Nonparametric Inference of the Neutron Star Equation of State from Gravitational Wave Observations

Reed Essick
reed.essick@gmail.com

(on behalf of an increasing number of people)

Kavli Institute for Cosmological Physics
University of Chicago

P. Landry and R. Essick. *Nonparametric inference of the neutron star equation of state from gravitational wave observations*.Phys. Rev. D **99**, 084049 (2019)

R. Essick, P. Landry, D. Holz. *Nonparametric Inference of Neutron Star Composition, Equation of State, and Maximum Mass with GW170817*. arXiv:1910.09740 (2019)

Goals

- Self-consistently incorporate information from arbitrary tabulated EOS models
- Automatically incorporate causality constraints and thermodynamic stability
- Allow for large amounts of model freedom
- Incorporate transparent priors

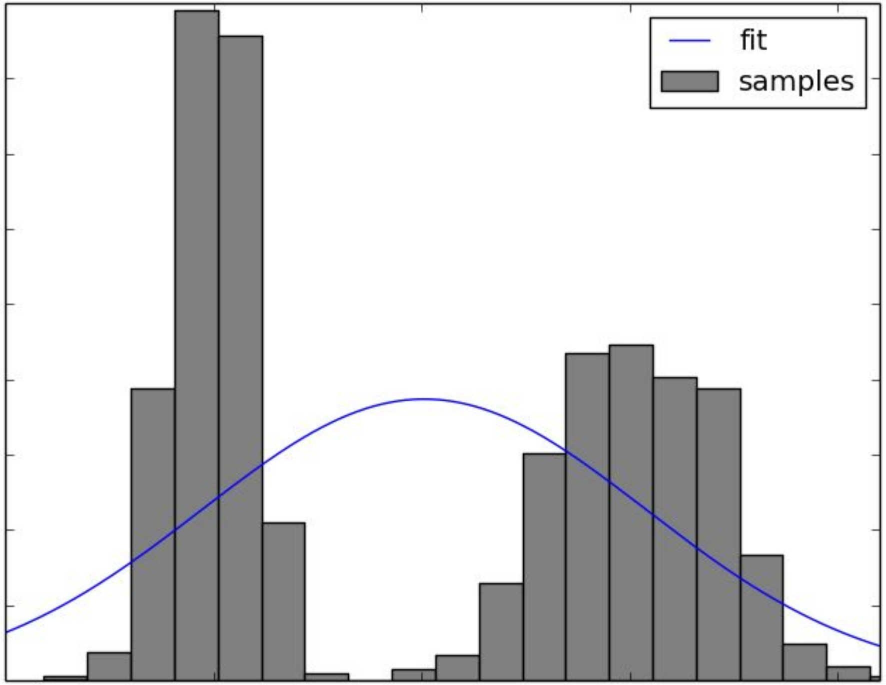
differences between Parametric and Nonparametric inference

Parametric vs. Nonparametric Inference

Parametric constructions:

- Typically, include a small number of parameters and claim they reproduce proposed EOS reasonably well
- Think of fitting a collection of data with a fixed function

Parametric analyses require the function to be of a specific form which may or may not faithfully represent the data.



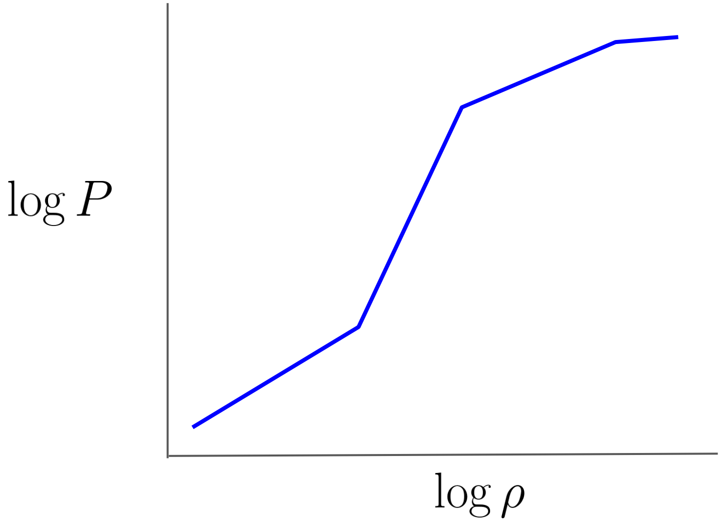
Parametric vs. Nonparametric Inference

Parametric constructions:

- Typically, include a small number of parameters and claim they reproduce proposed EOS reasonably well
- Think of fitting a collection of data with a fixed function

Piecewise polytrope

$$P = k_i \rho^{n_i} \quad \text{iff} \quad \rho_{i-1} \leq \rho \leq \rho_i$$



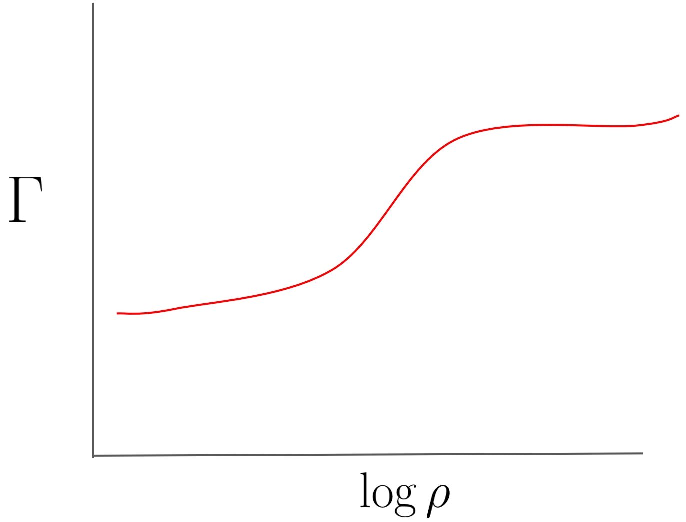
Parametric vs. Nonparametric Inference

Parametric constructions:

- Typically, include a small number of parameters and claim they reproduce proposed EOS reasonably well
- Think of fitting a collection of data with a fixed function

Piecewise polytrope $$P = k_i \rho^{n_i} \quad \text{iff} \quad \rho_{i-1} \leq \rho \leq \rho_i$$

Spectral decomposition $$\Gamma = \sum a_i \left(\log \rho\right)^i$$



Parametric vs. Nonparametric Inference

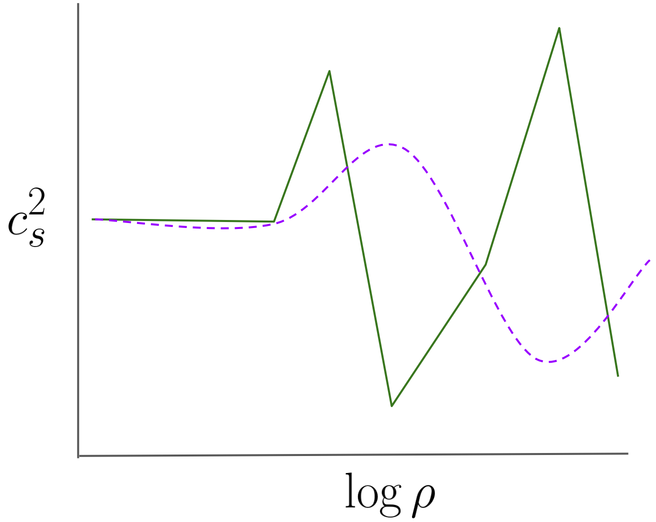
Parametric constructions:

- Typically, include a small number of parameters and claim they reproduce proposed EOS reasonably well
- Think of fitting a collection of data with a fixed function

Piecewise polytrope $$P = k_i \rho^{n_i} \quad \text{iff} \quad \rho_{i-1} \leq \rho \leq \rho_i$$

Spectral decomposition $$\Gamma = \sum a_i \left(\log \rho\right)^i$$

Sound-speed constructions $$c_s^2 = f_\theta(\rho)$$

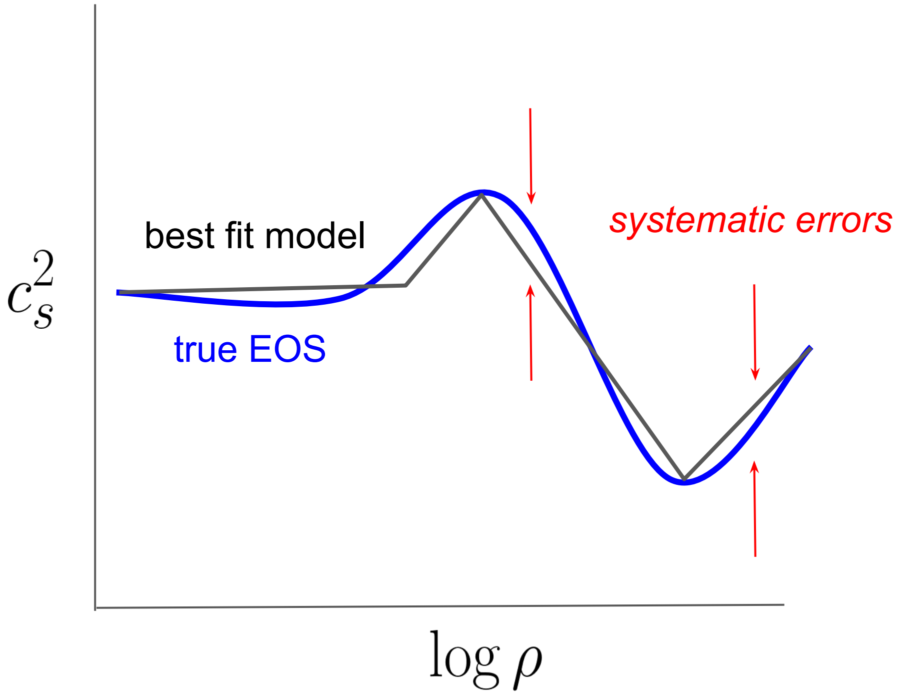


Parametric vs. Nonparametric Inference

a function-space perspective

Parametric constructions

- only allow for certain types of behavior (set of measure zero), and all expected behavior must be built into the model from the start
- If true EOS is not exactly described by the parameterized model, it can never be exactly recovered

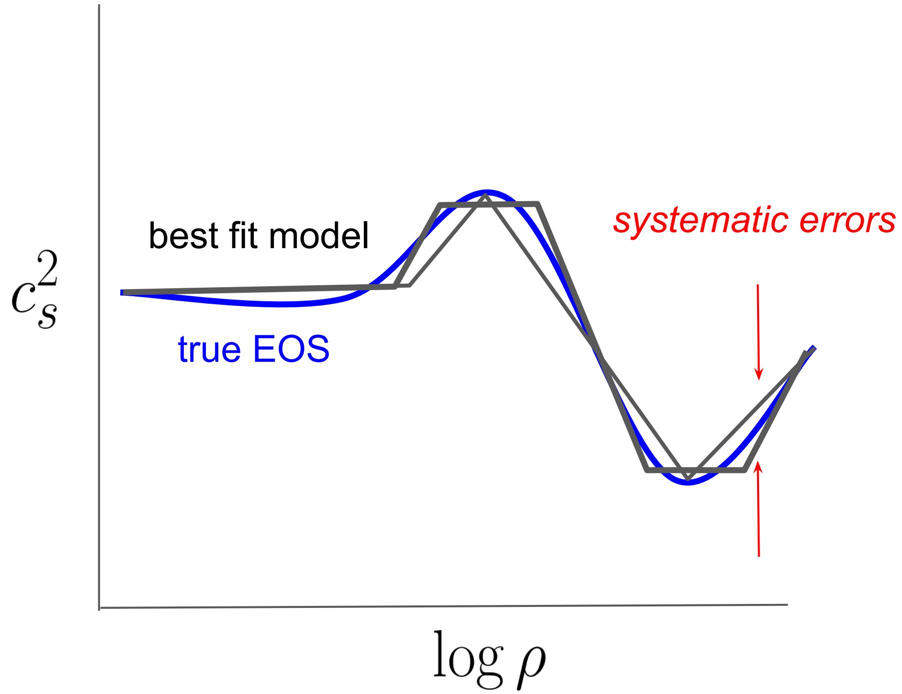


Parametric vs. Nonparametric Inference

a function-space perspective

Parametric constructions, adding more parameters

- May yield more model freedom, but any finite set will still have some systematic error
- Can further complicate unintuitive priors



Note!
adding more parameters does not make the analysis nonparametric!

drawbacks of parametric approaches

- Self-consistently incorporate information from arbitrary tabulated EOS models
- Automatically incorporate causality constraints and thermodynamic stability
- Allow for large amounts of model freedom
- Incorporate transparent priors

drawbacks of parametric approaches

- Self-consistently incorporate information from arbitrary tabulated EOS models
  - Can check for *faithfulness* between parameterized model and any proposed EOS
  - However, usually this is some sort of mean-square-error or other *ad hoc* statistic and more parameters may be needed

- Automatically incorporate causality constraints and thermodynamic stability

- Allow for large amounts of model freedom

- Incorporate transparent priors

drawbacks of parametric approaches

- Self-consistently incorporate information from arbitrary tabulated EOS models
    - Can check for *faithfulness* between parameterized model and any proposed EOS
    - However, usually this is some sort of mean-square-error or other *ad hoc* statistic and more parameters may be needed

- Automatically incorporate causality constraints and thermodynamic stability
    - This can be accomplished with clever prior bounds on the parameters
        - Can be hard to articulate
    - Some parameterizations build this in from the start (e.g., sound-speed parameterizations)

- Allow for large amounts of model freedom

- Incorporate transparent priors

drawbacks of parametric approaches

- Self-consistently incorporate information from arbitrary tabulated EOS models
  - Can check for *faithfulness* between parameterized model and any proposed EOS
  - However, usually this is some sort of mean-square-error or other *ad hoc* statistic and more parameters may be needed

- Automatically incorporate causality constraints and thermodynamic stability
  - This can be accomplished with clever prior bounds on the parameters
    - Can be hard to articulate
  - Some parameterizations build this in from the start (e.g., sound-speed parameterizations)

- Allow for large amounts of model freedom
  - From a functional-space perspective, any finite parametrization of the EOS constitutes a vanishingly small set of all possible causal and stable EOS
  - One can always add more parameters...

