Dynamical light vector mesons in low-energy scattering of Goldstone bosons

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The purpose of our work is to apply the novel scheme [A.Gasparyan, M.F.M.Lutz Nucl. Phys. A 848, 126 (2010)] to Goldstone boson scattering, based on the SU(3) chiral Lagrangian with light vector mesons.

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Resonance saturation mechanism

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Hadrogenesis conjecture

For instance the leading chiral interaction of Goldstone bosons with light vector mesons generates an axial-vector meson spectrum [M.F.M.Lutz, E.E.Kolomeitsev, Nucl. Phys. A **730** (2004) 392].

Chiral Lagrangian (No unknown parameters!)

The relevant terms of the chiral Lagrangian for the Goldstone bosons Φ (π, K, $\bar{K}$, η) and vector mesons $V_{\mu\nu}$ ($\rho_{\mu\nu}$, $\omega_{\mu\nu}$, $K_{\mu\nu}$, $\bar{K}_{\mu\nu}$, ϕ)

$$
\begin{aligned}
\mathcal{L} \;=\;& \frac{1}{4}\,\mathrm{tr}\left\{\partial^\mu\Phi\,\partial_\mu\Phi\right\} - \frac{1}{4}\,\mathrm{tr}\left\{\partial^\mu V_{\mu\alpha}\,\partial_\nu V^{\nu\alpha}\right\} + \frac{1}{8}\,m_{1^-}^2\,\mathrm{tr}\left\{V^{\mu\nu}\,V_{\mu\nu}\right\} \\
-\;& \frac{1}{4}\,\mathrm{tr}\left\{\Phi^2\,\chi_0\right\} + \frac{1}{48f^2}\,\mathrm{tr}\left\{\Phi^4\,\chi_0\right\} + \frac{1}{8}\,b_D\,\mathrm{tr}\left\{V^{\mu\nu}\,V_{\mu\nu}\,\chi_0\right\} \\
+\;& \frac{1}{48f^2}\,\mathrm{tr}\left\{[\Phi,\,\partial^\mu\Phi]_-\,[\Phi,\,\partial_\mu\Phi]_-\right\} - i\,\frac{m_V h_P}{8f^2}\,\mathrm{tr}\left\{\partial_\mu\Phi\,V^{\mu\nu}\,\partial_\nu\Phi\right\},
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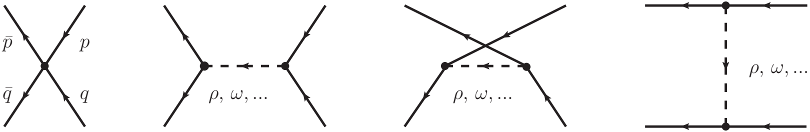
\end{aligned}
$$

All parameters were fixed before, for instance in [M.F.M.Lutz, S.Leupold, Nucl.Phys.A813 (2008), 51-71]

$$
\begin{aligned}
m_{1^-} \simeq 0.76\ \mathrm{GeV}, \quad & b_D \simeq 0.95\,, \\
m_V\, h_p \simeq 0.22\ \mathrm{GeV}, \quad & f \simeq 90\,\mathrm{MeV}\,.
\end{aligned}
$$

Partial-wave projection of the scatting amplitude



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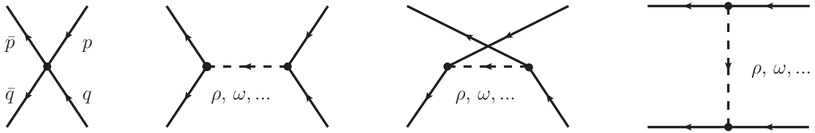

Partial-wave amplitudes are introduced by an average

$$T^J(s) = \int_{-1}^{+1} \frac{d\cos\theta}{2} \left(\frac{\bar{p}_{\rm cm}\, p_{\rm cm}}{s} \right)^J T(s,t,u)\, P_J(\cos\theta)\,,$$

over the center-of mass scattering angle θ.

