

Operator representation for realistic effective interactions

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Nuclear ab-initio calculations









Outline

Nuclear *ab-initio* calculations

- Argonne potential
- Unitary Correlation Operator Method
- Similarity Renormalization Group
- Fermionic Molecular Dynamics

Operator representation from partial wave matrix elements

3 Results

- UCOM with reduced set of operators
- SRG operator representation



Summary and conclusions

Argonne potential Wiringa, Stoks, Schiavilla; Phys.Rev.C 51 1995

Operator representation

$$\begin{split} \mathcal{V}_{Argonne} &= \sum_{S,T} V_{ST}^{Z}(\underline{r}) \prod_{ST} \langle k(LS) JM; TM_{T} | \mathcal{V}_{Argonne} | k'(L'S) JM; TM_{T} \rangle \\ &+ \sum_{S,T} V_{ST}^{L2}(\underline{r}) \vec{L}^{Z} \prod_{ST} \\ &+ \sum_{T} V_{1T}^{LS}(\underline{r}) \vec{L}^{Z} \prod_{ST} \\ &+ \sum_{T} V_{1T}^{T}(\underline{r}) \vec{S}_{12} \prod_{TT} \\ &+ \sum_{T} V_{1T}^{T}(\underline{r}) \vec{S}_{12} (\vec{L}, \vec{L}) \prod_{TT} \\ &+ \sum_{T} V_{1T}^{TLL}(\underline{r}) s_{12} (\vec{L}, \vec{L}) \prod_{TT} \\ &+ \sum_{T} V_{1T}^{TLL}(\underline{r}) s_{12} (\vec{L}, \vec{L}) \prod_{TT} \\ &+ \sum_{T} (\vec{L}, \vec{L}) \vec{S}_{12} (\vec{L}, \vec{L}) \vec{S}_{1T} \\ &+ \sum_{T} (\vec{L}, \vec{L}) \vec{S}_{12} (\vec{L}, \vec{L}) \vec{S}_{1T} \\ &+ \sum_{T} (\vec{L}, \vec{L}) \vec{S}_{12} (\vec{L}, \vec{L}) \vec{S}_{12} \vec{S}_{1T} \\ &+ \sum_{T} (\vec{L}, \vec{L}) \vec{S}_{12} (\vec{L}, \vec{L}) \vec{S}_{1T} \\ &+ \sum_{T} (\vec{L}, \vec{L}) \vec{S}_{12} (\vec{L}, \vec{L}) \vec{S}_{1T} \\ &+ \sum_{T} (\vec{L}, \vec{L}) \vec{S}_{12} (\vec{L}, \vec{L}) \vec{S}_{1T} \\ &+ \sum_{T} (\vec{L}, \vec{L}) \vec{S}_{12} (\vec{L}, \vec{L}) \vec{S}_{12} \vec{S}_{1T} \\ &+ \sum_{T} (\vec{L}, \vec{L}) \vec{S}_{12} (\vec{L}, \vec{L}) \vec{S}_{12} \vec{S}_{1T} \\ &+ \sum_{T} (\vec{L}, \vec{L}) \vec{S}_{12} (\vec{L}, \vec{L}) \vec{S}_{12} \vec{S}_{1T} \\ &+ \sum_{T} (\vec{L}, \vec{L}) \vec{S}_{12} (\vec{L}, \vec{L}) \vec{S}_{12} \vec{S}_{1T} \\ &+ \sum_{T} (\vec{L}, \vec{L}) \vec{S}_{12} \vec{S}_{1T} \vec{S}_{1T} \\ &+ \sum_{T} (\vec{L}, \vec{L}) \vec{S}_{1T} \vec{S}_{1T} \vec{S}_{1T} \vec{S}_{1T} \vec{S}_{1T} \\ &+ \sum_{T} (\vec{L}, \vec{L}) \vec{S}_{1T} \vec{S}_{$$

partial wave matrix elements

Unitary Correlation Operator Method

- > nuclear interaction induces strong short-range correlations which cannot be described by "simple" many-body states $\mid \psi \; \rangle$
- "Unitary Correlation Operator Method" (UCOM):
 Feldmeier, Neff, Roth, Schnack; Nucl.Phys. A632 1998
 Roth, Neff, Feldmeier; Prog.Part.Nucl.Phys. 65 2010
 - imprint correlations onto uncorrelated states by unitary operator <u>C</u>:

$$\left| \hat{\psi} \right\rangle = \mathop{\mathcal{C}}_{\sim} \left| \psi \right\rangle$$

• correlated operator: $\hat{\mathcal{Q}} = \mathcal{L}^{\dagger} \mathcal{Q} \mathcal{C}$:

$$\left\langle \left. \hat{\psi} \left| \underset{\sim}{\mathcal{O}} \right| \left. \hat{\psi}' \right. \right\rangle = \left\langle \left. \psi \left| \underset{\sim}{\mathcal{C}}^{\dagger} \underset{\sim}{\mathcal{O}} \underset{\sim}{\mathcal{C}} \right| \left. \psi' \right. \right\rangle = \left\langle \left. \psi \left| \underset{\sim}{\mathcal{O}} \right| \left. \psi' \right. \right\rangle \right.$$





