New Applications of Renormalization Group Methods in Nuclear Physics

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In collaboration with:
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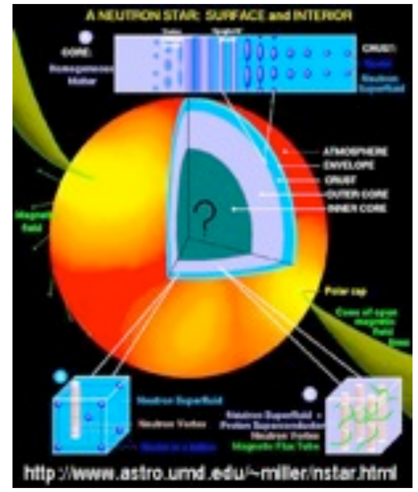
Facets of Strong-Interaction Physics

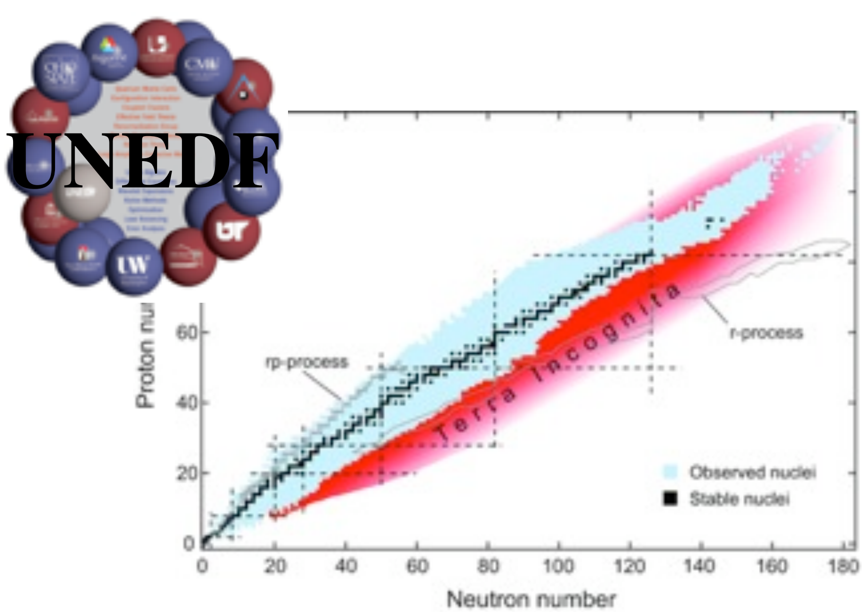
Hirschegg, January 18, 2012

THE OHIO STATE UNIVERSITY

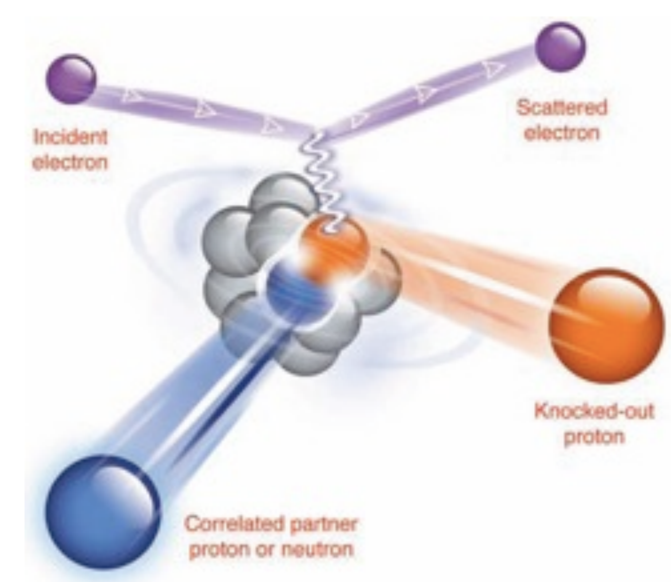


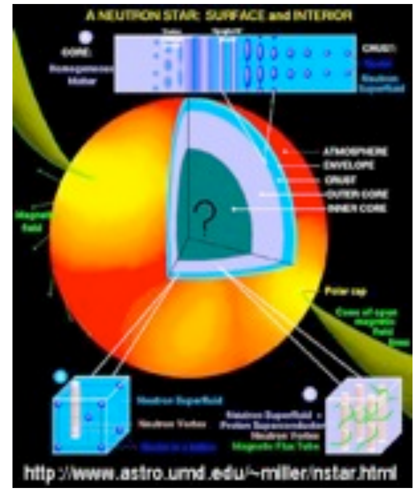
Nuclear equation of state
and astrophysical applications

Light and
neutron-rich nuclei



Correlations in
nuclear systems

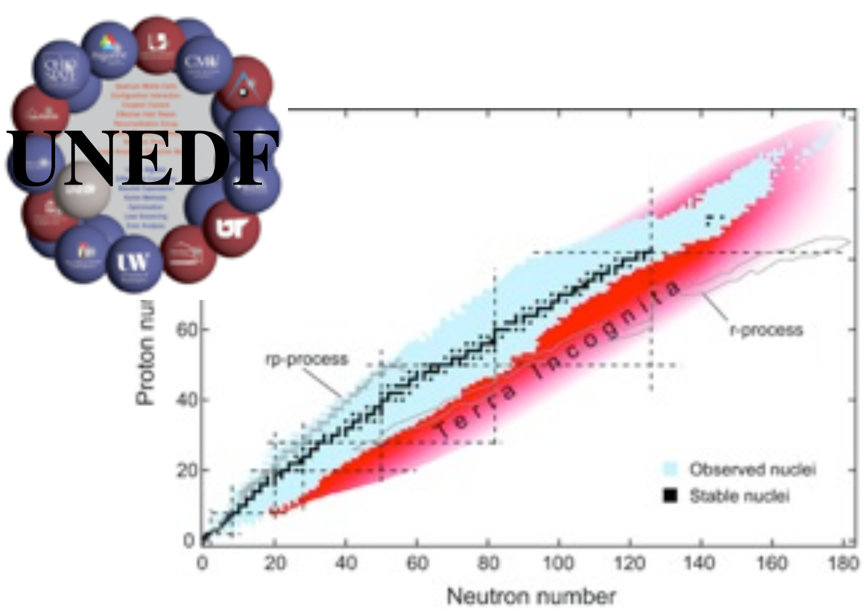




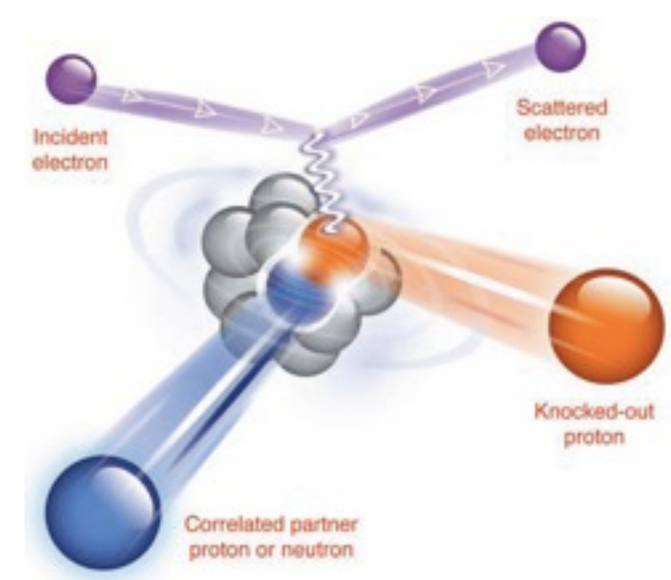
Nuclear equation of state
and astrophysical applications

Unified microscopic description.

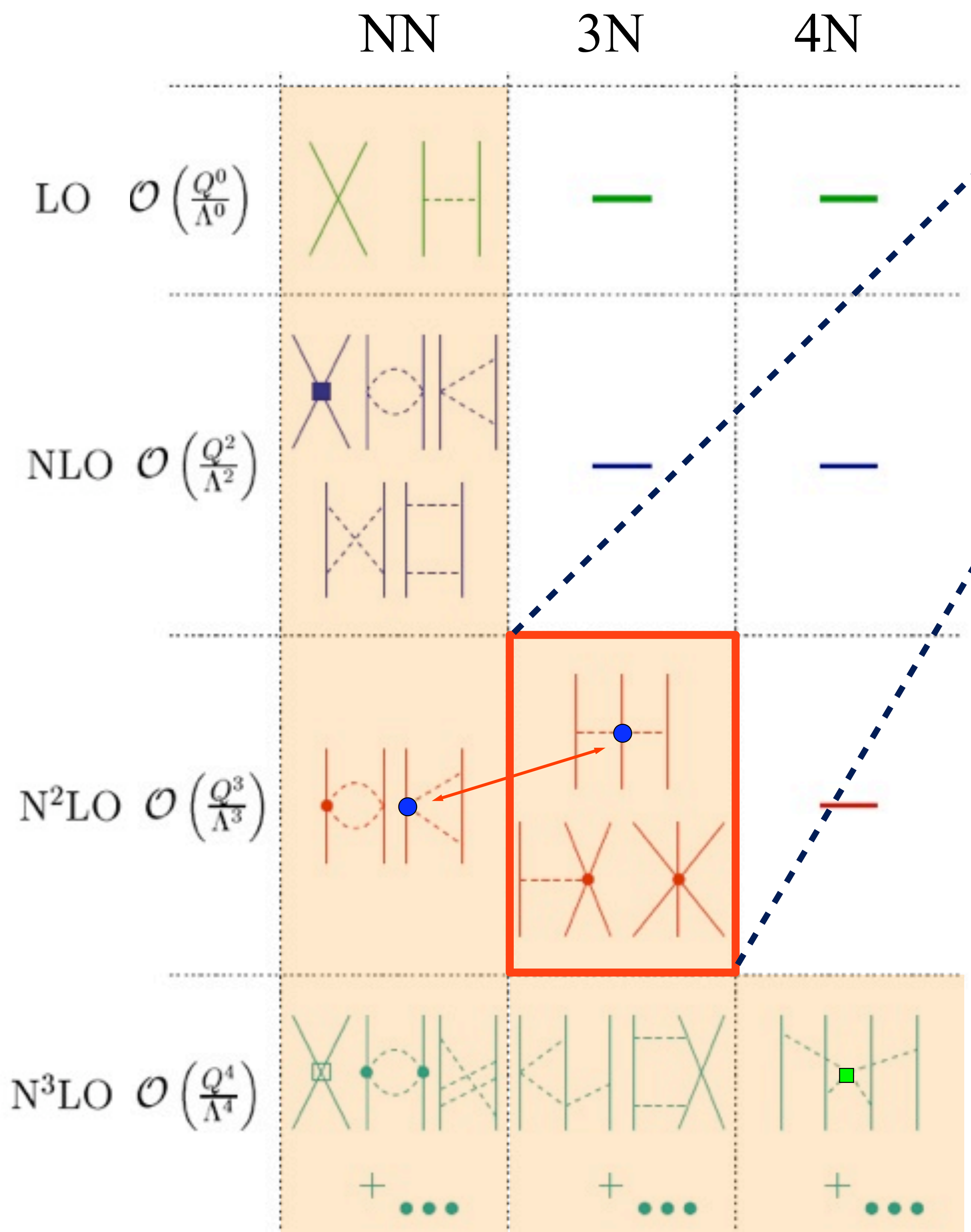
Light and
neutron-rich nuclei

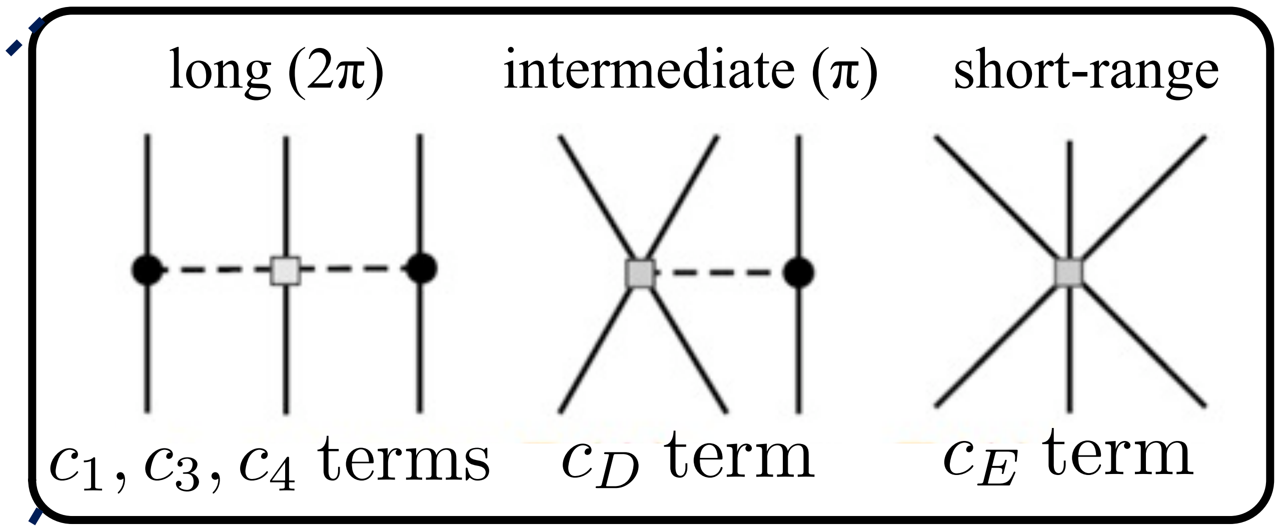


Correlations in
nuclear systems



Chiral EFT for nuclear forces, leading order 3N forces



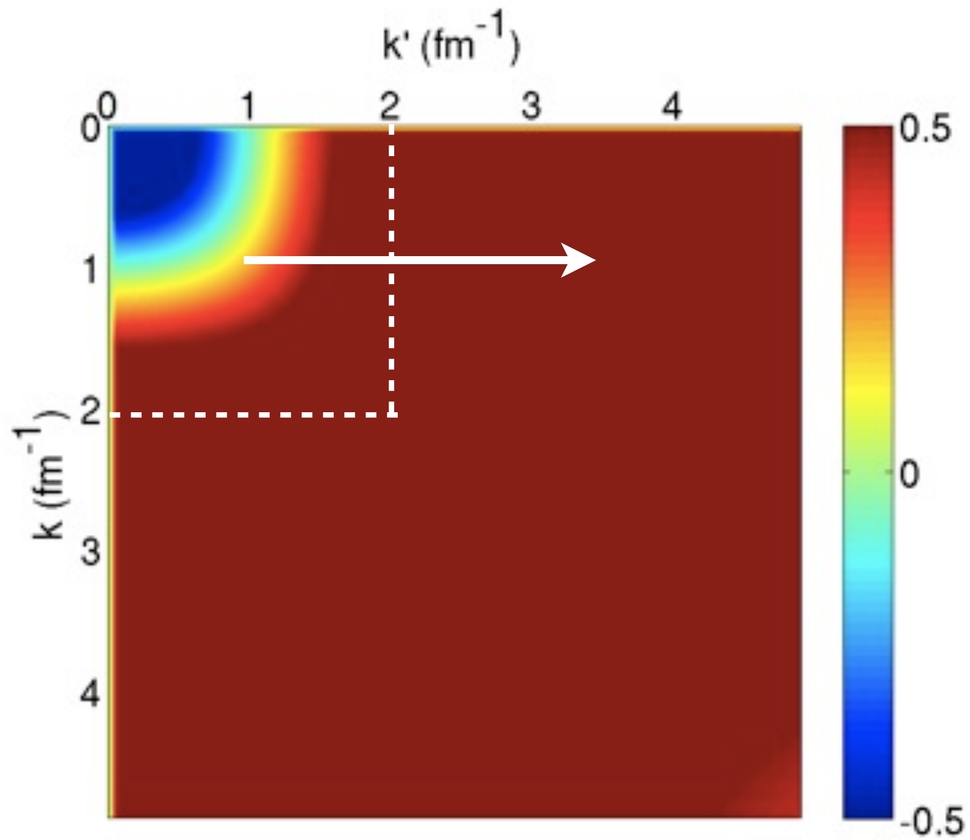


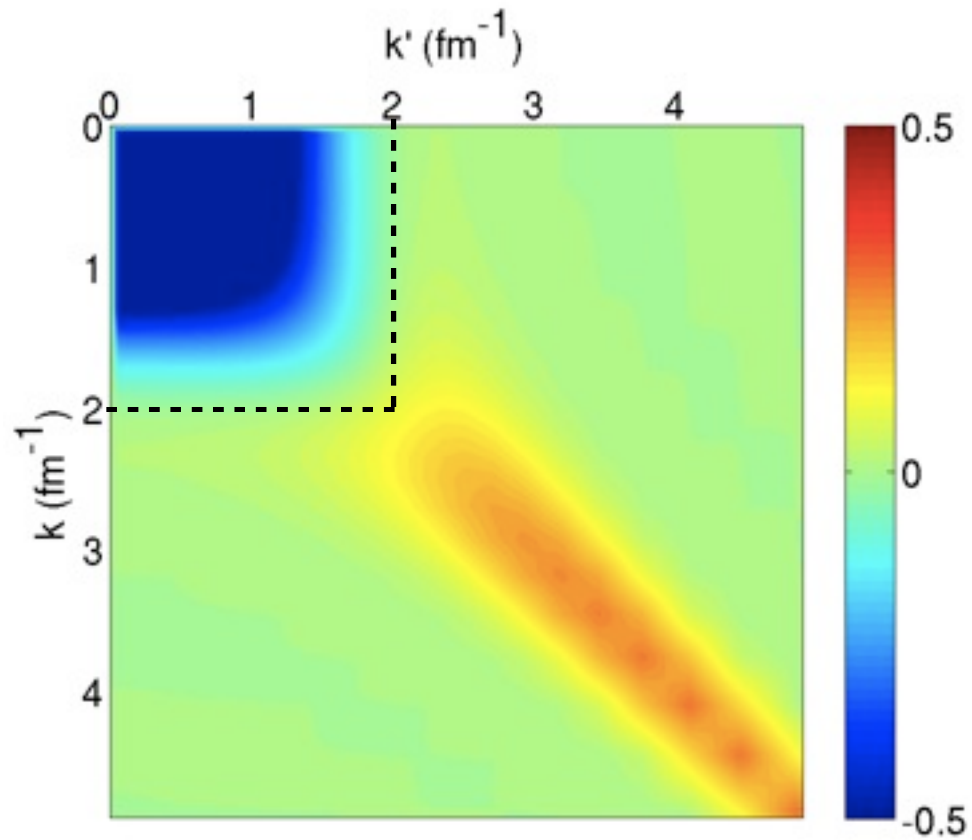
large uncertainties in coupling constants at present:

$$c_1 = -0.9^{+0.2}_{-0.5}\ ,\quad c_3 = -4.7^{+1.5}_{-1.0}\ ,\quad c_4 = 3.5^{+0.5}_{-0.2}$$

lead to theoretical uncertainties in many-body observables

Changing the resolution:
The (Similarity) Renormalization Group





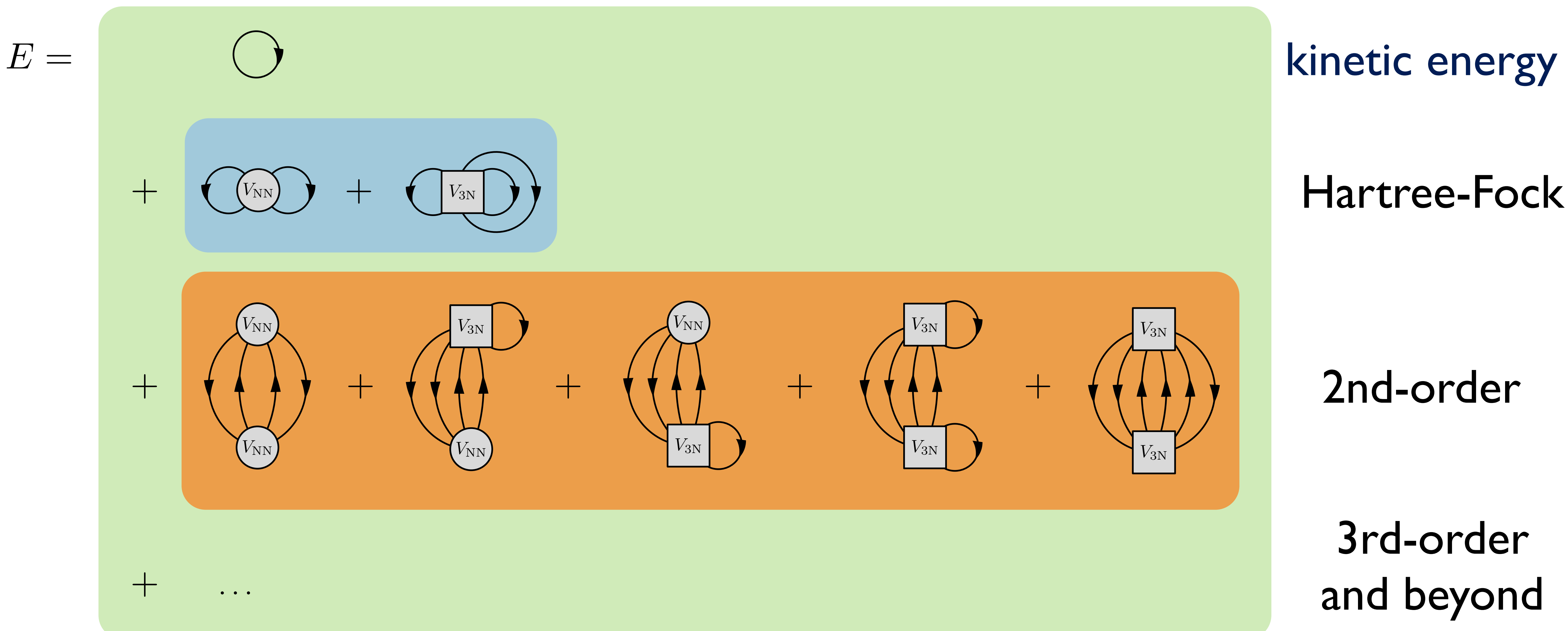
- elimination of coupling between low- and high momentum components, calculations much easier
- observables unaffected by resolution change (for exact calculations)
- residual resolution dependences can be used as tool to test calculations

Not the full story:
RG transformation also changes three-body (and higher-body) interactions.

Equation of state: Many-body perturbation theory

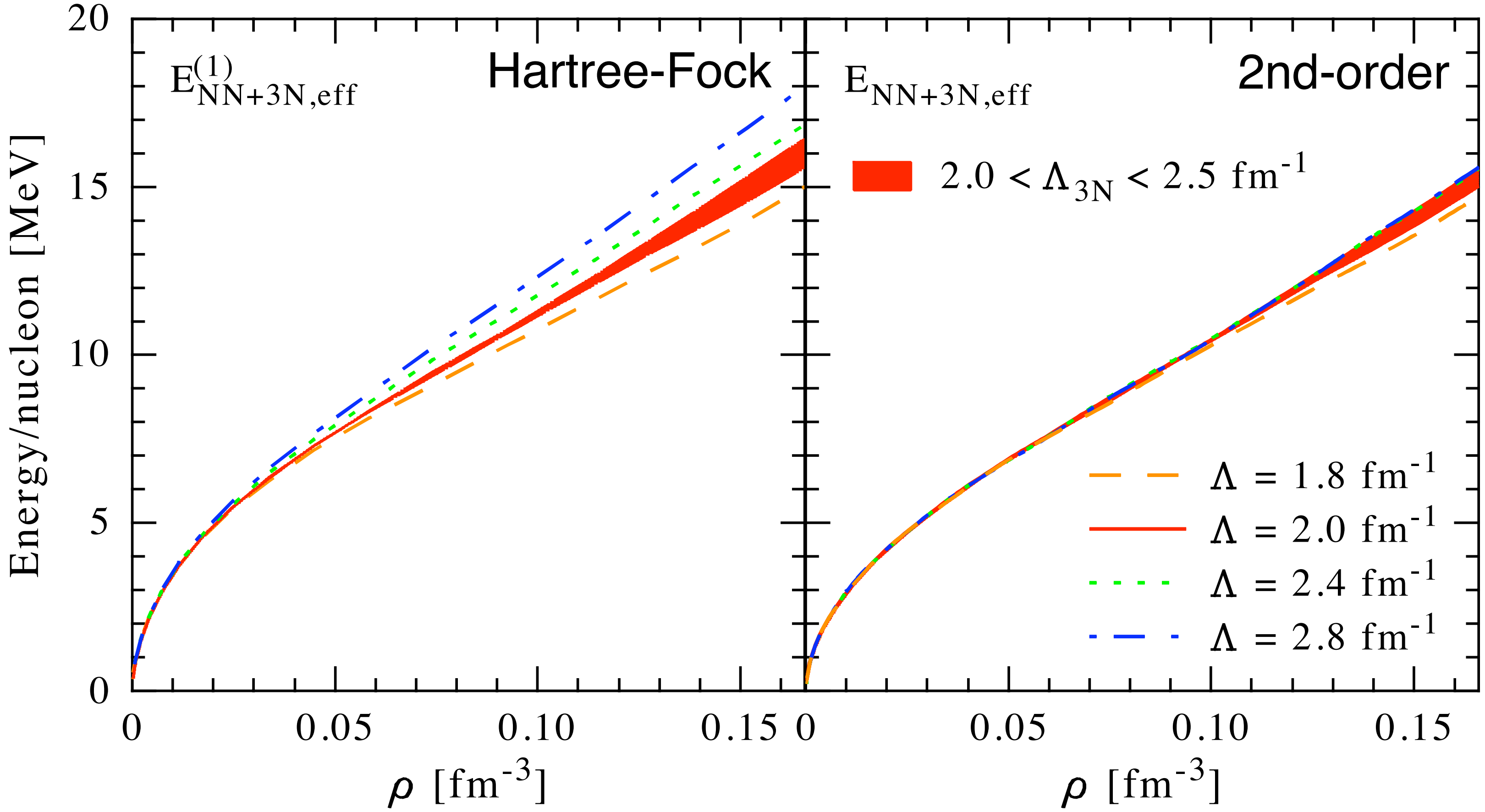
central quantity of interest: energy per particle E/N

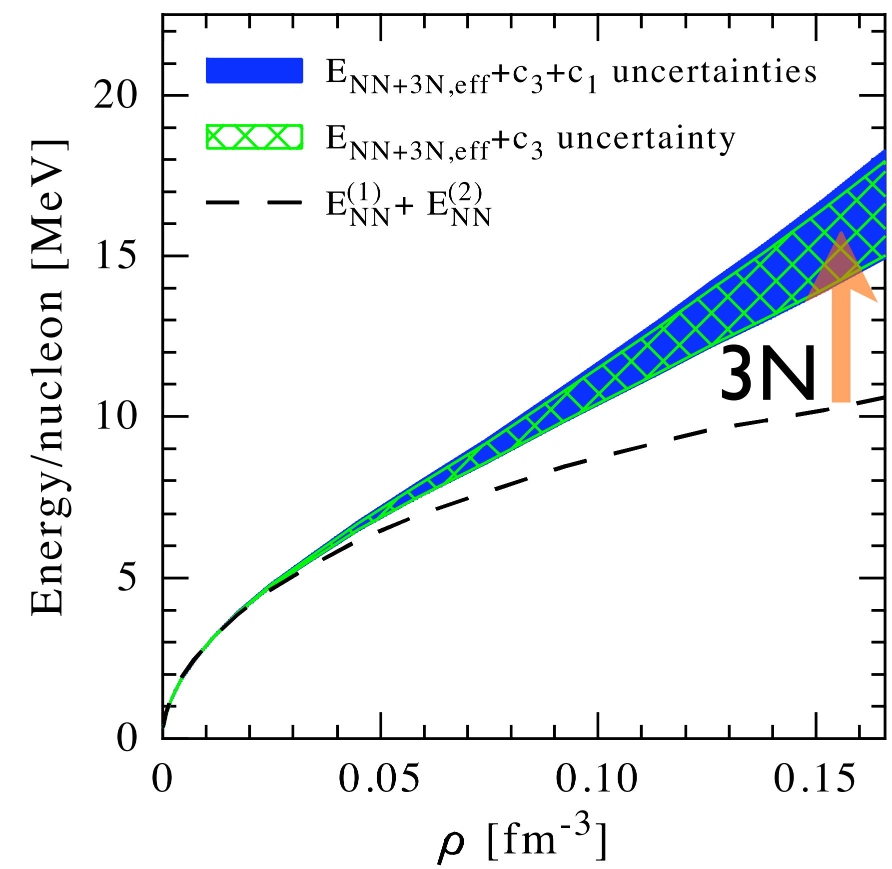
$$H(\lambda) = T + V_{\rm NN}(\lambda) + V_{\rm 3N}(\lambda) + \ldots$$



- "hard" interactions require non-perturbative summation of diagrams
- with low-resolution interactions much more perturbative
- inclusion of 3N interaction contributions crucial
- use chiral interactions as initial input for RG evolution

Equation of state of pure neutron matter





KH and Schwenk PRC 82, 014314 (2010)

- significantly reduced cutoff dependence at 2nd order perturbation theory
- small resolution dependence indicates converged calculation
- energy sensitive to uncertainties in 3N interaction
- variation due to 3N input uncertainty much larger than resolution dependence

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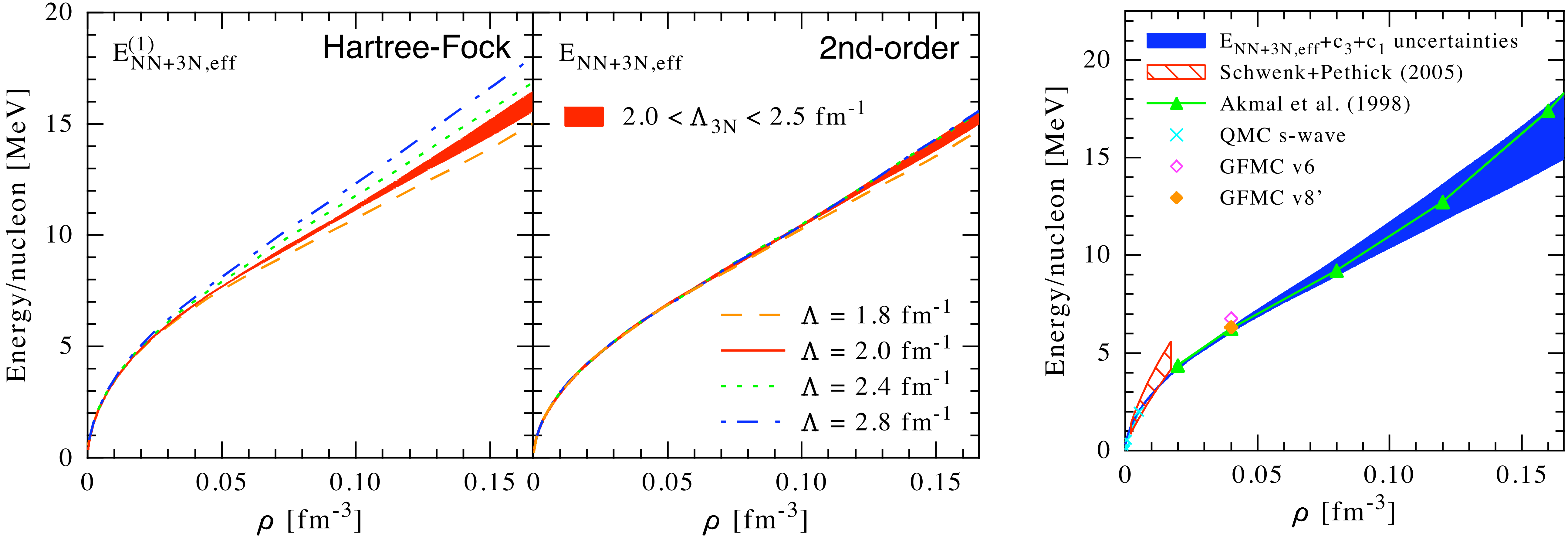

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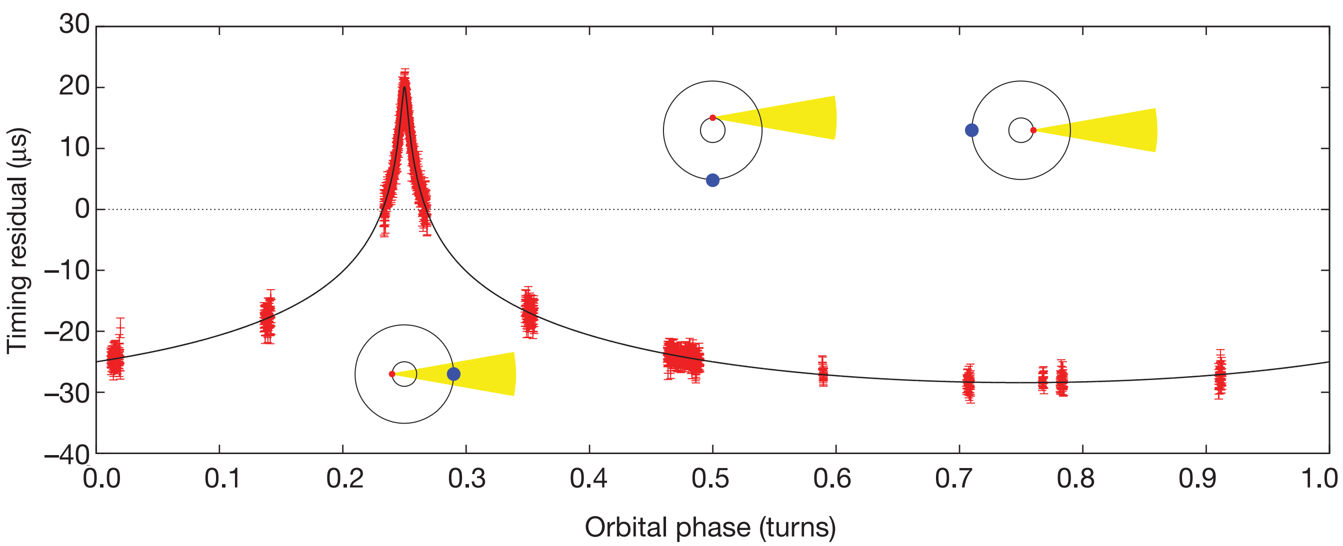
- significantly reduced cutoff dependence at 2nd order perturbation theory
- small resolution dependence indicates converged calculation
- energy sensitive to uncertainties in 3N interaction
- variation due to 3N input uncertainty much larger than resolution dependence
- good agreement with other approaches (different NN interactions)

Constraints on the nuclear equation of state (EOS)

nature

A two-solar-mass neutron star measured using Shapiro delay

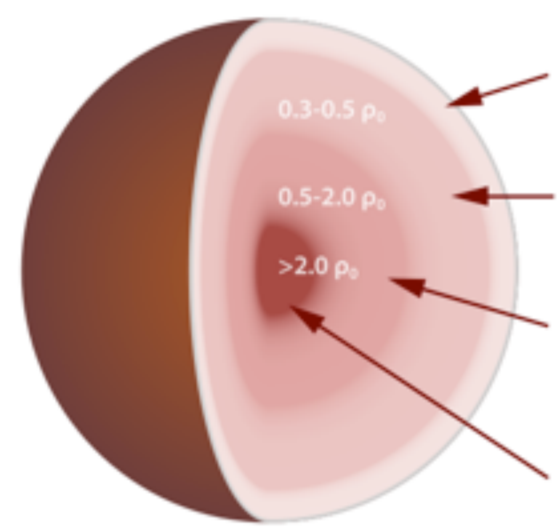
P. B. Demorest[1], T. Pennucci[2], S. M. Ransom[1], M. S. E. Roberts[3] & J. W. T. Hessels[4,5]



Demorest et al., Nature 467, 1081 (2010)

Credit: NASA/Dana Berry

$$M_{\max} = 1.65 M_\odot \rightarrow 1.97 \pm 0.04\, M_\odot$$



Calculation of neutron star properties requires EOS up to high densities.