Partial-wave dispersion relation

The partial-wave dispersion relation

$$T^J_{ab}(s) = U^J_{ab}(s) + \sum_{c,d} \int_{\mu^2_{thr}}^{\infty} \frac{d\bar{s}}{\pi} \frac{s-\mu^2_M}{\bar{s}-\mu^2_M} \frac{T^J_{ac}(\bar{s})\, \rho^J_{cd}(\bar{s})\, T^{J*}_{db}(\bar{s})}{\bar{s}-s-i\epsilon} \,,$$

- separate left and right-hand cuts
- the generalized potential $U^J_{ab}(s)$ contains all left hand cuts

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The phase space fuction

$$\Im\, T^J_{ab}(s) = \sum_{c,d} T^J_{ac}(s)\, \rho^J_{cd}(s)\, T^{J*}_{db}(\bar{s}), \quad \rho^J_{ab}(s) = \frac{1}{8\pi} \left(\frac{p_{cm}}{\sqrt{s}}\right)^{2J+1} \delta_{ab}$$

Approximation for the generalized potential $U_{ab}^{J}(s)$

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Reliable extrapolation is possible:

Conformal mapping techniques may be used to approximate $U^J_{ab}(s)$ for ($s > \mu^2_{thr}$), based on $U^J_{ab}(s)$ only around threshold μ^2_{thr}.

Conformal mapping technique

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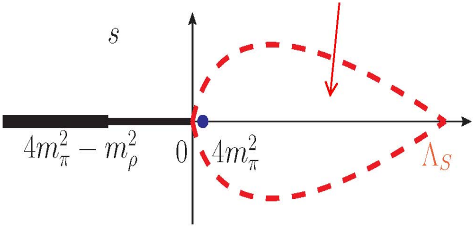
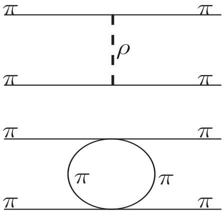
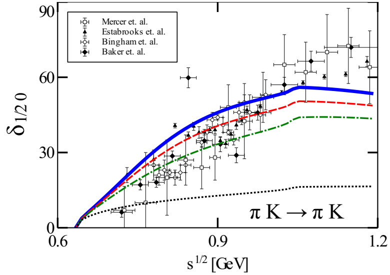
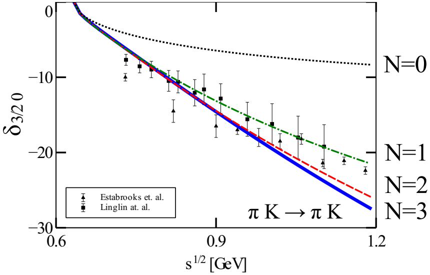
Reliable approximation is within this area

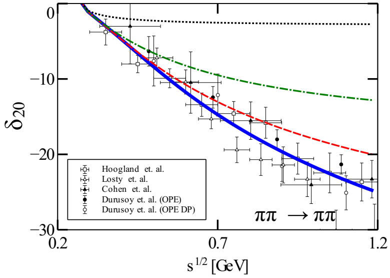




J=0: The role of the vector meson exchange







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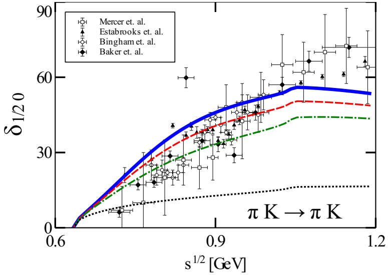

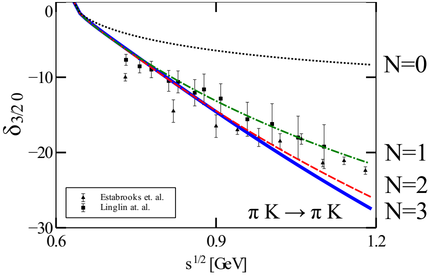

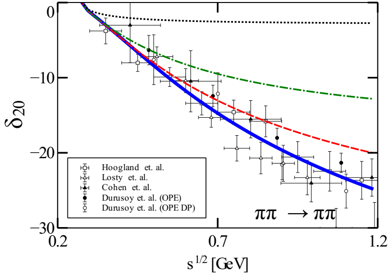

- $U^J_{ab}(s) = \sum_{k=0}^{N} C_k \, [\xi(s)]^k$

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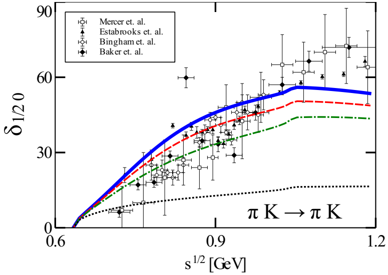

