Pions in Hot and Dense matter: Back to the Future

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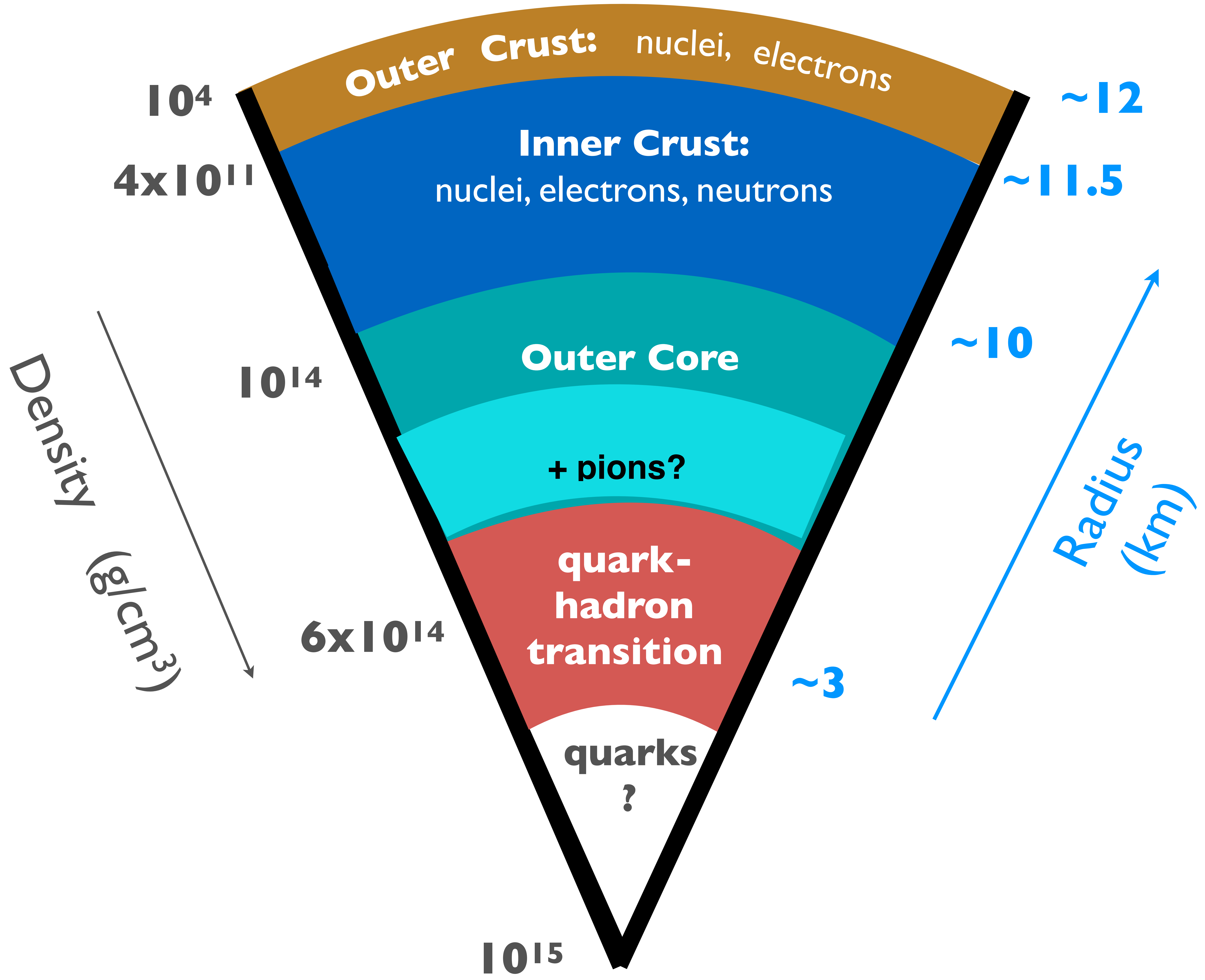
International Workshop XLVIII on Gross Properties of Nuclei and Nuclear Excitations
Hirschegg, Kleinwalsertal, Austria, January 12 - 18, 2020

Dense Matter in Neutron Stars: A Theorists View



Who are they? And what do they all have in common?

Pions in Cold Dense Matter

- There is a vast and daunting literature on pions in nuclei and nuclear matter.
- Several possible pion condensed states of matter were proposed and/or discarded.
- Negatively charged, p-wave pion condensate was the most popular.
- No compelling evidence was found for pion condensation in nuclei.
- Pions in neutron star matter was studied extensively with contradictory conclusions.
- Calculations were based on simple nucleon-pion potential models and approximate many-body methods.
- Interest in pions condensation seems to have waned after Kaplan and Nelson proposed kaon condensation in the late 1980s.

Bose-Einstein Condensation of Mesons in Dense Matter

At low temperature, bosons condense when

$$\mu_{\text{boson}} > E$$

chemical
potential

lowest energy
state of the boson

In dense neutron star matter, the relevant chemical potential is $\mu_{Q^-} = \mu_{e^-}$

Negatively charged mesons are preferentially sourced:

s-wave π^- condensation: $\mu_{e^-} > E_{\pi^-}(p = 0) = m^*_{\pi^-}$

s-wave K^- condensation: $\mu_{e^-} > E_{K-}(p = 0) = m^*_{K-}$

p-wave π^-condensation: $\mu_{e^-} > E_{\pi^-}(p \neq 0)$

S-wave Meson Condensation in Neutron Stars

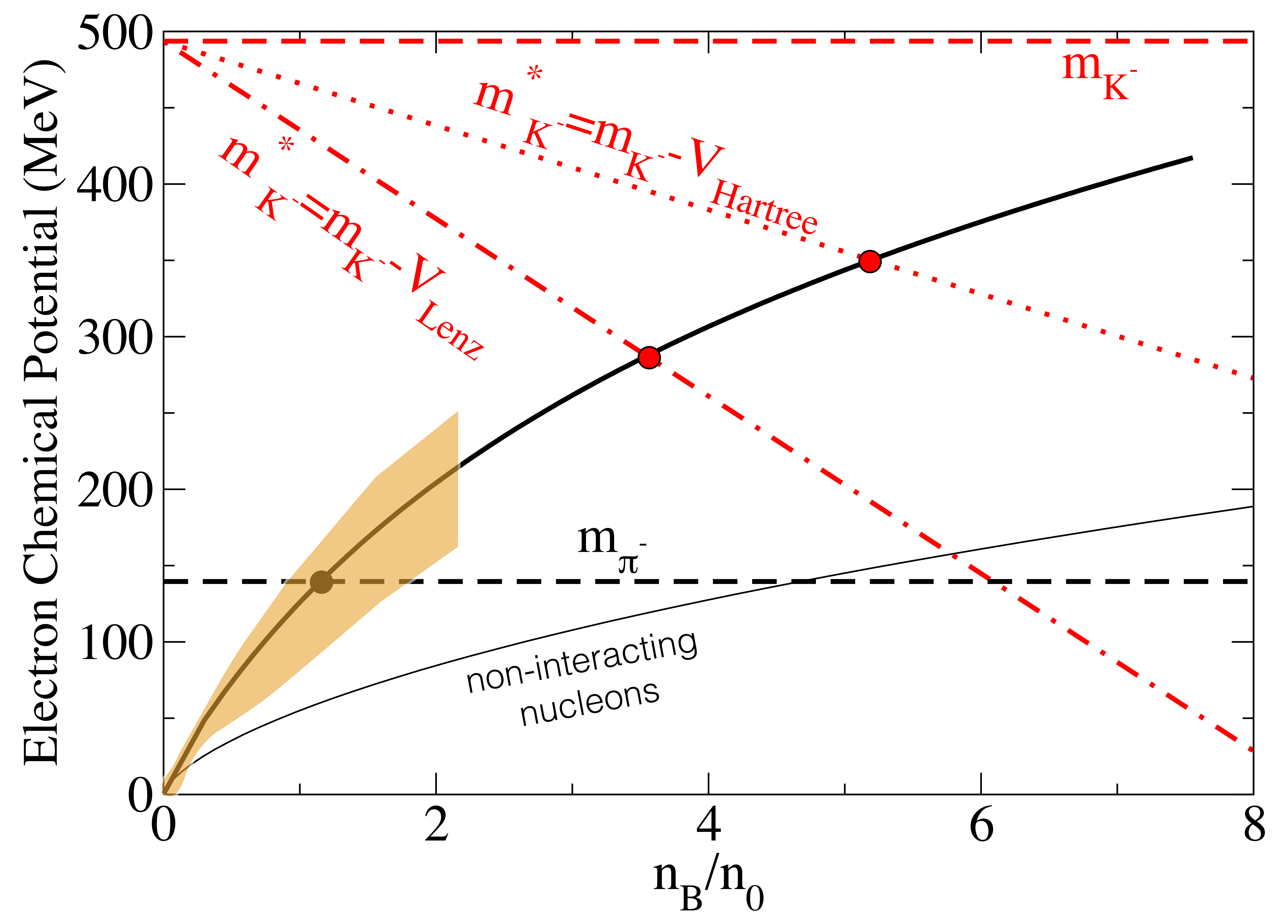
Need strong repulsion to prevent pion condensation.

Need very strong attraction to realize kaon condensation

Very simple models:

$$V_{\mathrm{Lenz}} = \frac{2\pi\ a}{\tilde{M}}\ n_B$$

$$V_{\mathrm{Hartree}} = \frac{4\pi}{3} V_0 R^3\ n_B$$



Pion-nucleon Interaction

Fairly well understood at low energy (Chiral Perturbation Theory)

Scattering lengths are very small:

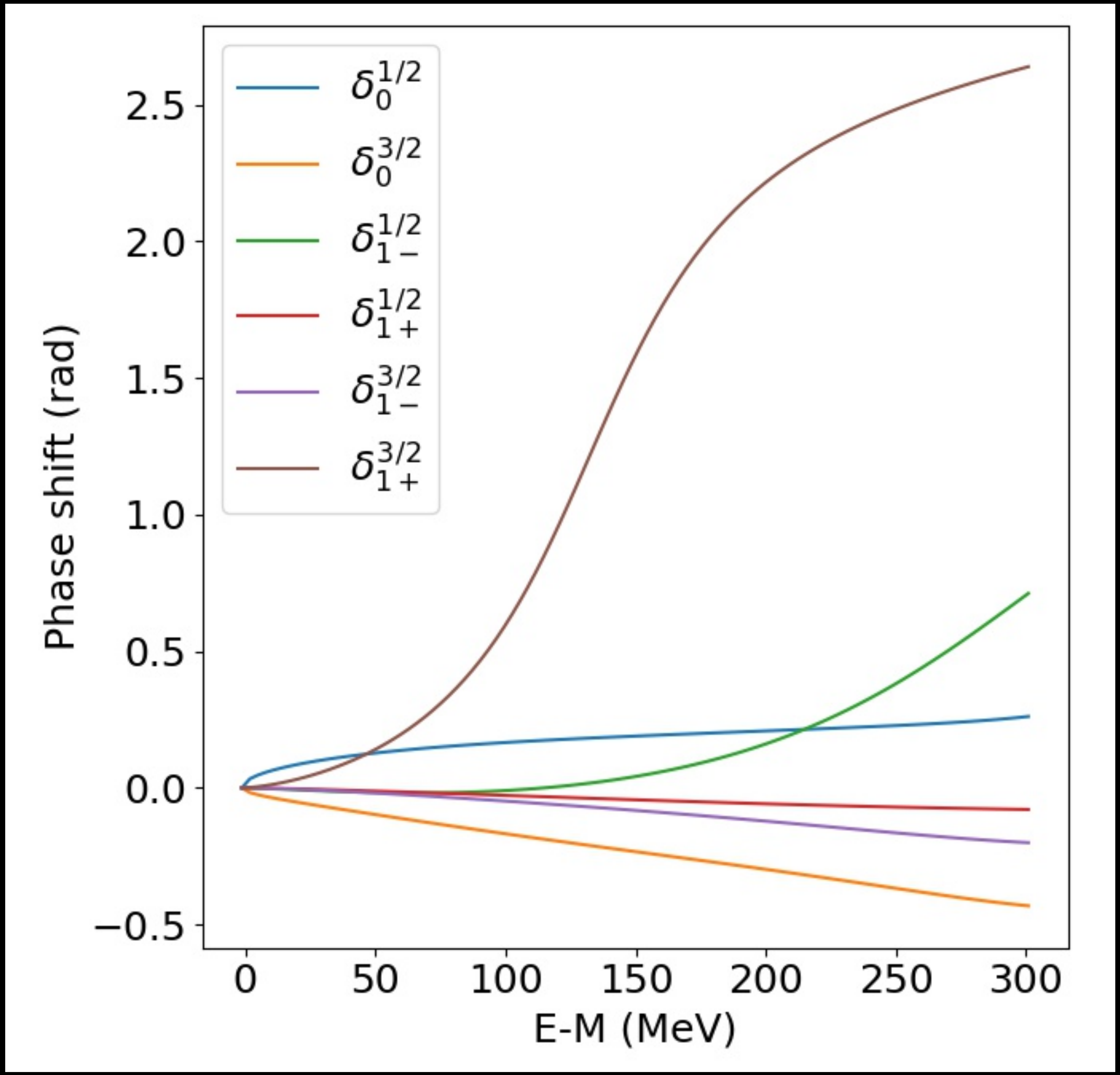
$$a_+ \simeq -0.004 \text{ fm}$$

$$a_- \simeq 0.09 \text{ fm}$$

Phase shifts have been reliably extracted from experiments.

