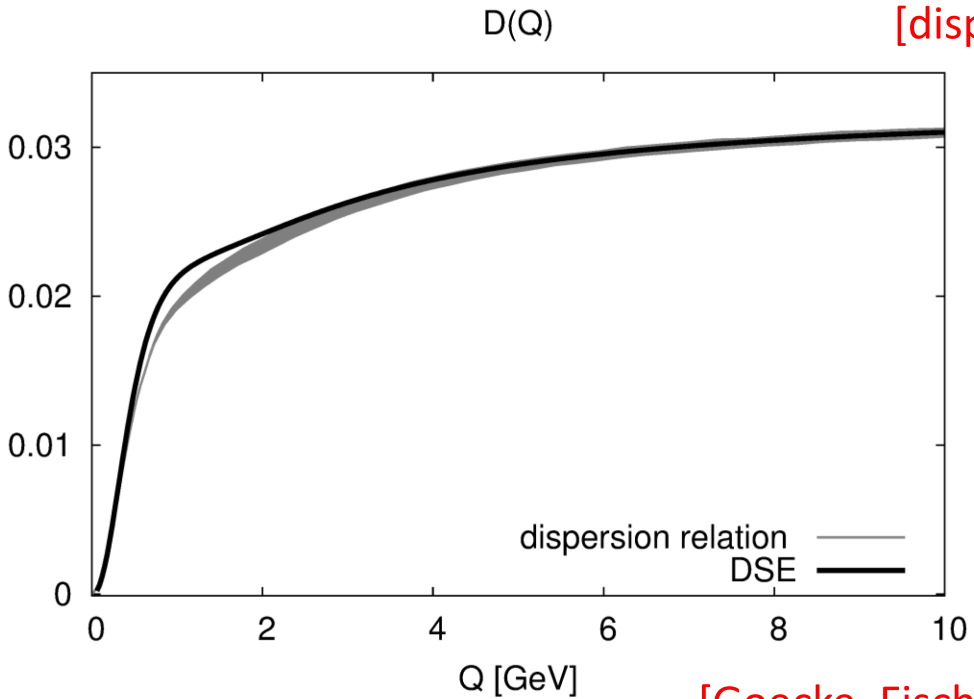
- Caveat
  - Requires 1PI quark propagator and quark-photon vertex



In turn require:

- Quark-antiquark interaction
- Quark-gluon vertex
- Gluon propagator
- ...

Hadronic Vacuum Polarisation: DSE



[dispersion data, Jegerlehner 2008]

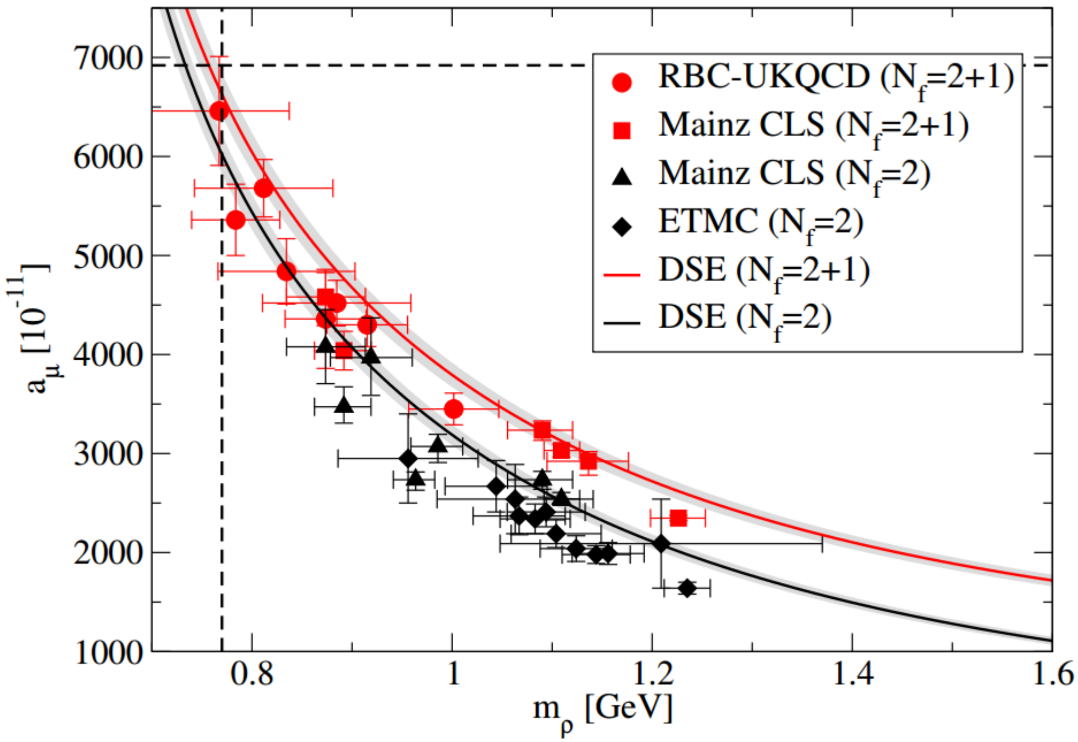
[Goecke, Fischer and RW, PLB 704 (2011) 211-217]

