Hadron structure in Lattice QCD

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Hadrons from Quarks and Gluons
International Workshop XLII on Gross Properties of Nuclei and Nuclear Excitations

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Outline

1. **Lattice QCD**
   - Introduction
   - Fermion actions
   - Dynamical simulations - Computational cost
2. **Recent achievements**
   - Hadron spectrum
   - Charmed baryons
   - Meson decay constants - see H. Wittig's talk
   - ρ-meson width
3. **Challenges**
   - Masses of excited states
   - Nucleon Structure
     - Axial charge g_A, momentum fraction $\langle x \rangle$ and σ-terms
     - Spin content of the nucleon
   - Hadron Structure
     - Resonance parameters for baryons
     - Axial charges and σ-terms
4. **Conclusions**

Quantum ChromoDynamics (QCD)

QCD-Gauge theory of the strong interaction
Lagrangian: formulated in terms of quarks and gluons

$$\mathcal{L}_{QCD} = -\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} + \sum_{f=u,d,s,c,b,t} \bar{\psi}_f \left(i\gamma^\mu D_\mu - m_f \right) \psi_f$$

$$D_\mu = \partial_\mu - ig\frac{\lambda^a}{2} A^a_\mu$$

Harald Fritzsch

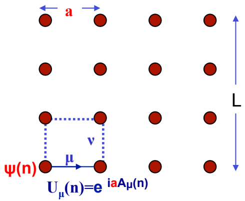
Murray Gell-Mann

Heinrich Leutwyler

This “simple” Lagrangian produces the amazingly rich structure of strongly interacting matter in the universe.

Numerical simulation of QCD provides essential input for a wide class of complex strong interaction phenomena
$\rightarrow$ In this talk: **Hadron structure with emphasis on the nucleon**

QCD on the lattice



Lattice QCD: K. Wilson, 1974 provided the formulation; M. Creutz, 1980 performed the first numerical simulation

- Discretization of space-time with lattice spacing a: quark fields $\psi(x)$ and $\bar{\psi}(x)$ on lattice sites and gauge field $U_\mu(x)$ on links
- Finite a provides an ultraviolet cutoff at $\pi/a \rightarrow$ non-perturbative regularization; Finite $L \rightarrow$ discrete momenta in units of $2\pi/L$ if periodic b.c.
- Construct an appropriate action S and rotate into imaginary time $\rightarrow$ Monte Carlo simulation to produce a representative ensemble of $\{U_\mu(x)\}$ using the largest supercomputers $\rightarrow$ Observables: $\langle \mathcal{O} \rangle = \sum_{\{U_\mu\}} O(D^{-1}, U_\mu)$, D^{-1} is the fermion propagator

5.0 Pflop/s, biggest in Europe and 7th in the world

Fermion action

Several $\mathcal{O}(a)$-improved fermion actions, K. Jansen, Lattice 2008

Action	Advantages	Disadvantages
Clover improved Wilson	computationally fast	breaks chiral symmetry needs operator improvement
Twisted mass (TM)	computationally fast automatic improvement	breaks chiral symmetry violation of isospin
Staggered	computational fast	four doublers (fourth root issue) complicated contractions
Domain wall (DW)	improved chiral symmetry	computationally demanding needs tuning
Overlap	exact chiral symmetry	computationally expensive

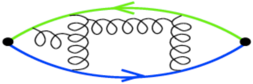
Several collaborations:

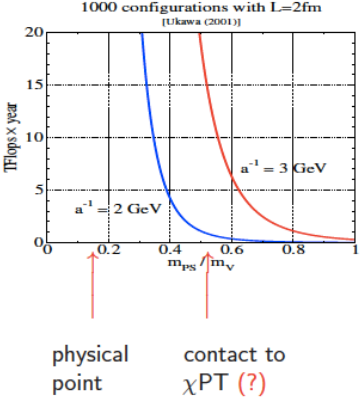
Clover	QCDSF, BMW, ALPHA, CLS, PACS-CS, NPQCD
Twisted mass	ETMC
Staggered	MILC
Domain wall	RBC-UKQCD
Overlap	JLQCD

Cost of simulations

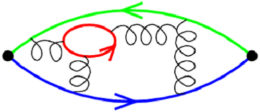
Main cost due to fermions

Cost of dynamical quark simulations in 2001





Before the 21st century: neglect pair creation (quenched QCD)



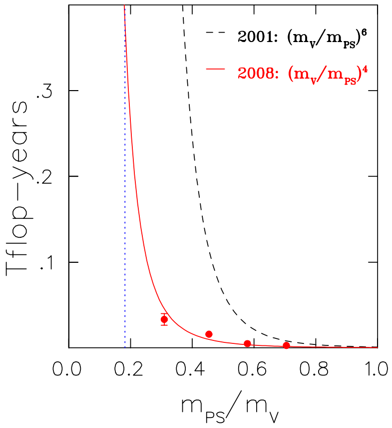
21st century: Dynamical quark simulations

Simulation cost: $C_{\rm sim} = C \left(\frac{300\text{MeV}}{m_\pi}\right)^{c_m} \left(\frac{L}{3\text{fm}}\right)^{c_L} \left(\frac{0.1\text{fm}}{a}\right)^{c_a}$

For $N_f = 2$ Wilson fermion simulations in 2001: Number of inversions, step size in molecular dynamics and autocorrelations $\sim \frac{1}{m_q}\frac{1}{m_q}\frac{1}{m_q} \rightarrow (m_\rho/m_\pi)^6$ scaling.

Computational cost

Simulation cost: $C_{\rm sim} = C\left(\frac{300\rm MeV}{m_\pi}\right)^{c_m}\left(\frac{L}{3\rm fm}\right)^{c_L}\left(\frac{0.1\rm fm}{a}\right)^{c_a}$



$N_f = 2$ twisted mass, L=2.1 fm, a=0.089 fm, K. Jansen and C. Urbach, arXiv:0905.3331

Current simulations use improved algorithms
$\Longrightarrow$ for twisted mass fermions: $c_m \sim 4 \rightarrow (m_\rho/m_\pi)^4$

- Simulations at physical quark masses, $a \sim 0.1$ fm and $L \sim 5$ fm are now feasible using Peta-scale computers
- The analysis to produce physics results also needs access to Peta-scale machines

