# On the thermodynamics and phase structure of QCD

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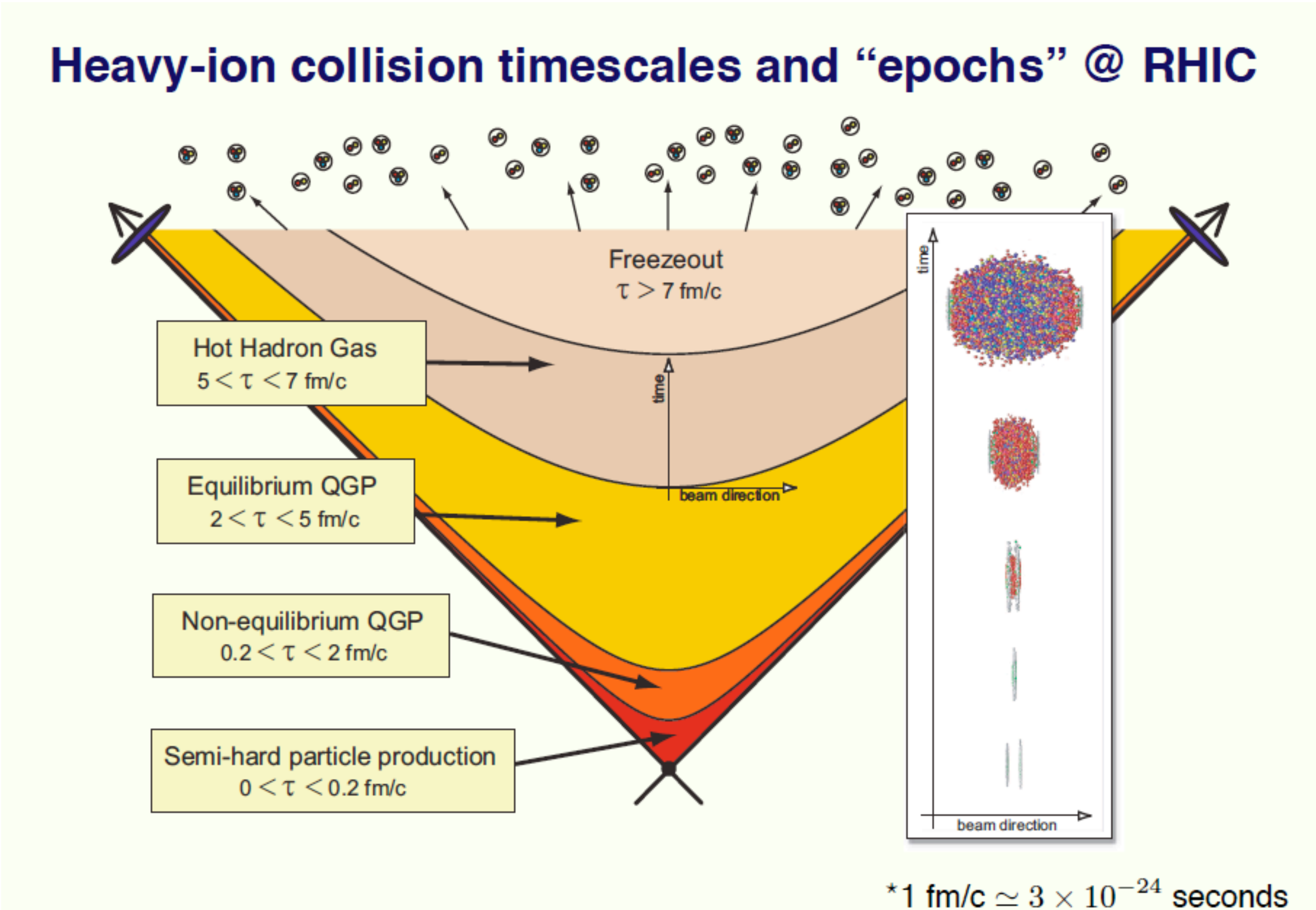
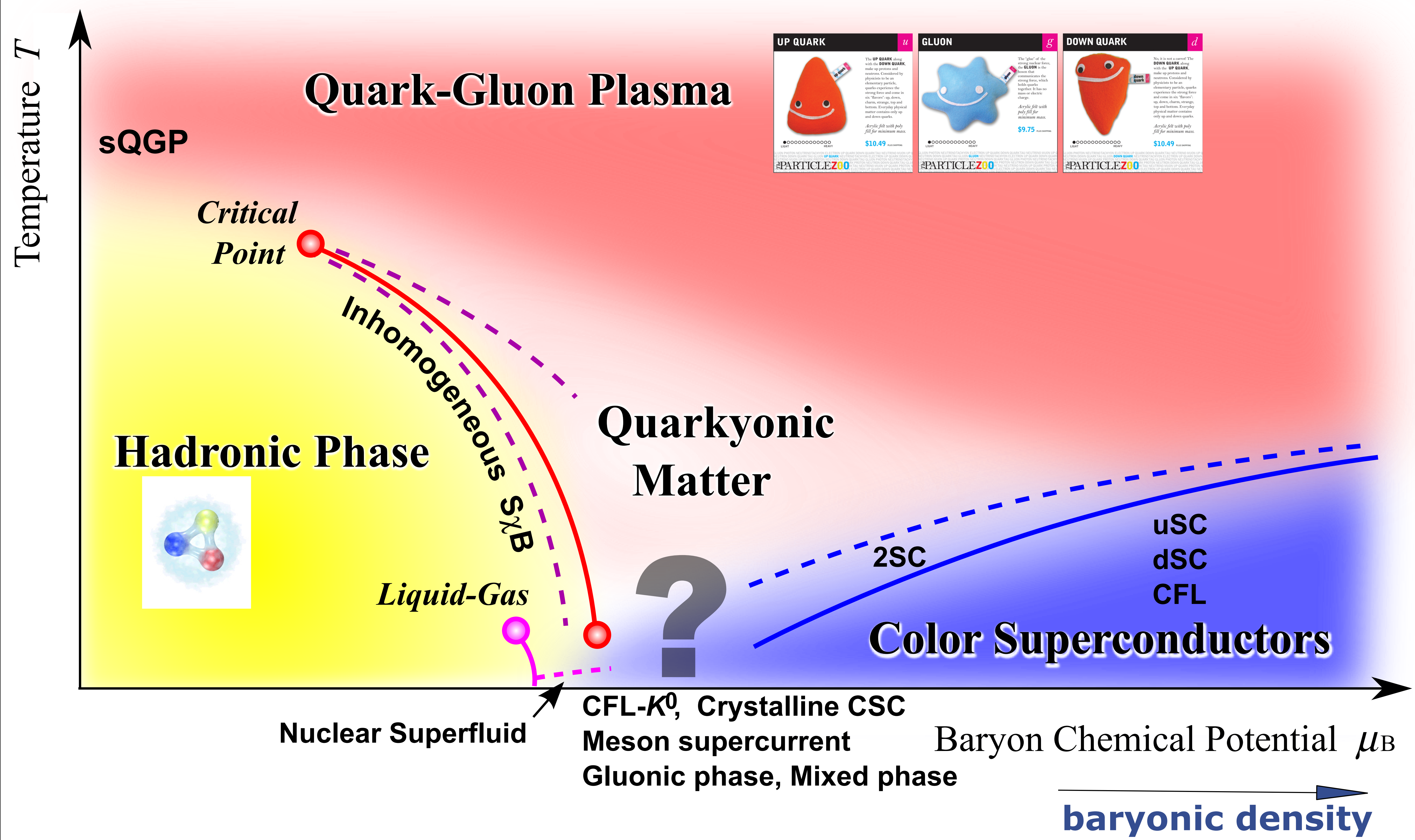
Hirschegg, January 17th 2012

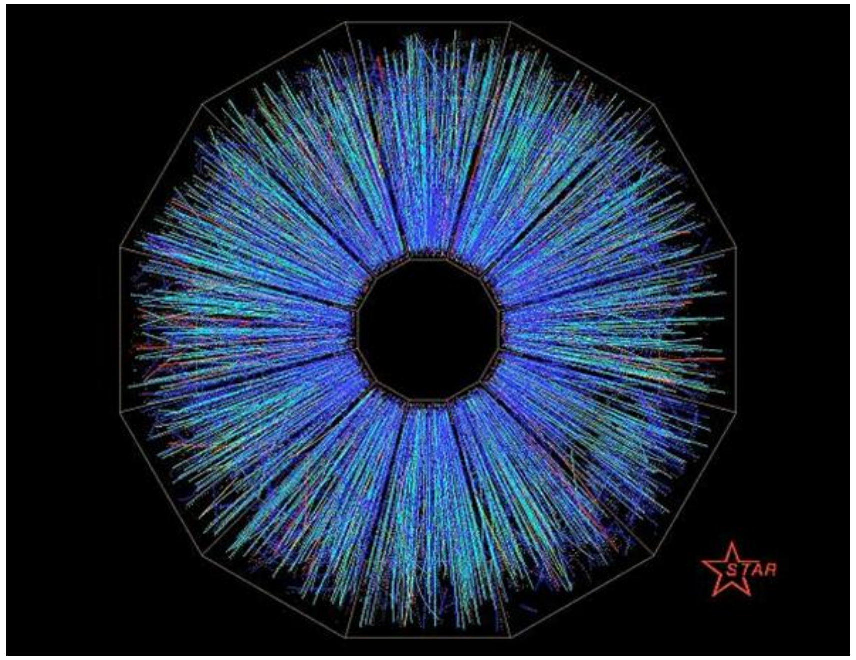
# Heavy ion collisions



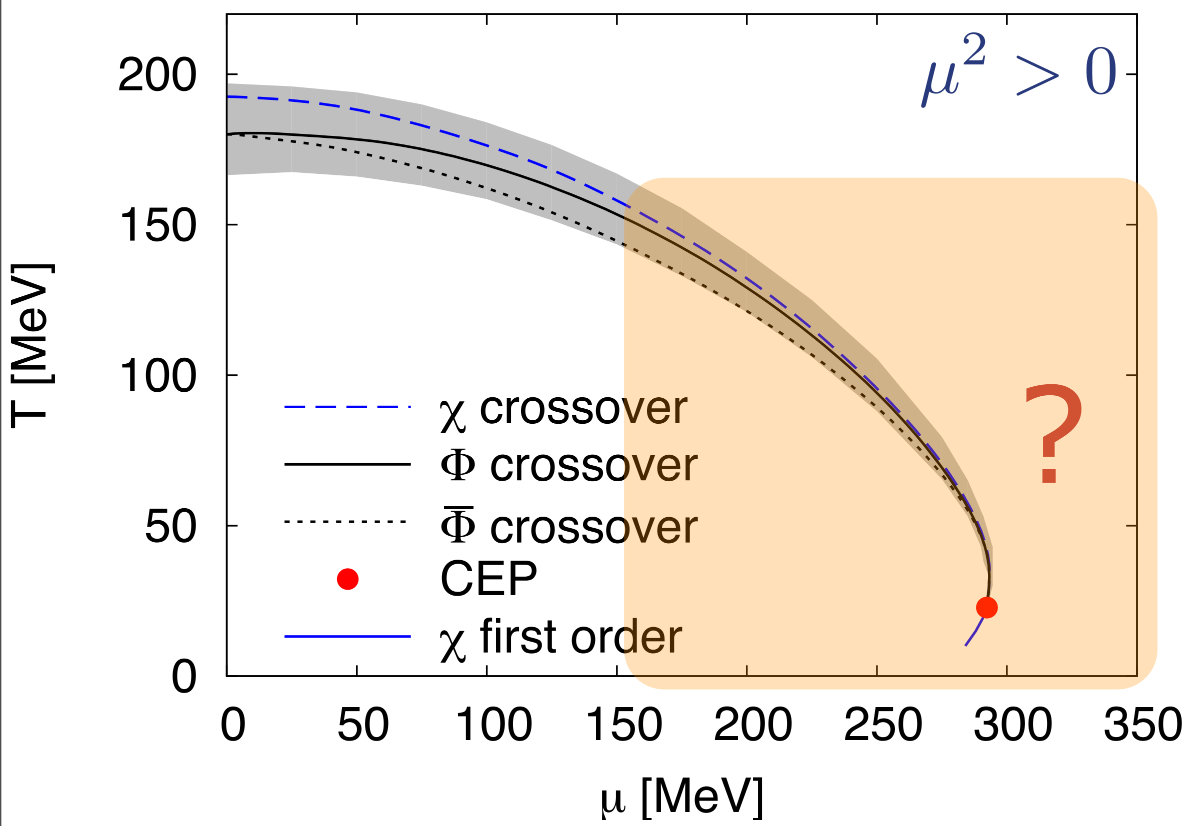
# Phase diagram of QCD



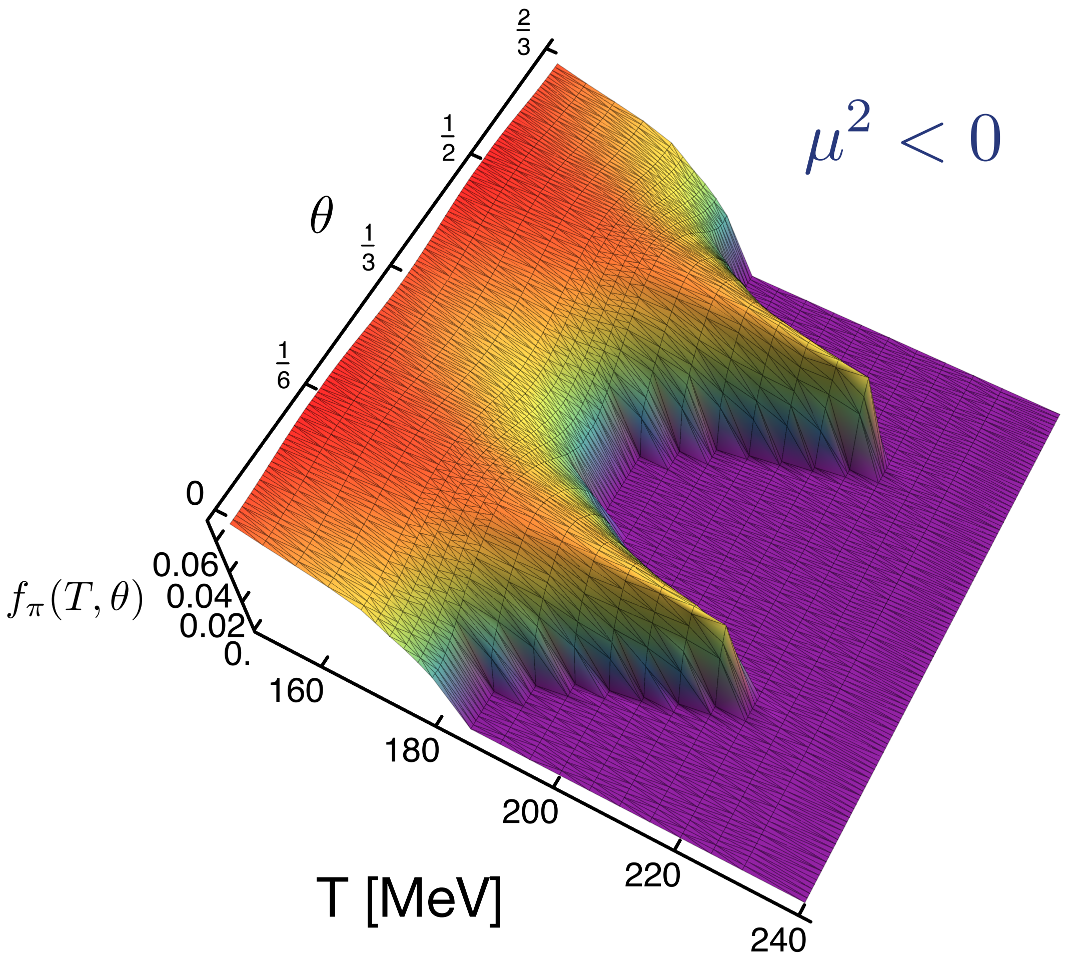

Fukushima




# From the quark-gluon plasma to the hadron gas




**Results**

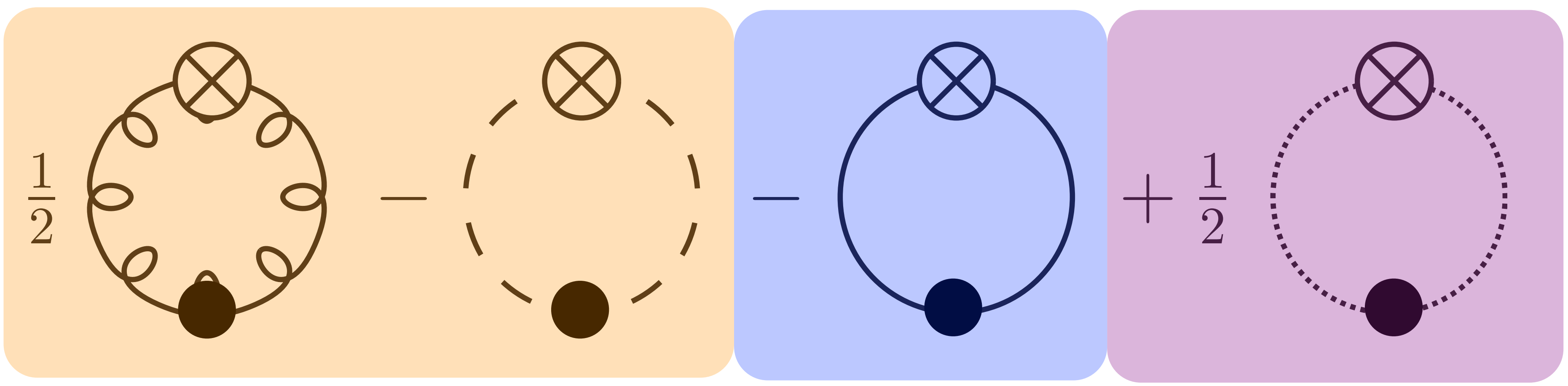


# Functional Methods for QCD

**for a short review see JMP, arXiv:1012.5075**

**glue quantum fluctuations** · **hadronic quantum fluctuations**

$\partial_t \Gamma_k[\phi] =$




**free energy** · **quark quantum fluctuations**

RG-scale k: $t = \ln k$

- **Gluons have cost us decades**
- **Fermions are straightforward** though 'physically' complicated
  - **no sign problem**
  - **chiral fermions**
- **bound states via dynamical hadronisation**

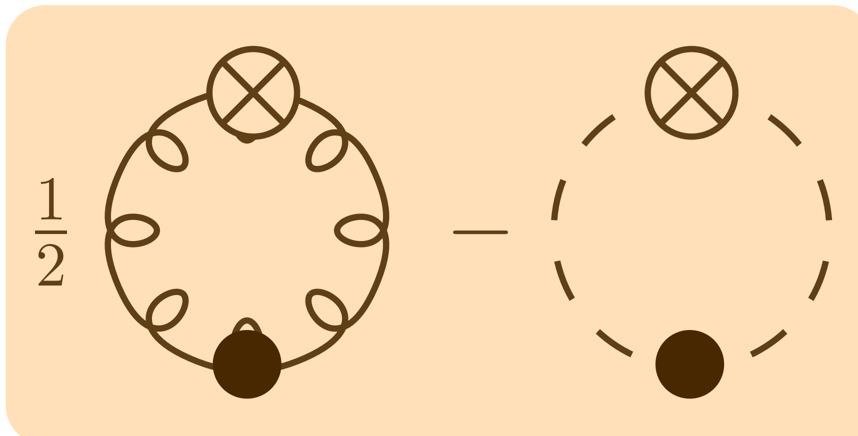
## Complementary to lattice!

**e.g. finite volume scaling: Braun, Klein, Piasecki, Schaefer '10-11**

# Functional Methods for QCD

glue
quantum fluctuations

$$\partial_t \Gamma_k[\phi] = $$



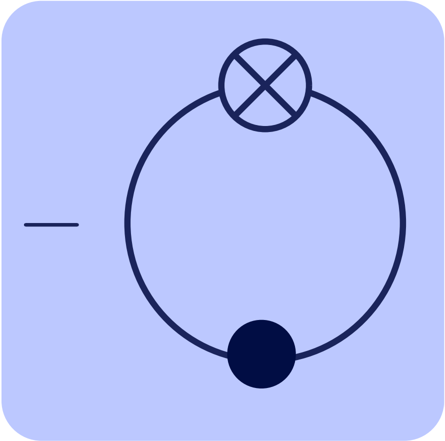
free energy

## Yang-Mills theory

# Functional Methods for QCD

$$\partial_t \Gamma_k[\phi] =$$

**free energy**



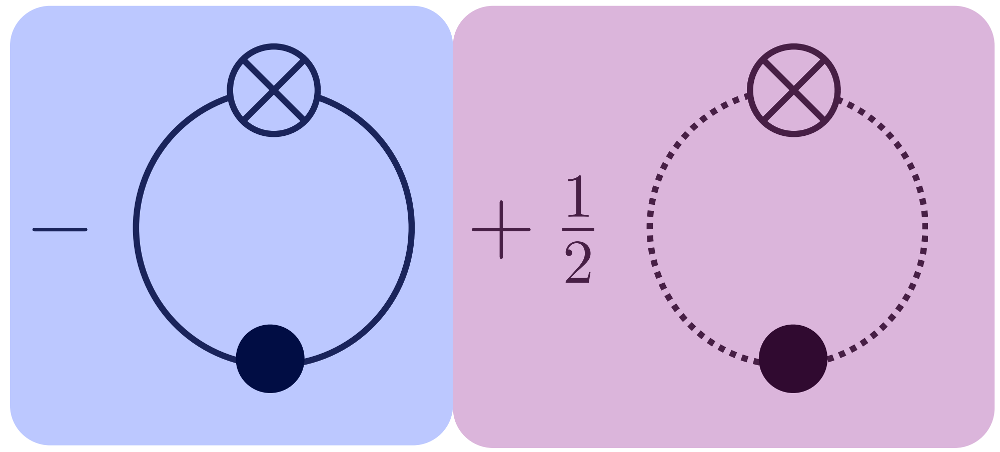

**quark**
**quantum fluctuations**

## NJL/PNJL model

# Functional Methods for QCD

**hadronic quantum fluctuations**

$$\partial_t \Gamma_k[\phi] =$$




**free energy**

**quark quantum fluctuations**

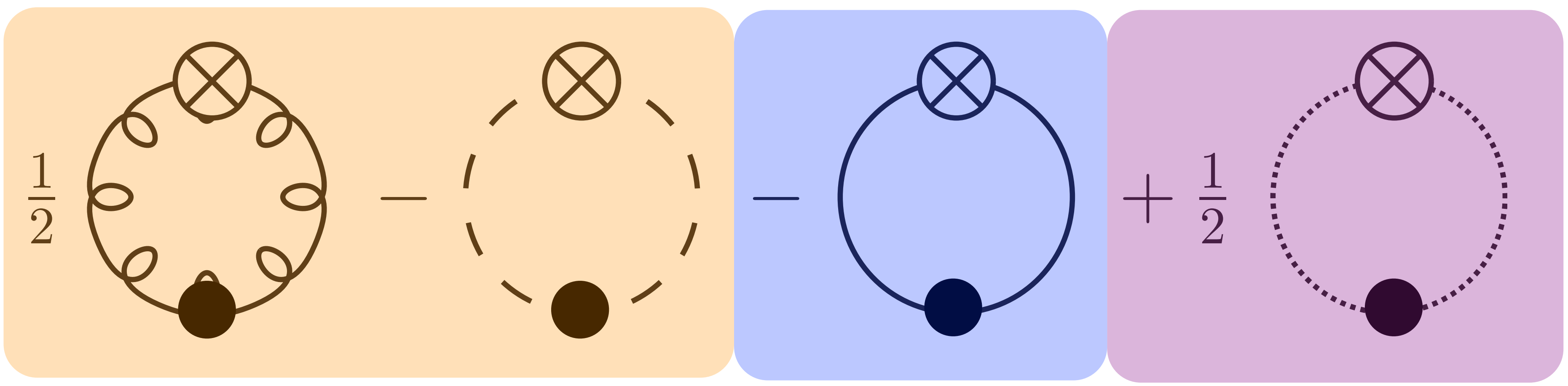
## Quark-hadron/PQH models

- **bound states via dynamical hadronisation**

# Functional Methods for QCD

glue quantum fluctuations — hadronic quantum fluctuations

$\partial_t \Gamma_k[\phi] =$

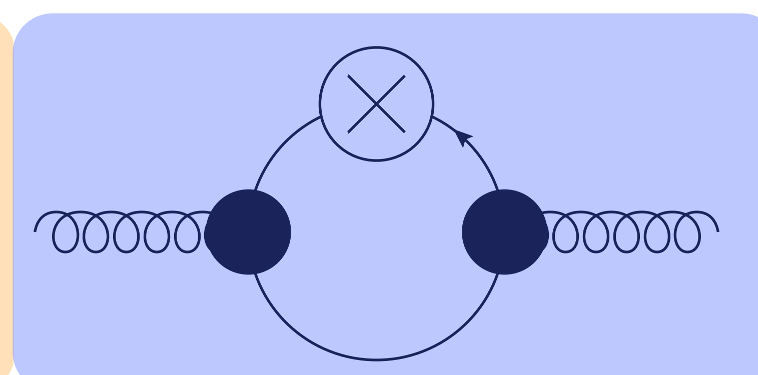



free energy

quark quantum fluctuations

**flow of gluon propagator**

**pure gauge theory flow** +

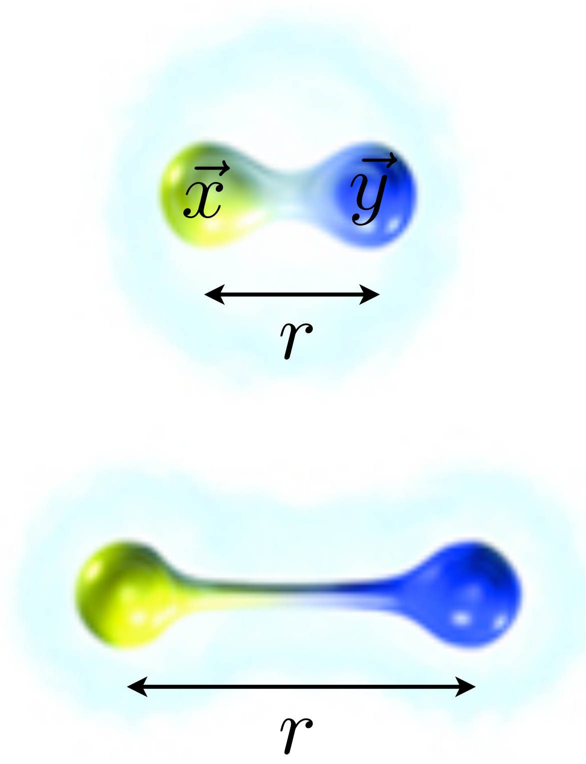


+ ...

