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Composite and elementary natures of hadron resonances

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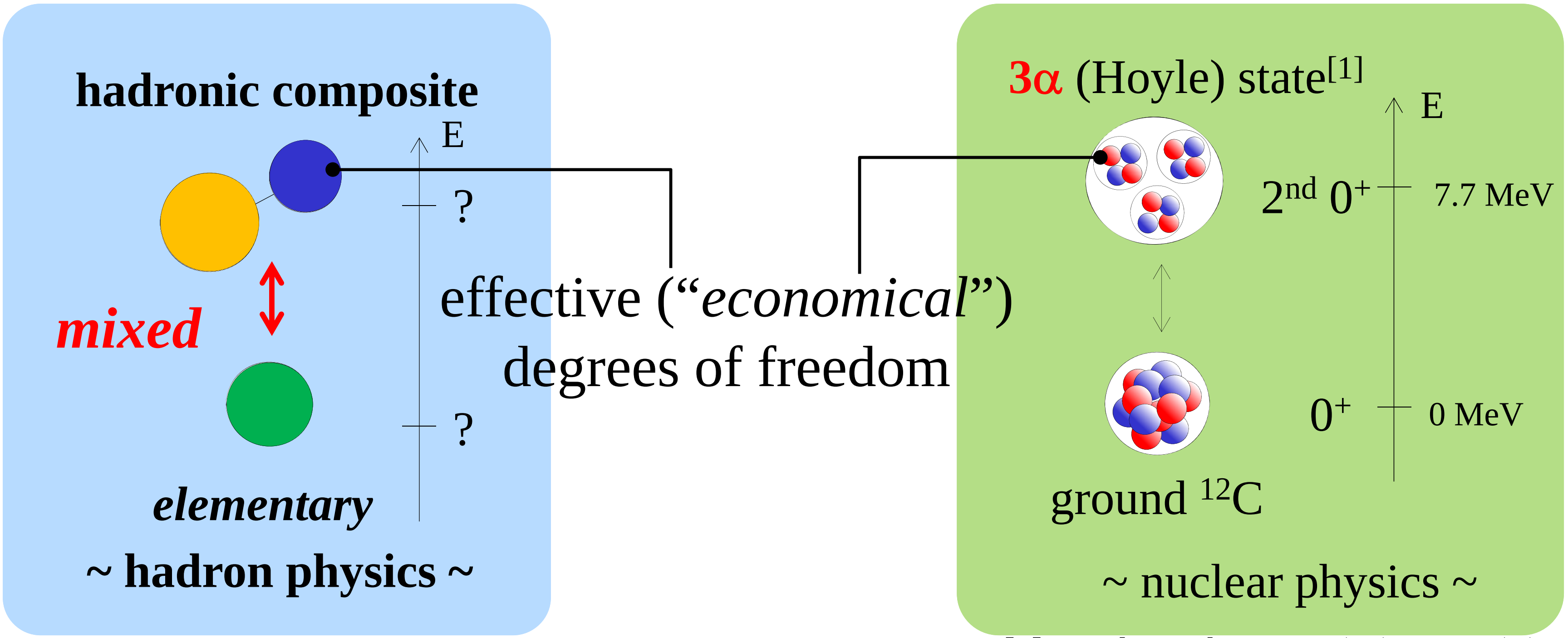
References:

H. Nagahiro, and A. Hosaka, PRC88(2013)055203
(as PRC Editors' Suggestions)
H. Nagahiro, and A. Hosaka, in progress

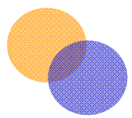
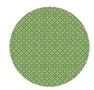
Introduction & motivation

Internal structure of hadrons

Complex mass spectrum



[1] Funaki *et al.*, PRC67(03)051306(R)

$$| \text{physical state} \rangle = C_1 \, |$$  $$\rangle + C_2 \, |$$  $$\rangle + \ldots$$

Simple question : "How and how much they are mixed ? Can we estimate it?"

Mixing nature of σ (or $f_0(500)$) meson

Mixing nature of σ meson consisting of $\pi\pi$ composite and elementary meson

» within the *nonlinear* representation of the sigma model

$$|\sigma\rangle_{\text{phys}} = C_1 \,|\pi\pi\rangle + C_2\,|\sigma\rangle$$

dynamically generated resonance

"elementary" ($q\bar{q}$) particle

in terms of **two-level problem** [Nagahiro-Hosaka, PRC88]

$$|a_1(1260)\rangle_{\text{phys}} = C_1\,|\rho\pi\rangle + C_2\,|a_1\rangle$$

Nagahiro *et al.*, PRD83(11)111504(R)

"Compositeness condition $Z = 0$" in the sigma model

» "compositeness condition $Z = 0$" [1-3]
$\Leftrightarrow$ "elementary component z^{22}"

[1] S.Weinberg, PR137(65)B672
[2] D. Lurie, A.J.Macfarlane, PR136(64)B816
[3] T. Hyodo, D.Jido, A. Hosaka, PRC85(12)015201

$\pi\pi$ scattering amplitude in s-wave isoscaler channel

model Lagrangian : the sigma model in the *nonlinear* representation

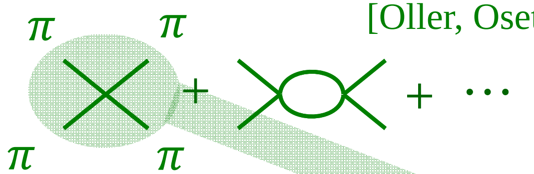
$$\mathcal{L} = \frac{1}{4}\,\mathrm{tr}\left(\partial_\mu \Sigma \partial^\mu \Sigma^\dagger\right) + \frac{\mu^2}{4}\,\mathrm{tr}\left(\Sigma^\dagger \Sigma\right) - \frac{\lambda}{16}\left(tr\left(\Sigma^\dagger \Sigma\right)\right)^2 + a\,\mathrm{tr}\left(\Sigma^\dagger \Sigma\right)$$

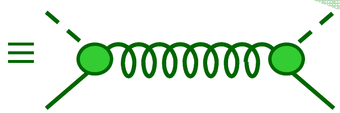
$$\Sigma = (f_\pi + \sigma)U, \quad U = \exp(i\vec{\tau}\cdot\vec{\pi}/f_\pi\,)$$

Composite σ

dynamically generated s-wave resonance
in chiral unitary approach

[Oller, Oset, NPA620(97)438, etc.]