Adler function

$$D(q) = -q^2 \frac{d\Pi(\mathrm{q}^2)}{dq^2}$$

$$a_\mu^{HVP} = \frac{\alpha}{\pi} \int_0^1 dx (1-x) \left[-e^2 \Pi \left(\frac{x^2}{1-x} m_\mu^2 \right) \right]$$

Hadronic Vacuum Polarisation: DSE



- DSE (5 flavours)

$$a_\mu^{HVP} = 7440(\,20) \times 10^{-11}$$

m_π mass fit

$$a_\mu^{HVP} = 6760(\,20) \times 10^{-11}$$

m_ρ mass fit

- *cf.* "experiment"

$$a_\mu^{HVP} = 6921(\,56) \times 10^{-11}$$

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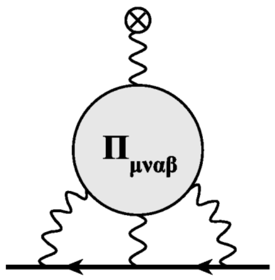
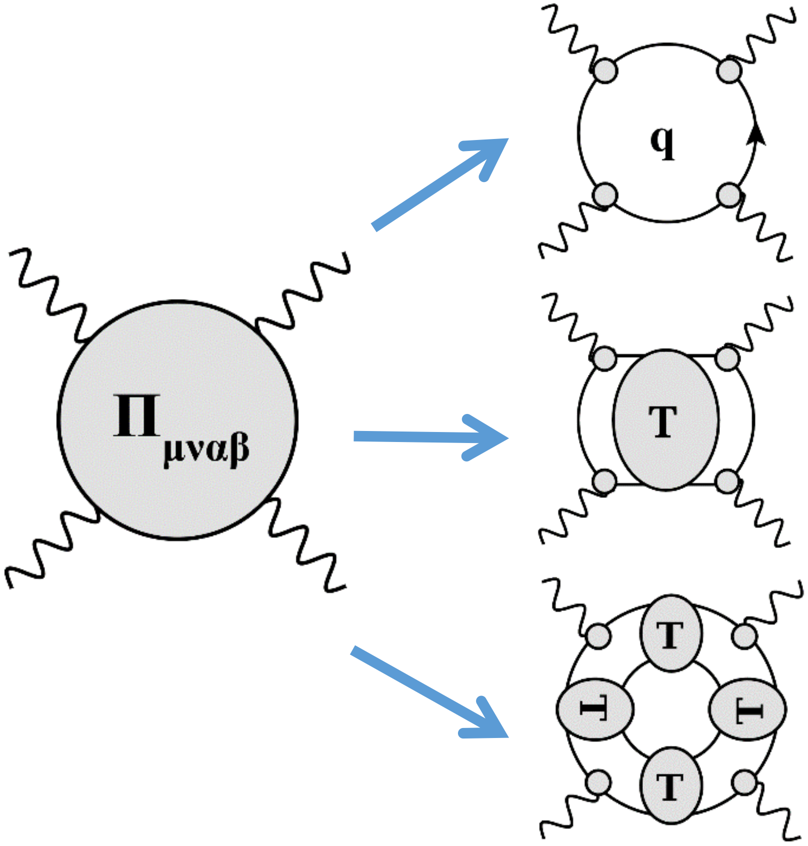
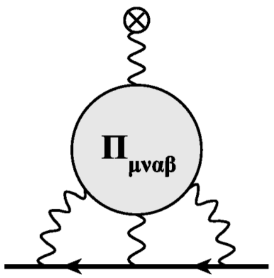
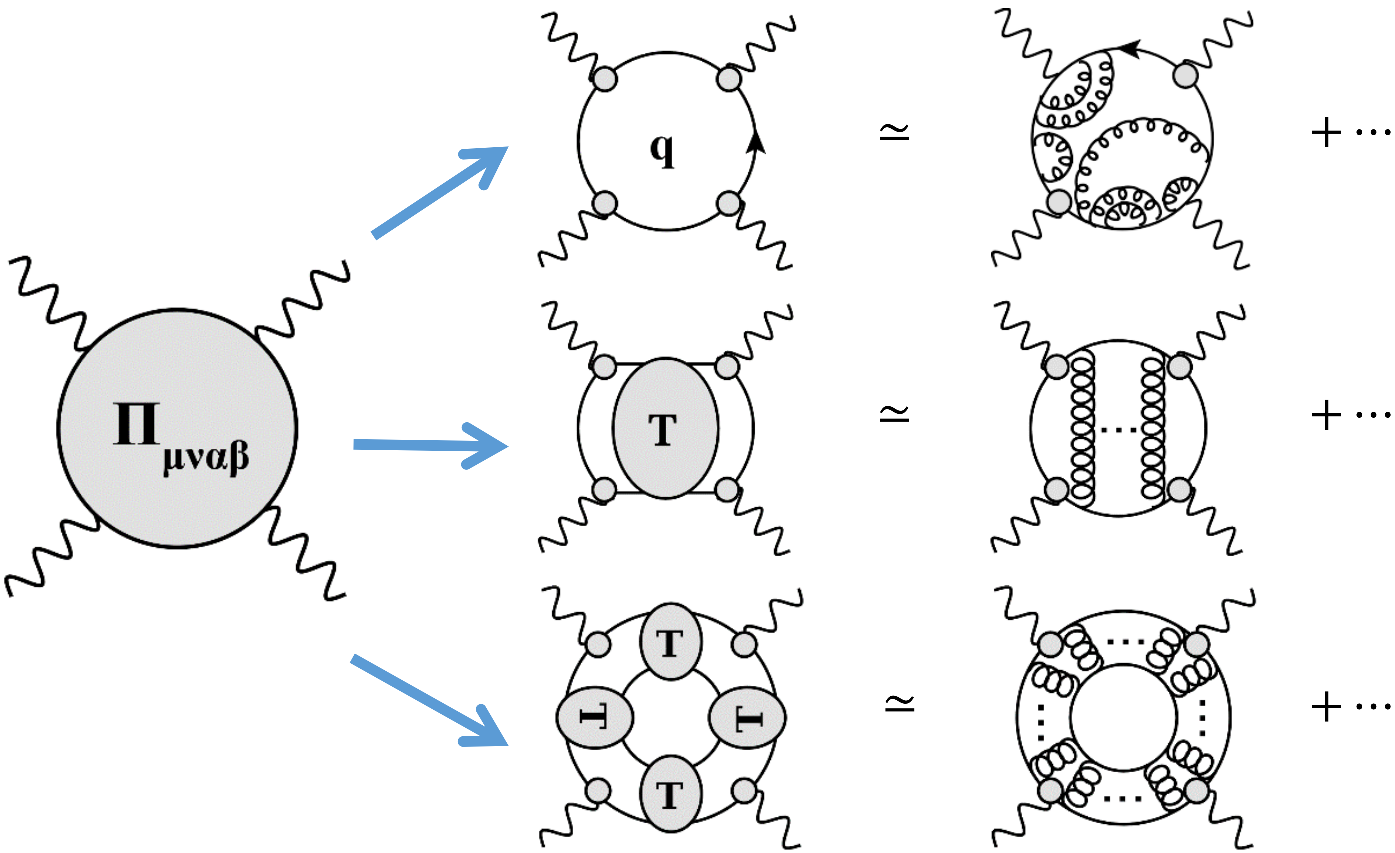
Hadronic light-by-light Scattering



