The Phase Diagram of Two-Color QCD

Hirschegg, 20 January 2012

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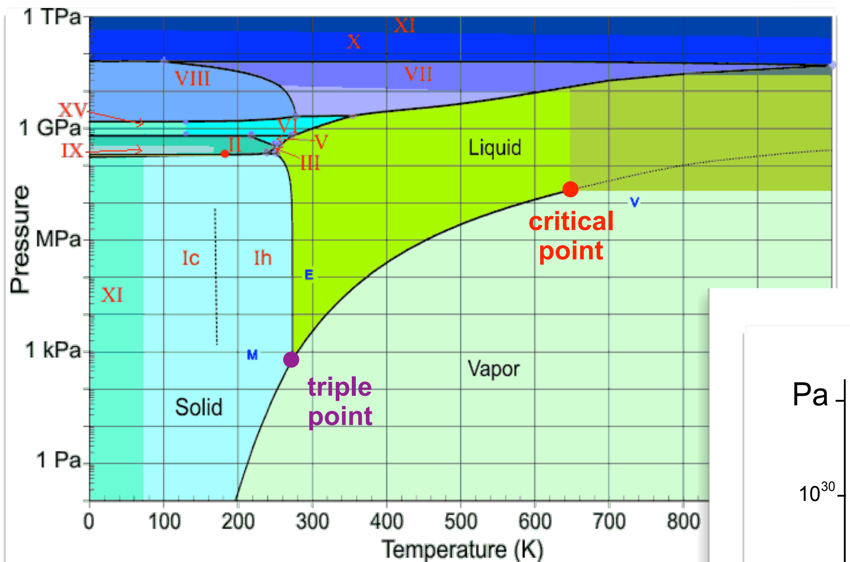
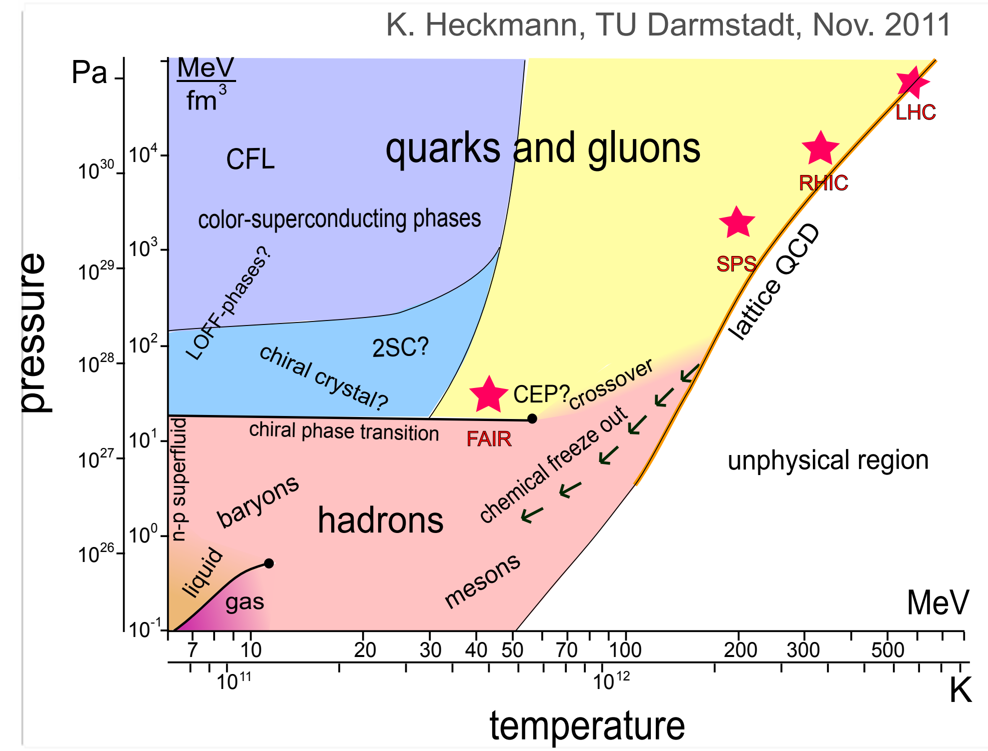
Contents