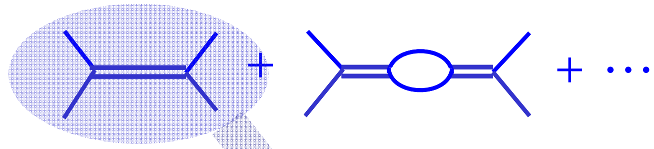




$$= g_R \frac{1}{s - s_p} g_R$$

elementary σ

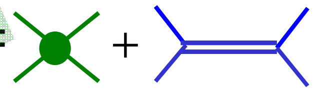
we keep the elementary σ field
with a finite mass m_0



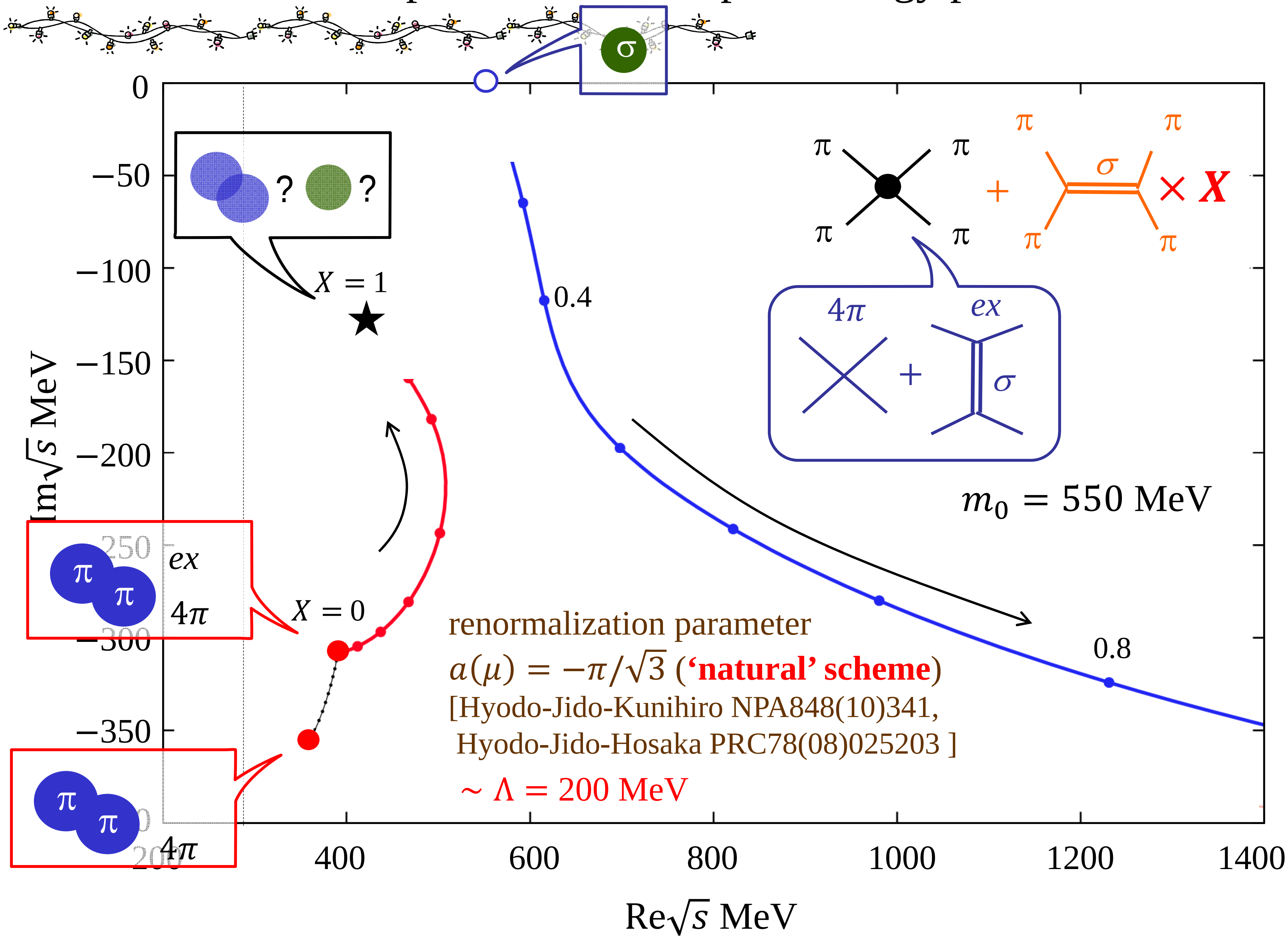
$$= g \frac{1}{s - m_0^2 - gGg} g$$

full $\pi\pi$ scattering amplitude

$$t_{\pi\pi\to\pi\pi} = \frac{v_{con} + v_{pole}}{1 - \left(v_{con} + v_{pole}\right)G},\qquad v_{con} + v_{pole} =$$



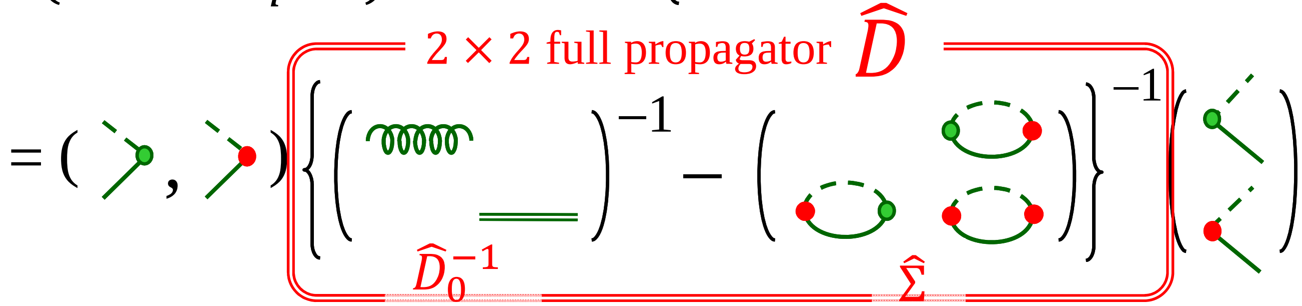
Numerical results : pole-flow in complex-energy plane



Reduction to the ***two-level problem*** : disentangle the mixing

Nagahiro-Hosaka, PRC88(2013)055203

$$T = \frac{v_{con} + v_{pole}}{1 - (v_{con} + v_{pole})G} = (g_R, g)\left\{\begin{pmatrix} s - s_p & \\ & s - m^2 \end{pmatrix} - \begin{pmatrix} & g_R G g \\ g G g_R & g G g \end{pmatrix}\right\}^{-1}\begin{pmatrix} g_R \\ g \end{pmatrix}$$



$$= \left(g_R\sqrt{z_a^{11}} + g\sqrt{z_a^{22}}\right)^2 \frac{1}{s - M_a^2} + \cdots$$

cf. in a simple Yukawa model $T_{\text{Yukawa}} = \left(g_0\sqrt{Z}\right)^2 \frac{1}{s - M^{*2}}$