Strategy:

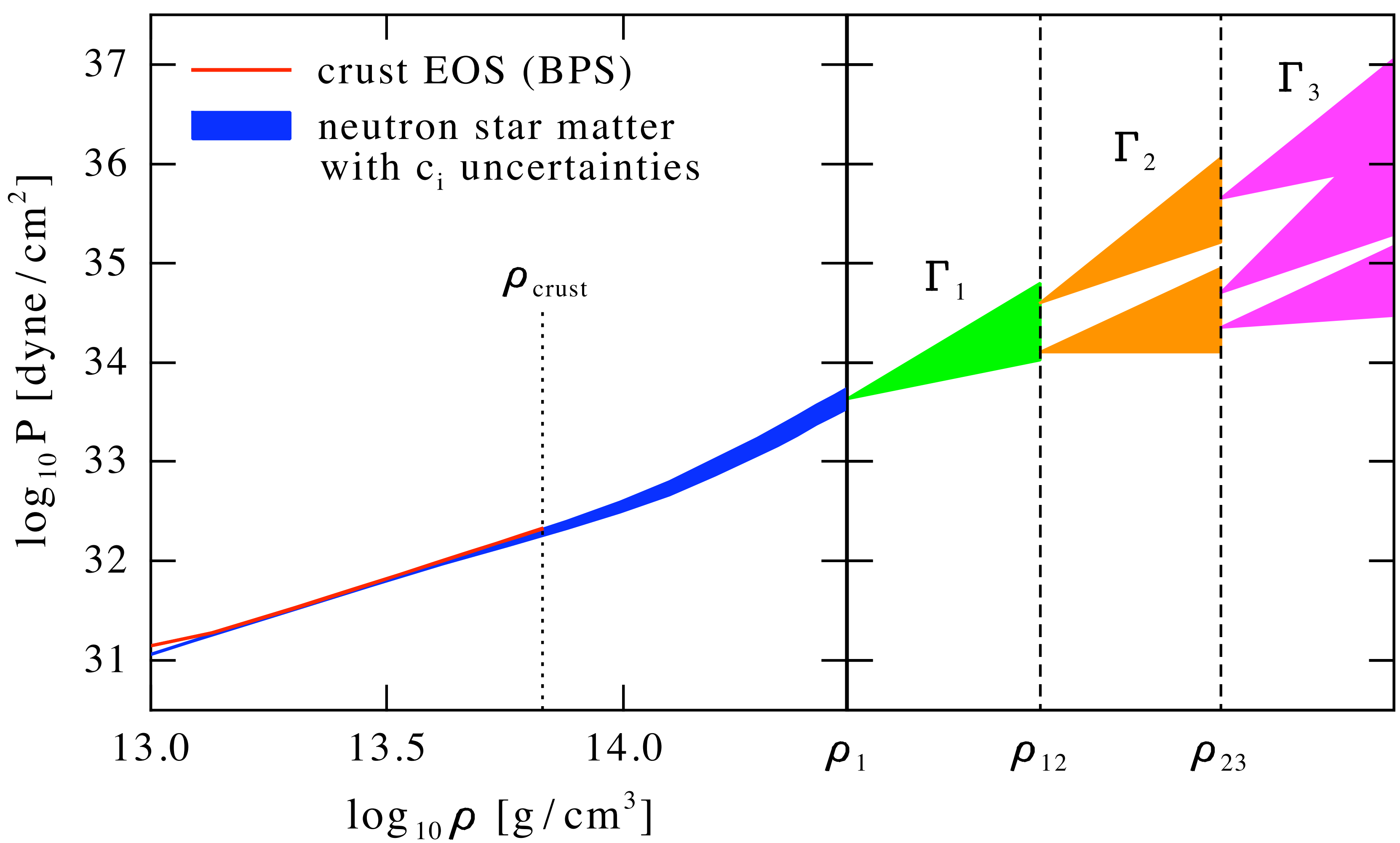
Use observations to constrain the high-density part of the nuclear EOS.

Neutron star radius constraints

incorporation of beta-equilibrium: neutron matter $\longrightarrow$ neutron star matter

parametrize piecewise high-density extensions of EOS:

- use polytropic ansatz $p \sim \rho^\Gamma$
- range of parameters $\Gamma_1, \rho_{12}, \Gamma_2, \rho_{23}, \Gamma_3$ limited by physics!



see also KH, Lattimer, Pethick, Schwenk, PRL 105, 161102 (2010)

Constraints on the nuclear equation of state

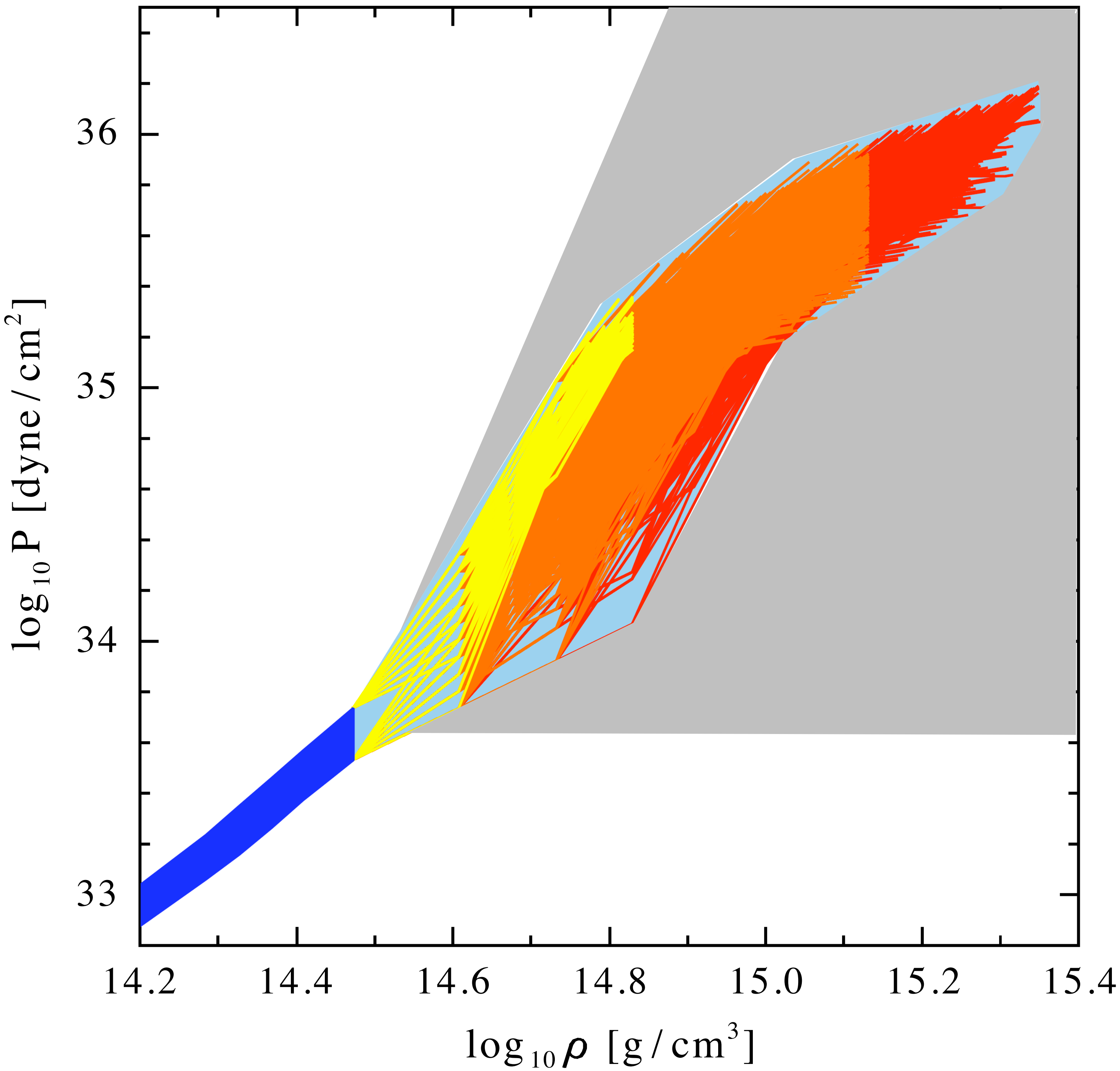
use the constraints:

recent NS observation

$$M_{\max} > 1.97\, M_{\odot}$$

causality

$$v_s(\rho) = \sqrt{dP/d\varepsilon} < c$$



significant reduction of possible equations of state

Constraints on the nuclear equation of state

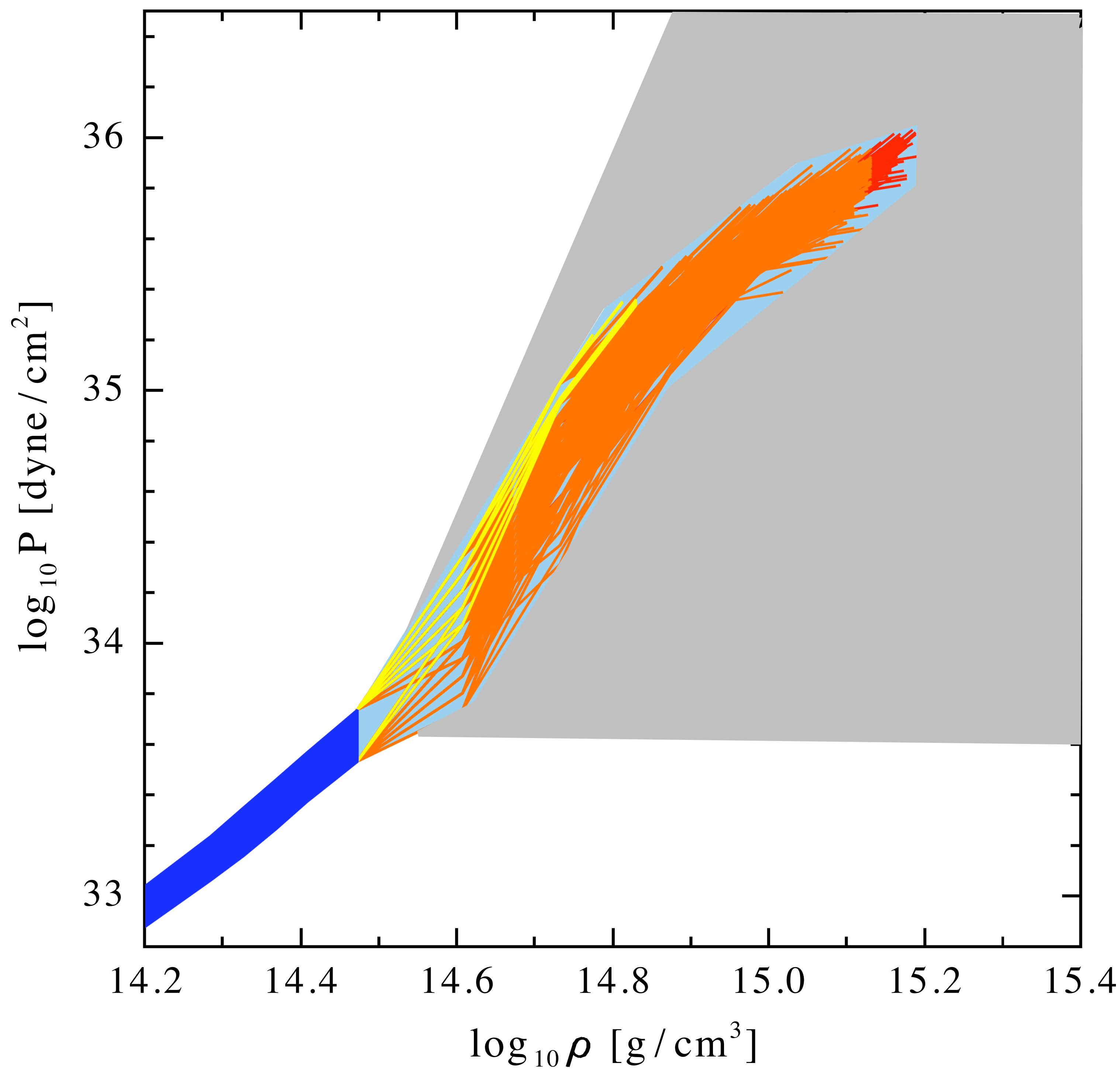
use the constraints:

NS mass

$M_{\rm max} > 2.4\, M_\odot$

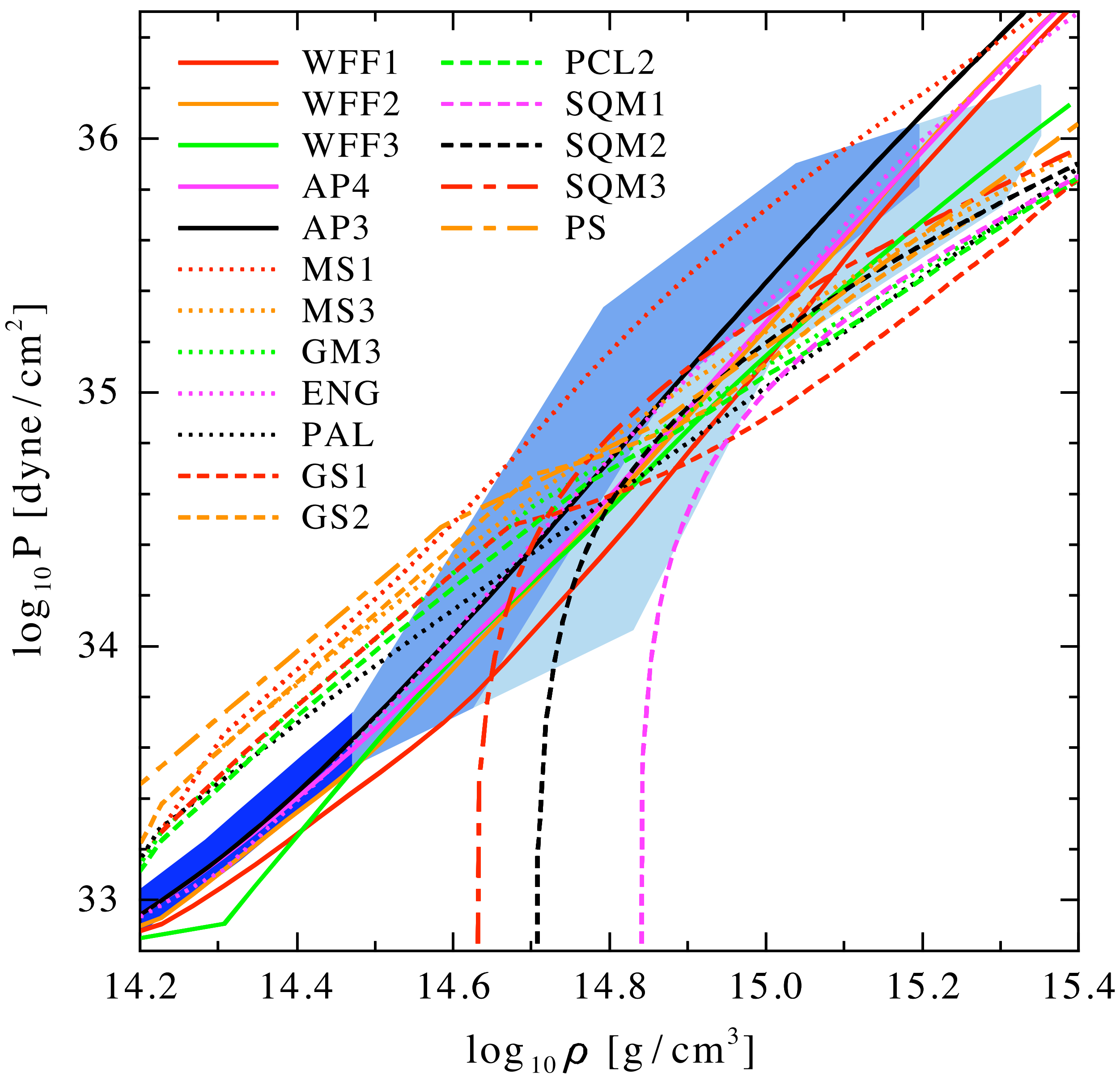
causality

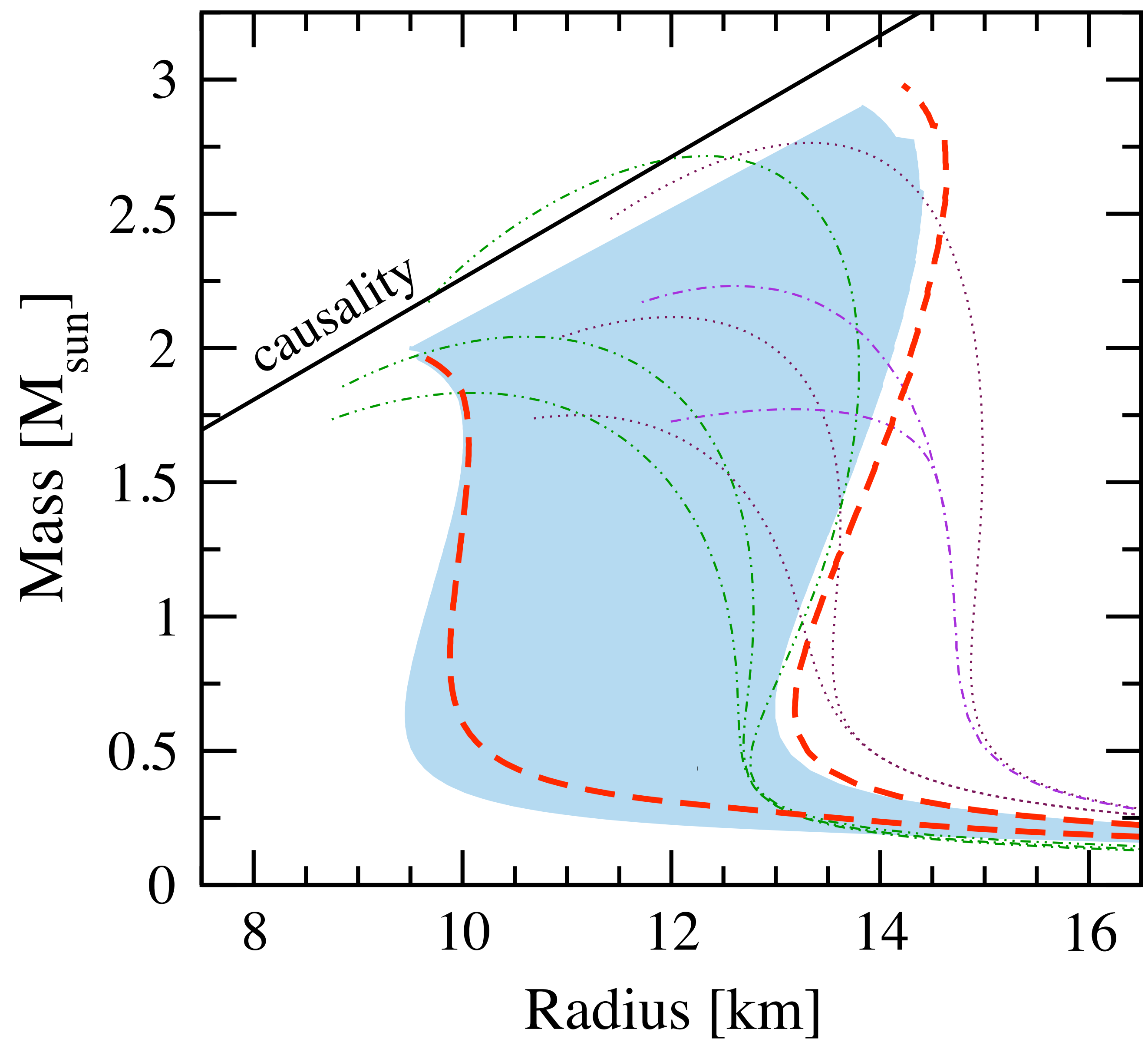
$v_s(\rho) = \sqrt{dP/d\varepsilon} < c$