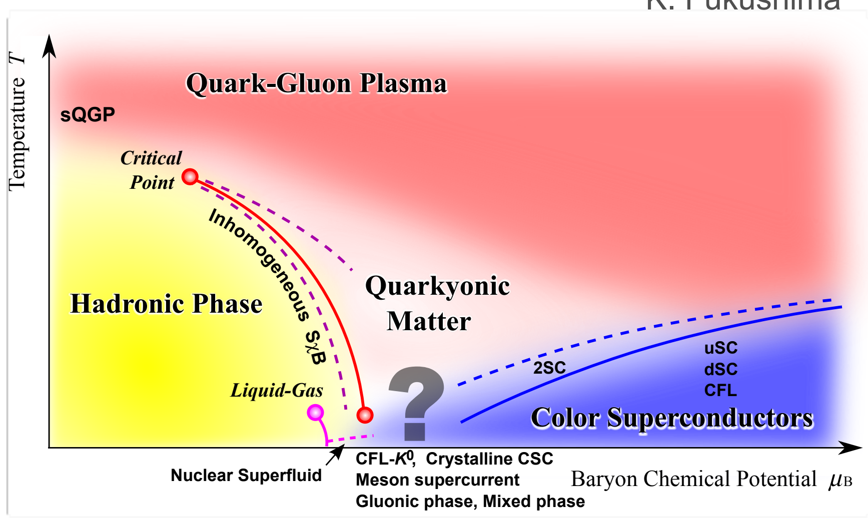
Phase Diagram

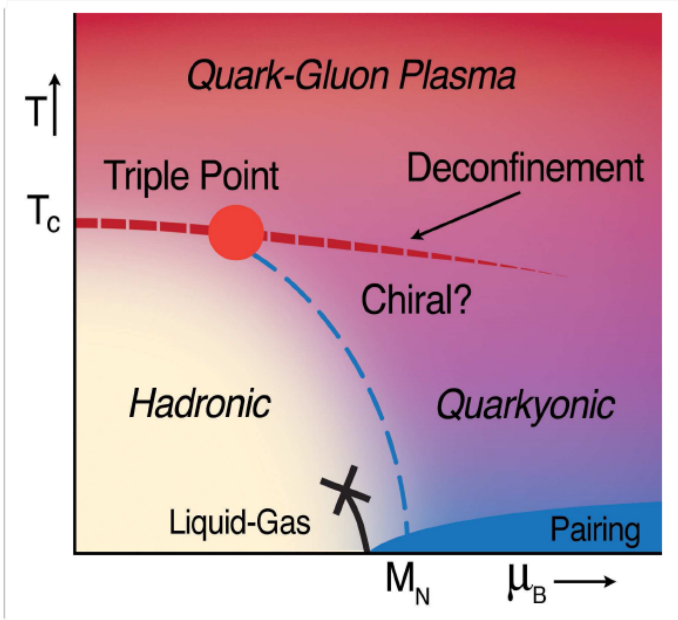


http://www.lsbu.ac.uk/water/phase.html



QCD Phase Diagram





A. Andronic *et al.*, Nucl. Phys. A837 (2010) 65

For large number of colors N_c with

fixed number of flavors N_f or $N_f \sim N_c$ also large

L. McLerran & R. Pisarski, Nucl. Phys. A796 (2007) 83

Y. Hidaka, L. McLerran, R. Pisarski, Nucl. Phys. A808 (2008) 117

Here, consider $N_f = N_c = 2\ \ldots$

2-Color QCD

SU(2) is pseudo-real:

Dirac operator $\mathcal{D}$ has antiunitary symmetry T, with $T^2 = 1$ (also at $\mu \neq 0$).

- **no sign problem**

χPT: Kogut, Stephanov, Toublan, Verbaarschot & Zhitnitsky, Nucl. Phys. B 582 (2000) 477

Lattice: Hands, Montvay, Scorzato & Skullerud, Eur. Phys. J. C 22 (2001) 451

Hands, Kenny, Kim & Skullerud, Eur. Phys. J. A 47 (2011) 60

NJL: Ratti & Weise, Phys. Rev. D 70 (2004) 054013

PNJL: Brauner, Fukushima & Hidaka, Phys. Rev. D 80 (2009) 074035

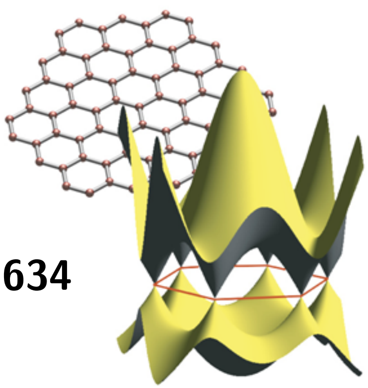
PQMD: Strodthoff, Schaefer, LvS, arXiv:1112.5401

- **extended flavor symmetry (Pauli-Gürsey), at $\mu = 0$**

$SU(N_f) \times SU(N_f) \times U(1)$ becomes $SU(2N_f)$

$N_f = 2$: connects pions and σ-meson with scalar (anti)diquarks.

Compare: even number of fermions in (2+1) dimensions:
QED_3 (semimetal-insulator transition, $N_f < 4$),
electronic properties of Graphene (half-filling, $N_f = 2$) – **SFB 634**
but also QCD_3 or generally SU(N_c) with $N_c \geq 3$.



Quark-Meson-Diquark model for QC_2D

- **color-singlet diquarks (bosonic baryons)**
- ***N_f*=2: Explicit (spontaneous) breaking pattern by Dirac mass (quark condensate),**