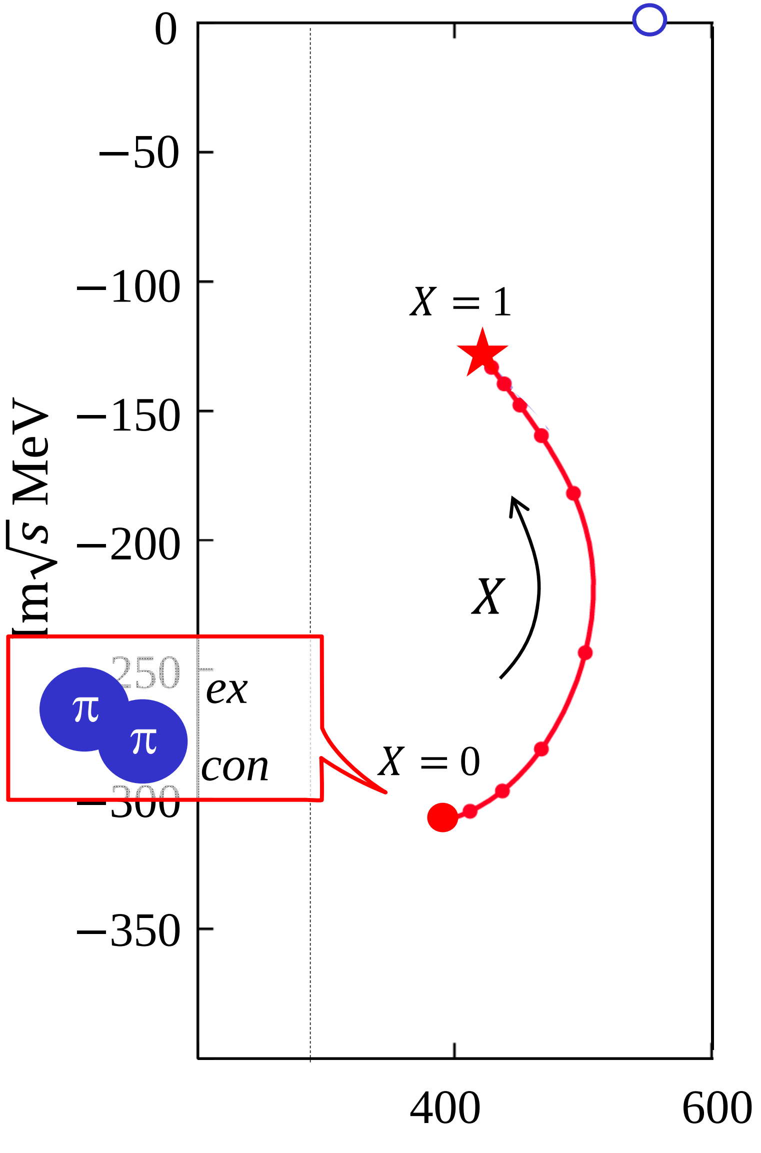
$\hat{D}$

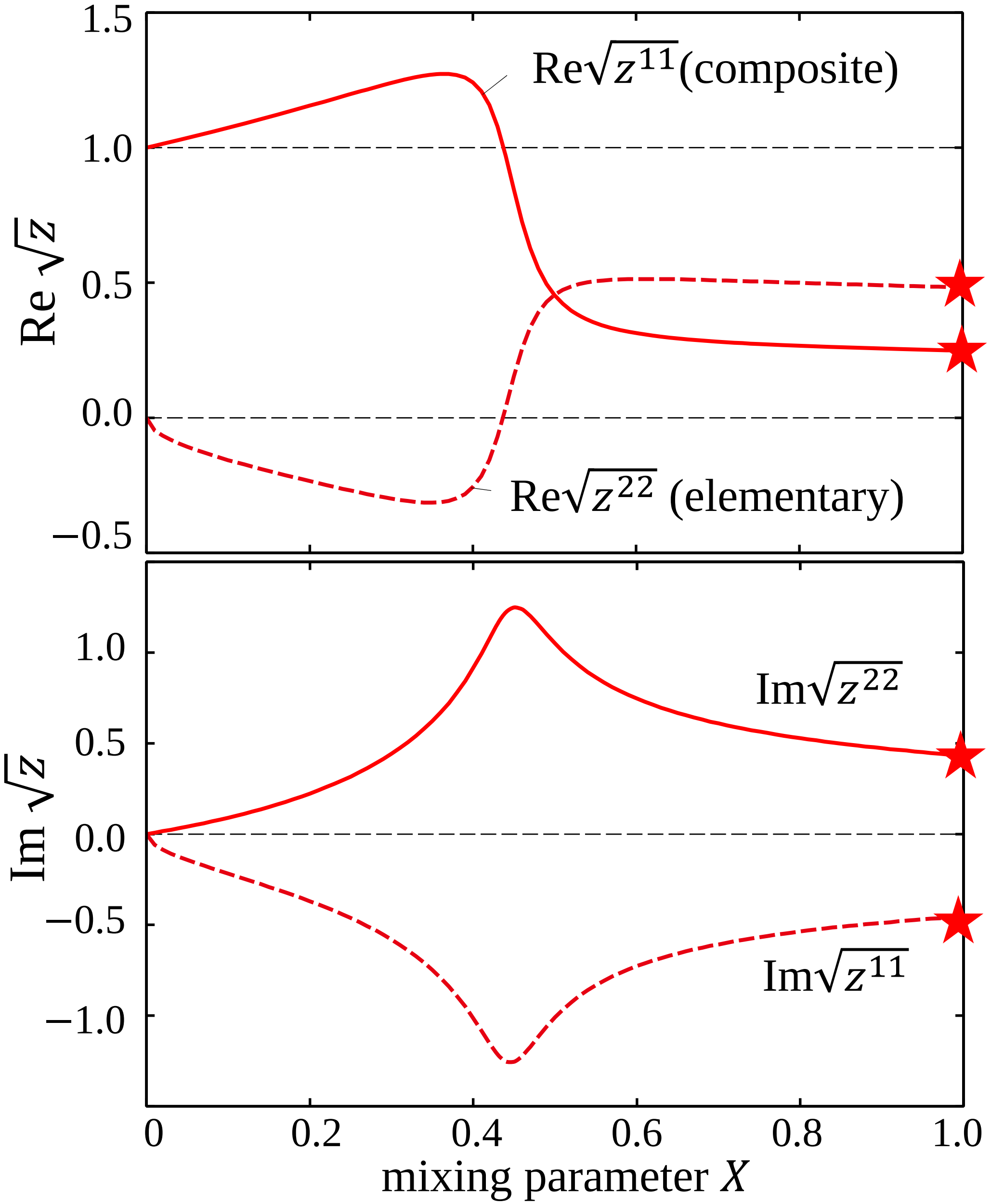
mixing rate

$$D^{11} = \frac{z_a^{11}}{s - M_a^2} + \frac{z_b^{11}}{s - M_b^2}, \qquad D^{22} = \frac{z_a^{22}}{s - M_a^2} + \frac{z_b^{22}}{s - M_b^2}$$