$$\mathscr{A}[\pi^- + n \to \pi^- + n] = \mathscr{A}^{3/2}$$

$$\mathscr{A}[\pi^- + p \to \pi^- + p] = \frac{2}{3}\mathscr{A}^{1/2} + \frac{1}{3}\mathscr{A}^{3/2}$$



Pions in Hot Neutron Star Matter

The virial expansion is a model independent approach to calculate thermodynamic properties at high temperature as an expansion in the particle fugacities $z_i = \exp(\beta\tilde{\mu}_i)$

$$n_i^{\text{int}} = n_i^{\text{ideal}} + \sum_j z_i z_j \; b_2^{(ij)}$$

second-viral coefficient depends on the scattering phase shifts

Virial expansion used in the heavy-ion context to describe hot meson gases.
[Dashen Ma, Bernstein (1969) Venugoplan and Prakash (1992), Houvinen and Petreczky (2018)]

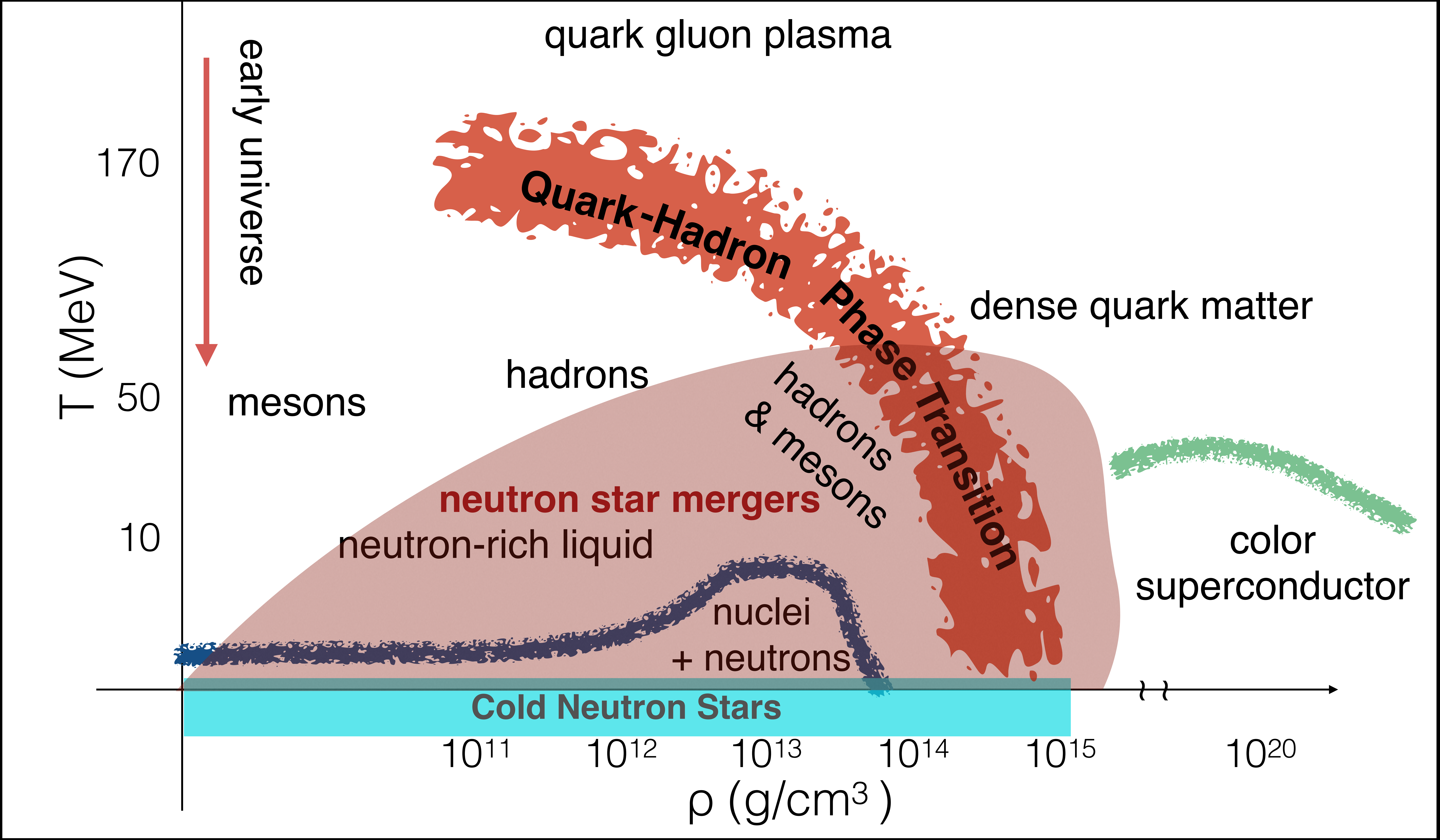
Also used to describe the high-temperature and low-density gas of nucleons in astrophysics.
[Horowitz and Schwenk (2006)]

The effect of interactions between pions and nucleons can be accounted for by the second virial coefficients when the fugacities are small.

$$b_2^{n\pi^-} = \frac{e^{\beta M}}{2\pi^3}\int_M^\infty dE E^2 K_1(\beta E)\sum_{l,\nu}(2l+1)\delta_{l,\nu}^{3/2}$$

$$b_2^{p\pi^-} = \frac{e^{\beta M}}{2\pi^3}\int_M^\infty dE E^2 K_1(\beta E)\sum_{l,\nu}(2l+1)(\delta_{l,\nu}^{3/2} + \delta_{l,\nu}^{1/2})$$