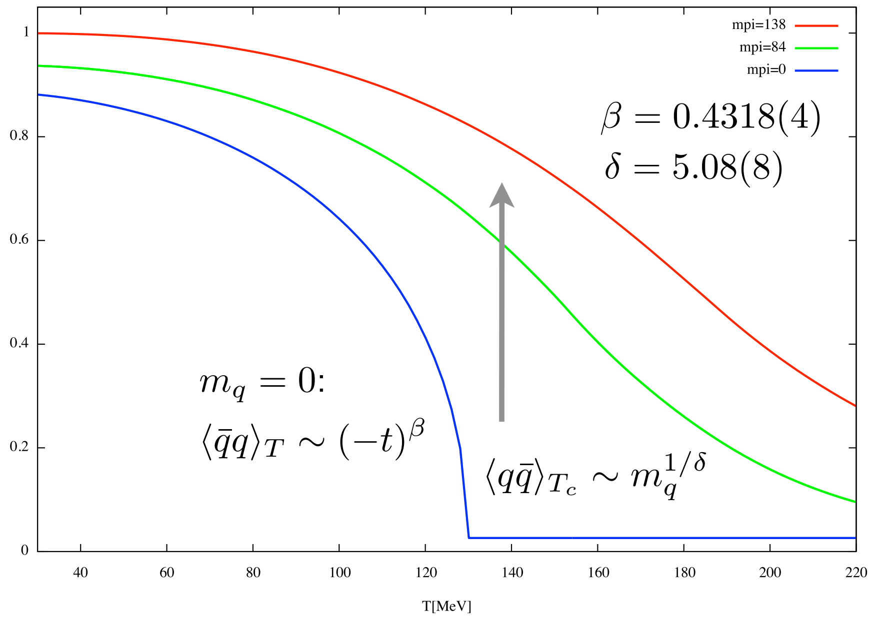
$$SU(4) \to Sp(2)$$

or $$SO(6) \to SO(5)$$

Parameter space: 15 dimensional $\to$ 10 dimensional

Coset: S^5

5 Goldstone bosons: pions and scalar (anti)diquarks



- ***O*(6) universality class:**

FRG, LPA: $\beta = 0.432,\ \delta = 5$

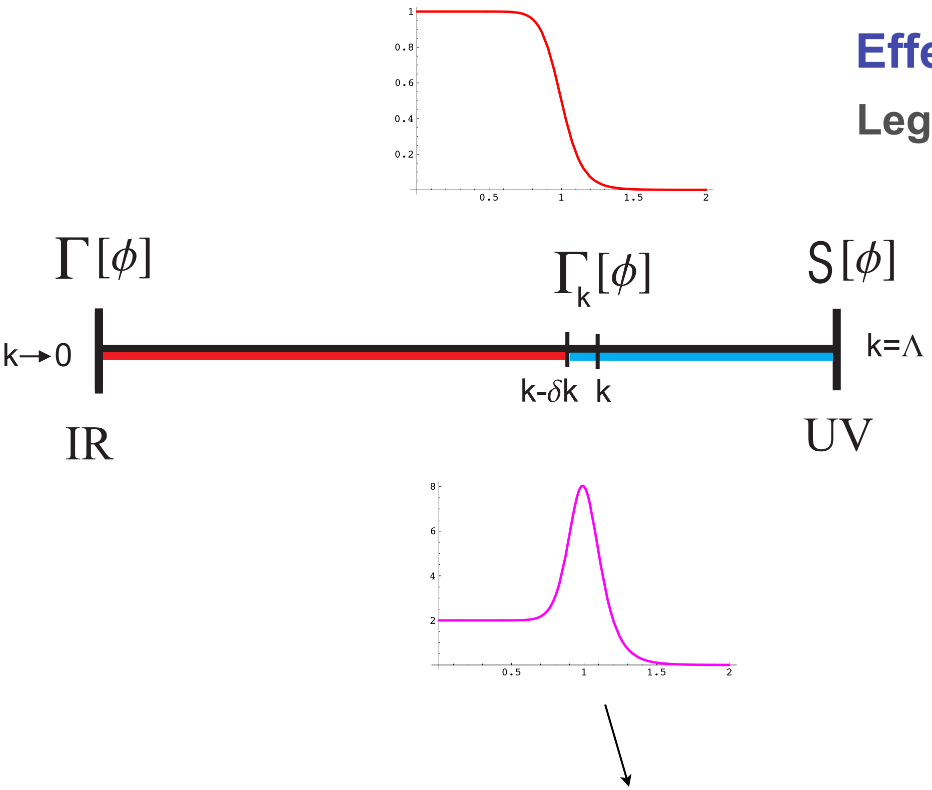
Litim, NPB 631 (2002) 128

Lattice: $\beta = 0.425(2),\ \delta = 4.77(4)$

Holtmann, Schulze, PRE 68 (2003) 036111

FRG beyond LPA in progress

Functional RG (Flow) Equations



Effective action: $\Gamma[\phi_j] = (j, \phi_j) - \ln Z[j]$

Legendre transform

→ **1PI vertex functions**

$\Gamma^{(n)}(x_1, \dots x_n)$

→ **free energy with**

$\phi_j = \langle \phi \rangle_j$

$$k\partial_k \Gamma_k[\Phi] = \frac{1}{2} \mathrm{Tr} \left\{ \partial_t R_k \, (\Gamma_k^{(2)} + R_k)^{-1} \right\}$$