$$|\sigma\rangle_{phys} = \sqrt{z_a^{11}}\,|\ \ \rangle + \sqrt{z_a^{22}}\,|\ \ \rangle$$

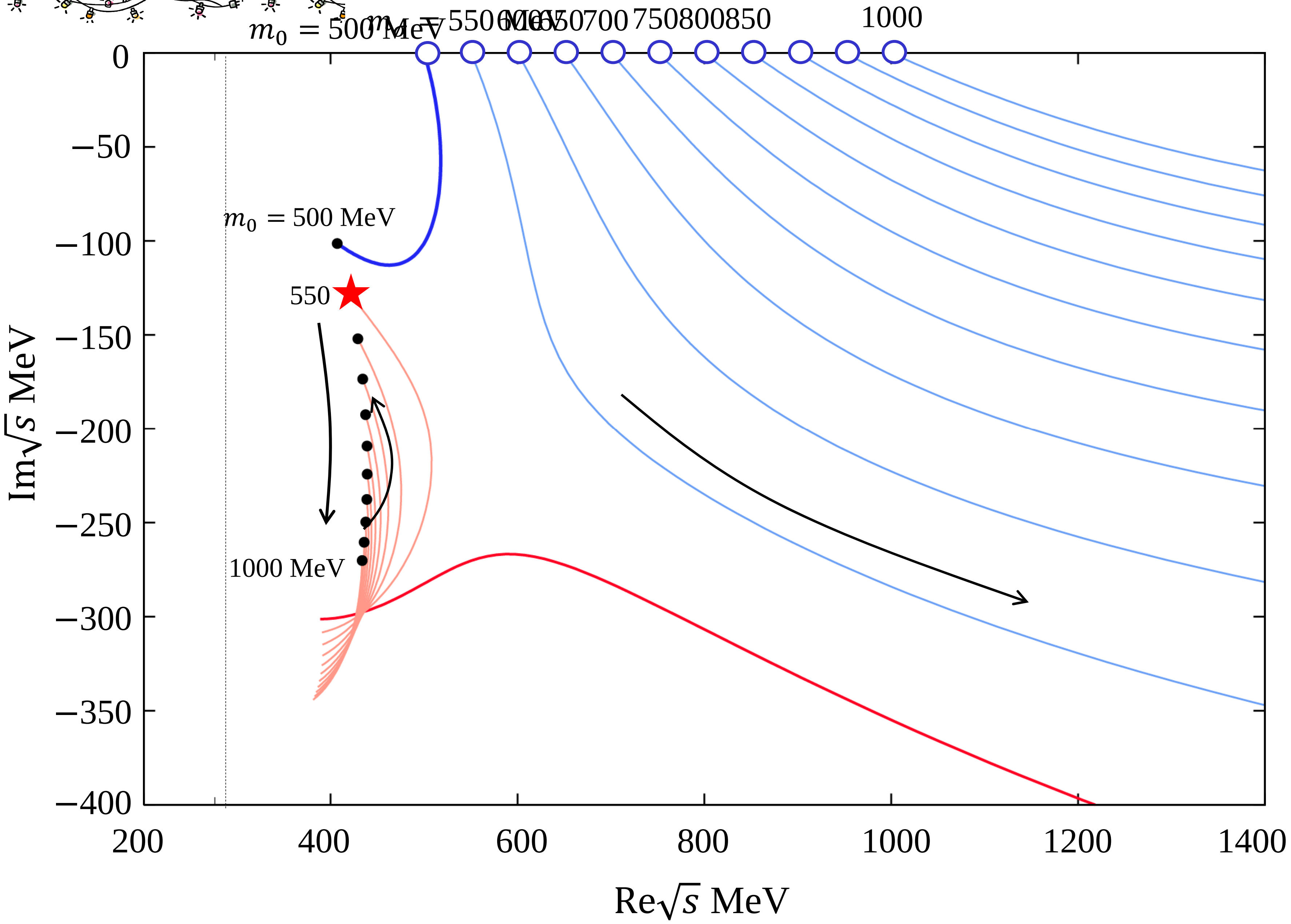
Numerical results : residues ($m_0 = 550$ MeV case)



