Constraints on the Equation of State at Higher Densities From the FRG

Marc Leonhardt

Hirschegg 2020
Nuclear equation of state and neutron stars
International Workshop XLVIII on Gross Properties of Nuclei and Nuclear Excitations
Hirschegg, Kleinwalsertal, Austria, January 12 - 18, 2020

[J. Braun, ML, M. Pospiech, PRD 96, 076003 (2017)]
[J. Braun, ML, M. Pospiech, PRD 97, 076010 (2018)]
[J. Braun, ML, J. M. Pawlowski, SciPost Phys. 6, 056 (2019)]
[ML, M. Pospiech, B. Schallmo, C. Drischler, K. Hebeler,
J. Braun, A. Schwenk, arXiv:1907.05814]
[J. Braun, ML, M. Pospiech; arXiv:1909.06298, accepted PRD]

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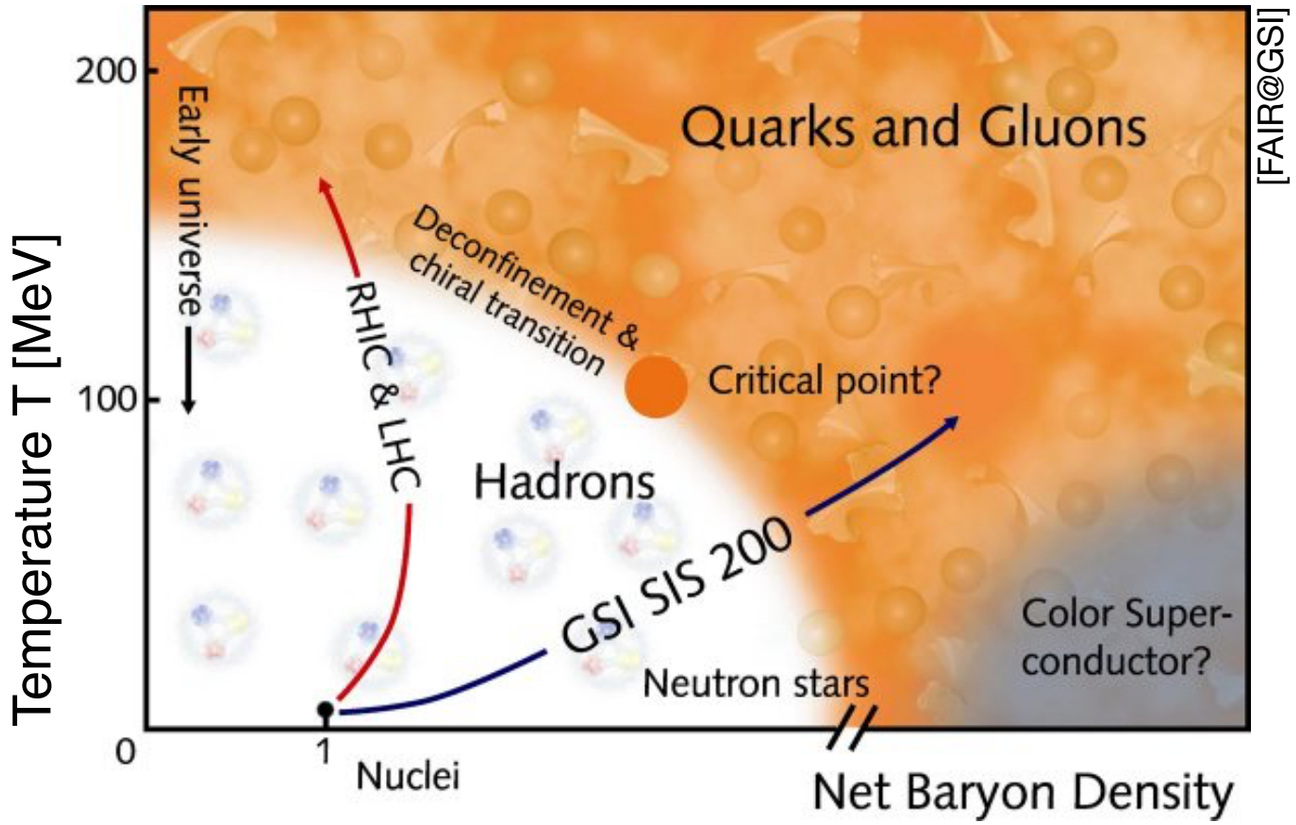
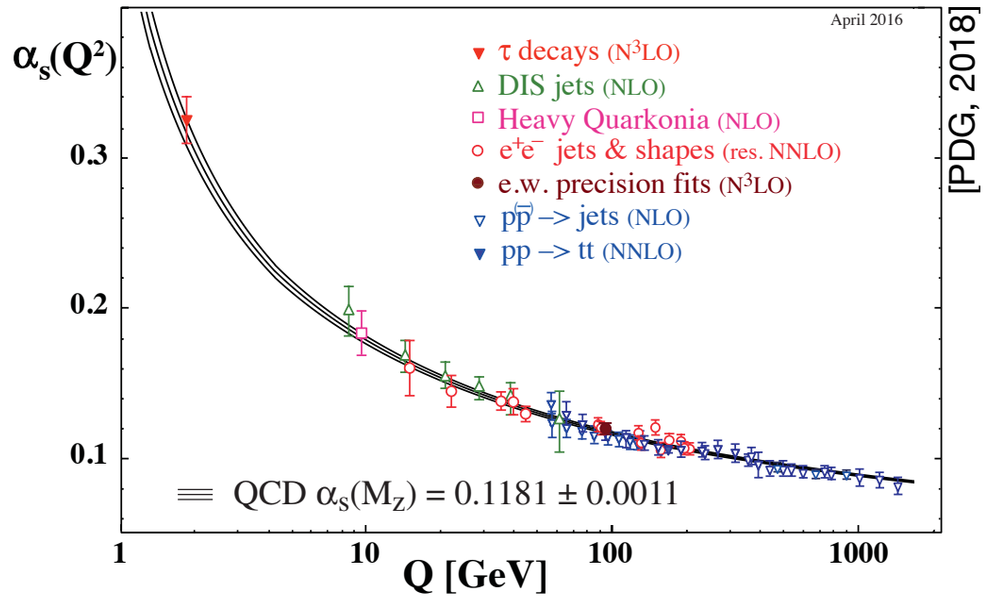
SFB 1245

The QCD phase diagram

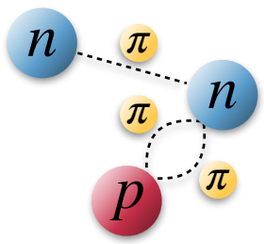
and the cold dense equation of state





Low-energy effective theories

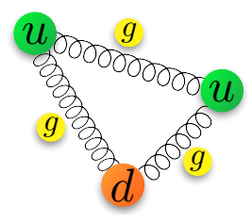
- effective degrees of freedom (d.o.f.'s)



$60\, n_0 \lesssim$ [Gorda et al., PRL (2018)]

Perturbative methods

- quarks and gluons as only d.o.f.'s
- weak coupling expansion



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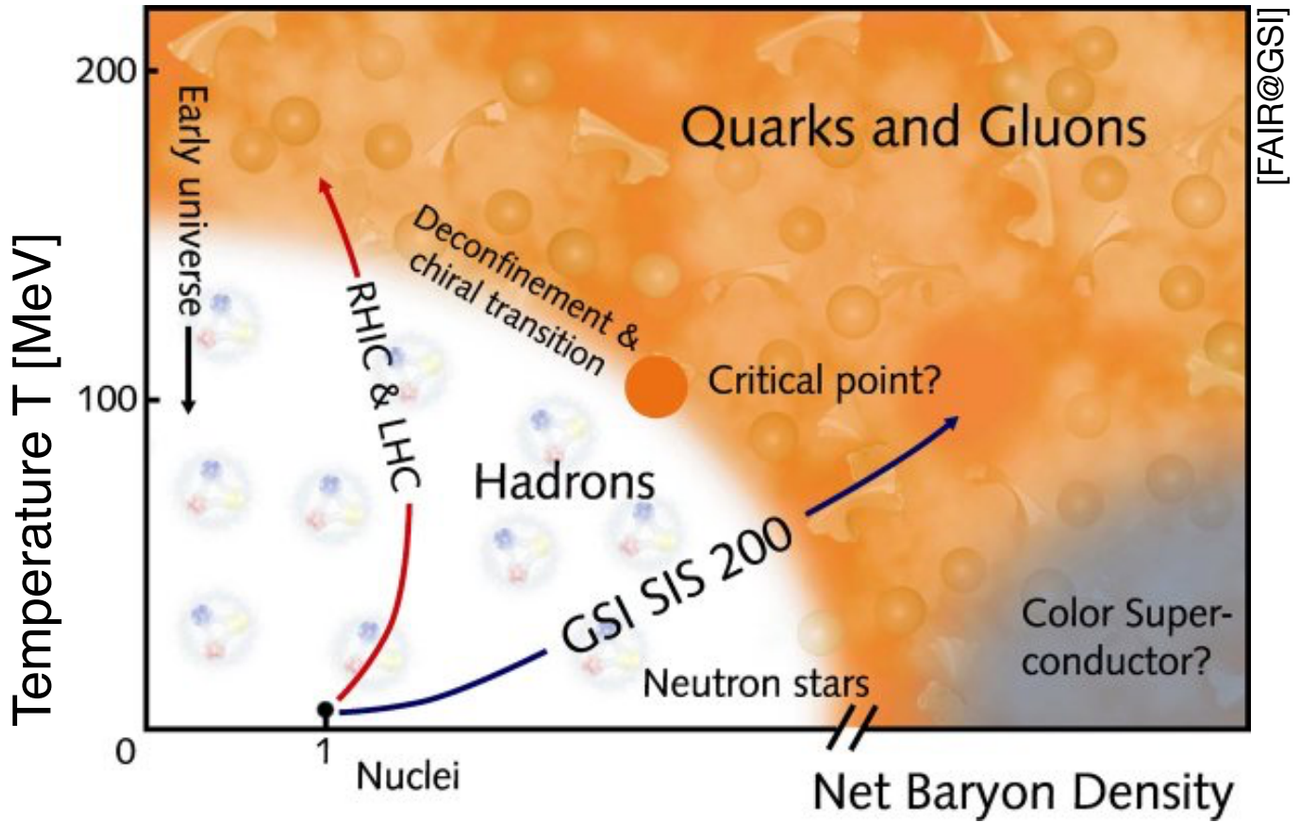

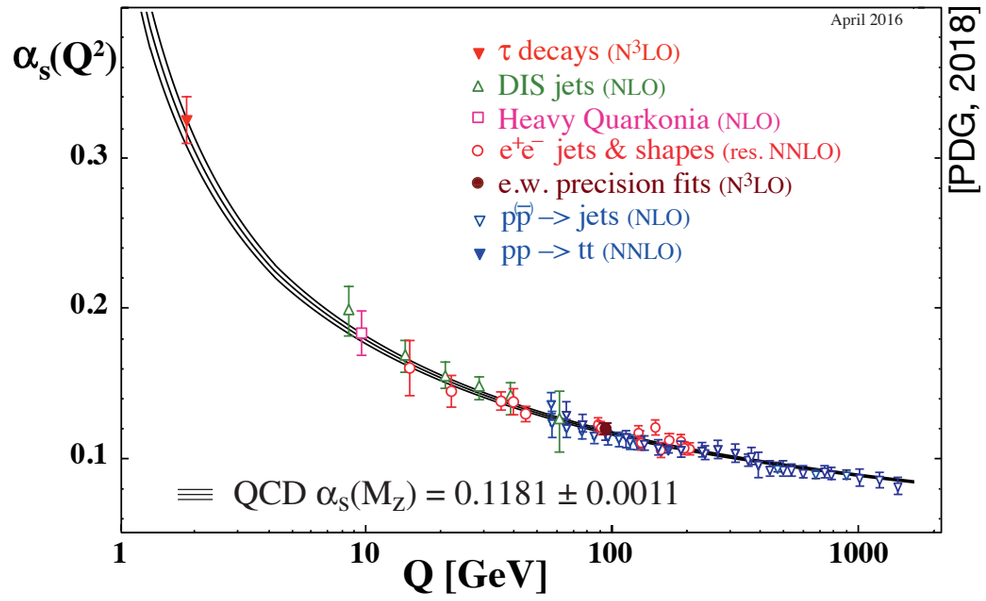

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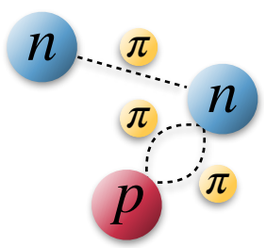