Naturally encorporates PQM/PNJL models as specific low order trunations

# Confinement

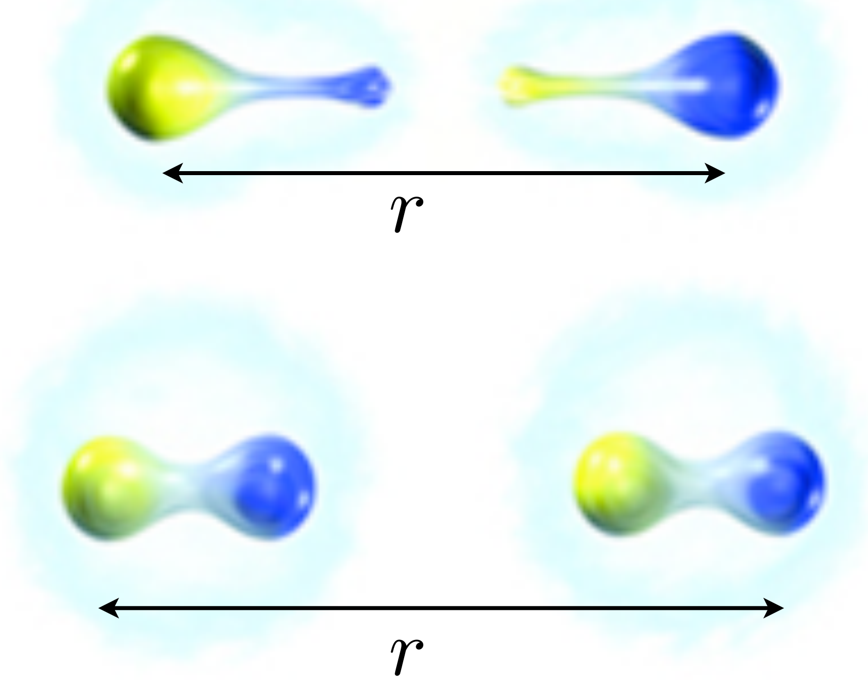
## Free energy $F_{q\bar{q}}$ of a quark - antiquark pair




$$F_{q\bar{q}} \simeq -\frac{1}{r}$$

$$F_{q\bar{q}} \simeq \sigma r$$

**string breaking at $\mathrm{r} \approx 1\mathrm{fm}$**




$$F_{q\bar{q}} \simeq \mathrm{const.}$$

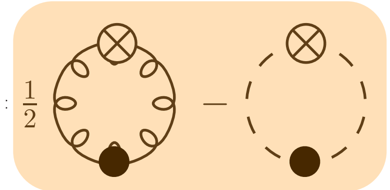
**Order parameter** $\sim\ '\langle q \rangle'$

$$\Phi = e^{-\frac{1}{2T} F_{q\bar{q}}(\infty)}$$

- **Confinement** $\Phi = 0$
- **Deconfinement** $\Phi \neq 0$

**Polyakov loop**

$$\Phi = \frac{1}{3} \langle \mathrm{Tr}\, \mathcal{P} \exp\{ig \int_0^{1/T} dx_0 A_0\}\rangle$$




# Confinement

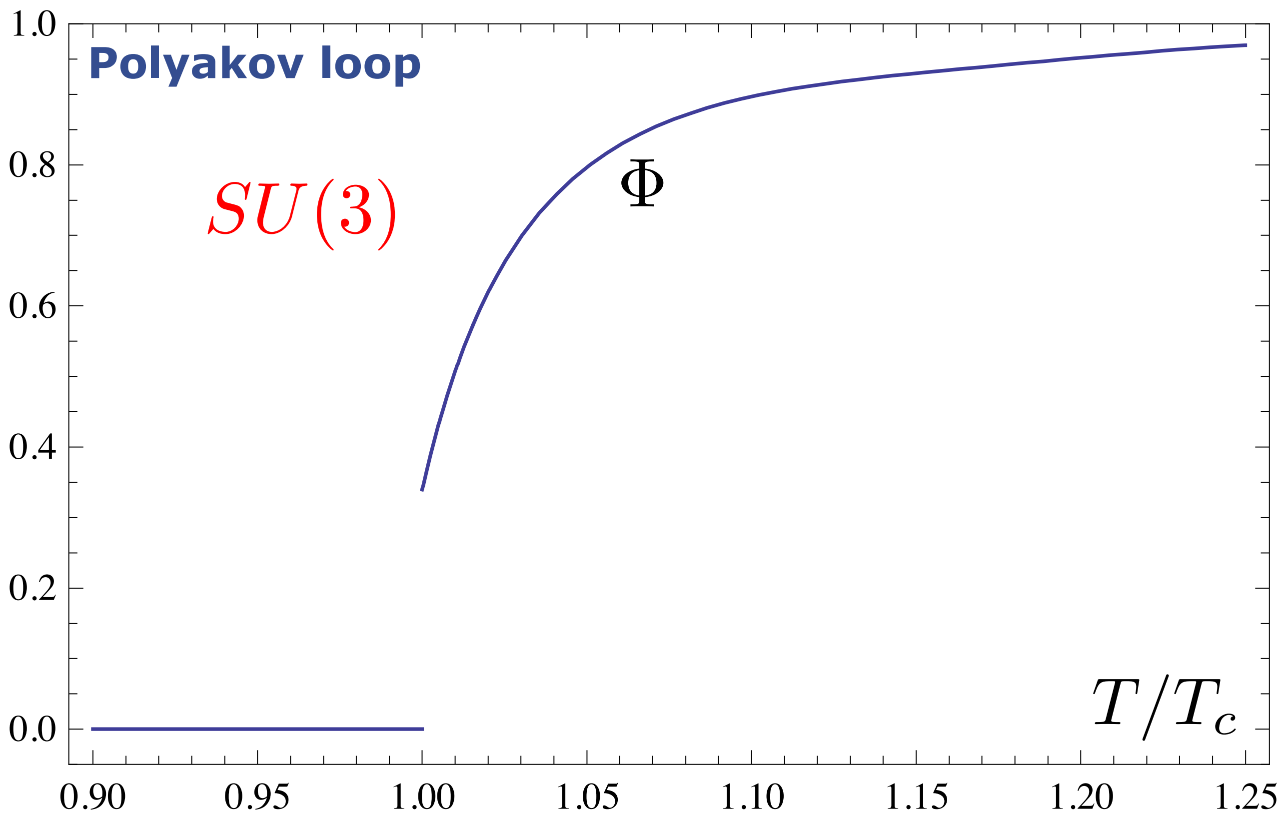
## Order parameter

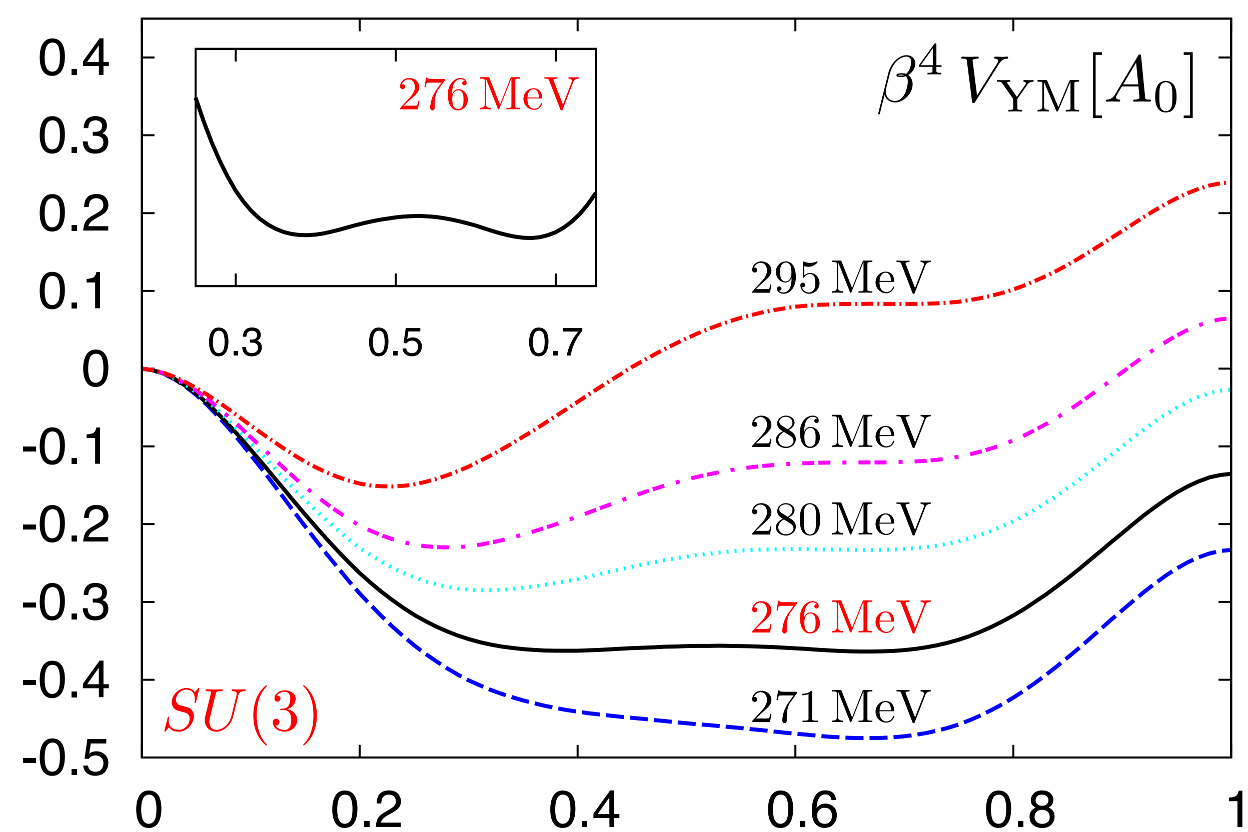
**Braun, Gies, JMP '07**

$$T_c = 276 \pm 10\,\mathrm{MeV}$$

$$T_c/\sqrt{\sigma} = 0.658 \pm 0.023$$

$$\mathrm{lattice}: T_c/\sqrt{\sigma} = 0.646$$

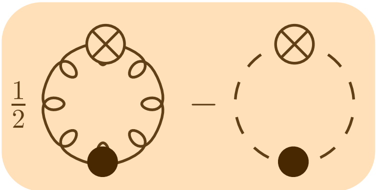



$$\Phi[A_0] = \frac{1}{3}\left(1 + 2\cos\frac{1}{2}\beta g A_0\right)$$

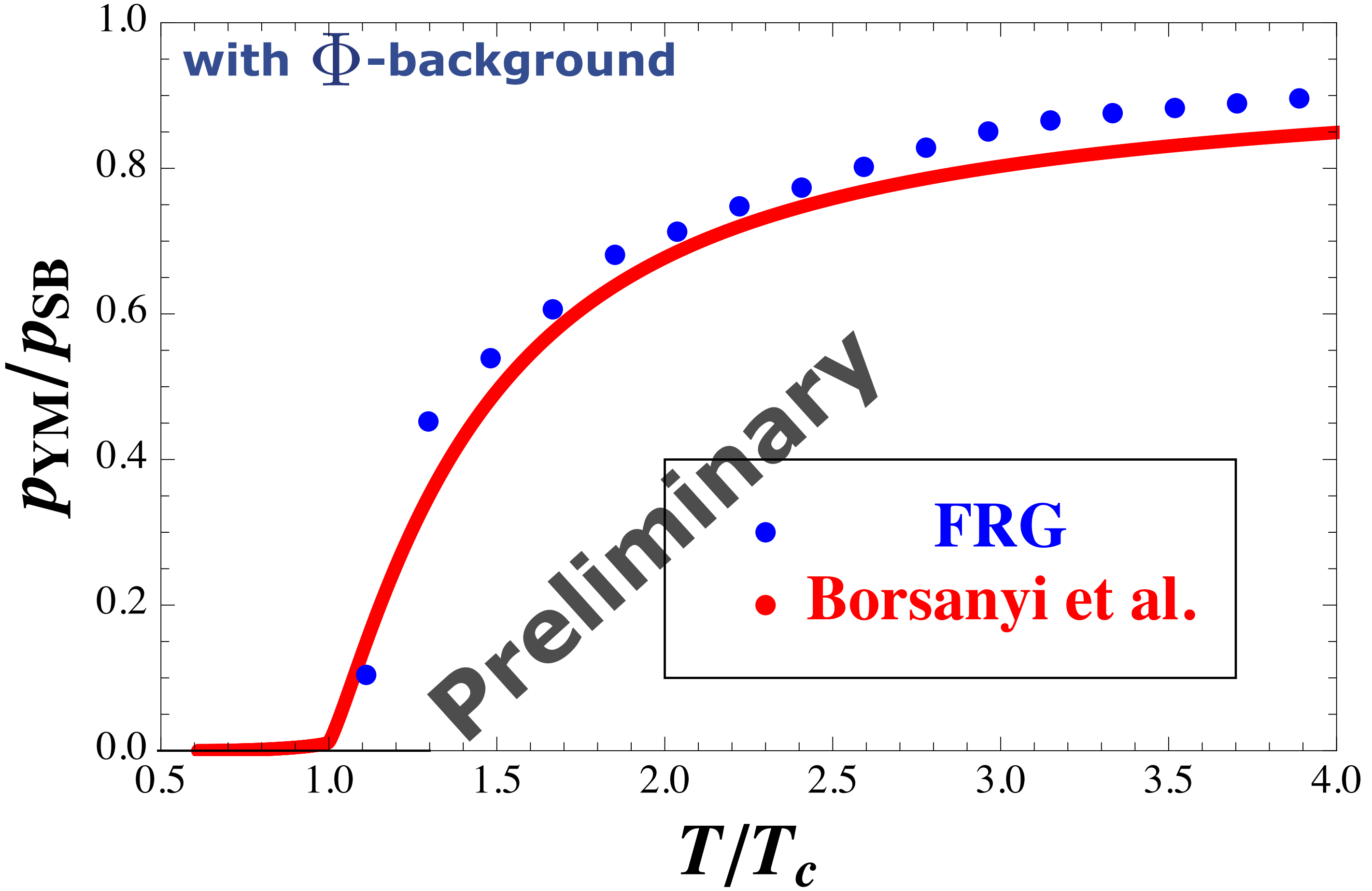
$$\Phi\left[\frac{4}{3}\pi\frac{1}{\beta g}\right] = 0$$