UCOM potential Operator representation

operator representation:

$$\begin{split} \mathcal{V}_{UCOM} &= \mathcal{L}^{\dagger} \mathcal{H} \mathcal{L} - \mathcal{I} &= \sum_{S,T} V_{ST}^{Z}(x) \prod_{ST} + \sum_{S,T} V_{ST}^{L2}(x) \vec{L}^{2} \prod_{ST} \\ &+ \sum_{S,T} \frac{1}{2} (\vec{p}^{2} V_{ST}^{p2}(x) + V_{ST}^{p2}(x) \vec{p}^{2}) \prod_{ST} \\ &+ \sum_{T} V_{1T}^{LS}(x) \vec{L} \vec{S} \prod_{1T} + \sum_{T} V_{1T}^{L2LS}(x) \vec{L}^{2} \vec{L} \vec{S} \prod_{1T} \\ &+ \sum_{T} V_{1T}^{T}(x) \mathcal{S}_{12} \prod_{1T} + \sum_{T} V_{1T}^{TL}(x) s_{12} (\vec{L}, \vec{L}) \prod_{1T} \\ &+ \sum_{T} V_{1T}^{T}(x) \mathcal{S}_{12} (p_{\Omega}, p_{\Omega}) \prod_{1T} \\ &+ \sum_{T} (p_{T} V_{1T}^{Tpp}(x) + V_{1T}^{Tpp}(x) p_{T}) s_{12} (x, p_{\Omega}) \prod_{1T} \\ &+ \sum_{T} V_{1T}^{L2Tpp}(x) [\vec{L}^{2} \vec{s}_{12} (p_{\Omega}, p_{\Omega}) + \vec{s}_{12} (p_{\Omega}, p_{\Omega}) \vec{L}^{2}] \prod_{TT} \\ &+ \cdots \end{split}$$

more complicated, nonlocal structure

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Similarity Renormalization Group

- decoupling of low and high momentum matrix elements by evolving potential to band-diagonal structure
- Similarity Renormalization Group (SRG) Bogner, Furnstahl, Perry; Phys.Rev.C 75 2007 starting from initial Hamiltonian H₀ = T + V₀ flow equation:

 $\frac{\mathsf{d}H_{s}}{\mathsf{d}s} = \left[\eta_{s}, H_{s}\right]$

with flow parameter s and generator $\eta_s = [\mathcal{I}, \mathcal{H}_s] = [\mathcal{I}, \mathcal{V}_s]$ • evolution towards $[\mathcal{I}, \mathcal{H}_s] = 0: \mathcal{H}_s \rightarrow \text{band-diagonal}$

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Similarity Renormalization Group SRG flow

SRG evolution



partial wave matrix elements, no operator representation

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Fermionic Molecular Dynamics (FMD)

Feldmeier, Nucl. Phys. A515 1990

- microscopic model to describe nuclei and nuclear reactions
- model states: antisymmetrized gaussian wave packets:

$$|Q\rangle = \mathcal{A}(|q_1\rangle \otimes \cdots \otimes |q_A\rangle)$$

$$\vec{r} |q\rangle = \sum_i c_i \exp\{\frac{(\vec{r} - \vec{b}_i)^2}{2a_i}\} \otimes |\chi_i^{\uparrow}, \chi_i^{\downarrow}\rangle \otimes |\xi\rangle$$

- complex parameters \vec{b}_i encode mean position and momentum
- width parameter a_i
- description of exotic phenomena like halos, cluster-states, ...
- interaction matrix elements in FMD basis calculated analytically: operator representation needed!

From partial wave matrix elements

to operator representation



operator representation

choose appropriate set of operators

$$\mathcal{V}_{ansatz} = \sum_{ST,i} \mathcal{V}_{ST}^{i}(\underline{r}, \underline{p}) \mathcal{Q}_{i} \prod_{ST} ST$$

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From partial wave matrix elements

to operator representation



operator representation

choose appropriate set of operators

$$\mathcal{V}_{ansatz} = \sum_{ST,i} \mathcal{V}^{i}_{ST}(\underline{r}, \underline{p}) \mathcal{Q}_{i} \prod_{ST} ST$$

 representation of (unknown!) radial functions by a sum of gaussians:

$$\mathcal{V}_{ST}^{i}(\underline{r}) = \sum_{k} \gamma_{ST, k}^{i} \cdot e^{-\frac{\overline{\zeta}^{2}}{2\kappa_{k}}}$$
$$p_{ST}^{i}(\underline{r}, \underline{p}) =$$

 $\sum_{k,l} \gamma_{ST,kl}^{i} \cdot e^{-\frac{\lambda_l}{4}\vec{p}^2} e^{\frac{-\vec{z}^2}{2(\kappa_k - \lambda_l/4)}} e^{-\frac{\lambda_l}{4}\vec{p}^2}$