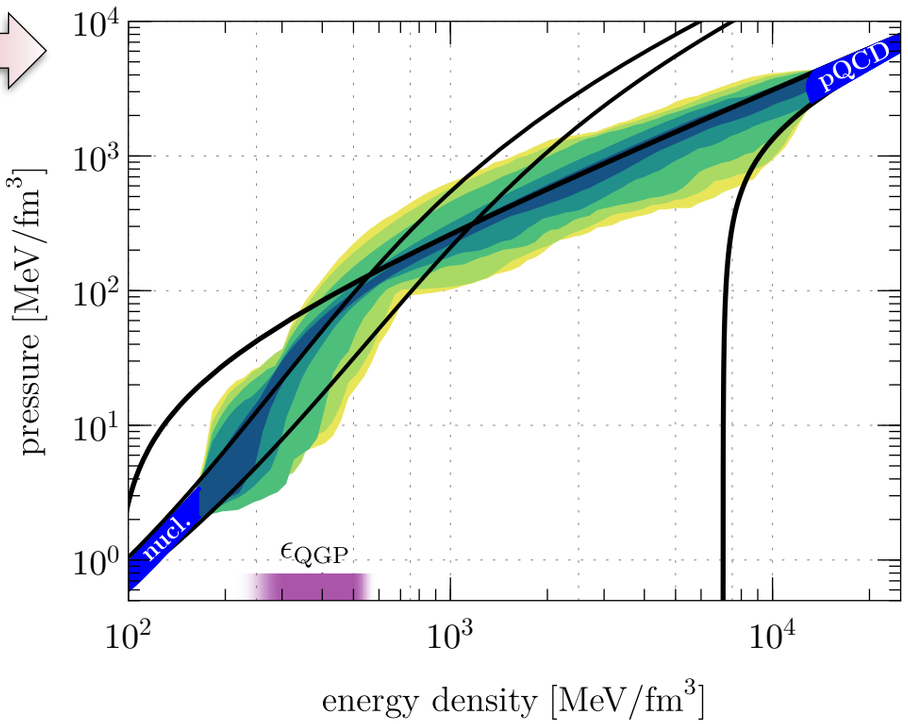

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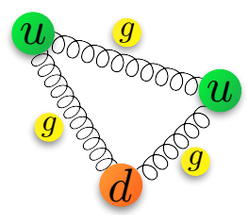

[Annala et al., 2019], see also, e.g., [Greif et al., 2019] [Tews et al., 2018] [Hebeler et al., 2010]

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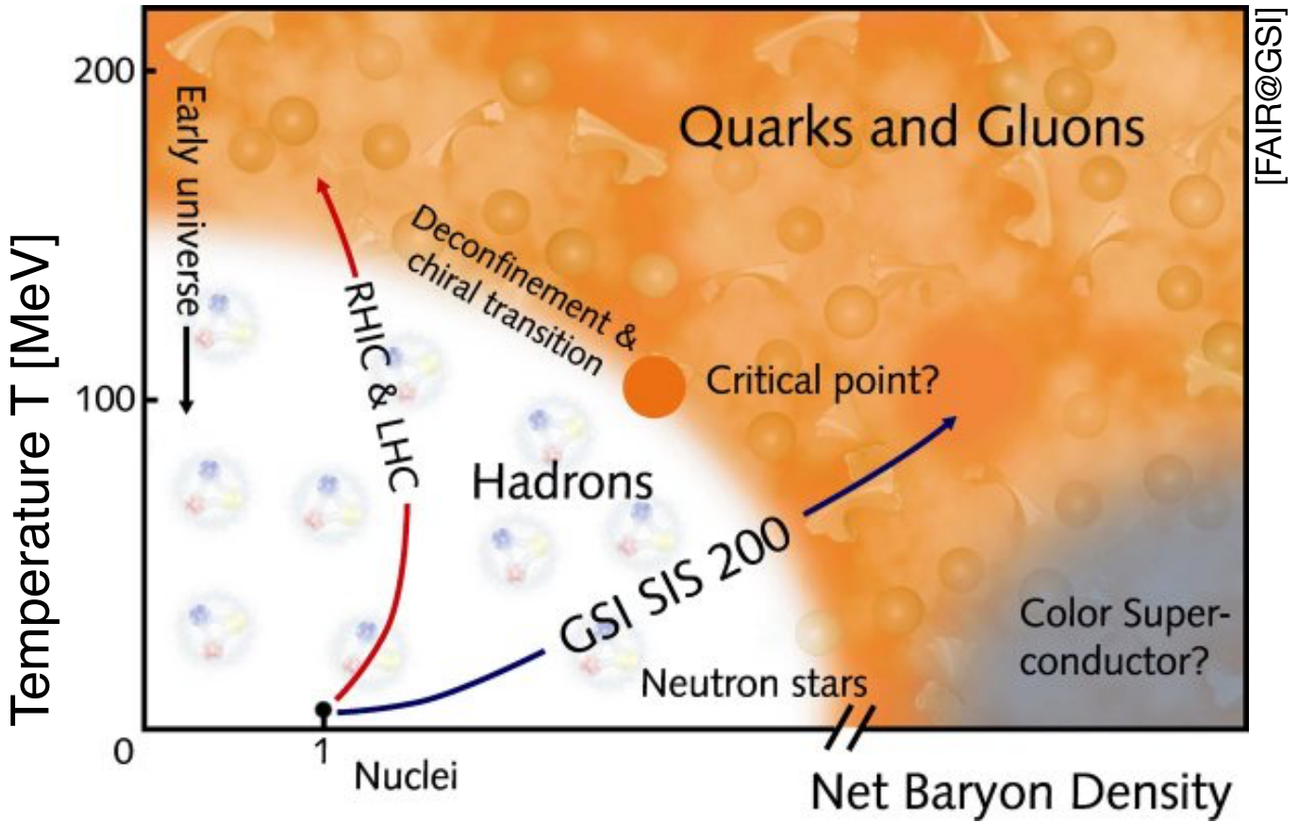

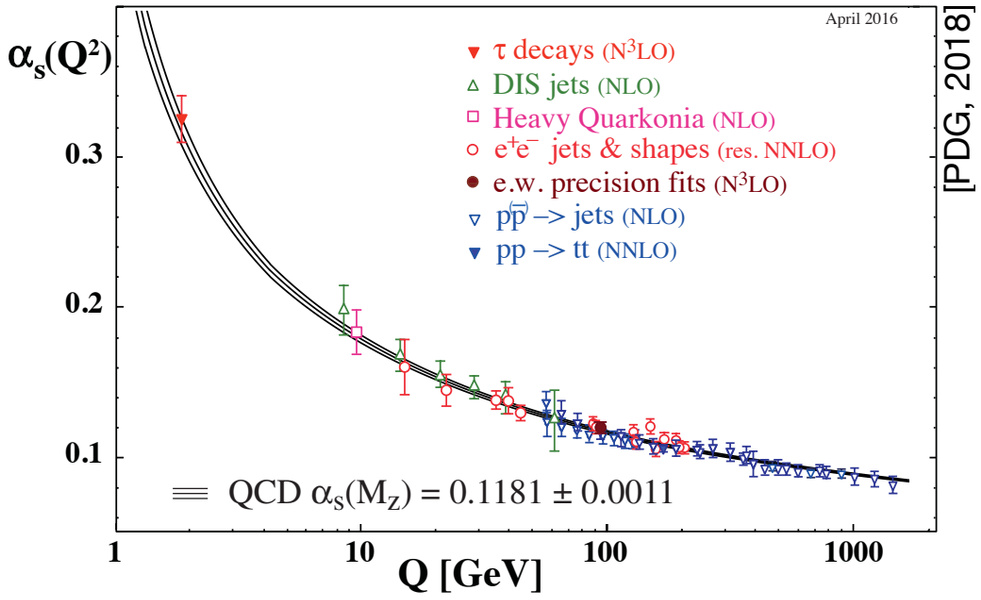

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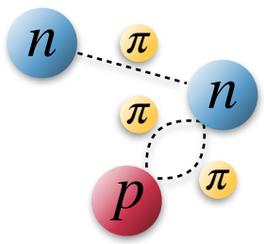

Model studies

- limited range of validity
- issue of parameter determination

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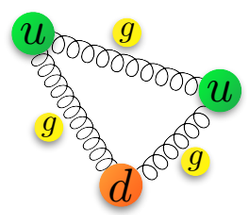

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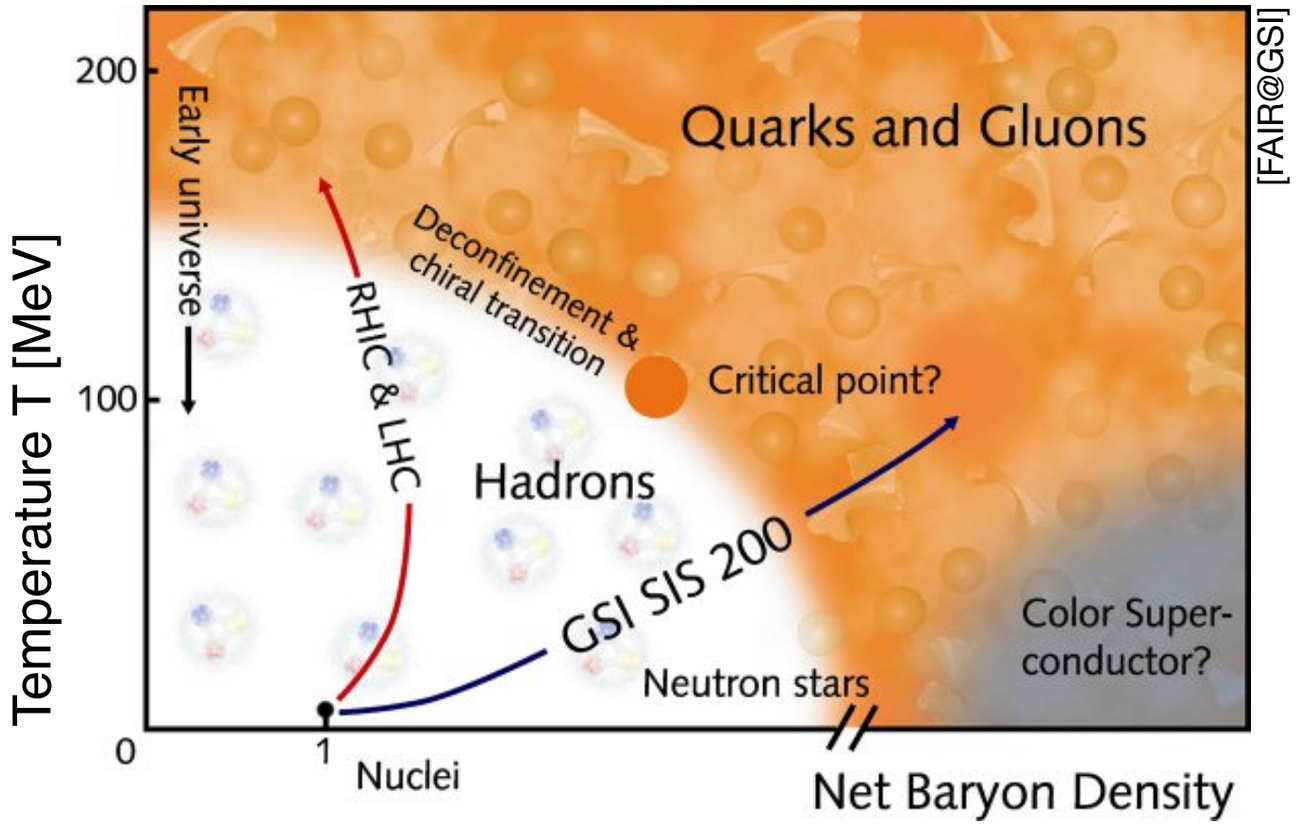

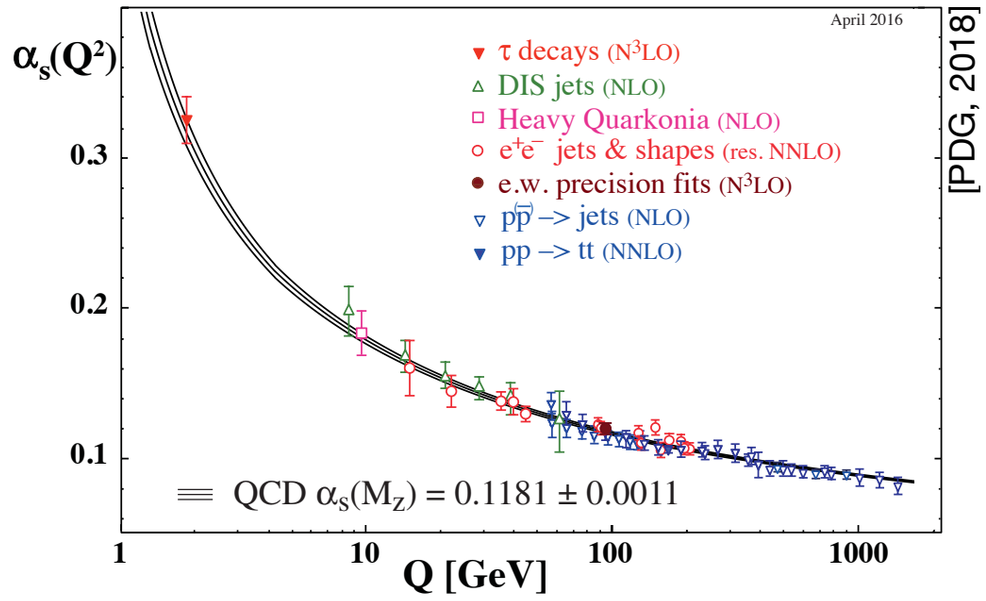

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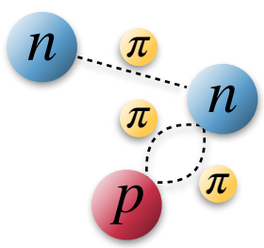

Functional methods

- strongly correlated matter at intermediate densities
- variety of condensates

Functional Renormalization Group

- non-perturbative approach from first principles
- capture physics over a wide range of scales