$\Longrightarrow$ progress depends crucially on access to Tier-0 machines

Systematic uncertainties

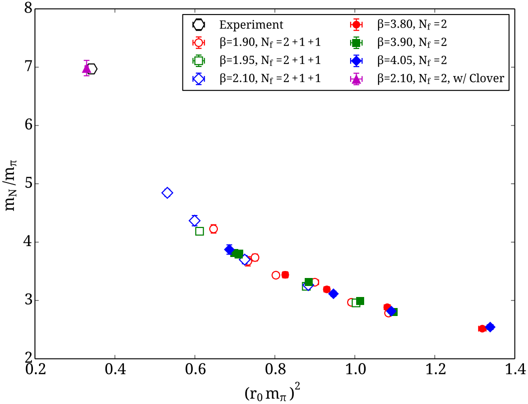
- Finite lattice spacing a - take the continuum limit $a \to 0$: $\star$
- Finite volume L - take infinite volume limit $L \to \infty$: $\star$
- Identification of hadron state of interest - g_A, $\langle x \rangle$ and nucleon σ-terms
- Simulation at physical quark masses - now feasible: $\star$
- Inclusion of quark loop contributions - now feasible

Recent achievements

Simulation with physical quark masses

A number of collaborations are producing simulations with physical values of the quark mass

ETM Collaboration:



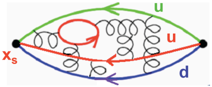
$L \sim 3$ fm and $a \sim 0.1$ fm; $r_0 \sim 0.5$ fm

Hadron masses

First goal: reproduce the low-lying masses

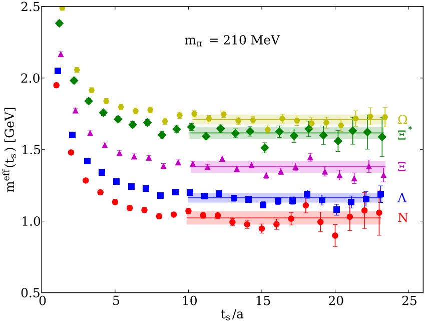
Use Euclidean correlation functions

$$G(\vec{q}, t_s) = \sum_{\vec{x}_s} e^{-i\vec{x}_s\cdot\vec{q}} \langle J(\vec{x}_s, t_s) J^\dagger(0)\rangle$$
$$= \sum_{n=0,\cdots,\infty} A_n e^{-E_n(\vec{q})t_s} \xrightarrow{t_s\to\infty} A_0 e^{-E_0(\vec{q})t_s}$$



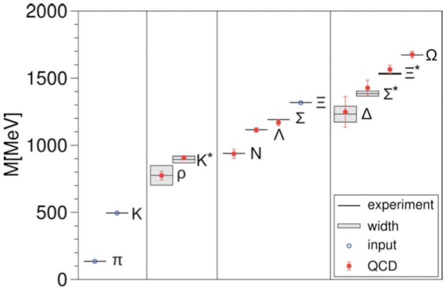
need D^{-1} per flavor

- $m_{\rm eff}(\vec{q}=0, t_s) \equiv -\frac{\partial}{\partial t_s} \ln\left[G(\vec{0}, t_s))\right]$
  $= m + \text{excited states}$
  $\xrightarrow{t_s\to\infty} m$
  Large Euclidean time evolution gives ground state
  Special techniques to extract excited states
- Noise to signal increases with t_s:$\sim e^{(m_h - \frac{3}{2}m_\pi)t_s}$



$N_f = 2+1+1$ TM fermions

Hadron spectrum



$N_f = 2 + 1$ Clover, BMW, Science 322 (2008)

Milestone calculation for lattice QCD → agreement with experiment is a success for QCD & LQCD

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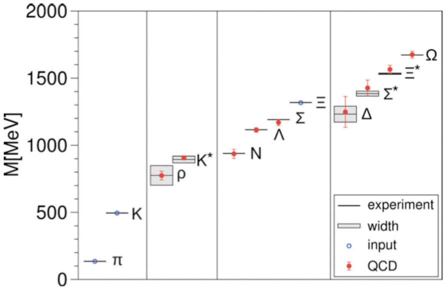

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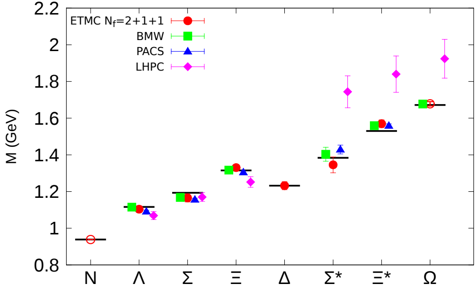
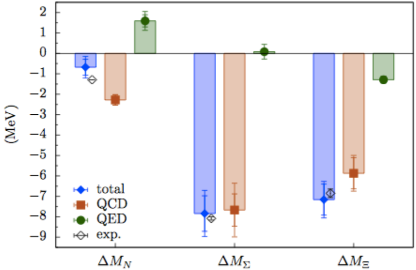


Several collaborations producing the hadron spectrum

Milestone calculation for lattice QCD $\rightarrow$ agreement with experiment is a success for QCD & LQCD

Isospin and electromagnetic mass splitting

RBC and BMW collaborations: Treat isospin and electromagnetic effects to LO



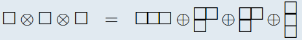
Baryon spectrum with mass splitting from BMW

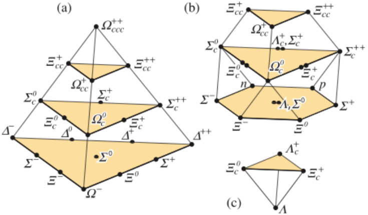
- Nucleon mass: isospin and electromagnetic effects with opposite signs
- Physical splitting reproduced

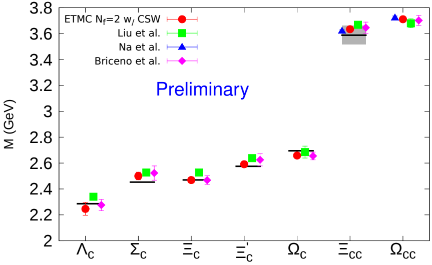
Charmed baryons

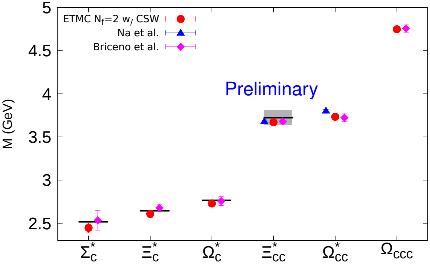
SU(4) representations:

$$4 \otimes 4 \otimes 4 = 20 \oplus 20 \oplus 20 \oplus \bar{4}$$







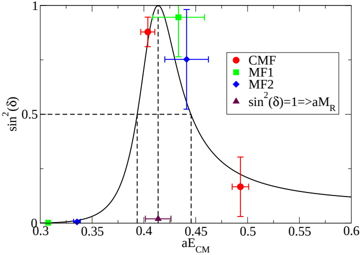


Results by ETMC using simulations with physical pion mass

ρ-meson width

- Consider $\pi^+\pi^-$ in the $I = 1$-channel
- Estimate P-wave scattering phase shift $\delta_{11}(k)$ using finite size methods
- Use Lüscher's relation between energy in a finite box and the phase in infinite volume
- Use Center of Mass frame and Moving frame
- Use effective range formula: $tan\delta_{11}(k) = \frac{g^2_{\rho\pi\pi}}{6\pi}\frac{k^3}{E\left(m_R^2 - E^2\right)}$, $k = \sqrt{E^2/4 - m_\pi^2}$ $\rightarrow$ determine m_R and $g_{\rho\pi\pi}$ and then extract $\Gamma_\rho = \frac{g^2_{\rho\pi\pi}}{6\pi}\frac{k_R^3}{m_R^2}$, $k_R = \sqrt{m_R^2/4 - m_\pi^2}$

$m_\pi = 309$ MeV, $L = 2.8$ fm



$N_F = 2$ twisted mass fermions, Xu Feng, K. Jansen and D. Renner, Phys. Rev. D83 (2011) 094505

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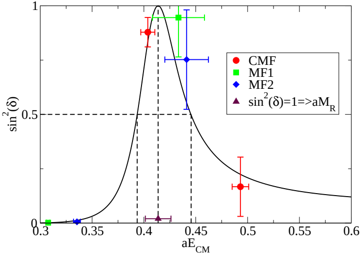

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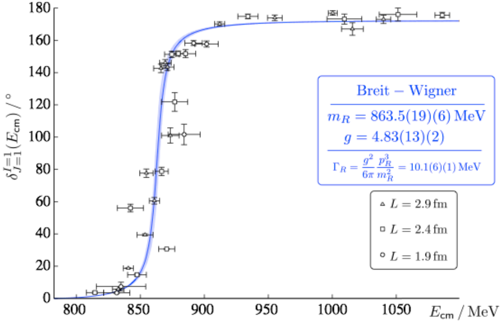


Impressive results using $N_f = 2 + 1$ clover fermions and 3 asymmetric lattices, J. J. Dudek, R. G. Edwards and C.E. Thomas, Phys. Rev. D 87 (2013) 034505

Challenges

Challenges: I. Excited states, multi-quark states & exotics

Variational approach: Enlarge basis of interpolating fields $\rightarrow$ correlation matrix

$$G_{jk}(\vec{q}, t_s) = \sum_{\vec{x}_s} e^{-i\vec{x}_s\cdot\vec{q}} \langle J_j(\vec{x}_s, t_s) J_k^\dagger(0)\rangle \,, j, k = 1, \ldots N$$

Solve the generalized eigenvalue problem (GEVP)

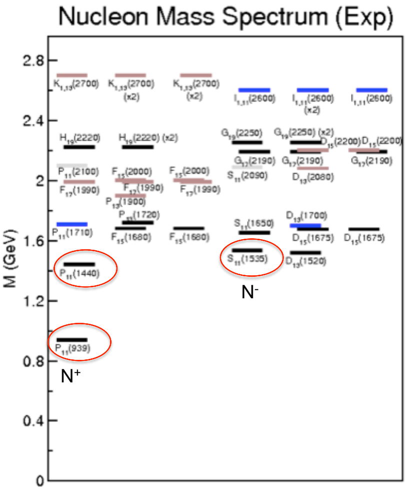
$G(t)v_n(t;t_0) = \lambda_n(t;t_0)G(t_0)v_n(t;t_0) \rightarrow \lambda_n(t;t_0) = e^{-E_n(t-t_0)}$ yields N lowest eigenstates, M. Lüscher & U. Wolff (1990)

Large effort to construct the appropriate basis using lattice symmetries, Hadron Spectrum Collaboration

- must extract all states lying below the state of interest
- as $m_\pi \rightarrow m_{\rm physical}$ need to consider multi-hadron states
- must include disconnected diagrams
- most excited states are unstable (resonances)

Where is the Roper?

First goal to obtain the known low-lying nucleon resonances



- R.G. Edwards, J. J. Dudek, D. G. Richards, S. J. Wallace. Phys. Rev. D 84 (2011) 074508
- M. Mahbub *et al.*, Phys. Lett. B679 (2009) 418
- G. P. Engel, C. Lang, D. Mohler, A. Schäfer, 1301.4318

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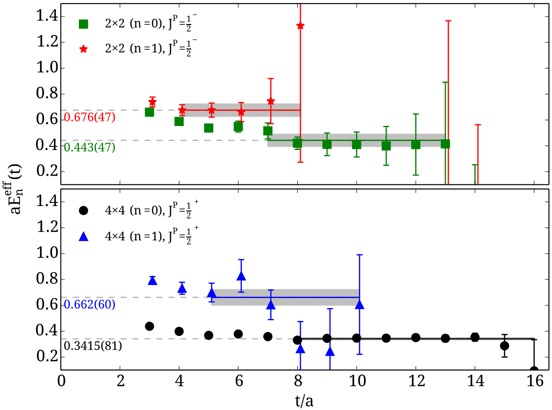

$N_f = 2$ Clover fermions, $m_\pi = 156$ MeV - configurations provided by QCDSF

- R.G. Edwards, J. J. Dudek, D. G. Richards, S. J. Wallace. Phys. Rev. D 84 (2011) 074508
- M. Mahbub *et al.*, Phys. Lett. B679 (2009) 418
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- $N_f = 2$ Twisted mass and clover, C.A., T. Korzec, G. Koutsou, T. Leontiou, arXiv:1302.4410

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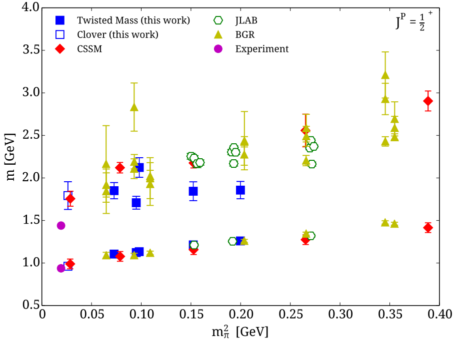
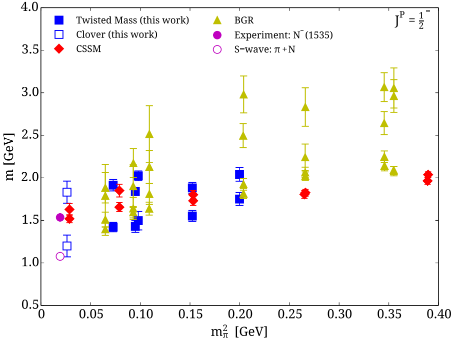


