Operator representation for realistic effective interactions

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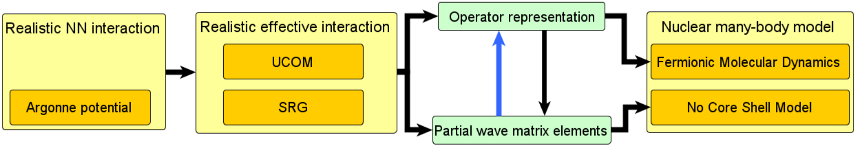
Nuclear *ab-initio* calculations



Outline

1. Nuclear *ab-initio* calculations
   - Argonne potential
   - Unitary Correlation Operator Method
   - Similarity Renormalization Group
   - Fermionic Molecular Dynamics
2. Operator representation from partial wave matrix elements
3. Results
   - UCOM with reduced set of operators
   - SRG operator representation
4. Summary and conclusions

Argonne potential

Wiringa, Stoks, Schiavilla; Phys.Rev.C 51 1995

- Operator representation

$$
\begin{aligned}
\underset{\sim}{V}_{Argonne} &= \sum_{S,T} V^{Z}_{ST}(\underset{\sim}{r})\,\underset{\sim}{\Pi}_{ST} \\
&+ \sum_{S,T} V^{L2}_{ST}(\underset{\sim}{r})\underset{\sim}{\vec{L}}^{2}\,\underset{\sim}{\Pi}_{ST} \\
&+ \sum_{T} V^{LS}_{1T}(\underset{\sim}{r})\underset{\sim}{\vec{L}}\underset{\sim}{\vec{S}}\,\underset{\sim}{\Pi}_{1T} \\
&+ \sum_{T} V^{T}_{1T}(\underset{\sim}{r})\underset{\sim}{S}_{12}\,\underset{\sim}{\Pi}_{1T} \\
&+ \sum_{T} V^{TLL}_{1T}(\underset{\sim}{r})s_{12}(\underset{\sim}{\vec{L}},\underset{\sim}{\vec{L}})\,\underset{\sim}{\Pi}_{1T}
\end{aligned}
$$

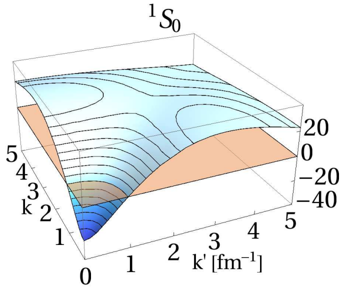
with

$$\underset{\sim}{S}_{12} = \frac{3}{\underset{\sim}{r}^2}\left(\underset{\sim}{\vec{r}}\cdot\underset{\sim}{\vec{\sigma}}_1\right)\left(\underset{\sim}{\vec{r}}\cdot\underset{\sim}{\vec{\sigma}}_2\right) - \underset{\sim}{\vec{\sigma}}_1\underset{\sim}{\vec{\sigma}}_2$$

$$s_{12}(\underset{\sim}{\vec{a}},\underset{\sim}{\vec{b}}) = \frac{3}{2}(\underset{\sim}{\vec{\sigma}}_1\cdot\underset{\sim}{\vec{a}})(\underset{\sim}{\vec{\sigma}}_2\cdot\underset{\sim}{\vec{b}}) - \frac{1}{2}(\underset{\sim}{\vec{\sigma}}_1\cdot\underset{\sim}{\vec{\sigma}}_2)\underset{\sim}{\vec{a}}\cdot\underset{\sim}{\vec{b}} + \underset{\sim}{\vec{a}} \leftrightarrow \underset{\sim}{\vec{b}}$$

- partial wave matrix elements

$$\left\langle\, k(LS)JM;\, TM_T \,\middle|\underset{\sim}{V}_{Argonne}\middle|\, k'(L'S)JM;\, TM_T \,\right\rangle$$



ME's in MeVfm^3

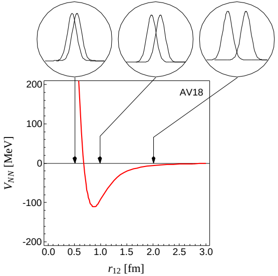
Unitary Correlation Operator Method

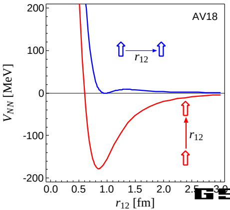
- nuclear interaction induces strong short-range correlations which cannot be described by "simple" many-body states $|\,\psi\,\rangle$
- "Unitary Correlation Operator Method" (UCOM):
  `Feldmeier, Neff, Roth, Schnack; Nucl.Phys. A632 1998`
  `Roth, Neff, Feldmeier; Prog.Part.Nucl.Phys. 65 2010`
  - imprint correlations onto uncorrelated states by unitary operator $\underset{\sim}{C}$:

$$|\,\hat{\psi}\,\rangle = \underset{\sim}{C}\,|\,\psi\,\rangle$$

- correlated operator: $\hat{\underset{\sim}{O}} = \underset{\sim}{C}^{\dagger}\underset{\sim}{O}\,\underset{\sim}{C}$:

$$\langle\,\hat{\psi}\,|\underset{\sim}{O}|\,\hat{\psi}'\,\rangle = \langle\,\psi\,|\underset{\sim}{C}^{\dagger}\underset{\sim}{O}\,\underset{\sim}{C}|\,\psi'\,\rangle = \langle\,\psi\,|\hat{\underset{\sim}{O}}|\,\psi'\,\rangle$$