• choose parameters κ_j and λ_l on a grid

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From partial wave matrix elements to operator representation

partial wave matrix elements ${}^{1}S_{0}$ 208 $-20 \\ -40$ 6 k4 8 2 6 "k' [fm⁻¹] 2 Fit ${}^{3}D_{2}$ $\gamma_{ST,i}^{i}$ 8 6 -4 k4 8 2 6 4 k' [fm⁻¹] 2

operator representation

choose appropriate set of operators

$$\mathcal{V}_{ansatz} = \sum_{ST,i} \mathcal{V}^{i}_{ST}(\underline{r}, \underline{p}) \mathcal{Q}_{i} \prod_{ST} ST$$

 representation of (unknown!) radial functions by a sum of gaussians:

$$\mathcal{V}_{ST}^{i}(\underline{r}) = \sum_{k} \gamma_{ST,k}^{i} \cdot e^{-\frac{\overline{r}^{2}}{2\kappa_{k}}}$$
$$\stackrel{i}{}_{ST}^{i}(\underline{r},\underline{p}) =$$

 $\sum_{k,l} \gamma_{ST, kl}^{i} \cdot e^{-\frac{\lambda_l}{4}\vec{p}^2} e^{\frac{-\vec{z}^2}{2(\kappa_k - \lambda_l/4)}} e^{-\frac{\lambda_l}{4}\vec{p}^2}$

- choose parameters κ_j and λ_l on a grid
- ► calculate analytically matrix elements $\langle k(LS)JT | \chi_{ansatz} | k'(L'S)JT \rangle$

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$$\mathcal{V}_{UCOM} = \sum_{S,T} V_{ST}^{Z}(\underline{r}) \prod_{ST} V_{ST}$$

- + $\sum_{C,T} V_{ST}^{L2}(\underline{r}) \overset{\mathcal{I}}{\sim} \overset{\mathcal{I}}{\sim} \overset{\mathcal{I}}{\sim} ST$
- + $\sum_{n=1}^{\infty} \frac{1}{2} (\vec{p}^{2} V_{ST}^{p2}(r) + V_{ST}^{p2}(r) \vec{p}^{2}) \prod_{n \in ST} V_{ST}^{p2}(r) \vec{p}^{2}$
- + $\sum_{T} V_{1T}^{LS}(\underline{r}) \overset{\vec{L}}{\sim} \overset{\vec{S}}{\sim} \overset{\vec{\Pi}}{\sim} \overset{1}{\sim} \overset{T}{\sim}$
- + $\sum_{r} V_{1T}^{L2LS}(r) \overset{\vec{L}^2}{\sim} \overset{\vec{L}S}{\sim} \overset{\vec{n}_{1T}}{\sim} V_{1T}^{L2LS}(r) \overset{\vec{L}^2}{\sim} \overset{\vec{L}S}{\sim} \overset{\vec{n}_{1T}}{\sim} V_{1T}^{L2LS}(r) \overset{\vec{n}_{1T}}{\sim} V_{1T}^{L2LS}(r) \overset{\vec{n}_{1T}}{\sim} \overset{\vec{n}_{1T}}{\sim} V_{1T}^{L2LS}(r) \overset{\vec{n}_{1T}}{\sim} \overset{\vec{n}_{1T}}{$
- + $\sum_{T} V_{1T}^{T}(\underline{r}) \underset{\sim}{S}_{12} \underset{\sim}{\Pi}_{1T}$
- + $\sum_{\tau} V_{1T}^{TLL}(\underline{r}) \underset{\sim}{s}_{12}(\overline{L}, \overline{L}) \underset{\sim}{\Pi}_{1T}$
- + $\sum V_{1T}^{Tpp}(\underline{r}) \overline{s}_{12}(p_{\Omega}, p_{\Omega}) \underset{\sim}{\Pi_{1T}}$
- + $\sum (\underset{\sim}{p_r} V_{1T}^{Trp}(\underline{r}) + V_{1T}^{Trp}(\underline{r}) \underset{\sim}{p_r}) \underset{\sim}{s_{12}} (r, p_{\Omega}) \underset{\sim}{\Pi_{1T}}$
- $+ \sum V_{1T}^{L2T\rhop}(\underline{r})[\underline{\vec{L}}^2 \bar{s}_{12}(\underline{\rho}_{\Omega}, \underline{\rho}_{\Omega}) + \bar{s}_{12}(\underline{\rho}_{\Omega}, \underline{\rho}_{\Omega})\underline{\vec{L}}^2]\underline{\prod}_{1T}$
- $\mathcal{V}_{ansatz} = \sum_{c \in T} \mathcal{V}_{ST}^{Z}(r) \prod_{ST} ST$ + $\sum_{C,T} \mathcal{V}_{ST}^{L2}(\underline{r}) \overset{\mathcal{I}}{\sim}$ + $\sum_{n=1}^{\infty} \frac{1}{2} (\vec{p}^2 \mathcal{V}_{ST}^{p2}(\vec{r}) + \mathcal{V}_{ST}^{p2}(\vec{r}) \vec{p}^2) \Pi_{ST}$ + $\sum \mathcal{V}_{1T}^{LS}(\underline{r}) \overset{\vec{L}S}{\sim} \overset{\vec{\Pi}_{1T}}{\sim} 1_T$ + $\sum_{r} \mathcal{V}_{1T}^{L2LS}(\underline{r}) \overset{\vec{L}}{\sim}$ + $\sum_{\tau} \mathcal{V}_{1T}^{T}(\underline{r}) S_{12} \prod_{\tau} \Pi_{1T}$ + $\sum_{\tau} \mathcal{V}_{1T}^{TLL}(r) \underset{\sim}{s}_{12}(\vec{L}, \vec{L}) \underset{\sim}{\Pi}_{1T}$ + $\sum_{r} \mathcal{V}_{1T}^{Tpp}(\underline{r}) \overline{\underline{s}}_{12}(p_{\Omega}, p_{\Omega}) \underset{\sim}{\square}_{1T}$ + $\sum_{\tau} (p_r \mathcal{V}_{1T}^{Trp}(r) + \mathcal{V}_{1T}^{Trp}(r) p_r) s_{12}(r, p_\Omega) \prod_{\tau} Trp$ + $\sum_{\boldsymbol{r}} V_{1T}^{L2Tpp}(\boldsymbol{r})[\boldsymbol{\vec{L}}^2 \bar{\boldsymbol{s}}_{12}(\boldsymbol{p}_{\Omega}, \boldsymbol{p}_{\Omega}) + \bar{\boldsymbol{s}}_{12}(\boldsymbol{p}_{\Omega}, \boldsymbol{p}_{\Omega})\boldsymbol{\vec{L}}^2] \boldsymbol{\prod}_1 \boldsymbol{\vec{r}}_{\boldsymbol{r}}$