Positive parity



Negative parity

- R.G. Edwards, J. J. Dudek, D. G. Richards, S. J. Wallace. Phys. Rev. D 84 (2011) 074508
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Challenges: II. Nucleon structure

Parton distribution functions - not directly accessible in lattice QCD calculations
Compute moments: $\langle x^n \rangle = \int dx\, x^n q(x) \rightarrow$ related to local operators:

- vector operator

$$\mathcal{O}_{Va}^{\mu_1 \cdots \mu_n} = \bar{\psi}(x)\gamma^{\{\mu_1} i \overleftrightarrow{D}^{\mu_2} \ldots i \overleftrightarrow{D}^{\mu_n\}} \frac{\tau^a}{2} \psi(x)$$

- axial-vector operator

$$\mathcal{O}_{Aa}^{\mu_1 \cdots \mu_n} = \bar{\psi}(x)\gamma^{\{\mu_1} i \overleftrightarrow{D}^{\mu_2} \ldots i \overleftrightarrow{D}^{\mu_n\}} \gamma_5 \frac{\tau^a}{2} \psi(x)$$

- tensor operator

$$\mathcal{O}_{Ta}^{\mu_1 \cdots \mu_n} = \bar{\psi}(x)\sigma^{\{\mu_1,\mu_2} i \overleftrightarrow{D}^{\mu_3} \ldots i \overleftrightarrow{D}^{\mu_n\}} \frac{\tau^a}{2} \psi(x)$$

Special cases:

- no-derivative $\rightarrow$ nucleon form factors
- one-derivative $\rightarrow$ first moments e.g. average momentum fraction $\langle x \rangle$
  Generalized form factor decomposition:

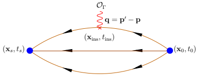
$$\langle N(p', s')|\mathcal{O}_{V3}^{\mu\nu}|N(p, s)\rangle = \bar{u}_N(p', s')\left[A_{20}(q^2)\gamma^{\{\mu} P^{\nu\}} + B_{20}(q^2)\frac{i\sigma^{\{\mu\alpha} q_\alpha P^{\nu\}}}{2m} + C_{20}(q^2)\frac{q^{\{\mu} q^{\nu\}}}{m}\right]\frac{1}{2} u_N(p, s)$$

Nucleon spin $J^q = \frac{1}{2}\left[A_{20}(0) + B_{20}(0)\right]$

Nucleon matrix elements

Evaluation of three-point functions:

$$G^{\mu\nu}(\Gamma, \vec{q}, t_s, t_{\rm ins}) = \sum_{\vec{x}_s, \vec{x}_{\rm ins}} e^{i\vec{x}_{\rm ins}\cdot\vec{q}}\, \Gamma_{\beta\alpha}\, \langle J_\alpha(\vec{x}_s, t_s)\mathcal{O}^{\mu\nu}(\vec{x}_{\rm ins}, t_{\rm ins})\overline{J}_\beta(\vec{x}_0, t_0)\rangle$$



connected contribution

Form ratio by dividing the three-point correlator by an appropriate combination of two-point functions:

$$R(t_s, t_{\rm ins}, t_0) \xrightarrow[(t_s - t_{\rm ins})\Delta \gg 1]{(t_{\rm ins} - t_0)\Delta \gg 1} \mathcal{M}[1 + \ldots e^{-\Delta(\mathbf{p})(t_{\rm ins} - t_0)} + \ldots e^{-\Delta(\mathbf{p}')(t_s - t_{\rm ins})}]$$

- $\mathcal{M}$ the desired matrix element
- t_s, $t_{\rm ins}$, t_0 the sink, insertion and source time-slices
- $\Delta(\mathbf{p})$ the energy gap with the first excited state

Nucleon matrix elements

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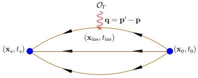

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- $\mathcal{M}$ the desired matrix element
- t_s, t_{ins}, t_0 the sink, insertion and source time-slices
- $\Delta(\mathbf{p})$ the energy gap with the first excited state

Summing over t_{ins}:

$$\sum_{t_{ins}=t_0}^{t_s} R(t_s, t_{ins}, t_0) = \text{Const.} + \mathcal{M}[(t_s - t_0) + \mathcal{O}(e^{-\Delta(\mathbf{p})(t_s - t_0)}) + \mathcal{O}(e^{-\Delta(\mathbf{p}')(t_s - t_0)})].$$

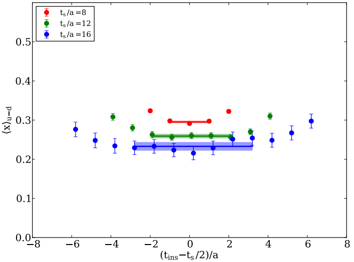
So the excited state contributions are suppressed by exponentials decaying with $t_s - t_0$, rather than $t_s - t_{ins}$ and/or $t_{ins} - t_0$.

However, one needs to fit the slope rather than to a constant

Connect lattice results to measurements:

$\mathcal{O}_{\overline{MS}}(\mu) = Z(\mu, a)\mathcal{O}_{latt}(a)$

$\Longrightarrow$ evaluate $Z(\mu, a)$ non-perturbatively

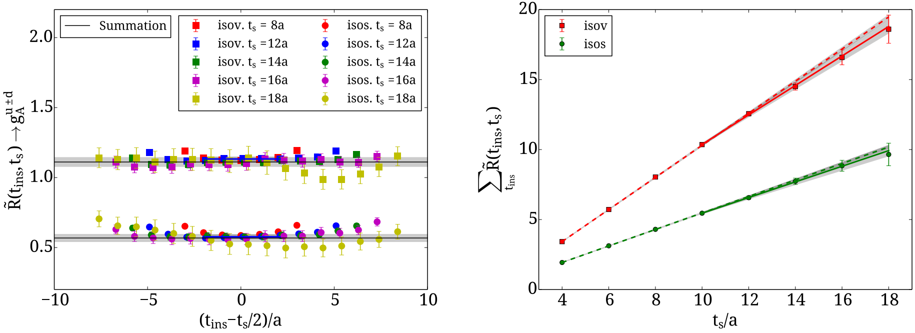


Axial charge g_A

Axial-vector FFs: $A^3_\mu = \bar\psi\gamma_\mu\gamma_5\frac{\tau^3}{2}\psi(x) \Longrightarrow \frac{1}{2}\bar{u}_N(\vec{p}')\left[\gamma_\mu\gamma_5 G_A(q^2) + \frac{q^\mu\gamma_5}{2m}G_p(q^2)\right]u_N(\vec{p})|_{q^2=0}$