UCOM potential

Operator representation

operator representation:

$$\underset{\sim}{V}_{UCOM} = \underset{\sim}{C}^\dagger \underset{\sim}{H}\underset{\sim}{C} - \underset{\sim}{T} = \sum_{S,T} V^{Z}_{ST}(\underset{\sim}{r})\,\underset{\sim}{\Pi}_{ST} + \sum_{S,T} V^{L2}_{ST}(\underset{\sim}{r})\underset{\sim}{\vec{L}}^2\underset{\sim}{\Pi}_{ST}$$
$$+ \sum_{S,T} \frac{1}{2}(\underset{\sim}{\vec{p}}^{\,2} V^{p2}_{ST}(\underset{\sim}{r}) + V^{p2}_{ST}(\underset{\sim}{r})\underset{\sim}{\vec{p}}^{\,2})\underset{\sim}{\Pi}_{ST}$$
$$+ \sum_{T} V^{LS}_{1T}(\underset{\sim}{r})\underset{\sim}{\vec{L}}\underset{\sim}{\vec{S}}\,\underset{\sim}{\Pi}_{1T} + \sum_{T} V^{L2LS}_{1T}(\underset{\sim}{r})\underset{\sim}{\vec{L}}^2\underset{\sim}{\vec{L}}\underset{\sim}{\vec{S}}\,\underset{\sim}{\Pi}_{1T}$$
$$+ \sum_{T} V^{T}_{1T}(\underset{\sim}{r})\underset{\sim}{S}_{12}\,\underset{\sim}{\Pi}_{1T} + \sum_{T} V^{TLL}_{1T}(\underset{\sim}{r})s_{12}(\underset{\sim}{\vec{L}},\underset{\sim}{\vec{L}})\,\underset{\sim}{\Pi}_{1T}$$
$$+ \sum_{T} V^{Tpp}_{1T}(\underset{\sim}{r})\bar{s}_{12}(\underset{\sim}{p}_\Omega,\underset{\sim}{p}_\Omega)\underset{\sim}{\Pi}_{1T}$$
$$+ \sum_{T} (\underset{\sim}{p}_r V^{Trp}_{1T}(\underset{\sim}{r}) + V^{Trp}_{1T}(\underset{\sim}{r})\underset{\sim}{p}_r)s_{12}(\underset{\sim}{r},\underset{\sim}{p}_\Omega)\underset{\sim}{\Pi}_{1T}$$
$$+ \sum_{T} V^{L2Tpp}_{1T}(\underset{\sim}{r})[\underset{\sim}{\vec{L}}^2\bar{s}_{12}(\underset{\sim}{p}_\Omega,\underset{\sim}{p}_\Omega) + \bar{s}_{12}(\underset{\sim}{p}_\Omega,\underset{\sim}{p}_\Omega)\underset{\sim}{\vec{L}}^2]\underset{\sim}{\Pi}_{1T}$$
$$+ \cdots$$

- more complicated, nonlocal structure

GSI

Similarity Renormalization Group

- decoupling of low and high momentum matrix elements by evolving potential to band-diagonal structure
- Similarity Renormalization Group (SRG)

  `Bogner, Furnstahl, Perry; Phys.Rev.C 75 2007`

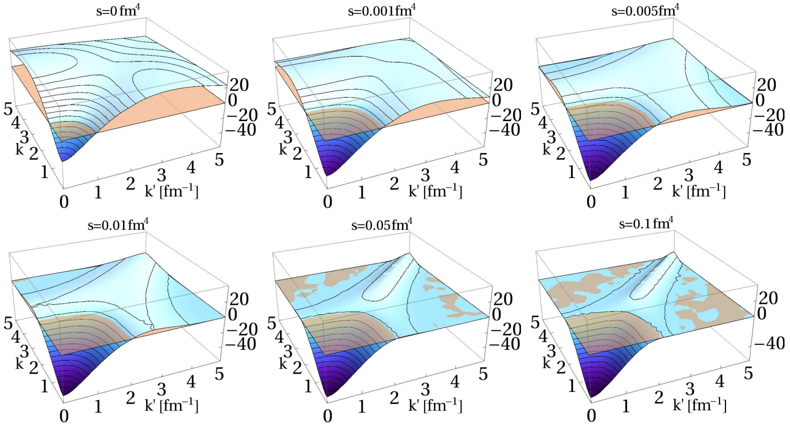
  starting from initial Hamiltonian $\underset{\sim}{H}_0 = \underset{\sim}{T} + \underset{\sim}{V}_0$

  flow equation:

$$\frac{\mathrm{d}\underset{\sim}{H}_s}{\mathrm{d}s} = \left[\underset{\sim}{\eta}_s, \underset{\sim}{H}_s\right]$$

  with flow parameter s and generator $\underset{\sim}{\eta}_s = \left[\underset{\sim}{T}, \underset{\sim}{H}_s\right] = \left[\underset{\sim}{T}, \underset{\sim}{V}_s\right]$
- evolution towards $\left[\underset{\sim}{T}, \underset{\sim}{H}_s\right] = 0$: $\underset{\sim}{H}_s \rightarrow$ band-diagonal

- SRG evolution



- partial wave matrix elements, no operator representation

Fermionic Molecular Dynamics (FMD)