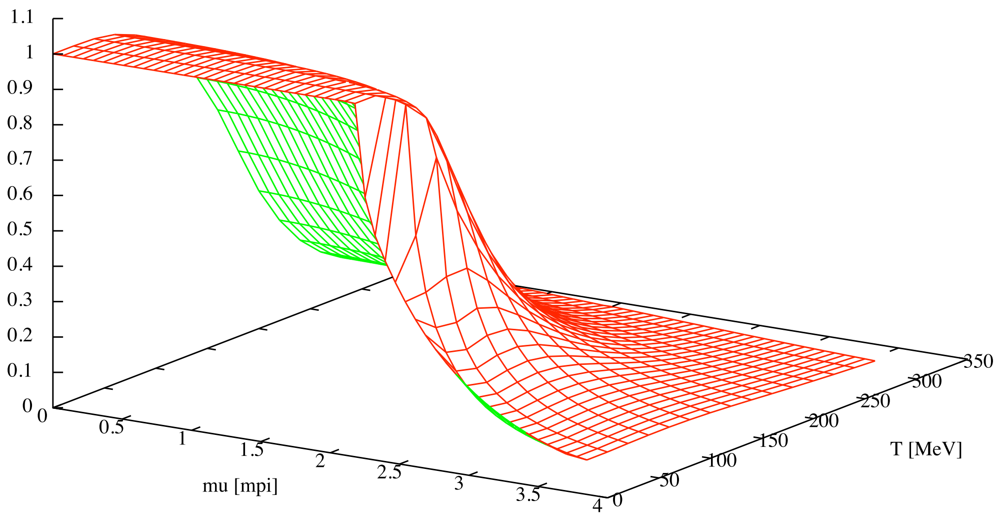
Wetterich, Phys. Lett. B 301 (1993) 90

Phase Diagram - SU(4) Symmetric

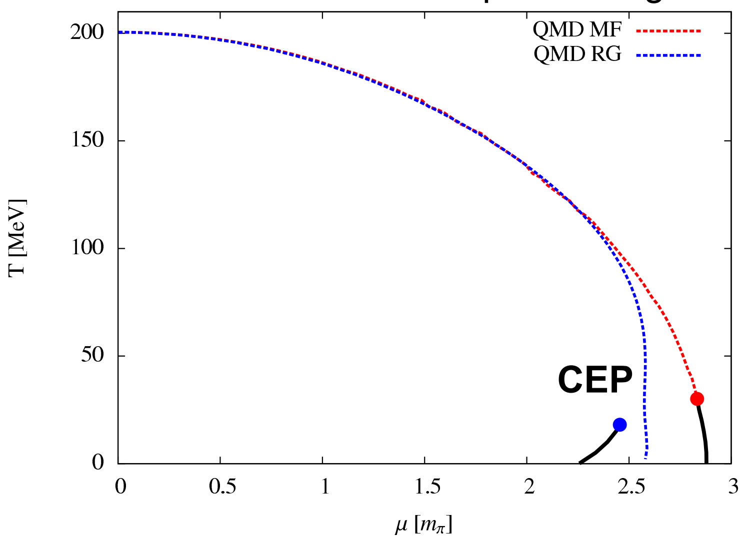
- **no diquark condensation, flow equation for 1 dim field variable, O(6) symmetric potential**

$$U = U(\phi^2) \text{ where } \vec{\phi} = (\sigma, \vec{\pi}, \text{Re}\Delta, \text{Im}\Delta)$$

$$\frac{\langle \bar{q}q \rangle}{\langle \bar{q}q \rangle_0}$$



phase diagram



- 1st order chiral transition and CEP at $\mu \approx 2.5\, m_\pi$

Phase Diagram - SU(4) Symmetric

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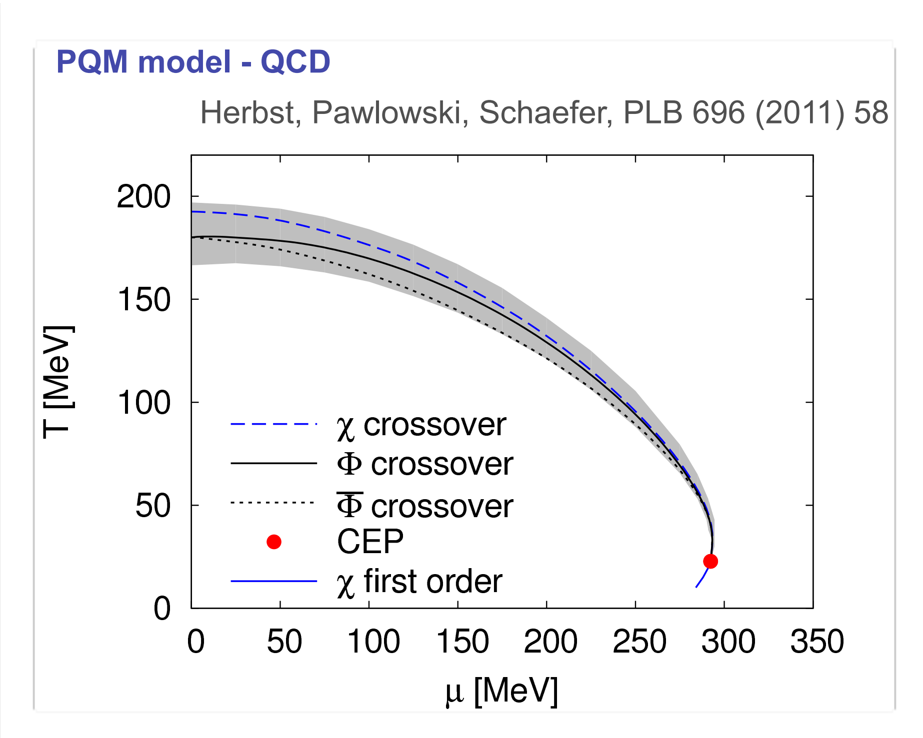
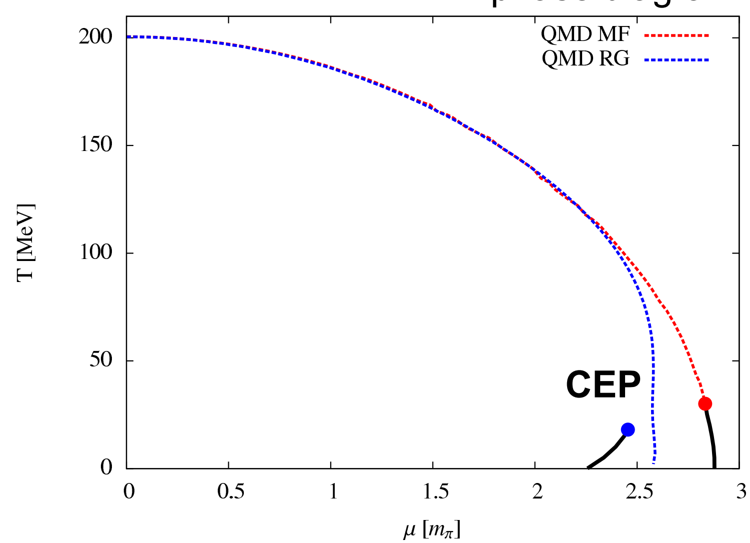


phase diagram



- 1st order chiral transition and CEP at $\mu \approx 2.5\, m_\pi$

- **but wait! need to properly include dynamics of our bosonic baryons....**

2-Color QCD at Finite Density

- **finite chemical potential μ:**

$N_f = 2,\ m_q = \mu = 0:$ $\quad SU(4) \simeq SO(6)$

$m_q \neq 0:$ $\quad SO(5)$ $\qquad\qquad$ $\mu \neq 0:$ $\quad SO(4) \times SO(2)$