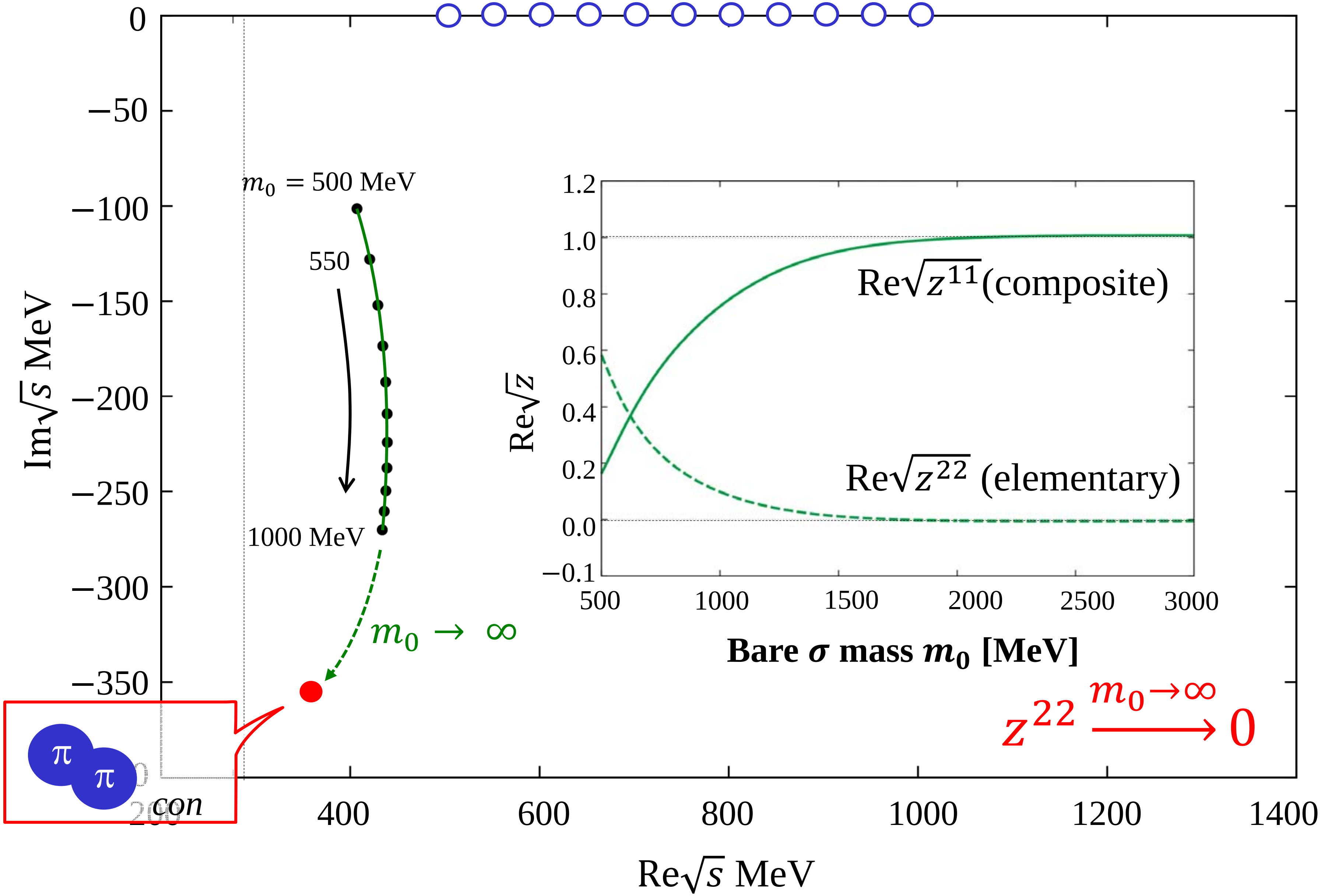


★ has larger ● contribution

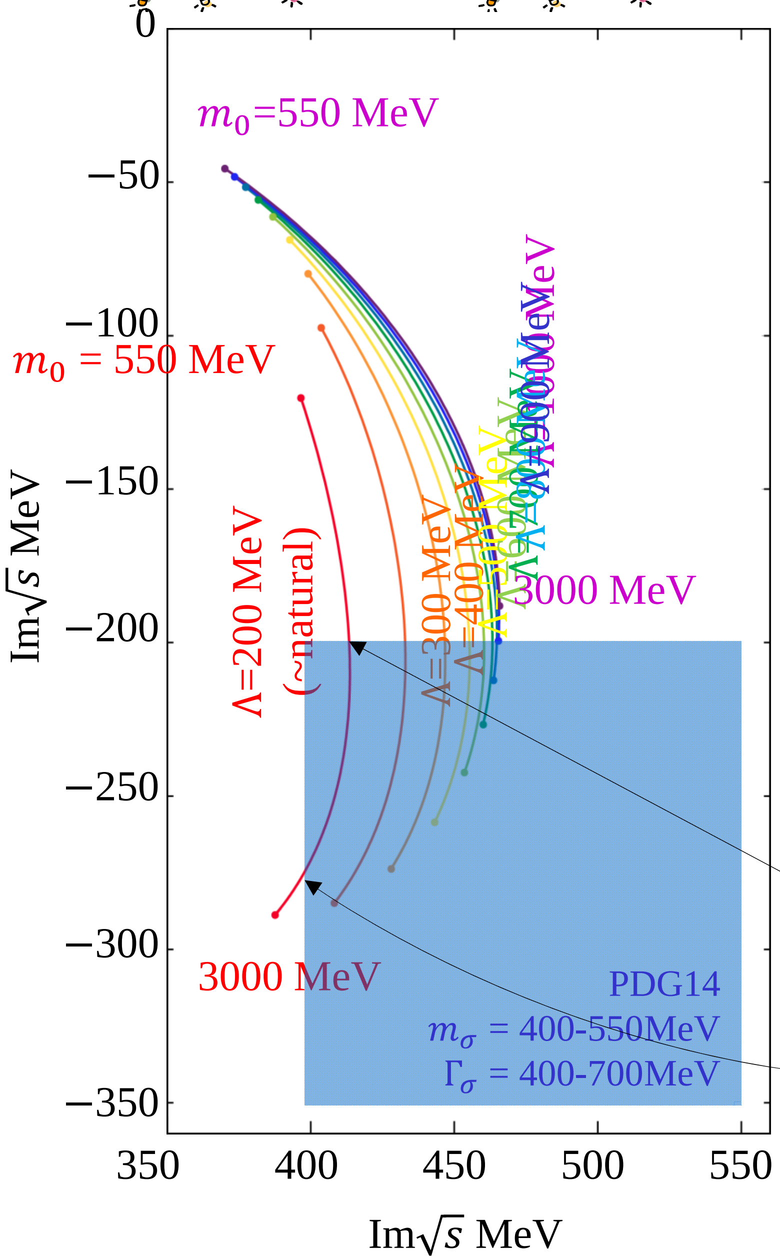
Numerical results : bare σ mass m_0 dependence