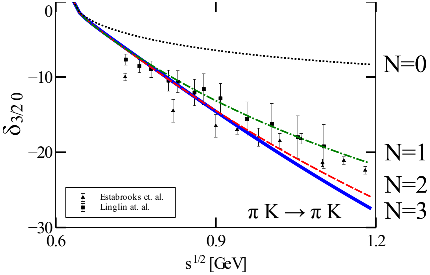

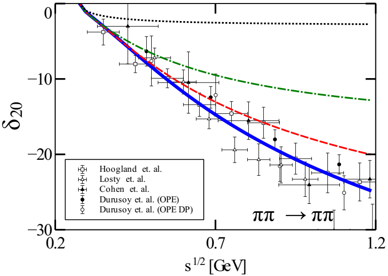

- $U^J_{ab}(s) = \sum_{k=0}^{N} C_k \, [\xi(s)]^k$
- No free parameters adjusted!

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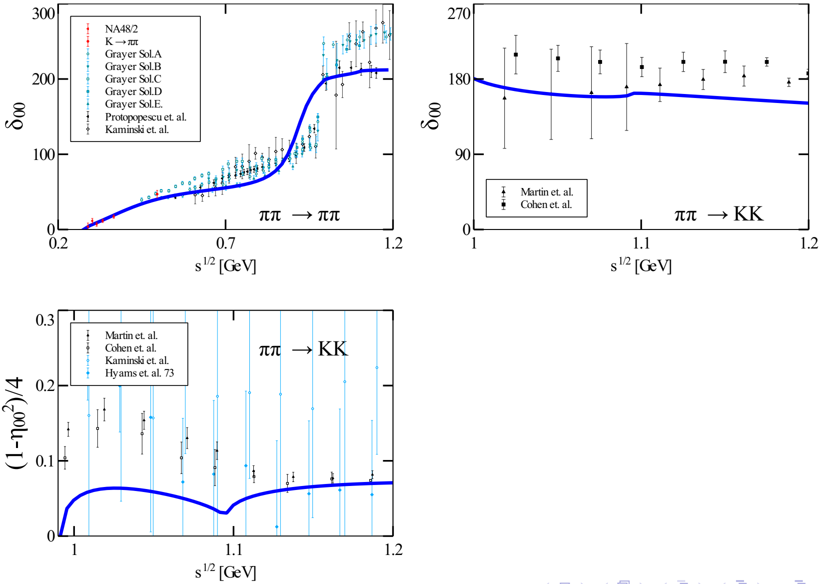

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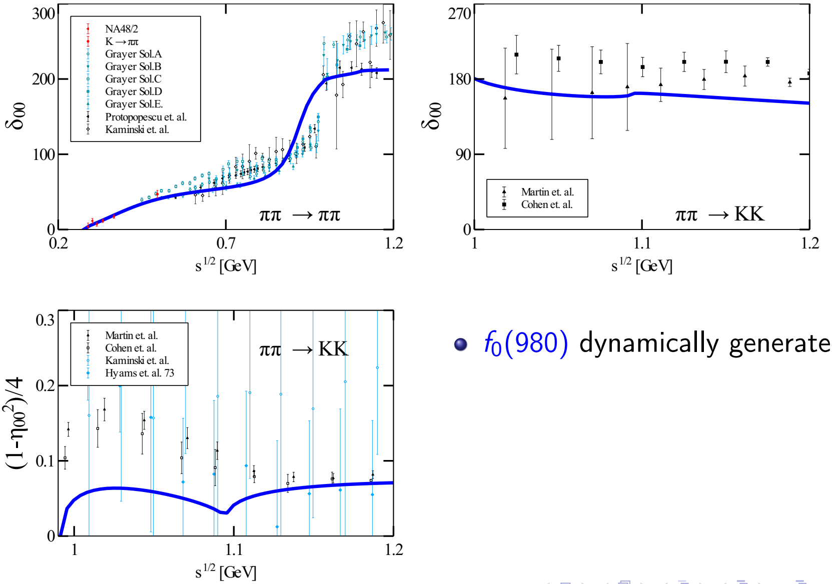

- $f_0(980)$ dynamically generated.