$m_q,\ \mu \neq 0:$ $\quad SO(3) \times SO(2)$

$\mu_B > m_\pi$: BEC $\to$ BCS

$U(1)_B$ breaks spontaneously at $\mu_B = m_\pi$

- **χPT: vacuum alignment,**

$2\mu = \mu_B < m_\pi$: $\quad \langle \bar{q}q \rangle \neq 0,\ \langle qq \rangle = 0,\ \langle \bar{q}q \rangle$-like,

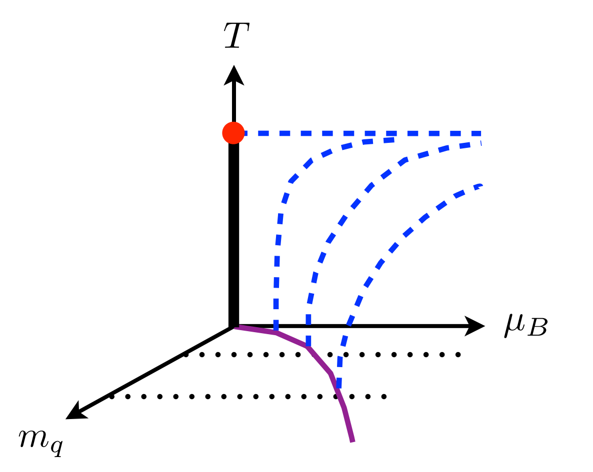
$2\mu = \mu_B > m_\pi$: $\quad \langle \bar{q}q \rangle \propto \left(\frac{m_\pi}{\mu_B}\right)^2,\ \langle qq \rangle \propto \sqrt{1 - \left(\frac{m_\pi}{\mu_B}\right)^4},$

turns $\langle qq \rangle$-like.

- **need 2 field variables in effective potential**

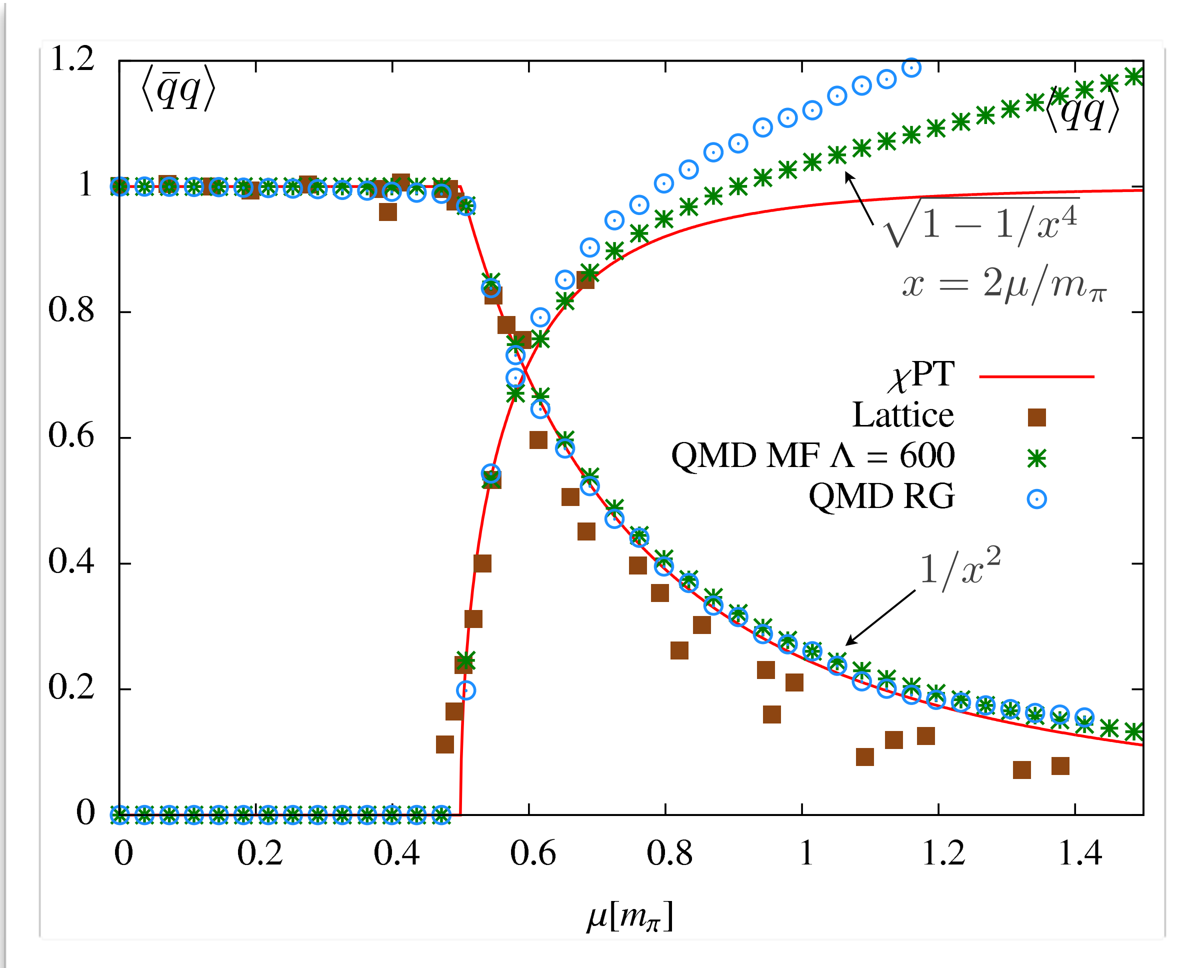
$U = U(\rho^2, d^2)$ where $\vec{\rho} = (\sigma, \vec{\pi})$ and $d^2 = |\Delta|^2$