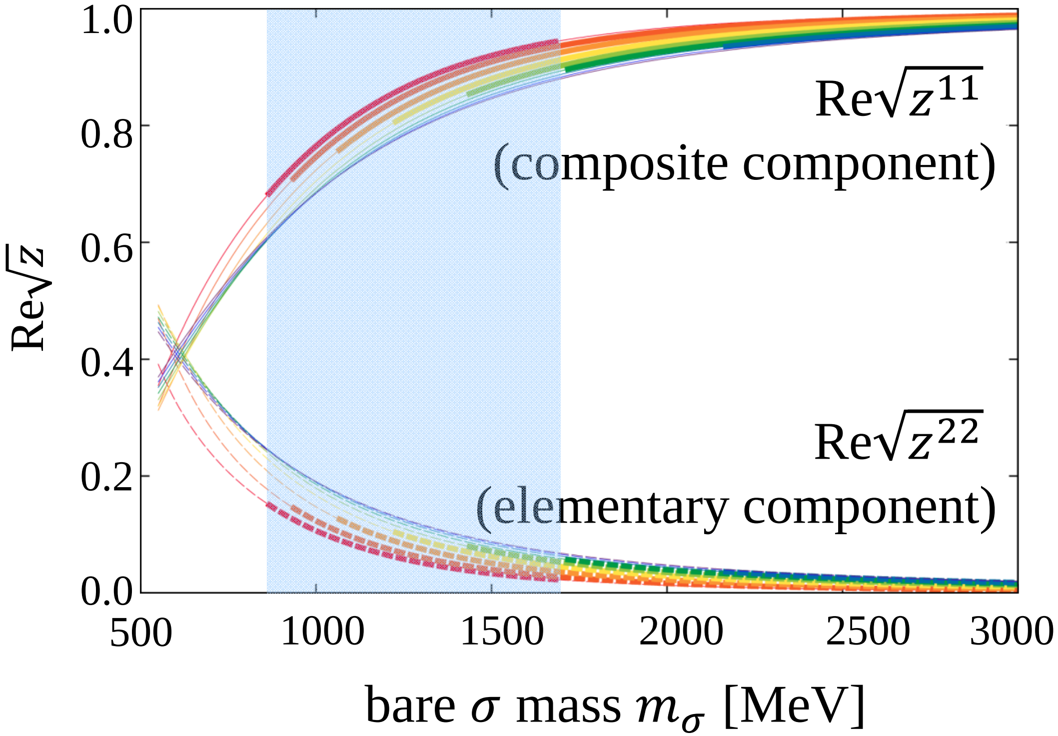


Numerical results : bare σ mass m_0 dependence



pole-position and z^{11}, z^{22} @ different cut-off Λ and bare mass m_0





in unit of MeV

Λ MeV	m_0 MeV	Re$\sqrt{z^{11}}$	Re$\sqrt{z^{22}}$	z^{22}
200	**860-1690**	0.68-0.94	0.15-0.02	$-0.04-0.08i$
600	**1430** $<$	0.85 $<$	< 0.08	$-0.008-0.02i$
1000	**9240** $<$	0.997 $<$	< 0.002	$< 10^{-5}$

my question …

How is z^{22} related to

the “compositeness condition $Z = 0$“[1-3] ?

Nagahiro-Hosaka, *in progress*

[1] S.Weinberg, PR137(65)B672
[2] D. Lurie, A.J.Macfarlane, PR136(64)B816
[3] T.Hyodo, D.Jido, A. Hosaka, PRC85(12)015201

Compositeness condition $Z = 0$?

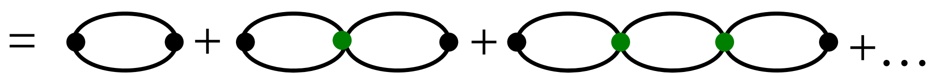
✓ Elementary component z^{22} is :

… nothing but the wave function renormalization Z for the σ field in $\mathcal{L}$

$$z^{22} = Z = \left(1 - \frac{d\Pi(s)}{ds}\bigg|_{s=m^{*2}}\right)^{-1}$$

where $$\Pi(s) = 3\frac{(s-m_\pi^2)^2}{f_\pi^2}\frac{G}{1-v_{con}G}$$

$$z^{22} \xrightarrow{m_0\to\infty} 0$$



✓ "compositeness condition $Z = 0$ [1-3]" is also the wave function renormalization

[1] S.Weinberg, PR137(65)B672
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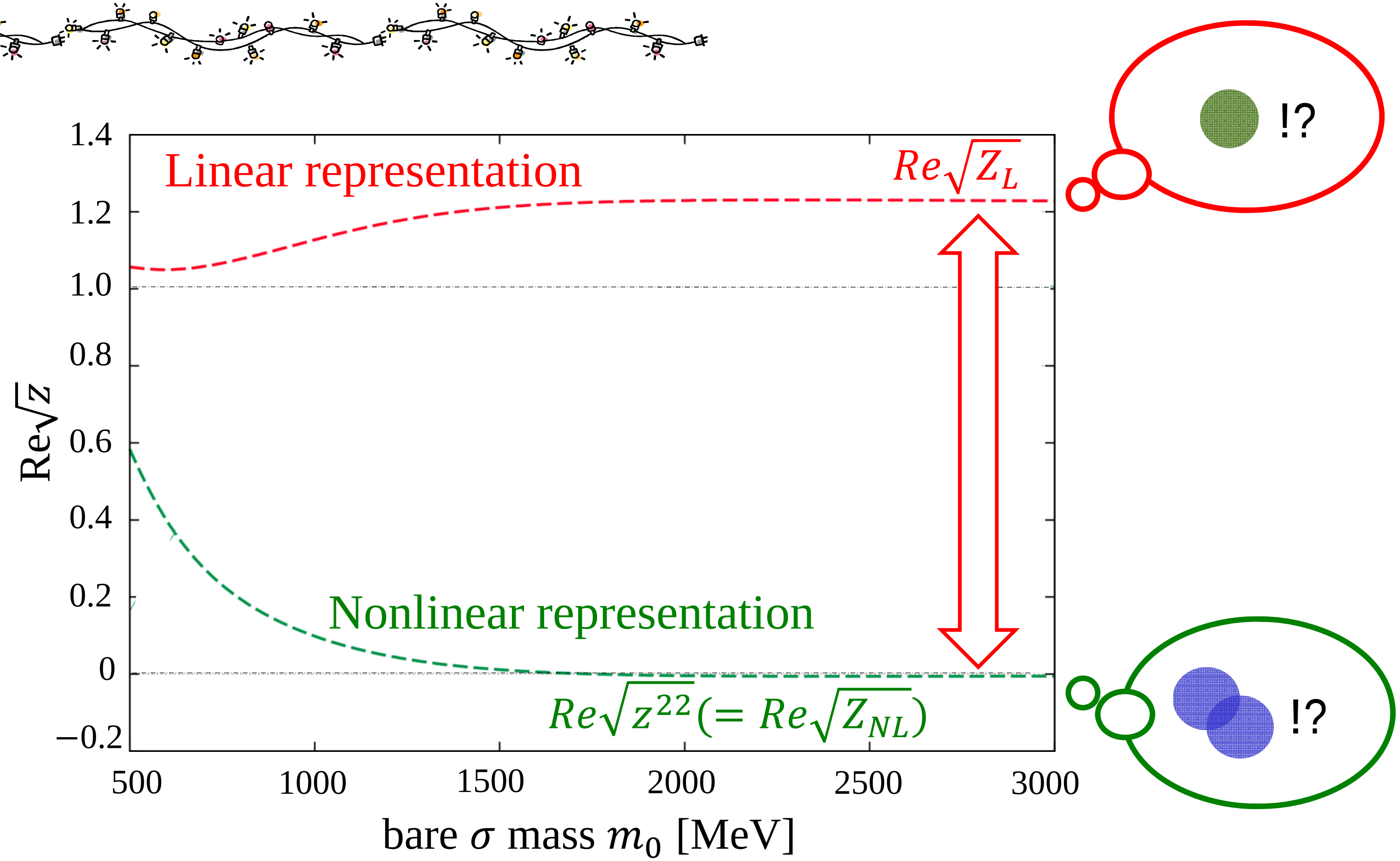
another question arises …