increased $M_{\rm max}$ systematically leads to stronger constraints

Neutron star radii

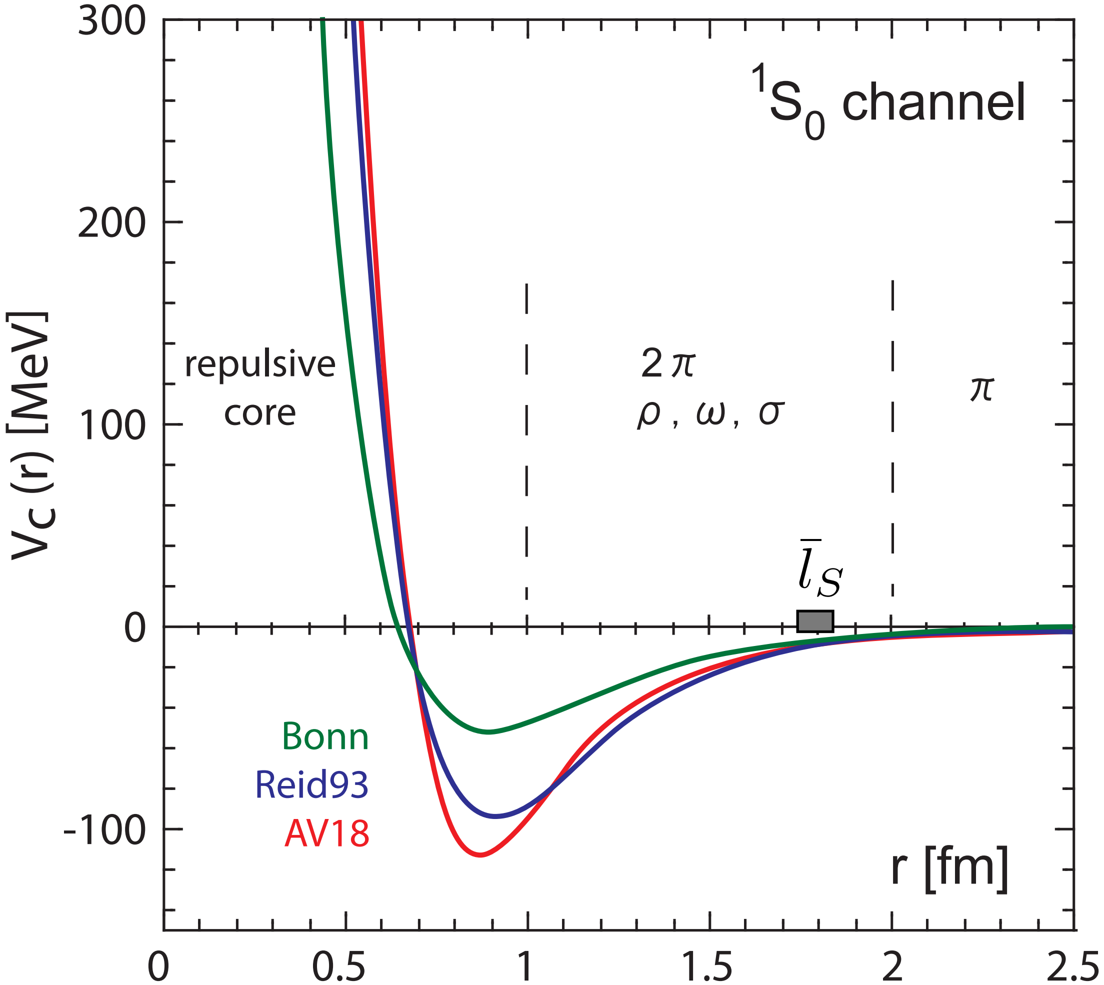


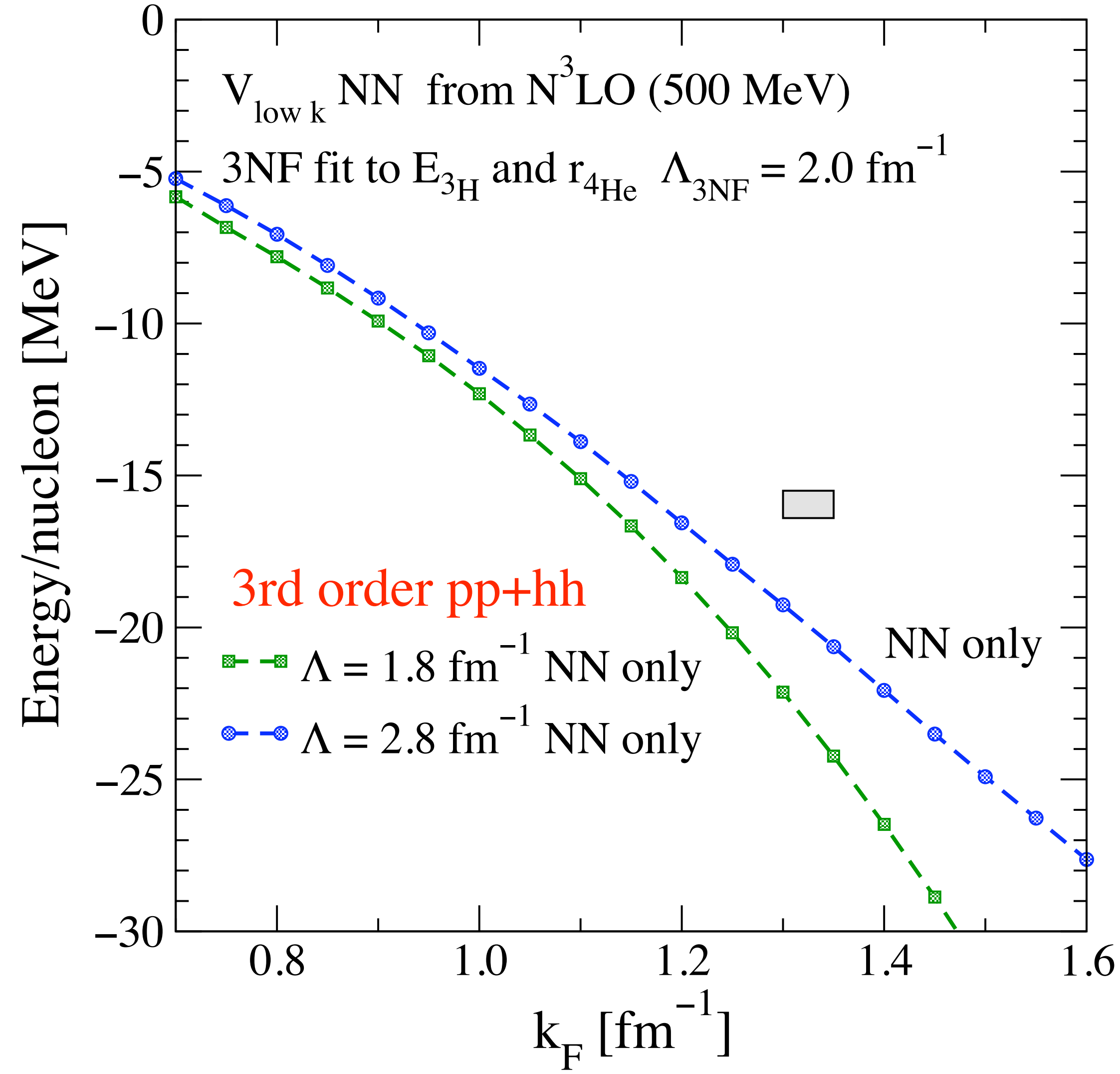


see also KH, Lattimer, Pethick, Schwenk, PRL 105, 161102 (2010)

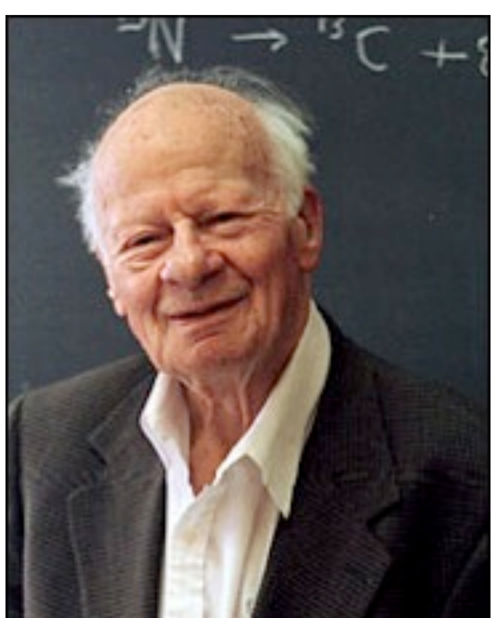
- low-density part of EOS sets scale for allowed high-density extensions
- radius constraint for typical $1.4\, M_\odot$ neutron star: $9.8 - 13.4\,\mathrm{km}$

Equation of state of symmetric nuclear matter, nuclear saturation





KH, Bogner, Furnstahl, Nogga, PRC(R) 83, 031301 (2011)



"Very soft potentials must be excluded because they do not give saturation; they give too much binding and too high density. In particular, a substantial tensor force is required."

Hans Bethe (1971)

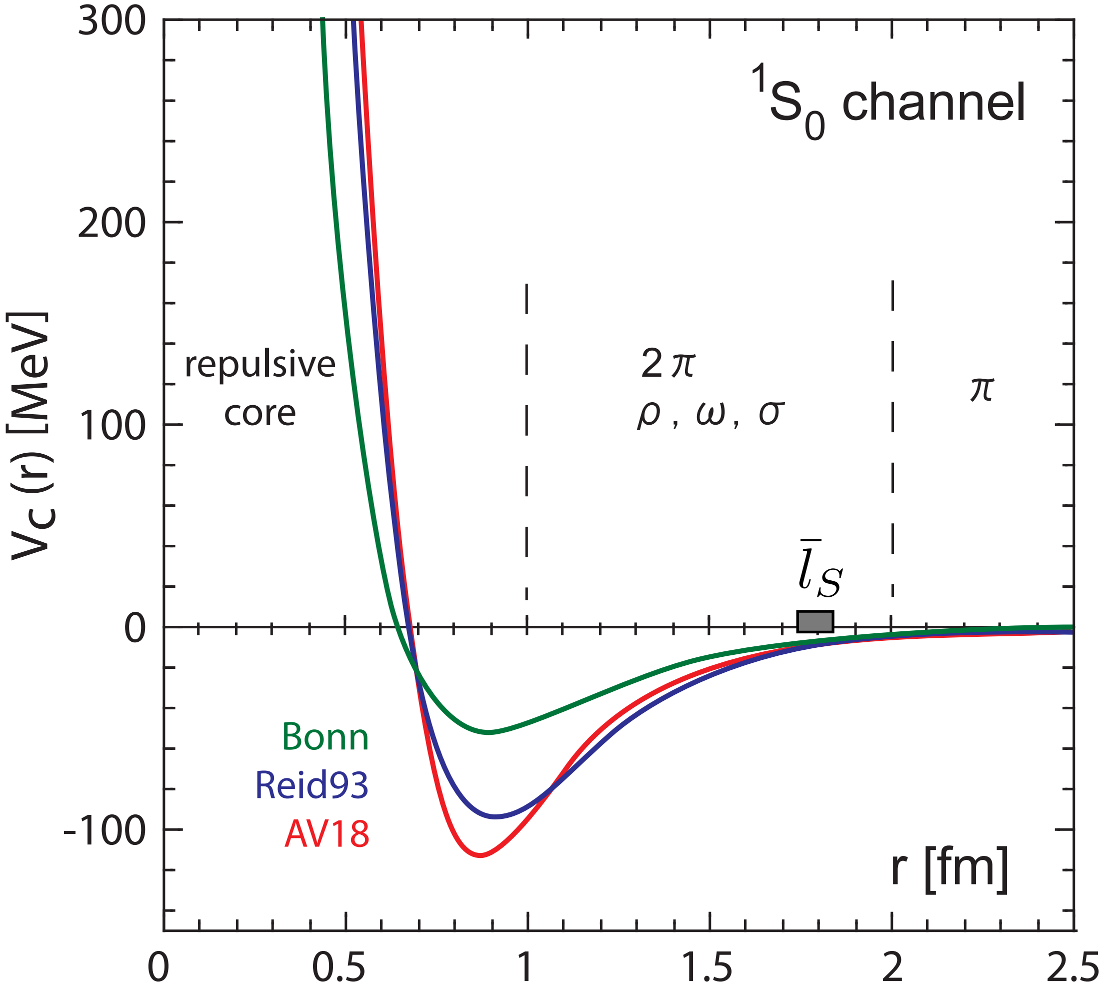
empirical nuclear saturation properties

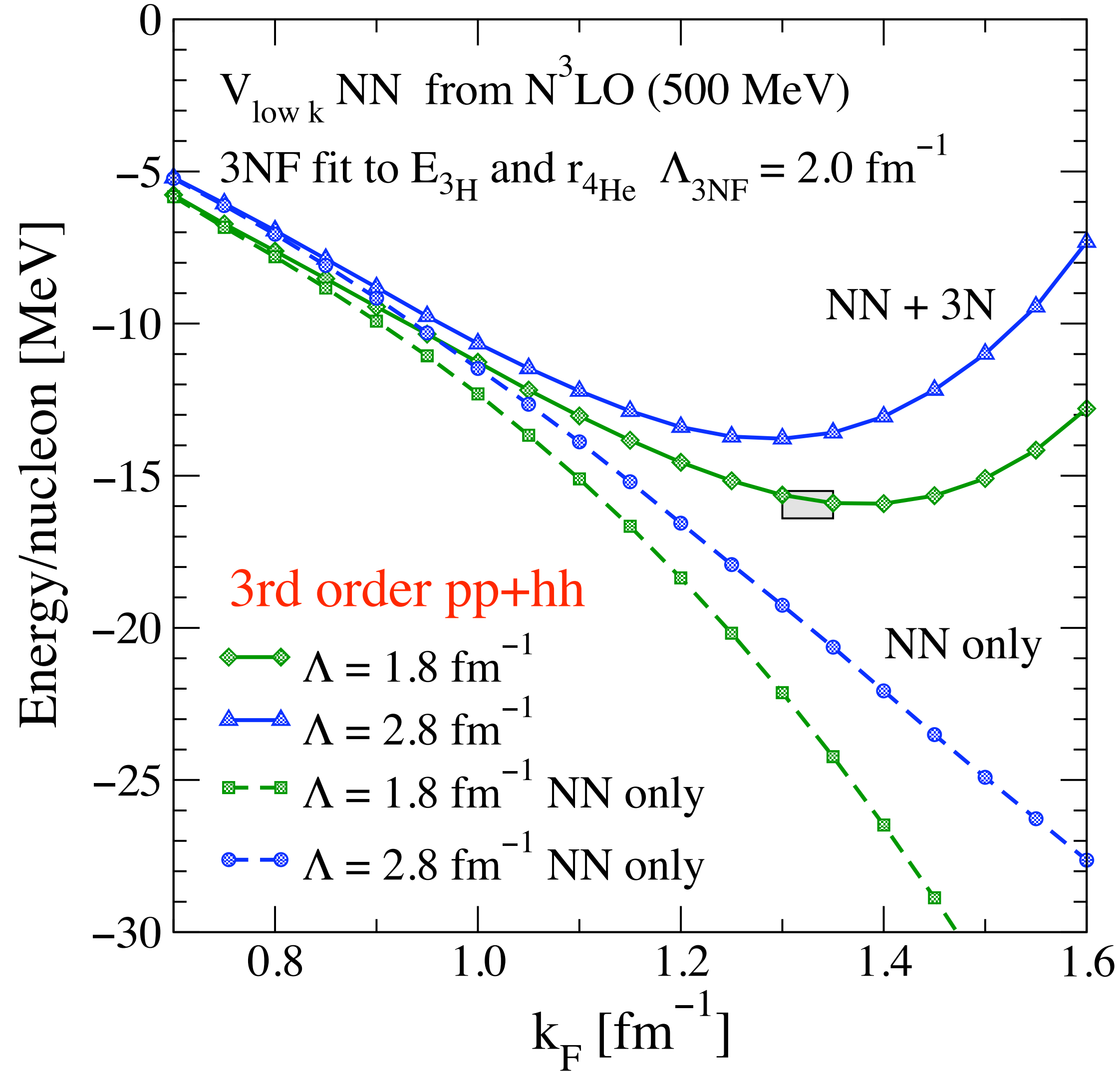
$$n_S \sim 0.16\,\mathrm{fm}^{-3}$$

$$E_{\mathrm{binding}}/N \sim -16\,\mathrm{MeV}$$

$$\bar{l}_S \sim 1.8\,\mathrm{fm}$$

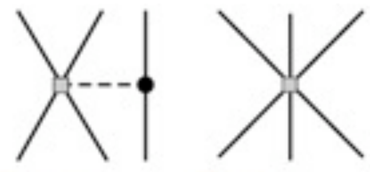
Equation of state of symmetric nuclear matter, Nuclear saturation



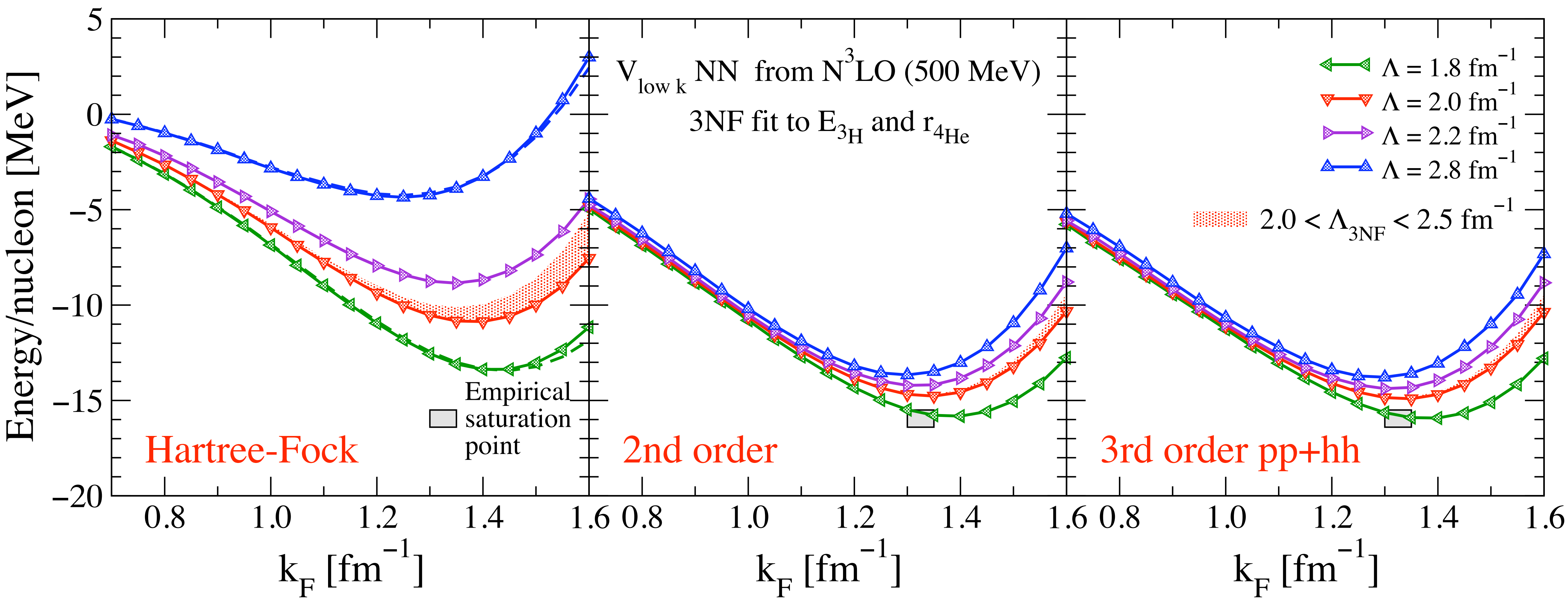


KH, Bogner, Furnstahl, Nogga, PRC(R) 83, 031301 (2011)