Hot Dense Matter Encountered in Neutron Star Mergers



A Simple Hybrid Model for Hot Neutron Star Matter

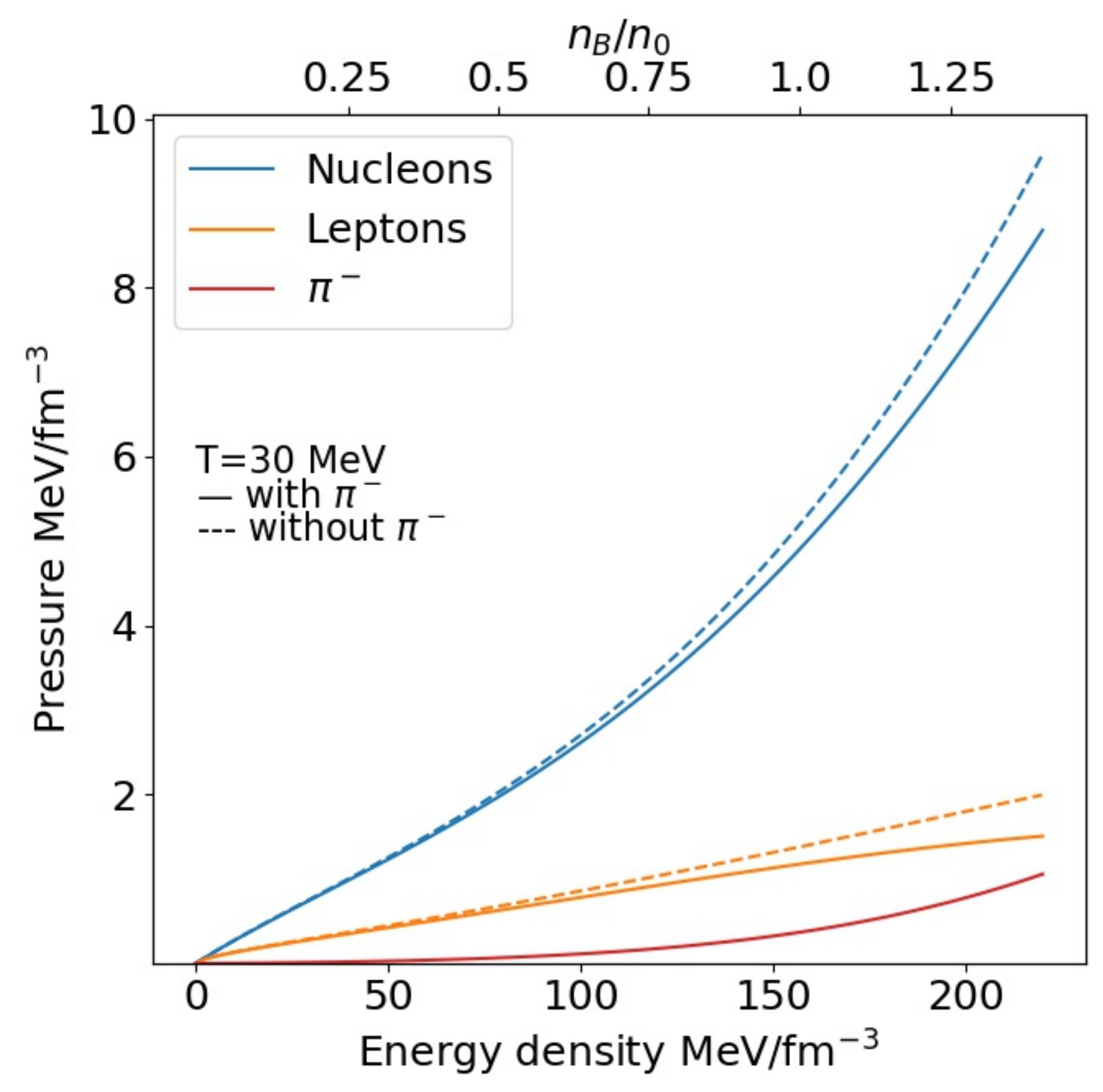
For matter with T=30-50 MeV and n_B = 0.1 n_{sat} - 2 n_{sat} the hybrid model:

1. Treats nucleon-nucleon interactions using a phenomenological mean field model (Skyrme fit to ab initio predictions and nuclear constraints)

2. Treat pion-nucleon interactions using the virial expansion.

At T~30 MeV, the number density of pions is comparable to that of the electrons.

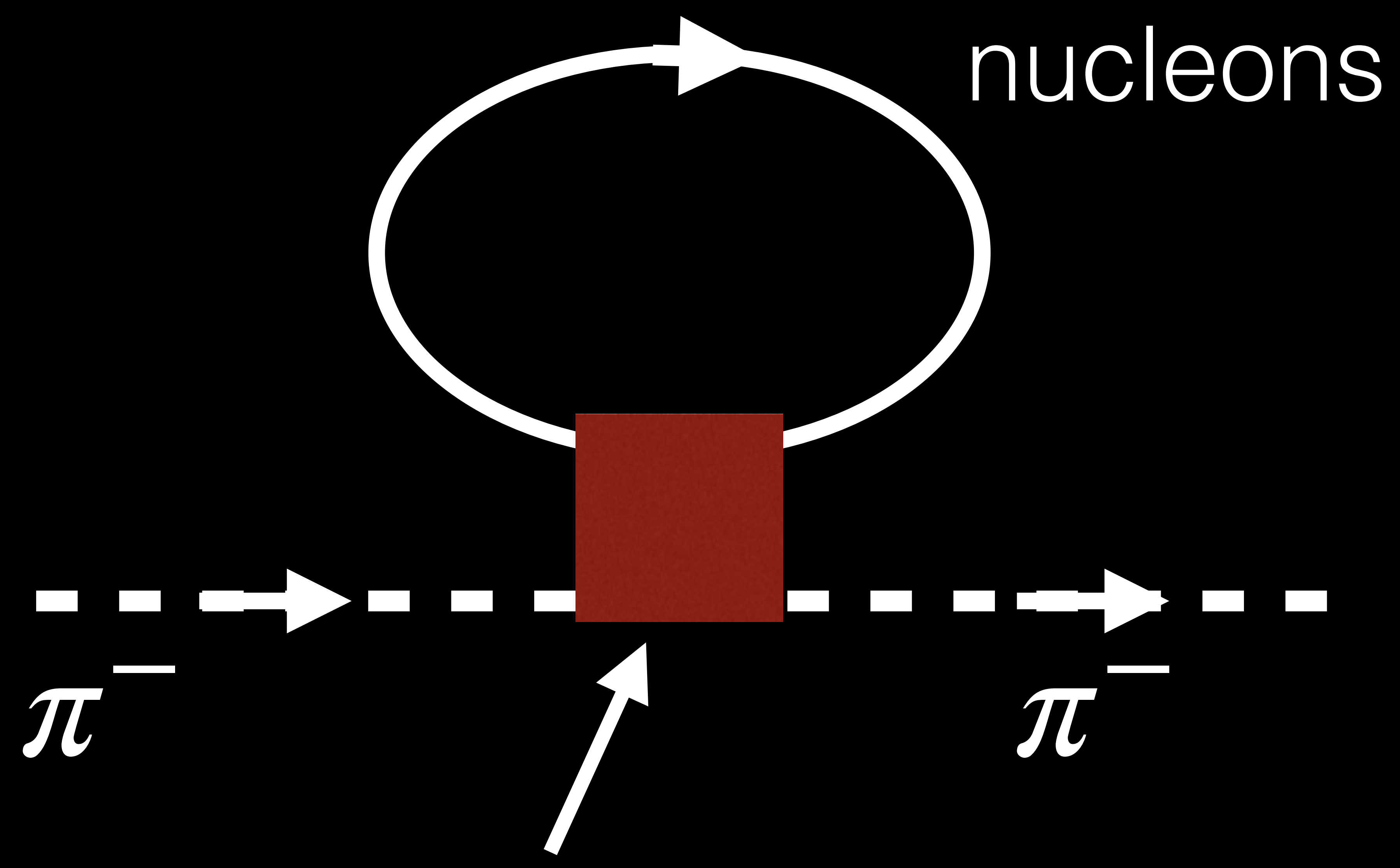
Pions soften the EOS (by increasing the proton fraction) . The correction is modest.