Fit

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$$\mathcal{V}_{UCOM} = \sum_{S,T} V_{ST}^{Z}(\underline{r}) \prod_{ST} V_{ST}$$

- + $\sum_{C,T} V_{ST}^{L2}(\underline{r}) \overset{\mathcal{I}}{\sim} \overset{\mathcal{I}}{\sim} \overset{\mathcal{I}}{\sim} ST$
- + $\sum_{n=1}^{\infty} \frac{1}{2} (\vec{p}^{2} V_{ST}^{p2}(\vec{r}) + V_{ST}^{p2}(\vec{r}) \vec{p}^{2}) \prod_{ST} V_{ST}^{p2}(\vec{r}) \vec{r}^{2}$
- + $\sum_{T} V_{1T}^{LS}(\underline{r}) \stackrel{\vec{LS}}{\sim} \stackrel{\vec{n}_{1T}}{\sim} \stackrel{\vec{n}$
- + $\sum_{r} V_{1T}^{L2LS}(r) \overset{\vec{L}^2}{\sim} \overset{\vec{L}S}{\sim} \overset{\vec{n}_{1T}}{\sim} V_{1T}^{L2LS}(r) \overset{\vec{L}^2}{\sim} \overset{\vec{L}S}{\sim} \overset{\vec{n}_{1T}}{\sim} V_{1T}^{L2LS}(r) \overset{\vec{n}_{1T}}{\sim} V_{1T}^{L2LS}(r) \overset{\vec{n}_{1T}}{\sim} \overset{\vec{n}_{1T}}{\sim} V_{1T}^{L2LS}(r) \overset{\vec{n}_{1T}}{\sim} \overset{\vec{n}_{1T}}{$
- + $\sum V_{1T}^{T}(\underline{r}) \underset{\sim}{S}_{12} \underset{\sim}{\Pi}_{1T}$
- + $\sum_{\tau} V_{1T}^{TLL}(\underline{r}) \underset{\sim}{s}_{12}(\overline{L}, \overline{L}) \underset{\sim}{\Pi}_{1T}$
- + $\sum V_{1T}^{Tpp}(\underline{r}) \overline{s}_{12}(p_{\Omega}, p_{\Omega}) \underset{\sim}{\Pi_{1T}}$
- + $\sum (\underset{\sim}{p_r} V_{1T}^{Trp}(\underline{r}) + V_{1T}^{Trp}(\underline{r}) \underset{\sim}{p_r}) \underset{\sim}{s_{12}} (r, p_{\Omega}) \underset{\sim}{\Pi_{1T}}$
- $+ \sum V_{1T}^{L2T\rhop}(\underline{r})[\underline{\vec{L}}^2 \bar{s}_{12}(\underline{\rho}_{\Omega}, \underline{\rho}_{\Omega}) + \bar{s}_{12}(\underline{\rho}_{\Omega}, \underline{\rho}_{\Omega})\underline{\vec{L}}^2]\underline{\prod}_{1T}$
- $\mathcal{V}_{ansatz} = \sum_{c \in T} \mathcal{V}_{ST}^{Z}(r) \prod_{ST} ST$ + $\sum_{C,T} \mathcal{V}_{ST}^{L2}(\underline{r}) \overset{\mathcal{I}}{\sim}$ + $\sum_{n=1}^{\infty} \frac{1}{2} (\vec{p}^2 \mathcal{V}_{ST}^{p2}(\vec{r}) + \mathcal{V}_{ST}^{p2}(\vec{r}) \vec{p}^2) \Pi_{ST}$ + $\sum \mathcal{V}_{1T}^{LS}(\underline{r}) \overset{\vec{L}S}{\sim} \overset{\vec{\Pi}_{1T}}{\sim} 1_T$ + $\sum_{r} \mathcal{V}_{1T}^{L2LS}(\underline{r}) \overset{\vec{L}}{\sim}$ + $\sum_{\tau} \mathcal{V}_{1T}^{T}(\underline{r}) S_{12} \prod_{\tau} \Pi_{1T}$ + $\sum_{\tau} \mathcal{V}_{1T}^{TLL}(r) \underset{\sim}{s}_{12}(\vec{L}, \vec{L}) \underset{\sim}{\Pi}_{1T}$ + $\sum_{r} \mathcal{V}_{1T}^{Tpp}(\underline{r}) \overline{\underline{s}}_{12}(p_{\Omega}, p_{\Omega}) \underset{\sim}{\prod}_{1T}$ + $\sum (\underbrace{p}_{r} \mathcal{V}_{1T}^{Trp}(\underline{r}) + \mathcal{V}_{1T}^{Trp}(\underline{r}) \underbrace{p}_{r} \underbrace{s}_{12}(r, p_{\Omega}) \prod_{\tau} \mathbf{r}_{\tau}$ $+ \sum V_{1T}^{L2Tpp}(\underline{r})[\underline{\vec{L}}^{2}\overline{s}_{12}(\underline{p}_{\Omega},\underline{p}_{\Omega}) + \overline{s}_{12}(\underline{p}_{\Omega},\underline{p}_{\Omega})\underline{\vec{L}}^{2}] \square_{1T}$