Perturbative methods

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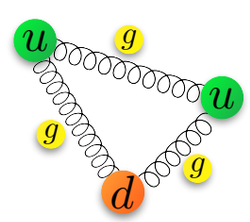

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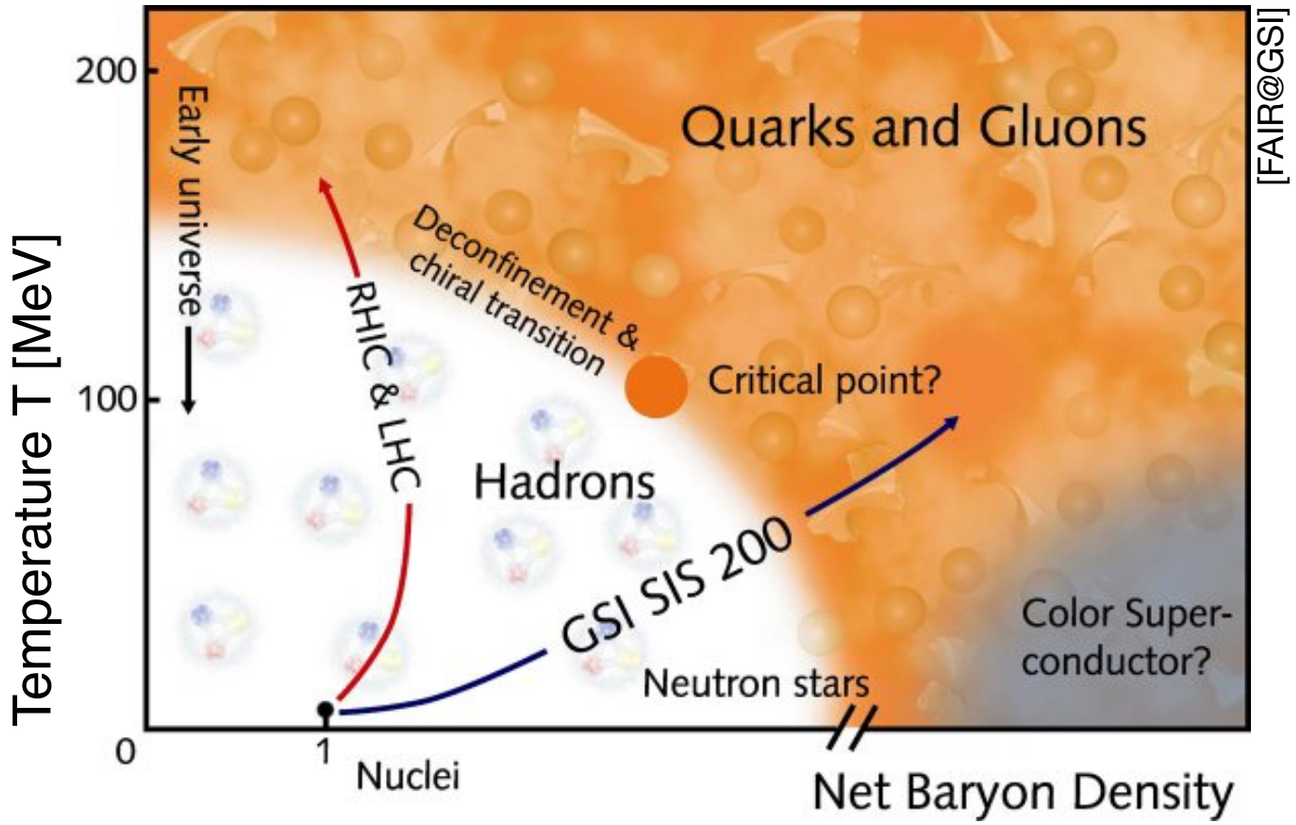

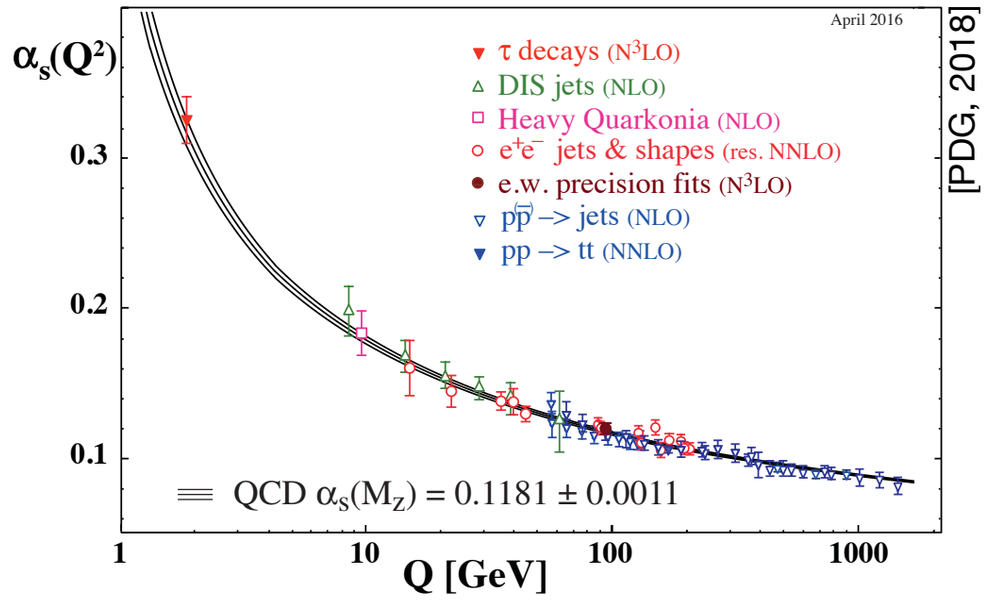

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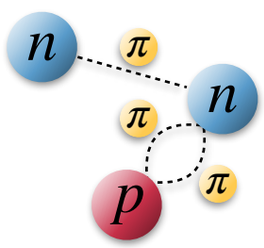
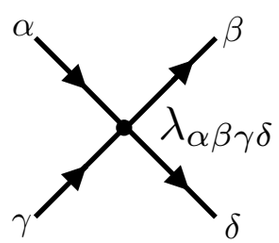


EOS

II



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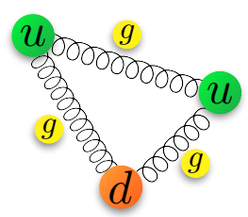

Outline

Constraints on the equation of State at Higher Densities from the FRG

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- Functional Renormalization Group
- Four-quark self-interactions

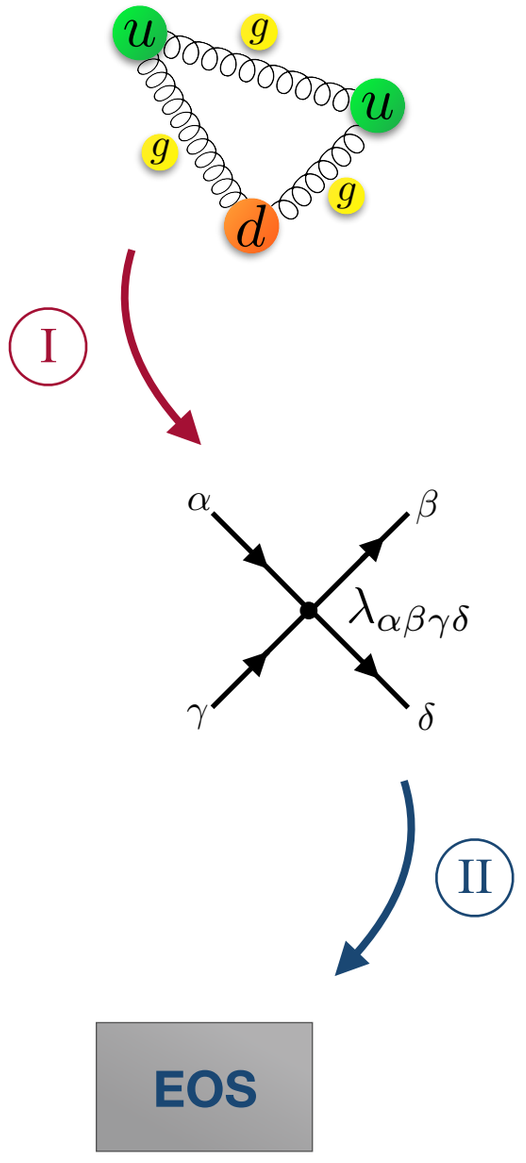
Ⓘ Ab initio: dynamic gauge fields

- Structure of β functions
- Symmetry breaking patterns at finite T
- Dominant low-energy d.o.f's at zero T

Ⓘⓘ Connecting to the low-energy regime

- Quark-meson-diquark truncation
- Findings on the equation of state (EOS)

Conclusions and outlook



Functional Renormalization Group (FRG)

From high to low energies in QCD

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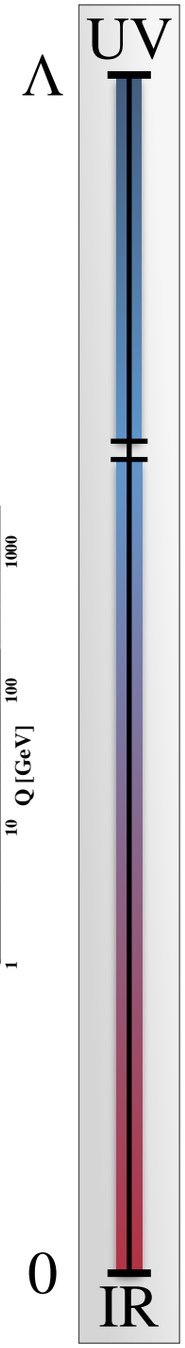
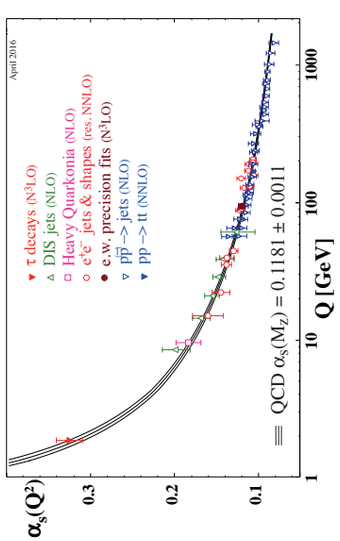
$$S_{\mathrm{QCD}} = \int \mathrm{d}^4x \left\{ \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{\psi} \left(\mathrm{i}\partial\!\!\!/ + \mathrm{i}\gamma_0 \mu \right) \psi + \bar{g} \bar{\psi} A\!\!\!/ \psi \right\}$$



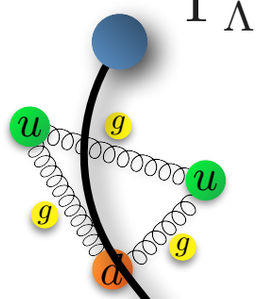
RG scale

$k \sim Q$



Classical action

$\Gamma_\Lambda = S$

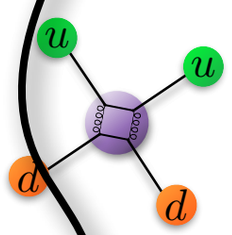


Γ_k

Effective average action

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{STr} \frac{\partial_t R_k}{\Gamma_k^{(2)} + R_k}$$

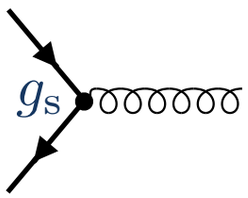
[Wetterich, 1993]



$\Gamma_{k\to 0} = \Gamma$

Full quantum effective action

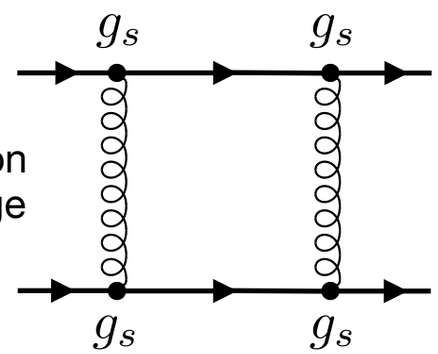
Gluodynamics



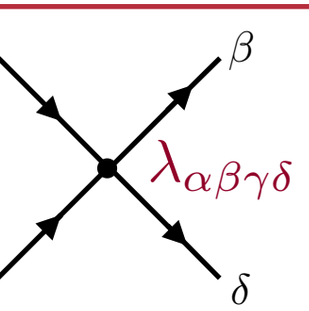
Strong coupling $g_s(\Lambda)$ only parameter

$g_s(\Lambda)$ at $\Lambda = 10\,\mathrm{GeV}$ within exp. error [Bethke, 2006]

two-gluon exchange



Four-quark interactions



Fierz-completeness $\longleftrightarrow$ variety of condensates

Matter sector

[J. Braun, ML, M. Pospiech, PRD 97, 076010 (2018)]
[J. Braun, ML, M. Pospiech; arXiv:1909.06298]