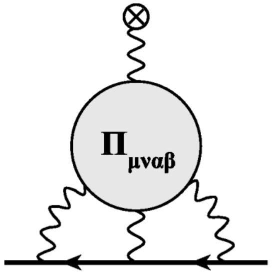


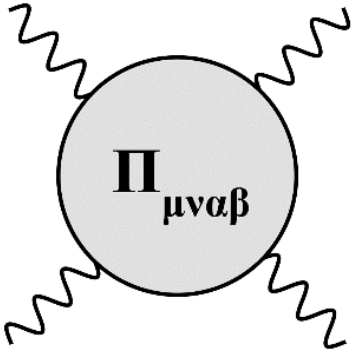
Hadronic light-by-light Scattering



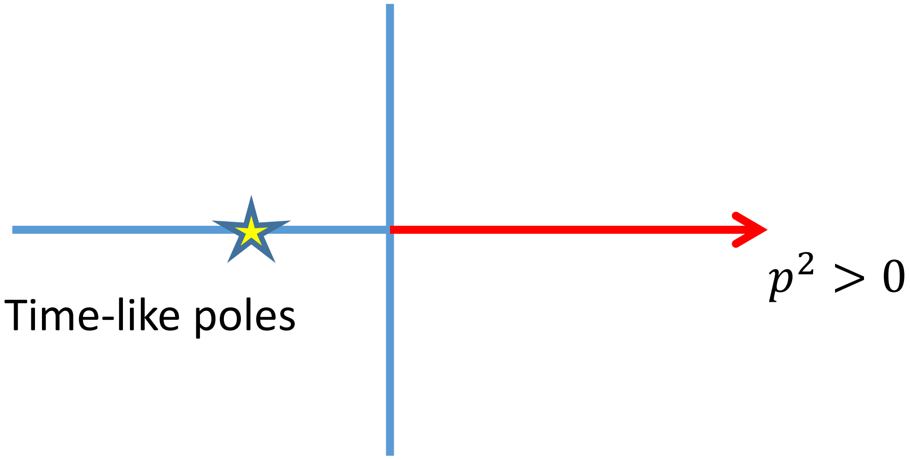