- nuclear saturation delicate due to cancellations of large kinetic and potential energy contributions
- 3N forces are essential! 3N interactions fitted to $^3\mathrm{H}$ and $^4\mathrm{He}$ properties



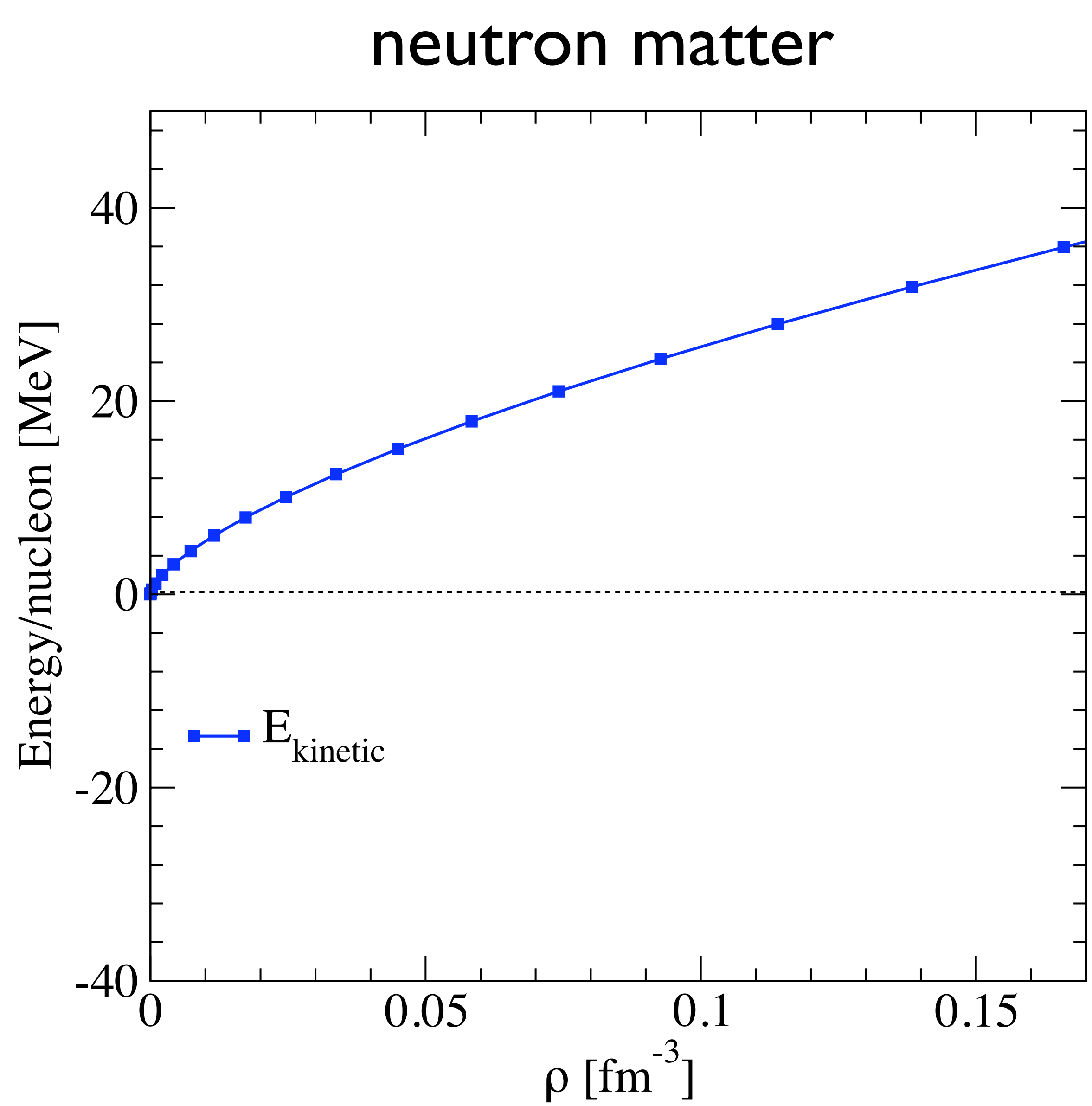
Equation of state of symmetric nuclear matter, Nuclear saturation

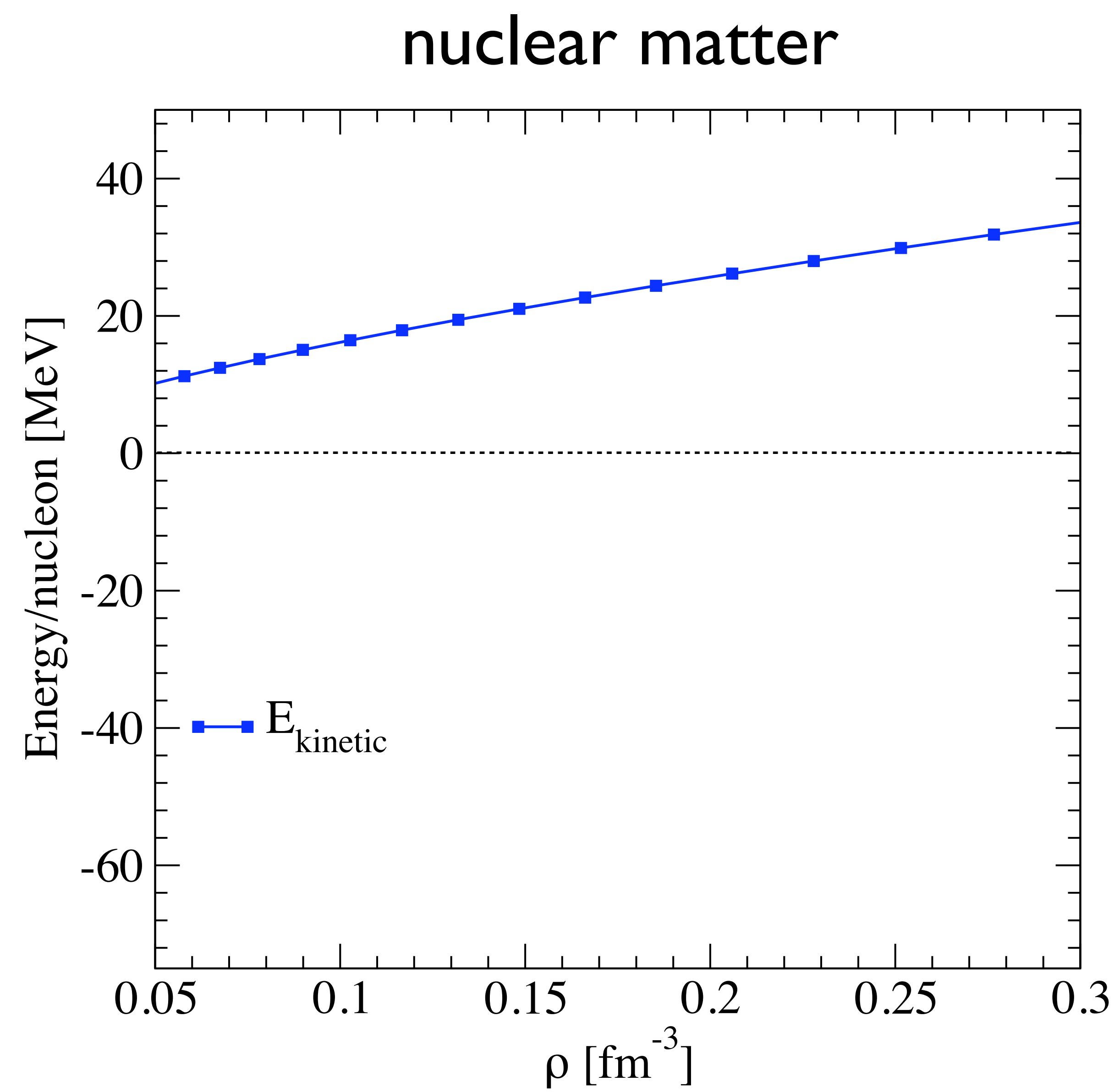


KH, Bogner, Furnstahl, Nogga, PRC(R) 83, 031301 (2011)

- saturation point consistent with experiment, without free parameters
- cutoff dependence at 2nd order significantly reduced
- 3rd order contributions small
- cutoff dependence consistent with expected size of 4N force contributions

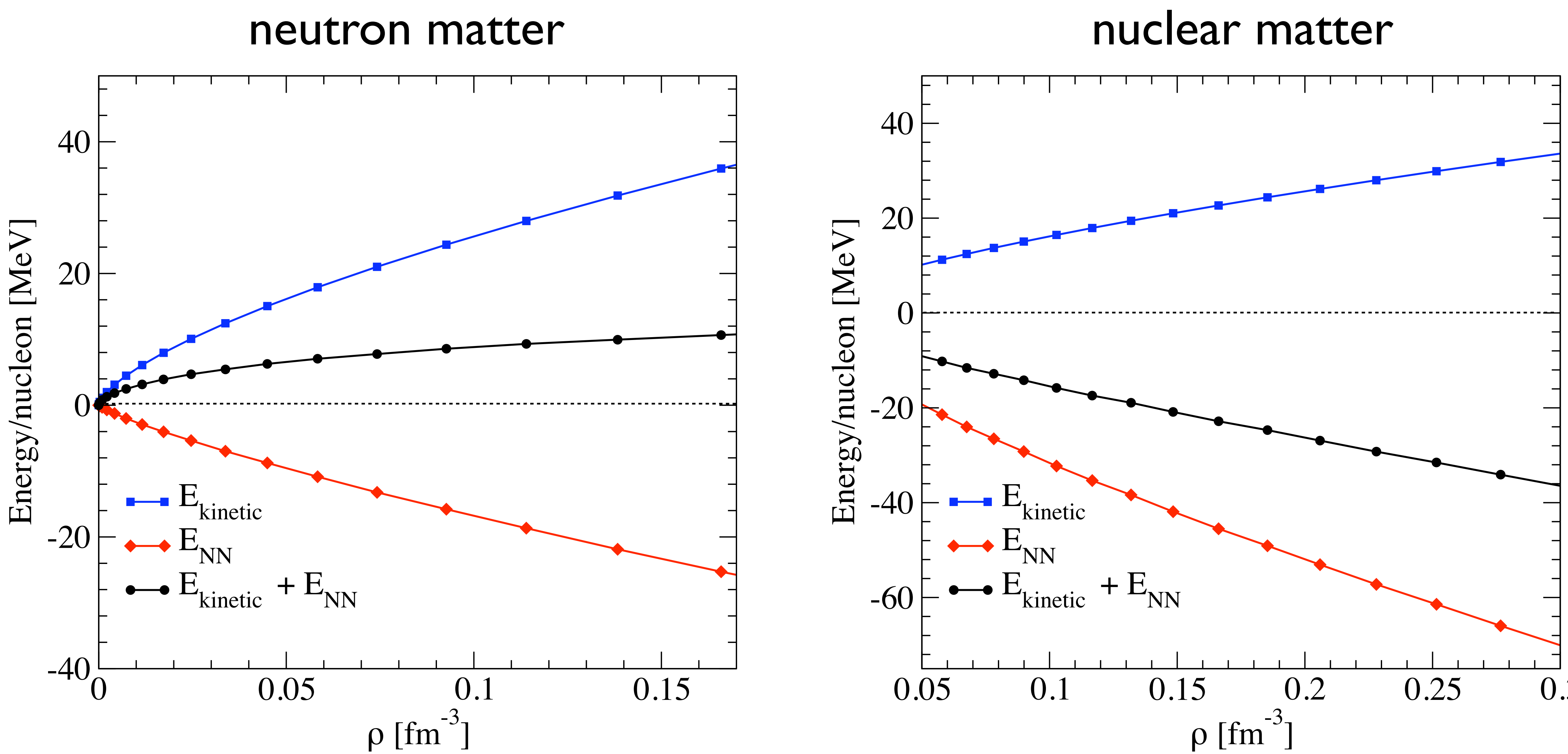
Hierarchy of many-body contributions





- binding energy results from cancellations of much larger kinetic and potential energy contributions
- chiral hierarchy of many-body terms preserved for considered density range
- cutoff dependence of natural size, consistent with chiral exp. parameter $\sim 1/3$

Hierarchy of many-body contributions



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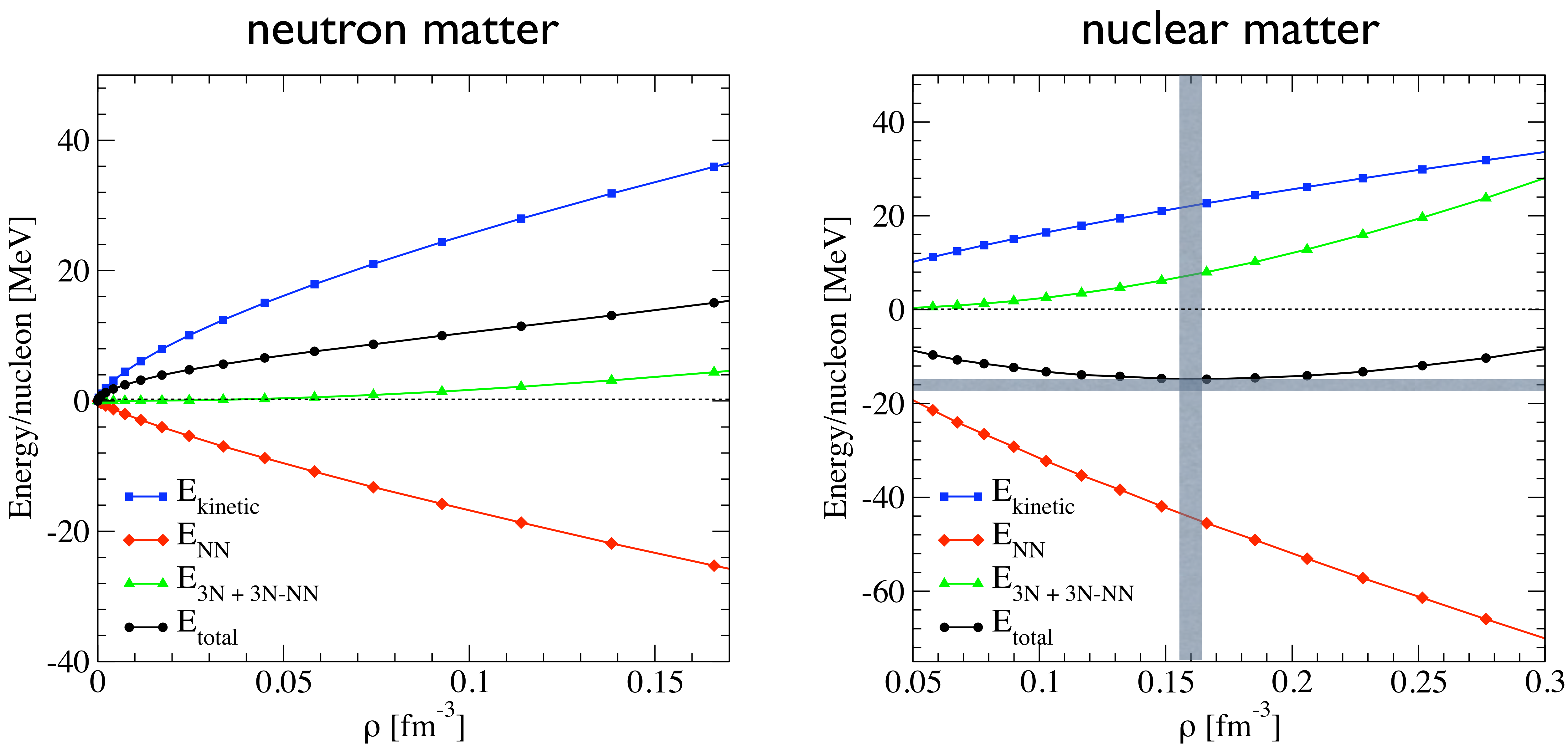

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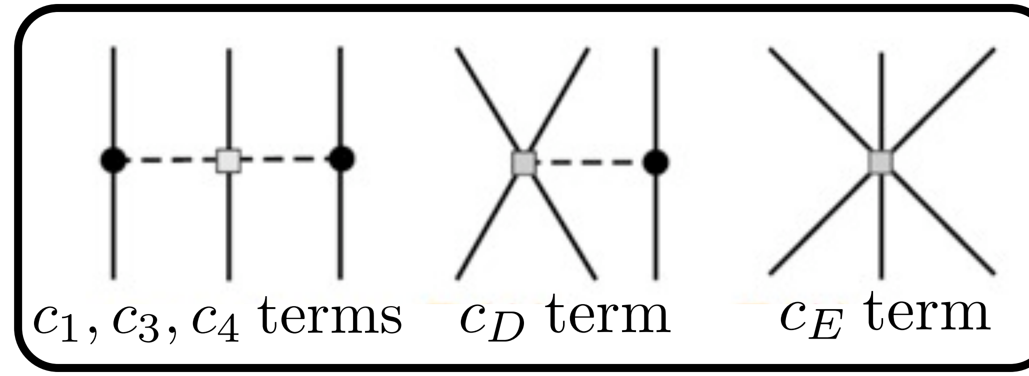
RG evolution of 3N interactions

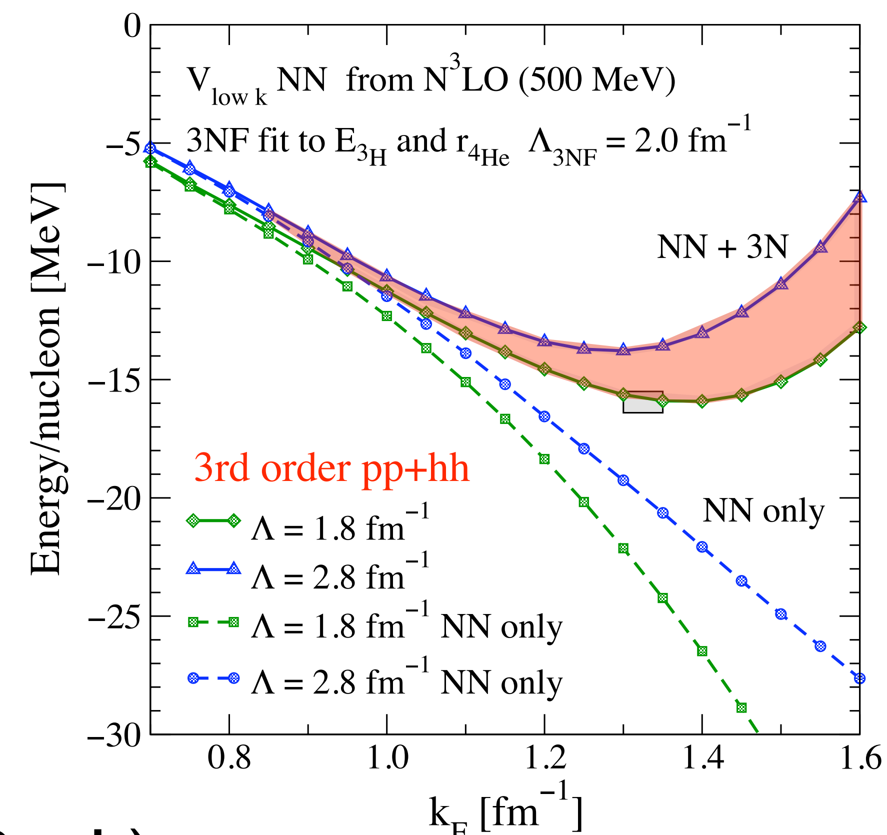


- So far:

intermediate (c_D) and short-range (c_E) 3NF couplings fitted to few-body systems at different resolution scales:

$$E_{^3\mathrm{H}} = -8.482\,\mathrm{MeV} \quad \text{and} \quad r_{^4\mathrm{He}} = 1.95 - 1.96\,\mathrm{fm}$$

$\longrightarrow$ coupling constants of natural size!