Feldmeier, Nucl.Phys. A515 1990

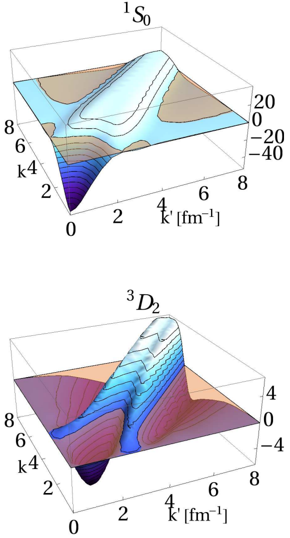
- microscopic model to describe nuclei and nuclear reactions
- model states: antisymmetrized gaussian wave packets:

$$\begin{aligned} \left| Q \right\rangle &= \underset{\sim}{\mathcal{A}}(\left| q_1 \right\rangle \otimes \cdots \otimes \left| q_A \right\rangle) \\ \left\langle \vec{r} \,\middle|\, q \right\rangle &= \sum_i c_i \exp\{\frac{(\vec{r}-\vec{b}_i)^2}{2a_i}\} \otimes \left| \chi_i^\uparrow, \chi_i^\downarrow \right\rangle \otimes \left| \xi \right\rangle \end{aligned}$$

  - complex parameters $\vec{b}_i$ encode mean position and momentum
  - width parameter a_i
- description of exotic phenomena like halos, cluster-states, …
- interaction matrix elements in FMD basis calculated analytically: operator representation needed!

From partial wave matrix elements
to operator representation

partial wave matrix elements



Fit

$\gamma^i_{ST,j}$

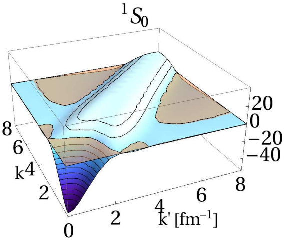
operator representation

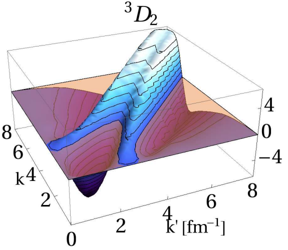
- choose appropriate set of operators

$$\underset{\sim}{V}_{ansatz} = \sum_{ST,i} \mathcal{V}^i_{ST}(\underset{\sim}{r}, \underset{\sim}{p})\, \underset{\sim}{\mathcal{O}}_i \underset{\sim}{\Pi}_{ST}$$

From partial wave matrix elements
to operator representation

partial wave matrix elements





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- representation of (unknown!) radial functions by a sum of gaussians:

$$\mathcal{V}^i_{ST}(\underset{\sim}{r}) = \sum_k \gamma^i_{ST,\,k} \cdot e^{-\frac{\underset{\sim}{\vec{r}}^{\,2}}{2\kappa_k}}$$

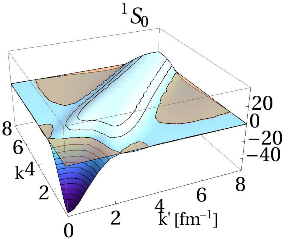
$$\mathcal{V}^i_{ST}(\underset{\sim}{r}, \underset{\sim}{p}) =$$

$$\sum_{k,l} \gamma^i_{ST,\,kl} \cdot e^{-\frac{\lambda_l}{4}\underset{\sim}{\vec{p}}^{\,2}}\; e^{\frac{-\underset{\sim}{\vec{r}}^{\,2}}{2(\kappa_k - \lambda_l/4)}}\; e^{-\frac{\lambda_l}{4}\underset{\sim}{\vec{p}}^{\,2}}$$

- choose parameters κ_j and λ_l on a grid

From partial wave matrix elements
to operator representation

partial wave matrix elements



Fit

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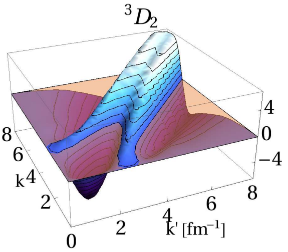

operator representation

- choose appropriate set of operators

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- representation of (unknown!) radial functions by a sum of gaussians:

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$$\mathcal{V}^i_{ST}(\underset{\sim}{r},\underset{\sim}{p}) = \sum_{k,l} \gamma^i_{ST,\,kl} \cdot e^{-\frac{\lambda_l}{4}\underset{\sim}{\vec{p}}^{\,2}}\; e^{\frac{-\underset{\sim}{\vec{r}}^{\,2}}{2(\kappa_k-\lambda_l/4)}}\; e^{-\frac{\lambda_l}{4}\underset{\sim}{\vec{p}}^{\,2}}$$

- choose parameters κ_j and λ_l on a grid
- calculate analytically matrix elements $\langle\, k(LS)JT \,|\underset{\sim}{V}_{ansatz}|\, k'(L'S)JT \,\rangle$