Four-quark self-interactions

Symmetries and spontaneous symmetry breaking

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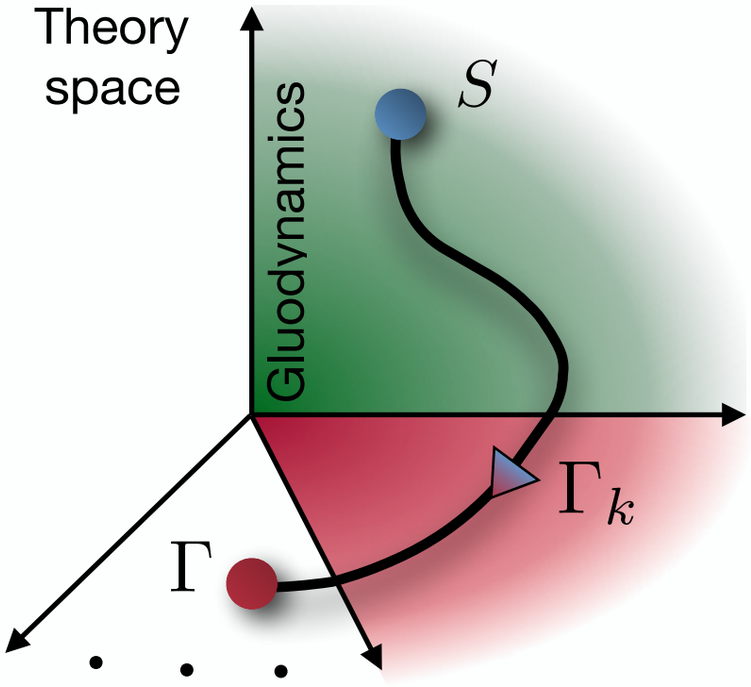

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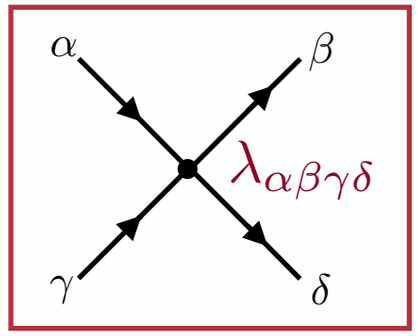


Partial bosonization

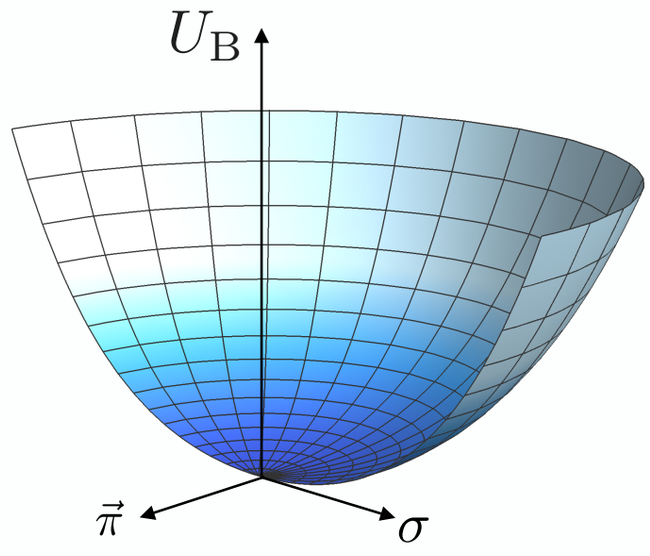
$$\sigma \sim \bar{\psi}\psi$$

$$\vec{\pi} \sim \bar{\psi}\gamma_5\vec{\tau}\psi$$

$$U_\mathrm{B} \sim \frac{1}{\bar{\lambda}_{\sigma\text{-}\pi}}\left(\sigma^2 + \vec{\pi}^2\right) + \ldots$$

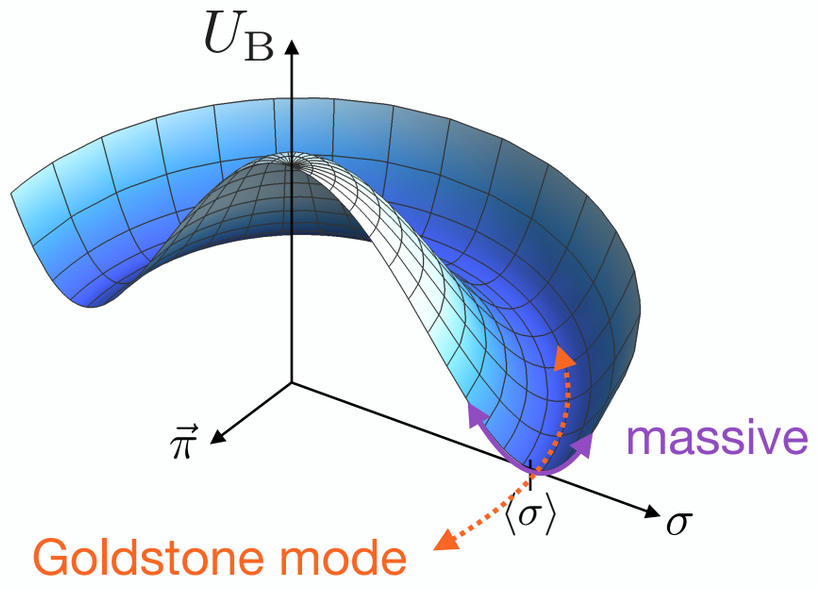
$$\bar{\lambda}_{(\sigma\text{-}\pi)}\left[\left(\bar{\psi}\psi\right)^2 - \left(\bar{\psi}\gamma_5\tau_i\psi\right)^2\right]$$

$\leftrightarrow$ chiral condensate



Symmetry of the ground state

$$\left.\frac{1}{\bar{\lambda}_{\sigma\text{-}\pi}}\right|_{k_\mathrm{cr}} \to 0$$



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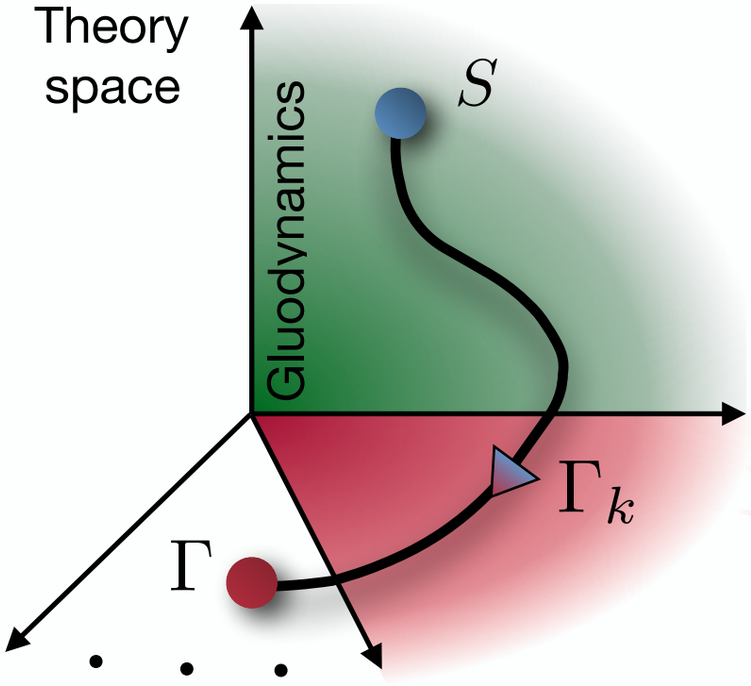

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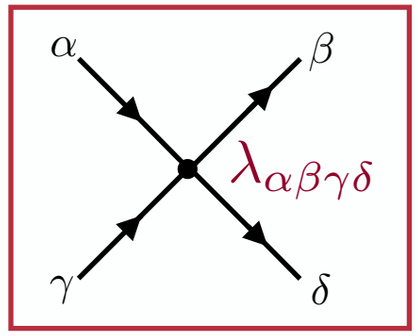

Symmetries at finite T and μ

Discrete:	$\hat{P}$, $\hat{T}$ ($\hat{C}$ broken by μ)
Lorentz group:	$SO(1,3) \to SO(3)$
Flavor space:	$SU_{\mathrm{L}}(2) \otimes SU_{\mathrm{R}}(2) \otimes U_{\mathrm{V}}(1)$
Color space:	$SU(N_{\mathrm{c}})$

Fierz-complete basis $\sum_{j \in B} \bar{\lambda}_j \left(\bar{\psi}\mathcal{O}_j\psi\right)^2$

$$\vdots$$

$$\bar{\lambda}_{(\sigma\text{-}\pi)} \left[\left(\bar{\psi}\psi\right)^2 - \left(\bar{\psi}\gamma_5\tau_i\psi\right)^2\right]$$

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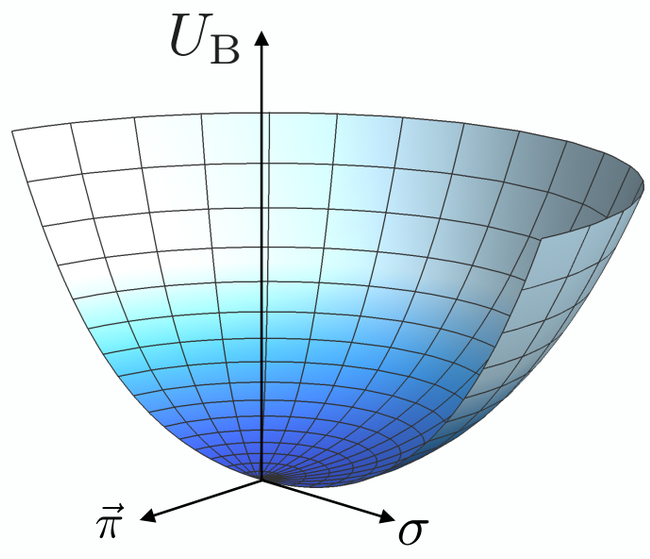

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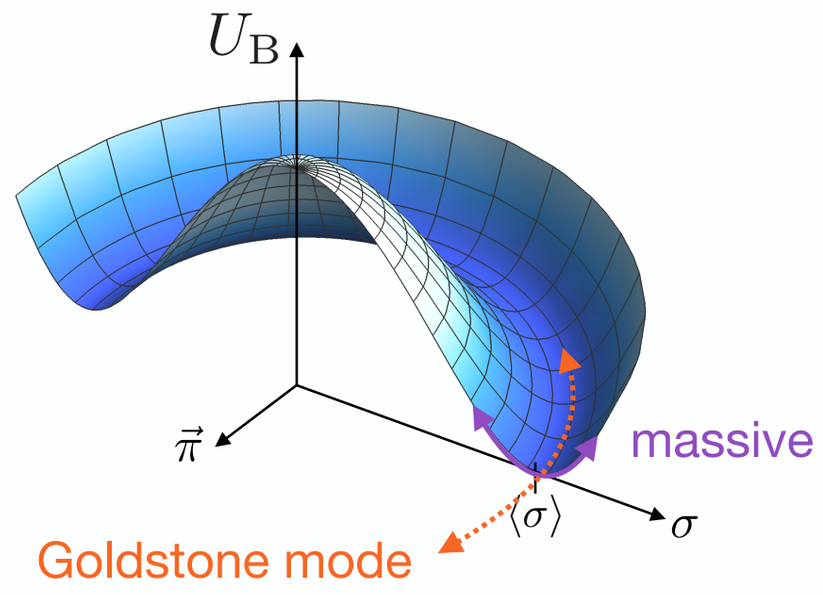

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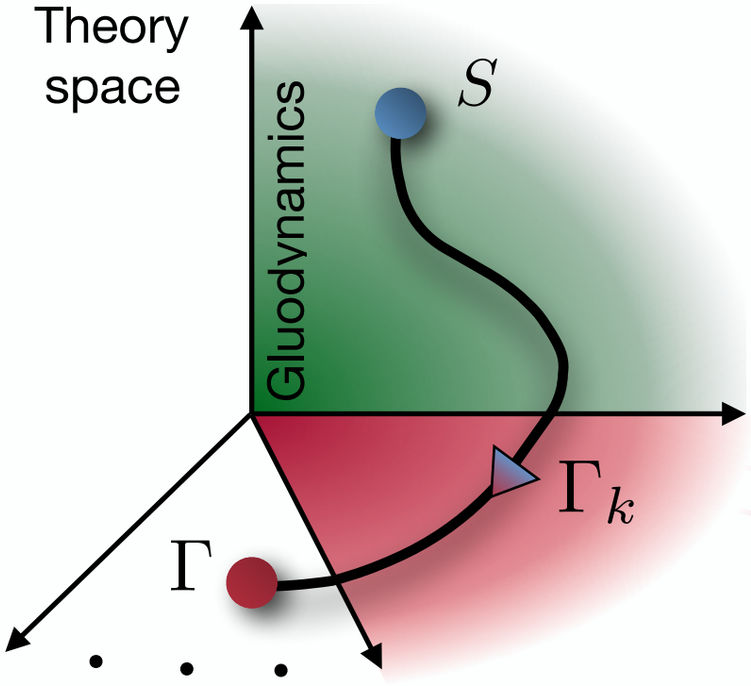

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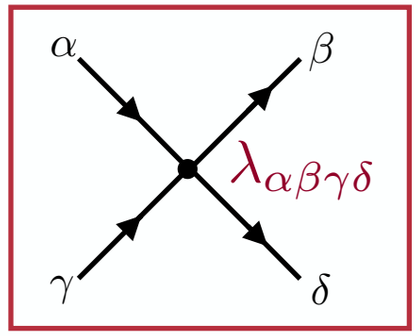