$\rightarrow$ yields $G_A(0) \equiv g_A$: i) well known experimentally, & ii) no quark loop contributions

$N_f = 2+1+1$ twisted mass, a = 0.082 fm, m_π = 373 MeV, 1200 statistics



- No detectable excited states contamination
- Consistent results between summation and plateau methods

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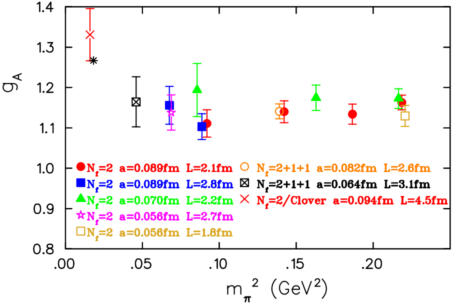


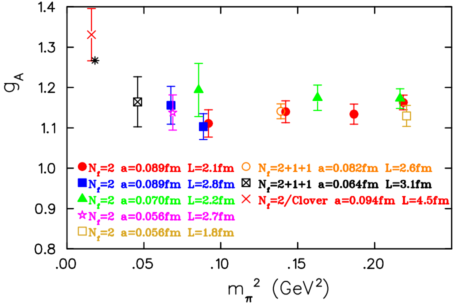
Results from ETMC, C.A., M. Constantinou, S. Dinter, V. Drach, K. Jansen, C. Kallidonis, G. Koutsou, arXiv:1303.5979

- Results at physical pion mass are now becoming available $\rightarrow$ need a dedicated study with high statistics, a larger volume and 3 lattice spacings

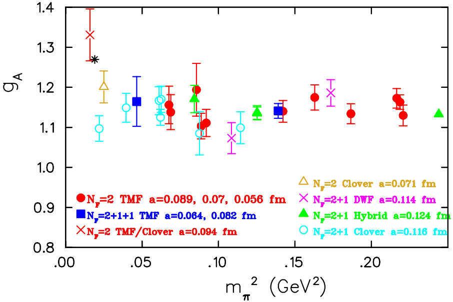
Axial charge g_A

Axial-vector FFs: $A^3_\mu = \bar\psi\gamma_\mu\gamma_5\frac{\tau^3}{2}\psi(x) \Longrightarrow \frac{1}{2}\bar u_N(\vec{p'})\left[\gamma_\mu\gamma_5 G_A(q^2) + \frac{q^\mu\gamma_5}{2m}G_p(q^2)\right]u_N(\vec p)|_{q^2=0}$

$\rightarrow$ yields $G_A(0) \equiv g_A$: i) well known experimentally, & ii) no quark loop contributions



Results from ETMC, C.A., M. Constantinou, S. Dinter, V. Drach, K. Jansen, C. Kallidonis, G. Koutsou, arXiv:1303.5979



Comparison with other groups using similar methods

- Results at physical pion mass are now becoming available $\rightarrow$ need a dedicated study with high statistics, a larger volume and 3 lattice spacings
- A number of collaborations are engaging in systematic studies, e.g.
  - $N_f = 2+1$ Clover, J. R. Green *et al.*, arXiv:1209.1687
  - $N_f = 2$ Clover, R.Hosley *et al.*, arXiv:1302.2233
  - $N_f = 2$ Clover, S. Capitani *et al.* arXiv:1205.0180
  - $N_f = 2+1$ Clover, B. J. Owen *et al.*, arXiv:1212.4668
  - $N_f = 2+1+1$ Mixed action (HISQ/Clover), T. Bhattacharya *et al.*, arXiv:1306.5435

Momentum fraction and the nucleon spin

What is the distribution of the nucleon momentum among the nucleon constituents?

$\rightarrow$ needs knowledge of the parton distribution functions (PDFs)

One measures moments of parton distributions,in DIS:

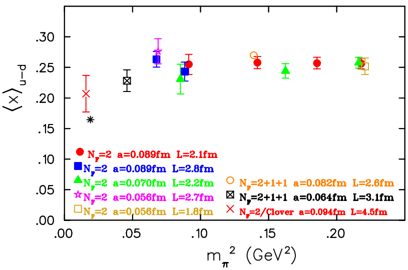
$$\langle x \rangle_q = \int_0^1 dx\, x\, [q(x) + \bar{q}(x)] \;, \qquad \langle x \rangle_{\Delta q} = \int_0^1 dx\, x\, [\Delta q(x) - \Delta \bar{q}(x)]$$

Unpolarized quark distribution $\quad q(x) = q(x)_{\downarrow} + q(x)_{\uparrow}$

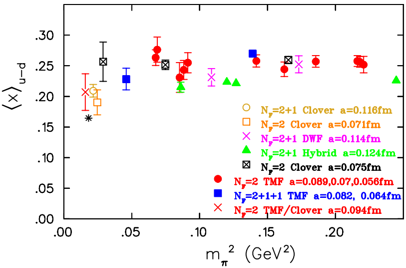
Polarized quark distribution $\quad \Delta q(x) = q(x)_{\downarrow} - q(x)_{\uparrow}$

Extracted from nucleon matrix elements of $\mathcal{O}_q^{\mu_1 \mu_2} = \bar{\psi}\gamma^{\{\mu_1} i \overleftrightarrow{D}^{\mu_2\}}\psi$ and $\mathcal{O}_{\Delta q}^{\mu_1 \mu_2} = \bar{\psi}\gamma^{\{\mu_1}\gamma_5 i \overleftrightarrow{D}^{\mu_2\}}\psi$

Results in the $\overline{MS}$ scheme at $\mu = 2$ GeV.