Hadronic light-by-light Scattering



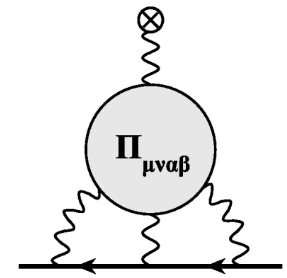


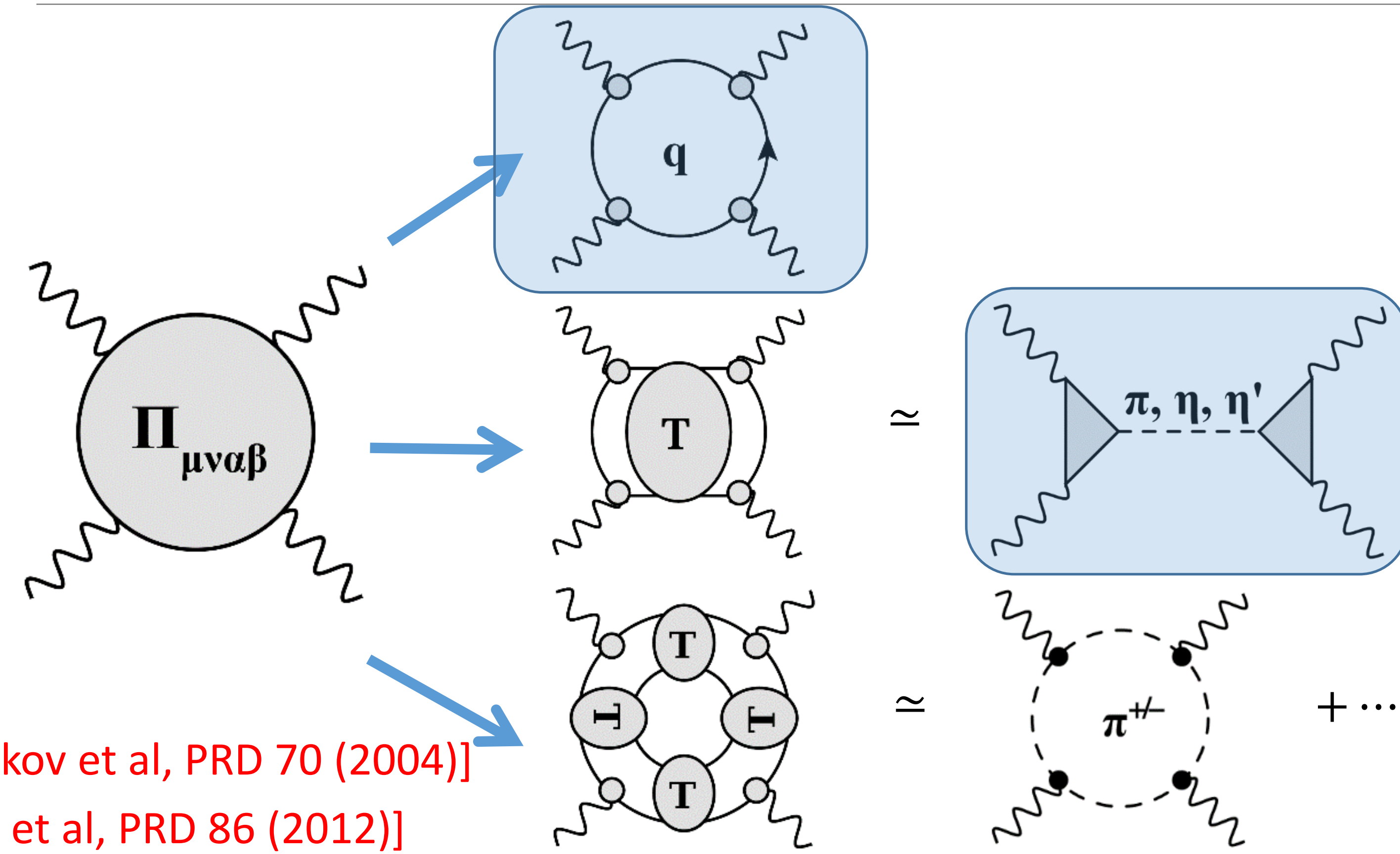
Evaluated at *spacelike* Euclidean momentum