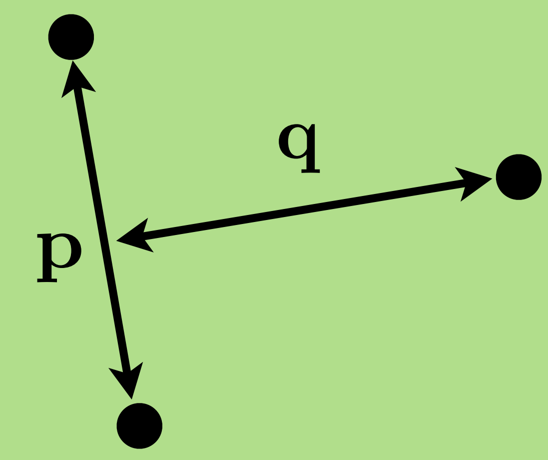
- Ideal case: evolve 3NF consistently with NN to lower resolution using the RG

- has been achieved in oscillator basis (Jurgenson, Roth)
- promising results in very light nuclei
- problems in heavier nuclei (see talk by R. Roth)
- not suitable for infinite systems

RG evolution of 3N interactions in momentum space

Three-body Faddeev basis:

$$|pq\alpha\rangle_i \equiv |p_i q_i; [(LS)J(ls_i)j]\,\mathcal{J}\mathcal{J}_z(Tt_i)\mathcal{T}\mathcal{T}_z\rangle$$



SRG flow equations for NN and 3N interactions:

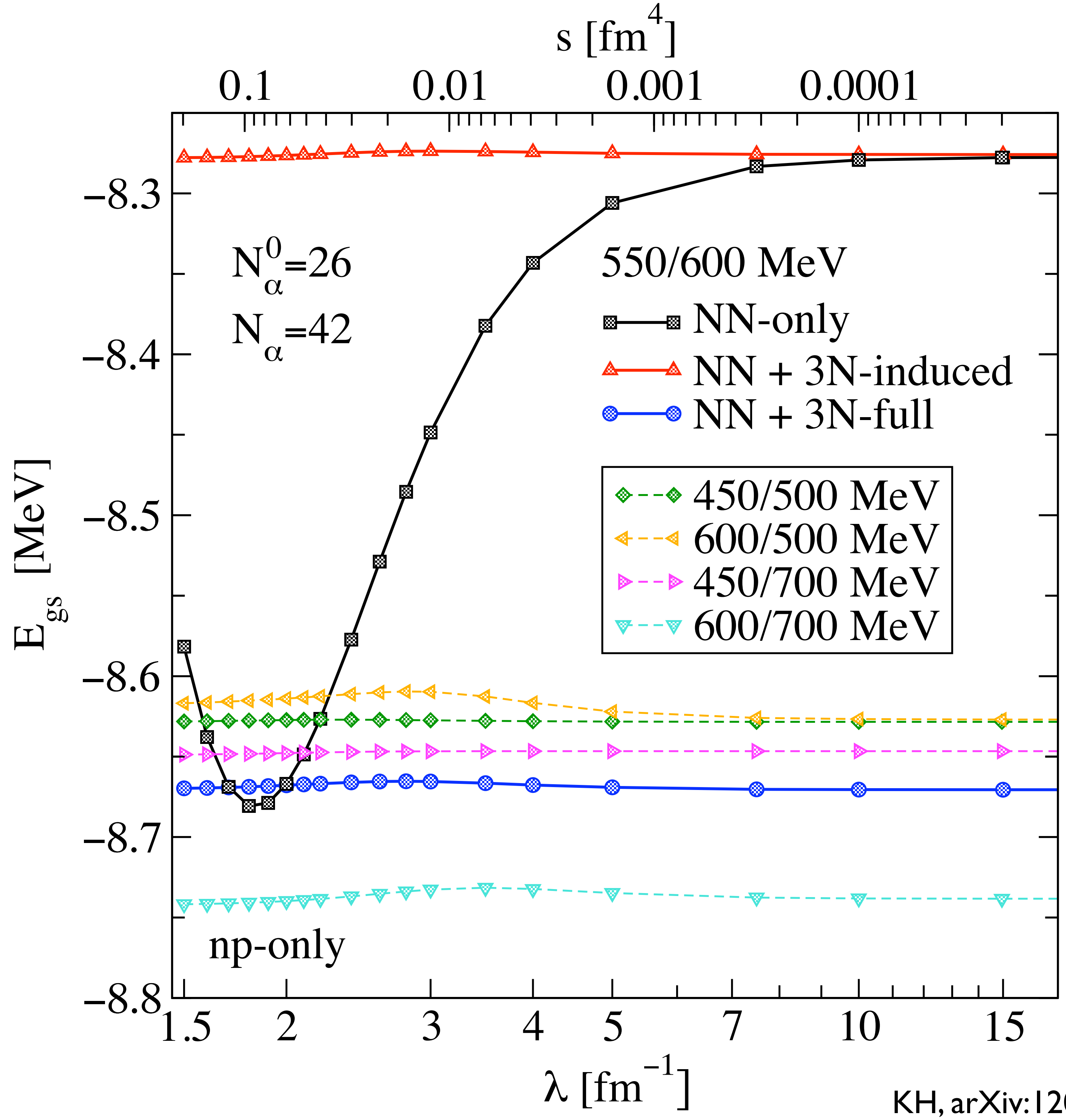
$$\frac{dH_s}{ds} = [\eta_s, H_s] \qquad \eta_s = [T_{\rm rel}, H_s]$$

$$\longrightarrow \quad \frac{dV_{ij}}{ds} = [[T_{ij}, V_{ij}], T_{ij} + V_{ij}],$$

$$\begin{aligned} \frac{dV_{123}}{ds} &= [[T_{12}, V_{12}], V_{13} + V_{23} + V_{123}] \\ &+ [[T_{13}, V_{13}], V_{12} + V_{23} + V_{123}] \\ &+ [[T_{23}, V_{23}], V_{12} + V_{13} + V_{123}] \\ &+ [[T_{\rm rel}, V_{123}], H_s] \end{aligned}$$

KH, arXiv:1201.0169 (2012)

RG evolution of 3N interactions in momentum space



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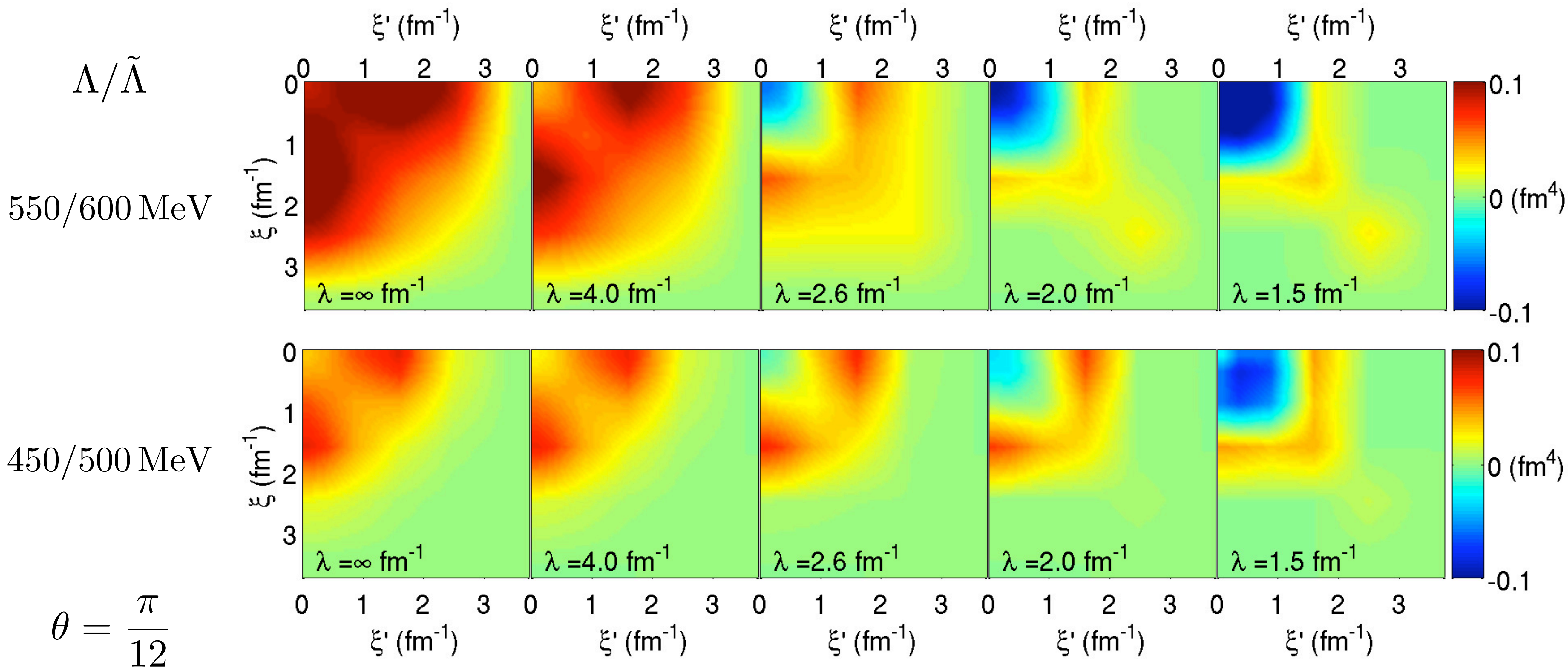
first study:

Invariance of E_{gs}^{3H} within $16\,\mathrm{keV}$ for consistent chiral interactions at $\mathrm{N^2LO}$

RG evolution of 3N interactions in momentum space



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hyperradius: $\xi^2 = p^2 + \frac{3}{4}q^2$

hyperangle: $\tan\theta = \frac{2\,p}{\sqrt{3}\,q}$

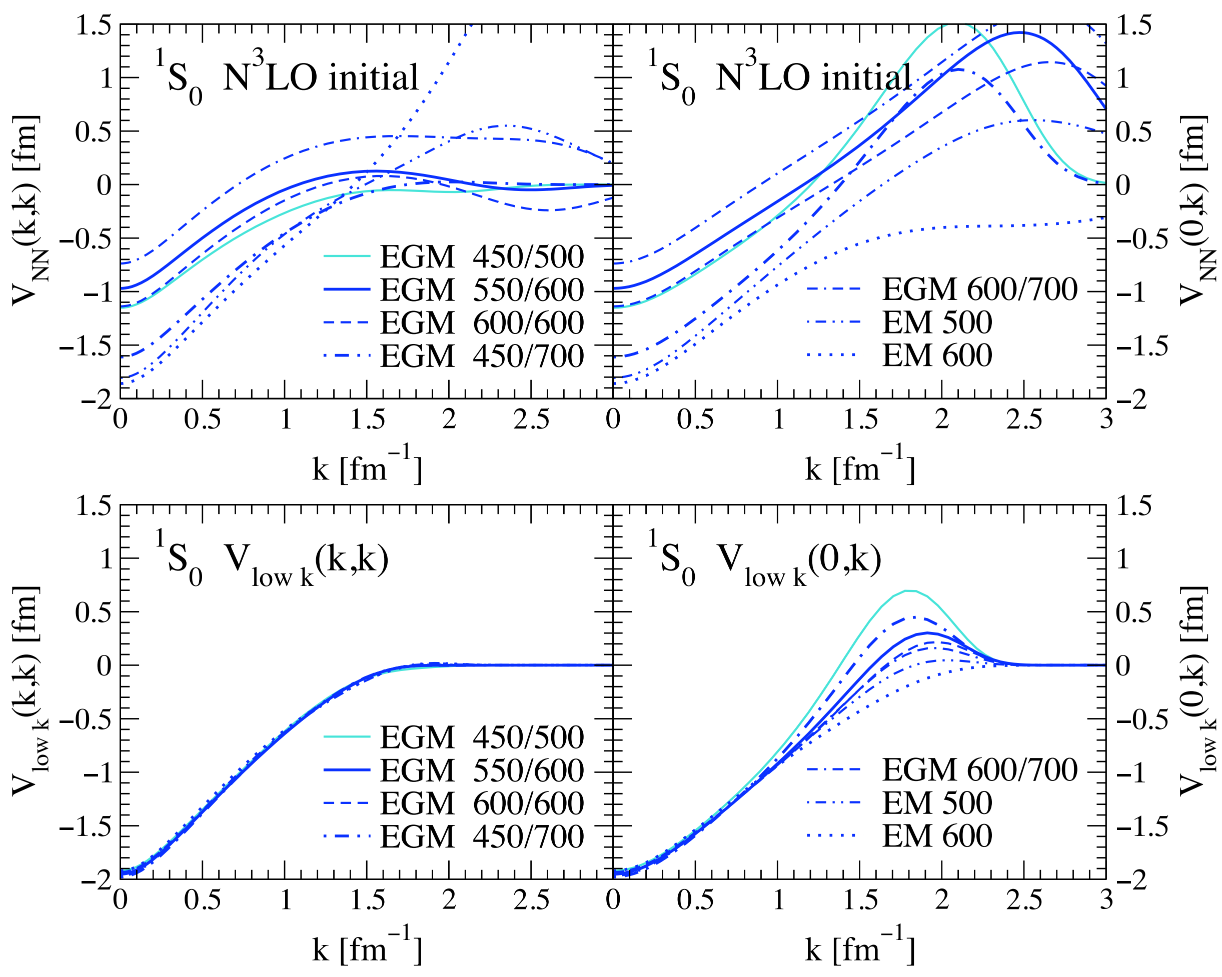
same decoupling patterns like in NN interactions

Universality in 3N interactions at low resolution

phase-shift equivalence

common long-range physics

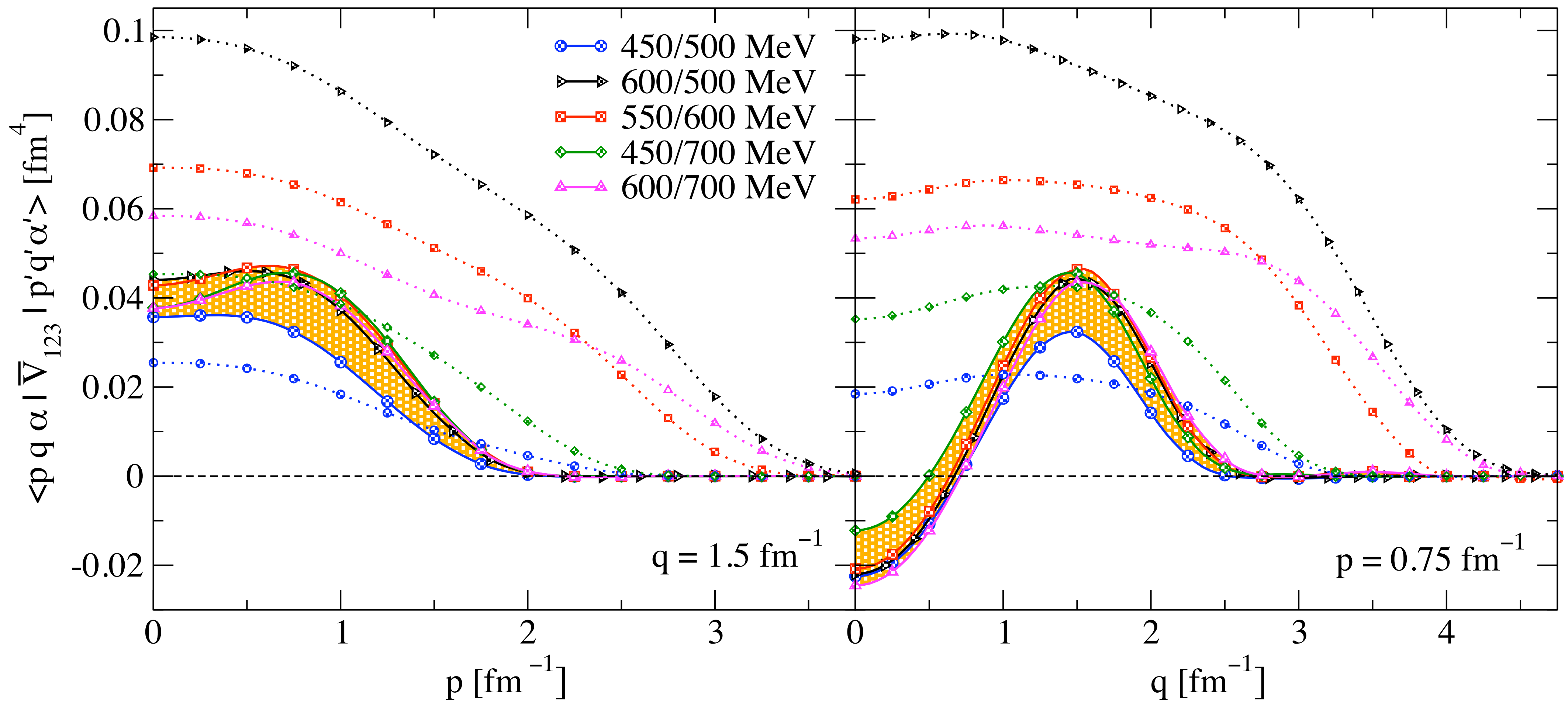
(approximate) universality of low-resolution NN interactions



To what extent are 3N interactions constrained at low resolution?