Results from ETMC



Comparison with other collaborations

Experimental values:

- $\langle x \rangle_{u-d}$ from S. Alekhin *et al.* arXiv:1202.2281

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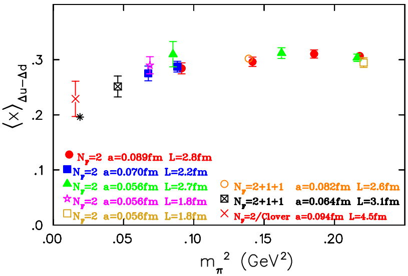

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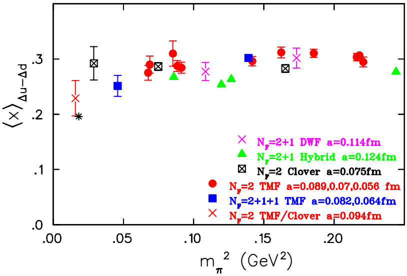
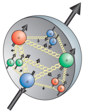


Comparison with other collaborations

Experimental values:

- $\langle x \rangle_{\Delta u - \Delta d}$ from Blumlein *et al.* arXiv:1005.3113

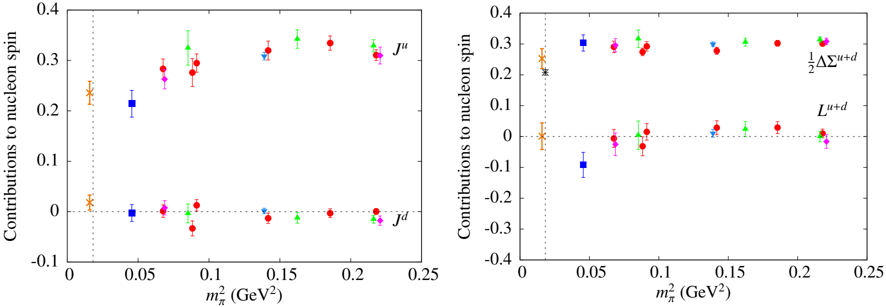
Where is the nucleon spin?



Spin sum: $\frac{1}{2} = \sum_q \underbrace{\left(\frac{1}{2}\Delta\Sigma^q + L^q\right)}_{J^q} + J^G$

$J^q = A^q_{20}(0) + B^q_{20}(0)$ and $\Delta\Sigma^q = g^q_A$

Connected contributions

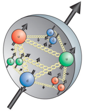


$\Longrightarrow$ Total spin for u-quarks $J^u \lesssim 0.25$ and for d-quark $J^d \sim 0$

- $L^{u+d} \sim 0$ at physical point
- $\Delta\Sigma^{u+d}$ in agreement with experimental value at physical point
- The total spin $J^{u+d} \sim 0.25 \Longrightarrow$ Where is the other half?

However, more statistics and checks of systematics are needed for final results at the physical point

Where is the nucleon spin?



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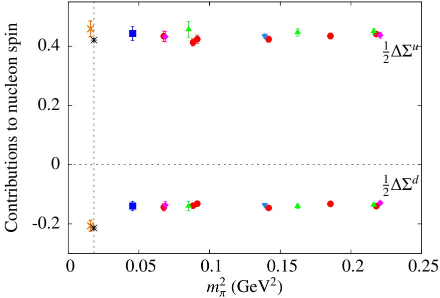
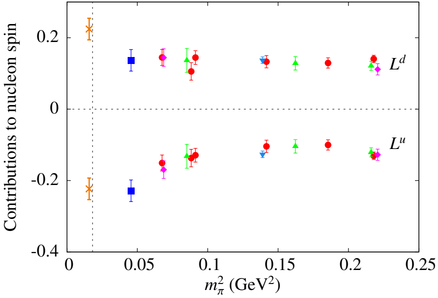
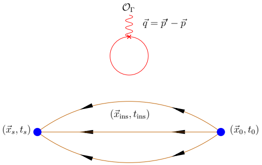




- $\Delta\Sigma^{u,d}$ consistent with experimental values
- $L^d \sim -L^u$

However, more statistics and checks of systematics are needed for final results at the physical point

Challenges: III. Disconnected quark loop contributions



Notoriously difficult

- $L(x_{\rm ins}) = Tr\,[\Gamma G(x_{\rm ins}; x_{\rm ins})]$ $\rightarrow$ need quark propagators from all $\vec{x}_{\rm ins}$ or L^3 more expensive as compared to the calculation of hadron masses
- Large gauge noise $\rightarrow$ large statistics
- Use special techniques that utilize stochastic noise on all spatial lattice sites $\rightarrow$ N_r more expensive that hadron masses with $N_r \ll L^3$
- Reduce noise by increasing statistics
  $\Longrightarrow$ take advantage of graphics cards (GPUs) $\rightarrow$ need to develop special multi-GPU codes

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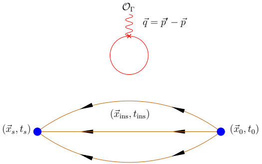

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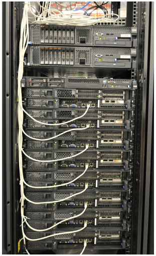
A Fermi card



Cluster of 8 nodes of Fermi GPUs at the Cyprus Institute

C. A., M. Constantinou, S. Dinter, V. Drach, K. Hadjiyiannakou, K. Jansen, G. Koutsou, A. Strelchenko, A. Vaquero arXiv:1211.0126
C.A., K. Hadjiyiannakou, G. Koutsou, A. O'Cais, A. Strelchenko, arXiv:1108.2473

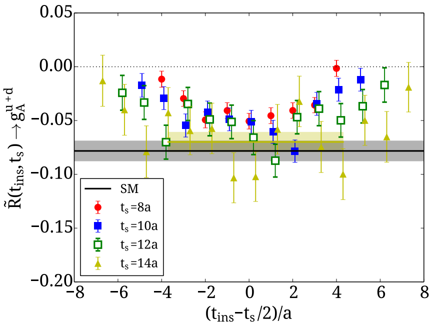
Axial charge g_A

To compute $\Delta\Sigma^q$ we need also the isoscalar g_A^{u+d}

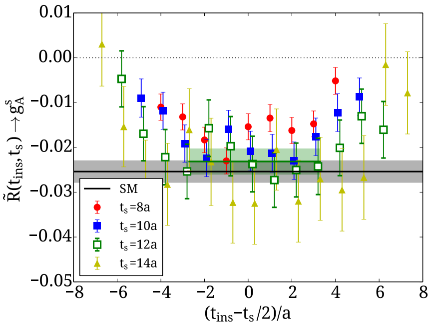
Dedicated high statistics study

Choose one ensemble to perform a high statistics analysis for all disconnected contributions to nucleon observables

$N_f = 2+1+1$ twisted mass, a = 0.082 fm, m_π = 373 MeV, $\sim$ **150, 000** statistics (on 4700 confs)



Disconnected isoscalar, agrees with Bali *et al.* (QCDSF), Phys.Rev.Lett. 108 (2012) 222001

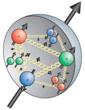


Strange quark loop

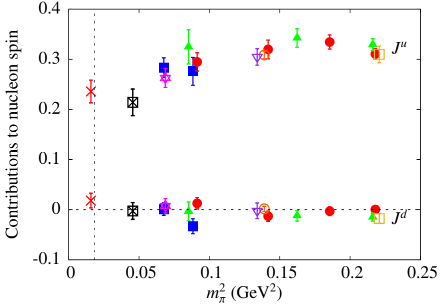
Where is the nucleon spin?