All bound-states/resonances are off-shell

Hadronic light-by-light Scattering



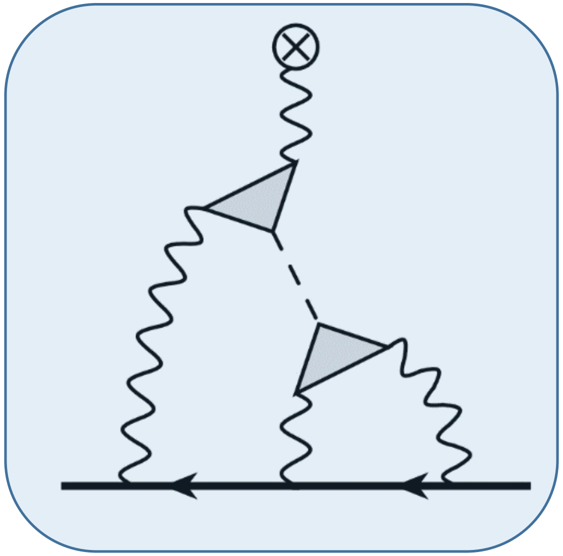


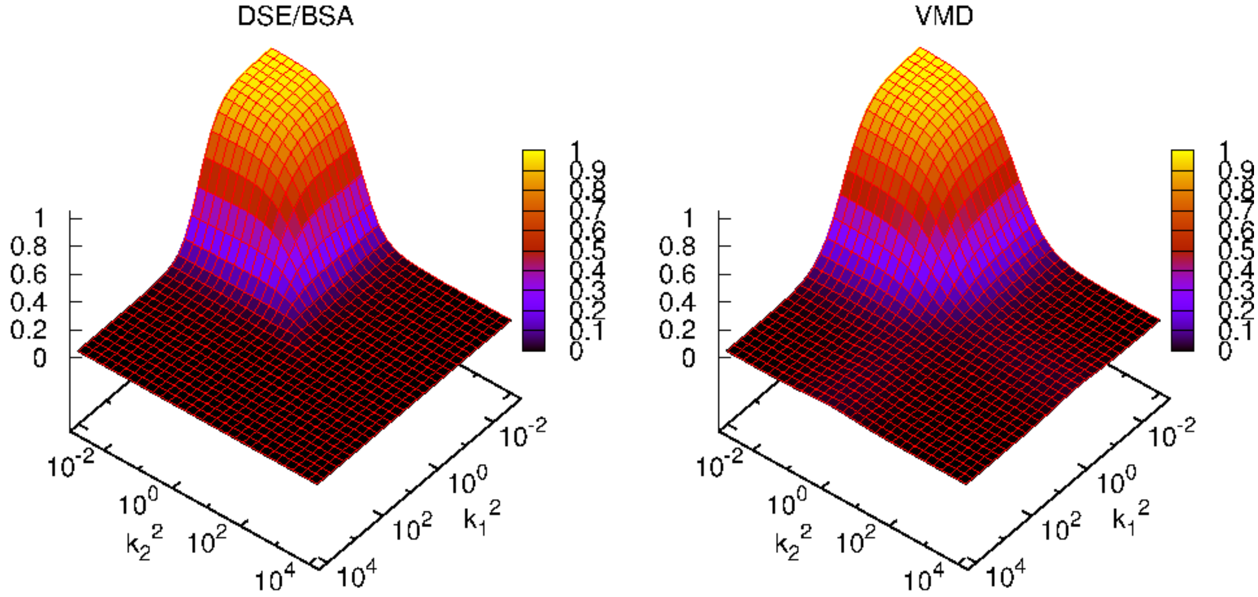
[Melnikov et al, PRD 70 (2004)]

[Engel et al, PRD 86 (2012)]

Small?

Hadronic light-by-light: *meson exchange*





Off shell form-factor:

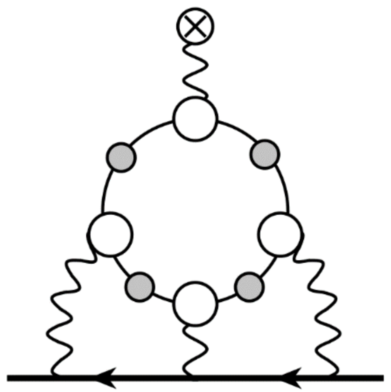
- Quark propagator
- Quark-photon vertex
- Bethe-Salpeter amplitude
- $\pi\gamma\gamma$ form factor

Off-shell prescription

$$a_\mu^{\pi,\eta,\eta'} = 81(13) \times 10^{-11}$$

[Goecke, Fischer and RW, PRD 83 (2011) 094006]

Hadronic light-by-light: *quark loop*



$$\Gamma^\mu = \sum_{i=1,4} \lambda_i L_i^\mu + \sum_{i=1,8} \tau_i T_i^\mu$$

Gauge part fixed by WTI

Transverse part contains vector mesons

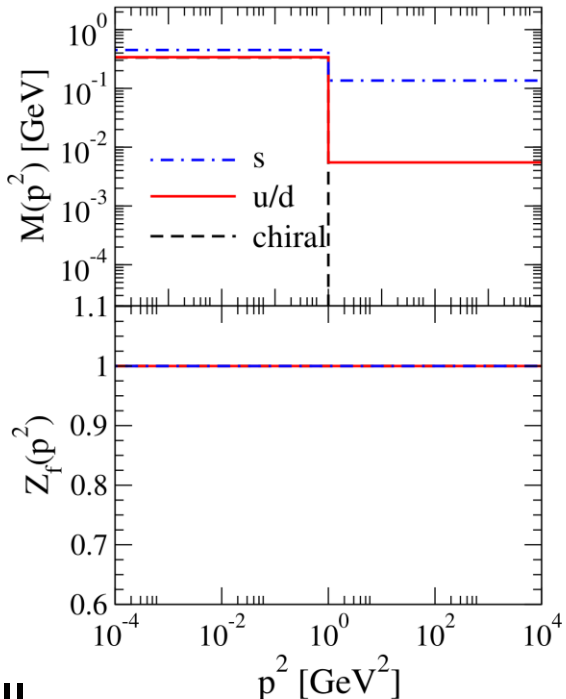
- ENJL result $L_1 + T_1 : (21 \pm 3) \times 10^{-11}$