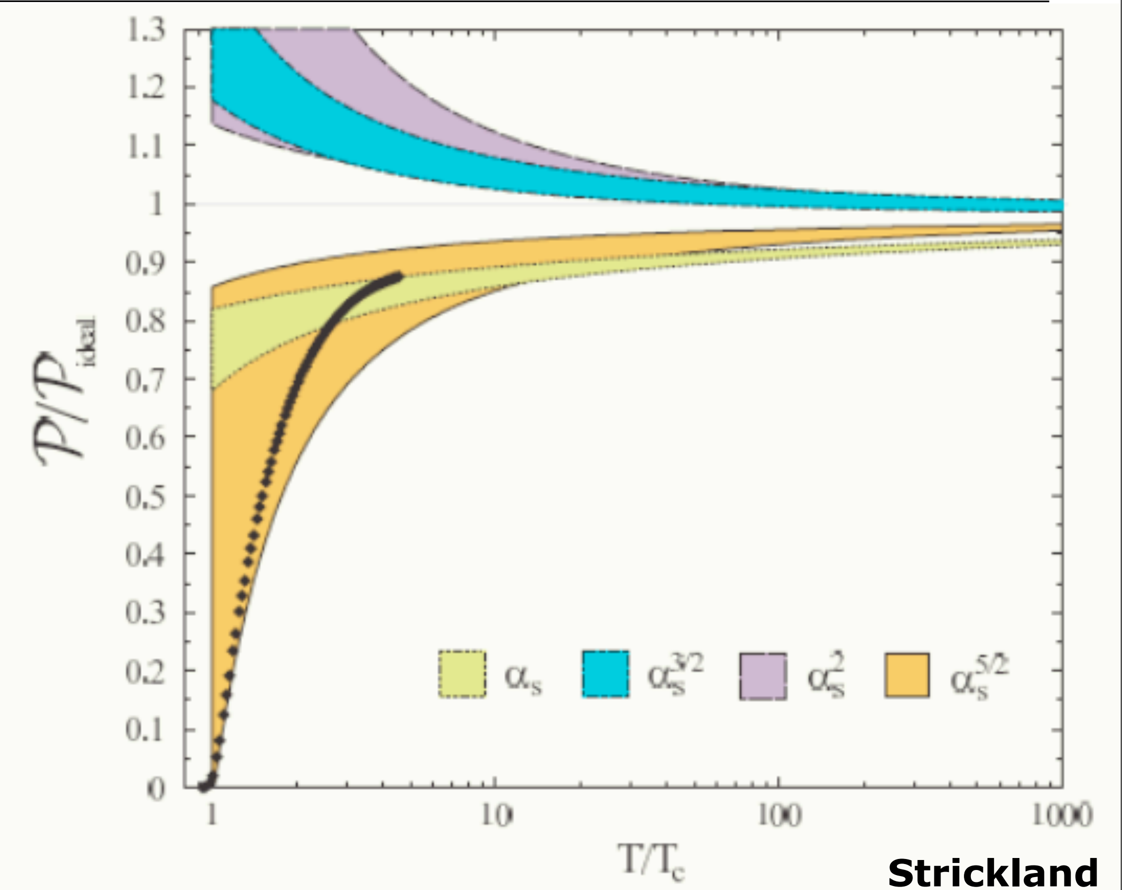
# thermodynamics



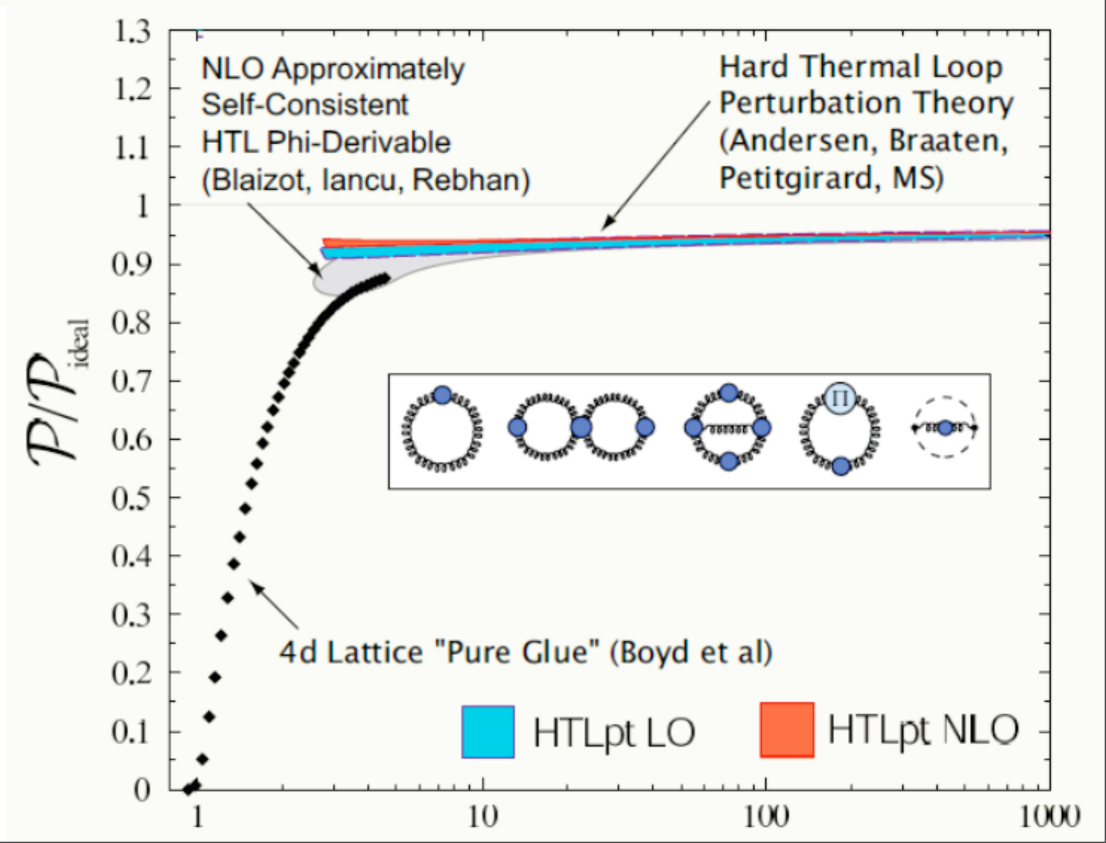
## Yang-Mills pressure

**Fister, JMP**

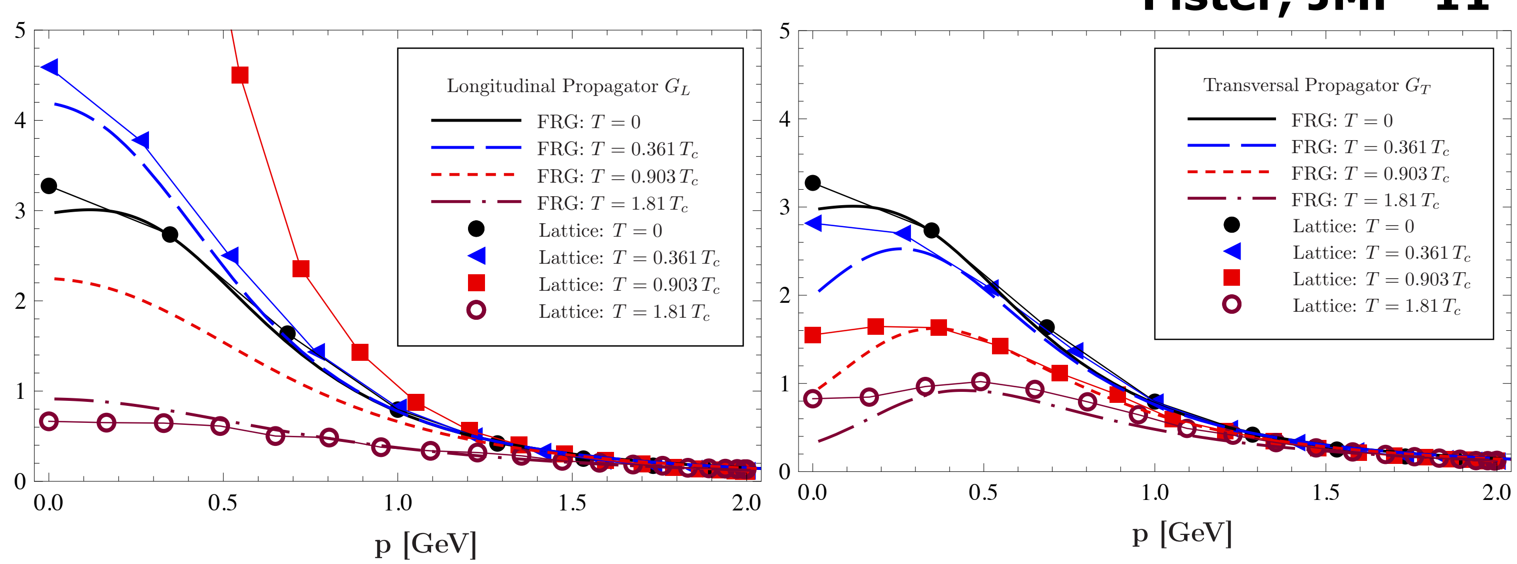


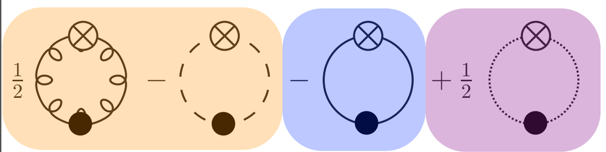

**Strickland**




**Fister, JMP '11**

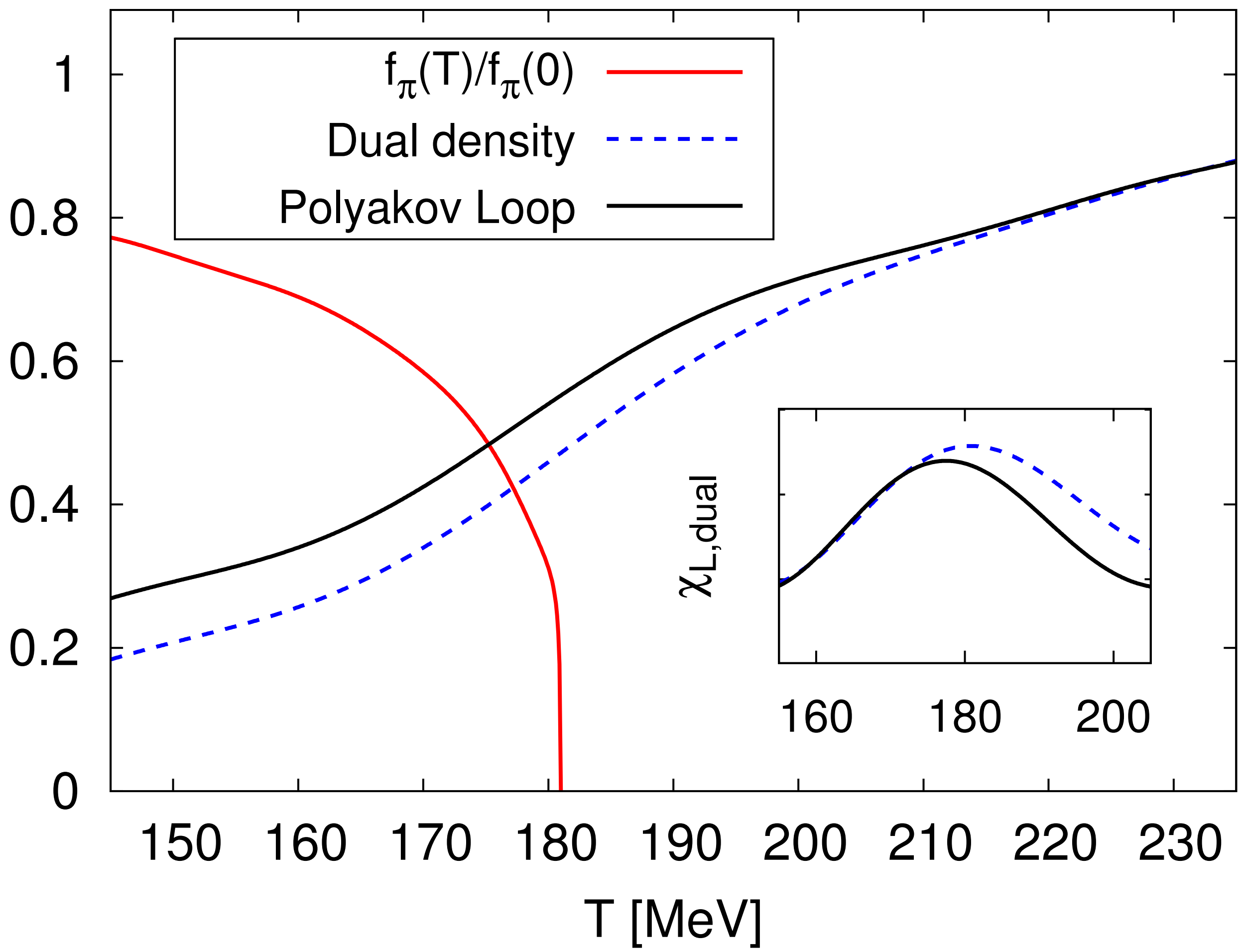


# Full dynamical QCD: $N_f = 2$ & chiral limit



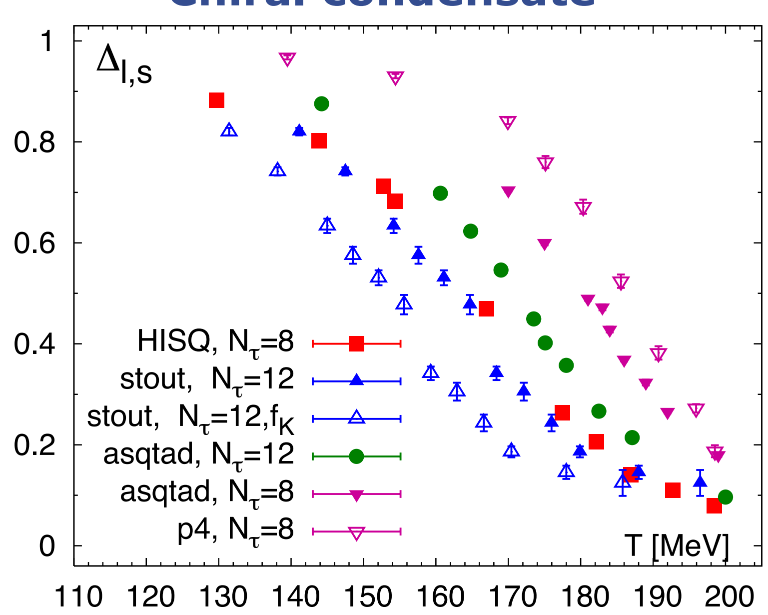
## Phase structure

Braun, Haas, Marhauser, JMP '09

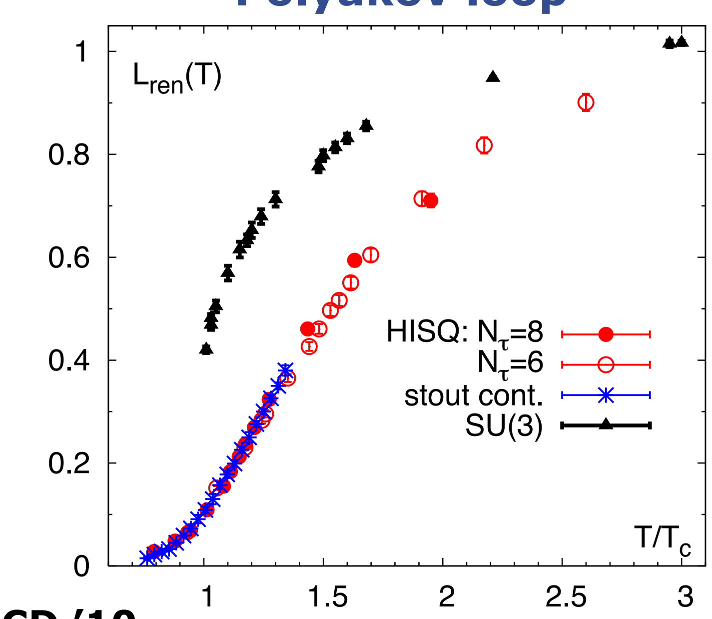



- $T_\chi \simeq T_{\rm conf} \simeq 180{\rm MeV}$
- **Width** $\Delta T_{\rm conf} \simeq \pm 20{\rm MeV}$
- $T_{\rm conf,FRG} \lesssim T_{\rm conf,lattice}$

**Chiral condensate**

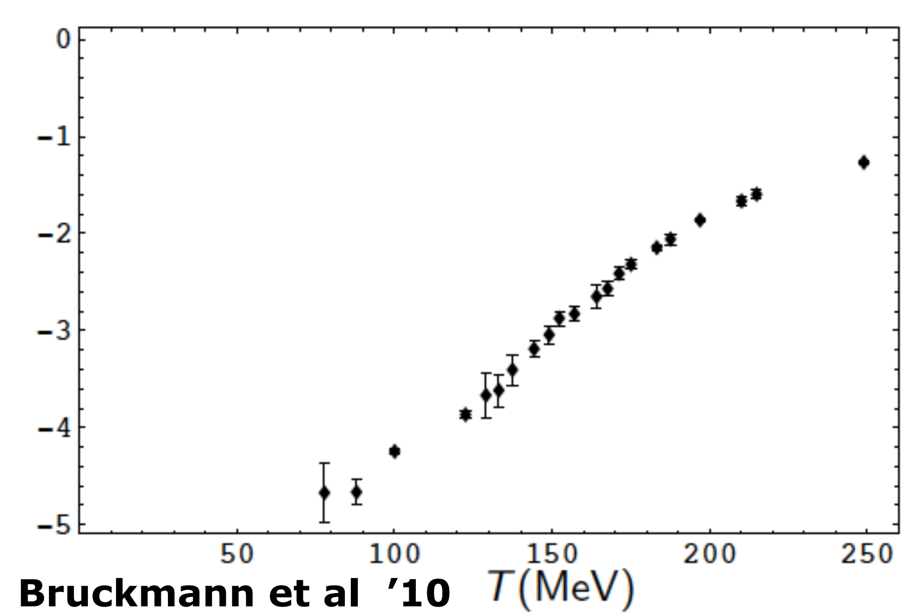



**Polyakov loop**



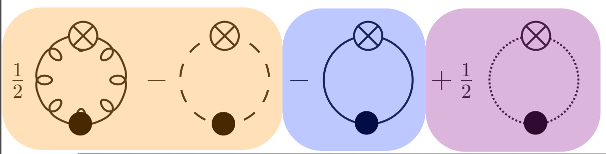

hotQCD '10

**Log of dual condensate, m=60 MeV**

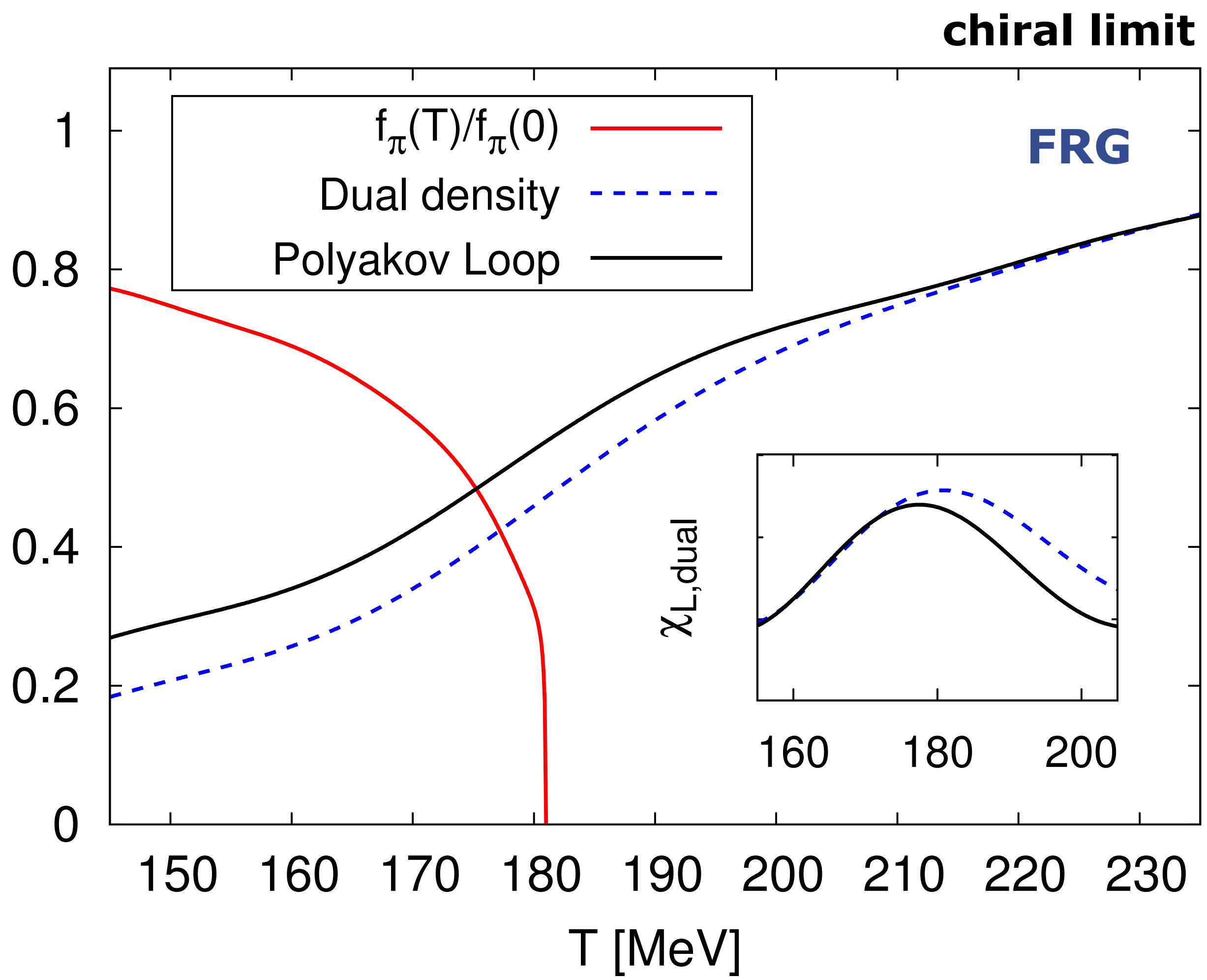



Bruckmann et al '10

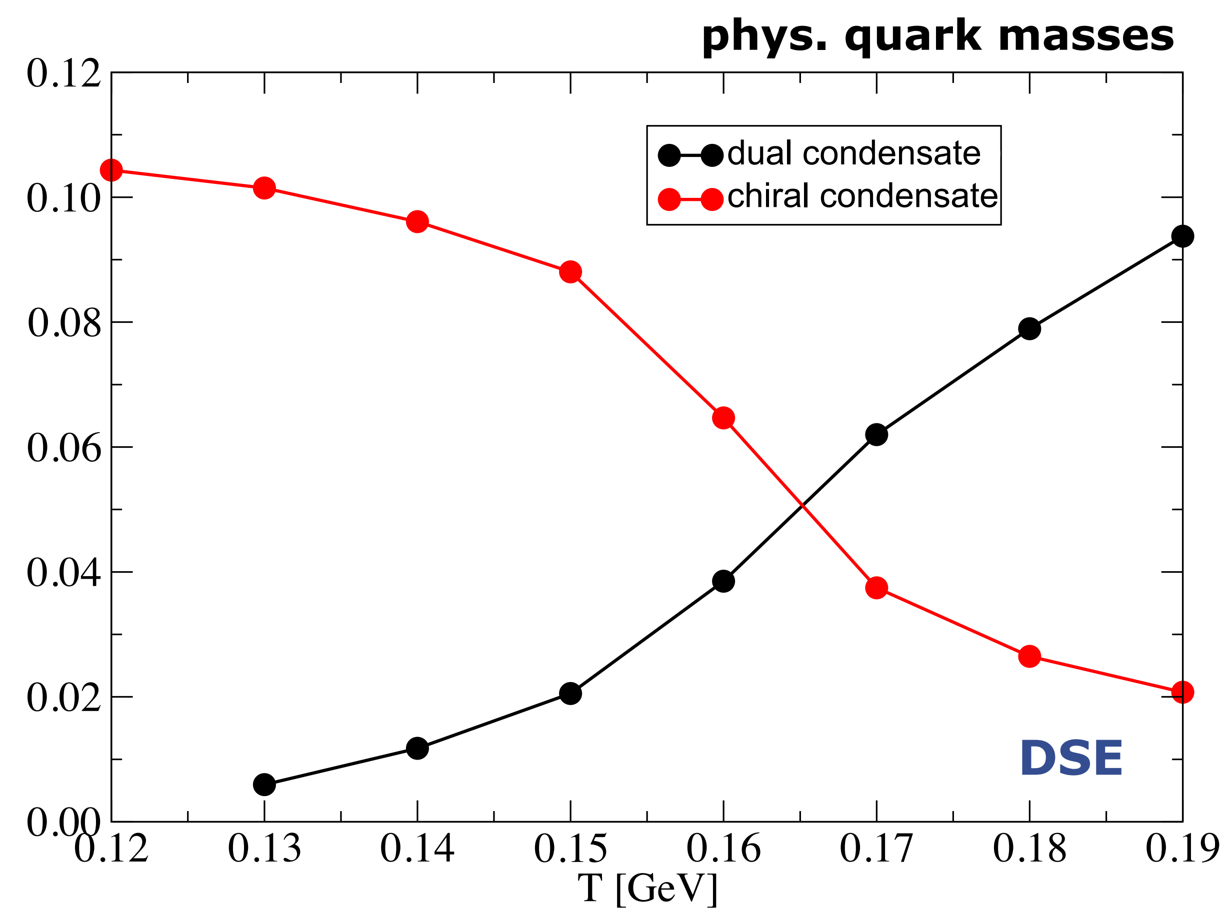
# Full dynamical QCD: $N_f = 2$ & chiral limit



## Phase structure



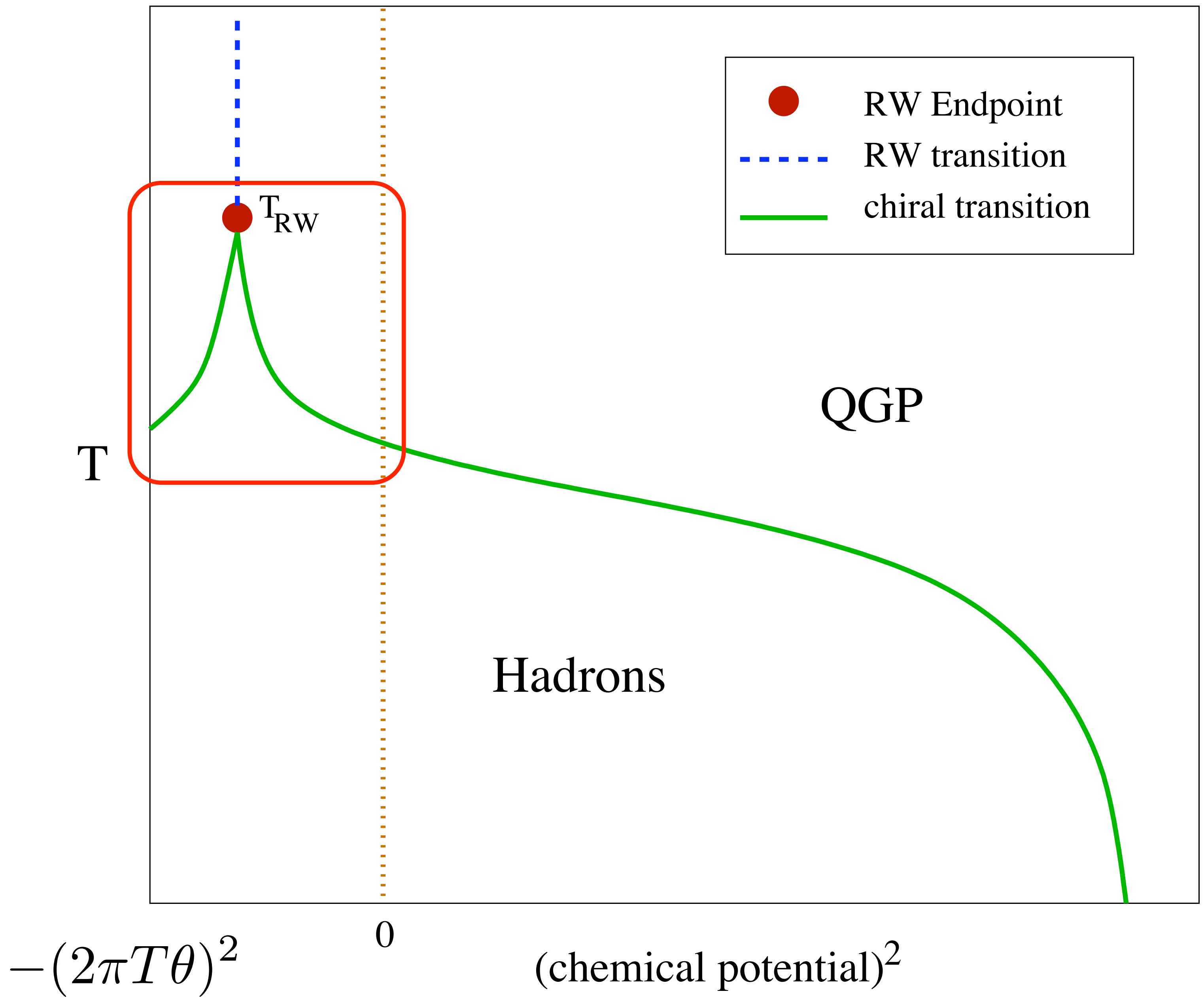

**Braun, Haas, Marhauser, JMP '09**




**Fischer, Lücker, Mueller '11**

# Imaginary chemical potential

$$\psi_\theta(t+\beta,\vec{x}) = -e^{2\pi i\theta}\psi_\theta(t,x) \qquad \textbf{with} \qquad \mu = \mathbf{2\pi i\,T\,\theta}$$