UCOM - with a reduced set of operators

$$
\begin{aligned}
\underset{\sim}{V}_{UCOM} = & \sum_{S,T} V^{Z}_{ST}(\underset{\sim}{r})\,\underset{\sim}{\Pi}_{ST} \\
+ & \sum_{S,T} V^{L2}_{ST}(\underset{\sim}{r})\,\underset{\sim}{\vec{L}}^{2}\,\underset{\sim}{\Pi}_{ST} \\
+ & \sum_{S,T} \frac{1}{2}\left(\underset{\sim}{\vec{p}}^{\,2}\, V^{p2}_{ST}(\underset{\sim}{r}) + V^{p2}_{ST}(\underset{\sim}{r})\,\underset{\sim}{\vec{p}}^{\,2}\right)\underset{\sim}{\Pi}_{ST} \\
+ & \sum_{T} V^{LS}_{1T}(\underset{\sim}{r})\,\underset{\sim}{\vec{L}}\underset{\sim}{\vec{S}}\,\underset{\sim}{\Pi}_{1T} \\
+ & \sum_{T} V^{L2LS}_{1T}(\underset{\sim}{r})\,\underset{\sim}{\vec{L}}^{2}\,\underset{\sim}{\vec{L}}\underset{\sim}{\vec{S}}\,\underset{\sim}{\Pi}_{1T} \\
+ & \sum_{T} V^{T}_{1T}(\underset{\sim}{r})\,\underset{\sim}{S}_{12}\,\underset{\sim}{\Pi}_{1T} \\
+ & \sum_{T} V^{TLL}_{1T}(\underset{\sim}{r})\,\underset{\sim}{s}_{12}(\vec{L},\vec{L})\,\underset{\sim}{\Pi}_{1T} \\
+ & \sum_{T} V^{Tpp}_{1T}(\underset{\sim}{r})\,\underset{\sim}{\bar{s}}_{12}(p_{\Omega},p_{\Omega})\,\underset{\sim}{\Pi}_{1T} \\
+ & \sum_{T} \left(\underset{\sim}{p}_{r}\, V^{Trp}_{1T}(\underset{\sim}{r}) + V^{Trp}_{1T}(\underset{\sim}{r})\,\underset{\sim}{p}_{r}\right)\underset{\sim}{s}_{12}(r,p_{\Omega})\,\underset{\sim}{\Pi}_{1T} \\
+ & \sum_{T} V^{L2Tpp}_{1T}(\underset{\sim}{r})\left[\underset{\sim}{\vec{L}}^{2}\,\bar{s}_{12}(\underset{\sim}{p}_{\Omega},\underset{\sim}{p}_{\Omega}) + \bar{s}_{12}(\underset{\sim}{p}_{\Omega},\underset{\sim}{p}_{\Omega})\,\underset{\sim}{\vec{L}}^{2}\right]\underset{\sim}{\Pi}_{1T} \\
+ & \cdots
\end{aligned}
$$

Fit

$$
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+ & \cdots
\end{aligned}
$$

UCOM - with a reduced set of operators

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+ & \sum_{S,T} \frac{1}{2}(\underset{\sim}{\vec{p}}^{\,2} V^{p2}_{ST}(\underset{\sim}{r}) + V^{p2}_{ST}(\underset{\sim}{r})\underset{\sim}{\vec{p}}^{\,2})\underset{\sim}{\Pi}_{ST} \\
+ & \sum_{T} V^{LS}_{1T}(\underset{\sim}{r})\underset{\sim}{\vec{L}}\underset{\sim}{\vec{S}}\,\underset{\sim}{\Pi}_{1T} \\
+ & \sum_{T} V^{L2LS}_{1T}(\underset{\sim}{r})\underset{\sim}{\vec{L}}^{2}\underset{\sim}{\vec{L}}\underset{\sim}{\vec{S}}\,\underset{\sim}{\Pi}_{1T} \\
+ & \sum_{T} V^{T}_{1T}(\underset{\sim}{r})\underset{\sim}{S}_{12}\,\underset{\sim}{\Pi}_{1T} \\
+ & \sum_{T} V^{TLL}_{1T}(\underset{\sim}{r})\underset{\sim}{s}_{12}(\vec{L},\vec{L})\,\underset{\sim}{\Pi}_{1T} \\
+ & \sum_{T} V^{Tpp}_{1T}(\underset{\sim}{r})\underset{\sim}{\bar{s}}_{12}(p_{\Omega},p_{\Omega})\underset{\sim}{\Pi}_{1T} \\
+ & \sum_{T} (\underset{\sim}{p}_{r} V^{Trp}_{1T}(\underset{\sim}{r}) + V^{Trp}_{1T}(\underset{\sim}{r})\underset{\sim}{p}_{r})\underset{\sim}{s}_{12}(r,p_{\Omega})\underset{\sim}{\Pi}_{1T} \\
+ & \sum_{T} V^{L2Tpp}_{1T}(\underset{\sim}{r})[\underset{\sim}{\vec{L}}^{2}\bar{s}_{12}(\underset{\sim}{p}_{\Omega},\underset{\sim}{p}_{\Omega}) + \bar{s}_{12}(\underset{\sim}{p}_{\Omega},\underset{\sim}{p}_{\Omega})\underset{\sim}{\vec{L}}^{2}]\underset{\sim}{\Pi}_{1T} \\
+ & \cdots
\end{aligned}
$$