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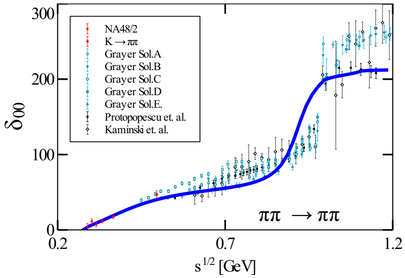

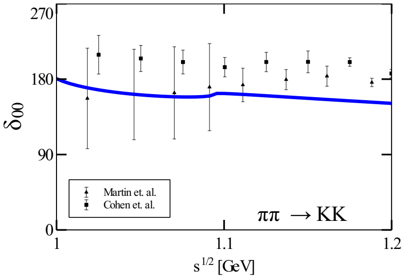

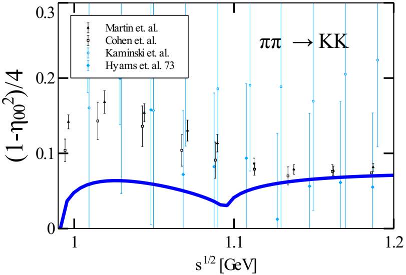

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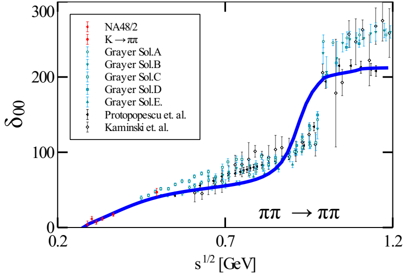

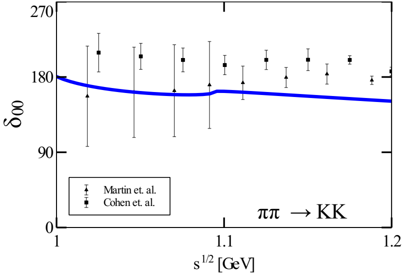

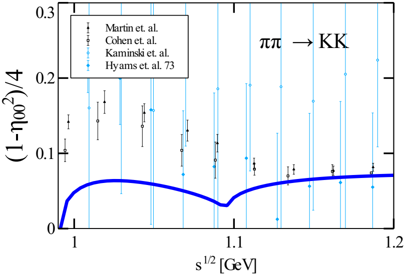

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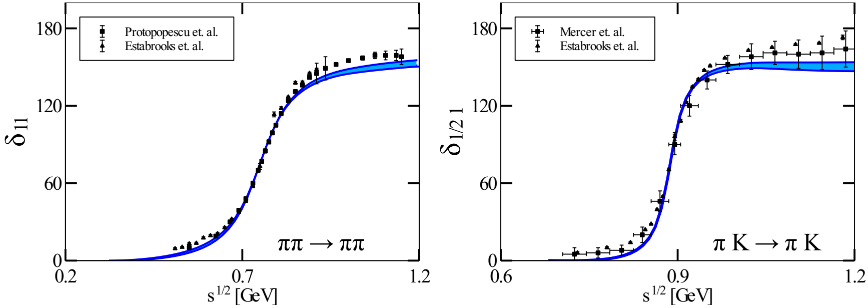
J=1: The role of the vector meson exchange