**Roberge-Weiss symmetry:** $\theta \to \theta + 1/3$

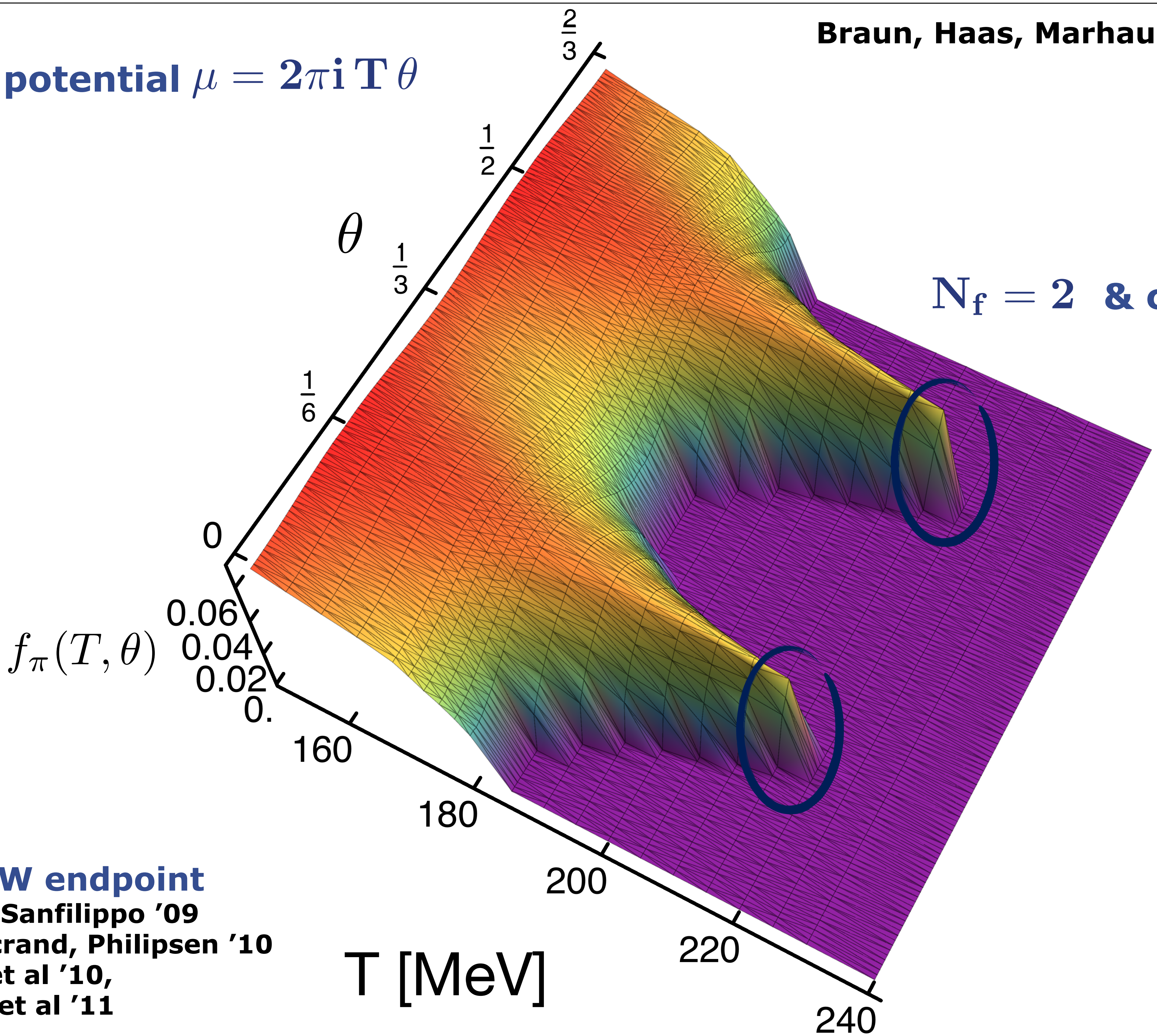
# Imaginary chemical potential

## Chiral phase structure

Braun, Haas, Marhauser, JMP '09

**chemical potential** $\mu = 2\pi \mathrm{i}\, T\, \theta$

$\mathrm{N_f} = 2$ **& chiral limit**



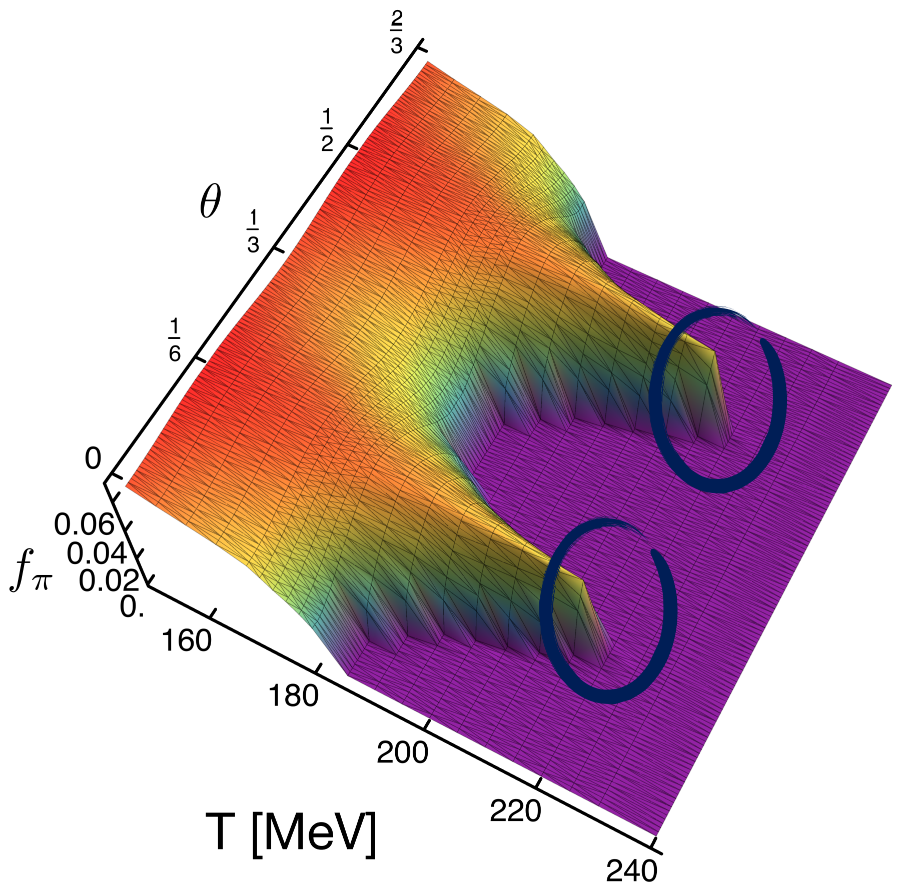

**Nature of RW endpoint**

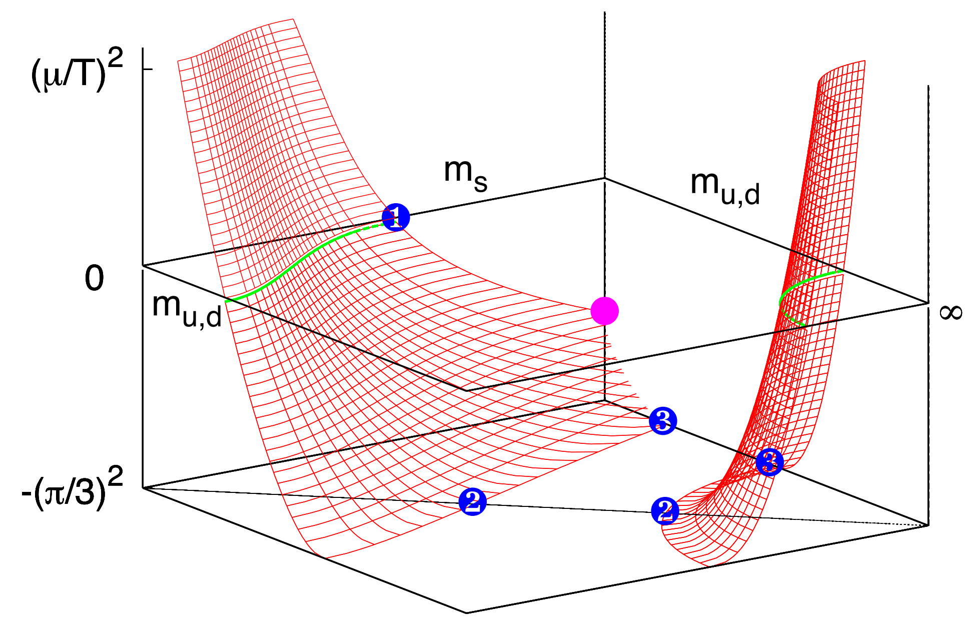
**lattice: D'Elia, Sanfilippo '09**
**de Forcrand, Philipsen '10**
**PNJL: Sakai et al '10,**
**Morita et al '11**

# Imaginary chemical potential

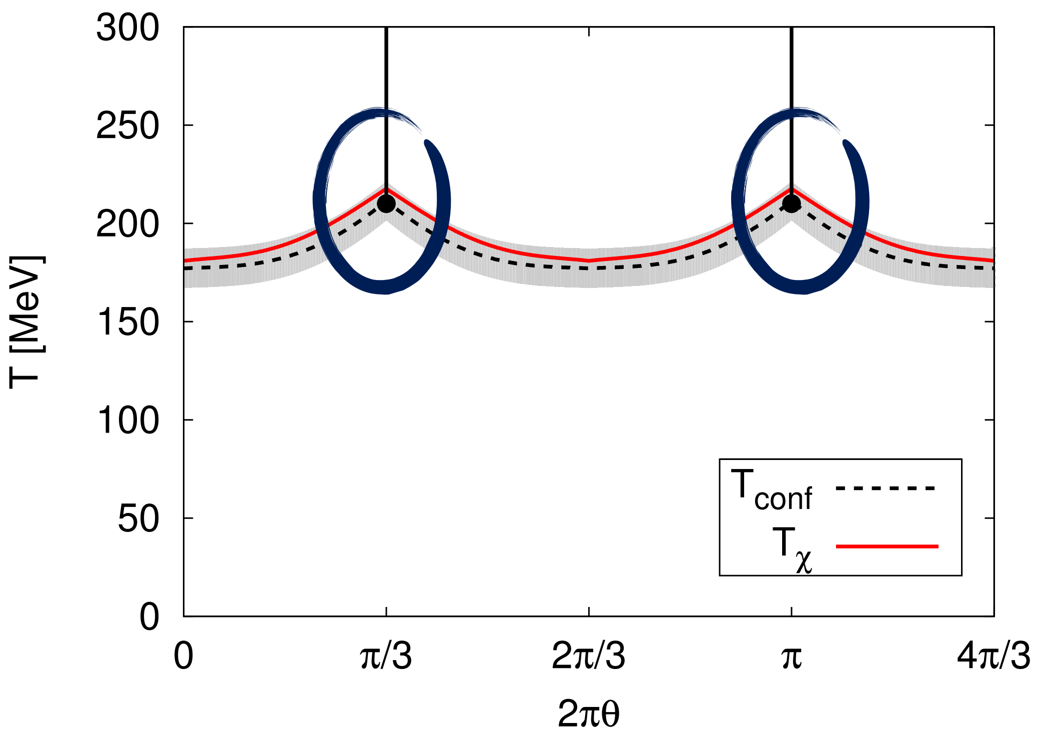
## Nature of the RW endpoint

**chemical potential $\mu = 2\pi i\, T\, \theta$**

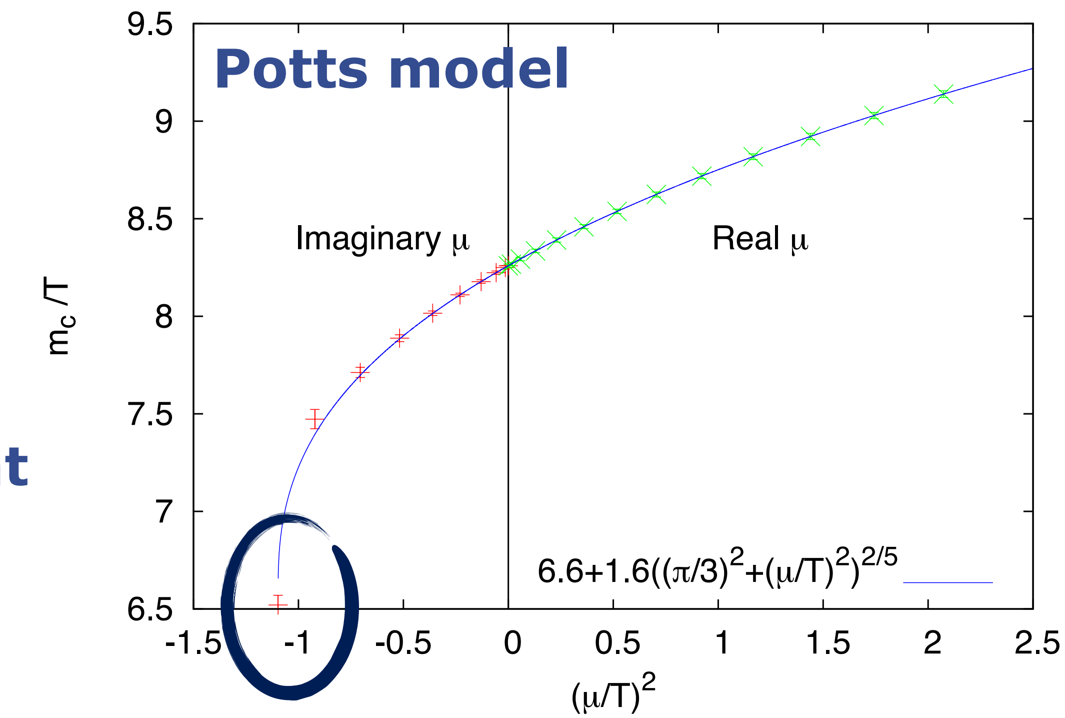



O. Philipsen '11




**RW endpoint**

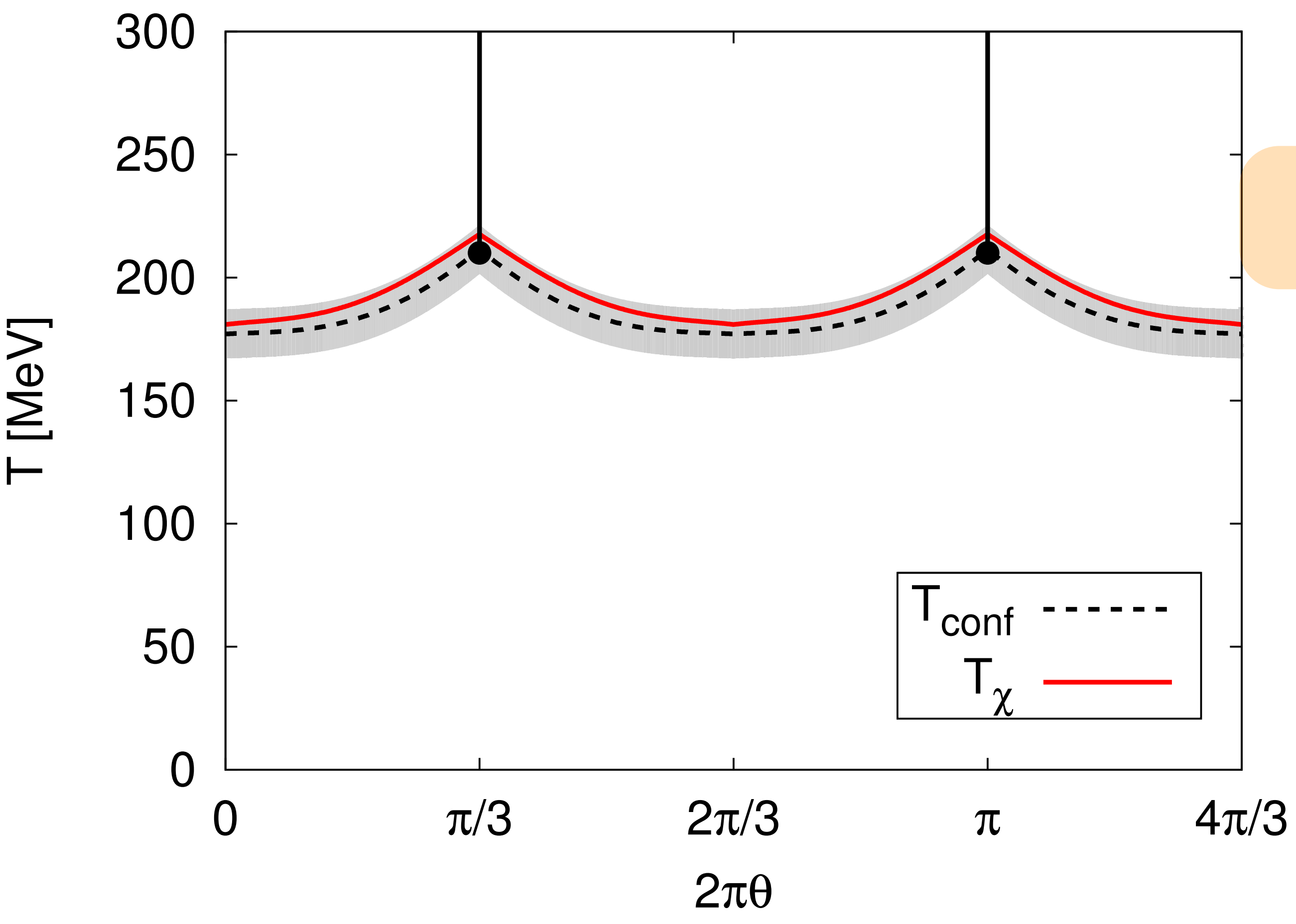


# Imaginary chemical potential

## Phase structure

**chemical potential** $\mu = 2\pi \mathrm{i}\, T\, \theta$

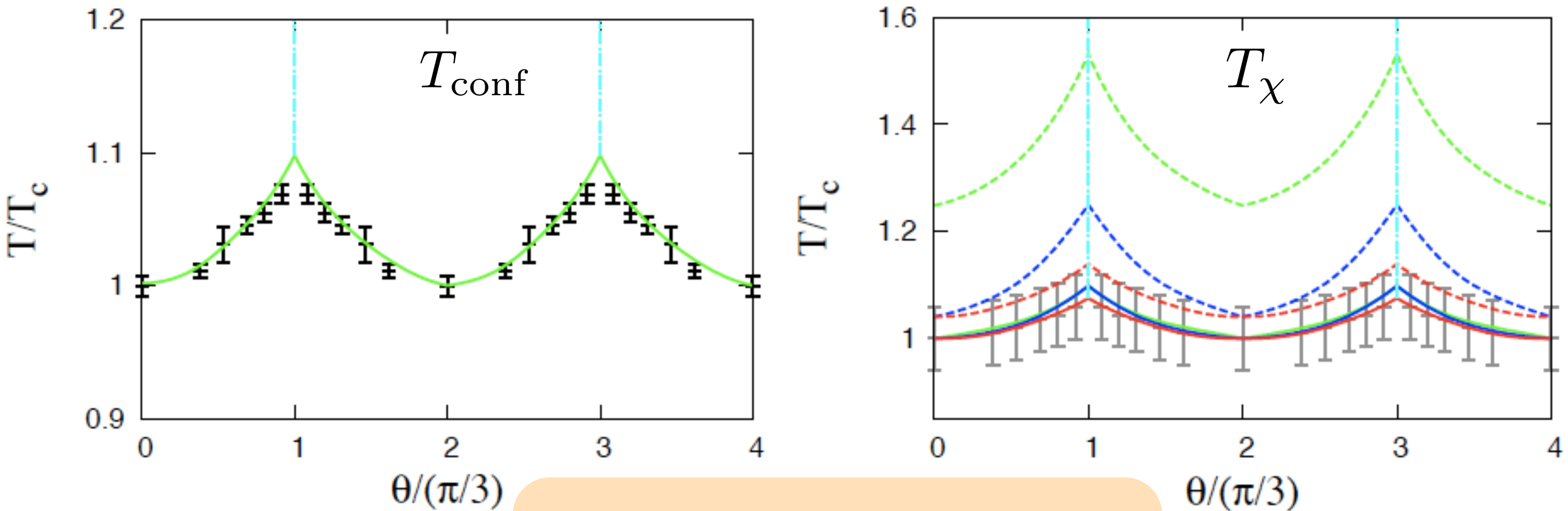
**Braun, Haas, Marhauser, JMP '09**




**compatibility**

**lattice results, e.g.**
**Kratochvila et al '06,**
**Wu et al '06,**
**D'Elia et al '07, ....**

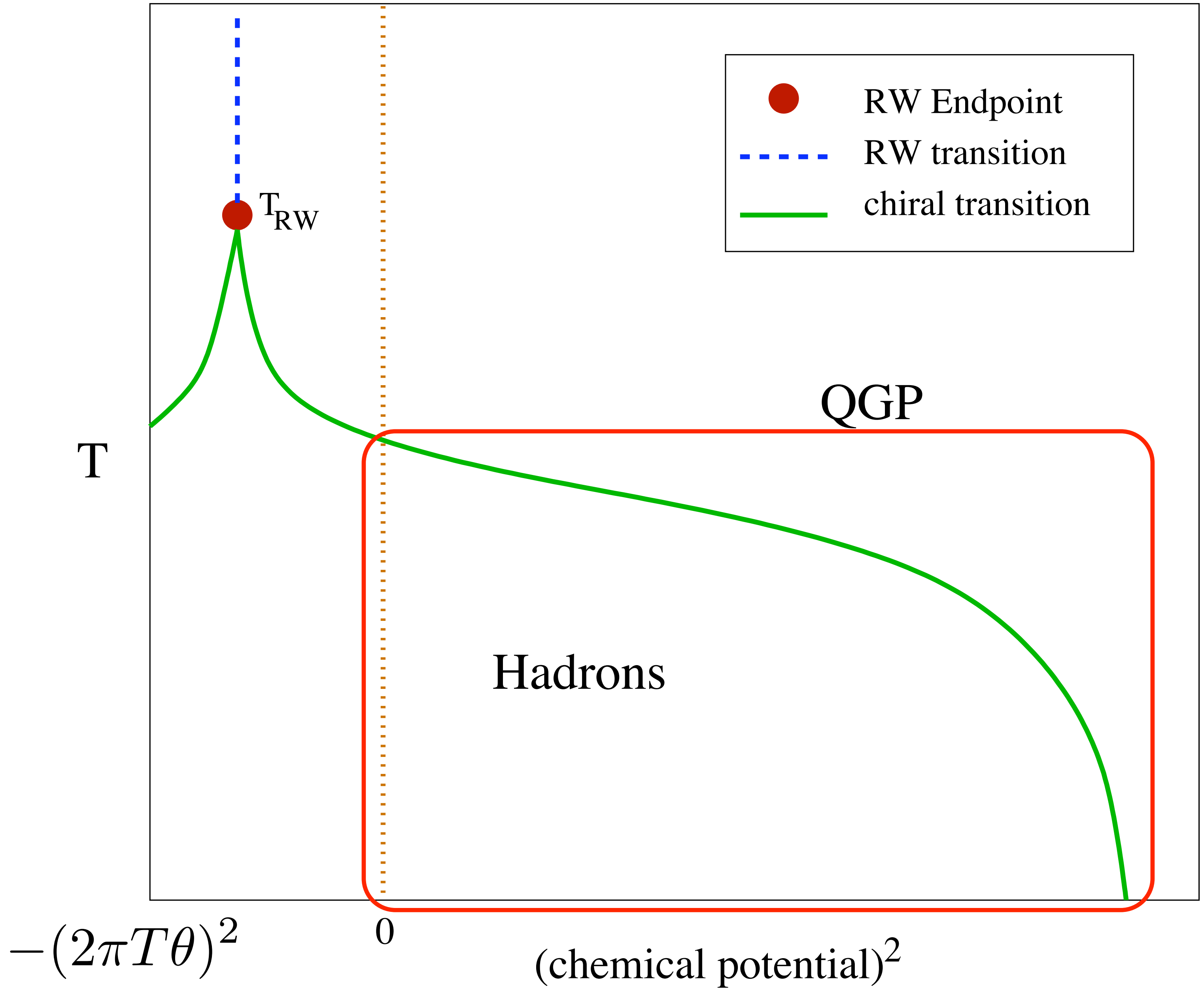
**Polyakov-NJL model**
**Sakai et al '09, ...**