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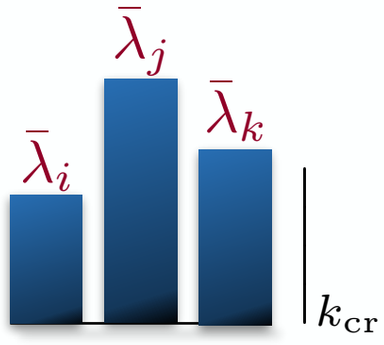
$\leftrightarrow$ chiral condensate

Assessing relative interaction strengths



Symmetry of the ground state

Formation of specific condensates

- chiral condensate
- diquark condensate

  $\leftrightarrow \bar{\lambda}_{\mathrm{csc}} \left(\mathrm{i}\bar{\psi}\gamma_5\tau_2\lambda^A\psi^C\right)\left(\mathrm{i}\bar{\psi}^C\gamma_5\tau_2\lambda^A\psi\right)$

  e.g., [Rapp, Schäfer, Shuryak, Velkovsky, 1998], see also [Son, 1999], [Pisarski and Rischke, 1999]
- …

Relevance of Fierz completeness

at large chemical potential

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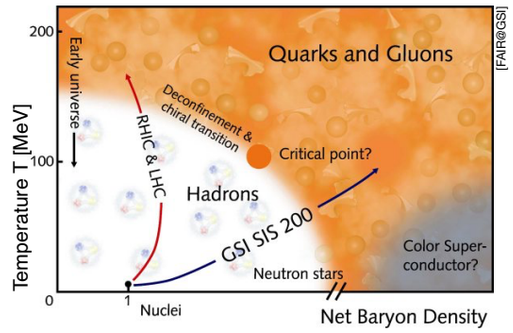
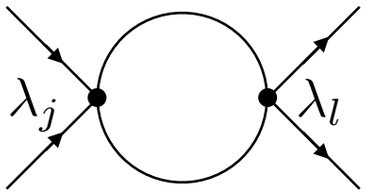
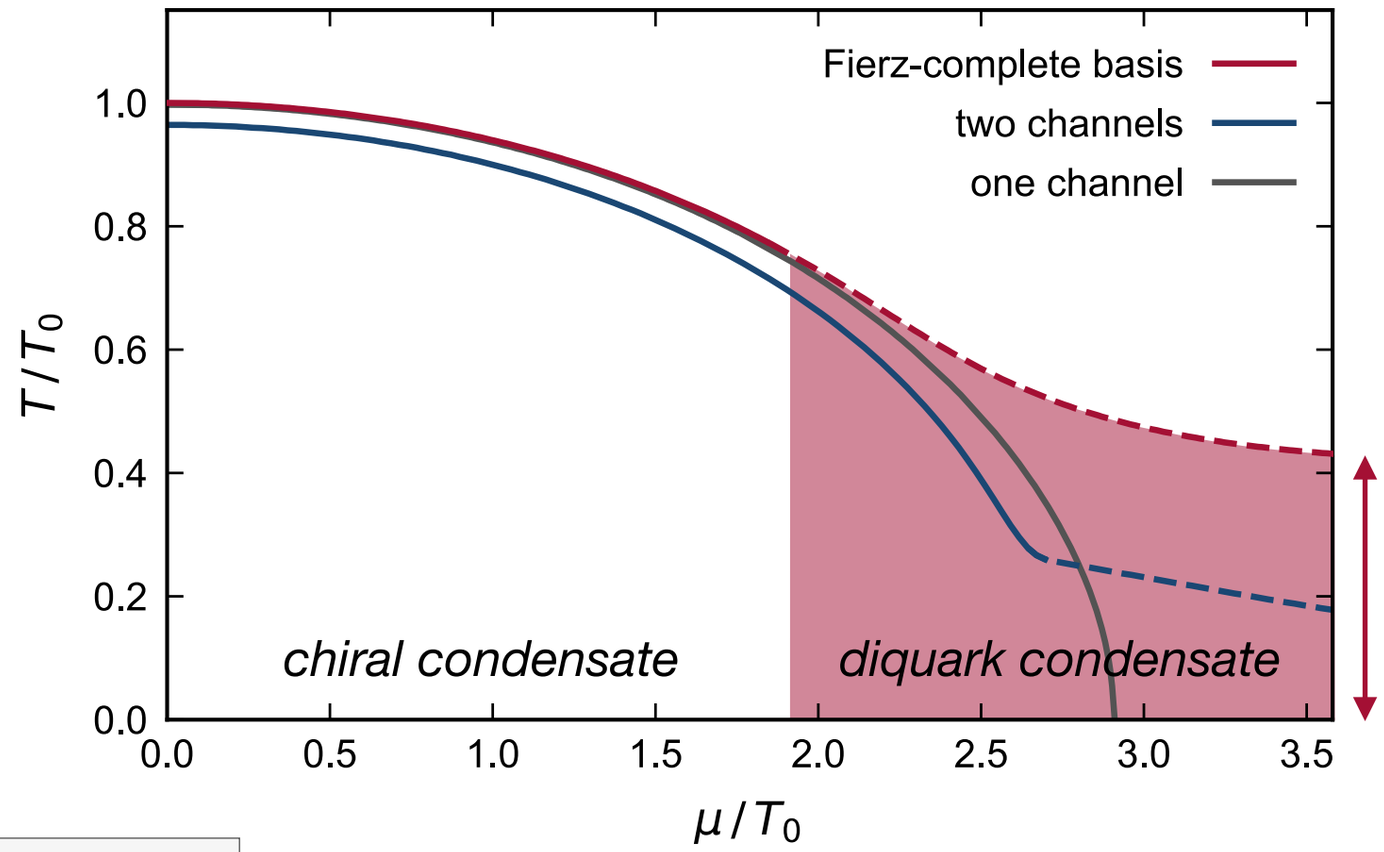
$$\Gamma_{\mathrm{NJL}}[\bar{\psi},\psi] = \int_0^{\frac{1}{T}} \mathrm{d}\tau \int \mathrm{d}^3x \left\{ \bar{\psi}\left(\mathrm{i}\partial\!\!\!/ - \mathrm{i}\mu\gamma_0\right)\psi + \frac{1}{2}\sum_{j\in\mathcal{B}} Z_j \bar{\lambda}_j \mathcal{L}_j \right\}$$



→ RG flow equation: $k\frac{\partial}{\partial k}\lambda_i = 2\lambda_i - \sum_{j,l}$  λ_l



$$T_{\mathrm{cr}} \sim \Delta_{\mathrm{csc}}(T=0)$$

[BCS theory]
[Schmitt, Wang, Rischke, 2002]

Implications for EOS

Fierz completeness crucial at large chemical potential!

[J. Braun, ML, M. Pospiech, PRD 97, 076010 (2018)]

Outline

Constraints on the equation of State at Higher Densities from the FRG

TECHNISCHE UNIVERSITÄT DARMSTADT

- Functional Renormalization Group
- Four-quark self-interactions

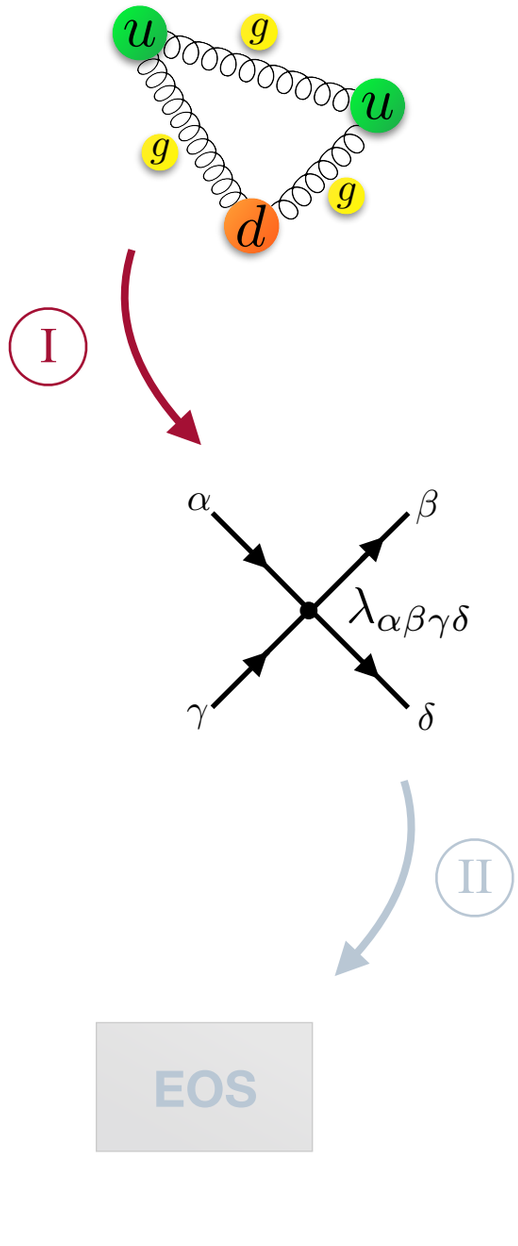
Ⅰ Ab initio: dynamic gauge fields

- Structure of β functions
- Symmetry breaking patterns at finite T
- Dominant low-energy d.o.f's at zero T

Ⅱ Connecting to the low-energy regime

- Quark-meson-diquark truncation
- Findings on the equation of state (EOS)

Conclusions and outlook



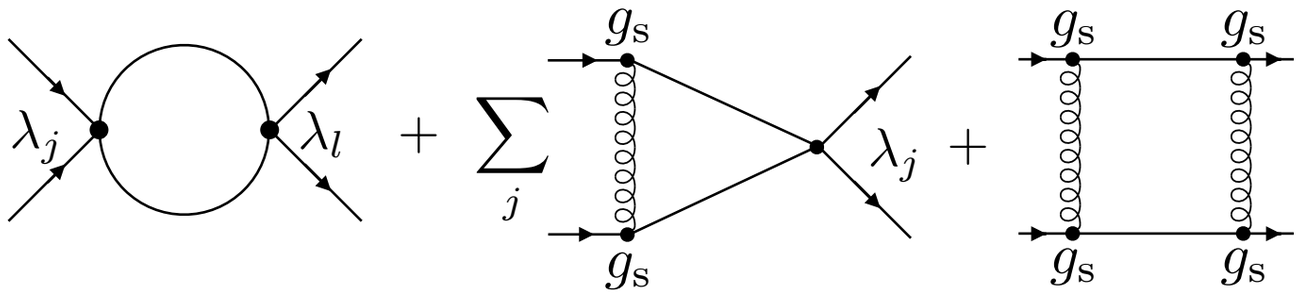
Emergence from quark-gluon dynamics

Structure of the β functions and formation of condensates

TECHNISCHE UNIVERSITÄT DARMSTADT

RG flow equation:

$$k\frac{\partial}{\partial k}\lambda_i = 2\lambda_i - \sum_{j,l} \lambda_j \quad \lambda_l + \sum_j \quad \lambda_j + \quad$$



Running gauge coupling:

$$k\frac{\partial}{\partial k} g_\mathrm{s}^2(k) = \eta_{A,k}\, g_\mathrm{s}^2(k)$$

[Abbott, Nucl. Phys. B (1981)]
[Braun, Gies, Phys. Lett. B (2007)]

Emergence from quark-gluon dynamics

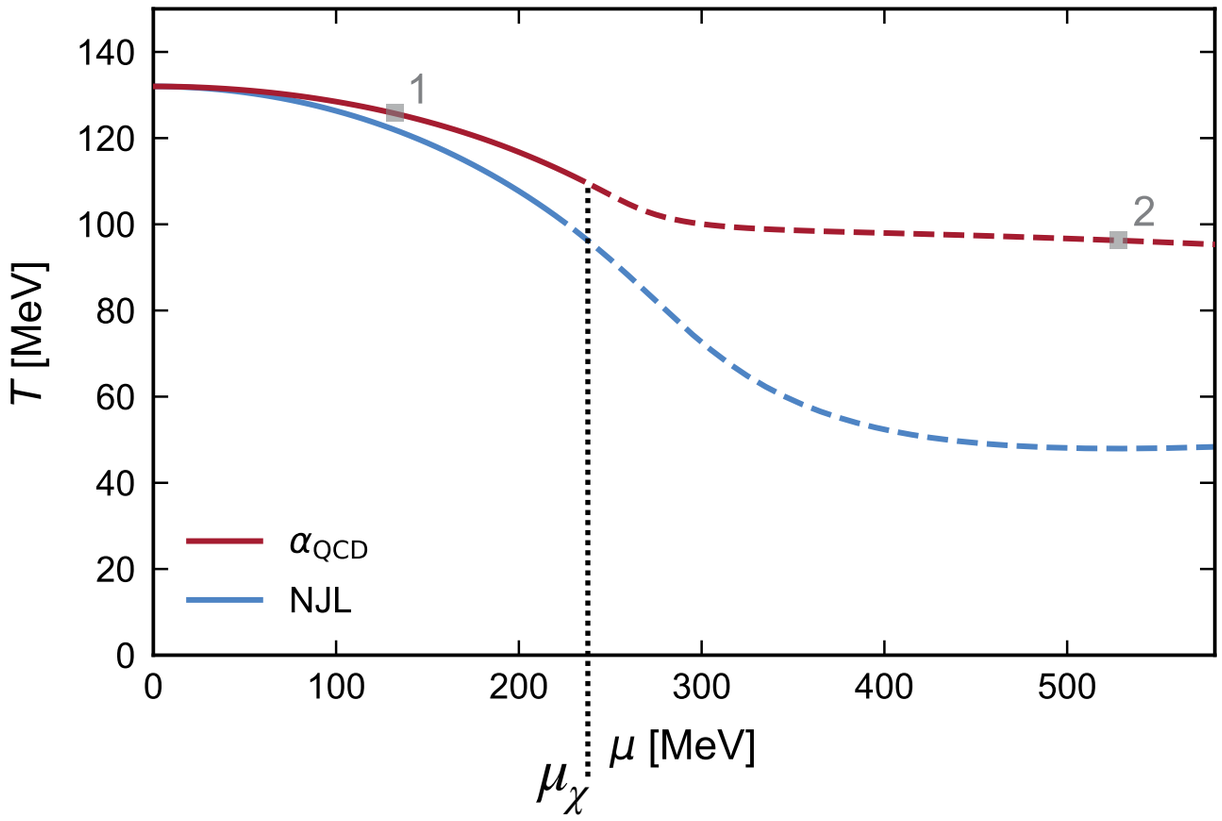
Structure of the β functions and formation of condensates