- Incorporate transparent priors

drawbacks of parametric approaches

- Self-consistently incorporate information from arbitrary tabulated EOS models
  - Can check for *faithfulness* between parameterized model and any proposed EOS
  - However, usually this is some sort of mean-square-error or other *ad hoc* statistic and more parameters may be needed

- Automatically incorporate causality constraints and thermodynamic stability
  - This can be accomplished with clever prior bounds on the parameters
    - Can be hard to articulate
  - Some parameterizations build this in from the start (e.g., sound-speed parameterizations)

- Allow for large amounts of model freedom
  - From a functional-space perspective, any finite parametrization of the EOS constitutes a vanishingly small set of all possible causal and stable EOS
  - One can always add more parameters...

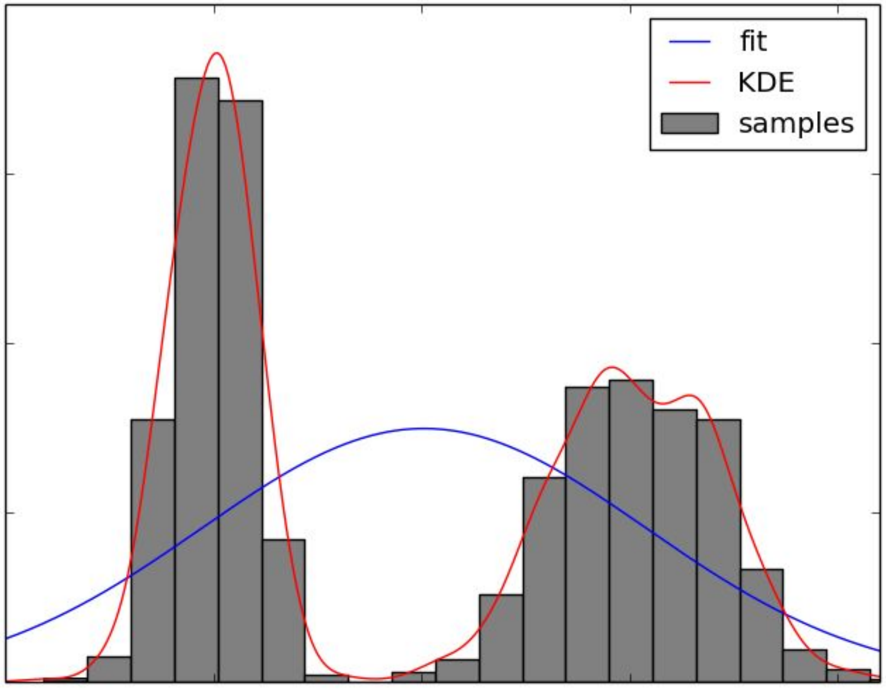
- Incorporate transparent priors
  - Depending on the parameterization, priors can be unintuitive
  - “Flat priors” may in fact be quite informative

Parametric vs. Nonparametric Inference

Nonparametric constructions:

- Do not assume a functional form for the EOS a prior
- Think of making a histogram or kernel density estimate instead of using a fixed functional form

Nonparametric analyses assume things about the type of correlations within a function but do not require the function to have any specific form!



Parametric vs. Nonparametric Inference

a function-space perspective

Parameterized models

- only allow for certain types of behavior (set of measure zero), and all expected behavior must be built into the model from the start
  - If true EOS is not exactly described by the parameterized model, it can never be recovered without some bias

Nonparametric constructions (with Gaussian processes)

- Assign non-zero prior probability to ***all causal and thermodynamically stable EOS***
  - No modeling systematics (in the limit of infinite data)
  - How do we assign relative probabilities to different possible EOS?
  - Surely we have some prior knowledge?

What is a Gaussian Process?

A "distribution over functions" described by a mean and covariance matrix over infinitely many degrees of freedom

$$\langle \phi(p) \rangle = \mu(p)$$
$$\big\langle \phi(p_i)\phi(p_j) \big\rangle = K(p_i, p_j)$$

Assuming these cumulants describe all statistical properties of the unknown function implies a *Gaussian process*.

What is a Gaussian Process?

A “distribution over functions” described by a mean and covariance matrix over infinitely many degrees of freedom

$$\langle \phi(p) \rangle = \mu(p)$$
$$\big\langle \phi(p_i)\phi(p_j) \big\rangle = K(p_i, p_j)$$

Assuming these cumulants describe all statistical properties of the unknown function implies a *Gaussian process*.

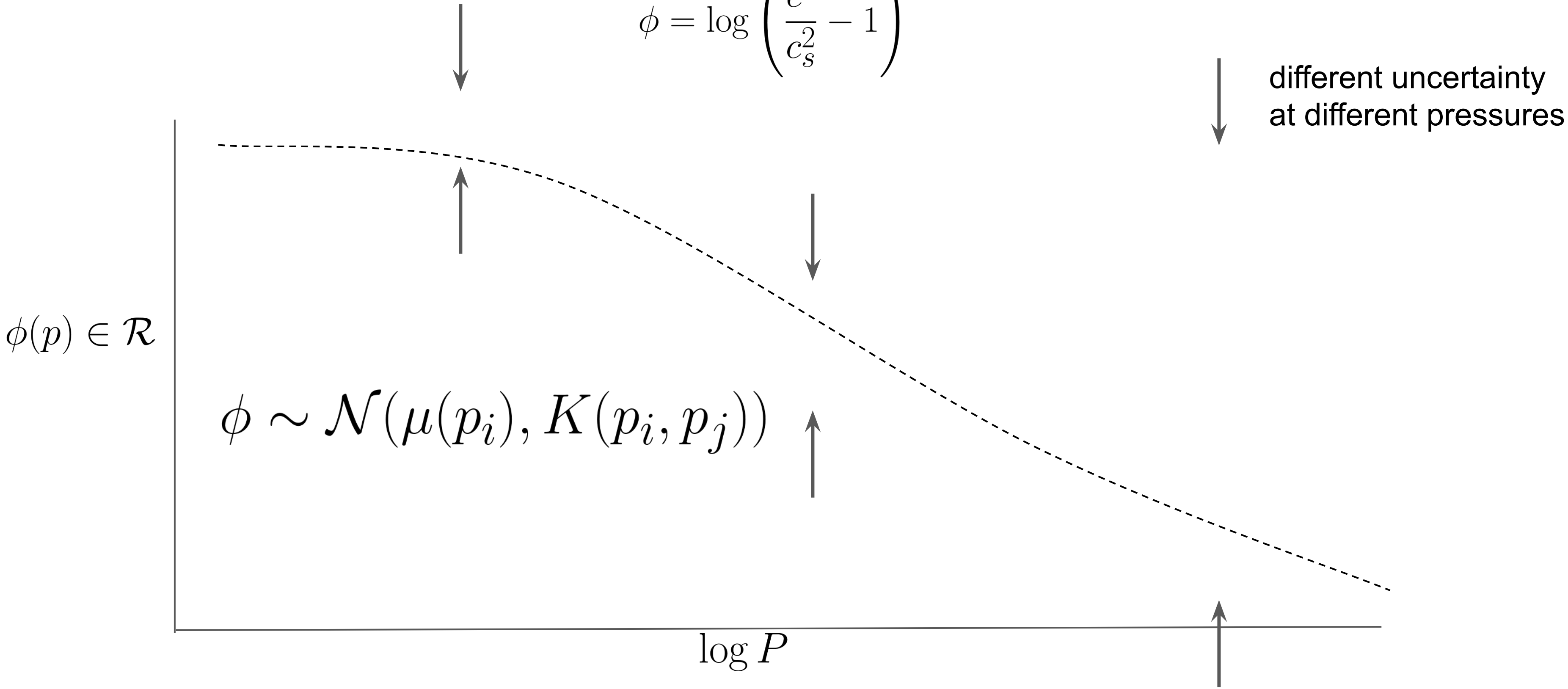
$$K(p_i, p_j) \sim \sigma^2 \exp\left(-\frac{(p_i - p_j)^2}{2l^2} \right) + \sigma_n^2(p_i)\delta(p_i - p_j) + \cdots$$

We parameterize the type of correlations preferred by the function, but not the specific functions themselves. This means we assign (and can compute!) a non-zero probability of obtaining *any function specified at arbitrary precision!*

Our Gaussian Processes

Gaussian processes generate functions that have support along the entire real line, so we map the sound speed to an *auxiliary variable* that spans this range

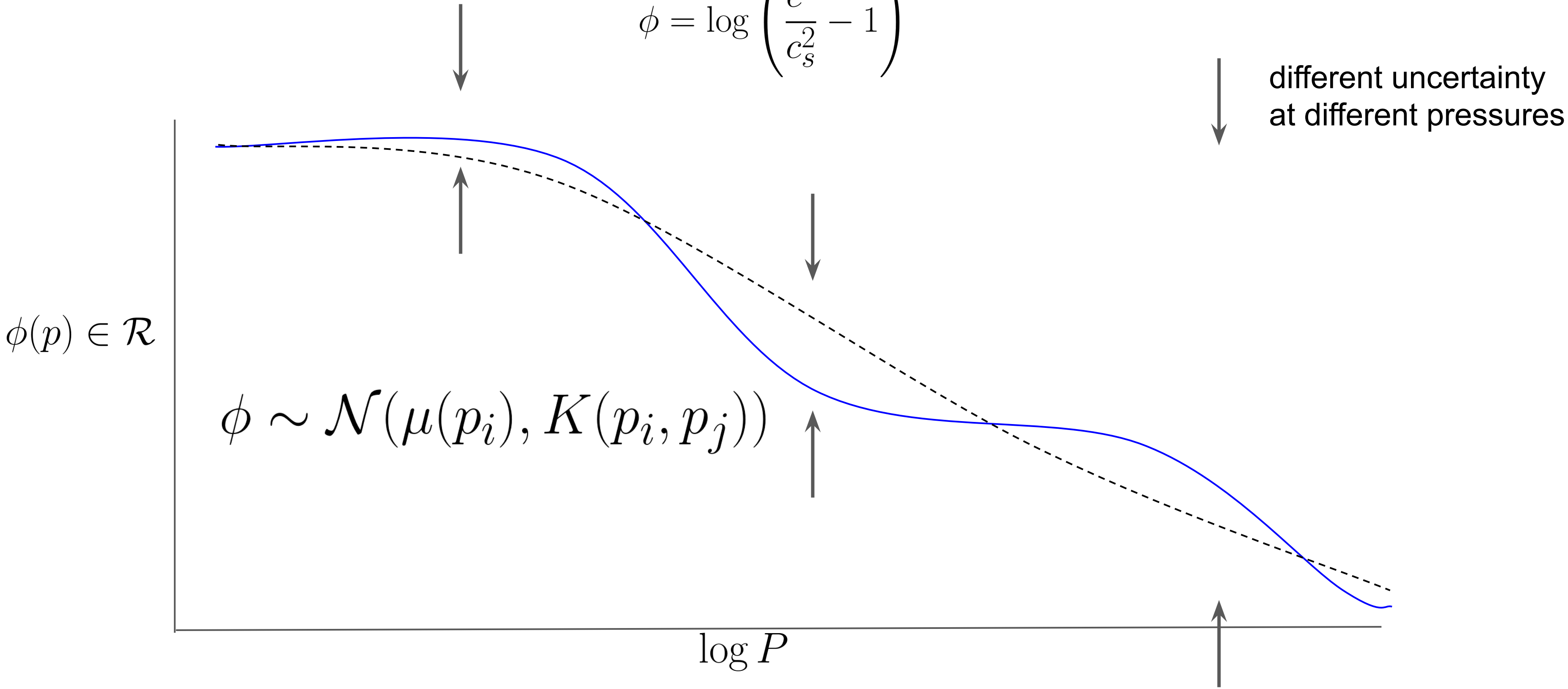
$$\phi = \log\left(\frac{c^2}{c_s^2} - 1\right)$$



Our Gaussian Processes

Gaussian processes generate functions that have support along the entire real line, so we map the sound speed to an *auxiliary variable* that spans this range