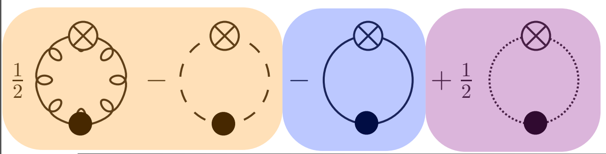

**adjust 8-fermi interaction**

# Real chemical potential

$$\psi_\theta(t+\beta,\vec{x}) = -\psi(t,x)$$

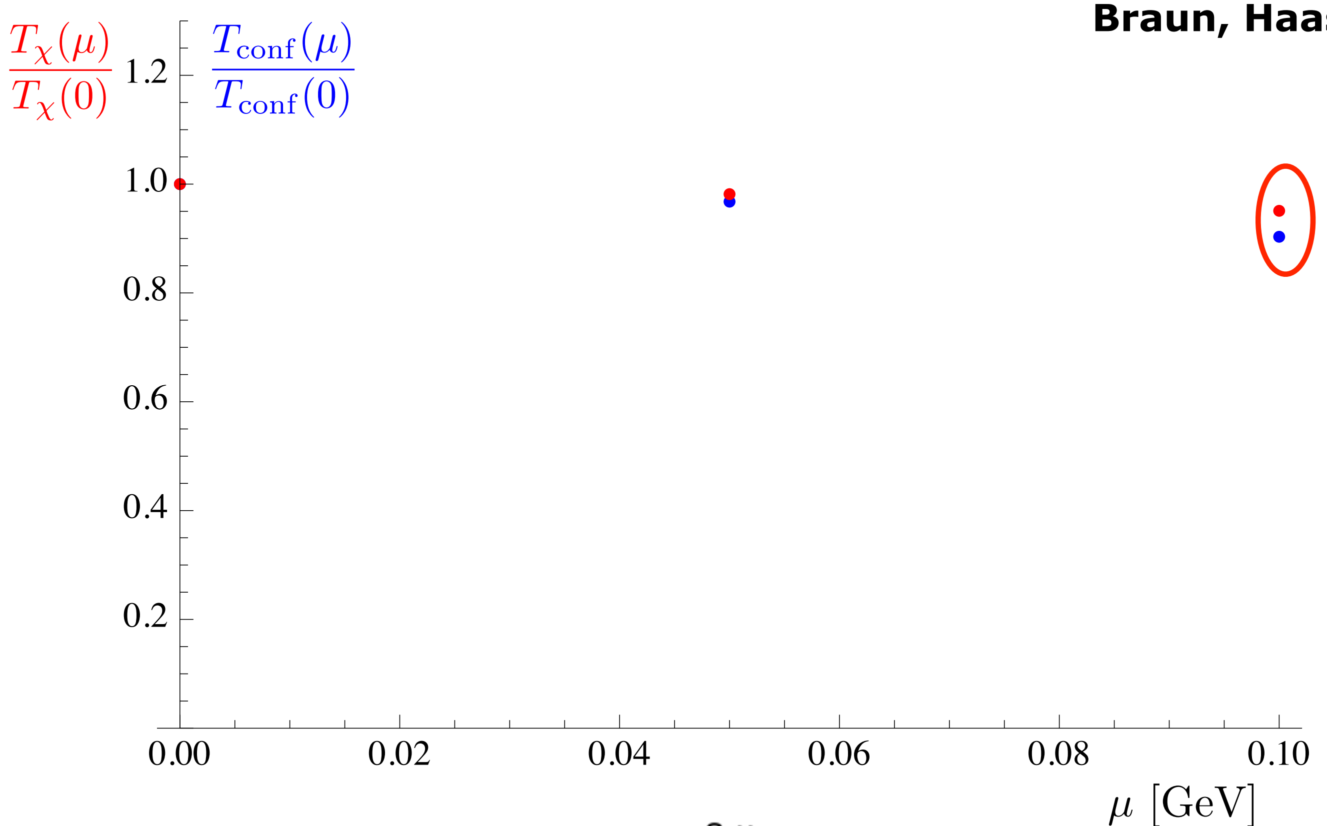


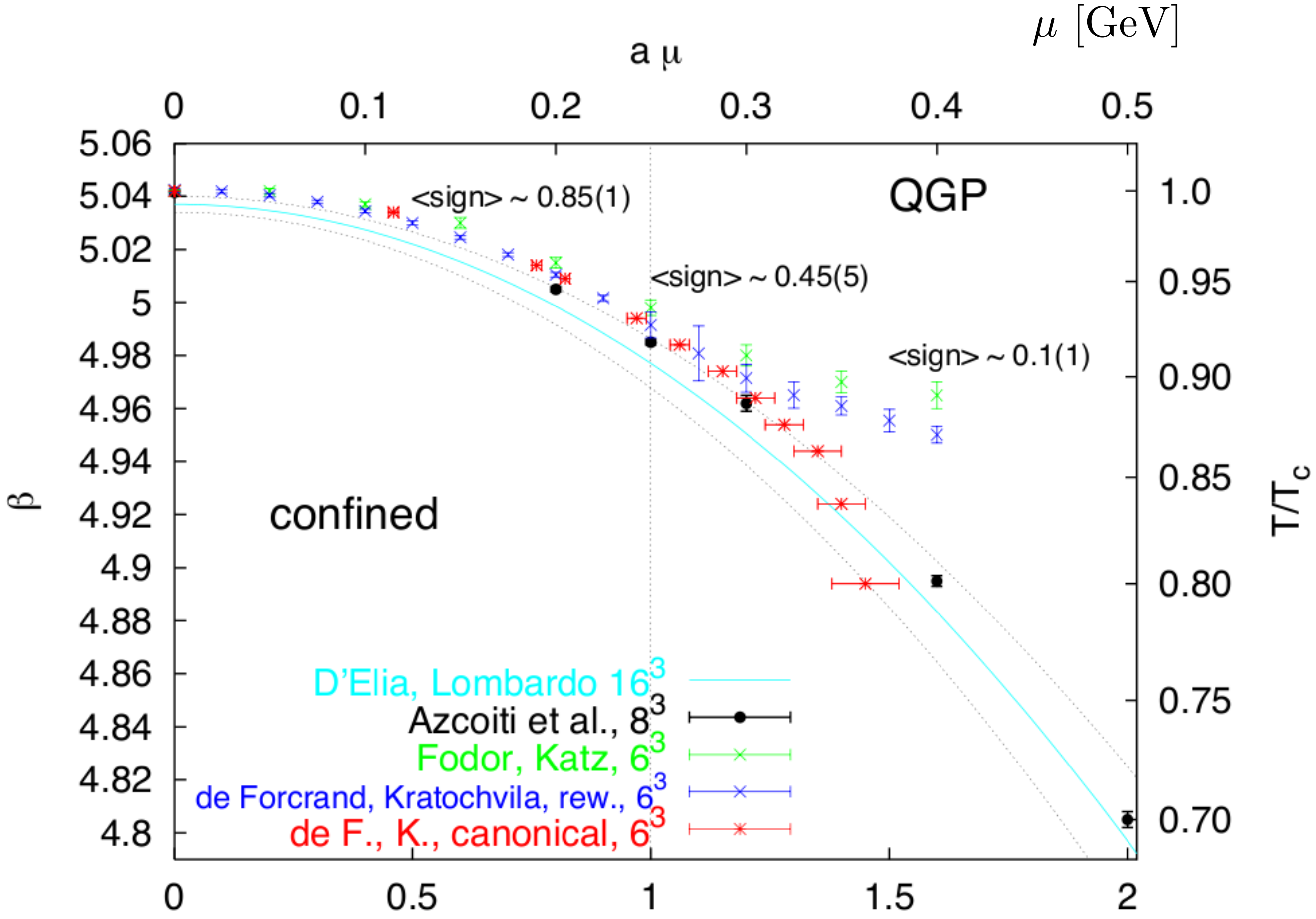
# Real chemical potential

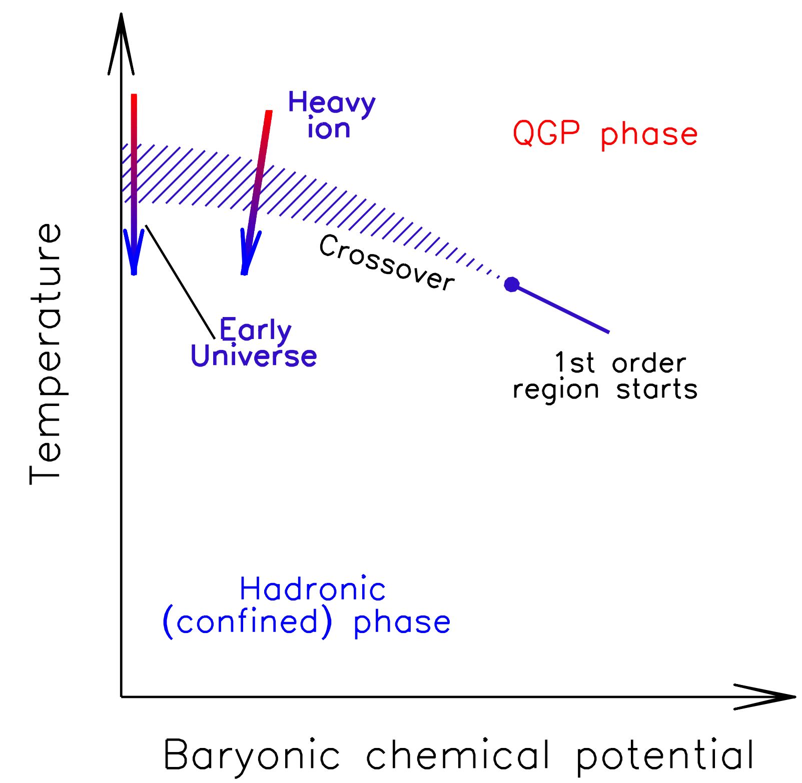


## Full dynamical QCD

**Braun, Haas, Fister, JMP**

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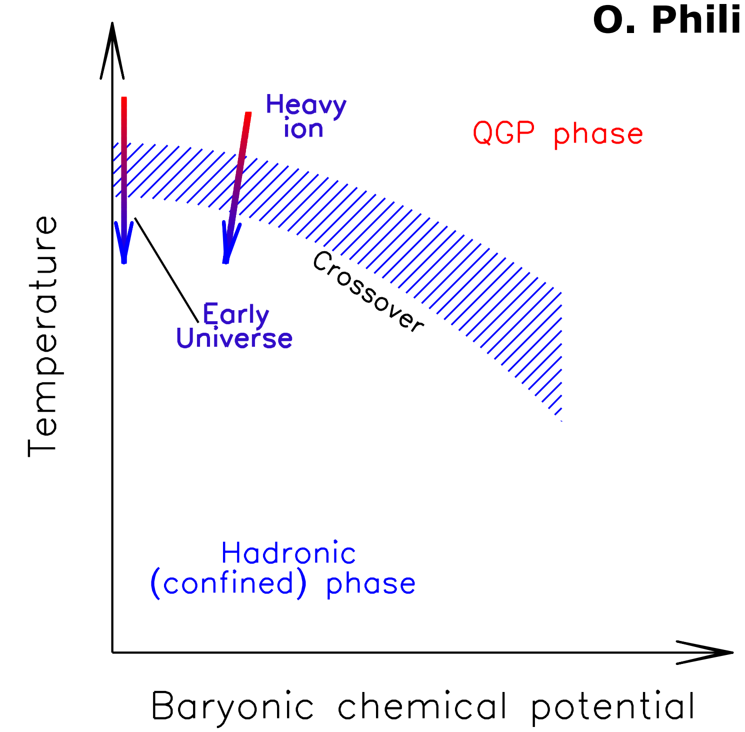
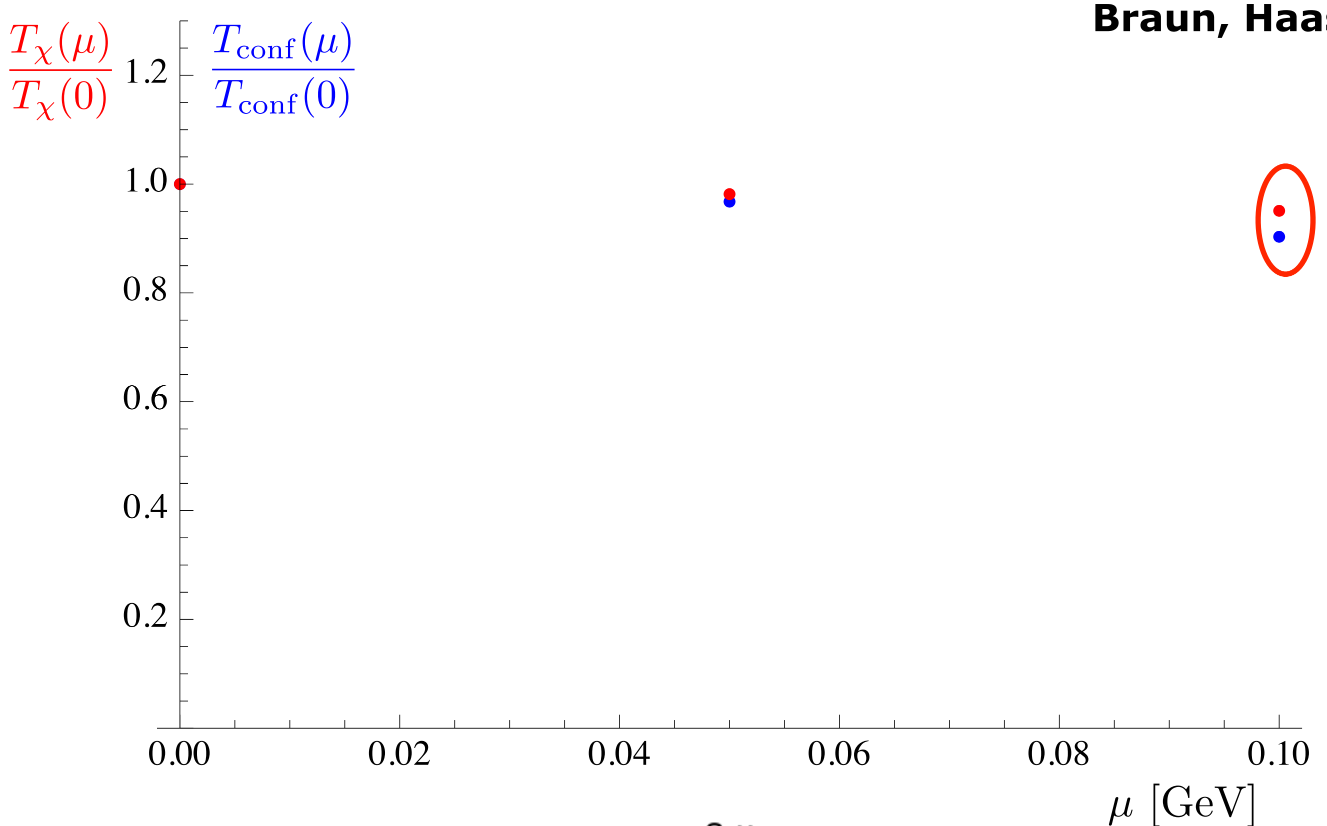