- only two low-energy constants c_D and c_E
- 3N interactions give only subleading contributions to observables

Universality in 3N interactions at low resolution



KH, arXiv:1201.0169 (2012)

- remarkably reduced model dependence for typical momenta $\sim 1\,\mathrm{fm}^{-1}$, matrix elements with significant phase space well constrained at low resolution
- new momentum structures induced at low resolution
- study based on $\mathrm{N^2LO}$ chiral interactions, improved universality at $\mathrm{N^3LO}$?

Applications

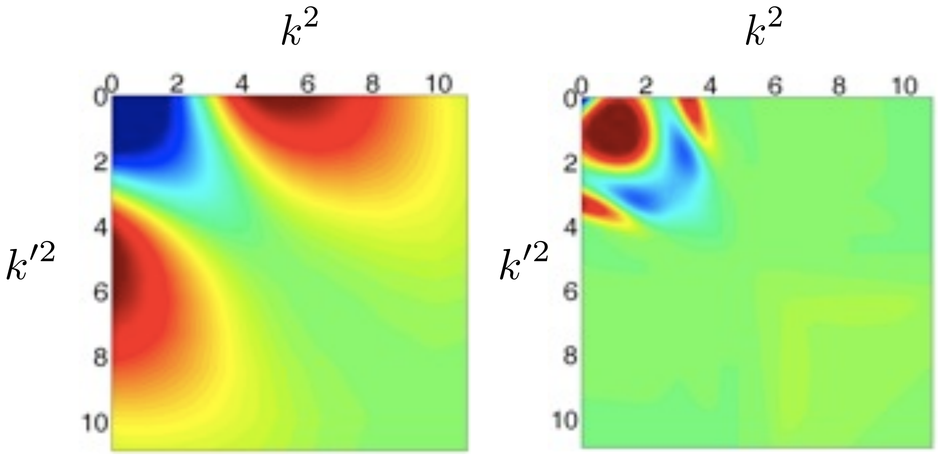
- application to infinite systems

  ⟶ equation of state, systematic study of induced many-body contributions

- transformation of evolved interactions to oscillator basis

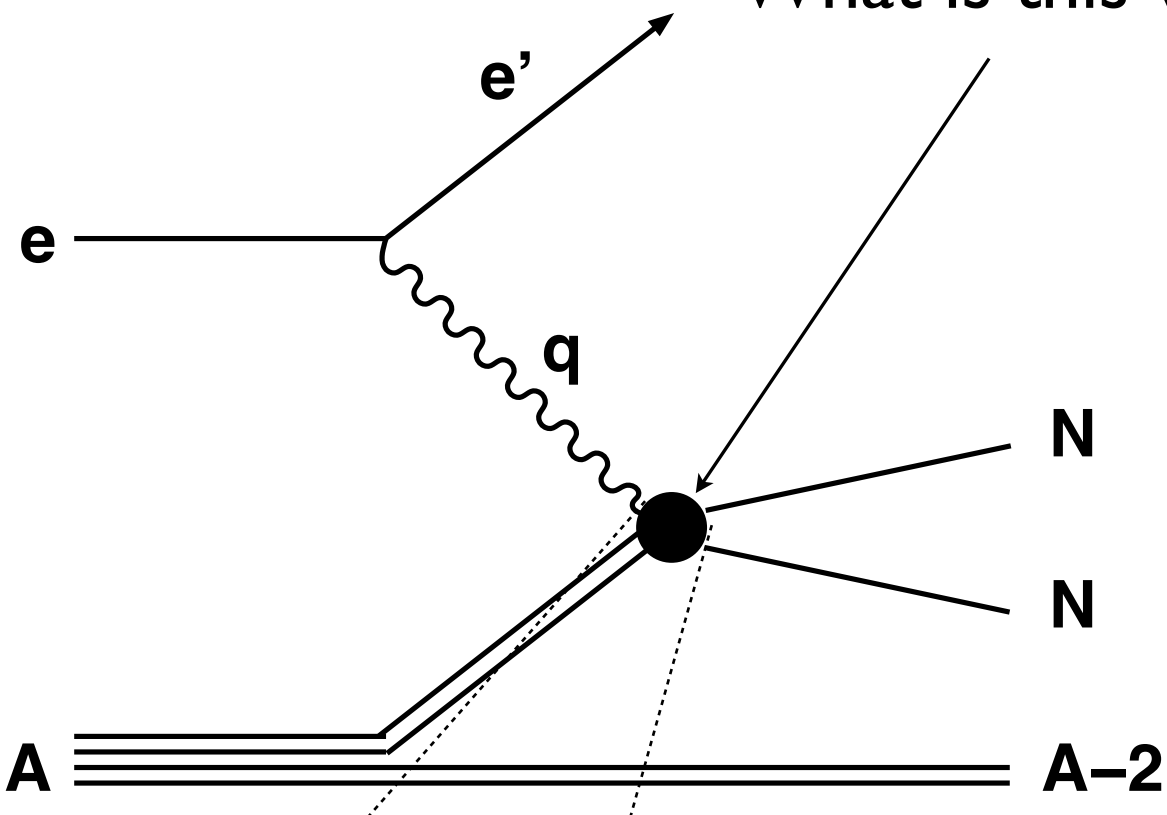
  ⟶ application to finite nuclei, complimentary to HO evolution (no core shell model, coupled cluster)

- study of alternative generators
  - different decoupling patterns (e.g. $V_{\mathrm{low}\,k}$)
  - improved efficiency of evolution
  - suppression of many-body forces

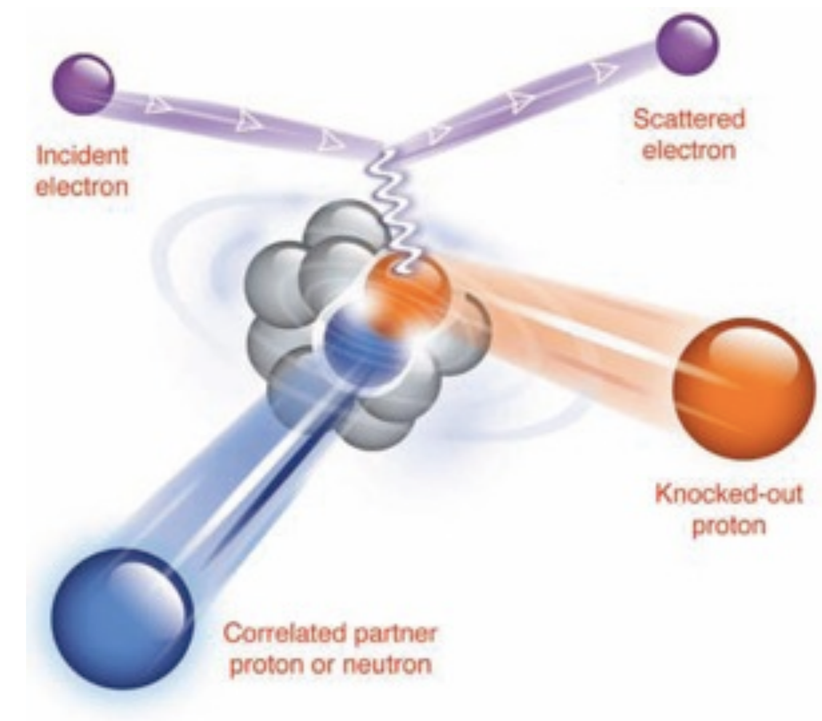


Correlations in nuclear systems

What is this vertex?



Higinbotham, arXiv:1010.4433

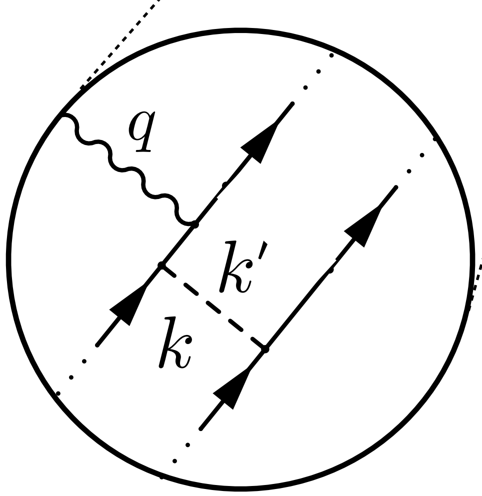


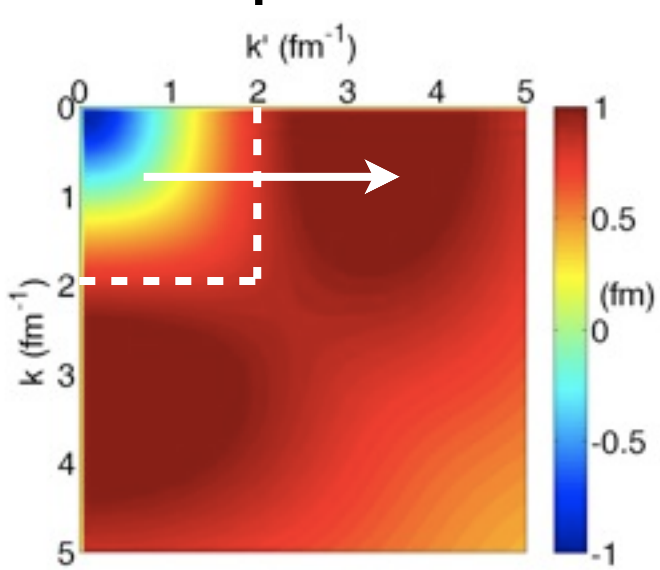
- detection of knocked out pairs with large relative momenta
- excess of np pairs over pp pairs

Subedi et al., Science 320, 1476 (2008)

Explanation in terms of low-momentum interactions?

Short-range-correlation interpretation:





k : low rel. momentum

k' : high rel. momentum

Correlations in nuclear systems

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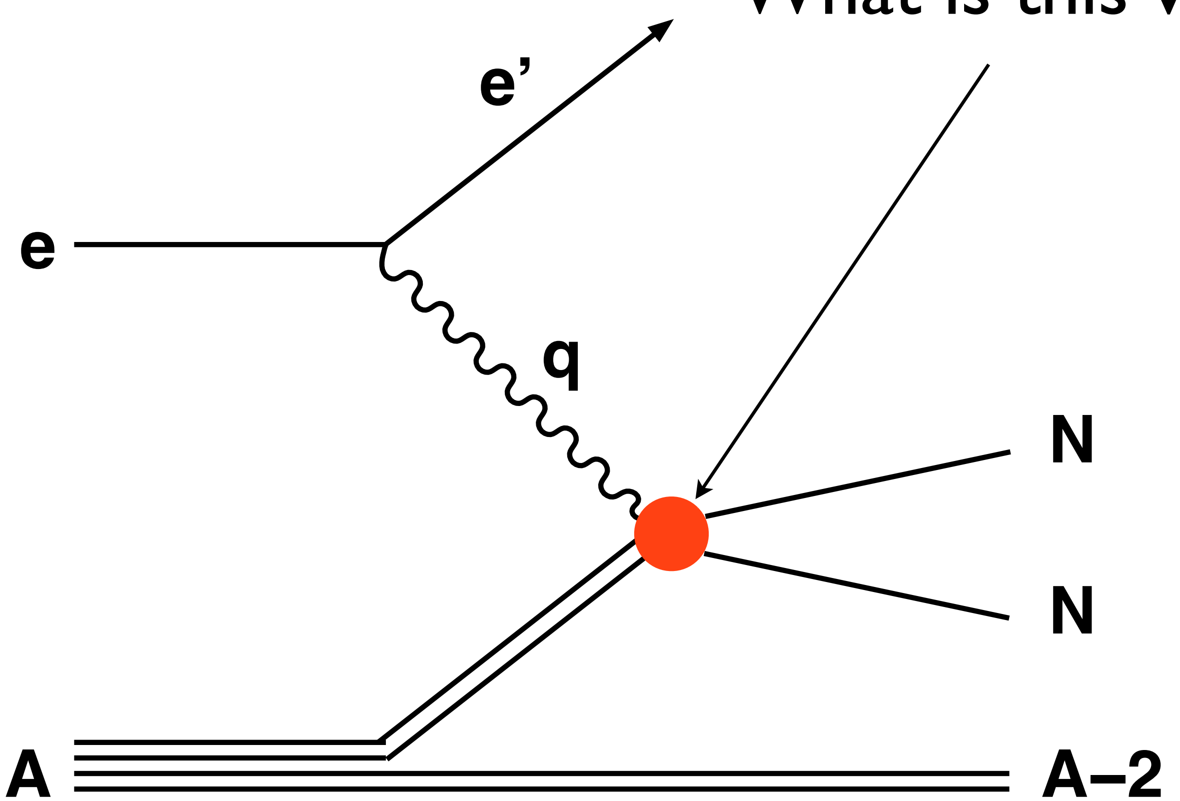

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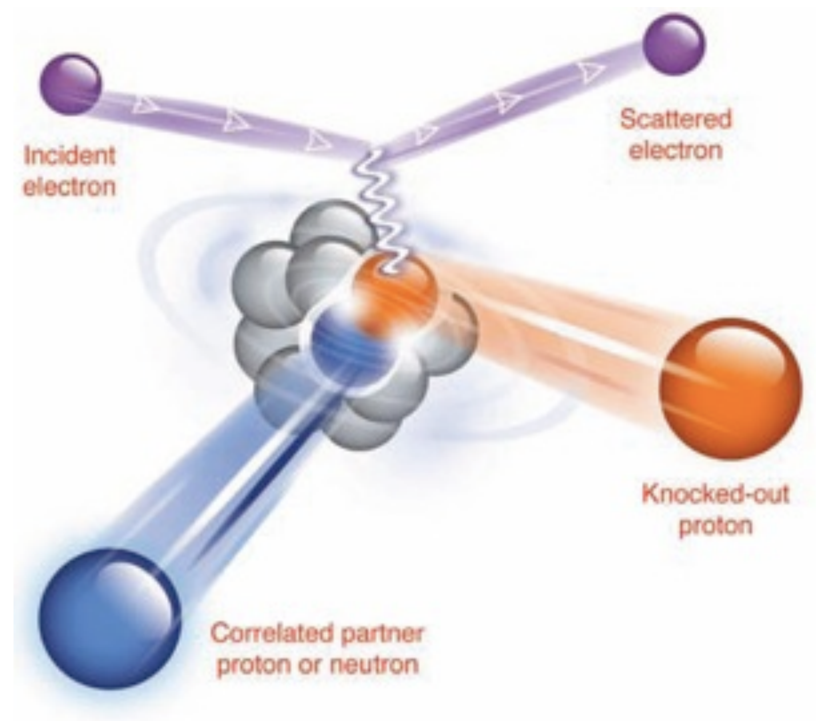

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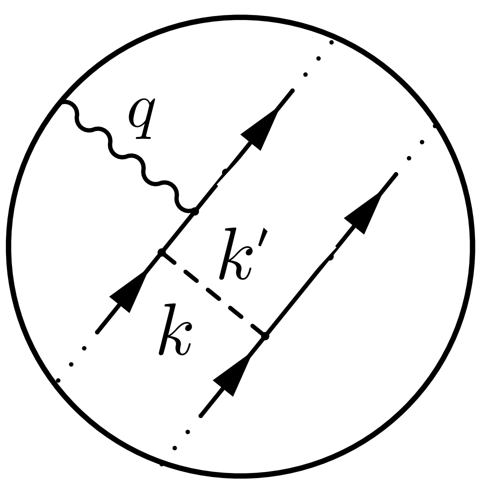

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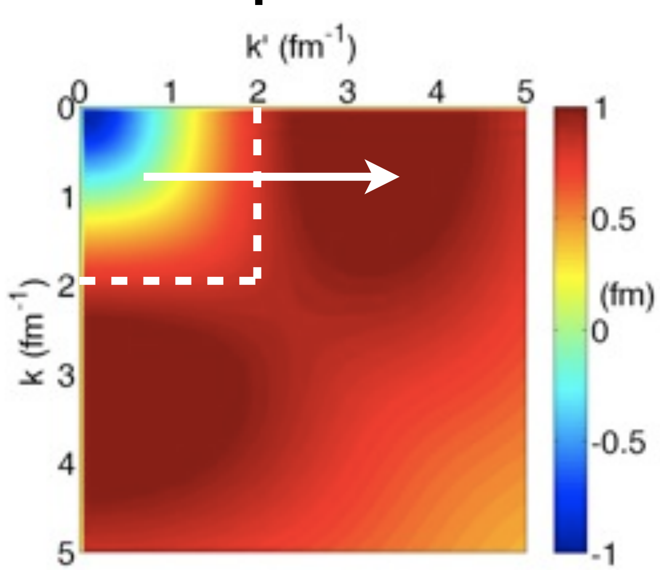
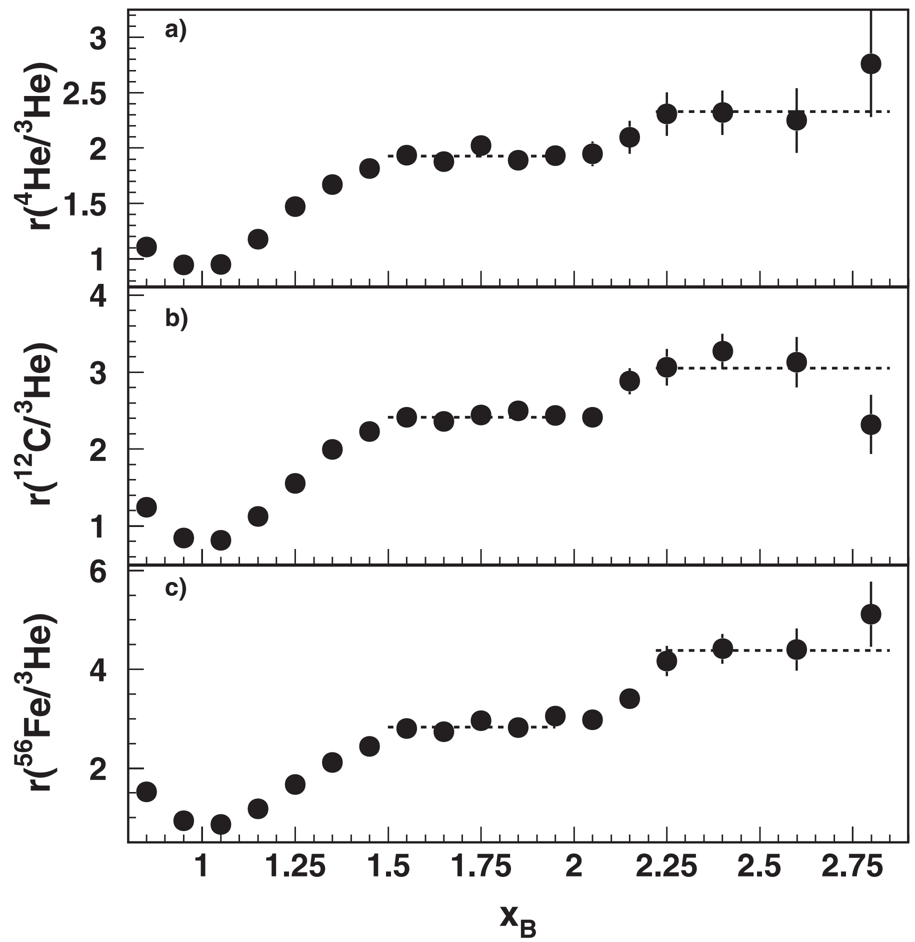