Fit

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$$V_{UCOM} = \sum_{S,T} V_{ST}^{Z}(r) \prod_{i \in ST} V_{ST}$$

- $+ \sum_{S,T} V_{ST}^{L2}(\underline{r}) \widetilde{\underline{L}}^2 \widetilde{\Pi}_{ST}$
- + $\sum_{S,T} \frac{1}{2} (\vec{p}^2 V_{ST}^{p2}(\underline{r}) + V_{ST}^{p2}(\underline{r}) \vec{p}^2) \prod_{ST} ST$
- + $\sum_{T} V_{1T}^{LS}(\underline{r}) \underset{\sim}{\overset{\sim}{\sim}} \underset{\sim}{\overset{\sim}{\sim}} \underset{\sim}{\overset{\Pi}{\sim}} \Pi_{1T}$
- + $\sum_{T} V_{1T}^{L2LS}(\underline{r}) \vec{L}^2 \vec{L} \vec{S} \prod_{1T} \Pi_1 T$
- + $\sum_{T} V_{1T}^{T}(\underline{r}) \underset{\sim}{S}_{12} \prod_{T} T$
- + $\sum_{T}^{\prime} V_{1T}^{TLL}(\underline{r}) \underline{s}_{12}(\vec{L}, \vec{L}) \underline{\Pi}_{1T}$
- $+ \sum_{T} V_{1T}^{T\rho\rho}(\underline{r}) \bar{\underline{s}}_{12}(p_{\Omega}, p_{\Omega}) \bar{\underline{n}}_{1T}$
- $+ \sum_{T} (\underbrace{p_r}_{T} V_{1T}^{Trp}(\underline{r}) + V_{1T}^{Trp}(\underline{r}) \underbrace{p_r}_{\Sigma_{12}} (r, p_{\Omega}) \underbrace{\prod}_{T_T}$
- $+ \sum_{T} V_{1T}^{L2Tpp}(\underline{r})[\underline{\vec{L}}^2 \bar{s}_{12}(\underline{p}_{\Omega}, \underline{p}_{\Omega}) + \bar{s}_{12}(\underline{p}_{\Omega}, \underline{p}_{\Omega})\underline{\vec{L}}^2] [\Pi_{1T}$
- $\mathcal{V}_{ansatz} = \sum_{c \in T} \mathcal{V}_{ST}^{Z}(r) \prod_{ST} ST$ + $\sum_{C,T} \mathcal{V}_{ST}^{L2}(\underline{r}) \overset{\mathcal{I}}{\sim}$ $+ \sum_{\alpha,\tau} \frac{1}{2} (\vec{p}^{2} \mathcal{V}_{ST}^{p2}(r) + \mathcal{V}_{ST}^{p2}(r) \vec{p}^{2}) \Pi_{ST}$ + $\sum_{T} \mathcal{V}_{1T}^{LS}(\underline{r}) \stackrel{\vec{L}}{\sim} \stackrel{\vec{S}}{\sim} \stackrel{\Pi_{1T}}{\sim}$ + $\sum_{r} \mathcal{V}_{1T}^{L2LS}(r) \overset{\vec{L}}{\sim} \overset{\vec$ + $\sum_{\tau} \mathcal{V}_{1T}^{T}(\underline{r}) S_{12} \prod_{\tau} \Pi_{1T}$ + $\sum_{\tau} \mathcal{V}_{1T}^{TLL}(r) \underset{\sim}{s}_{12}(\vec{L}, \vec{L}) \underset{\sim}{\Pi}_{1T}$ + $\sum_{r} \mathcal{V}_{1T}^{Tpp}(\underline{r}) \overline{\underline{s}}_{12}(p_{\Omega}, p_{\Omega}) \underset{\sim}{\prod}_{1T}$ + $\sum (\underbrace{p}_{r} \mathcal{V}_{1T}^{Trp}(\underline{r}) + \mathcal{V}_{1T}^{Trp}(\underline{r}) \underbrace{p}_{r} \underbrace{s}_{12}(r, p_{\Omega}) \prod_{\tau} \mathbf{r}_{\tau}$ + $\sum V_{1T}^{L2Tpp}(\underline{r})[\underline{\vec{L}}^{2}\bar{s}_{12}(\underline{p}_{\Omega},\underline{p}_{\Omega}) + \bar{s}_{12}(\underline{p}_{\Omega},\underline{p}_{\Omega})\underline{\vec{L}}^{2}]\prod_{1T}$