Fit

$$
\begin{aligned}
\underset{\sim}{V}_{ansatz} = & \sum_{S,T} \mathcal{V}^{Z}_{ST}(\underset{\sim}{r})\,\underset{\sim}{\Pi}_{ST} \\
+ & \sum_{S,T} \mathcal{V}^{L2}_{ST}(\underset{\sim}{r})\underset{\sim}{\vec{L}}^{2}\underset{\sim}{\Pi}_{ST} \\
+ & \sum_{S,T} \frac{1}{2}(\underset{\sim}{\vec{p}}^{\,2} \mathcal{V}^{p2}_{ST}(\underset{\sim}{r}) + \mathcal{V}^{p2}_{ST}(\underset{\sim}{r})\underset{\sim}{\vec{p}}^{\,2})\underset{\sim}{\Pi}_{ST} \\
+ & \sum_{T} \mathcal{V}^{LS}_{1T}(\underset{\sim}{r})\underset{\sim}{\vec{L}}\underset{\sim}{\vec{S}}\,\underset{\sim}{\Pi}_{1T} \\
+ & \sum_{T} \mathcal{V}^{L2LS}_{1T}(\underset{\sim}{r})\underset{\sim}{\vec{L}}^{2}\underset{\sim}{\vec{L}}\underset{\sim}{\vec{S}}\,\underset{\sim}{\Pi}_{1T} \\
+ & \sum_{T} \mathcal{V}^{T}_{1T}(\underset{\sim}{r})\underset{\sim}{S}_{12}\,\underset{\sim}{\Pi}_{1T} \\
+ & \sum_{T} \mathcal{V}^{TLL}_{1T}(\underset{\sim}{r})\underset{\sim}{s}_{12}(\vec{L},\vec{L})\,\underset{\sim}{\Pi}_{1T} \\
+ & \sum_{T} \mathcal{V}^{Tpp}_{1T}(\underset{\sim}{r})\underset{\sim}{\bar{s}}_{12}(p_{\Omega},p_{\Omega})\underset{\sim}{\Pi}_{1T} \\
+ & \sum_{T} (\underset{\sim}{p}_{r} \mathcal{V}^{Trp}_{1T}(\underset{\sim}{r}) + \mathcal{V}^{Trp}_{1T}(\underset{\sim}{r})\underset{\sim}{p}_{r})\underset{\sim}{s}_{12}(r,p_{\Omega})\underset{\sim}{\Pi}_{1T} \\
+ & \sum_{T} V^{L2Tpp}_{1T}(\underset{\sim}{r})[\underset{\sim}{\vec{L}}^{2}\bar{s}_{12}(\underset{\sim}{p}_{\Omega},\underset{\sim}{p}_{\Omega}) + \bar{s}_{12}(\underset{\sim}{p}_{\Omega},\underset{\sim}{p}_{\Omega})\underset{\sim}{\vec{L}}^{2}]\underset{\sim}{\Pi}_{1T} \\
+ & \cdots
\end{aligned}
$$

UCOM - with a reduced set of operators

$$
\begin{aligned}
\underset{\sim}{V}_{UCOM} = & \sum_{S,T} V^{Z}_{ST}(\underset{\sim}{r})\,\underset{\sim}{\Pi}_{ST} \\
& + \sum_{S,T} V^{L2}_{ST}(\underset{\sim}{r})\,\underset{\sim}{\vec{L}}^{2}\,\underset{\sim}{\Pi}_{ST} \\
& + \sum_{S,T} \frac{1}{2}\left(\underset{\sim}{\vec{p}}^{\,2} V^{p2}_{ST}(\underset{\sim}{r}) + V^{p2}_{ST}(\underset{\sim}{r})\,\underset{\sim}{\vec{p}}^{\,2}\right)\underset{\sim}{\Pi}_{ST} \\
& + \sum_{T} V^{LS}_{1T}(\underset{\sim}{r})\,\underset{\sim}{\vec{L}}\underset{\sim}{\vec{S}}\,\underset{\sim}{\Pi}_{1T} \\
& + \sum_{T} V^{L2LS}_{1T}(\underset{\sim}{r})\,\underset{\sim}{\vec{L}}^{2}\,\underset{\sim}{\vec{L}}\underset{\sim}{\vec{S}}\,\underset{\sim}{\Pi}_{1T} \\
& + \sum_{T} V^{T}_{1T}(\underset{\sim}{r})\,\underset{\sim}{S}_{12}\,\underset{\sim}{\Pi}_{1T} \\
& + \sum_{T} V^{TLL}_{1T}(\underset{\sim}{r})\,\underset{\sim}{s}_{12}(\vec{L},\vec{L})\,\underset{\sim}{\Pi}_{1T} \\
& + \sum_{T} V^{Tpp}_{1T}(\underset{\sim}{r})\,\underset{\sim}{\bar{s}}_{12}(p_{\Omega},p_{\Omega})\,\underset{\sim}{\Pi}_{1T} \\
& + \sum_{T} \left(\underset{\sim}{p}_{r} V^{Trp}_{1T}(\underset{\sim}{r}) + V^{Trp}_{1T}(\underset{\sim}{r})\,\underset{\sim}{p}_{r}\right)\underset{\sim}{s}_{12}(r,p_{\Omega})\,\underset{\sim}{\Pi}_{1T} \\
& + \sum_{T} V^{L2Tpp}_{1T}(\underset{\sim}{r})\left[\underset{\sim}{\vec{L}}^{2}\,\bar{s}_{12}(\underset{\sim}{p}_{\Omega},\underset{\sim}{p}_{\Omega}) + \bar{s}_{12}(\underset{\sim}{p}_{\Omega},\underset{\sim}{p}_{\Omega})\,\underset{\sim}{\vec{L}}^{2}\right]\underset{\sim}{\Pi}_{1T} \\
& + \cdots
\end{aligned}
$$