We have another model Lagrangian : the sigma model in the ***linear representation***

Do we get the same conclusion ($Z \to 0$ as $m_0 \to \infty$) ?

The answer is “NO”.



- ✓ Pole position, scattering amplitude, … etc. are the same in both models (= representation-independent)

- ✓ Which result should we believe ?
- ✓ Which "Z" corresponds to “compositeness condition Z” ?

z^{22} ↔ "Compositeness condition Z=0" ?

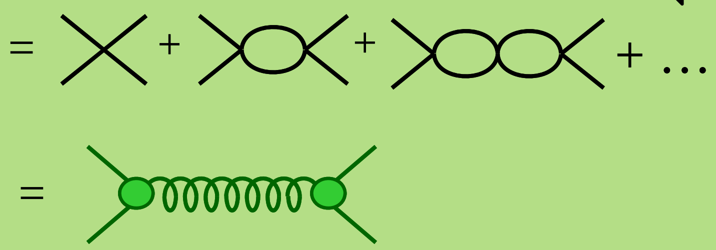
[1] S.Weinberg, PR137(65)B672
[2] D. Lurie, A.J.Macfarlane, PR136(64)B816
D. Lurie, *Particle and Fields*, 1968
[3] T.Hyodo, D.Jido, A. Hosaka, PRC85(12)015201

Compositeness condition

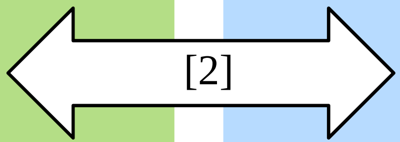
four-Fermi theory w/o "elementary"

$$\mathcal{L}_F = -g_0 \bar{\psi}\gamma_5\psi\,\bar{\psi}\gamma_5\psi$$

$$T_F = \frac{v_F}{1 - v_F \Pi(s)}$$



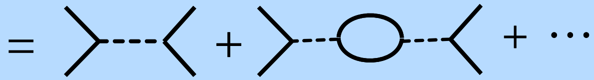
$Z = 0$



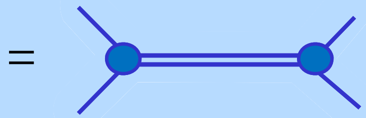
Yukawa theory w/o four-Fermi

$$\mathcal{L}_Y = iG_0 \bar{\psi}\gamma_5\psi\,\boldsymbol{\phi}$$

$$T_Y = G_0 \frac{1}{s - m_0^2 - G_0^2 \Pi(s)} G_0$$



$$= G_R \frac{1}{s - {m^*}^2 - G_R^2\, \Pi_c'(s)} G_R$$



compositeness condition

$$Z = 1 + G_R^2 \Pi'(m^*) = 0$$

w.f. renormalization of Yukawa theory

Yukawa theory is used as a
tool to measure the "elementarity"

$z^{22} \leftrightarrow$ "compositeness condition $Z = 0$" ?

the σ model in *non-linear* rep.

- ✓ $\sigma\pi\pi$ is energy dependent
- ✓ 4π contact is **large** (attractive)

the σ model in *linear* rep.

- ✓ $\sigma\pi\pi$ is energy independent
- ✓ 4π contact is **large** (repulsive)

$\rightarrow$ neither $z^{22}(Z_{NL})$ nor Z_L
$\neq$ "Compositeness condition"
of Weinberg/Lurie's definition

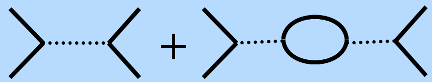
We don't have a Yukawa theory
equivalent with the sigma model.

…but we have a Yukawa-*like* theory…

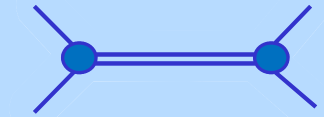
Yukawa theory w/o four π ?

$$\mathcal{L}_Y = G_0\, \phi_\pi \phi_\pi \phi_\sigma$$

$$T_Y = G_0 \frac{1}{s - m_0^2 - G_0^2 \Pi(s)} G_0$$

$=$  $+ \cdots$

$$= G_R \frac{1}{s - {m^*}^2 - G_R^2\, \Pi_c'(s)} G_R$$

$=$ 

compositeness condition

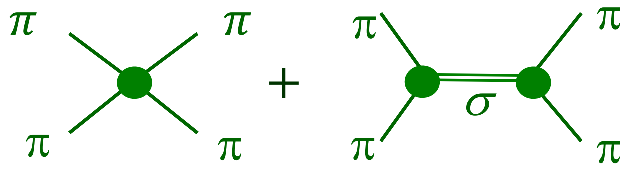
$$Z_Y = 1 + G_R^2 \Pi'(m^*) = 0$$

w.f. renormalization of Yukawa theory

Yukawa-*like* model : “quasi-particle” representation

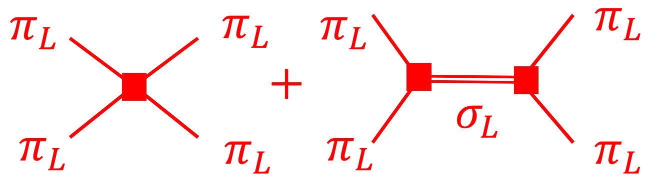
Nonlinear representation

$$A^{NL}(s) = -\frac{1}{f_\pi^2}(\boldsymbol{s} - m_\pi^2) + \frac{(\boldsymbol{s} - m_\pi^2)^2}{f_\pi^2}\boxed{\frac{1}{s - m_\sigma^2}}$$