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- $\bigvee_{\sim} UCOM = \sum_{S,T} V_{ST}^{Z}(\underline{r}) \prod_{\sim} ST$
 - $+ \sum_{S,T} V_{ST}^{L2}(\underline{r}) \widetilde{\underline{L}}^2 \widetilde{\Pi}_{ST}$
 - + $\sum_{S,T} \frac{1}{2} (\vec{p}^2 V_{ST}^{p2}(\underline{r}) + V_{ST}^{p2}(\underline{r}) \vec{p}^2) \prod_{ST} ST$
 - $+ \sum_{T} V_{1T}^{LS}(\underline{r}) \widetilde{\underline{LS}} \widetilde{\underline{S}}_{1T}^{\Pi_{1T}}$
 - + $\sum_{T} V_{1T}^{L2LS}(\underline{r}) \overset{\mathcal{I}}{\sim} \overset{\mathcal$
 - + $\sum_{T} V_{1T}^{T}(\underline{r}) \underset{\sim}{S}_{12} \underset{\sim}{\Pi}_{1T}$
 - $+ \sum_{\mathcal{T}} V_{1\mathcal{T}}^{\mathcal{TLL}}(\underline{r}) \underline{s}_{12}(\vec{L},\vec{L}) \underline{\bigcap}_{1\mathcal{T}}$
 - + $\sum_{T} V_{1T}^{Tpp}(\underline{r}) \overline{\underline{s}}_{12}(p_{\Omega}, p_{\Omega}) \underset{\sim}{\prod}_{1T}$
 - + $\sum_{T} (\underline{p}_{r} V_{1T}^{Trp}(\underline{r}) + V_{1T}^{Trp}(\underline{r})\underline{p}_{r}) \underline{s}_{12}(r, p_{\Omega}) \prod_{n \in T} V_{n}$
 - $+ \sum_{T} V_{1T}^{L2Tpp}(\underline{r})[\underline{\vec{L}}^2 \bar{s}_{12}(\underline{p}_{\Omega}, \underline{p}_{\Omega}) + \bar{s}_{12}(\underline{p}_{\Omega}, \underline{p}_{\Omega})\underline{\vec{L}}^2] [\Pi_{1T}$

- $\mathcal{V}_{ansatz} = \sum_{c \in T} \mathcal{V}_{ST}^{Z}(r) \prod_{ST} ST$ + $\sum_{C,T} \mathcal{V}_{ST}^{L2}(\underline{r}) \overset{\mathcal{I}}{\sim}$ + $\sum_{n=1}^{\infty} \frac{1}{2} (\vec{p}^2 \mathcal{V}_{ST}^{p2}(\vec{r}) + \mathcal{V}_{ST}^{p2}(\vec{r}) \vec{p}^2) \Pi_{ST}$ + $\sum \mathcal{V}_{1T}^{LS}(\underline{r}) \overset{\vec{L}S}{\sim} \overset{\vec{\Pi}_{1T}}{\sim} 1_T$ + $\sum_{r} \mathcal{V}_{1T}^{L2LS}(r) \overset{\vec{L}}{\sim} \overset{\vec$ + $\sum \mathcal{V}_{1T}^{T}(\underline{r}) \underset{\sim}{S}_{12} \underset{\sim}{\Pi}_{1T}$ + $\sum_{\tau} \mathcal{V}_{1T}^{TLL}(r) \underset{\sim}{s}_{12}(\vec{L}, \vec{L}) \underset{\sim}{\Pi}_{1T}$
 - + $\sum_{T} \mathcal{V}_{1T}^{Tpp}(\underline{r}) \overline{\underline{s}}_{12}(p_{\Omega}, p_{\Omega}) \underset{\Pi 1T}{\prod}$
 - + $\sum_{T} (\underset{\sim}{p_r} \mathcal{V}_{1T}^{Trp}(\underline{r}) + \mathcal{V}_{1T}^{Trp}(\underline{r}) \underset{\sim}{p_r}) \underset{\sim}{s_{12}} (r, p_{\Omega}) \underset{\sim}{\square}_{1T}$
 - $+ \sum_{T} V_{1T}^{L2T\rhop}(r) [\vec{L}^2 \bar{s}_{12}(p_{\Omega}, p_{\Omega}) + \bar{s}_{12}(p_{\Omega}, p_{\Omega}) \vec{L}^2] [\Pi_1 T]_{T}$