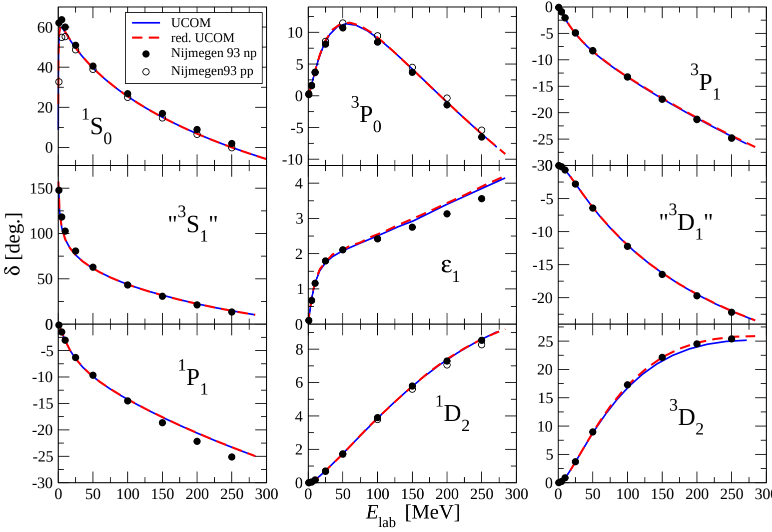
Fit

$$
\begin{aligned}
\underset{\sim}{V}_{ansatz} = & \sum_{S,T} \mathcal{V}^{Z}_{ST}(\underset{\sim}{r})\,\underset{\sim}{\Pi}_{ST} \\
& + \sum_{S,T} \mathcal{V}^{L2}_{ST}(\underset{\sim}{r})\,\underset{\sim}{\vec{L}}^{2}\,\underset{\sim}{\Pi}_{ST} \\
& + \sum_{S,T} \frac{1}{2}\left(\underset{\sim}{\vec{p}}^{\,2} \mathcal{V}^{p2}_{ST}(\underset{\sim}{r}) + \mathcal{V}^{p2}_{ST}(\underset{\sim}{r})\,\underset{\sim}{\vec{p}}^{\,2}\right)\underset{\sim}{\Pi}_{ST} \\
& + \sum_{T} \mathcal{V}^{LS}_{1T}(\underset{\sim}{r})\,\underset{\sim}{\vec{L}}\underset{\sim}{\vec{S}}\,\underset{\sim}{\Pi}_{1T} \\
& + \sum_{T} \mathcal{V}^{L2LS}_{1T}(\underset{\sim}{r})\,\underset{\sim}{\vec{L}}^{2}\,\underset{\sim}{\vec{L}}\underset{\sim}{\vec{S}}\,\underset{\sim}{\Pi}_{1T} \\
& + \sum_{T} \mathcal{V}^{T}_{1T}(\underset{\sim}{r})\,\underset{\sim}{S}_{12}\,\underset{\sim}{\Pi}_{1T} \\
& + \sum_{T} \mathcal{V}^{TLL}_{1T}(\underset{\sim}{r})\,\underset{\sim}{s}_{12}(\vec{L},\vec{L})\,\underset{\sim}{\Pi}_{1T} \\
& + \sum_{T} \mathcal{V}^{Tpp}_{1T}(\underset{\sim}{r})\,\underset{\sim}{\bar{s}}_{12}(p_{\Omega},p_{\Omega})\,\underset{\sim}{\Pi}_{1T} \\
& + \sum_{T} \left(\underset{\sim}{p}_{r} \mathcal{V}^{Trp}_{1T}(\underset{\sim}{r}) + \mathcal{V}^{Trp}_{1T}(\underset{\sim}{r})\,\underset{\sim}{p}_{r}\right)\underset{\sim}{s}_{12}(r,p_{\Omega})\,\underset{\sim}{\Pi}_{1T} \\
& + \sum_{T} \mathcal{V}^{L2Tpp}_{1T}(\underset{\sim}{r})\left[\underset{\sim}{\vec{L}}^{2}\,\bar{s}_{12}(\underset{\sim}{p}_{\Omega},\underset{\sim}{p}_{\Omega}) + \bar{s}_{12}(\underset{\sim}{p}_{\Omega},\underset{\sim}{p}_{\Omega})\,\underset{\sim}{\vec{L}}^{2}\right]\underset{\sim}{\Pi}_{1T} \\
& + \cdots
\end{aligned}
$$

- fitting ansatz with reduced set of operators to the matrix elements containing full set of operators
- optimized for 'low' angular momenta, contributions from neglected terms 'absorbed' in other terms
- reduced set of operators has to describe two-nucleon data correctly