Fore and Reddy arXiv:1911.02632

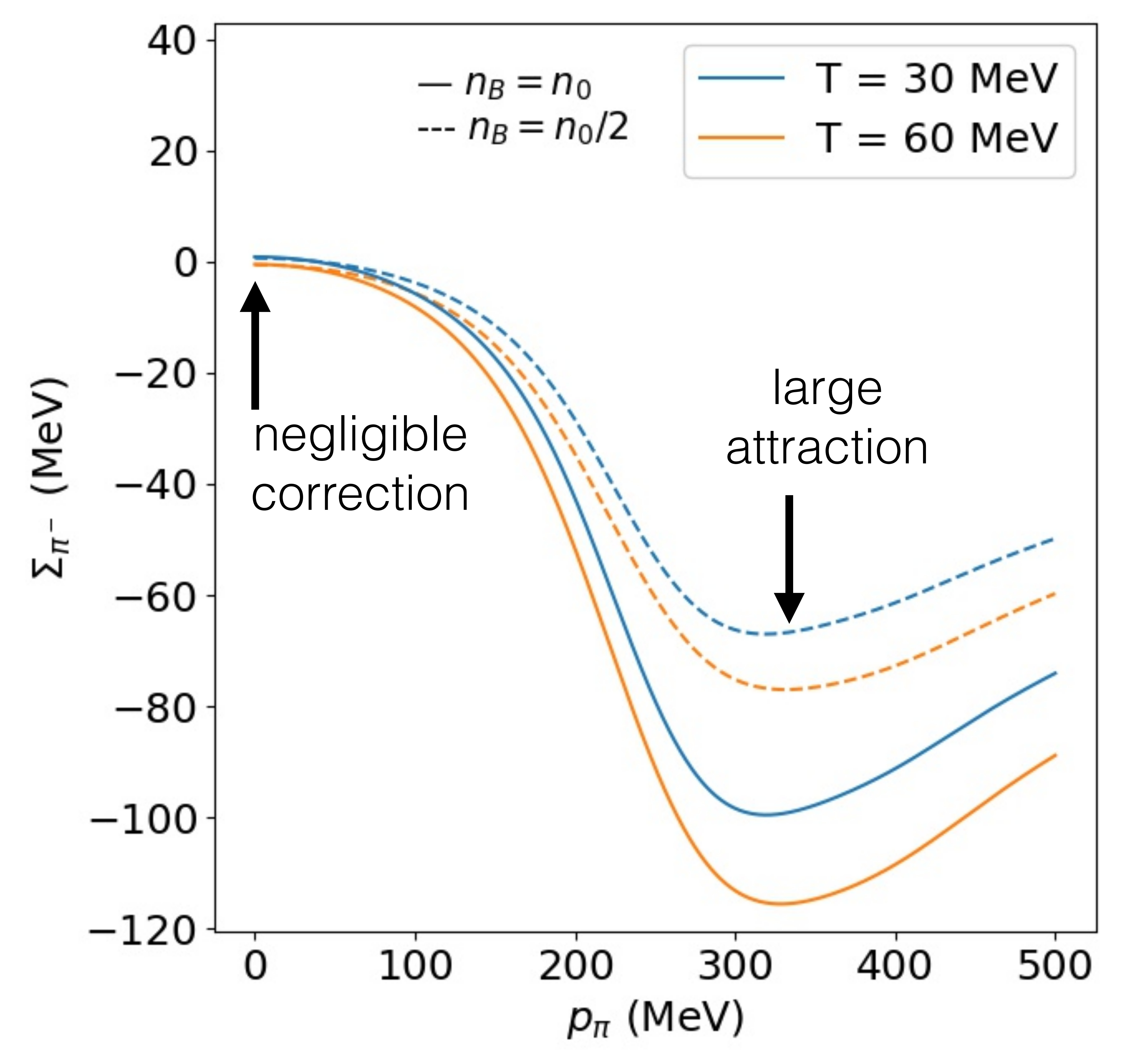
Pion single-particle energy in hot dense matter

$$E_{\pi^-}(p) = \sqrt{p^2 + m_\pi^2} + \Sigma_{\pi^-}(p)$$



Effective (pseudo) potential

$$V_{\text{pseudo}}(p_{\text{cm}}) = \sum_{I,l,\nu} (2l+1)\frac{\delta^I_{l,\mu}}{\tilde{m}\ p_{\text{cm}}}$$



negligible correction

large attraction

Fore and Reddy arXiv:1911.02632

Thermal Pions Alter Neutrino Mean Free Paths

Important reactions:

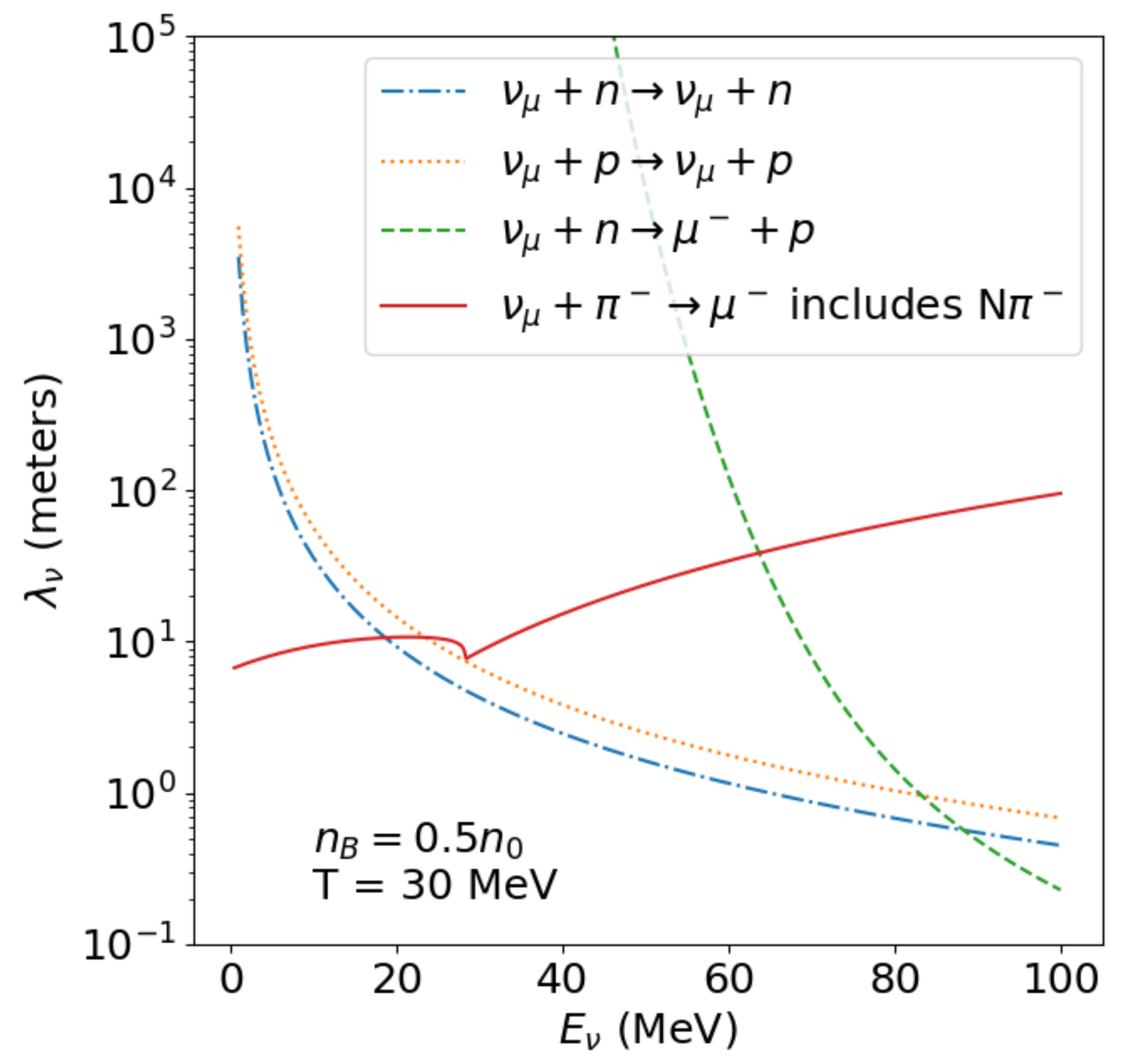
$$\bar{\nu}_\mu + \mu^- \rightarrow \pi^-$$

$$\nu_\mu + \pi^- \rightarrow \mu^-$$

(possible due to in-medium effects)

$$\mathscr{L} = \frac{G_F \cos\theta_c}{\sqrt{2}} f_\pi \, \partial^\alpha \pi^- \, \bar{\psi}_{\nu_\mu}(\gamma_\alpha(1-\gamma_5))\psi_\mu$$

Introduces new reaction channels. The reduced muon neutrino mean free paths may be relevant to the evolution of neutron star mergers and proto-neutron stars.



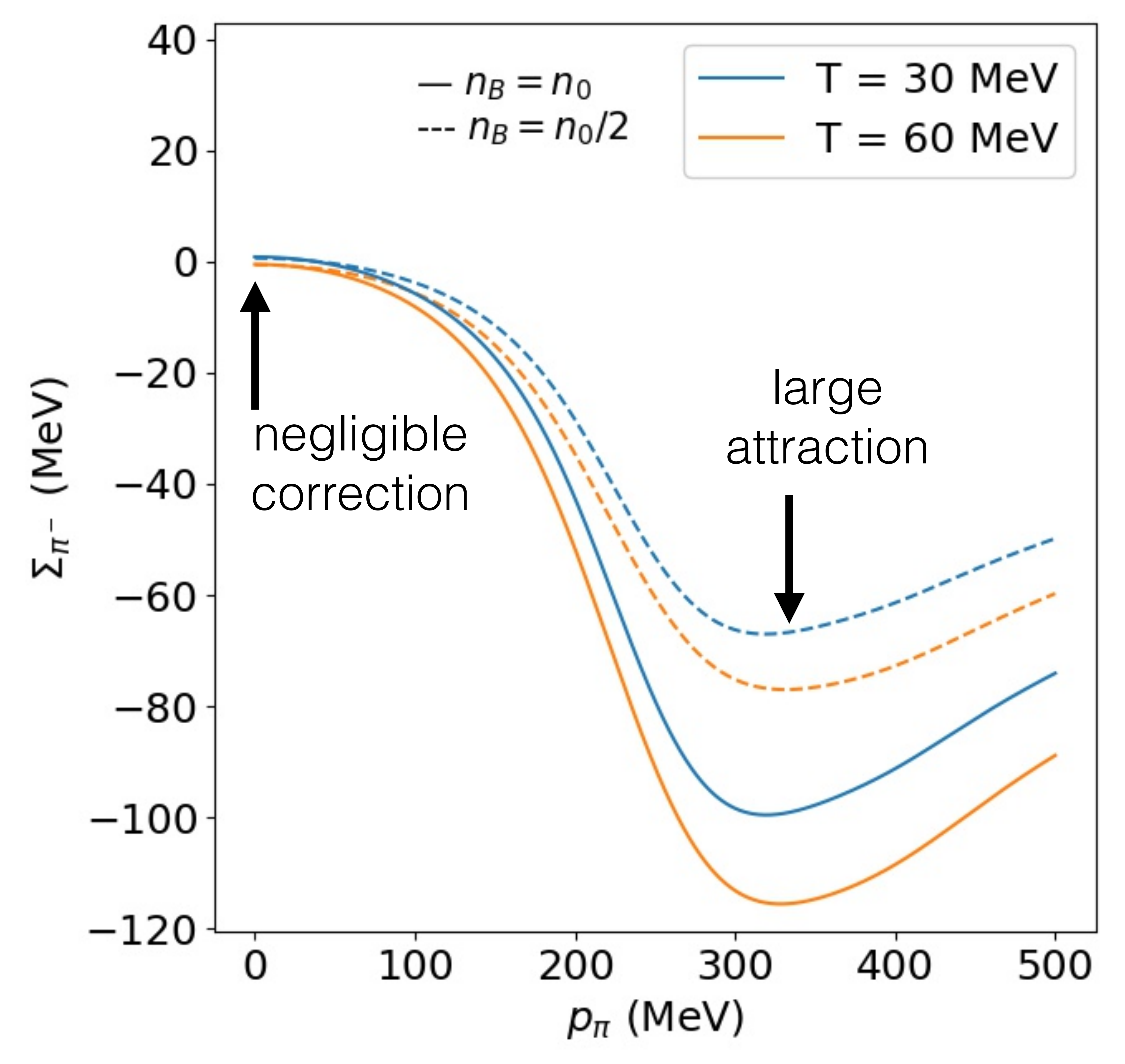
Back to Pion Condensation

The pseudo-potential provides useful insights:

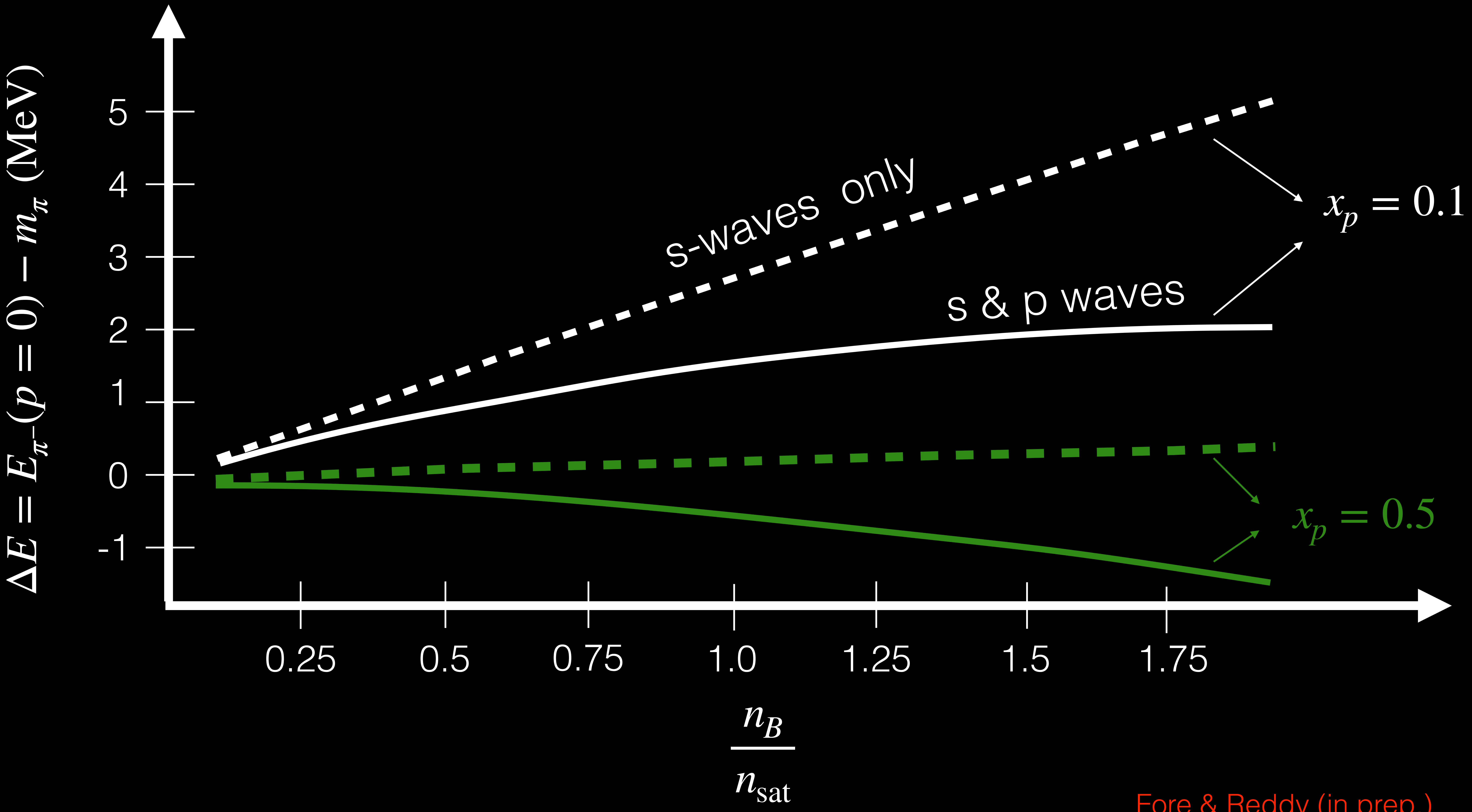
Energy shift of low momentum pions is small.

Shift has a weak density dependence.

The large repulsion required to prevent pion condensation is not to be found.



Energy Shift of a Zero Momentum π^- in Dense Matter



Fore & Reddy (in prep.)