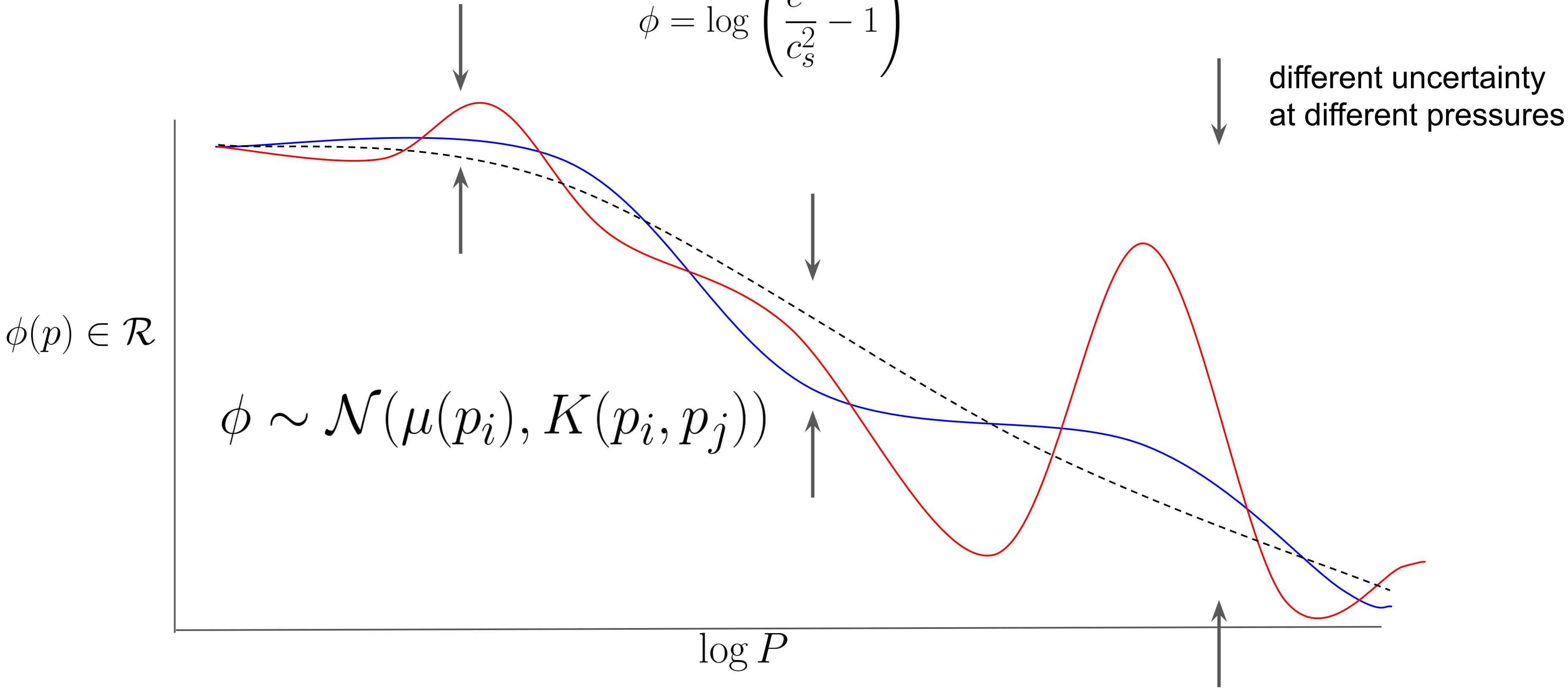
$$\phi = \log\left(\frac{c^2}{c_s^2} - 1\right)$$



Our Gaussian Processes

Gaussian processes generate functions that have support along the entire real line, so we map the sound speed to an *auxiliary variable* that spans this range

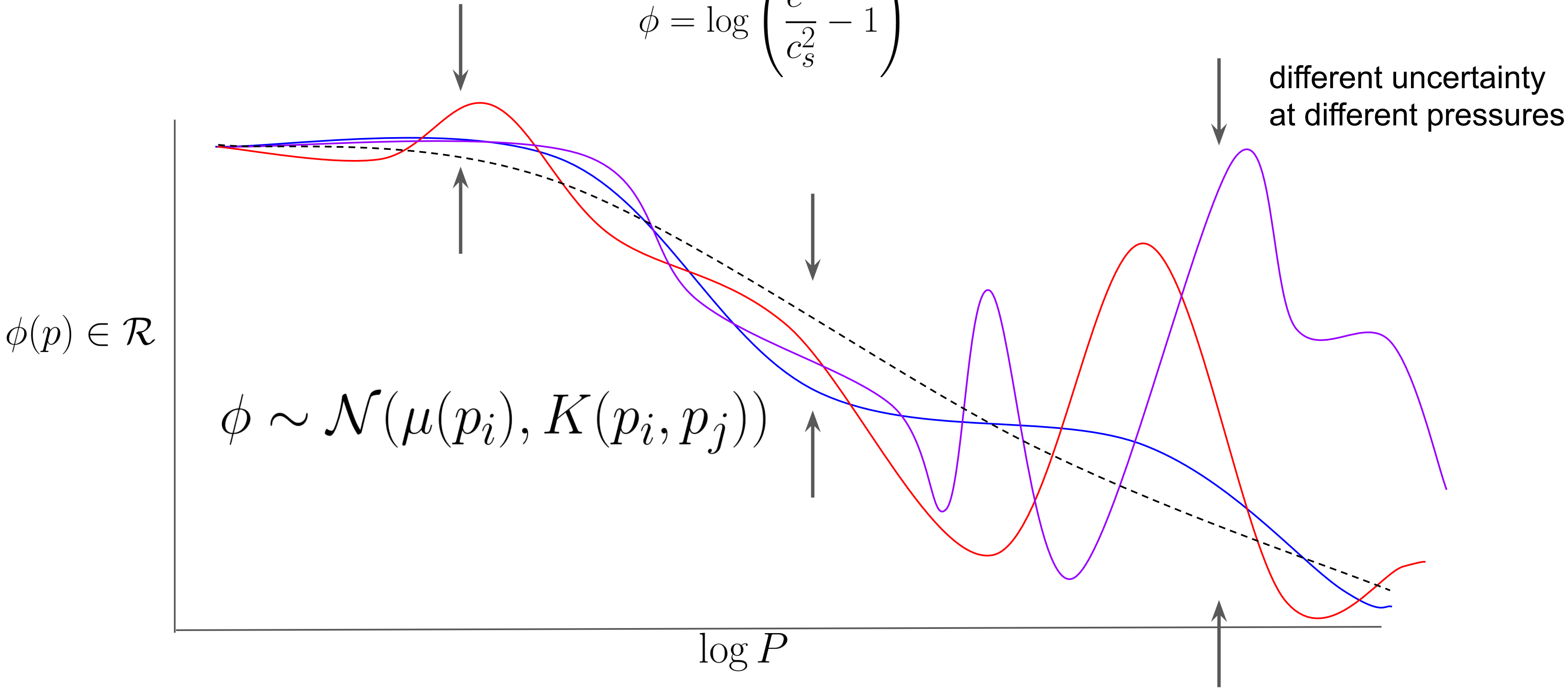
$$\phi = \log\left(\frac{c^2}{c_s^2} - 1\right)$$



Our Gaussian Processes

Gaussian processes generate functions that have support along the entire real line, so we map the sound speed to an *auxiliary variable* that spans this range

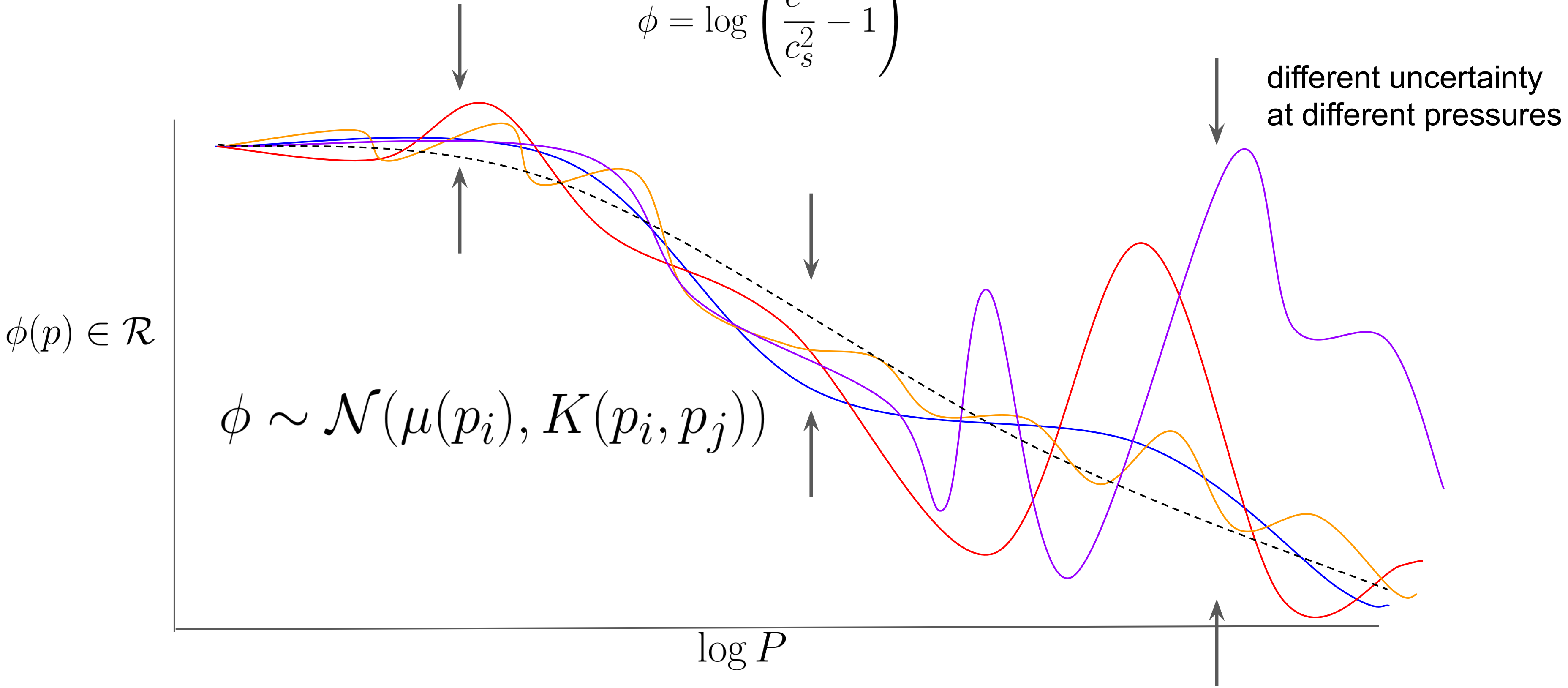
$$\phi = \log\left(\frac{c^2}{c_s^2} - 1\right)$$



Our Gaussian Processes

Gaussian processes generate functions that have support along the entire real line, so we map the sound speed to an *auxiliary variable* that spans this range

$$\phi = \log\left(\frac{c^2}{c_s^2} - 1\right)$$



Advantages of Nonparametric approaches

- Self-consistently incorporate information from arbitrary tabulated EOS models
- Automatically incorporate causality constraints and thermodynamic stability
- Allow for large amounts of model freedom
- Incorporate transparent priors

Advantages of Nonparametric approaches

- Self-consistently incorporate information from arbitrary tabulated EOS models
  - condition prior directly on proposed EOS
  - Gaussian processes can be trained to emulate the behavior seen in an arbitrary collection of proposed EOS without knowing what that behavior is a priori
  - What's more, we can *tune* the amount we want to believe theoretical models easily

- Automatically incorporate causality constraints and thermodynamic stability

- Allow for large amounts of model freedom

- Incorporate transparent priors

Advantages of Nonparametric approaches

- Self-consistently incorporate information from arbitrary tabulated EOS models
  - condition prior directly on proposed EOS
  - Gaussian processes can be trained to emulate the behavior seen in an arbitrary collection of proposed EOS without knowing what that behavior is a priori
  - What's more, we can *tune* the amount we want to believe theoretical models easily

- Automatically incorporate causality constraints and thermodynamic stability
  - A modified sound-speed prescription via an auxiliary variable

$$\phi = \log\left(\frac{c^2}{c_s^2} - 1\right)$$

- Allow for large amounts of model freedom

- Incorporate transparent priors

Advantages of Nonparametric approaches

- Self-consistently incorporate information from arbitrary tabulated EOS models
  - condition prior directly on proposed EOS
  - Gaussian processes can be trained to emulate the behavior seen in an arbitrary collection of proposed EOS without knowing what that behavior is a priori
  - What's more, we can *tune* the amount we want to believe theoretical models easily

- Automatically incorporate causality constraints and thermodynamic stability
  - A modified sound-speed prescription via an auxiliary variable

$$\phi = \log\left(\frac{c^2}{c_s^2} - 1\right)$$

- Allow for large amounts of model freedom
  - Gaussian process formally supports any possible EOS, although our priors favor a subset
  - i.e., they emulate "reasonable behavior" ***but also produce behavior not seen in training set if desired***

- Incorporate transparent priors

Advantages of Nonparametric approaches

- Self-consistently incorporate information from arbitrary tabulated EOS models
  - condition prior directly on proposed EOS
  - Gaussian processes can be trained to emulate the behavior seen in an arbitrary collection of proposed EOS without knowing what that behavior is a priori
  - What's more, we can *tune* the amount we want to believe theoretical models easily