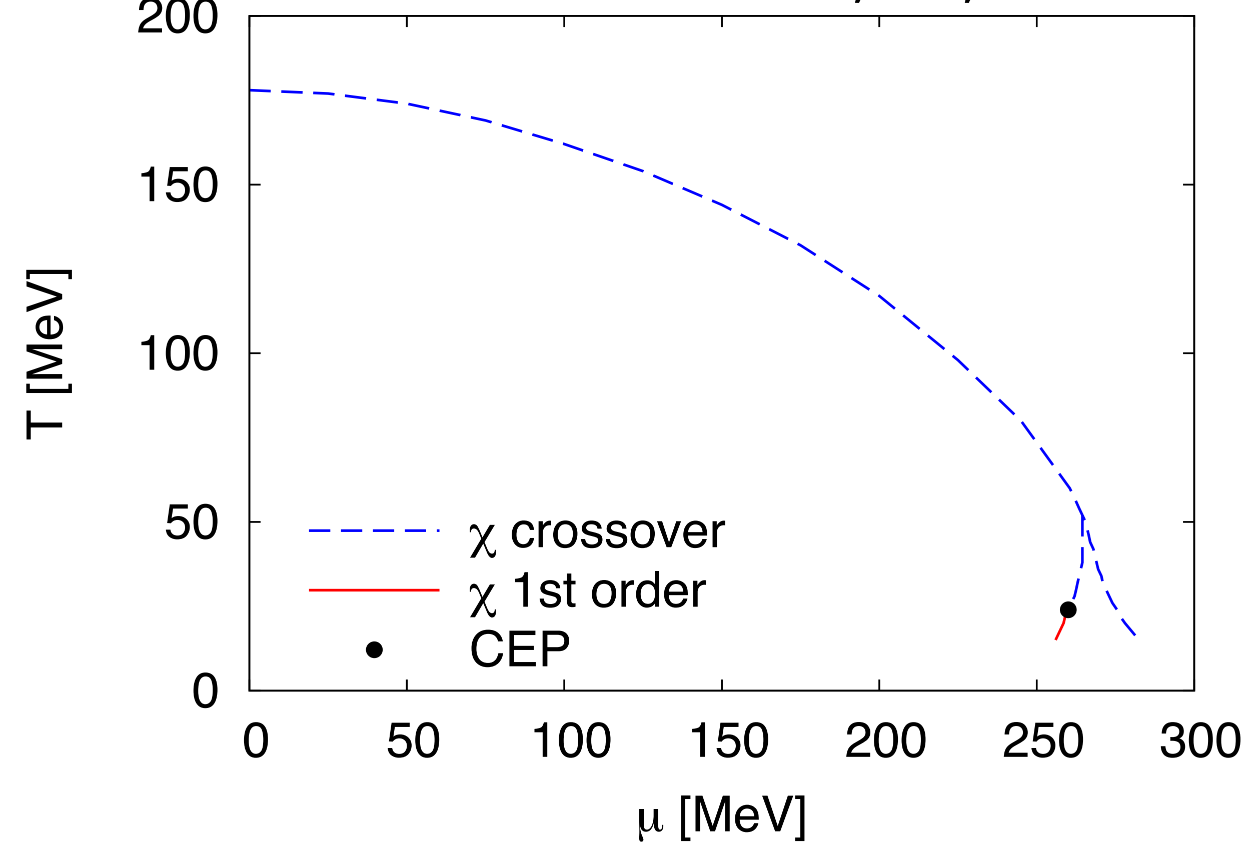
# Real chemical potential

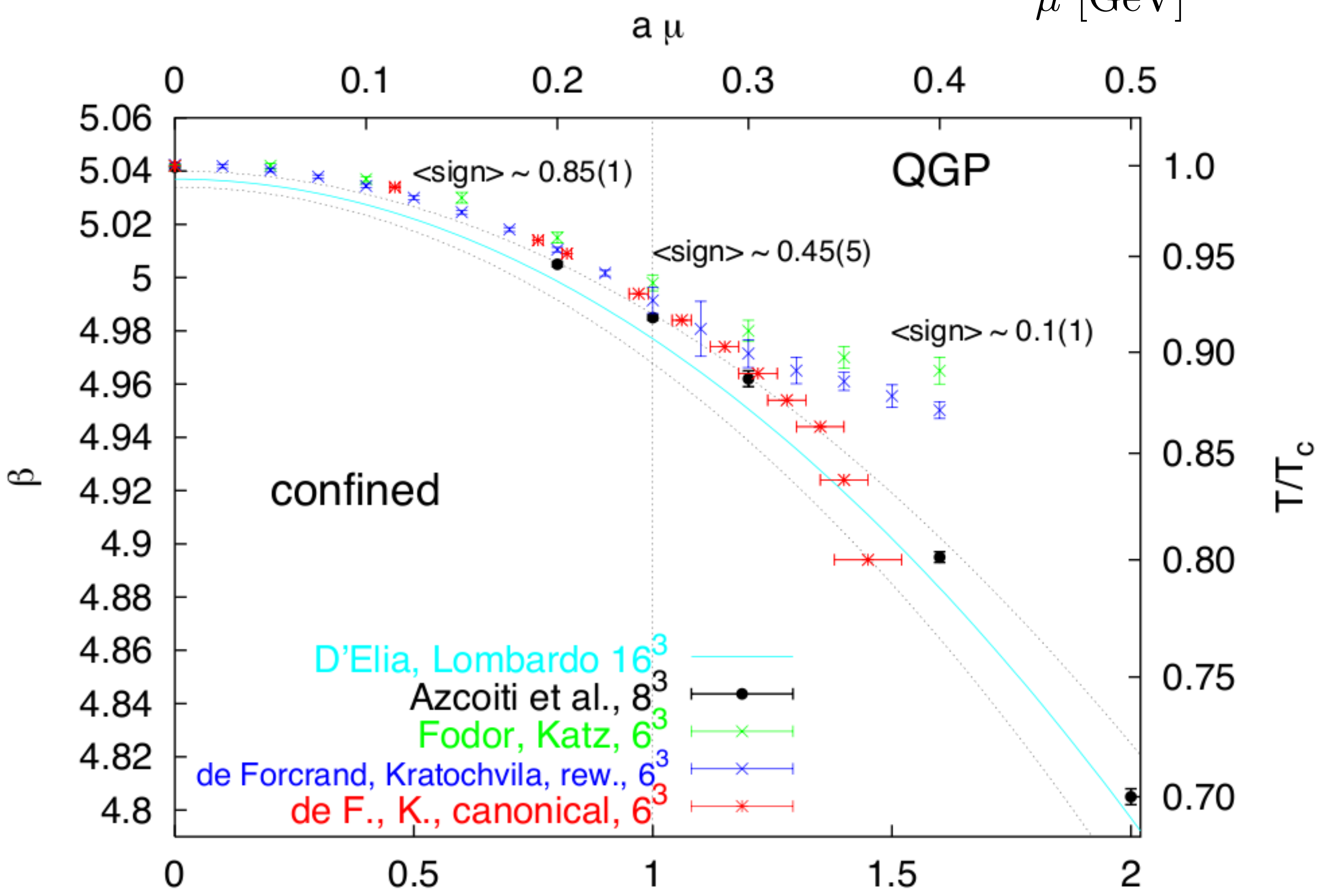
## Full dynamical QCD

Braun, Haas, Fister, JMP

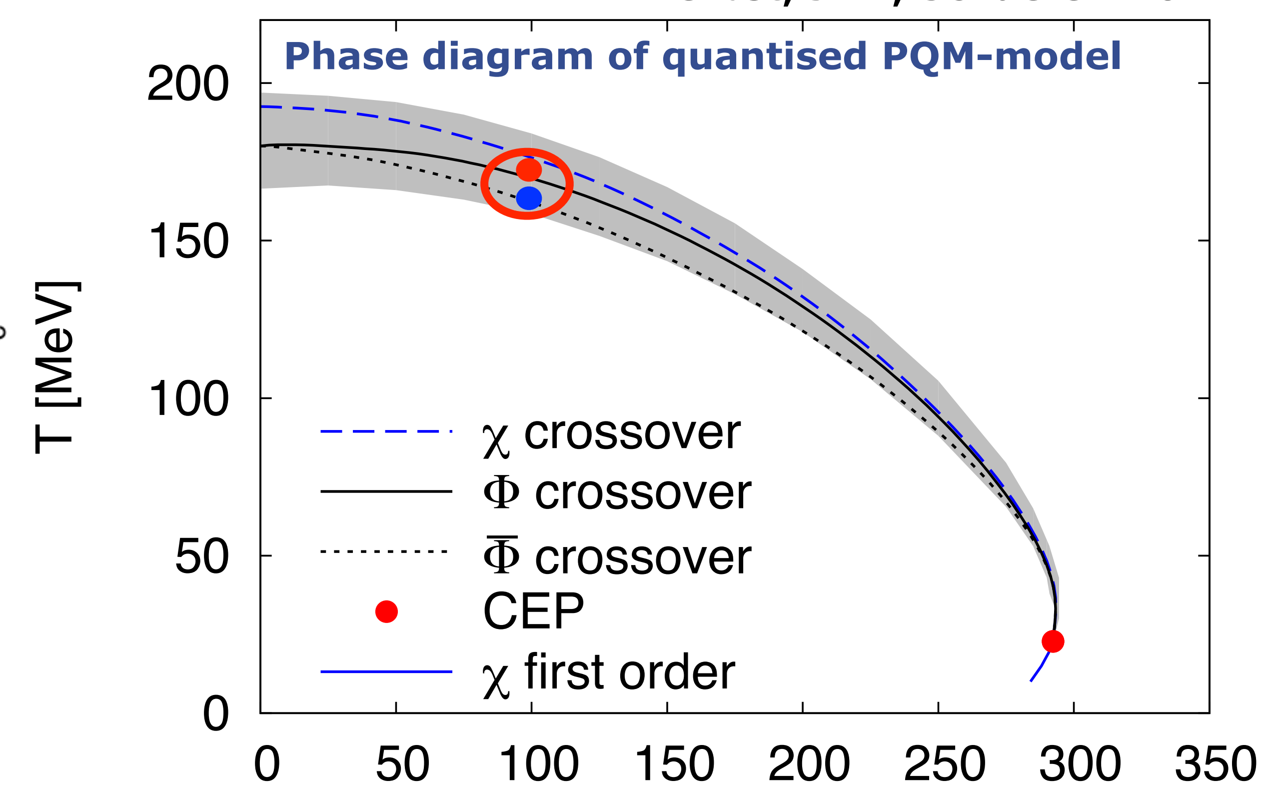



Herbst, JMP, Schaefer



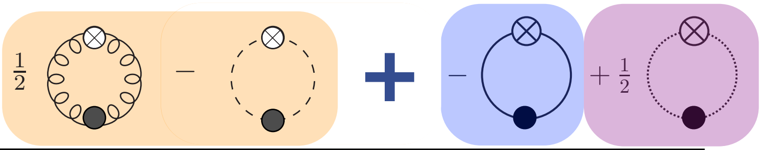




Herbst, JMP, Schaefer '10

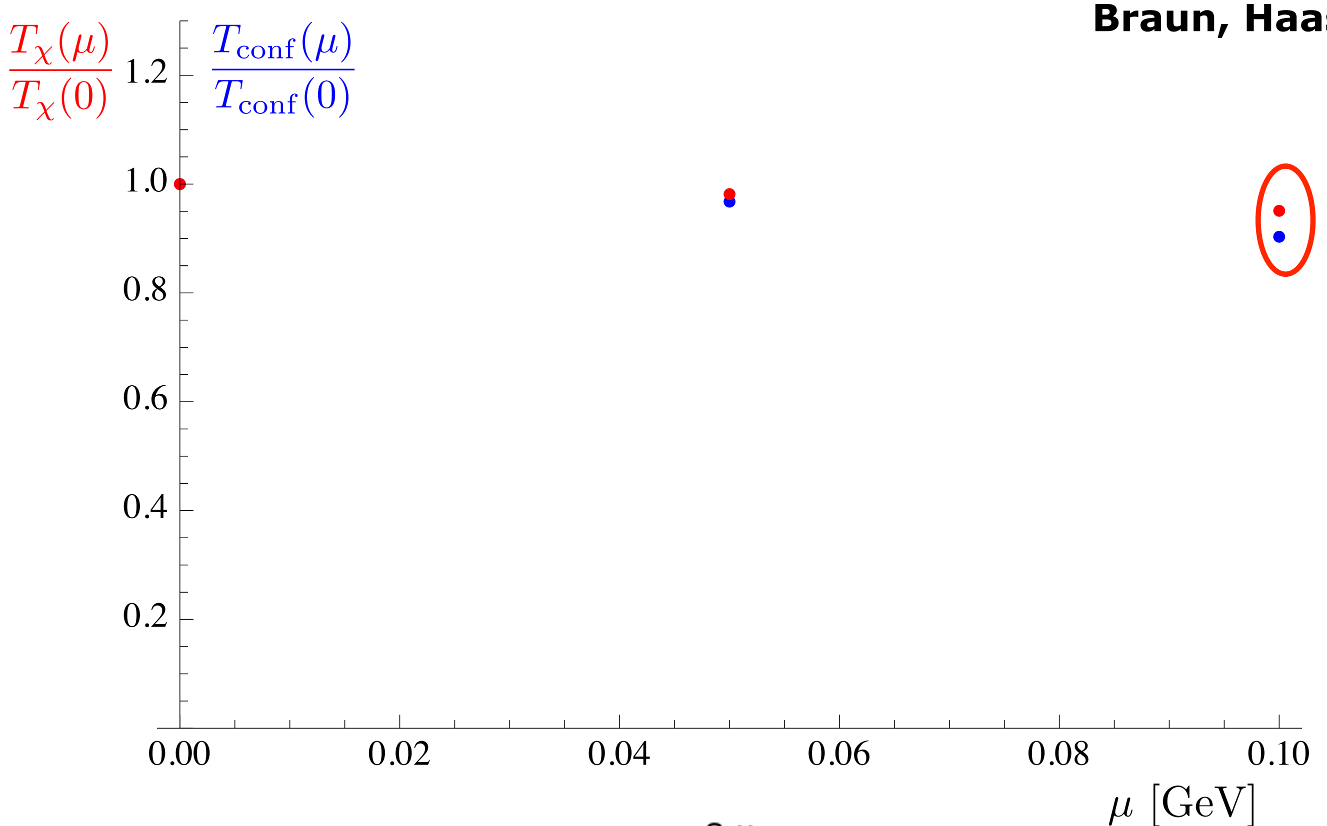


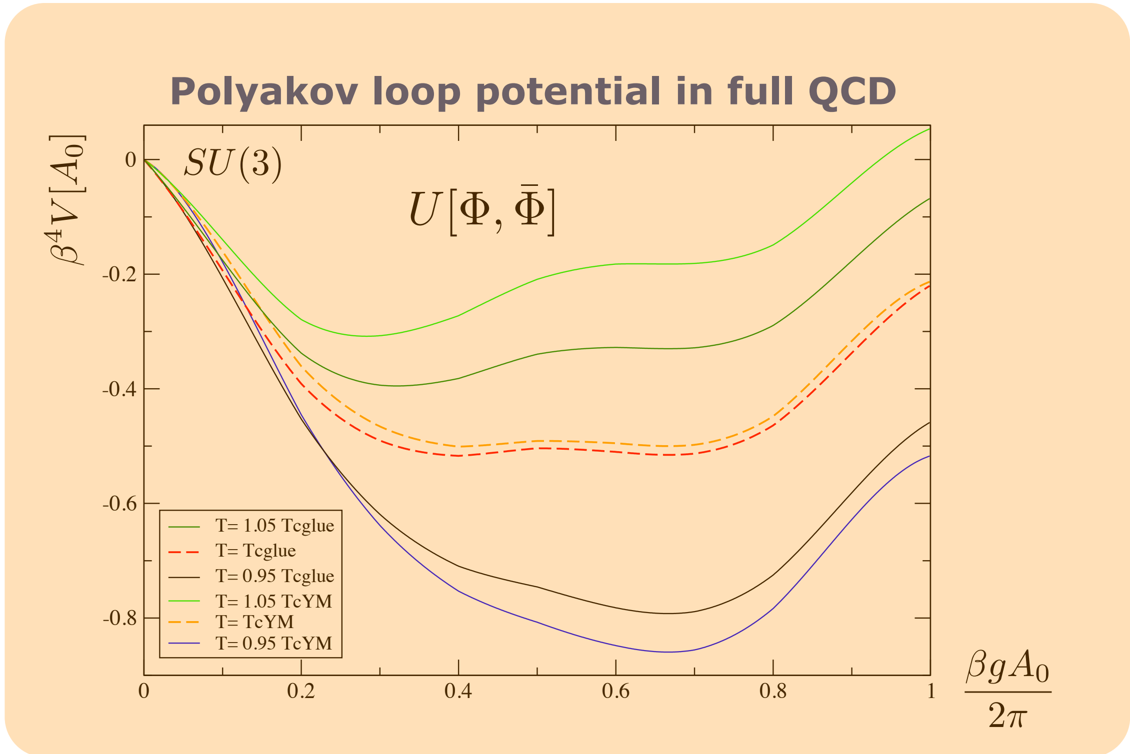
# Real chemical potential

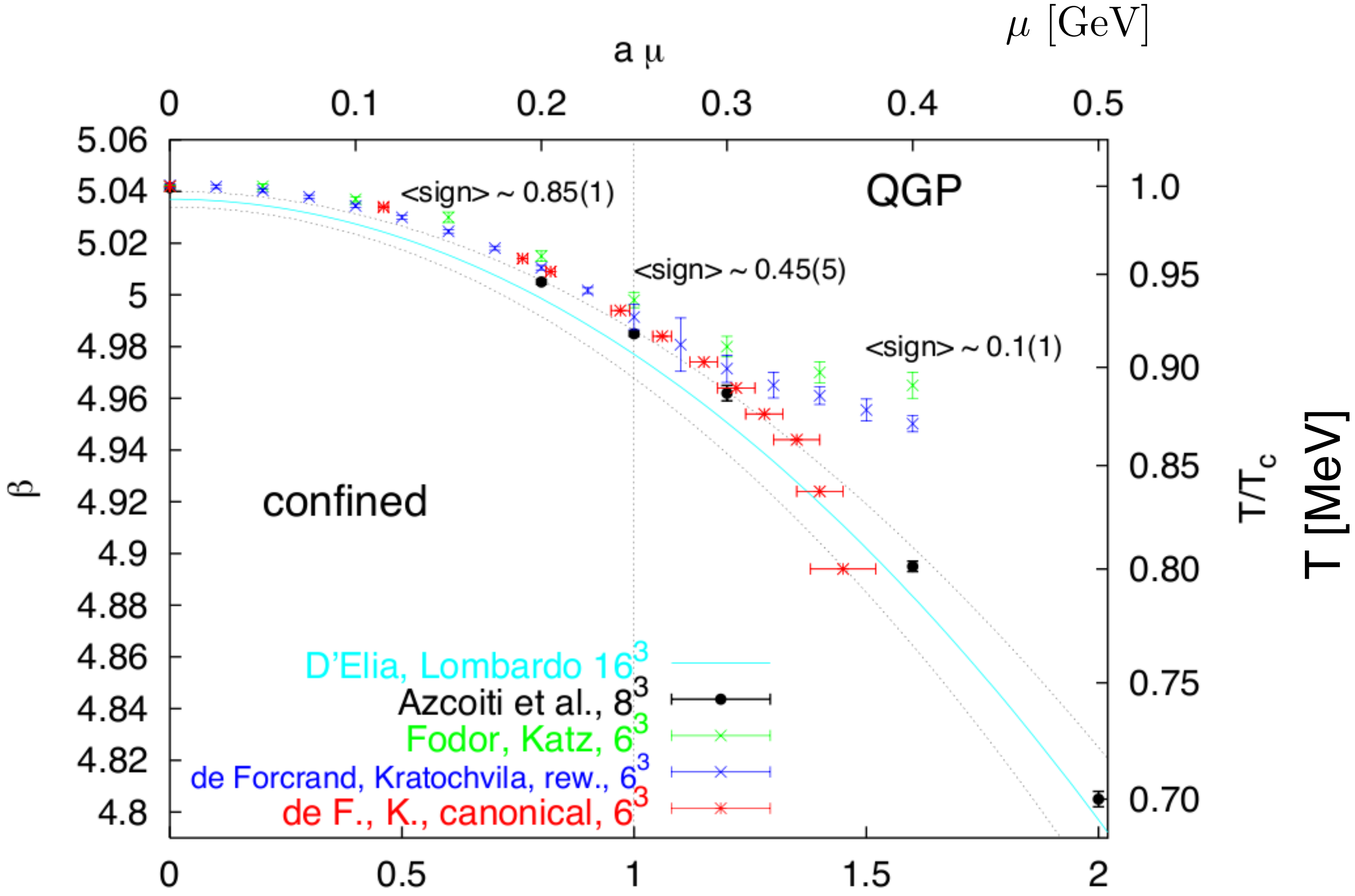
## Full dynamical QCD



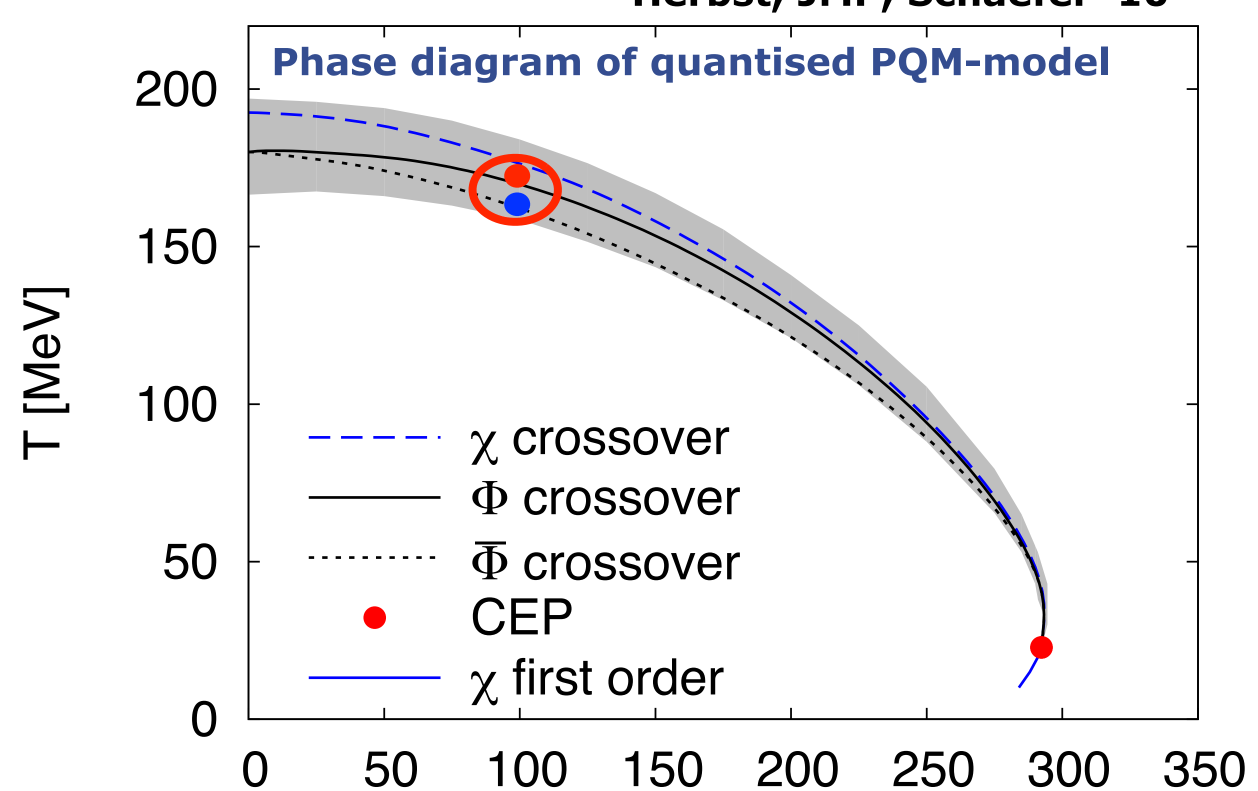

**Braun, Haas, Fister, JMP**



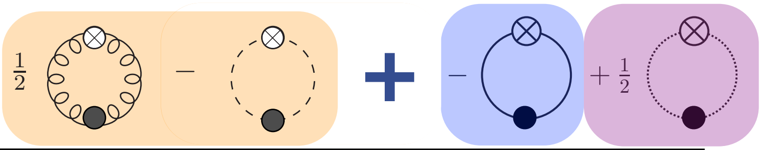




**Herbst, JMP, Schaefer '10**



# Real chemical potential

## Full dynamical QCD

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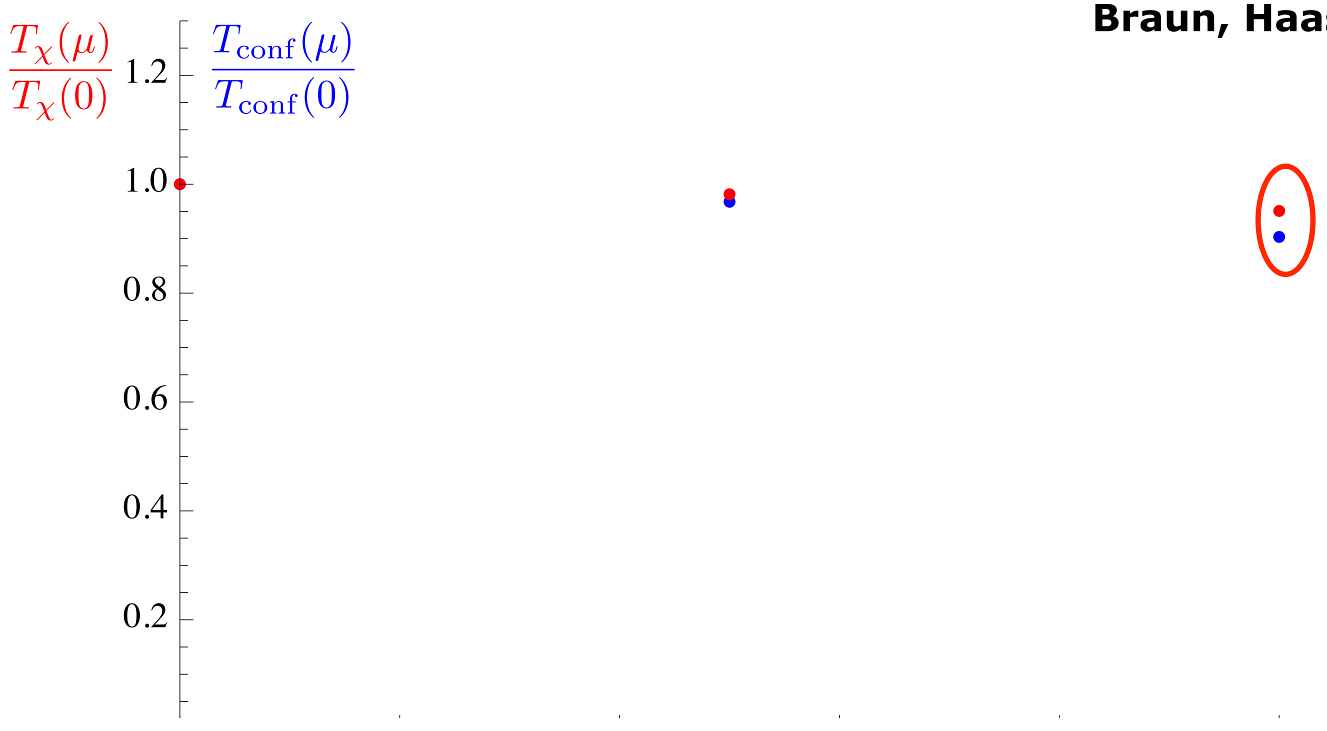
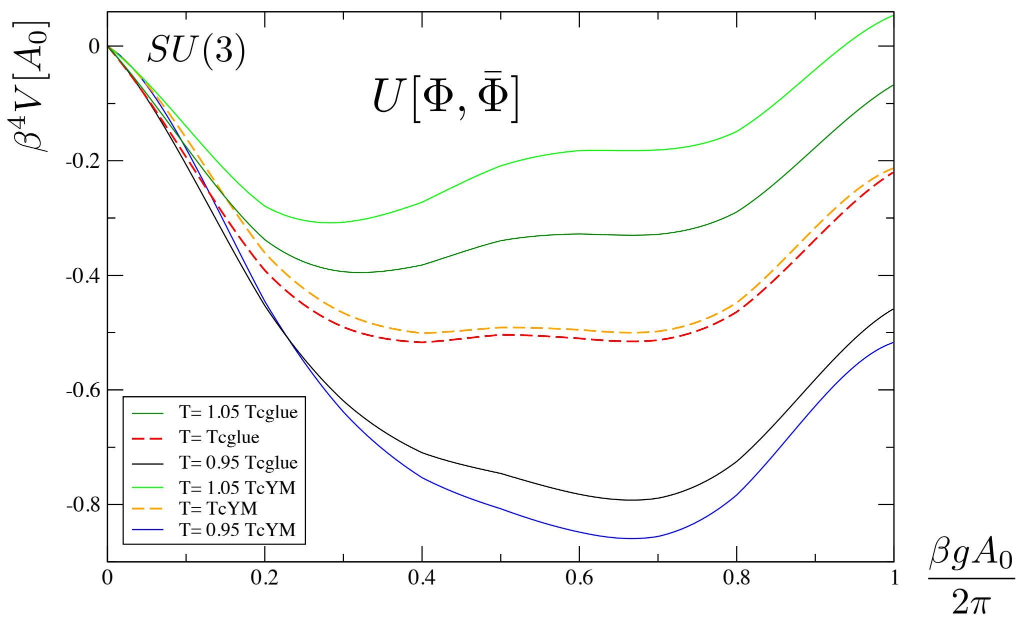
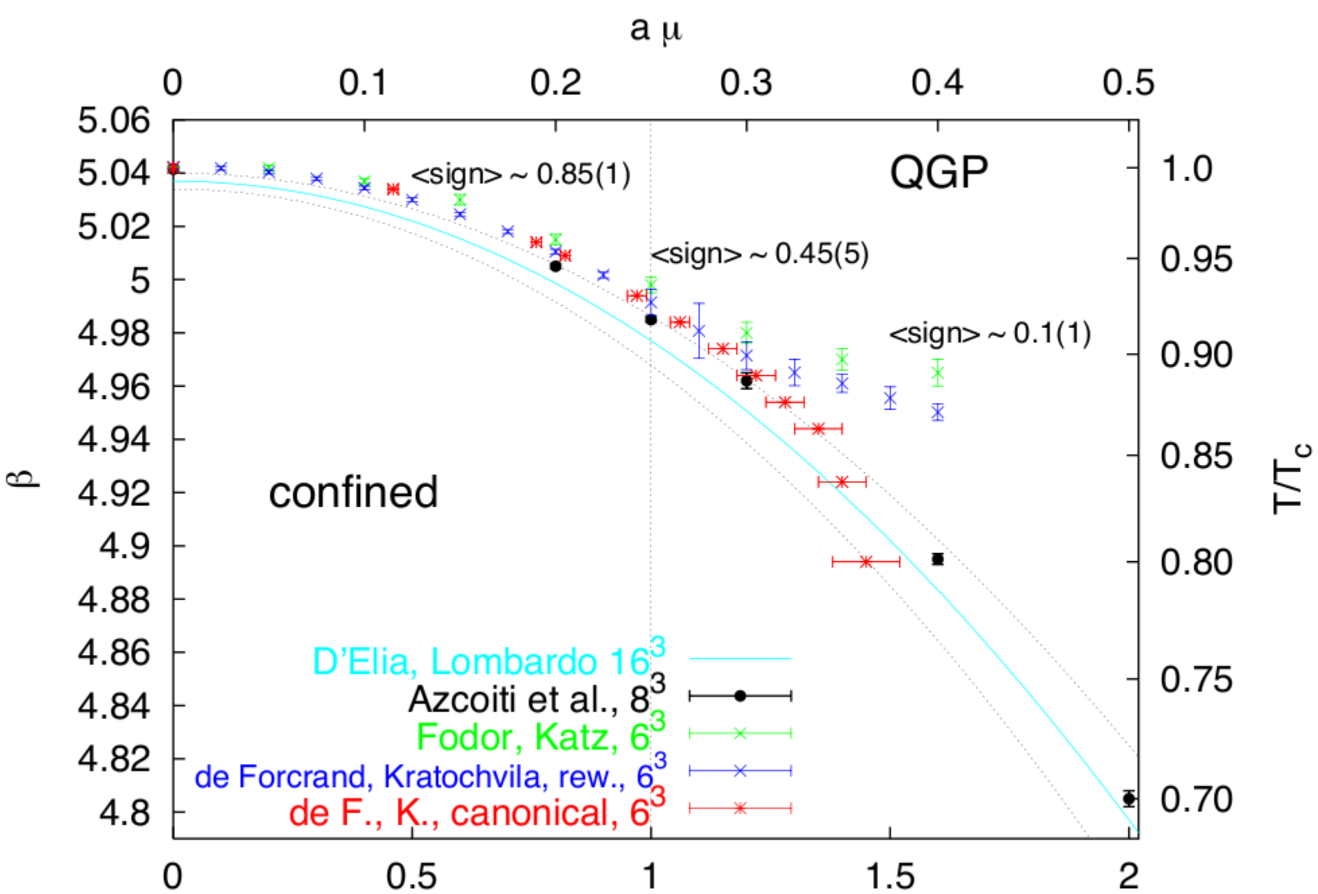


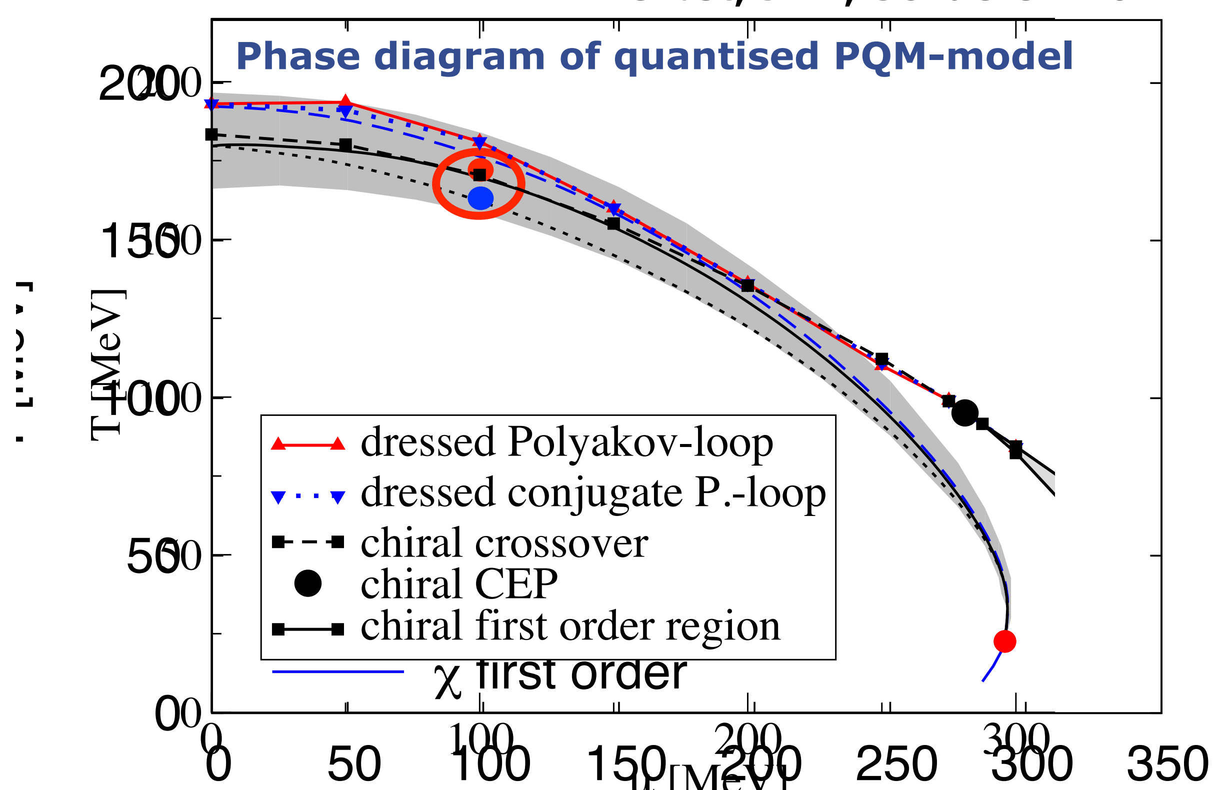

**Polyakov loop potential in full QCD**




**DSE: Fischer, Lücker, Mueller '11**

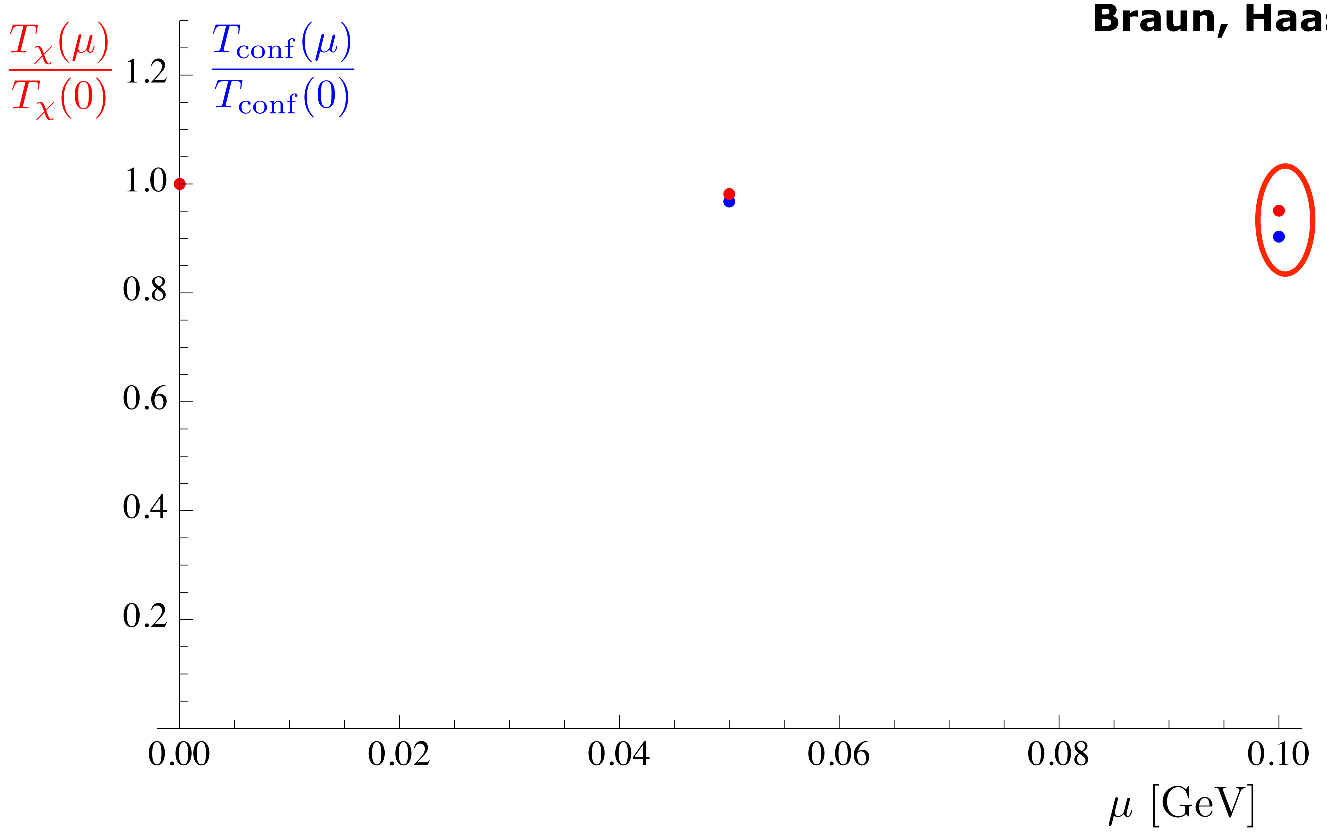
**Herbst, JMP, Schaefer '10**



# Real chemical potential

## Full dynamical QCD

**Braun, Haas, Fister, JMP**

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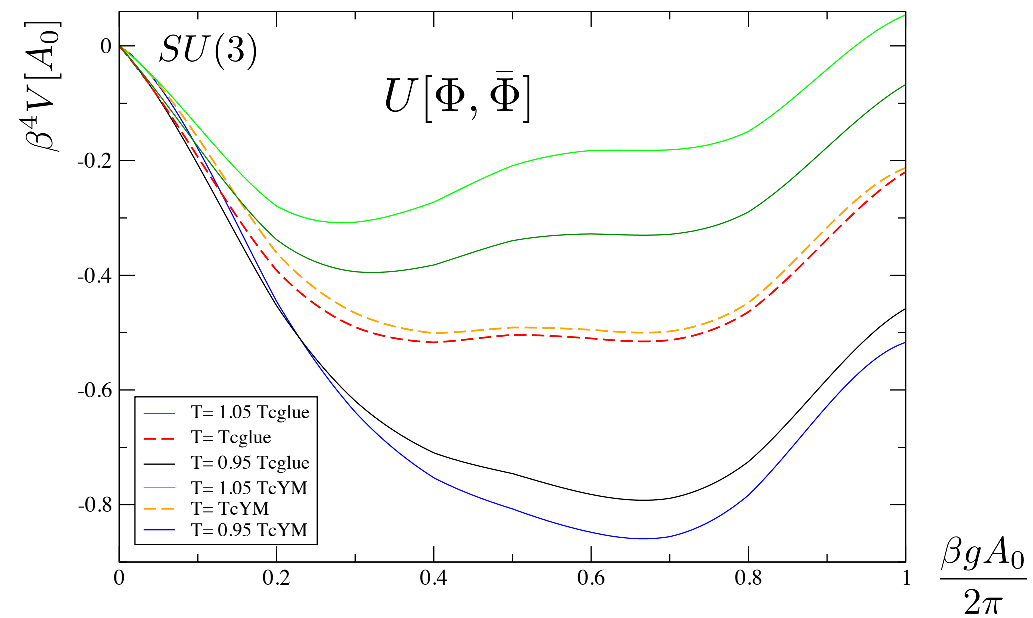