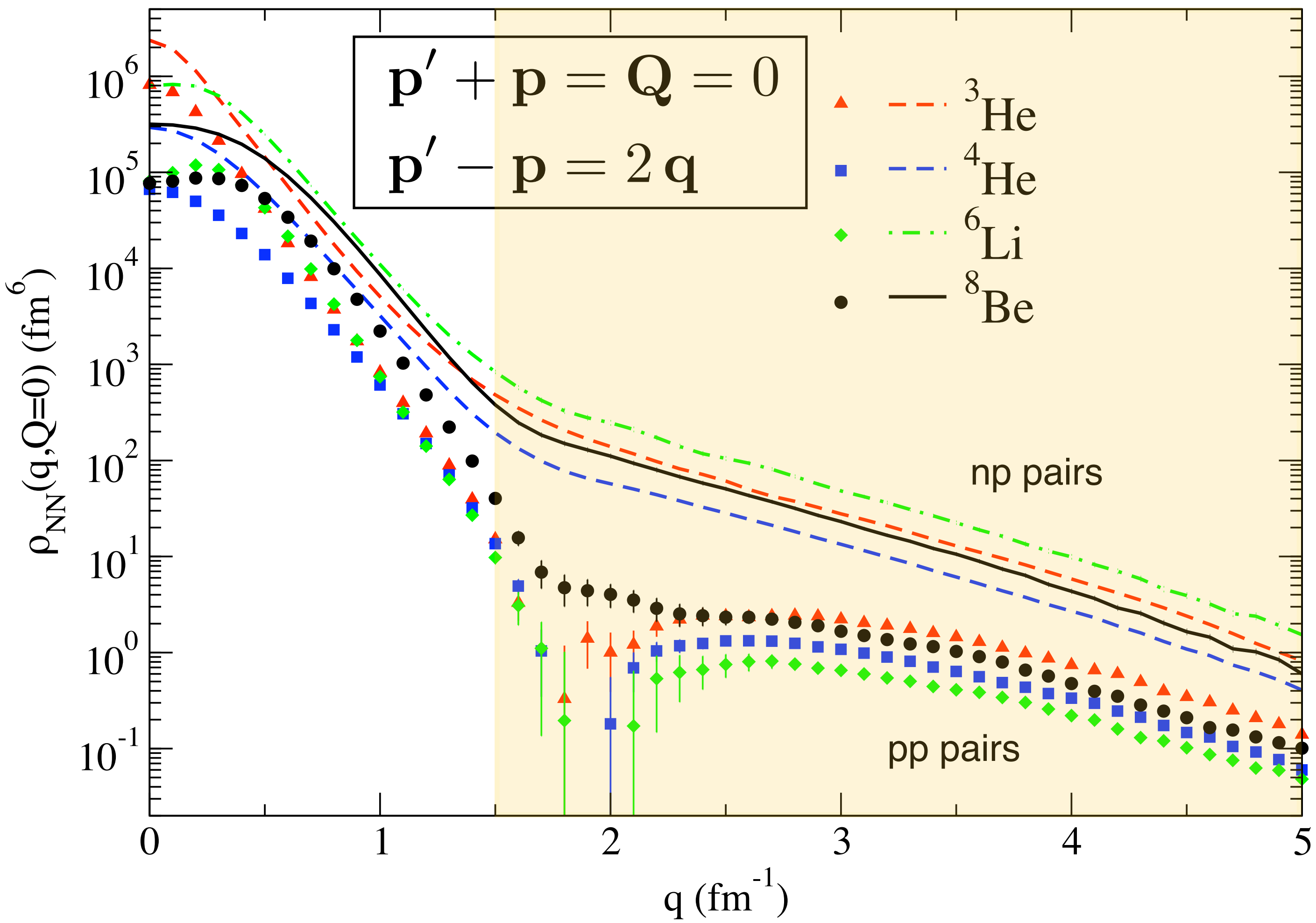
Vertex depends on the resolution!
RG provides systematic way to calculate such processes at low resolution.

Scaling in nuclear systems



$1.4 < Q^2 < 2.6\, GeV^2$

Egiyan et al. PRL 96, 1082501 (2006)



Schiavilla et al., PRL 98, 132501 (2007)

- scaling behavior of momentum distribution function:

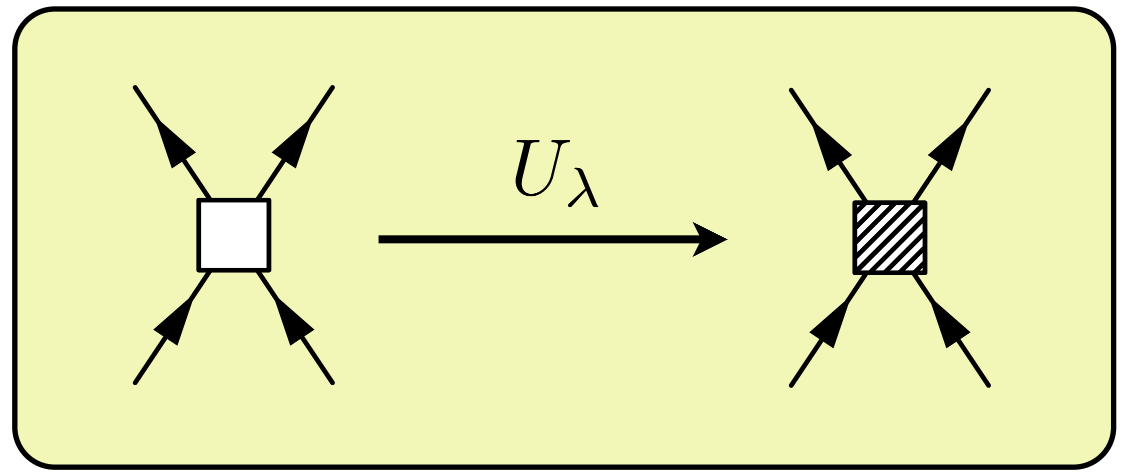
  $\rho_{\mathrm{NN}}(q, Q=0) \approx C_A \times \rho_{\mathrm{NN,Deuteron}}(q, Q=0)$ at large q

- dominance of np pairs over pp pairs
- “hard” (high resolution) interaction used, calculations hard!
- dominance explained by short-range tensor forces

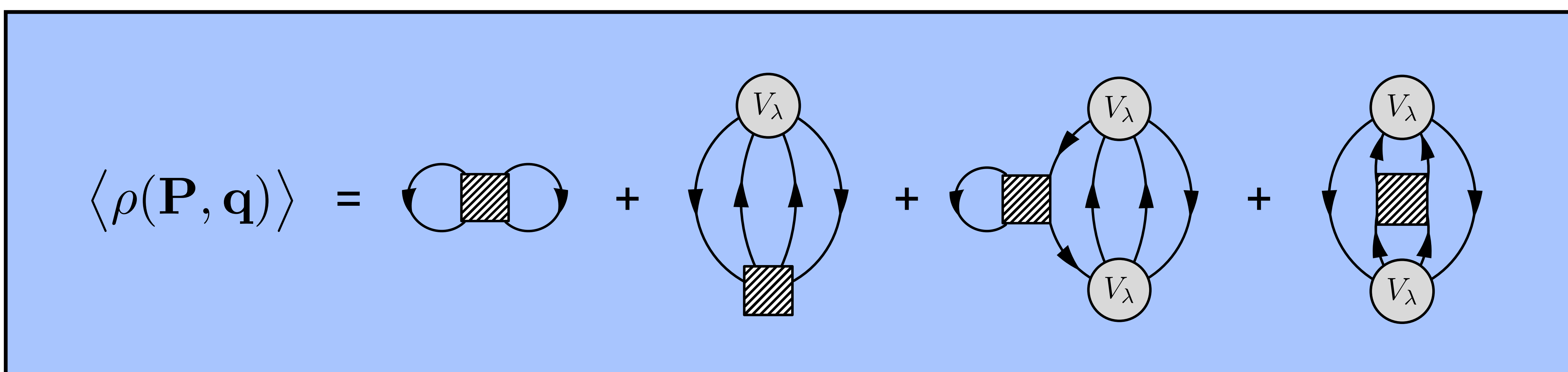
Nuclear scaling at low resolution

$\langle \psi_\lambda | O_\lambda | \psi_\lambda \rangle$ **factorizes** into a low-momentum structure and a **universal** high momentum part if the initial operator only weakly couples low and high momenta ⟶ explains scaling!

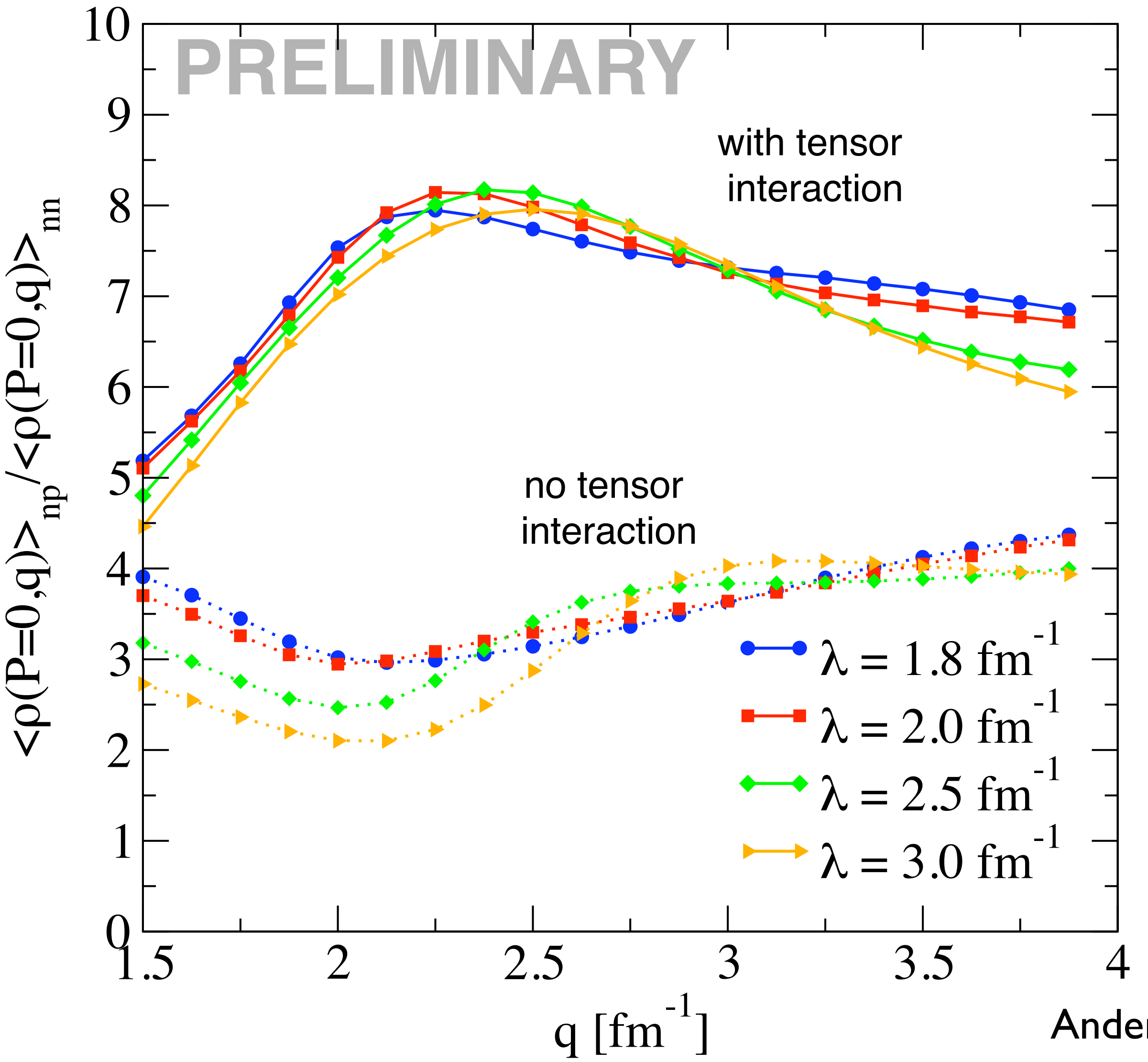
RG transformation of pair density operator (induced many-body terms neglected):

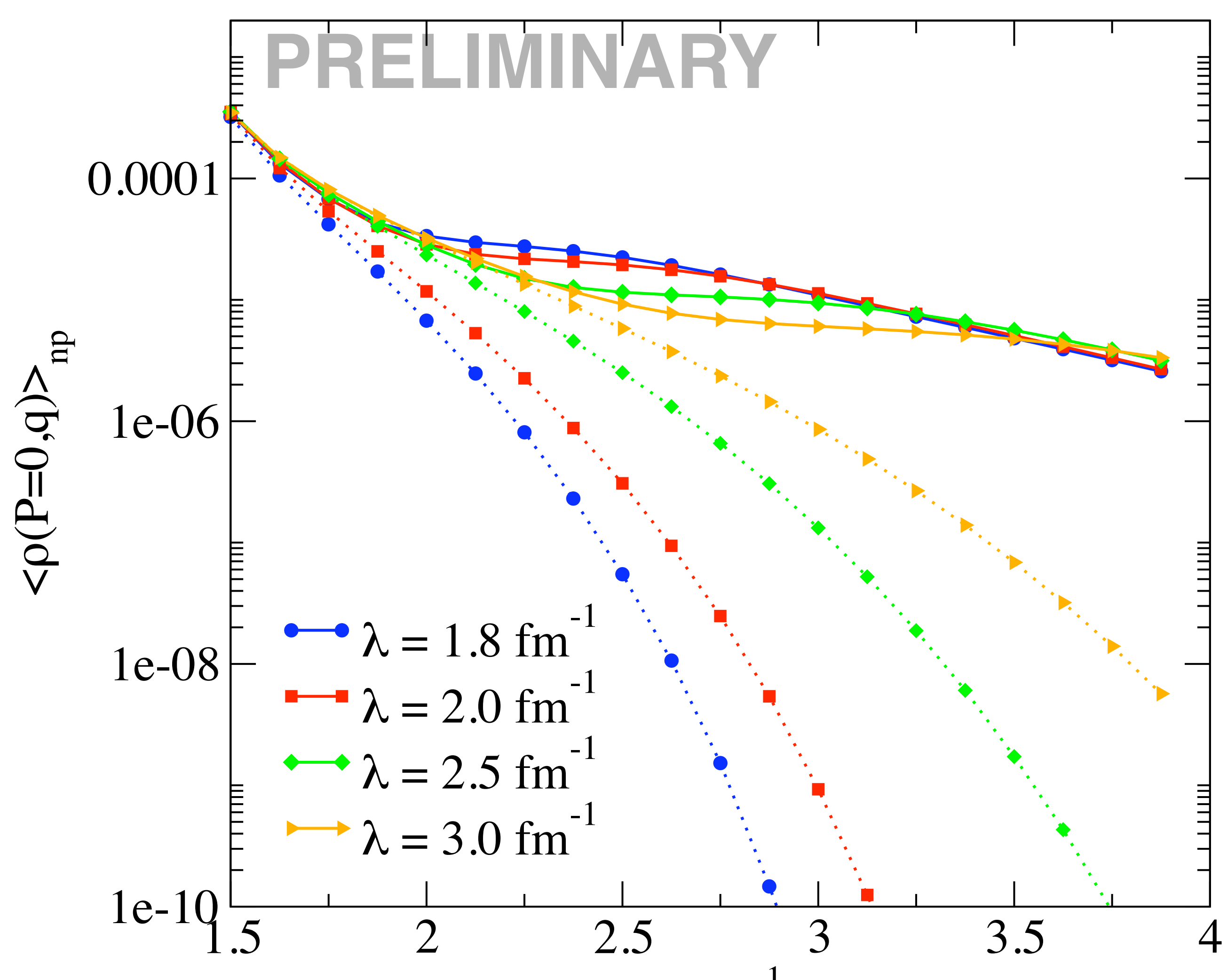


“simple” calculation of pair density at low resolution in nuclear matter:



Nuclear scaling at low resolution





Anderson, KH, Furnstahl, in preparation

- pair-densities approximately resolution independent
- significant enhancement of np pairs over nn pairs due to tensor force
- reproduction of previous results using a “simple” calculation at low resolution!

High-resolution experiments can be explained by low-resolution methods!
Opens door to study other electro-weak processes and higher-body correlations.

Summary

- low-resolution interactions allow simpler calculations for nuclear systems
- observables invariant under resolution changes, interpretation of results can change!
- chiral EFT provides systematic framework for constructing nuclear Hamiltonians
- 3N interactions are essential at low resolution
- nucleonic matter equation of state based on low-resolution interactions consistent with empirical constraints
- constraints for the nuclear equation of state and structure of neutron stars

Outlook

- RG evolution of three-nucleon interactions: microscopic study of light nuclei and nucleonic matter using chiral nuclear interactions at low resolution
- RG evolution of operators: nuclear scaling and correlations in nuclear systems