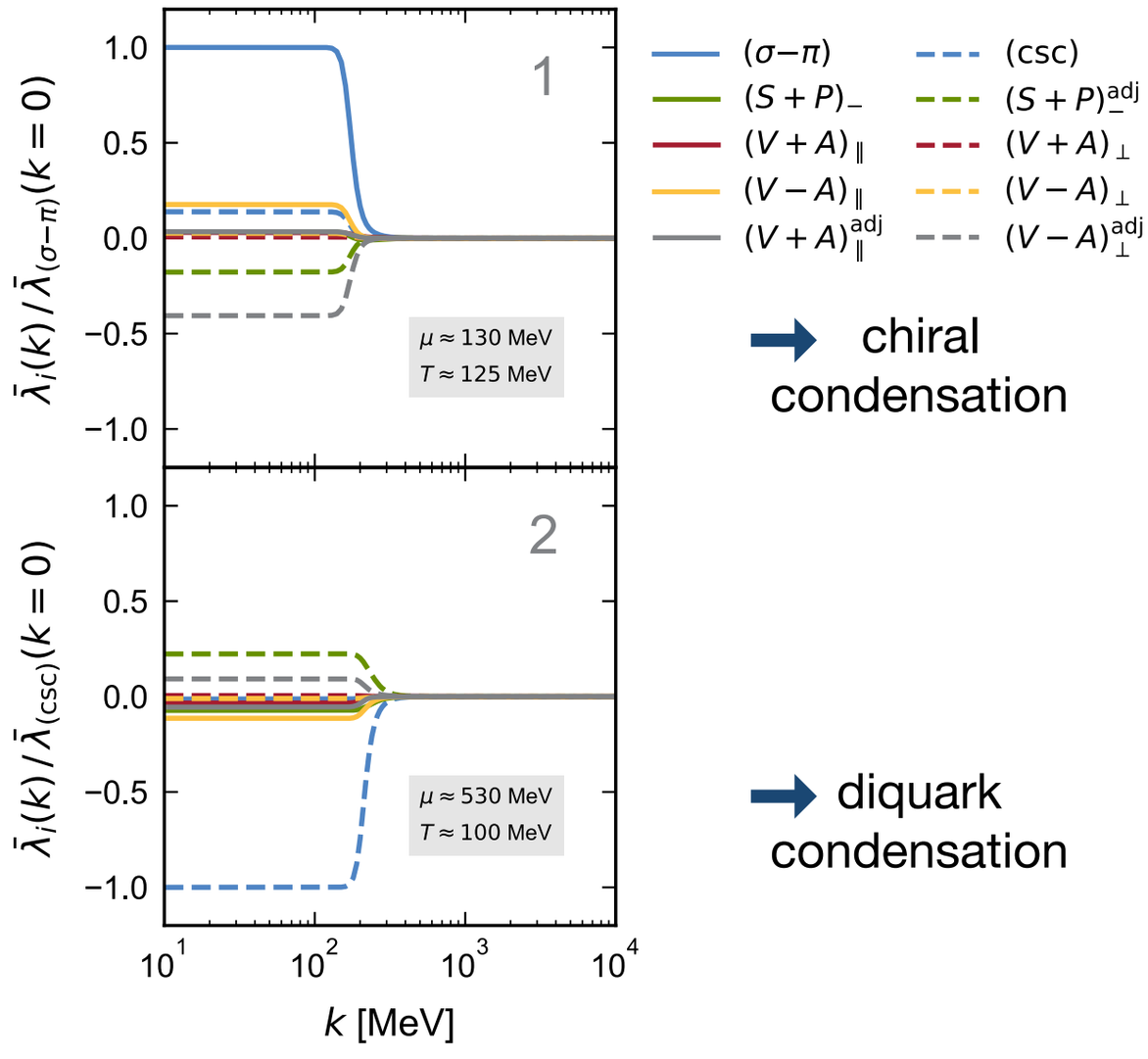
TECHNISCHE UNIVERSITÄT DARMSTADT

RG flow equation:

$$k\frac{\partial}{\partial k}\lambda_i = 2\lambda_i - \sum_{j,l} \lambda_j \lambda_l + \sum_j \lambda_j + \; g_s^2 \; g_s^2$$

At $k = \Lambda$:
$\lambda_i^{(\mathrm{UV})} = 0$
$g_s(\Lambda)$ only parameter





➡ chiral condensation

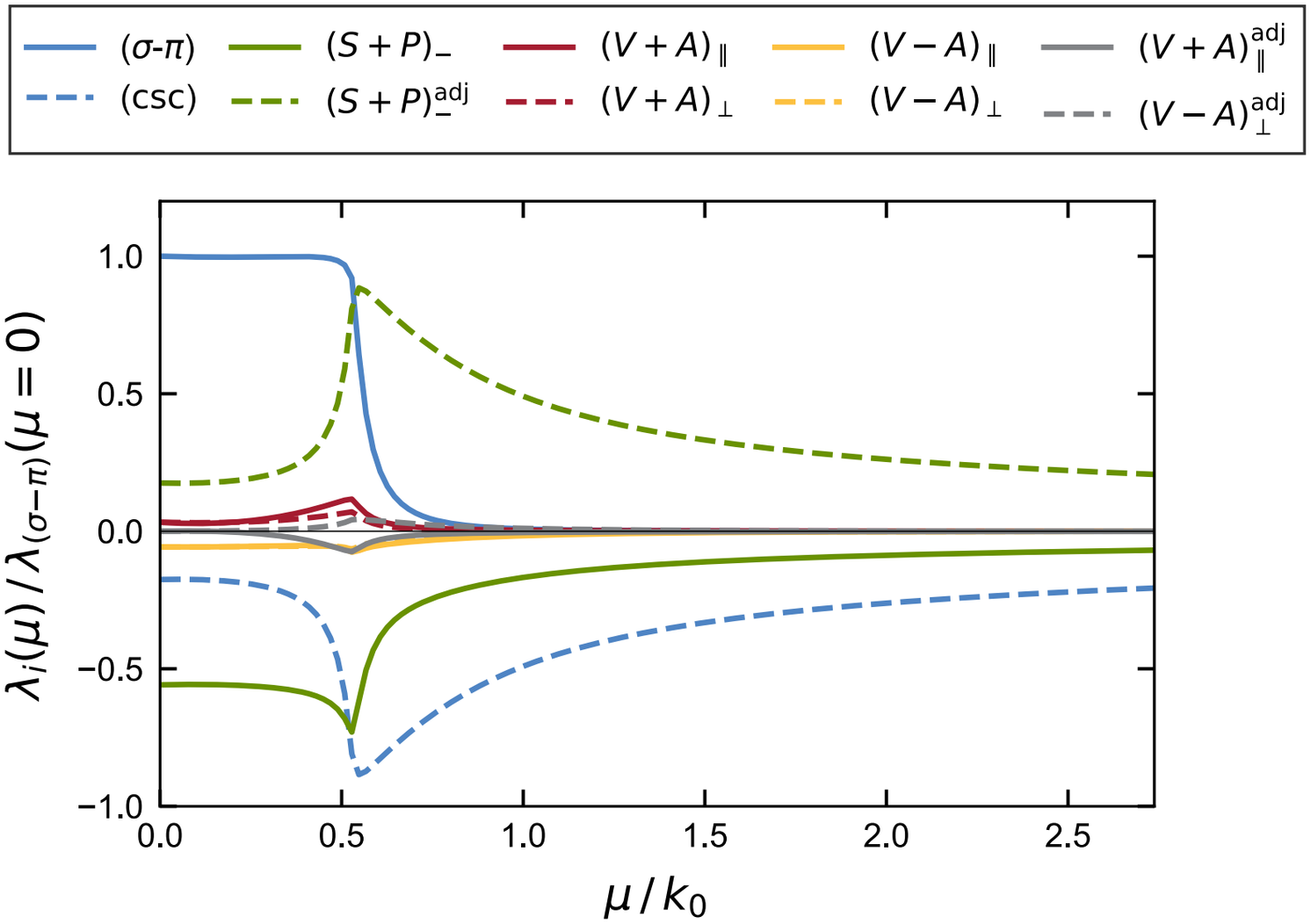
➡ diquark condensation

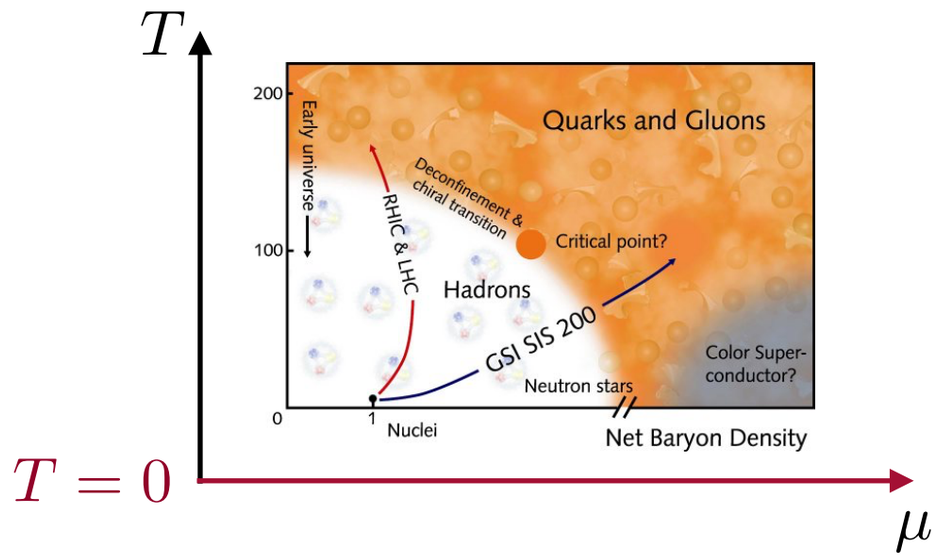
- Gluodynamics essential at high μ
- Natural emergence of dominances
- Remarkable robustness of phase boundary, dominances and μ_χ ➡ quark dynamics

[Braun, ML, Pospiech, arXiv: 1909.06298]

Dominance pattern

in the zero-temperature limit at finite densities





$$\lambda_i(\mu)\Big|_{k=k_{\mathrm{cr}}+\epsilon}$$

- $U_A(1)$ symmetry artifact ⟶ green lines suppressed in case of explicit $U_A(1)$ breaking
- Dominant d.o.f.'s: chiral fields $\phi \sim \langle\bar{\psi}\psi\rangle$

  diquark fields $\Delta^a \sim \langle \mathrm{i}\bar{\psi}^C\gamma_5\epsilon_{(f)}\varepsilon^a_{(c)}\psi\rangle$

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- Findings on the equation of state (EOS)

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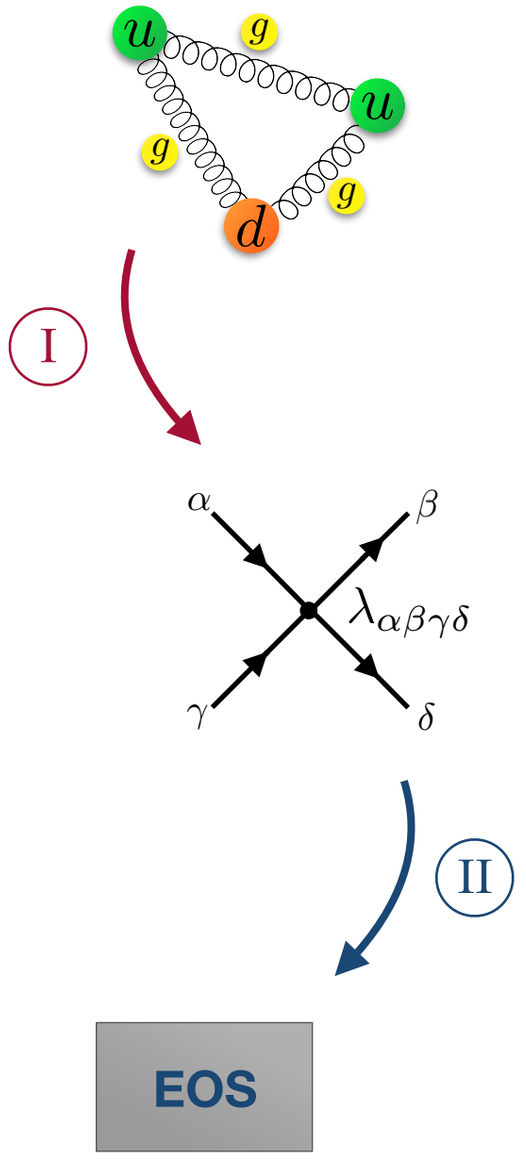
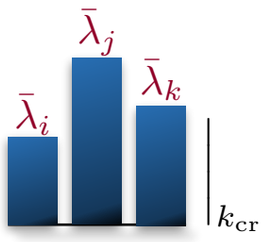