Fit

- $\bigvee_{\sim} UCOM = \sum_{S,T} V_{ST}^{Z}(\underline{r}) \prod_{\sim} ST$
 - $+ \sum_{S,T} V_{ST}^{L2}(\underline{r}) \widetilde{\underline{L}}^2 \widetilde{\Pi}_{ST}$
 - + $\sum_{S,T} \frac{1}{2} (\vec{p}^2 V_{ST}^{p2}(\underline{r}) + V_{ST}^{p2}(\underline{r}) \vec{p}^2) \prod_{ST} ST$
 - $+ \sum_{T} V_{1T}^{LS}(\underline{r}) \overset{\vec{L}}{\sim} \overset{\vec{n}}{\sim} \overset{\vec{n}}{\sim} 1T$
 - + $\sum_{T} V_{1T}^{L2LS}(\underline{r}) \overset{\mathcal{I}}{\sim} \overset{\mathcal$
 - + $\sum_{T} V_{1T}^{T}(\underline{r}) \underset{\sim}{S}_{12} \underset{\sim}{\Pi}_{1T}$
 - $+ \sum_{\mathcal{T}} V_{1\mathcal{T}}^{\mathcal{TLL}}(\underline{r}) \underline{s}_{12}(\vec{L},\vec{L}) \underline{\bigcap}_{1\mathcal{T}}$
 - $+ \sum_{T} V_{1T}^{Tpp}(\underline{r}) \underline{\bar{s}}_{12}(p_{\Omega}, p_{\Omega}) \underline{\bar{n}}_{1T}$
 - + $\sum_{T} (\underline{p}_{r} V_{1T}^{Trp}(\underline{r}) + V_{1T}^{Trp}(\underline{r})\underline{p}_{r}) \underline{s}_{12}(r, p_{\Omega}) \prod_{\tau} V_{1T}$
 - $+ \sum_{T} V_{1T}^{L2Tpp}(\underline{r})[\underline{\vec{L}}^2 \bar{s}_{12}(\underline{p}_{\Omega}, \underline{p}_{\Omega}) + \bar{s}_{12}(\underline{p}_{\Omega}, \underline{p}_{\Omega})\underline{\vec{L}}^2] [\Pi_{1T}$

- $\begin{aligned} \underbrace{\mathcal{V}_{ansatz}}_{S,T} &= \sum_{S,T} \underbrace{\mathcal{V}_{ST}^{Z}(\underline{r})}_{ST} \underbrace{\Pi_{ST}}_{S,T} \\ &+ \sum_{S,T} \underbrace{\mathcal{V}_{ST}^{L2}(\underline{r})}_{ST} \underbrace{\overline{\mathcal{L}}_{ST}^{2}}_{ST} \end{aligned}$
 - $+ \sum_{S,T} \frac{1}{2} (\vec{p}^{\,2} \mathcal{V}_{ST}^{p2}(\underline{r}) + \mathcal{V}_{ST}^{p2}(\underline{r}) (\vec{p}^{\,2}) (\underline{p}^{\,2}) (\underline{r}) ($
 - $+ \sum_{T} \mathcal{V}_{1T}^{LS}(\underline{r}) \underset{\sim}{\vec{\mathcal{L}}} \overset{\sigma}{\overset{}_{\sim}} \overset{\Pi}{\underset{\sim}_{\sim}} \overset{\sigma}{\underset{\sim}_{\sim}} \overset{\sigma}{\underset{\sim}} \overset{\sigma}{\underset{\sim}_{\sim}} \overset{\sigma}{\underset{\sim}_{\sim}} \overset{\sigma}{\underset{\sim}_{\sim}} \overset{\sigma}{\underset{\sim}} \overset{\sigma}{} \overset{\sigma}{\underset{\sim}} \overset{\sigma}{\underset{\sim}} \overset{\sigma}{\underset{\sim}} \overset{\sigma}{\underset{\sim}} \overset{\sigma}{} \overset{\sigma}{\underset{\sim}} \overset{\sigma}{\underset{\sim}} \overset{\sigma}{} \overset{\sigma}{\underset{\sim}} \overset{\sigma}{} \overset{\sigma}$
 - $+ \sum_{T} \mathcal{V}_{1T}^{L2LS}(\underline{r}) \overleftarrow{L}^2 \vec{L} \vec{S} \prod_{n \neq T} \Pi_{1T}$
 - + $\sum_{T} \mathcal{V}_{1T}^{T}(\underline{r}) \underset{\sim}{\lesssim} 12 \underset{\sim}{\Pi}_{1T}$
 - + $\sum_{T} \mathcal{V}_{1T}^{TLL}(\underline{r}) \underline{s}_{12}(\vec{L}, \vec{L}) \prod_{\tau} T$
 - + $\sum_{T} \mathcal{V}_{1T}^{Tpp}(\underline{r}) \overline{\underline{s}}_{12}(p_{\Omega}, p_{\Omega}) \prod_{1T}$
 - $+ \sum_{T} (\underset{\sim}{p_r} \mathcal{V}_{1T}^{Trp}(\underline{r}) + \mathcal{V}_{1T}^{Trp}(\underline{r}) \underset{\sim}{p_r}) \underset{\sim}{s_{12}} (r, p_{\Omega}) \underset{\sim}{\Pi_{1T}}$
 - $+ \sum_{T} V_{1T}^{L2T\rhop}(\underline{r})[\underline{\vec{L}}^2 \bar{\mathbf{s}}_{12}(\underline{p}_{\Omega}, \underline{p}_{\Omega}) + \bar{\mathbf{s}}_{12}(\underline{p}_{\Omega}, \underline{p}_{\Omega})\underline{\vec{L}}^2] \underline{\vec{l}}_{1T} \mathbf{T}$