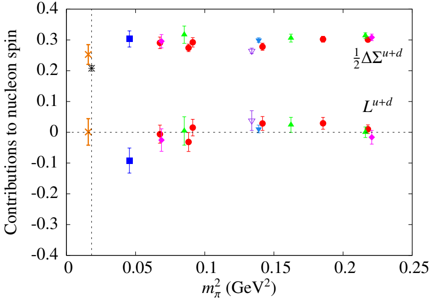
Spin sum: $\frac{1}{2} = \sum_q \underbrace{\left(\frac{1}{2}\Delta\Sigma^q + L^q\right)}_{J^q} + J^G$

$J^q = A^q_{20}(0) + B^q_{20}(0)$ and $\Delta\Sigma^q = g^q_A$



For one ensemble at $m_\pi = 373$ MeV we have the disconnected contribution $\rightarrow$ we can check the effect on the observables, $\mathcal{O}(200,000)$ statistics





- Disconnected quark loop contributions non-zero for $\Delta\Sigma^{u,d,s}$
- Consistent with zero for $J^{u,d}$
- The total spin $J^{u+d} \sim 0.25 \implies$ Where is the other half?
- Contributions from J^G? $\rightarrow$ on-going efforts to compute them, K.-F. Liu *et al.* (χQCD), arXiv:1203.6388

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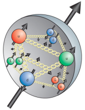

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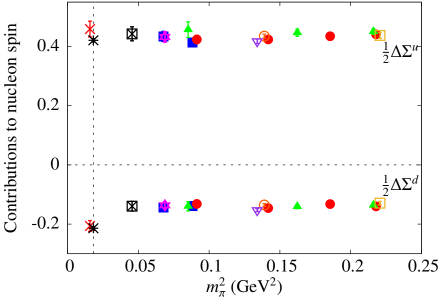

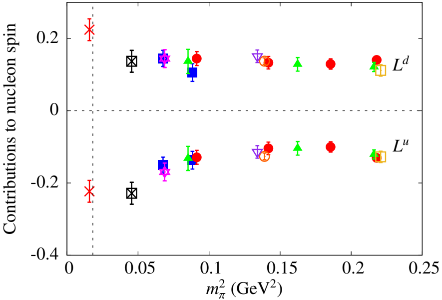

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The quark content of the nucleon

- $\sigma_{\pi N} \equiv m_l \langle N|\bar{u}u + \bar{d}d|N\rangle$: measures the explicit breaking of chiral symmetry
  Extracted from analysis of low-energy pion-proton scattering data
  Largest uncertainty in interpreting experiments for dark matter searches - Higgs-nucleon coupling depends on σ-terms, J. Ellis, K. Olive, C. Savage, arXiv:0801.3656
- In lattice QCD it can be obtained via the Feynman-Hellman theorem: $\sigma_{\pi N} = m_l \frac{\partial m_N}{\partial m_l}$
- Similarly $\sigma_s \equiv m_s \langle N|\bar{s}s|N\rangle >= m_s \frac{\partial m_N}{\partial m_s}$
- The strange quark content of the nucleon: $y_N = \frac{2\langle N|\bar{s}s|N\rangle}{\langle N|\bar{u}u+\bar{d}d|N\rangle} = 1 - \frac{\sigma_0}{\sigma_{\pi N}}$, where
  $\sigma_0 = m_l \langle N|\bar{u}u + \bar{d}d - 2\bar{s}s|N\rangle$
- A number of groups have used the spectral method to extract the σ-terms, R. Young, Lattice 2012
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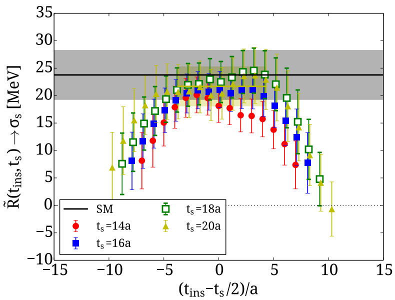

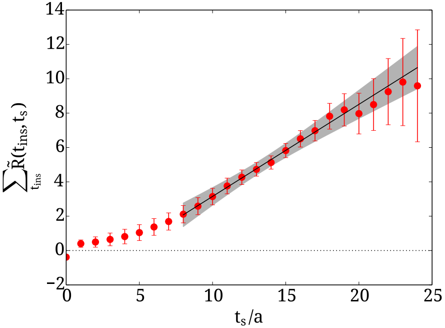

$m_\pi = 373$ MeV, **150,000** measurements

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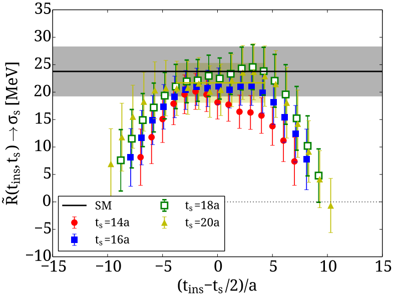


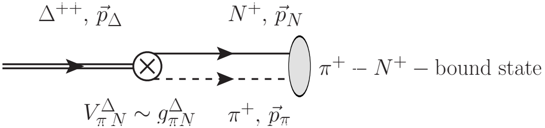
At the physical point:
$y_N = 0.135\,\underbrace{(22)}_{\text{stat.}}\,\underbrace{(33)}_{\text{ch.extr.}}\,\underbrace{(22)}_{\text{exc.stat.}}\,\underbrace{(9)}_{\text{fin.a}}$

$\rightarrow$ using $\sigma_s = \frac{1}{2}\frac{m_s}{m_l} y_N \sigma_{\pi N}$ we find σ_s to be less ~ 200 MeV

$m_\pi = 373$ MeV, **150,000** measurements

Challenges: IV. Decay width of baryons

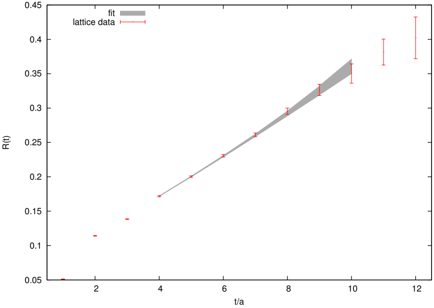
Use the transition amplitude method, C. McNeile, C. Michael, P. Pennanen, PRD 65 094505 (2002)
Test the method for the Δ



$\Longrightarrow$ compute transition amplitude $x = \langle \Delta | N\pi \rangle$ from the correlator $G^{\Delta \to N\pi}$