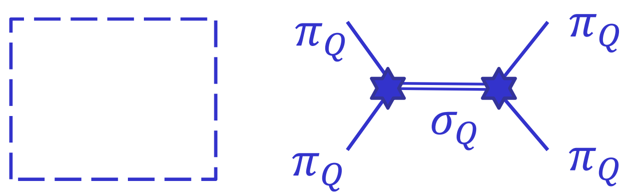
linear representation

$$A^{L}(s) = +\frac{1}{f_\pi^2}\left(\boldsymbol{m_\sigma^2} - m_\pi^2\right) + \frac{\left(\boldsymbol{m_\sigma^2} - m_\pi^2\right)^2}{f_\pi^2}\boxed{\frac{1}{s - m_\sigma^2}}$$



“quasi-particle” representation ~ Yukawa-*like*

$$A^{Q}(s) = \boldsymbol{0} + \frac{(\boldsymbol{s} - m_\pi^2)(\boldsymbol{m_\sigma^2} - m_\pi^2)}{f_\pi^2}\boxed{\frac{1}{s - m_\sigma^2}}$$

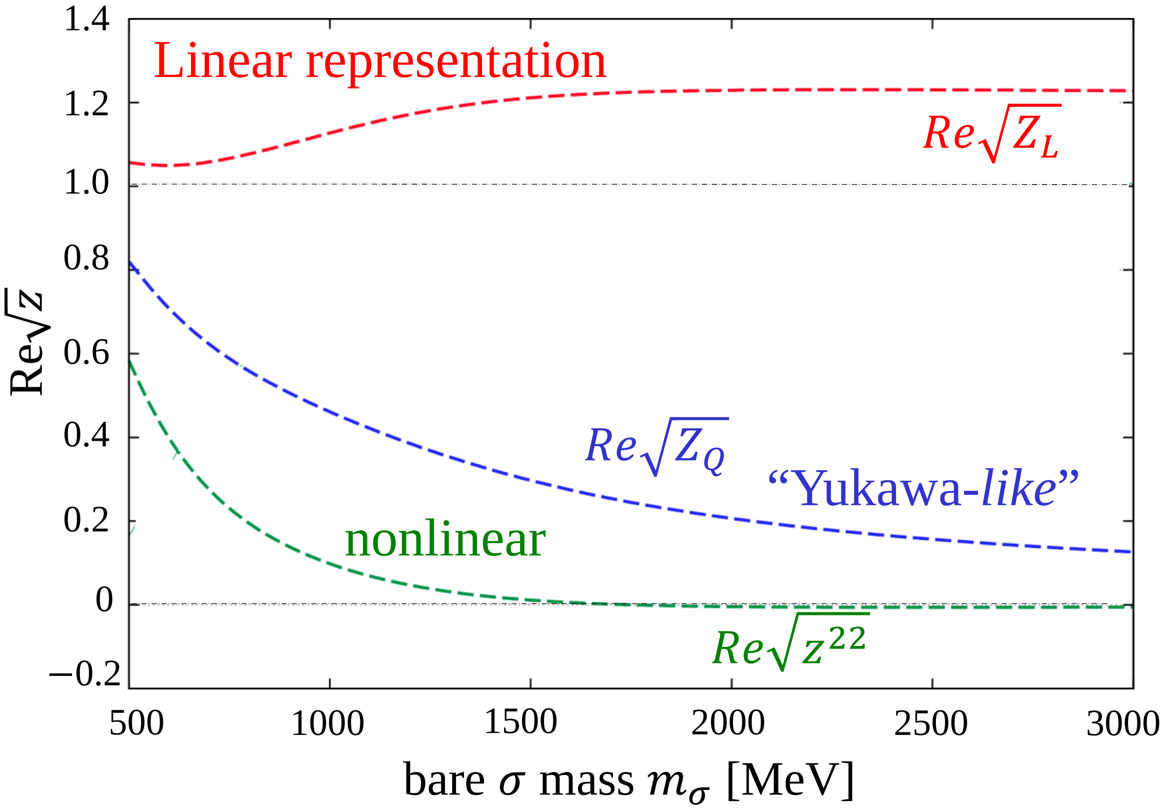


“elementary” and no contact

$$A^{NL}(s) = A^{L}(s) = A^{Q}(s)$$

- ✓ All “representations” give the same scattering amplitude
- ✓ Definitions of the elementary σ field are different

Yukawa-*like* σ model



- ✓ Z $\nrightarrow$ 0 as $m_\sigma \to \infty$ in linear rep. / $Z \to 0$ in nonlinear and "quasi-particle"
- ✓ **"*Z*" is not a universal measure**
  → it depends on the definition of "elementary" particle
- ✓ We first need to define "what is the elementary particle" .

Interpretations of the physical sigma pole m_σ^*

cut-off $\Lambda = 1$ GeV : $m_0 \sim 9$ GeV $\rightarrow$ $m_\sigma^* = 465 - i200$ MeV

[(400– 550) – i(200– 350) MeV PDG14]

Nonlinear rep.

- ✓ Composite σ mixes with the elementary σ (two basis states)
- ✓ physical pole position is very close to composite one
- ✓ physical σ is almost composite : $Z_{NL} \sim 0$

Linear rep.

- ✓ No composite σ
- ✓ elementary σ ($m_0 \sim 9$ GeV) goes down to 465 MeV by the quantum effect
- ✓ $Z_L \sim 1$ in the limit of $m_0 \rightarrow \infty$?

"quasi-particle" (Yukawa-*like* theory)

- ✓ No composite σ
- ✓ elementary σ ($m_0 \sim 9$ GeV) goes down to 465 MeV by the quantum effect
- ✓ Z is small
- ✓ similar to Weinberg/Lurie's definition
- ✓ Yukawa-*like* sigma model … *What is this "elementary σ"…?*

Summary

» **Mixing property of σ meson in nonlinear rep. by means of two level prob.**

› Mixture of a $\pi\pi$ composite and "elementary" σ

› Physical σ is almost "$\pi\pi$ composite" and the component of "elementary" is small within the present model setting.

$\Leftrightarrow$ $a_1(1260)$ with hidden local symmetry, concluded that the physical a_1 has comparable amounts of $\pi\rho$ composite to elementary a_1 (PRD83(11)111504(R))

» **Representation dependence of wave function renormalization Z**

› **"Compositeness condition $Z = 0$" $\leftrightarrow$ z^{22}**

› **"Z" is not a universal measure**

› **Generally**, it depends on the definition of "elementary particle" and it may not be a specific problem of σ

» **What is the most "economical" basis ?**

› Or maybe we need to approach from different axis (such as behavior expected in finite T/ρ)