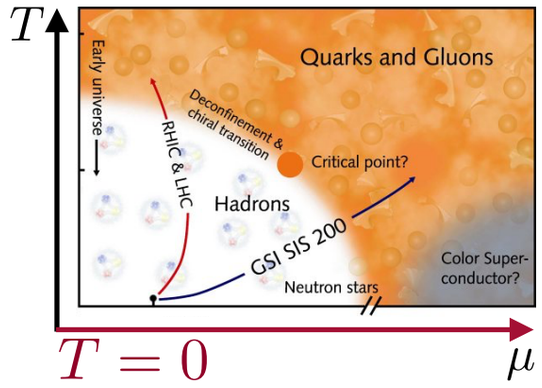
EOS of dense strong-interaction matter

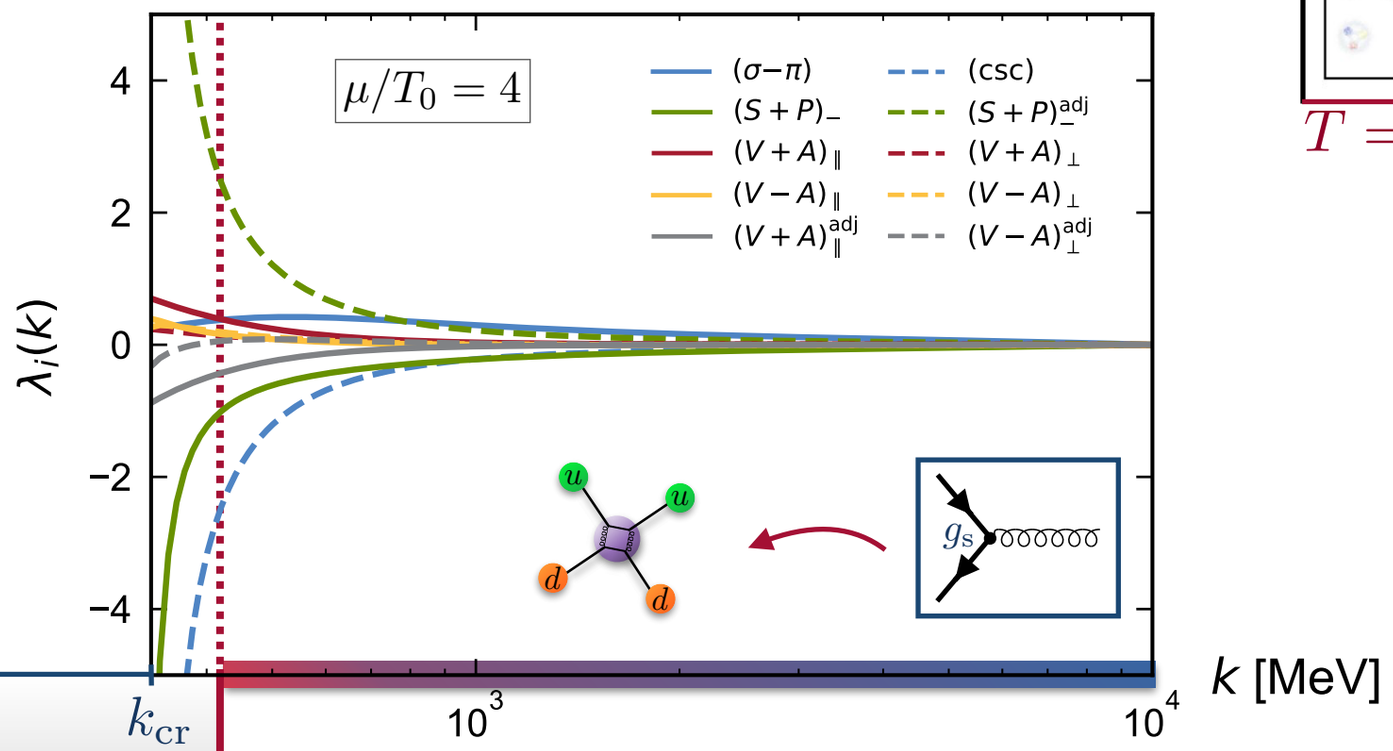
Connecting to the low-energy regime

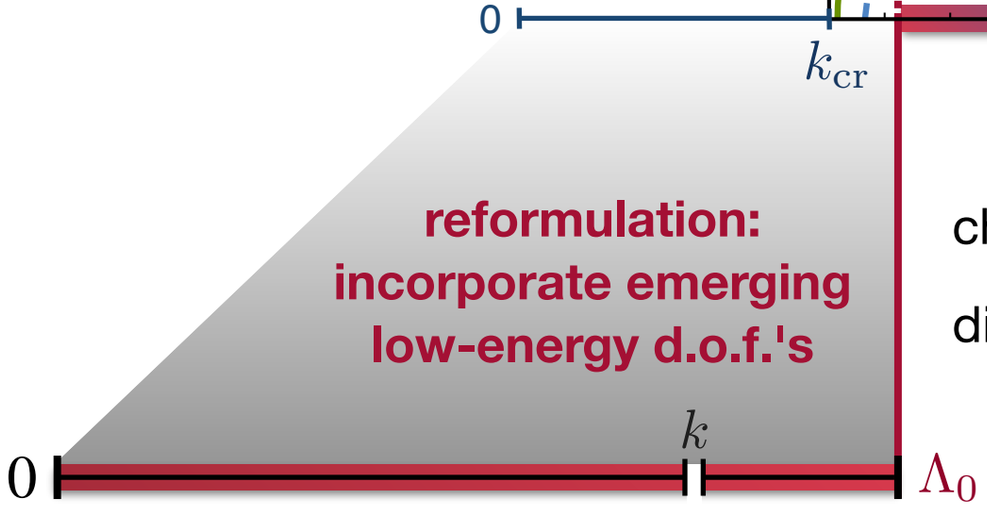
TECHNISCHE UNIVERSITÄT DARMSTADT

- Diverging couplings signal onset of condensate formation
- Assessing relative interaction strengths









chiral fields $\phi \sim \langle \bar{\psi}\psi \rangle$

diquark fields $\Delta^a \sim \langle \mathrm{i}\bar{\psi}^C \gamma_5 \epsilon_{(f)} \varepsilon^a_{(c)} \psi \rangle$

$S_{\rm QMD}$

quark-meson-diquark truncation

Determining parameters at high densities

Connecting to the low-energy regime

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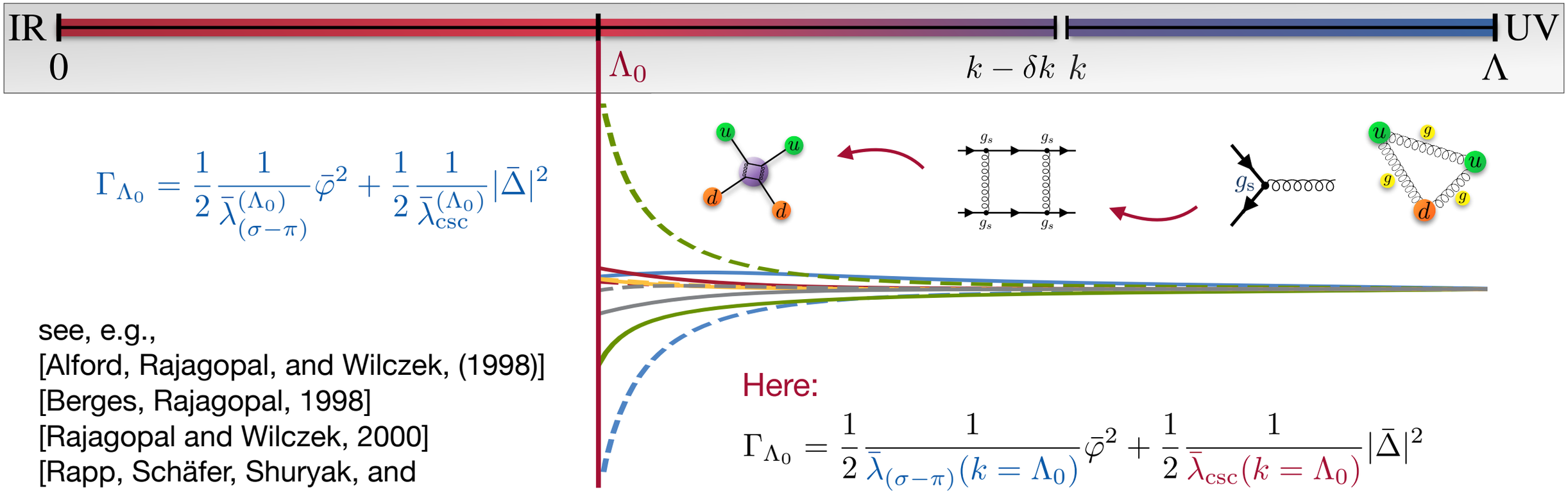
Classical action of the quark-meson-diquark model (QMD)

$$S_{\mathrm{QMD}} = \int_x \Big\{ \bar{\psi}\Big(\mathrm{i}\partial\!\!\!/ - \mathrm{i}\mu\gamma_0 + \mathrm{i}(\sigma + \mathrm{i}\vec{\tau}\cdot\vec{\pi}\gamma_5)\Big)\psi + \frac{1}{2}\frac{1}{\bar{\lambda}_{(\sigma-\pi)}}\varphi^2 \Big\} \quad \text{QM part}$$

$$+\bar{\psi}\gamma_5\tau_2\Delta_A^* T^A \mathcal{C}\bar{\psi}^T - \psi^T\mathcal{C}\gamma_5\tau_2\Delta_A T^A\mathcal{C}\psi + \frac{1}{2}\frac{1}{\bar{\lambda}_{\mathrm{csc}}}|\Delta|^2 \Big\} \quad \text{diquarks}$$



$$\Gamma_{\Lambda_0} = \frac{1}{2}\frac{1}{\bar{\lambda}^{(\Lambda_0)}_{(\sigma-\pi)}}\bar{\varphi}^2 + \frac{1}{2}\frac{1}{\bar{\lambda}^{(\Lambda_0)}_{\mathrm{csc}}}|\bar{\Delta}|^2$$

see, e.g.,
[Alford, Rajagopal, and Wilczek, (1998)]
[Berges, Rajagopal, 1998]
[Rajagopal and Wilczek, 2000]
[Rapp, Schäfer, Shuryak, and Velkovsky, 1998]
see also [Buballa, 2005],
[Shovkovy, 2005], [Alford et al., 2008]

Here:

$$\Gamma_{\Lambda_0} = \frac{1}{2}\frac{1}{\bar{\lambda}_{(\sigma-\pi)}(k=\Lambda_0)}\bar{\varphi}^2 + \frac{1}{2}\frac{1}{\bar{\lambda}_{\mathrm{csc}}(k=\Lambda_0)}|\bar{\Delta}|^2$$

Determining parameters at high densities

Connecting to the low-energy regime

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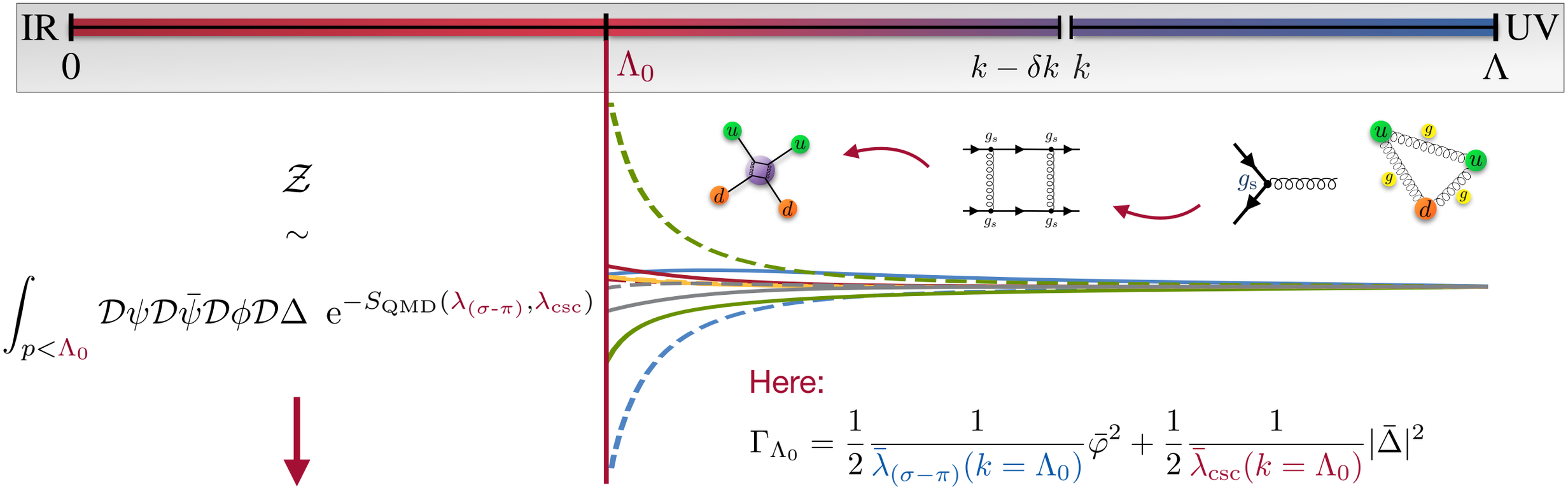
- Pass on values of four-quark couplings at transition scale Λ_0: determine parameters of QMD
- Parameters density dependent
- Circumvent limit of accessible range of external parameters
- Minimization of thermodynamic potential: size of gaps, pressure, density, …



$$\mathcal{Z} \sim \int_{p<\Lambda_0} \mathcal{D}\psi\mathcal{D}\bar{\psi}\mathcal{D}\phi\mathcal{D}\Delta \; \mathrm{e}^{-S_{\mathrm{QMD}}(\lambda_{(\sigma\text{-}\pi)},\lambda_{\mathrm{csc}})}$$