phase diagram



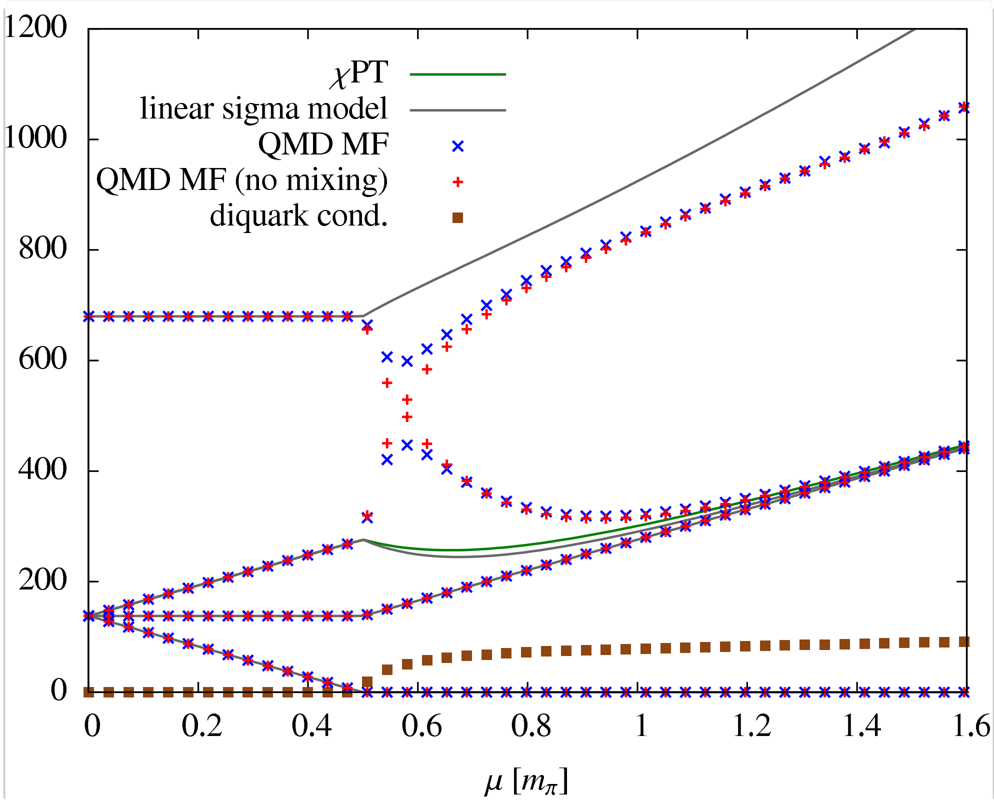
Vacuum Alignment, *T*=0

- Diquark condensation at $2\mu = \mu_B = m_\pi$



Vacuum Alignment, *T*=0

- **RPA pole masses, QMD model:**



- **PNJL model:**

  Brauner, Fukushima & Hidaka, Phys. Rev. D 80 (2009) 074035

- **NJL with isospin chemical potential:**

  He, Jin & Zhuang, Phys. Rev. D 71 (2005) 116001

  Xiong, Jin & Li, J. Phys. G 36 (2009) 125005

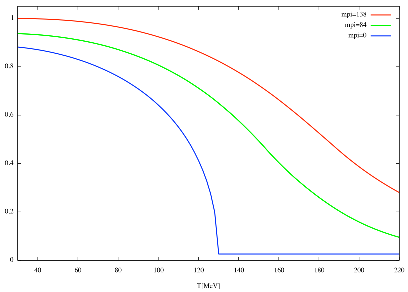