- Automatically incorporate causality constraints and thermodynamic stability
  - A modified sound-speed prescription via an auxiliary variable

$$\phi = \log\left(\frac{c^2}{c_s^2} - 1\right)$$

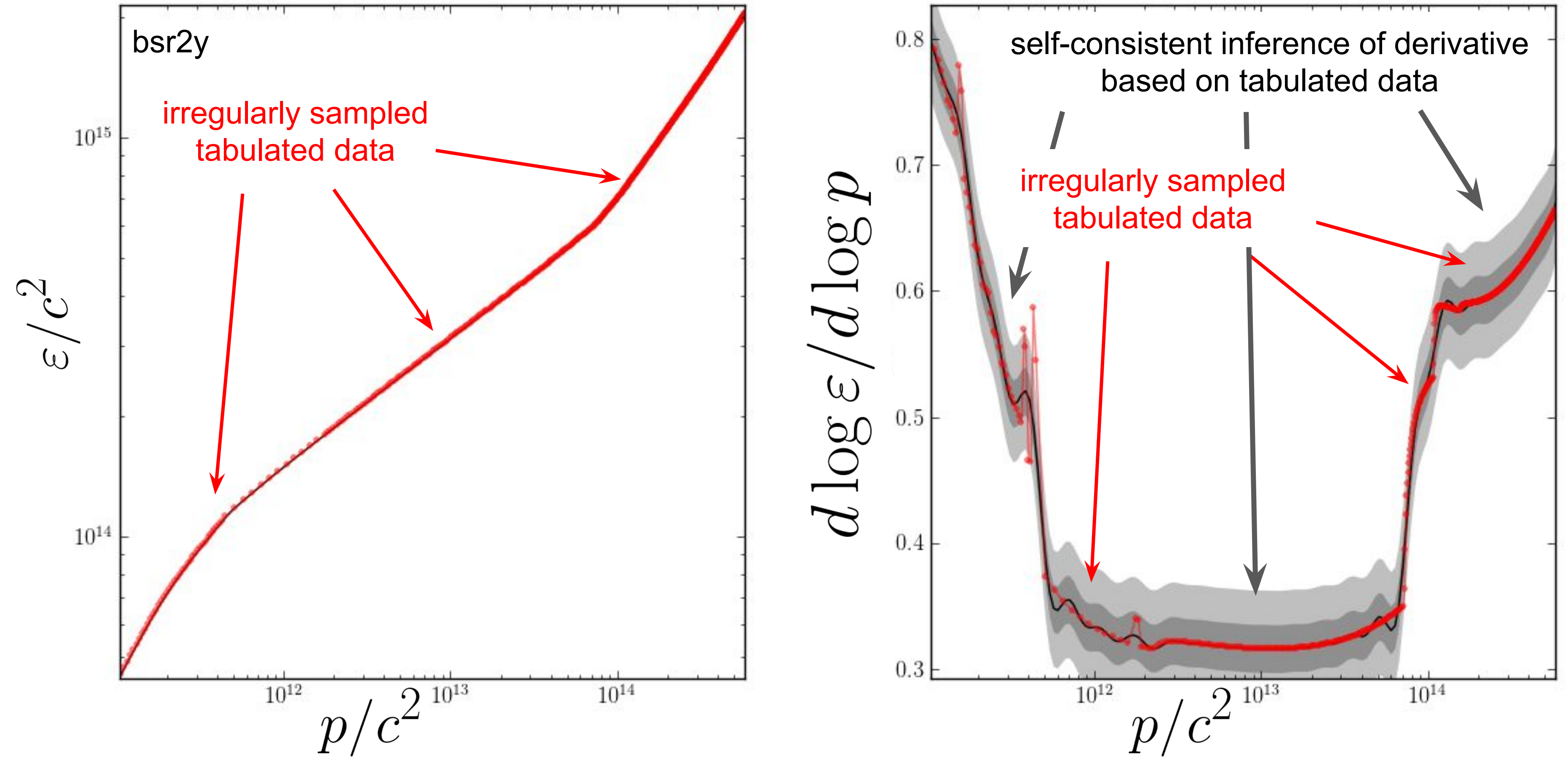
- Allow for large amounts of model freedom
  - Gaussian process formally supports any possible EOS, although our priors favor a subset
  - i.e., they emulate "reasonable behavior" ***but also produce behavior not seen in training set if desired***

- Incorporate transparent priors
  - configurable "confidence" in tabulated EOS
  - different uncertainty at different pressures
    - small uncertainty near the crust
    - large uncertainty near the central core

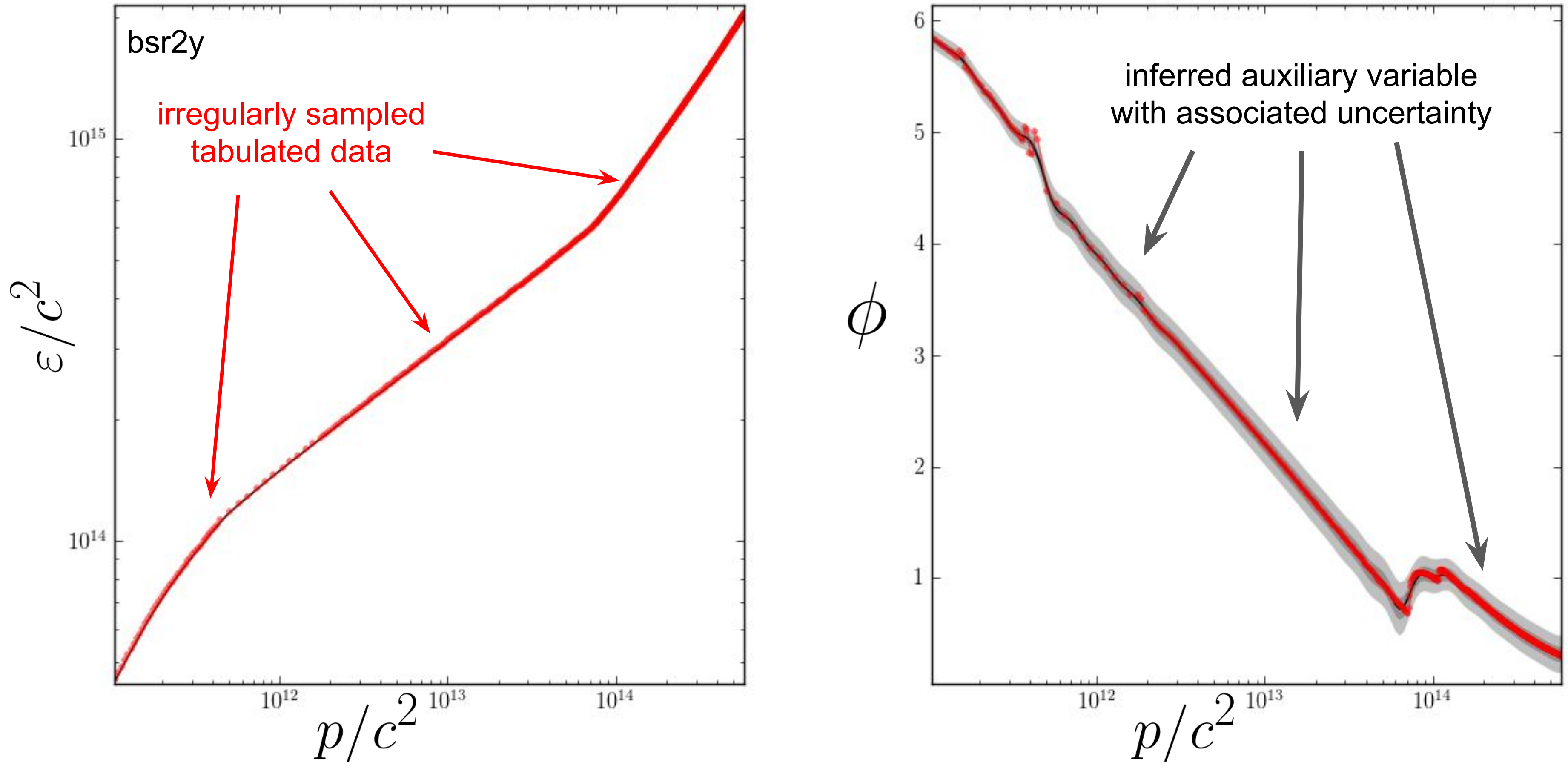
} determined by the *covariance kernel* and *hyperparameters*, but come automatically from the training set