[Bijnens, Pallante and Prades, PRL 75 (1995)]

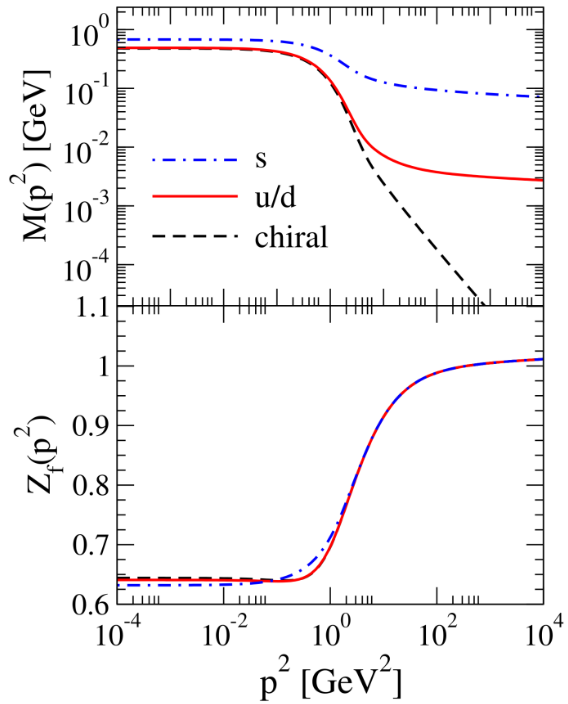
- DSE result $L_1 + T_1 : (107 \pm 2) \times 10^{-11}$

[Goecke, Fischer and RW, PRD 87 (2013) 034013]

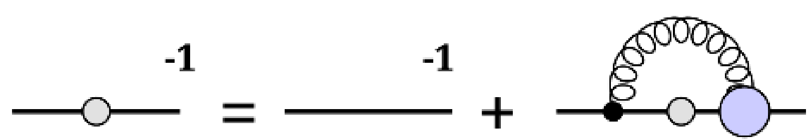
Hadronic light-by-light: *quark loop*



ENJL



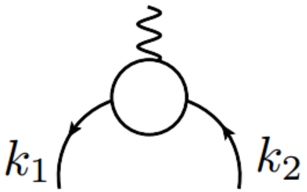
DSE



ENJL vs DSE: *gauge part*

$$\Gamma^\mu = \sum_{i=1,4} \lambda_i L_i^\mu + \sum_{i=1,8} \tau_i T_i^\mu$$

$$L_1^\mu = \gamma^\mu$$

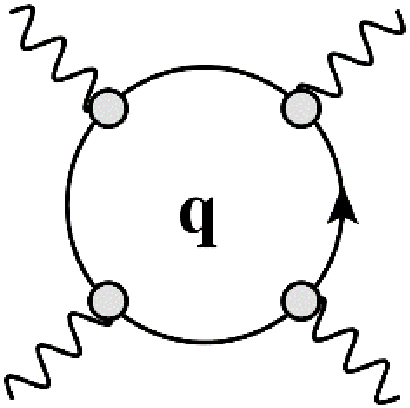


ENJL/DSE both satisfy the Ward-Takahashi identity

$$\lambda_1(k_1, k_2) = \frac{1}{2}\left(\frac{1}{Z_f(k_1^2)} + \frac{1}{Z_f(k_2^2)}\right)$$

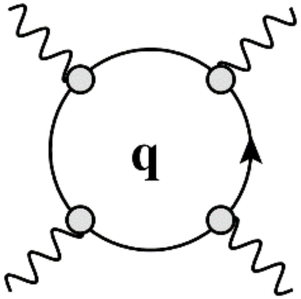
ENJL: $Z\_f \;=\; 1$
DSE: $Z\_f \;<\; 1$