- **Functional RG:**

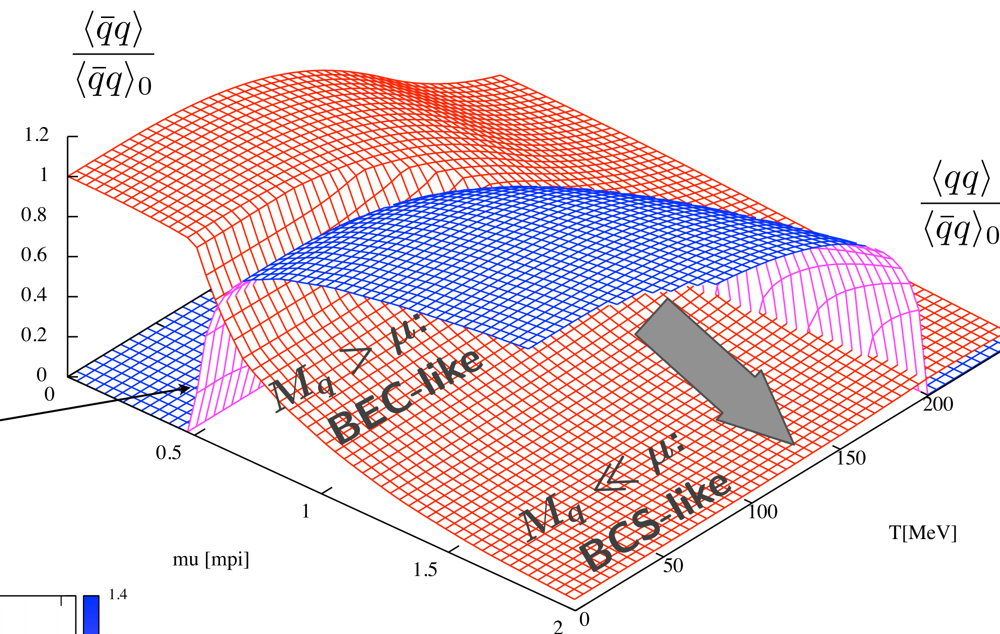
  Kazuhiko Kamikado's talk on Tue

Quark-Meson-Diquark Model

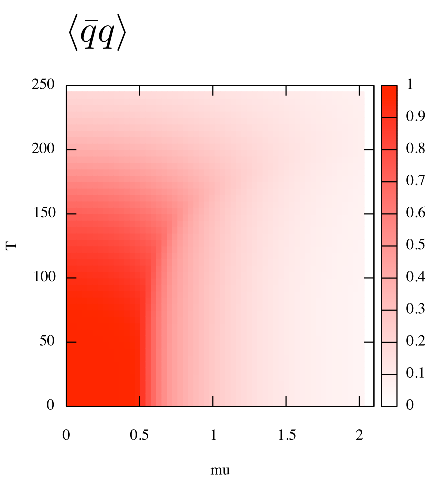
- **Finite T and μ:**

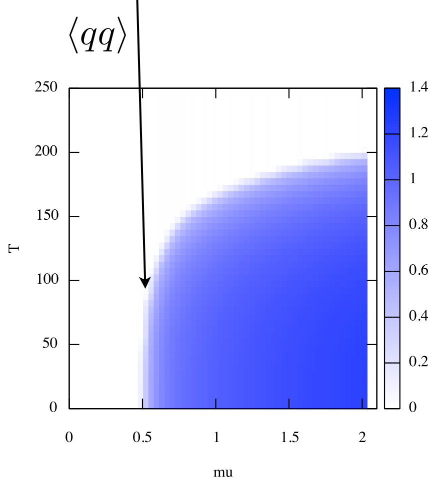
along $\mu = 0$ axis as before





baryon density
rises abruptly

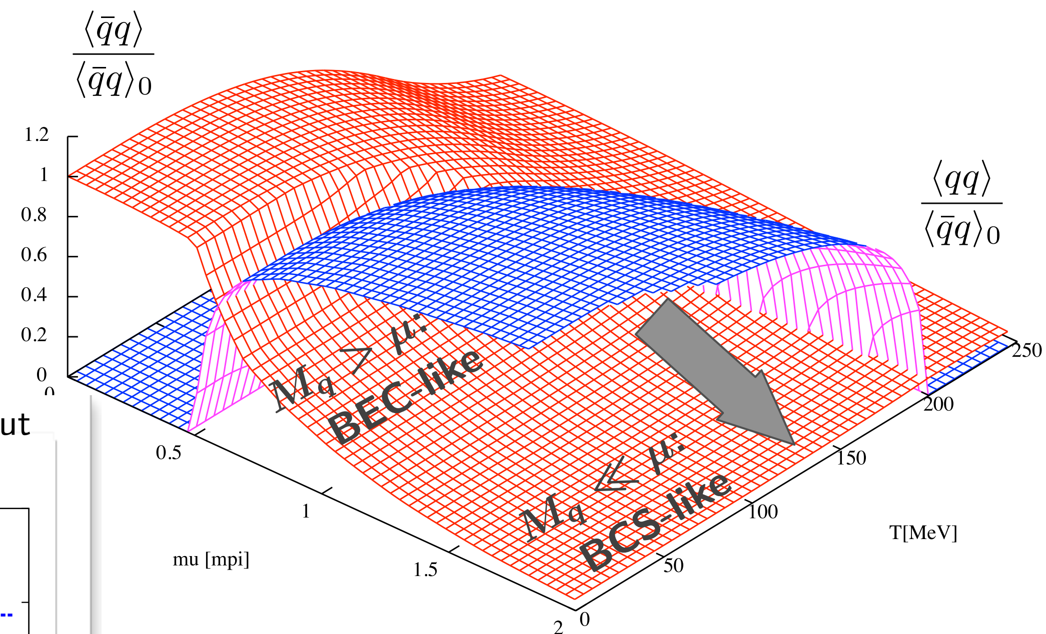




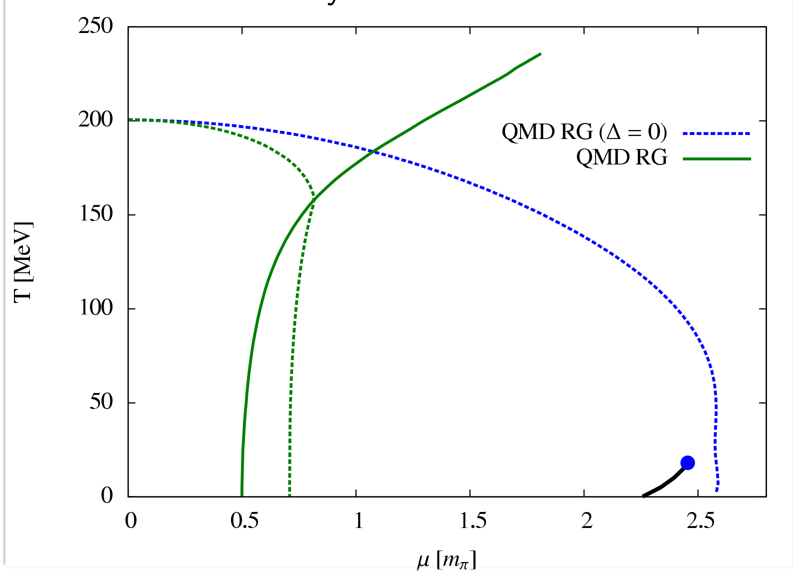
baryon density:
order parameter for $N_c = N_f = 2$
as it is for $N_c = N_f \to \infty$