constructing Nonparametric priors

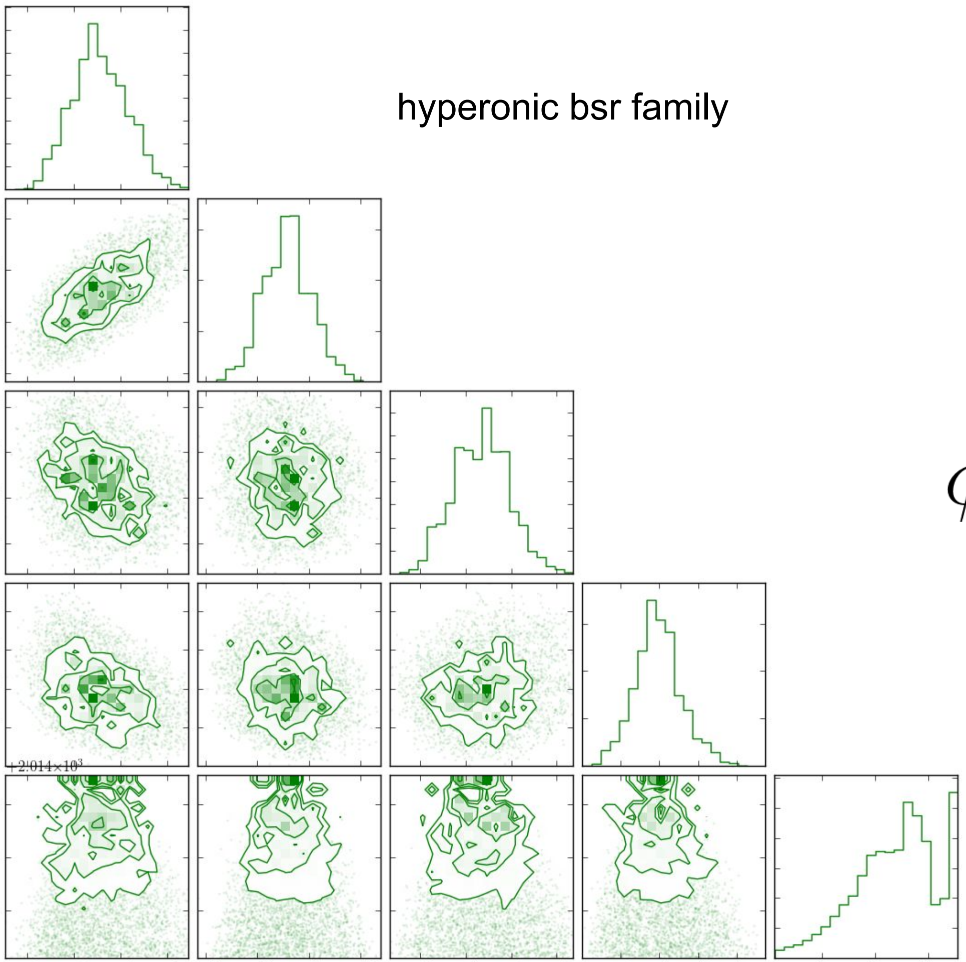
Emulating Proposed EOS

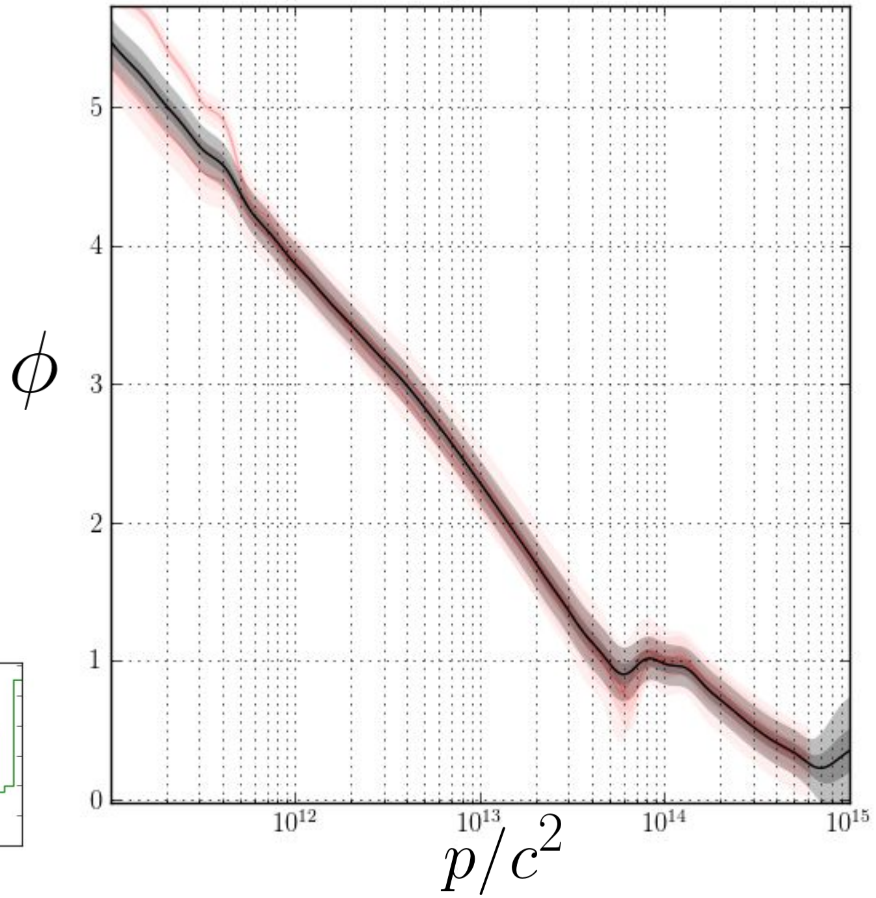


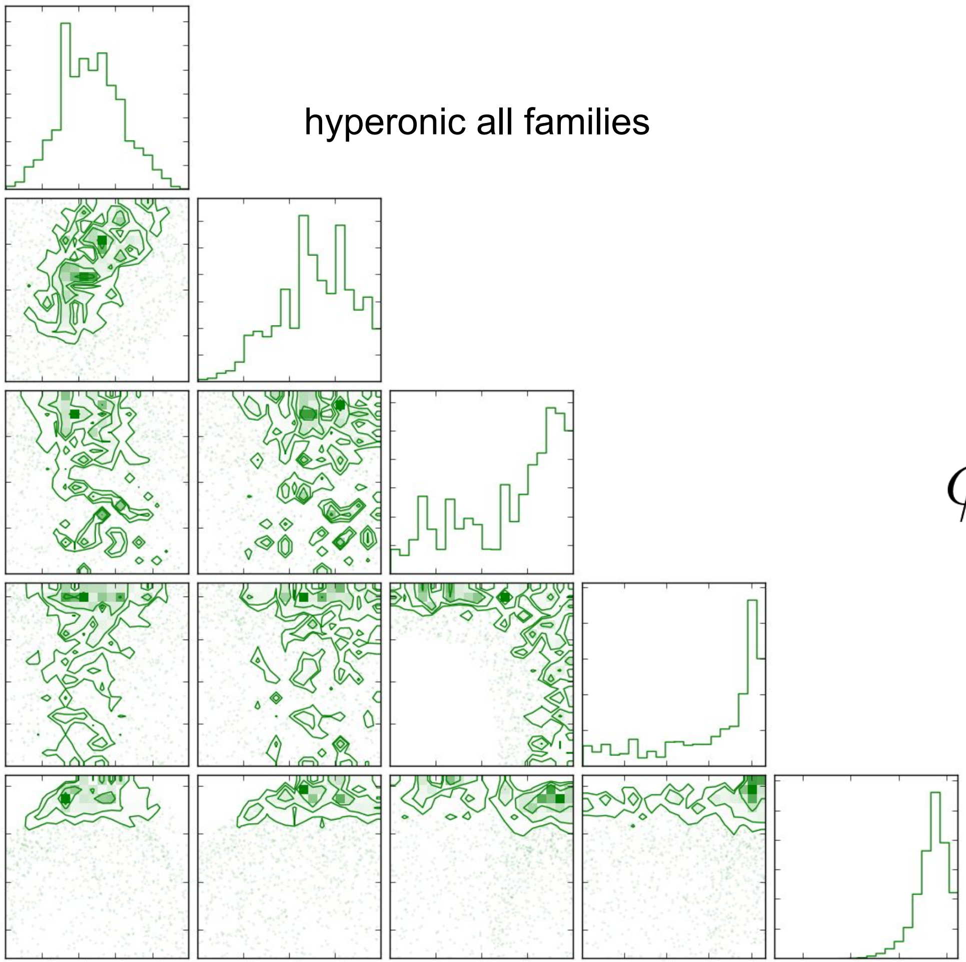
Emulating Proposed EOS

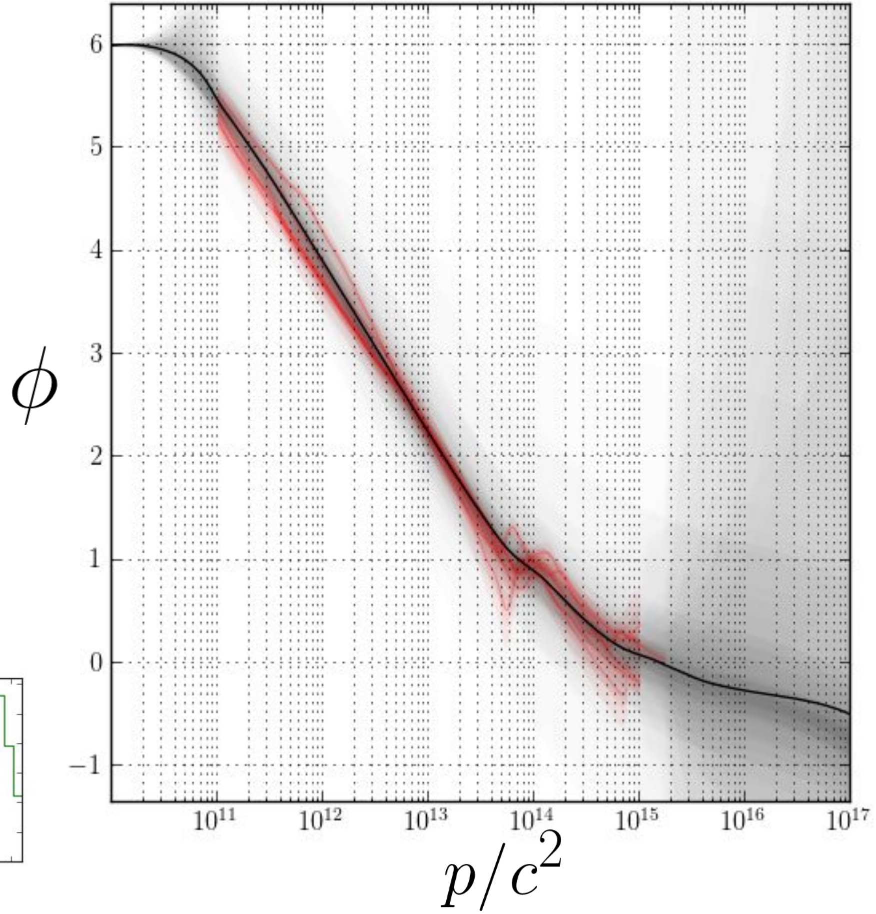


hyperonic bsr family

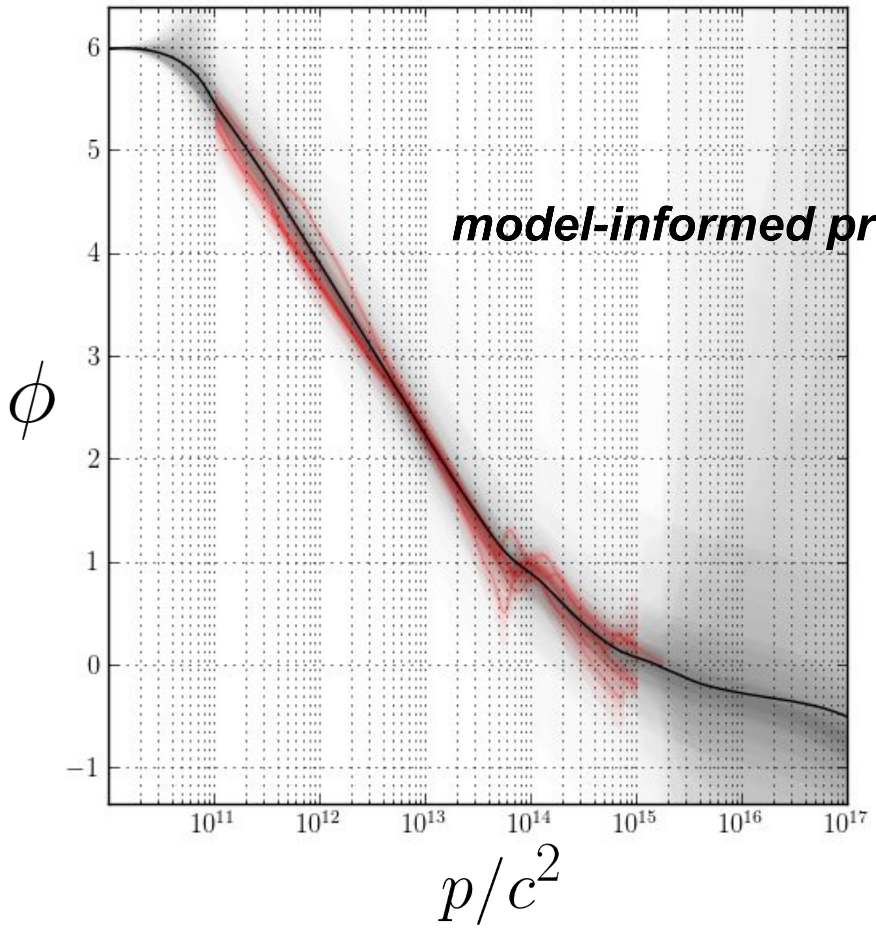




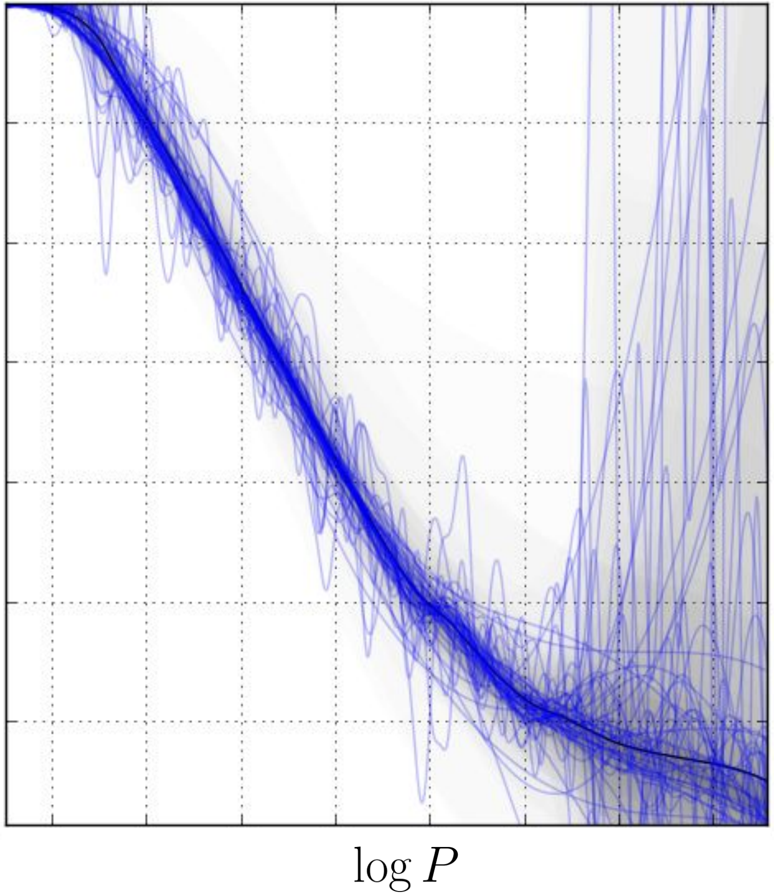

hyperonic all families


ϕ
p/c^2
6
5
4
3
2
1
0
−1
10^{11}
10^{12}
10^{13}
10^{14}
10^{15}
10^{16}
10^{17}

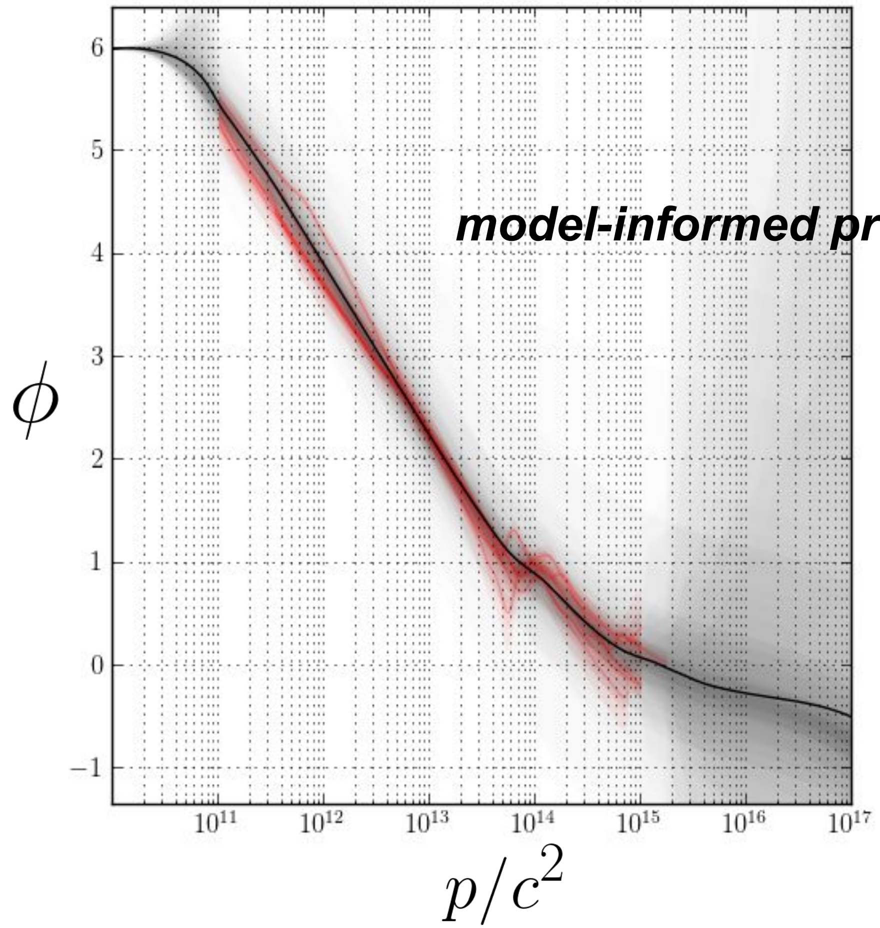
Emulating Proposed EOS



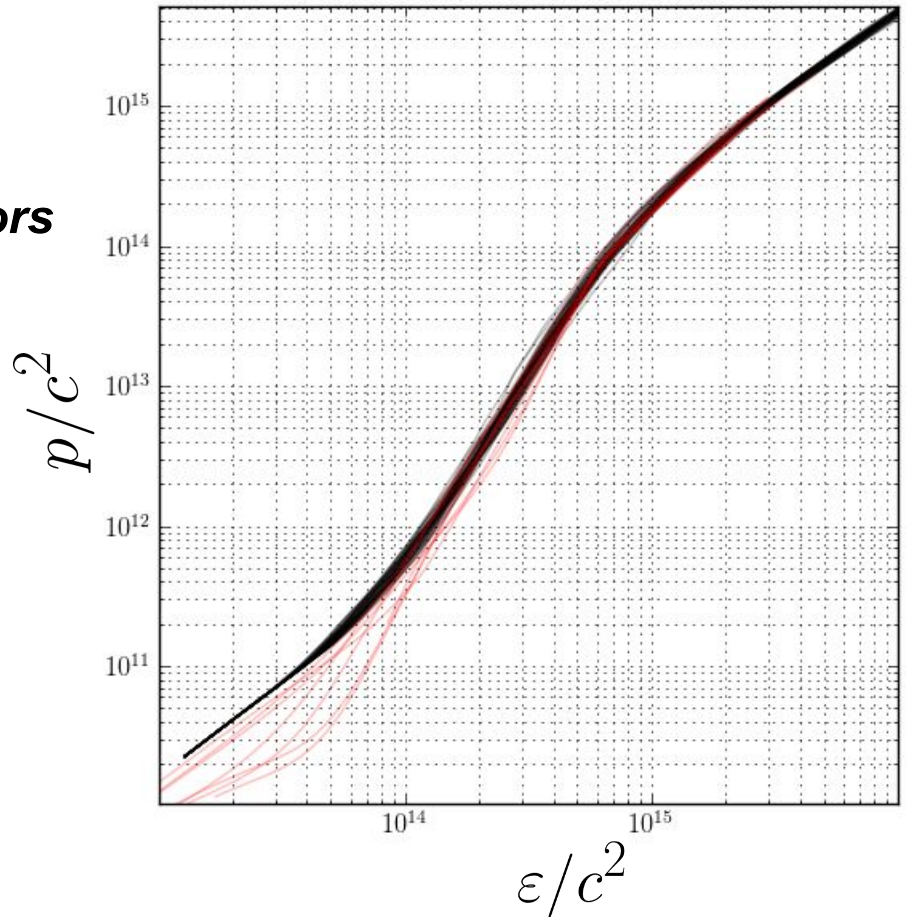
model-informed priors



Emulating Proposed EOS

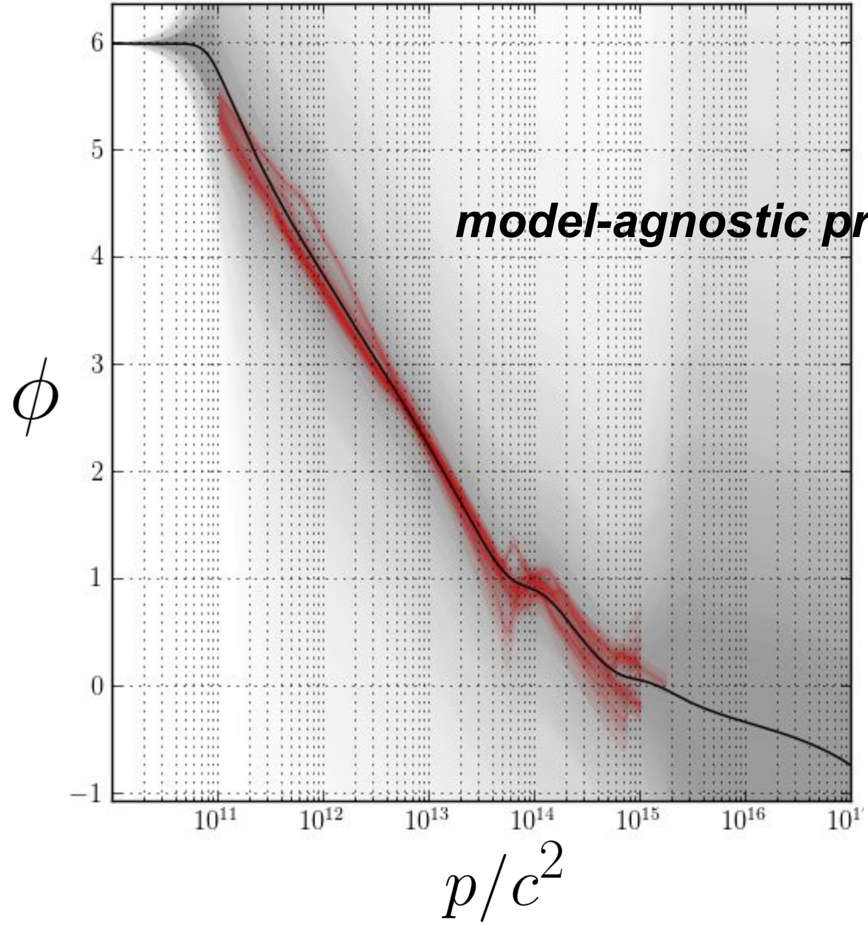


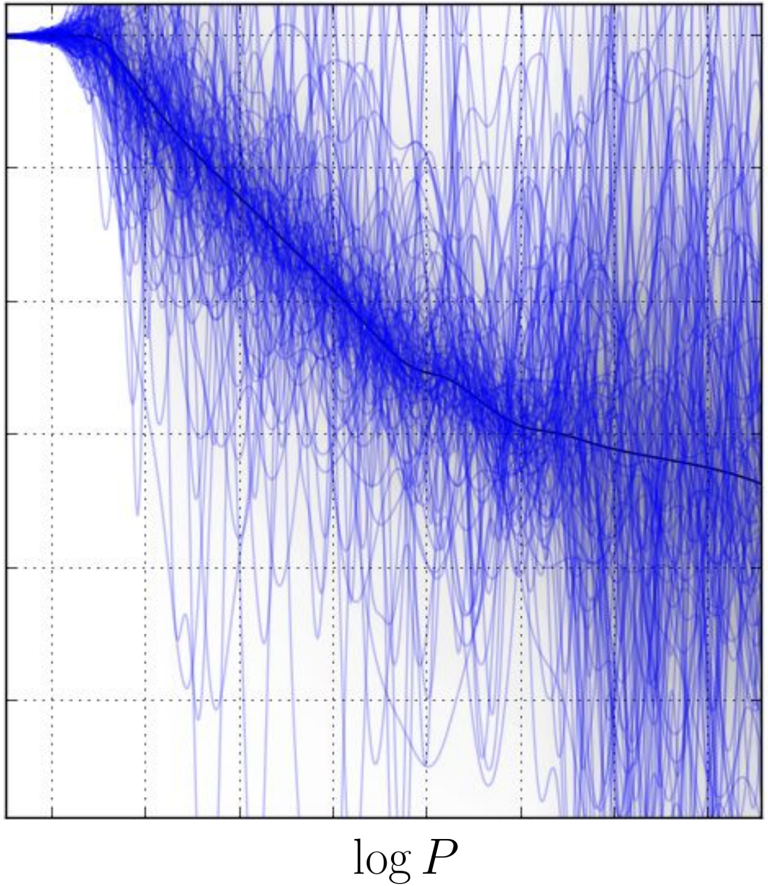
model-informed priors



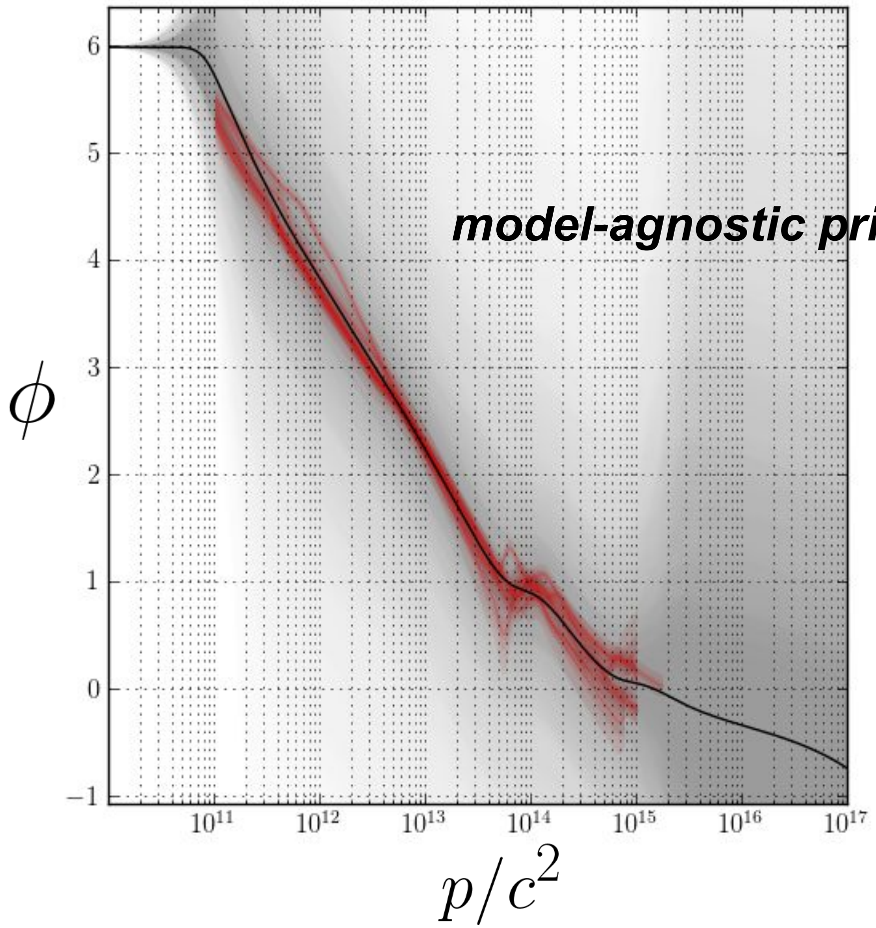
Exploring all possible EOS

model-agnostic priors