Gauge part roughly cancels Z_f from quarks

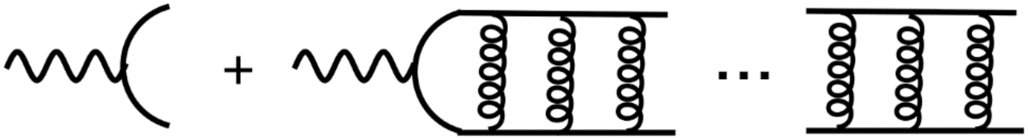


Enhancement in due to momentum dependent mass function

ENJL vs DSE: *transverse part*



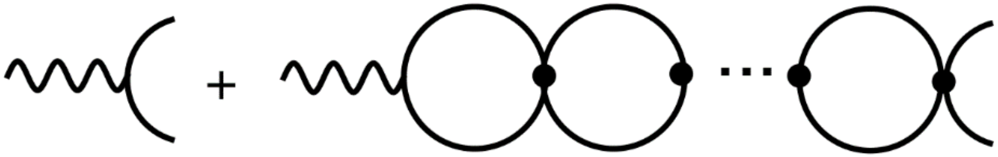
DSE



Couples to vector meson: SUPPRESSION
Non-contact interaction: relative momentum; DAMPS suppression

$(111 \to 107) \times 10^{-11}$

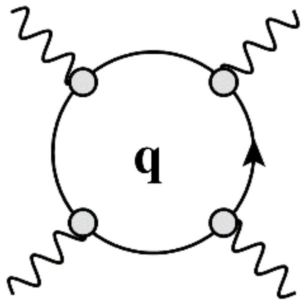
ENJL



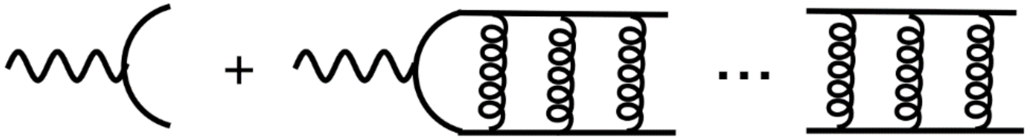
Point-like vector meson: SUPPRESSION
Contact interaction: no relative momentum

$(60 \to 22) \times 10^{-11}$

ENJL vs DSE: *transverse part*



DSE



Couples to vector meson: SUPPRESSION

Non-contact interaction: relative momentum; DAMPS suppression

$(111 \rightarrow 107) \times 10^{-11}$

Large contributions from the quark loop arises due to the full momentum dependence that is considered here

Hadronic light-by-light: *results*

Group	Model	π	π, η, η'	Quark loop	a_μ^{LBL}
BPP	ENJL	59(11)	85(13)	21(3)	83(32)
HKS	HLS	57(4)	83(6)	10(11)	89(16)
KN	LMD+V	58(10)	83(11)	–	83(12)
MV	LMD+V	77(5)	114(10)	–	114(10)
DB	NLχQ	65(2)	–	–	–
N	LMD+V	72(12)	99(16)	–	99(16)
GR	CχQM	68(3)	–	82	150(5)
GFW	**DSE**	58(1)	81(2)	107(3)	188(5)

BPP: Bijnens et al, PRL 75 (1995) 1447
HKS: Hayakawa et al, PRL 75 (1995) 790
KN: Knecht et al, PRD 65 (2002) 073034
MV: Melnikov et al, PRD 70 (2004) 113006
DB: Dorokhov et al, PRD 78 (2008) 073011
N: Nyffeler, PRD 79 (2009) 073012
GR: Greynat et al, JHEP 1207 (2012) 020
GFW: PRD 87 (2013) 034013

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Conclusions

Dynamical *ab initio* calculation of muon $g-2$ desirable

- Model approximations can **underestimate** contributions
- **Lattice QCD** (LO now, NLO future)
- **Functional methods** (now, but more work needed)

QCD corrections from **theory not necessarily robust**

- Possible underestimation of errors (overconfidence?)

No statement regarding beyond-the-standard model from muon $g-2$

Thank you!

Conclusions:
beyond standard model?

Anomalous magnetic moment of the muon as the harbinger of new physics?

first get QCD under control

Experiment	$a_\mu = (11\,659\,208.9 \pm 6.3) \times 10^{-10}$
	[Benett et al, PRD 73 (2006) 072003]
Theory	$a_\mu = (11\,659\,182.8 \pm 4.9) \times 10^{-10}$
	[Hagiwara et al, JPG 38 (2011) 085003]
Our value	$a_\mu = (11\,659\,191.4 \pm 10) \times 10^{-10}$

Error Estimate? Just a guess …