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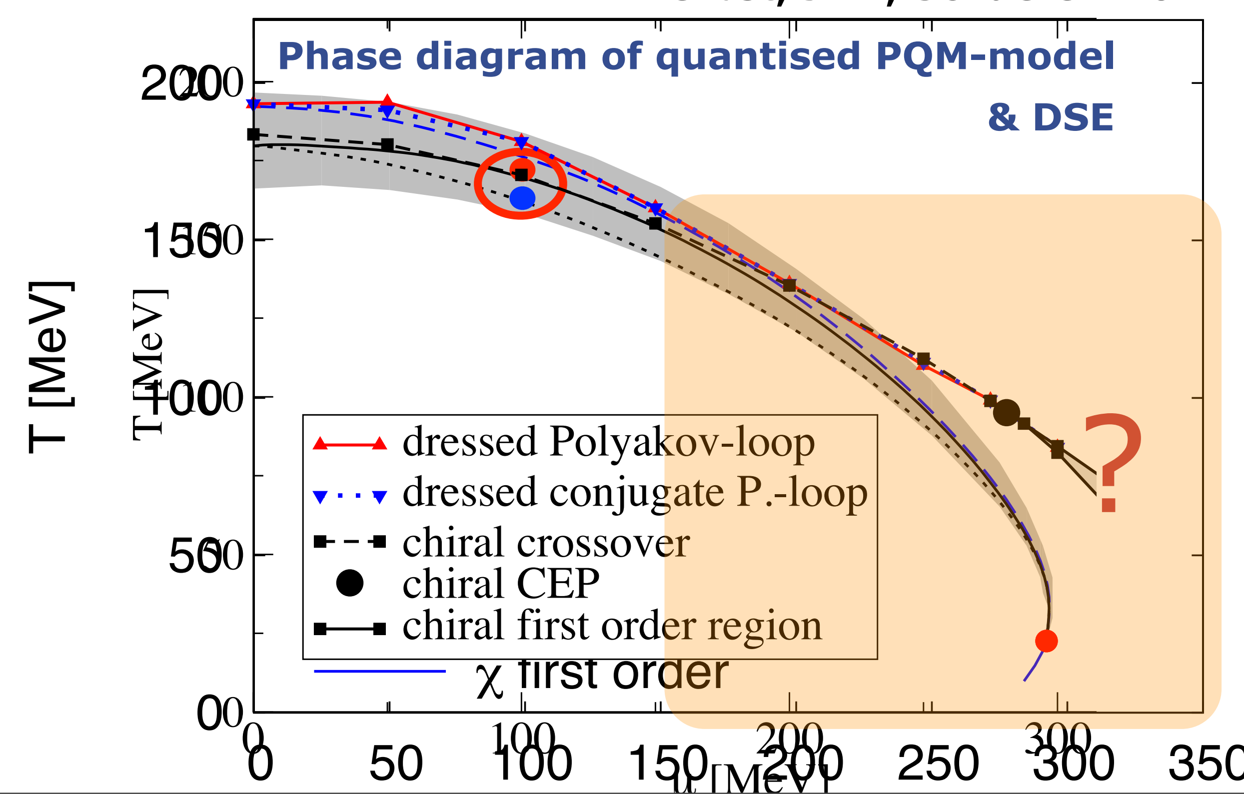