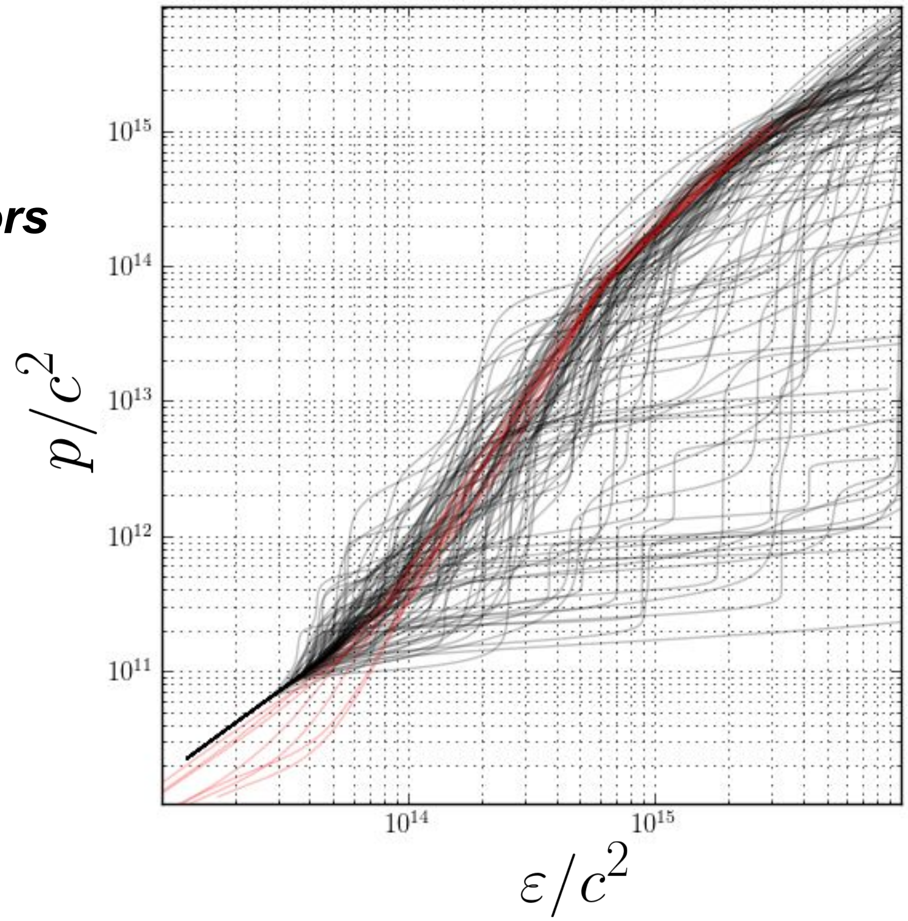




Exploring all possible EOS



model-agnostic priors



results from GW170817 and massive Pulsars

Constraints on macroscopic observables

GW data favors small radii, but can be overwhelmed by strong theory prior

Central densities between 2x-4x saturation

Prior ($\mathcal{H}_i$)	M_1 [$M_\odot$]	M_2 [$M_\odot$]	Λ_1	Λ_2	$\tilde{\Lambda}$	R_1 [km]	R_2 [km]	$\rho_{c,1}$ [10^{14} g/cm^3]	$\rho_{c,2}$ [10^{14} g/cm^3]
informed	$1.46^{+0.11}_{-0.10}$	$1.28^{+0.08}_{-0.09}$	380^{+249}_{-231}	844^{+553}_{-405}	572^{+254}_{-212}	$12.50^{+0.98}_{-0.87}$	$12.51^{+1.02}_{-0.96}$	$7.43^{+1.48}_{-1.23}$	$6.65^{+1.03}_{-1.04}$
agnostic	$1.49^{+0.13}_{-0.13}$	$1.25^{+0.10}_{-0.10}$	148^{+274}_{-125}	430^{+519}_{-301}	245^{+361}_{-160}	$10.88^{+1.99}_{-1.37}$	$10.82^{+2.14}_{-1.55}$	$9.79^{+3.21}_{-3.72}$	$8.85^{+2.52}_{-3.18}$