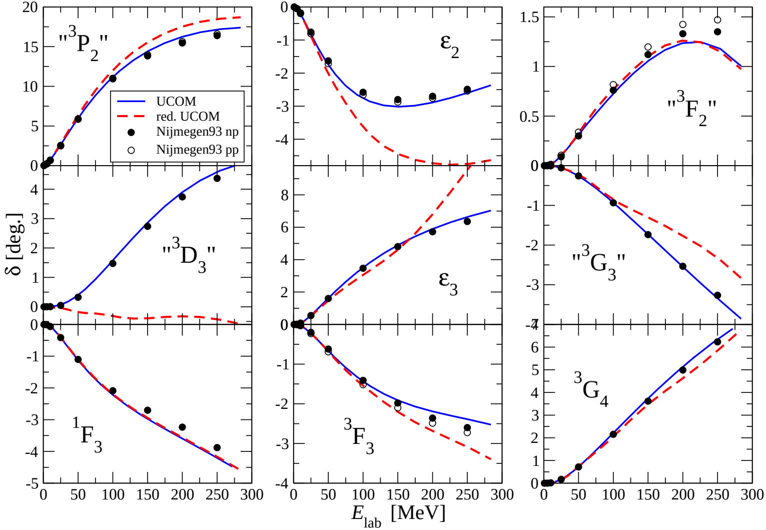
Reduced UCOM potential

NN phase shifts



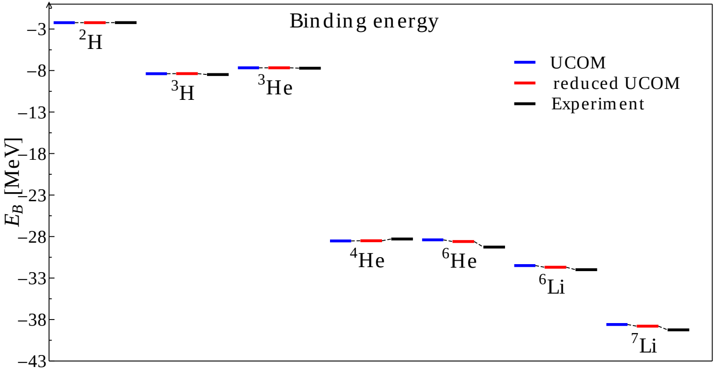
Reduced UCOM potential

NN phase shifts



Reduced UCOM potential

Binding energies

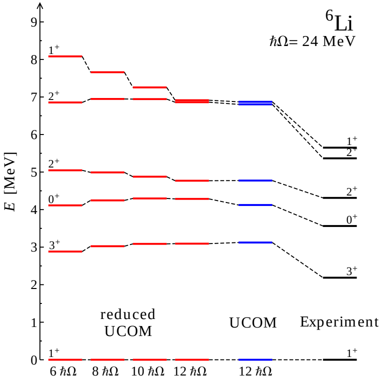


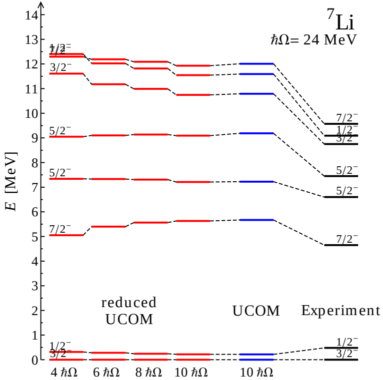
Binding energy [MeV]

	^{2}H	^{3}H	^{3}He	^{4}He	^{6}He	^{6}Li	^{7}Li
UCOM	2.23	8.38(1)	7.67(1)	28.53(1)	28.4(2)	31.5(2)	38.6(4)
red. UCOM	2.23	8.37(1)	7.67(1)	28.51(2)	28.6(2)	31.7(2)	38.8(4)
Experiment	2.2246	8.482	7.718	28.296	29.269	31.995	39.245

Reduced UCOM potential

Spectra and other observables from the NCSM





Properties of ^{6}Li

	R_p [fm]	μ [μ_N]	Q [e fm^2]
UCOM	2.1(1)	0.843(2)	-0.04(2)
red. UCOM	2.1(1)	0.842(1)	-0.03(3)
Experiment	2.41(3)	0.8220	-0.0818(17)

Properties of ^{7}Li

	R_p [fm]	μ [μ_N]	Q [e fm^2]
UCOM	2.0(1)	2.988(3)	-2.6(3)
red. UCOM	2.0(1)	2.987(2)	-2.5(3)
Experiment	2.26(2)	3.2564	-4.06(8)

SRG operator representation

Ansatz

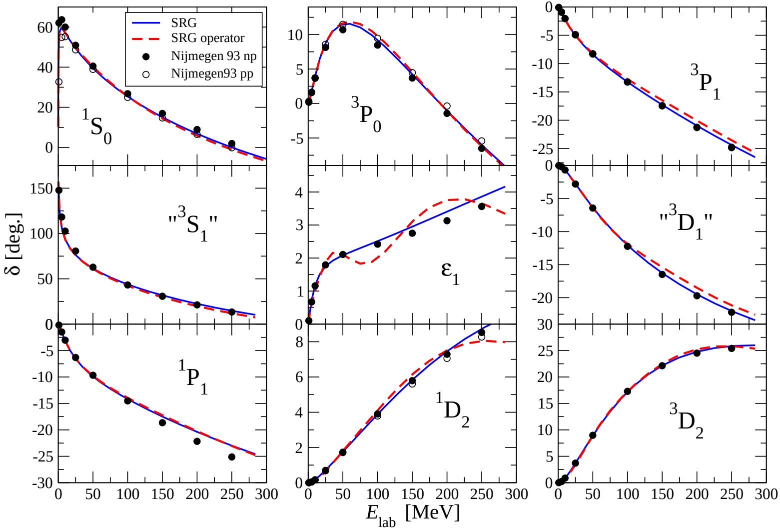
- complicated momentum dependence
- operators of Argonne potential, but $\mathcal{V}^{i}_{ST}(\underset{\sim}{r}) \to \mathcal{V}^{i}_{ST}(\underset{\sim}{r},\underset{\sim}{p})$

$$
\begin{aligned}
\underset{\sim}{V}_{ansatz} &= \sum_{S,T} \mathcal{V}^{Z}_{ST}(\underset{\sim}{r},\underset{\sim}{p})\,\underset{\sim}{\Pi}_{ST} \\
&+ \sum_{S,T} \mathcal{V}^{L2}_{ST}(\underset{\sim}{r},\underset{\sim}{p})\underset{\sim}{\vec{L}}^{2}\,\underset{\sim}{\Pi}_{ST} \\
&+ \sum_{T} \mathcal{V}^{LS}_{1T}(\underset{\sim}{r},\underset{\sim}{p})\underset{\sim}{\vec{L}}\underset{\sim}{\vec{S}}\,\underset{\sim}{\Pi}_{1T} \\
&+ \sum_{T} \frac{1}{2}\left(\underset{\sim}{S}_{12}\mathcal{V}^{T}_{1T}(\underset{\sim}{r},\underset{\sim}{p}) + \mathcal{V}^{T}_{1T}(\underset{\sim}{r},\underset{\sim}{p})\,\underset{\sim}{S}_{12}\right)\underset{\sim}{\Pi}_{1T} \\
&+ \sum_{T} \mathcal{V}^{TLL}_{1T}(\underset{\sim}{r},\underset{\sim}{p})s_{12}(\underset{\sim}{\vec{L}},\underset{\sim}{\vec{L}})\,\underset{\sim}{\Pi}_{1T}
\end{aligned}
$$

- SRG transforms each partial wave differently for a given flow parameter: more complicated L and J dependence?