Condensate Amplitude

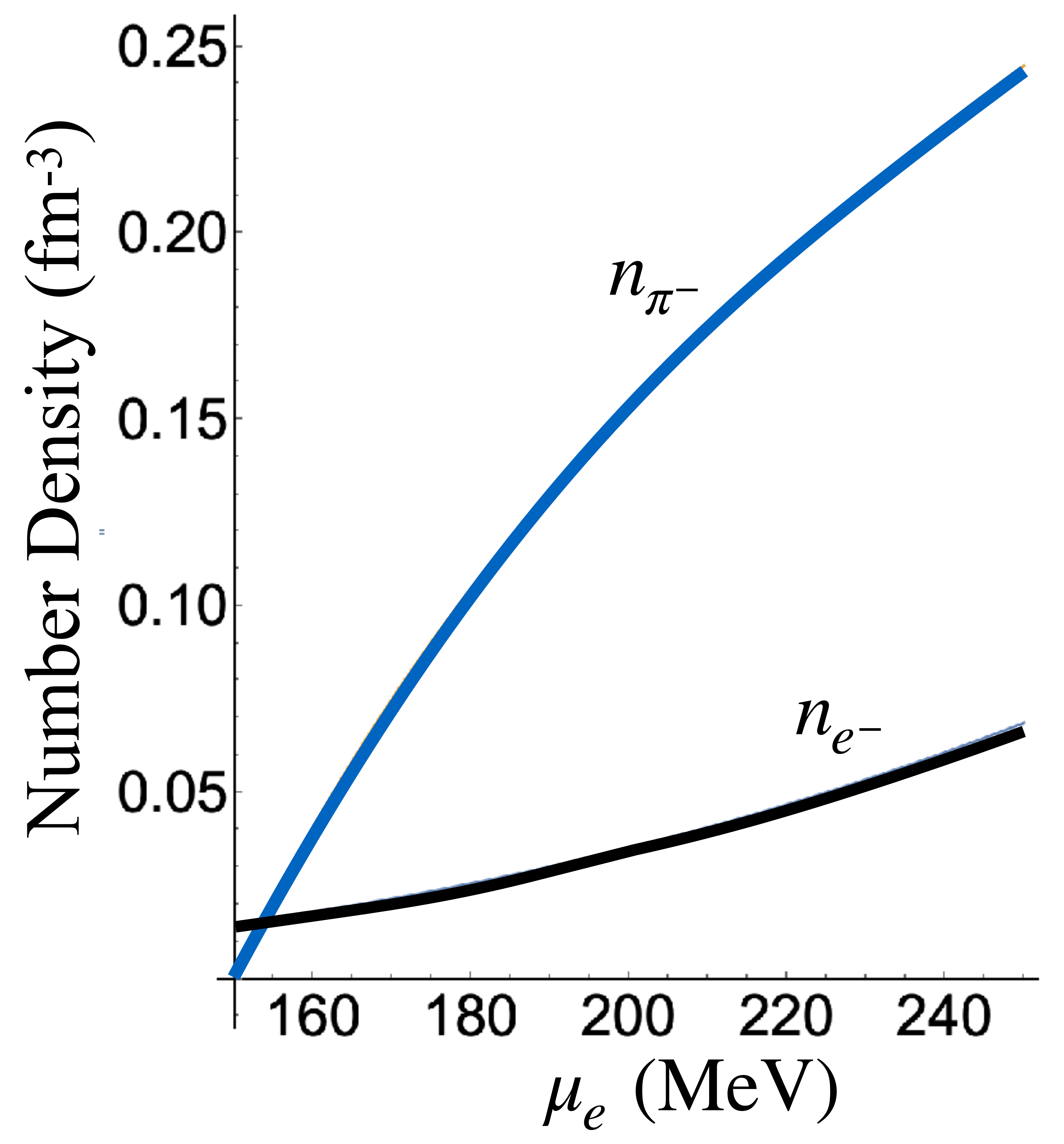
If the pion effective mass m* in the dense matter is not strongly altered by interactions, the condensate will grow quickly

$$n_{\pi^-} = f_\pi^2 \ \mu_e \left(1 - \left(\frac{m_\pi^*}{\mu_e} \right)^4 \right)$$

Condensate amplitude is set by pion-pion interactions.

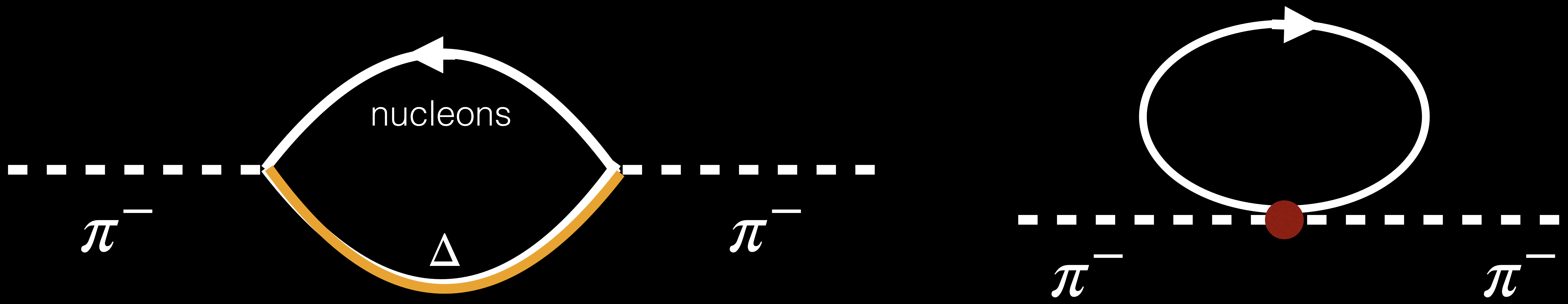
Implications:

- Soft EOS (smaller radii and deformability)
- Superconductivity (magnetic field evolution)
- Mixing between neutron and proton states

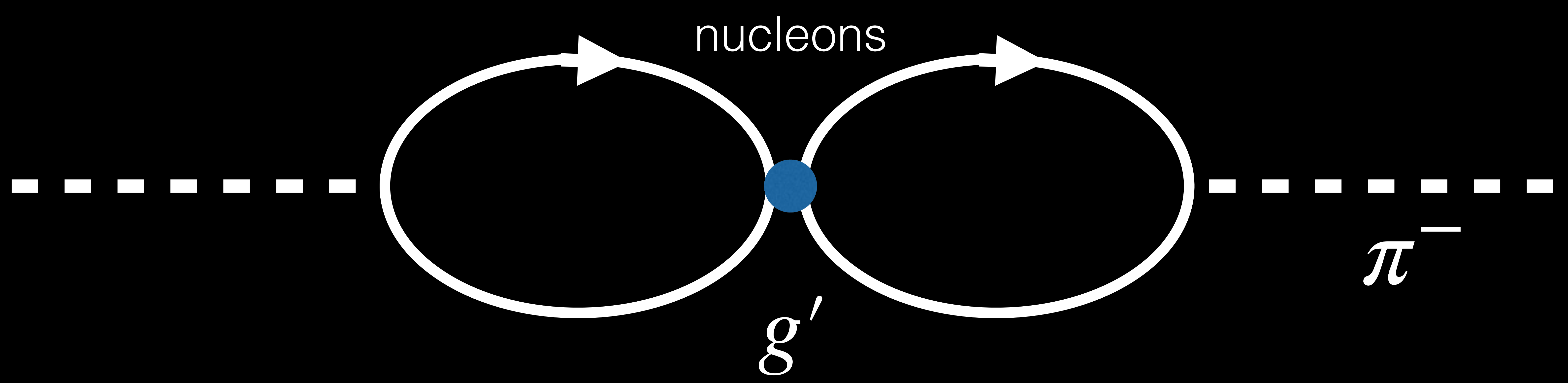


Need to Revisit Pion Condensation with Modern Nuclear Interactions

- More detailed studies that incorporate realistic pion-nucleon potentials in many-body theory are needed. Chiral perturbation theory with explicit pions and Deltas?



- Study the role of (spin-isospin) correlations in neutron-rich matter:



- Reanalyze deeply bound pionic atoms.

Conclusions

- Thermal pions can be incorporated into the EOS of hot and dense matter using the virial expansion for a range of densities and temperatures encountered in astrophysics.

- Even relatively small populations of pions can greatly alter the neutrino mean free paths.

- The pseudo-potential model suggests the pion condensation is likely in neutron stars. The energy shift of a low-momentum pion in dense neutron rich matter is small.

- The softening of the EOS around twice saturation density has implications for masses, radii and tidal deformabilties.