Prior ($\mathcal{H}_i$)	$\Lambda_{1.4}$	$R_{1.4}$ [km]	$I_{1.4}$ [10^{45} g cm^2]	$M_{b,1.4}$ [$M_\odot$]	$M_{\max}$ [$M_\odot$]
informed	491^{+216}_{-181}	$12.51^{+1.00}_{-0.88}$	$1.55^{+0.17}_{-0.16}$	$1.566^{+0.025}_{-0.021}$	$2.017^{+0.238}_{-0.087}$
agnostic	211^{+312}_{-137}	$10.86^{+2.04}_{-1.42}$	$1.25^{+0.35}_{-0.22}$	$1.591^{+0.049}_{-0.053}$	$2.064^{+0.260}_{-0.134}$

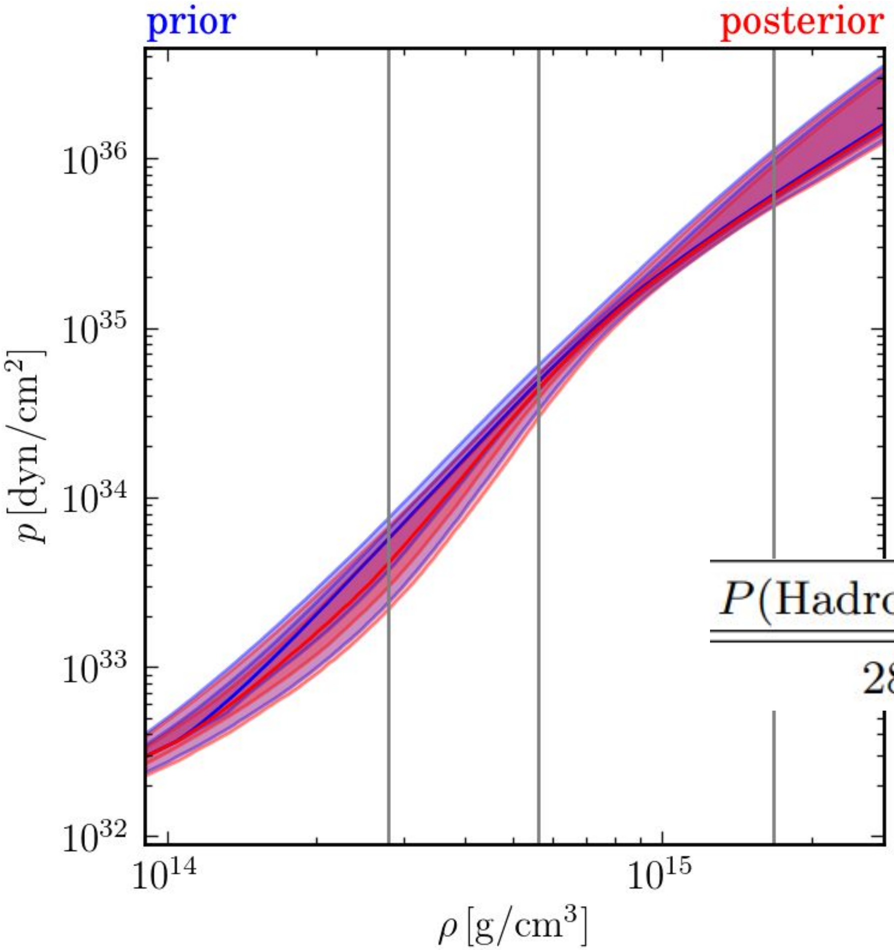
Neither informed nor agnostic prior is strongly preferred

$$B^{\mathcal{A}}_{\mathcal{I}} = 1.7952 \pm (1.1 \times 10^{-3})$$

Spin Prior	EOS Prior ($\mathcal{H}_i$)	$P(\text{BNS}\|\text{data};\mathcal{H}_i)$	$P(\text{BHNS}\|\text{data};\mathcal{H}_i)$	$P(\text{NSBH}\|\text{data};\mathcal{H}_i)$	$P(\text{BBH}\|\text{data};\mathcal{H}_i)$
$\|\chi_i\| \leq 0.05$	*informed*	$(14.3 \pm 4.5)\%$	$(23.6 \pm 0.5)\%$	$(54.4 \pm 1.2)\%$	$(7.8 \pm 2.7)\%$
	agnostic	$(25.9 \pm 7.1)\%$	$(27.6 \pm 1.8)\%$	$(38.3 \pm 2.2)\%$	$(8.1 \pm 3.1)\%$
$\|\chi_i\| \leq 0.89$	*informed*	$(11.2 \pm 3.7)\%$	$(18.1 \pm 0.1)\%$	$(61.0 \pm 0.3)\%$	$(9.7 \pm 3.3)\%$
	agnostic	$(23.9 \pm 6.9)\%$	$(25.2 \pm 1.4)\%$	$(40.4 \pm 1.7)\%$	$(10.5 \pm 3.9)\%$

BBH disfavored by the GW data alone

Constraints on EOS and phenomenology

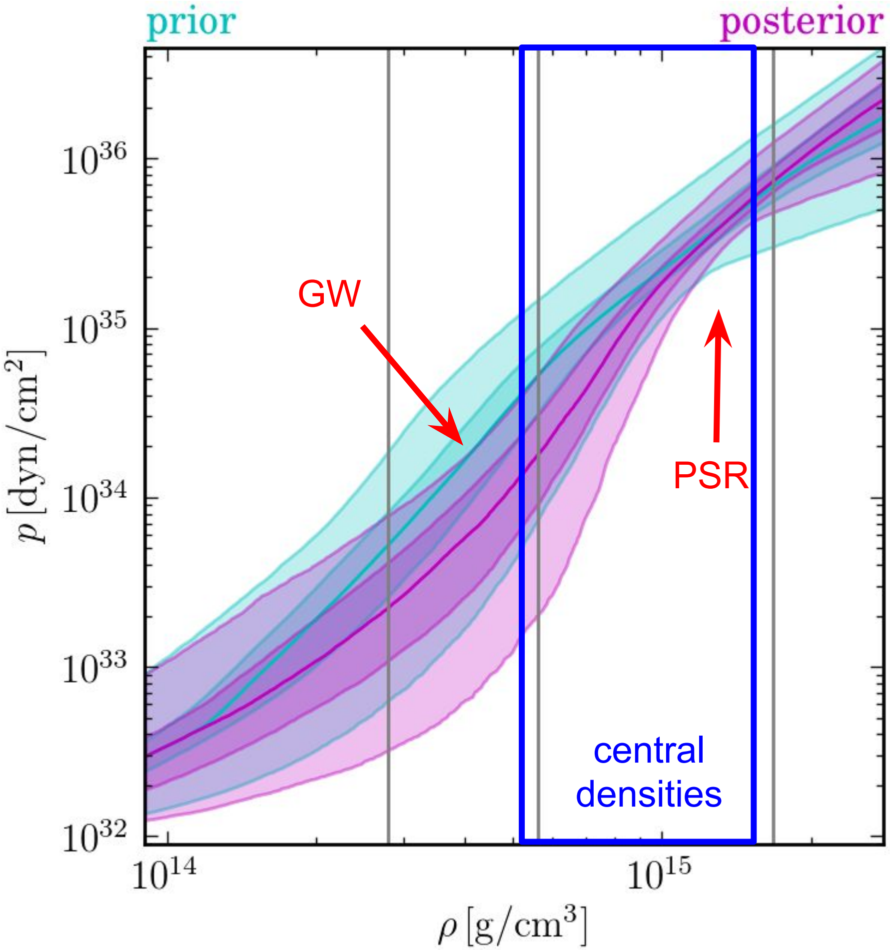


Posterior process essentially recovers the prior process, but there is ***a weak preference for the GP models conditioned on Quark EOS***

model-informed priors

$P(\mathrm{Hadronic}\vert\mathrm{data})$	$P(\mathrm{Hyperonic}\vert\mathrm{data})$	$P(\mathrm{Quark}\vert\mathrm{data})$
28%	16%	56%

Constraints on EOS and phenomenology



model-agnostic priors

GW data mostly influences EOS between 1x and 2x saturation

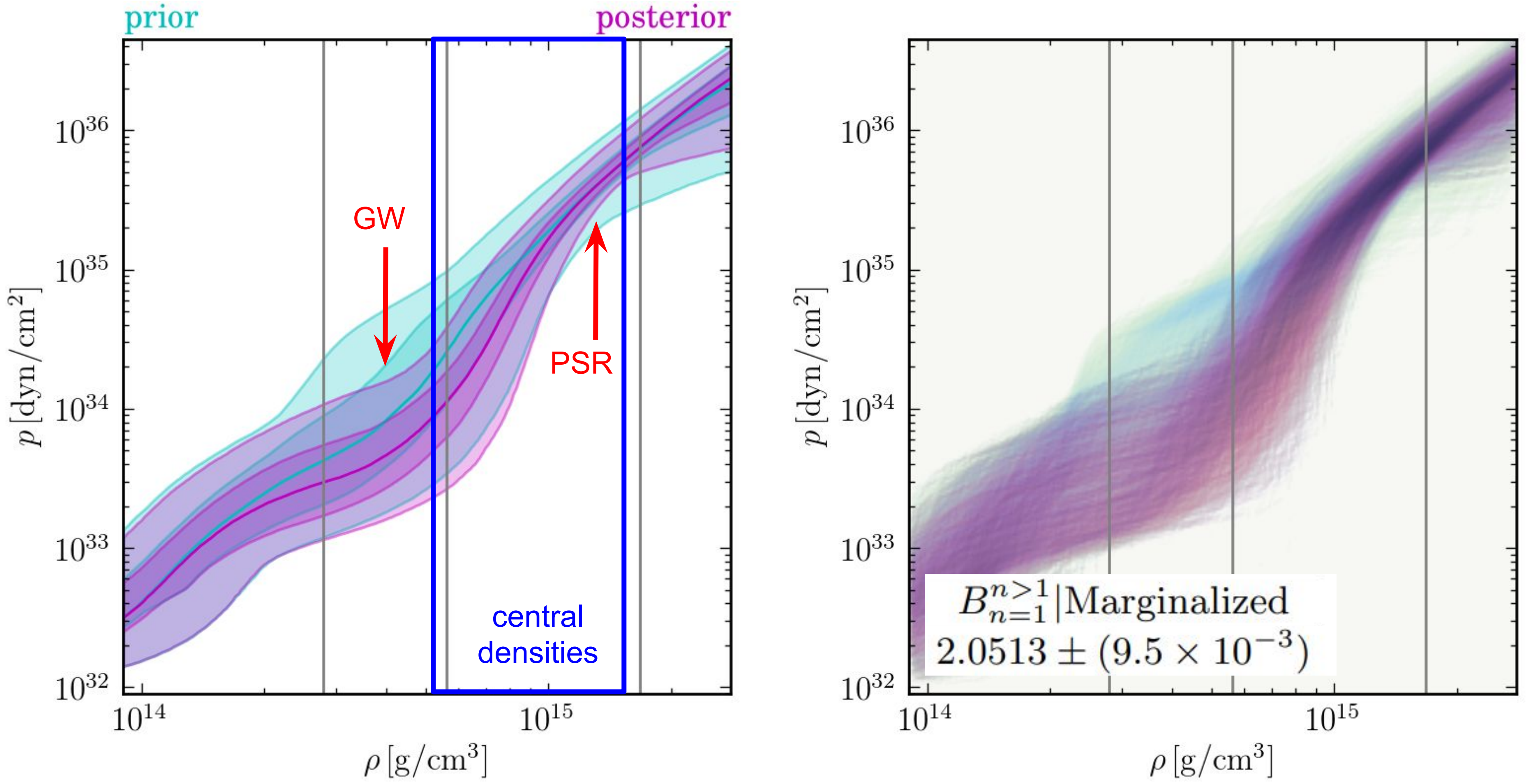
Essentially no constraint below ~1x saturation

Tightened constraints at high density are mostly due to truncating the tail of the prior distribution (which includes PSR data)

Note! Model-agnostic priors produce EOS behavior not seen in any of the training EOS