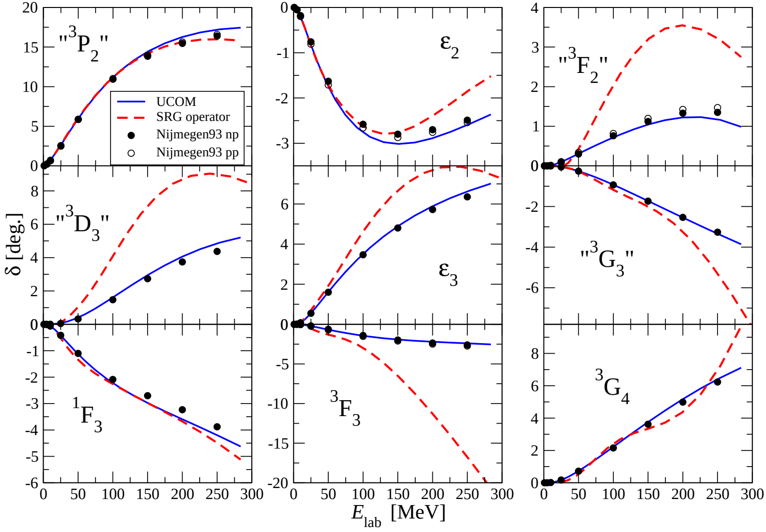
SRG operator representation

NN phase shifts



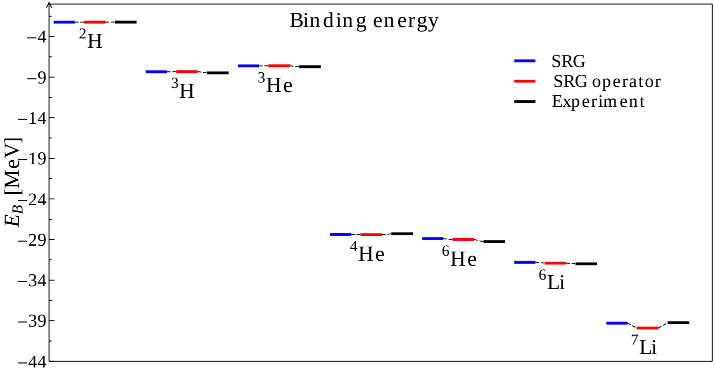
SRG operator representation

NN phase shifts



SRG operator representation

Binding energies

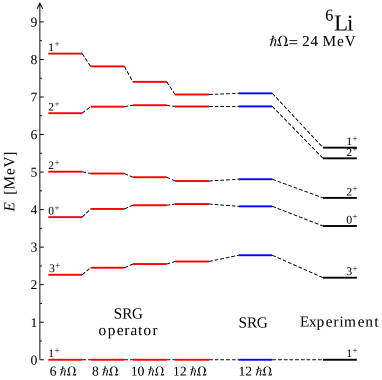


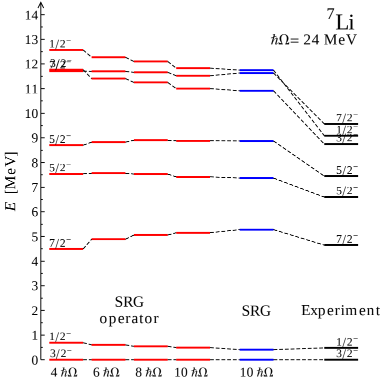
Binding energy [MeV]

	^{3}H	^{3}H	^{3}He	^{4}He	^{6}He	^{6}Li	^{7}Li
SRG	2.23	8.35(1)	7.62(1)	28.38(1)	28.9(4)	31.8(3)	39.3(5)
SRG operator	2.23	8.33(1)	7.61(1)	28.41(2)	29.0(5)	31.9(3)	39.9(5)
Experiment	2.2246	8.482	7.718	28.296	29.269	31.995	39.245

SRG operator representation

Spectra and other observables from the NCSM





Properties of ^{6}Li

	R_p [fm]	μ [μ_N]	Q [e fm^2]
SRG	2.0(2)	0.839(1)	0.01(1)
SRG operator	2.0(1)	0.840(2)	0.02(2)
Experiment	2.41(3)	0.8220	-0.0818(17)

Properties of ^{7}Li

	R_p [fm]	μ [μ_N]	Q [e fm^2]
SRG	2.0(1)	2.983(3)	-2.4(2)
SRG operator	2.0(1)	2.997(2)	-2.4(3)
Experiment	2.26(2)	3.2564	-4.06(8)

Summary and conclusions

- Fermionic (Antisymmetric) Molecular Dynamics require operator representation of the interaction
- method to derive an operator representation starting from the partial wave matrix elements of the interaction
- UCOM potential with a reduced set of operators
  - operator form with less operators, but same accuracy in few-nucleon calculations
  - quadratic momentum dependent operators
- operator representation for SRG transformed Argonne potential
  - nonlocal radial functions
  - exact description of low angular momentum phase shifts
  - good agreement with few-nucleon properties calculated with the exact SRG interaction