- fitting ansatz with reduced set of operators to the matrix elements containing full set of operators
- optimized for 'low' angular momenta, contributions from neglected terms 'absorbed' in other terms
- reduced set of operators has to describe two-nucleon data correctly

Reduced UCOM potential NN phase shifts



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Reduced UCOM potential NN phase shifts



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Reduced UCOM potential

Binding energies



Binding energy [MeV]

	² H	³ Н	³ He	⁴ He	⁶ He	⁶ Li	⁷ Li
UCOM	2.23	8.38(1)	7.67(1)	28.53(1)	28.4(2)	31.5(2)	38.6(4)
red. UCOM	2.23	8.37(1)	7.67(1)	28.51(2)	28.6(2)	31.7(2)	38.8(4)
Experiment	2.2246	8.482	7.718	28.296	29.269	31.995	39.245

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Reduced UCOM potential

Spectra and other observables from the NCSM



Properties of ⁶Li

	R _p [fm]	$\mu [\mu_N]$	$Q[e \mathrm{fm}^2]$
UCOM	2.1(1)	0.843(2)	-0.04(2)
red. UCOM	2.1(1)	0.842(1)	-0.03(3)
Experiment	2.41(3)	0.8220	-0.0818(17)

Properties	of ⁷ Li			
	R_p [fm]	$\mu [\mu_N]$	$Q[e \text{ fm}^2]$	L
red. UCOM	2.0(1) 2.0(1) 2.26(2)	2.987(2)	-2.5(3)	

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SRG operator representation Ansatz

- complicated momentum dependence
- ▶ operators of Argonne potential, but $\mathcal{V}_{ST}^{i}(\underline{r}) \rightarrow \mathcal{V}_{ST}^{i}(\underline{r}, p)$

$$\begin{split} \mathcal{V}_{ansatz} &= \sum_{S,T} \mathcal{V}_{ST}^{Z}(\underline{r},\underline{\rho}) \prod_{ST} \\ &+ \sum_{S,T} \mathcal{V}_{ST}^{L2}(\underline{r},\underline{\rho}) \vec{L}^{2} \prod_{ST} \\ &+ \sum_{T} \mathcal{V}_{1T}^{LS}(\underline{r},\underline{\rho}) \vec{L} \vec{S} \prod_{1T} \\ &+ \sum_{T} \frac{1}{2} \left(\sum_{12} \mathcal{V}_{1T}^{T}(\underline{r},\underline{\rho}) + \mathcal{V}_{1T}^{T}(\underline{r},\underline{\rho}) \sum_{12} \sum_{11} \right) \prod_{1T} \\ &+ \sum_{T} \mathcal{V}_{1T}^{TLL}(\underline{r},\underline{\rho}) \mathfrak{s}_{12}(\vec{L},\vec{L}) \prod_{1T} \end{split}$$

SRG transforms each partial wave differently for a given flow parameter: more complicated L and J dependence?

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SRG operator representation NN phase shifts



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SRG operator representation NN phase shifts



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SRG operator representation Binding energies



Binding energy [MeV]

	³ H	³ Н	³ He	⁴ He	⁶ He	⁶ Li	⁷ Li
SRG	2.23	8.35(1)	7.62(1)	28.38(1)	28.9(4)	31.8(3)	39.3(5)
SRG operator	2.23	8.33(1)	7.61(1)	28.41(2)	29.0(5)	31.9(3)	39.9(5)
Experiment	2.2246	8.482	7.718	28.296	29.269	31.995	39.245

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SRG operator representation

Spectra and other observables from the NCSM



Properties of ⁶Li

	R_p [fm]	$\mu [\mu_N]$	$Q[e \mathrm{fm}^2]$
SRG	2.0(2)	0.839(1)	0.01(1)
SRG operator	2.0(1)	0.840(2)	0.02(2)
Experiment	2.41(3)	0.8220	-0.0818(17)

SRG	R_p [fm] 2.0(1)	$\mu [\mu_N]$ 2.983(3)	$Q[e {\rm fm}^2]$
SRG operator Experiment	2.0(1) 2.26(2)	2.997(2) 3.2564	-2.4(3) -4.06(8)

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Summary and conclusions

- Fermionic (Antisymmetric) Molecular Dynamics require operator representation of the interaction
- method to derive an operator representation starting from the partial wave matrix elements of the interaction
- UCOM potential with a reduced set of operators
 - operator form with less operators, but same accuracy in few-nucleon calculations
 - quadratic momentum dependent operators
- operator representation for SRG transformed Argonne potential
 - nonlocal radial functions
 - exact description of low angular momentum phase shifts
 - good agreement with few-nucleon properties calculated with the exact SRG interaction

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