- Need $E_\Delta \sim E_\pi + E_N$
- Applicable for $xt \ll 1$

Hybrid calculation at $m_\pi \sim 360$ MeV with $L = 3.6$ fm



$$R(t) = \frac{G^{\Delta \to \pi N}(t, \vec{Q}, \vec{q})}{\sqrt{G^{\Delta}(t, \vec{q})\, G^{N\pi}(t, \vec{Q}, \vec{q})}} \sim A + B \frac{sinh(\Delta t/2)}{sin(\Delta/2)}$$

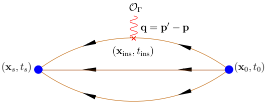
$$\sim A + Bt$$

$g_{\Delta \pi N} = 27.0(0.6)(1.5) \rightarrow \Gamma_\Delta = 99(12)$ MeV

C. A., J. W. Negele, M. Petschlies, A. Tsapalis, PRD 88 031501 (2013)

Challenges: V. Axial charge for hyperons & Charmed baryons

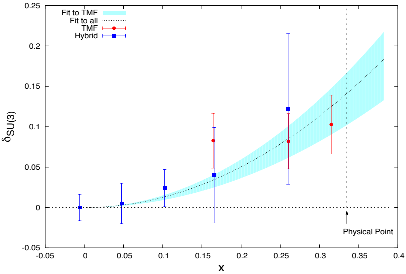
- Hyperon axial charges: $g_{\Lambda\Sigma} \sim 0.60$, $g_{\Sigma\Sigma}$, $g_{\Xi\Xi}$ not known experimentally
- Calculation equivalent to g_A:$\langle h|\bar{\psi}\gamma_\mu\gamma_5\psi|h\rangle|_{q^2=0}$ - Efficient to calculate with fixed current method



If exact SU(3) flavor symmetry:

- $g_A^N = F + D$, $g_A^\Sigma = 2F$, $g_A^\Xi = -D + F \Longrightarrow g_A^N - g_A^\Sigma + g_A^\Xi = 0$

Probe deviation: $\delta_{\rm SU(3)} = g_A^N - g_A^\Sigma + g_A^\Xi$ versus $x = (m_K^2 - m_\pi^2)/4\pi^2 f_\pi^2$, H.- W. Lin and K. Orginos, PRD 79, 034507 (2009)



Breaking $\sim x^2$ leads to about 15% at the physical point $x_{\rm phy} = 0.33$

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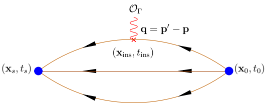

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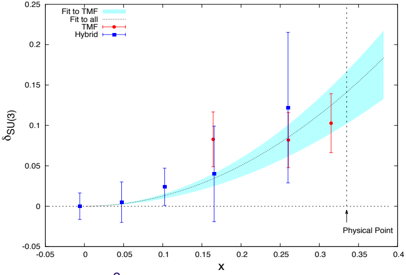
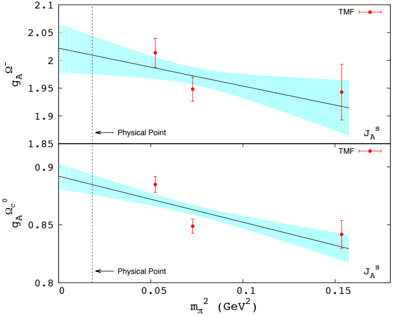




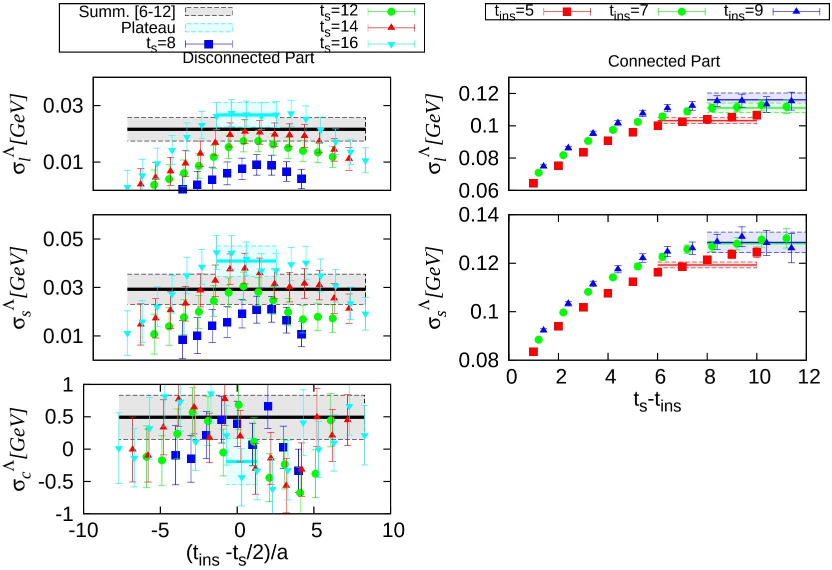
Breaking $\sim x^2$ leads to about 15% at the physical point

$x_{\rm phy} = 0.33$

σ-terms for hyperons and charmed baryons

Need both connected and disconnected pieces

First results using $N_F = 2+1+1$ TMF at $m_\pi = 373$ MeV



Conclusions

- Nucleon structure is a benchmark for the Lattice QCD approach
  Investigation of g_A, $\langle x \rangle_{u-d}$ etc is now feasible at physical pion mass and larger volumes $\rightarrow$ need high statistics and careful cross-checks
- Evaluation of disconnected quark loop diagrams has become feasible addressing an up to now unknown systematic error
- Predictions for other hadron observables are beginning to emerge e.g. masses and axial charges of hyperons and charmed baryons
- The study of excited states and resonances is under way $\rightarrow$ provide insight into the structure of hadrons and input that is crucial for new physics
- Many challenges ahead ...

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- Nucleon structure is a benchmark for the Lattice QCD approach
  Investigation of g_A, $\langle x \rangle_{u-d}$ etc is now feasible at physical pion mass and larger volumes $\rightarrow$ need high statistics and careful cross-checks
- Evaluation of disconnected quark loop diagrams has become feasible addressing an up to now unknown systematic error
- Predictions for other hadron observables are beginning to emerge e.g. masses and axial charges of hyperons and charmed baryons
- The study of excited states and resonances is under way $\rightarrow$ provide insight into the structure of hadrons and input that is crucial for new physics
- Many challenges ahead ...

As simulations at the physical pion mass and more computer resources are becoming available we expect many physical results on key hadron observables that will impact both experiments and phenomenology

Acknowledgments

European Twisted Mass Collaboration (ETMC)

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