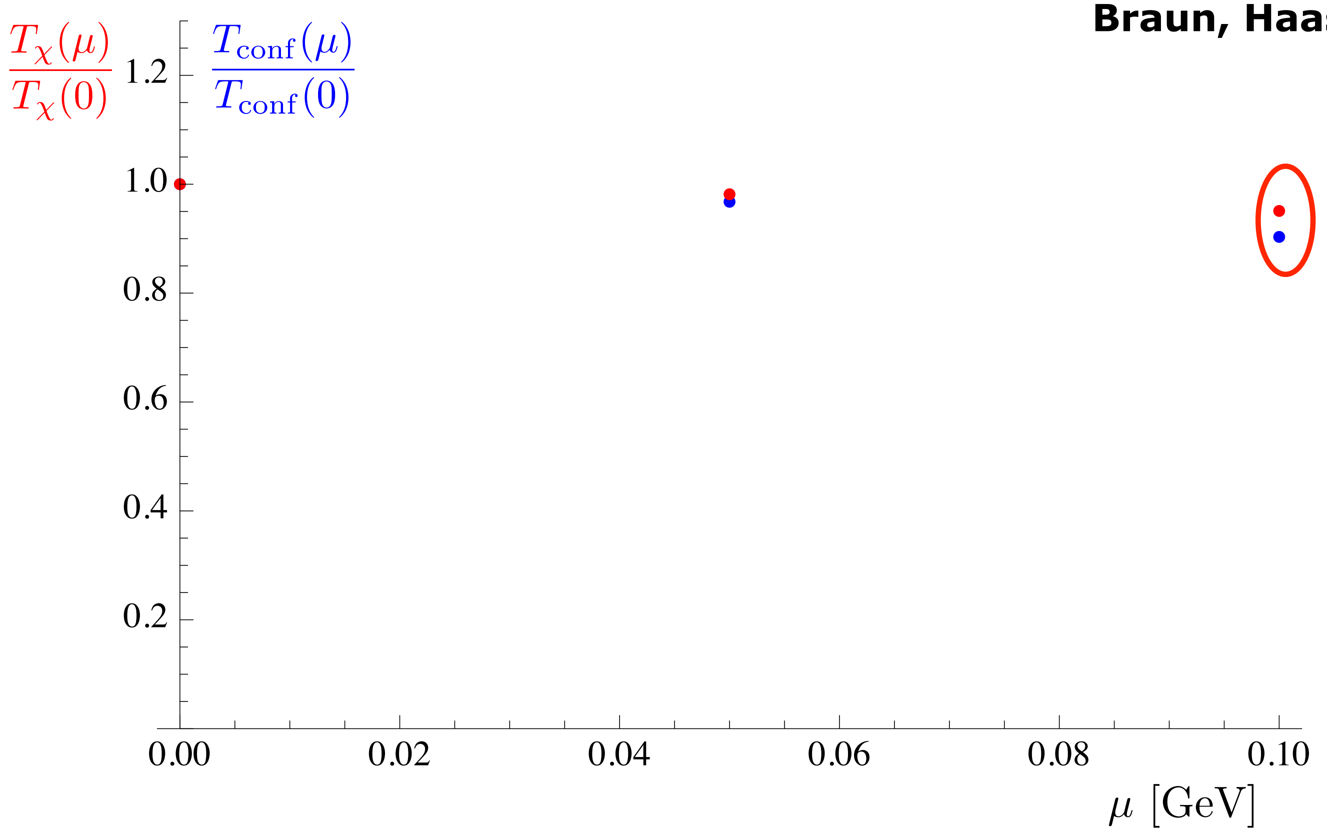

**Critical point unlikely for**

$$\frac{\mu_B}{T} < 2$$

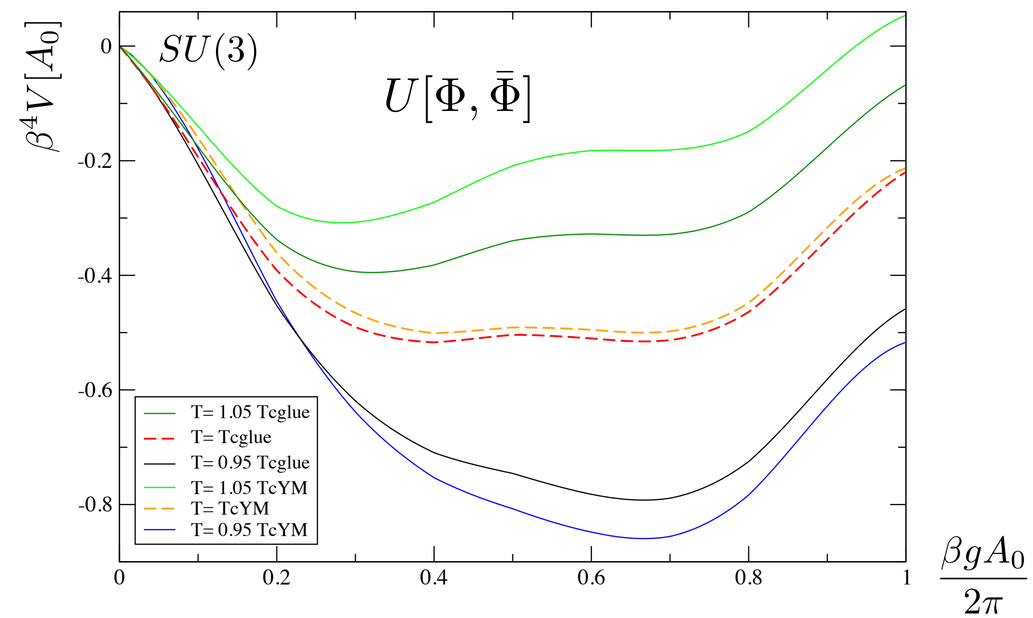
# Real chemical potential

## Full dynamical QCD

**Braun, Haas, Fister, JMP**

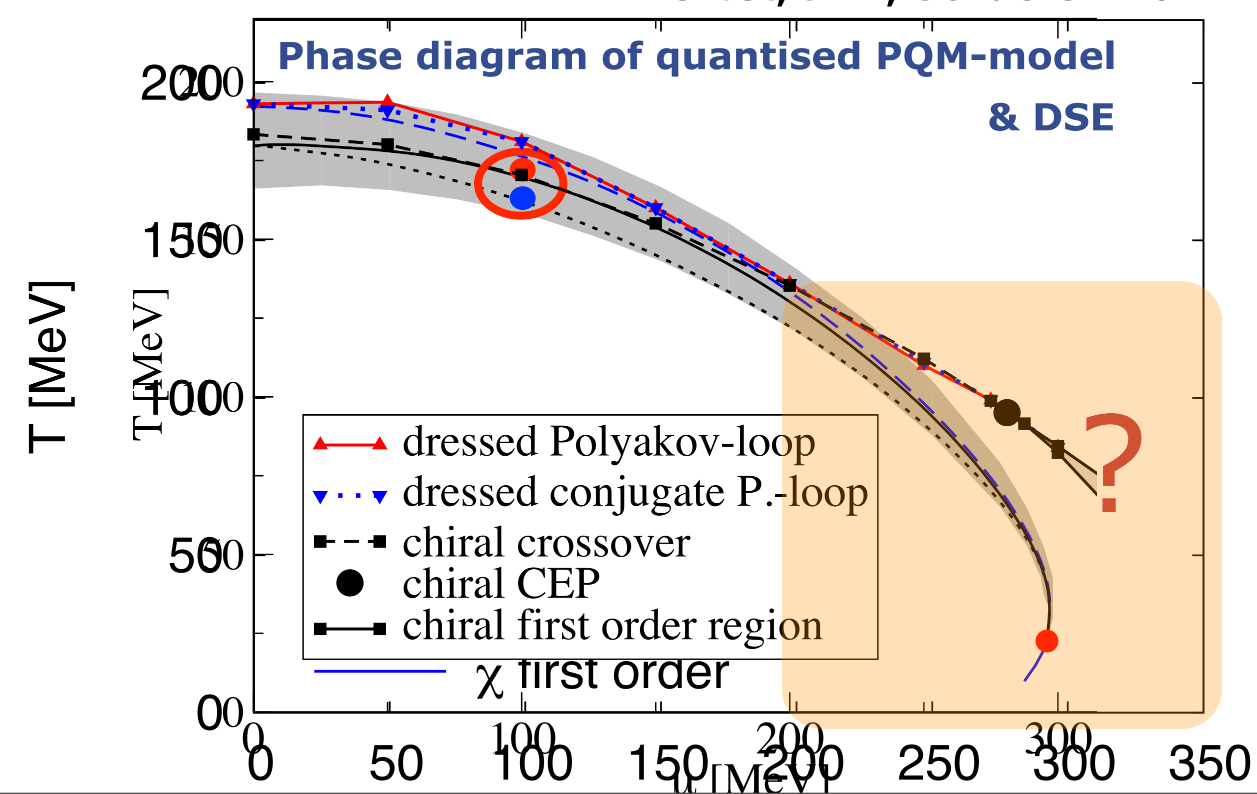



**Polyakov loop potential in full QCD**




**DSE: Fischer, Lücker, Mueller '11**

**Herbst, JMP, Schaefer '10**



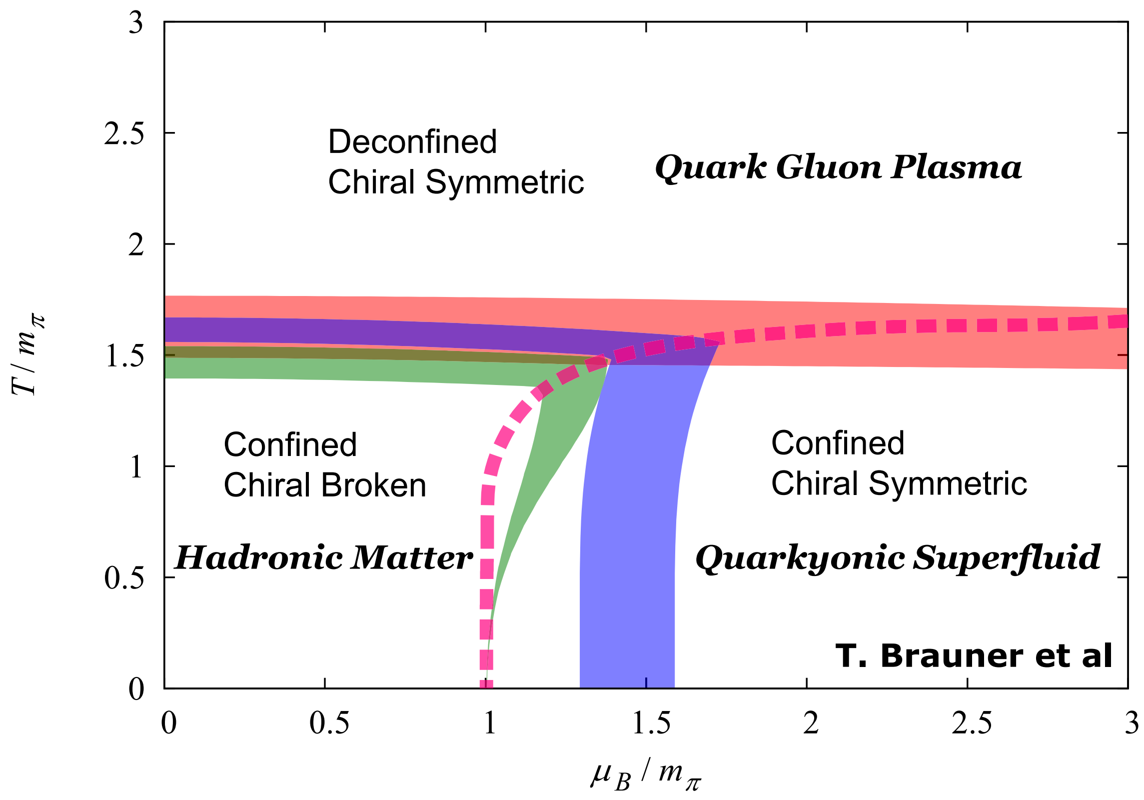

**Critical point unlikely for**

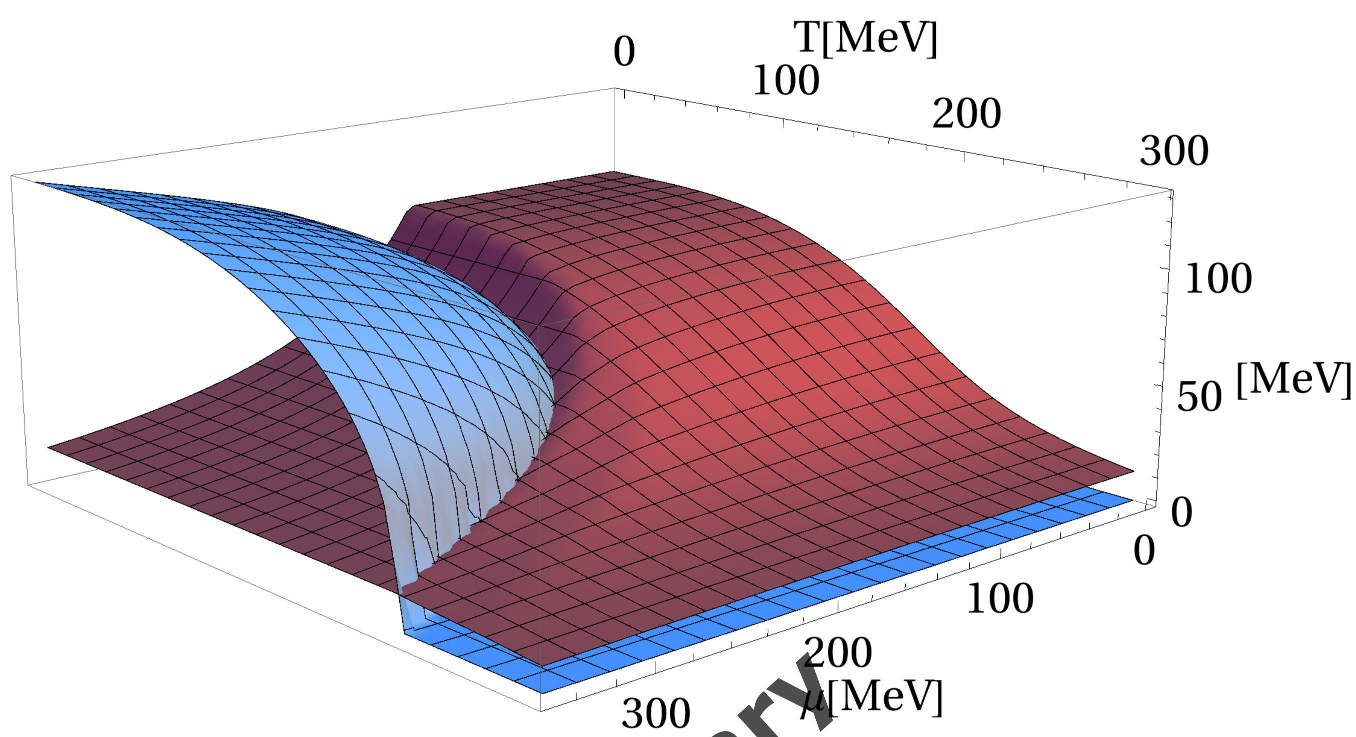
$$\frac{\mu_B}{T} < 4.5$$

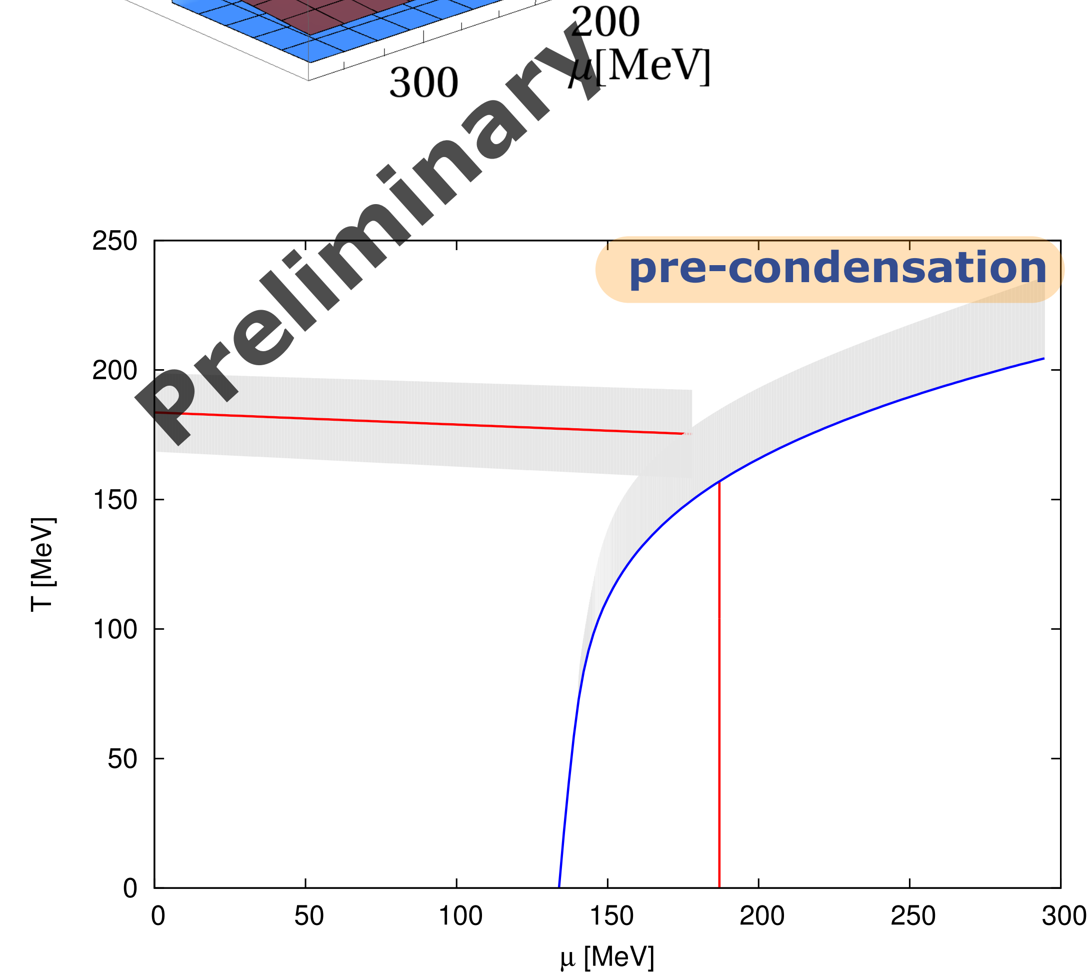
# Real chemical potential

## a glimpse at baryons

**Haas, Khan, JMP, Rennecke, Scherer**




## two colour QCD

**…, Ratti et al '04, …, Brauner et al '08, ….**

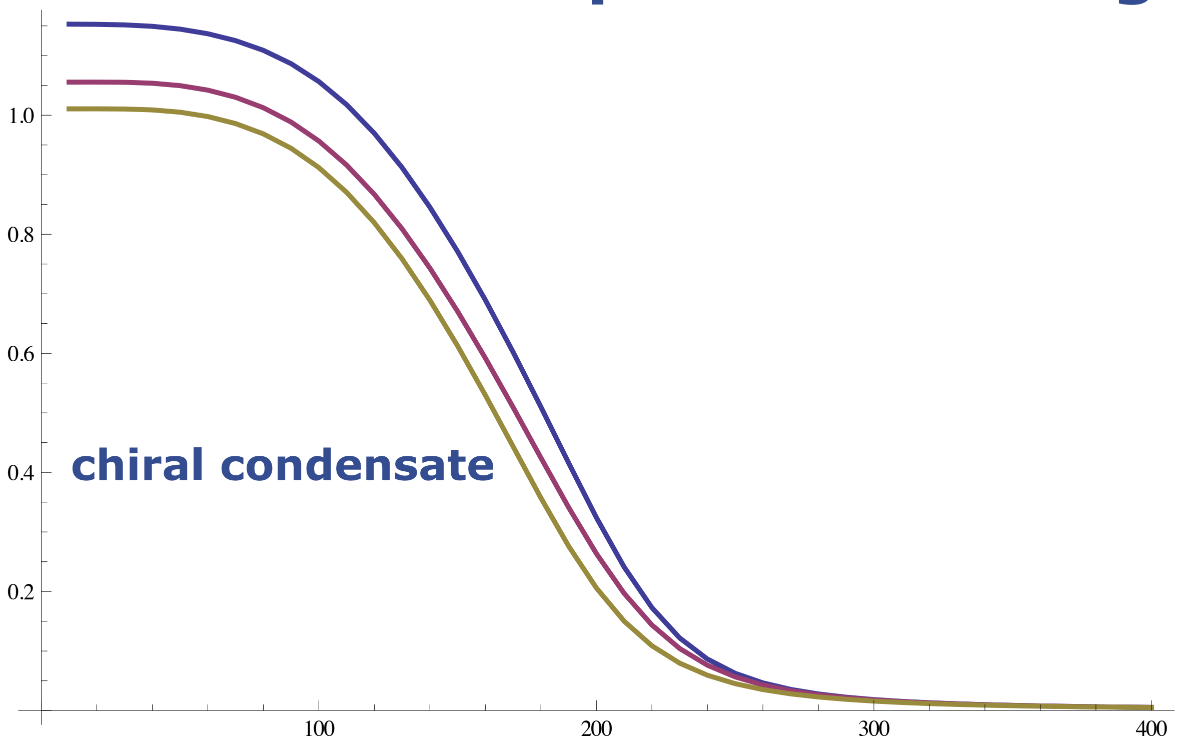
**see talk of Lorenz von Smekal**

# Real chemical potential

## a glimpse at multi-scatterings

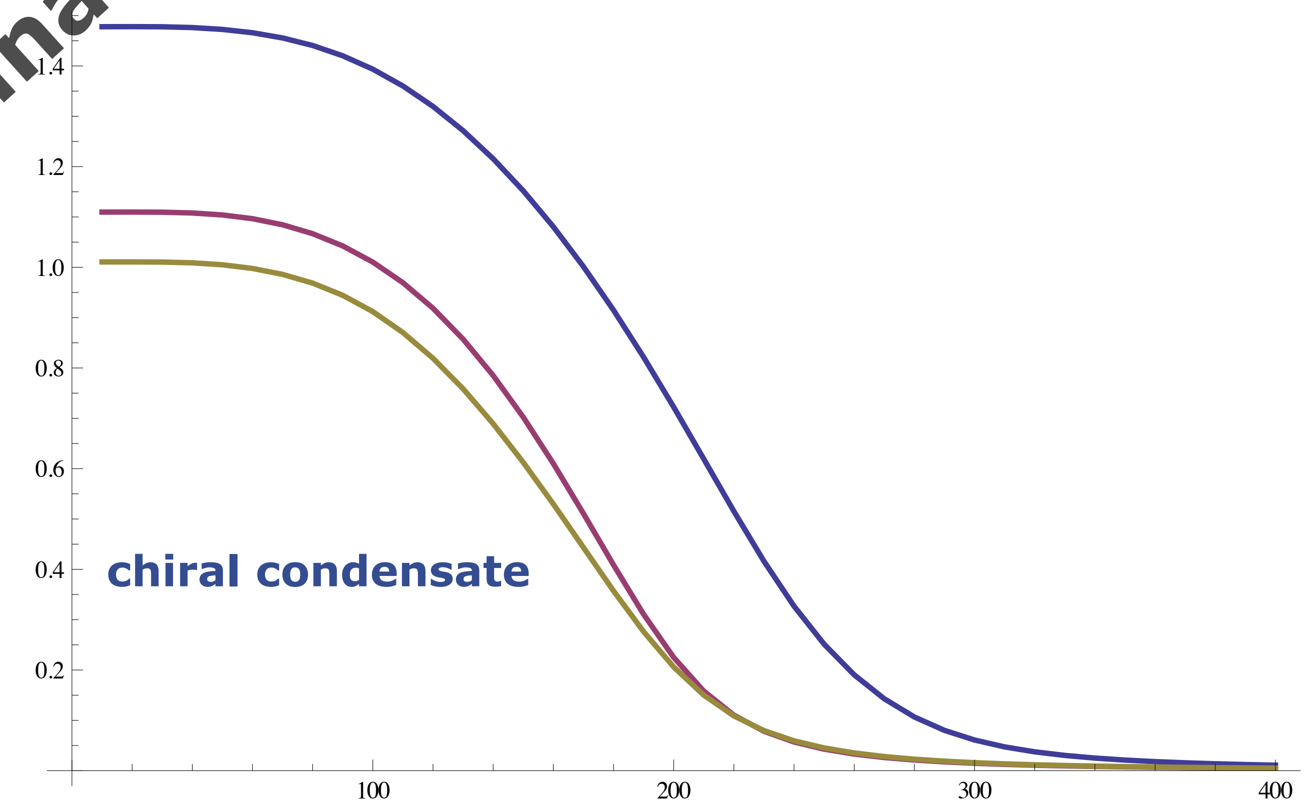
JMP, Rennecke

### Multi-meson-quark scatterings




Preliminary

### Multi-meson scatterings

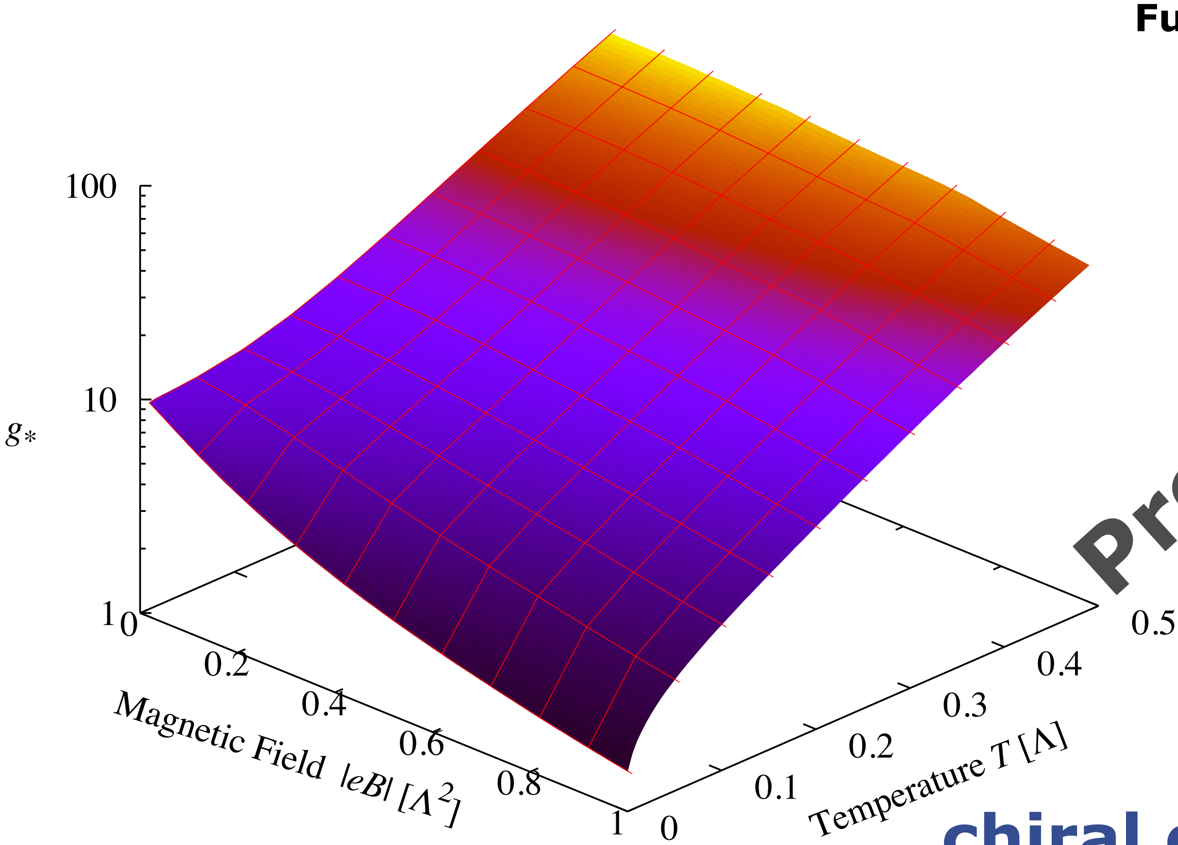


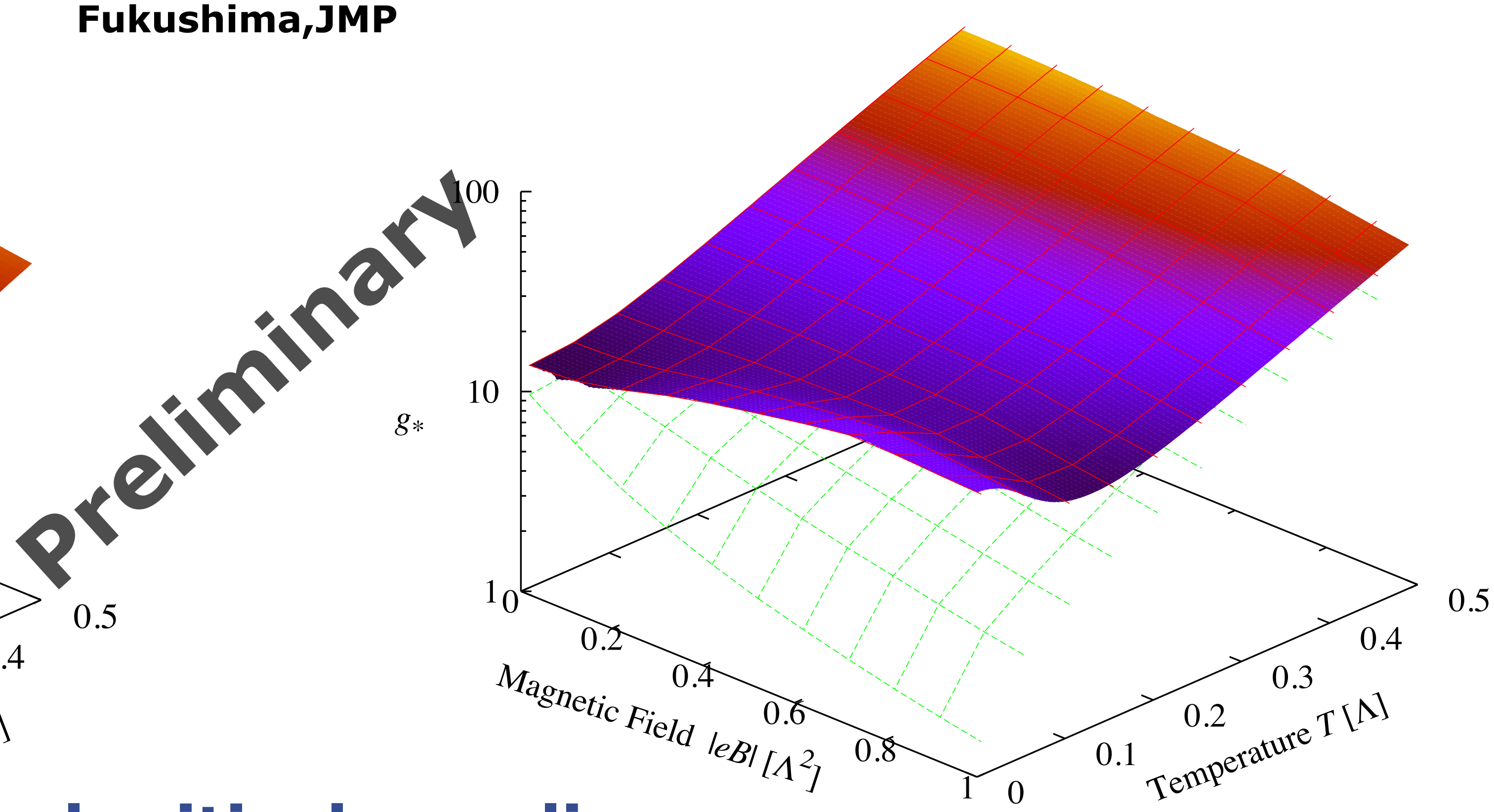

**PQM model**

# Real chemical potential

## a glimpse at strong magnetic fields

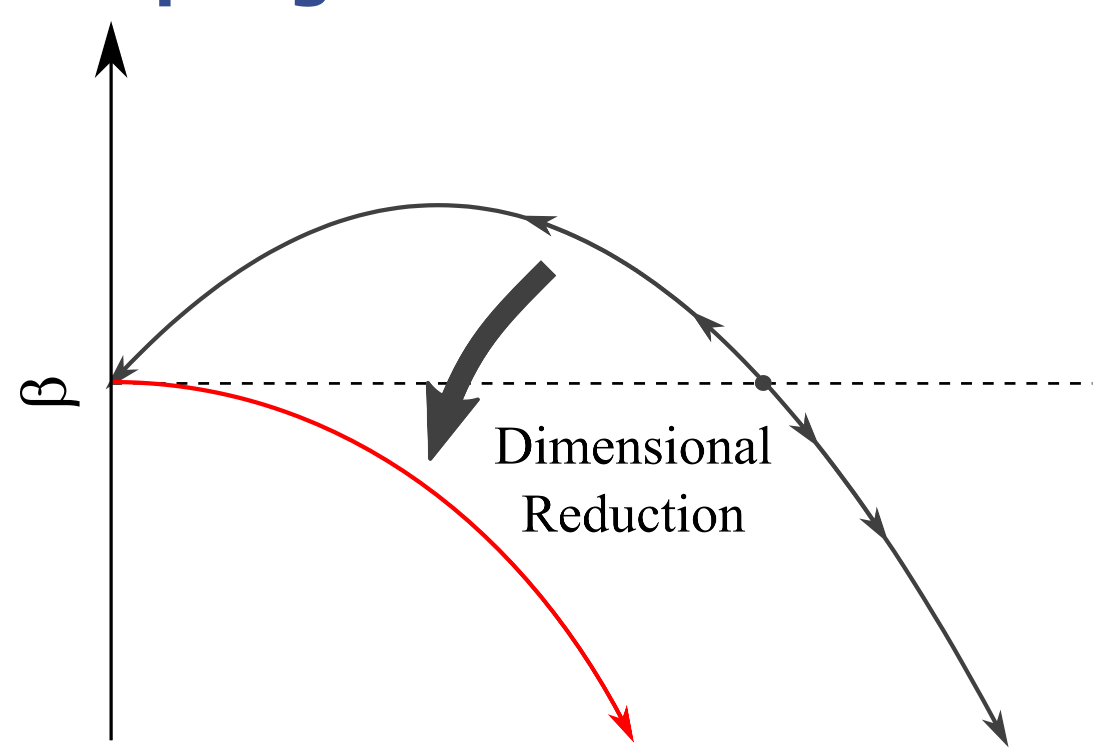
**Fukushima,JMP**




## chiral critical coupling

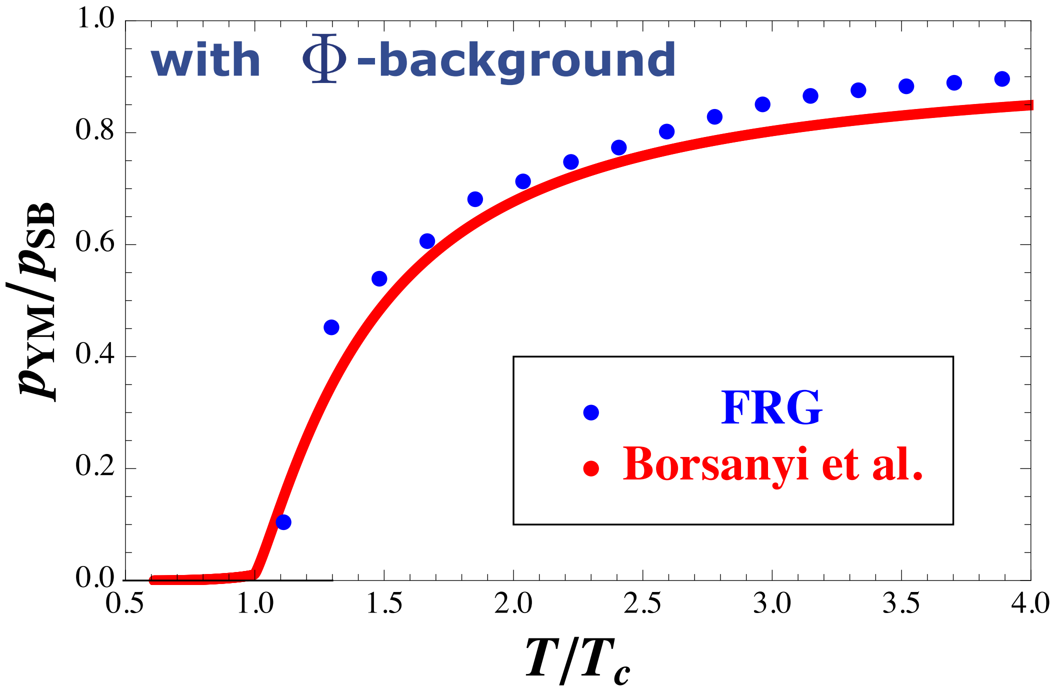
## Matter sector

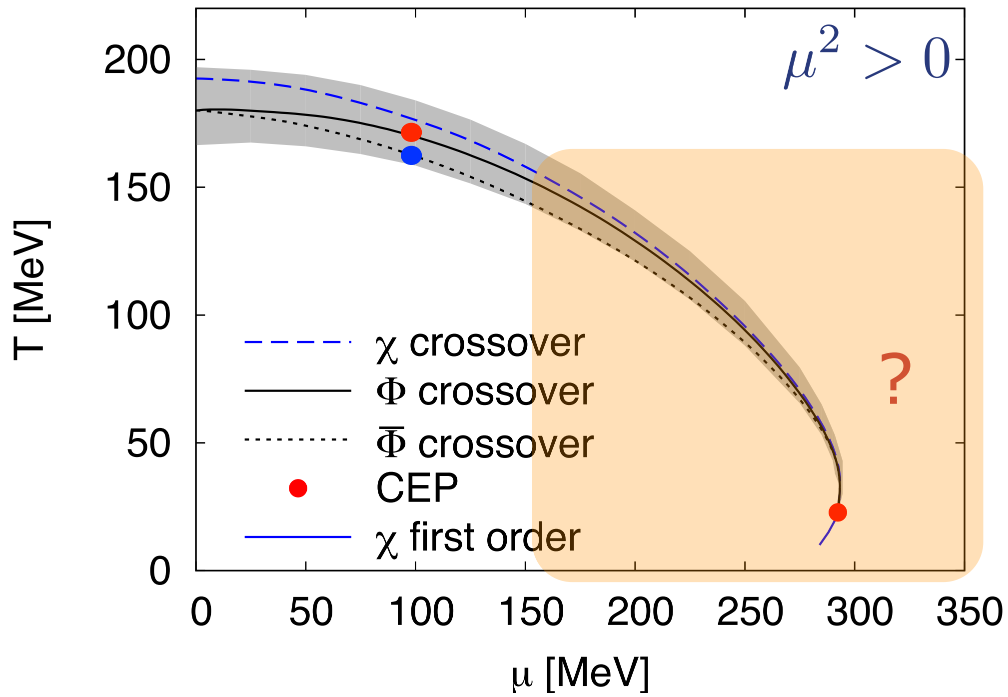


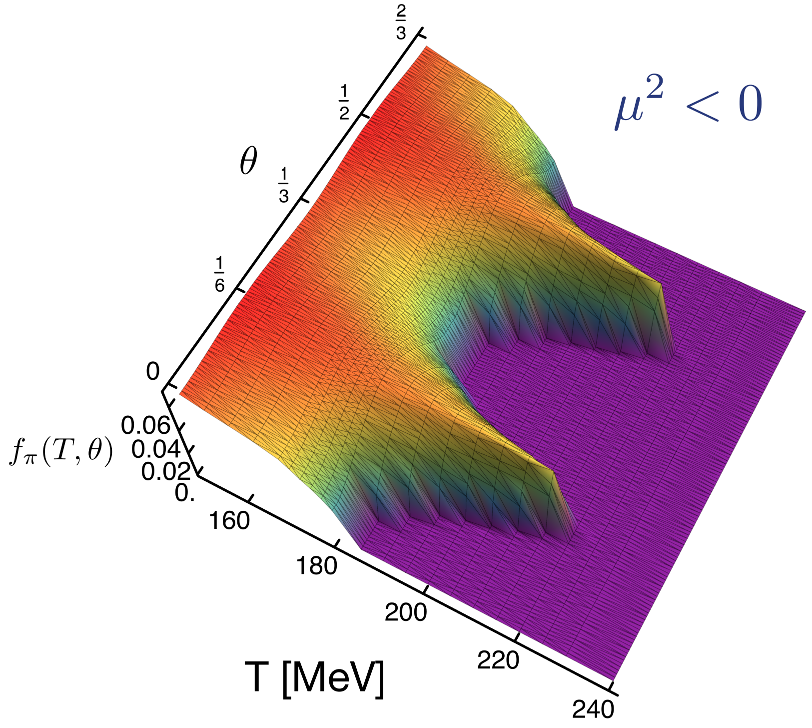
# Summary & outlook

- **Phase diagram of QCD**

  - **Phase structure and thermodynamics at finite $T$ & $\mu$**



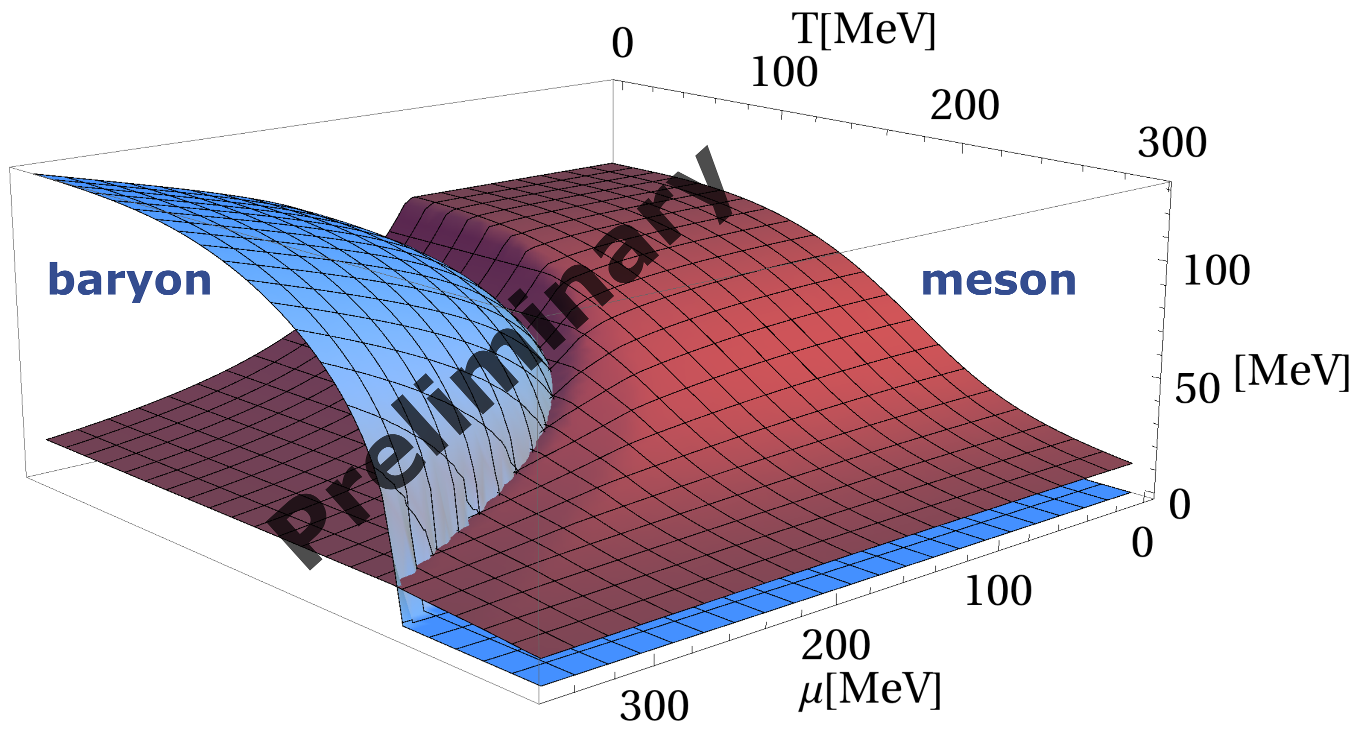

**Critical point unlikely for**

$$\frac{\mu_B}{T} < 2$$

# Summary & outlook

- **Phase diagram of QCD**
  - **Phase structure and thermodynamics at finite $T$ & $\mu$**
  - **2+1 flavours, baryons, phenomenology, dynamics**
  - **QCD meets cold quantum gases: two-colour QCD**

**Haas, Khan, JMP, Rennecke, Scherer**




**diquark condensate** **chiral condensate**

# Summary & outlook

- **Phase diagram of QCD**
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  - **QCD meets cold quantum gases: two-colour QCD**

**EpisodeIII: QGP meets ultracold atoms  (Hirschegg August 25th -31st)**

- **Hadronic properties**
  - **dynamical hadronisation**
  - **dynamics**