Here:

$$\Gamma_{\Lambda_0} = \frac{1}{2}\frac{1}{\bar{\lambda}_{(\sigma-\pi)}(k=\Lambda_0)}\bar{\varphi}^2 + \frac{1}{2}\frac{1}{\bar{\lambda}_{\mathrm{csc}}(k=\Lambda_0)}|\bar{\Delta}|^2$$

Thermodynamic potential $\Omega(\mu) = -\frac{T}{V}\log\mathcal{Z}\Big|_{\mathrm{g.s.}}$

EOS of dense strong-interaction matter

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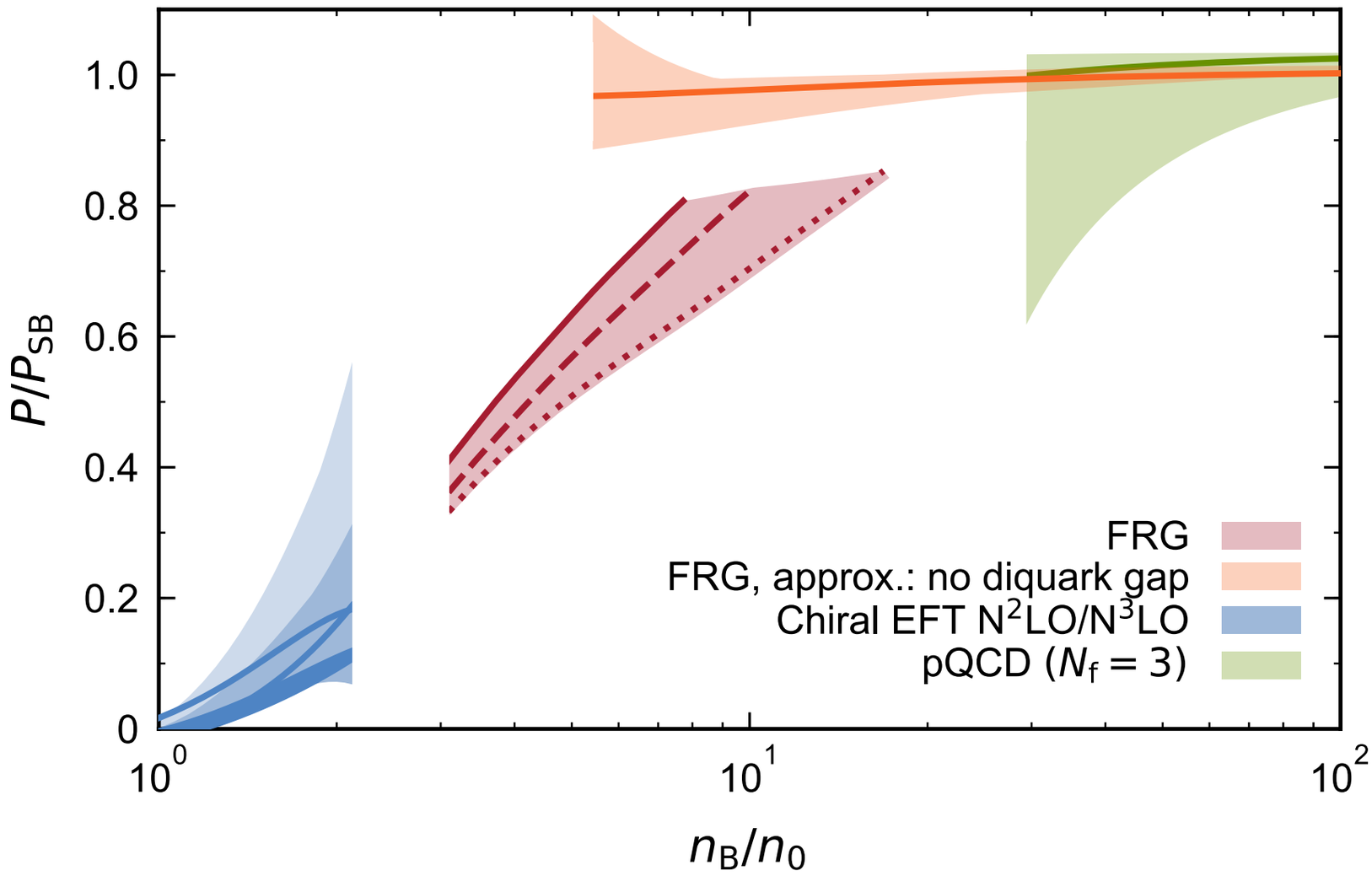
Pressure as a function of the density



- EOS computation within FRG framework directly based on fundamental quark-gluon dynamics
- Diquark condensation as non-perturbative effect essential at intermediate densities
- Compatible with calculations based on chiral EFT interactions [Drischler et al., PRL (2019)] toward lower densities and with perturbative QCD [Gorda et al., PRL (2018)] at high density

[ML, Pospiech, Schallmo, Braun, Drischler, Hebeler, Schwenk, arXiv:1907.05814]

EOS of dense strong-interaction matter

Speed of sound at intermediate densities

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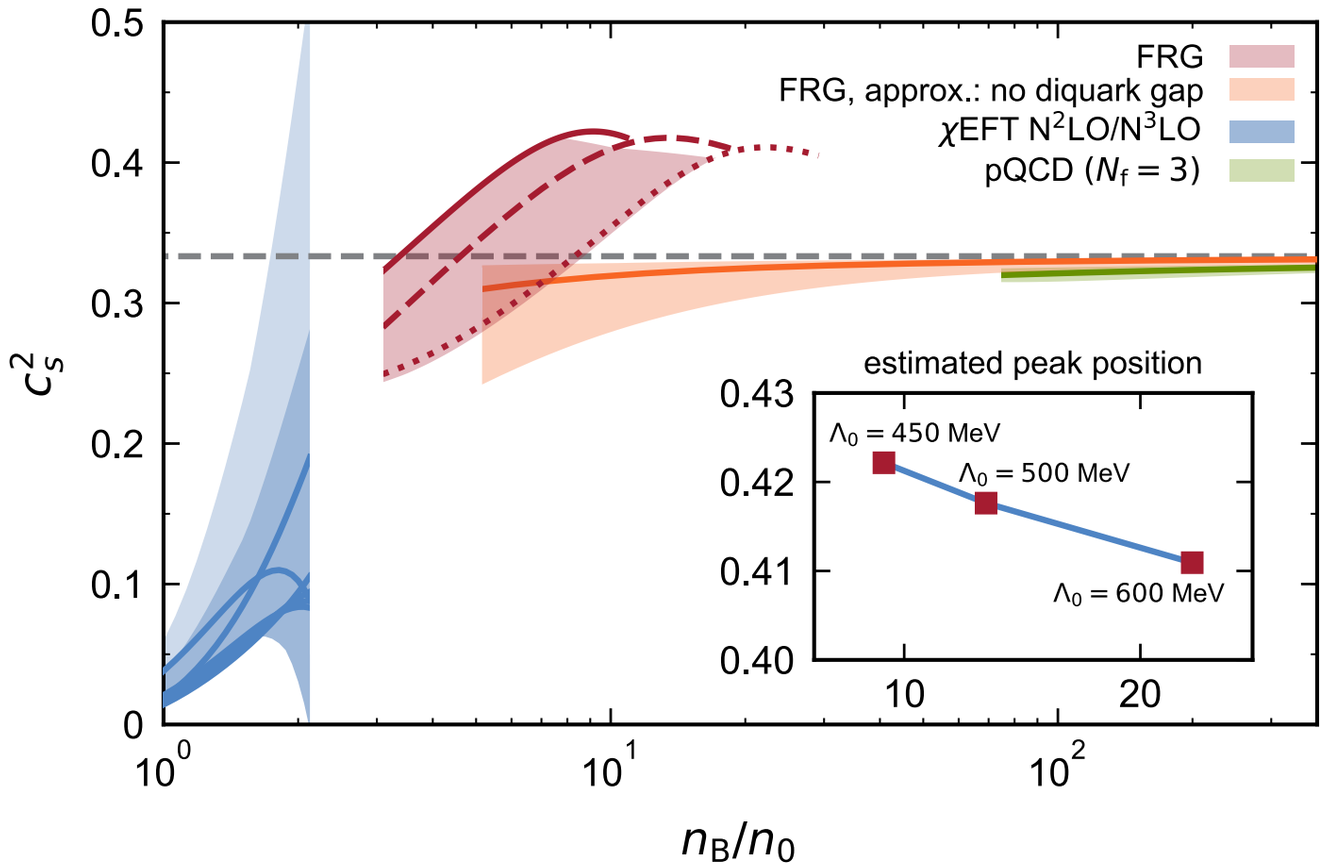

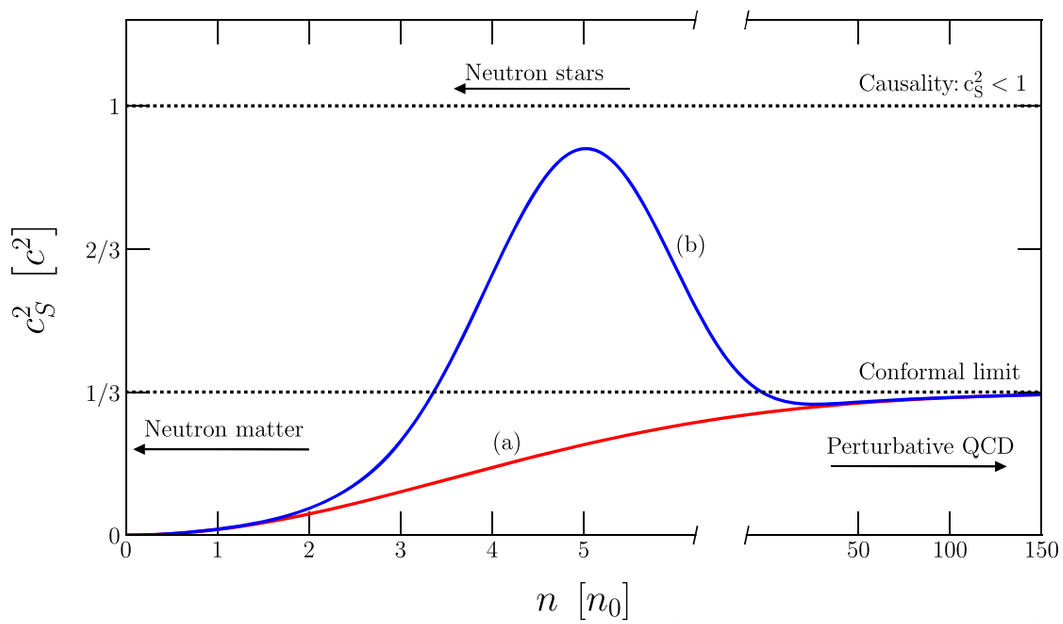

[Tews et al., ApJ (2018)]

- Natural emergence of maximum in speed of sound
- Peak position shows dependence on transition scale Λ_0, magnitude insensitive
- Maximum in speed of sound of neutron matter necessary to support observed heavy neutron star masses [e.g. Tews et al., ApJ (2018)]

[ML, Pospiech, Schallmo, Braun, Drischler, Hebeler, Schwenk, arXiv:1907.05814]

Conclusions

Thank you very much for your attention!

TECHNISCHE UNIVERSITÄT DARMSTADT

Phase structure

- Dynamical generation of four-quark interactions by two-gluon exchange
- Importance of Fierz completeness at high quark chemical potential
- Natural emergence of dominances, formation of diquark condensate favored at high densities
- Remarkable robustness of phase boundary and dominances, dynamics within quark sector play decisive role

[Braun, ML, Pospiech, PRD (2017)]
[Braun, ML, Pospiech, PRD (2018)]
[Braun, ML, Pospiech, arXiv:1909.06298]

Equation of state

- Higher density regime accessed by utilizing RG flow of four-quark couplings in QCD
- Incorporation of low-energy fluctuations with quark-meson-diquark truncation
- EOS of symmetric nuclear matter from first principles
  - Compatible with chiral EFT data as well as perturbative QCD
  - Condensation effects becoming essential at intermediate densities
- Maximum in the speed of sound exceeding the non-interacting limit

[Braun, ML, Pawlowski, SciPost (2019)]
[ML, Pospiech, Schallmo, Braun, Drischler, Hebeler, Schwenk, arXiv:1907.05814]