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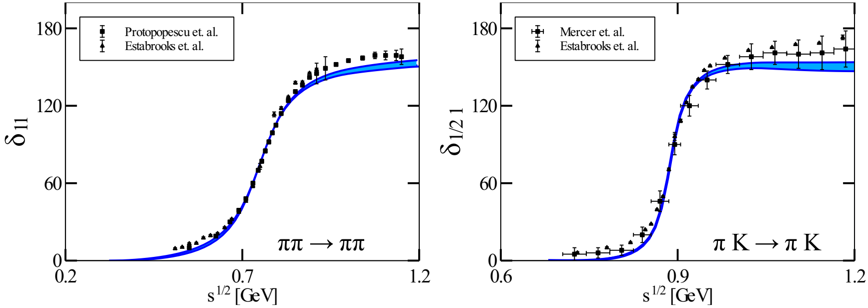

- Resonance fields in the chiral Lagrangian are incorporated by the CDD poles

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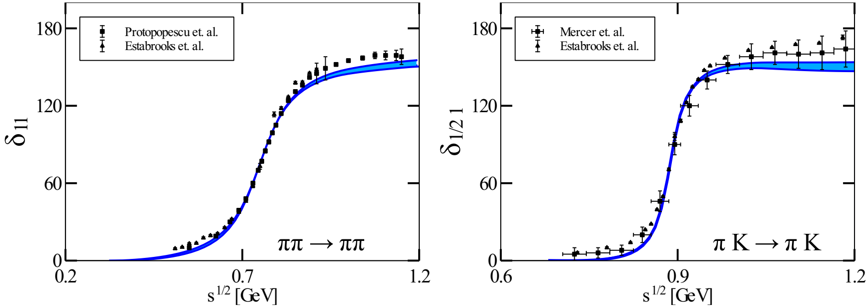

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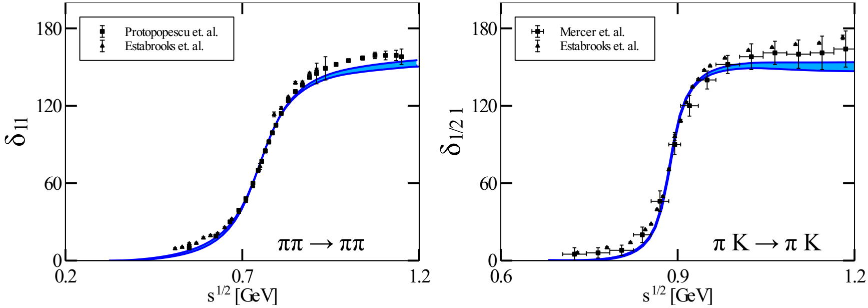

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Conclusions

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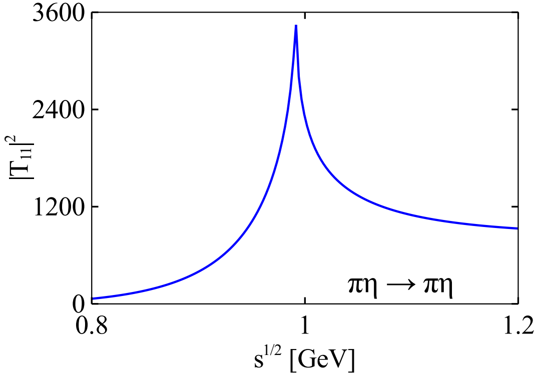
We conclude that the dynamics depends sensitively on the details of the vector meson exchange.

$a_0(980)$

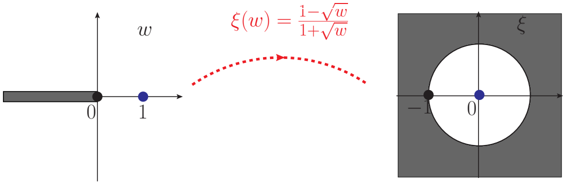


$J = 0$ and $(I^G, S) = (1^-, 0)$

Conformal mapping technique

Typical example: $U(w) = ln\,(w) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k!} [w-1]^k$

$$= \sum_{k=0}^{\infty} C_k \, [\xi(w)]^k$$



$$ln\,(w): \quad \sum_{k=1}^{N} \frac{(-1)^{k+1}}{k!}\,[w-1]^k \quad vs. \quad \sum_{k=0}^{N} C_k\,[\xi(w)]^k$$