Quark-Meson-Diquark Model

- **Finite T and μ:**



compare phase diagram with and without collective baryonic fluctuations



no low-T 1^{st} order transition,

no CEP at $\mu \sim 2.5\ m_\pi$!

PhD projects: Nils Strodthoff (FRG), David Scheffler (Lattice MC, GPUs), with C. Schmidt & D. Smith

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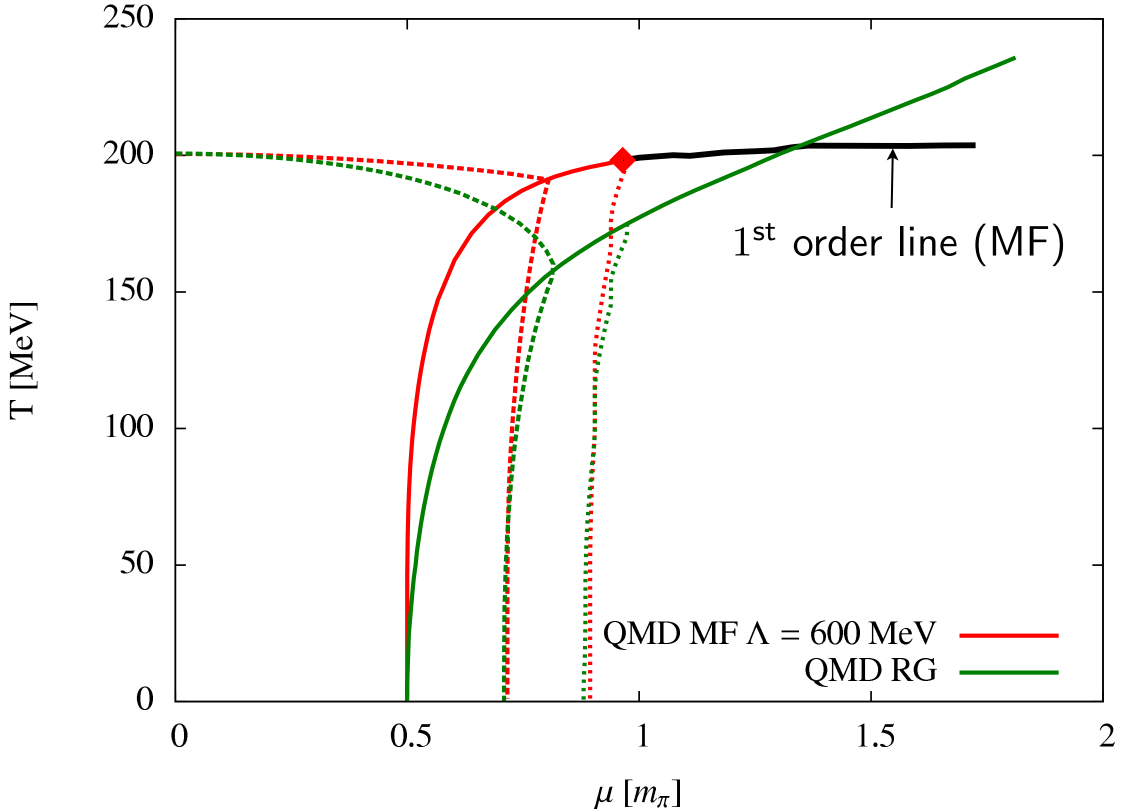
Quark-Meson-Diquark Model

- **Functional RG vs mean field:**

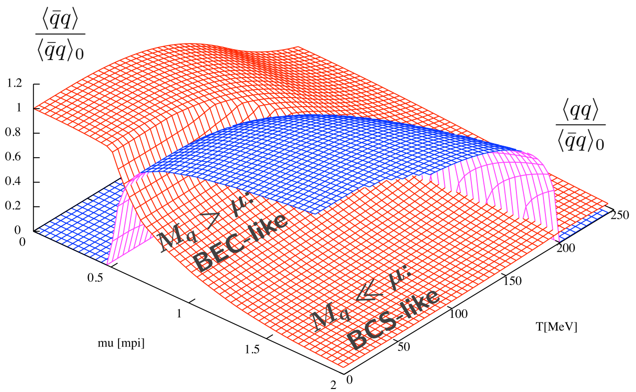


- Tricritical point predicted in:

  Splittorff, Toublan & Verbaarschot, Nucl. Phys. B 620 (2002) 290

Summary & Outlook

- **Phase Diagram of Two-Color QCD**
  - need to include baryonic fluctuations
  - functional methods and lattice MC



- **QCD with isospin chemical potential**
  - equivalent problem
- **Phase Diagram of G_2 Gauge Theory**
  - no sign problem − fermionic baryons
- **Fermions in 2+1 Dimensions**
  - quantum phase transitions, transport properties, topological aspects...
- **QCD Phase Diagram**
  - refined functional methods & models, baryonic dofs, finite volume...

Thank You for Your Attention!