Constraints on EOS and phenomenology

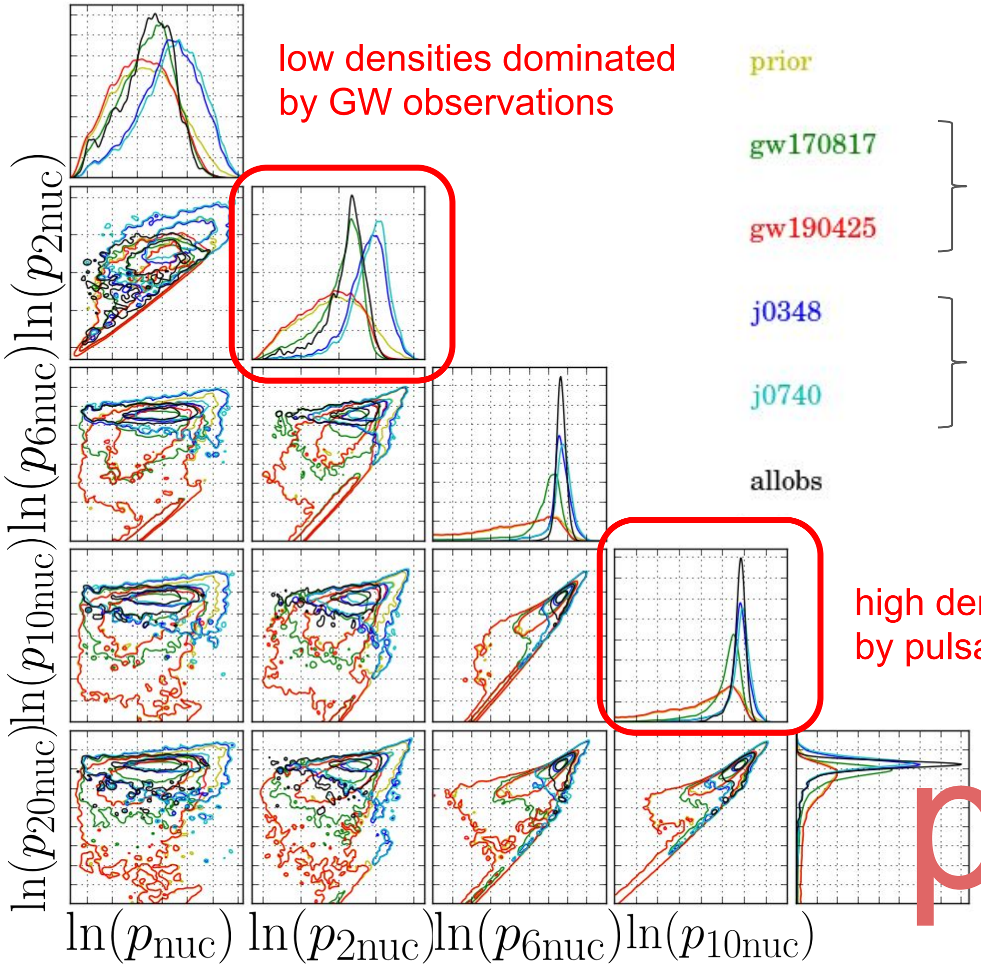
model-agnostic priors



next steps

Including more types of astrophysical observations (and population models)

and NICER results…
and more GW data...



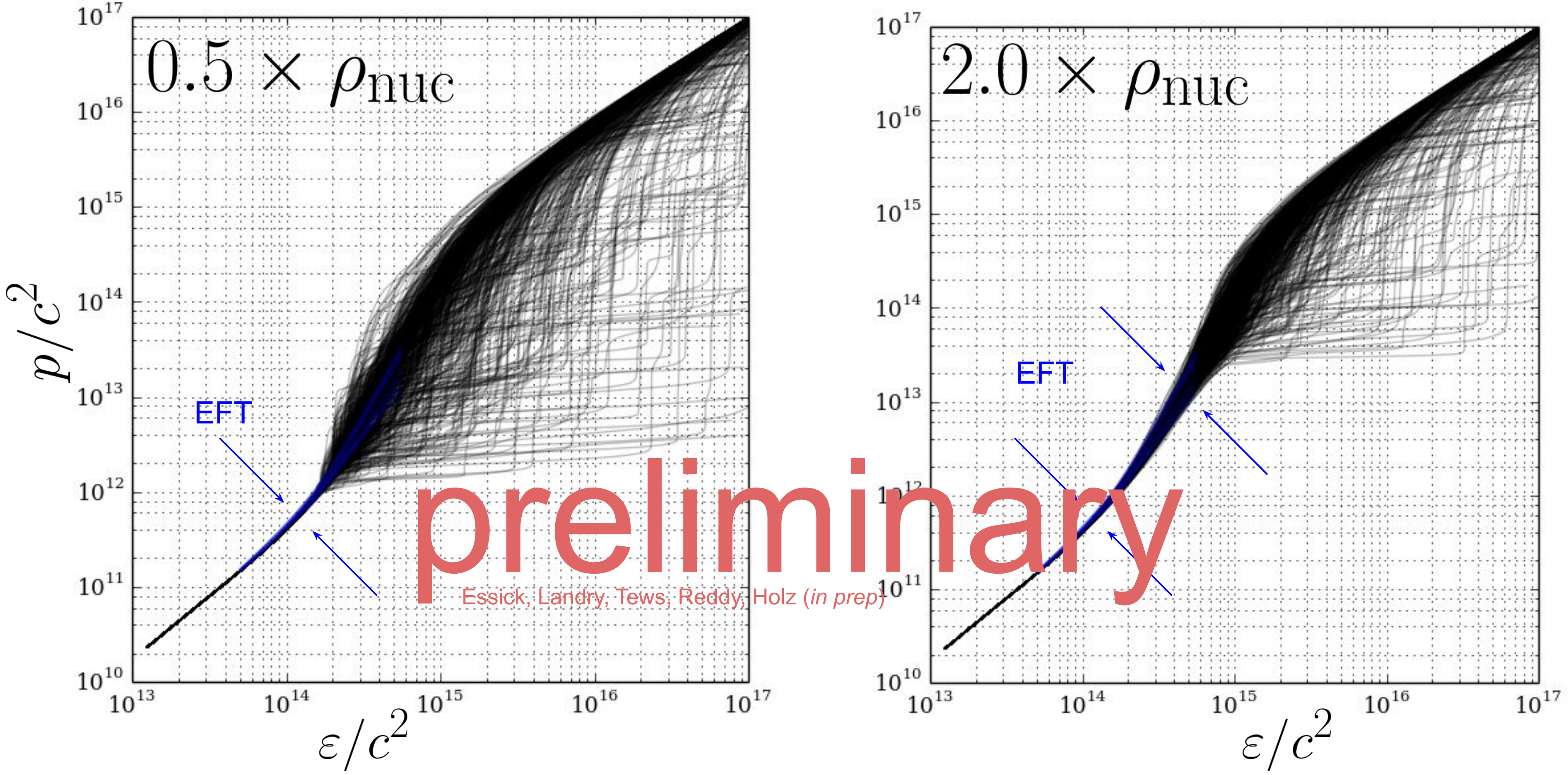
GW events
(only informative if SNR≳20)

heavy pulsars
(dominated by heaviest observed object)

preliminary

Landry, Essick, and Chatziioannou (*in prep*)

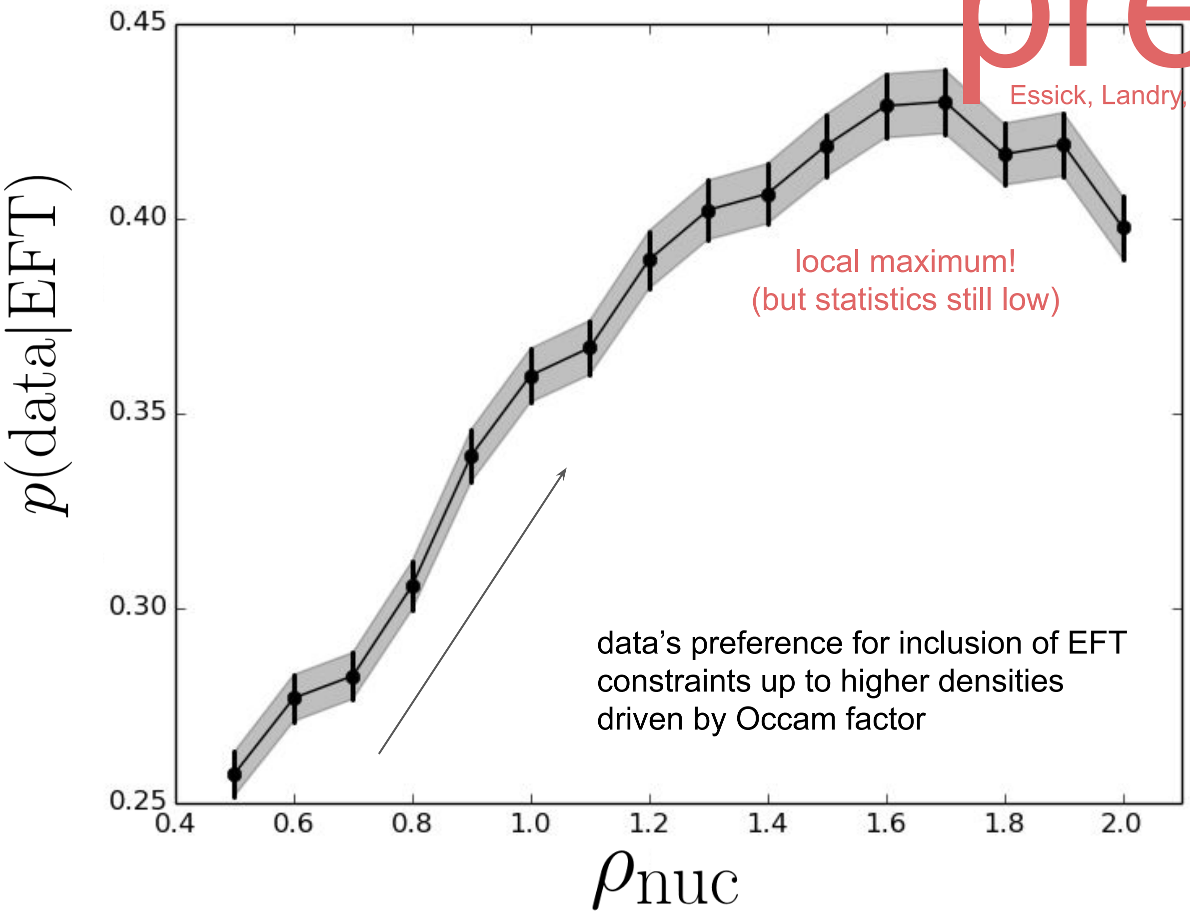
Including more rigorous estimates theoretical uncertainty (chiral EFT)



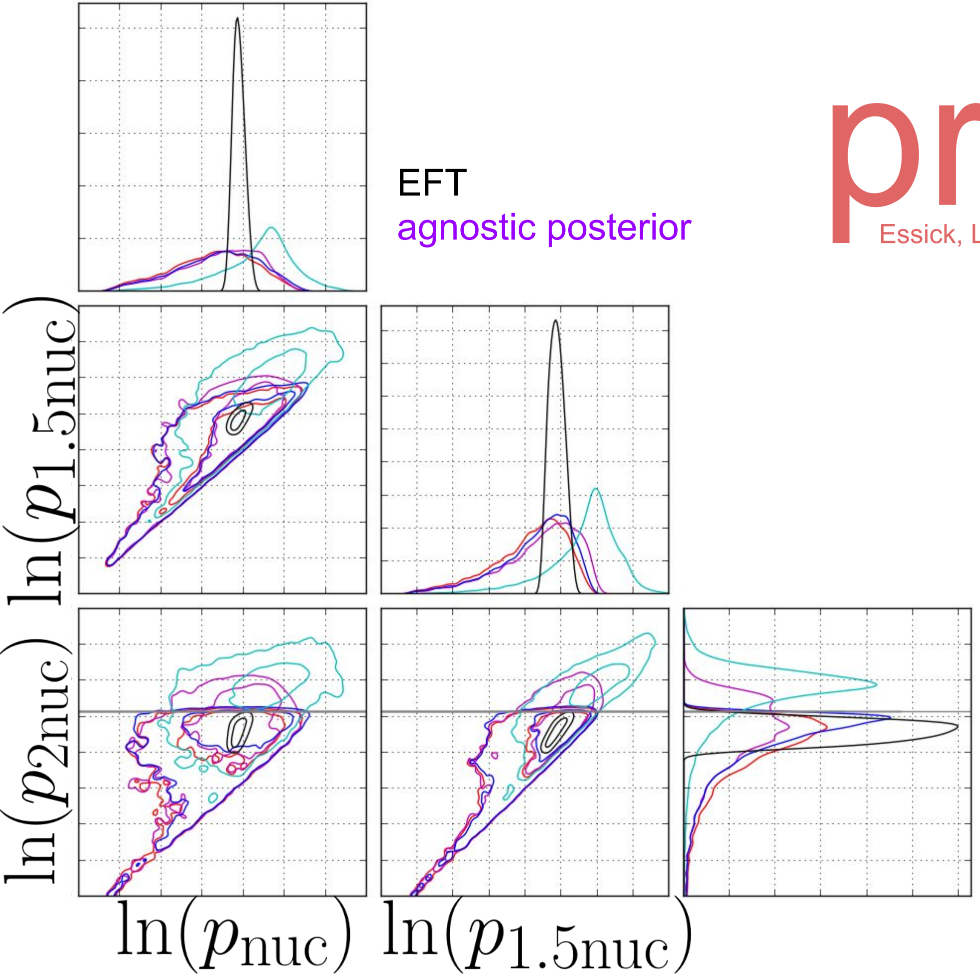
Including more rigorous estimates theoretical uncertainty (chiral EFT)

preliminary

Essick, Landry, Tews, Reddy, Holz (*in prep*)



Including more rigorous estimates theoretical uncertainty (chiral EFT)

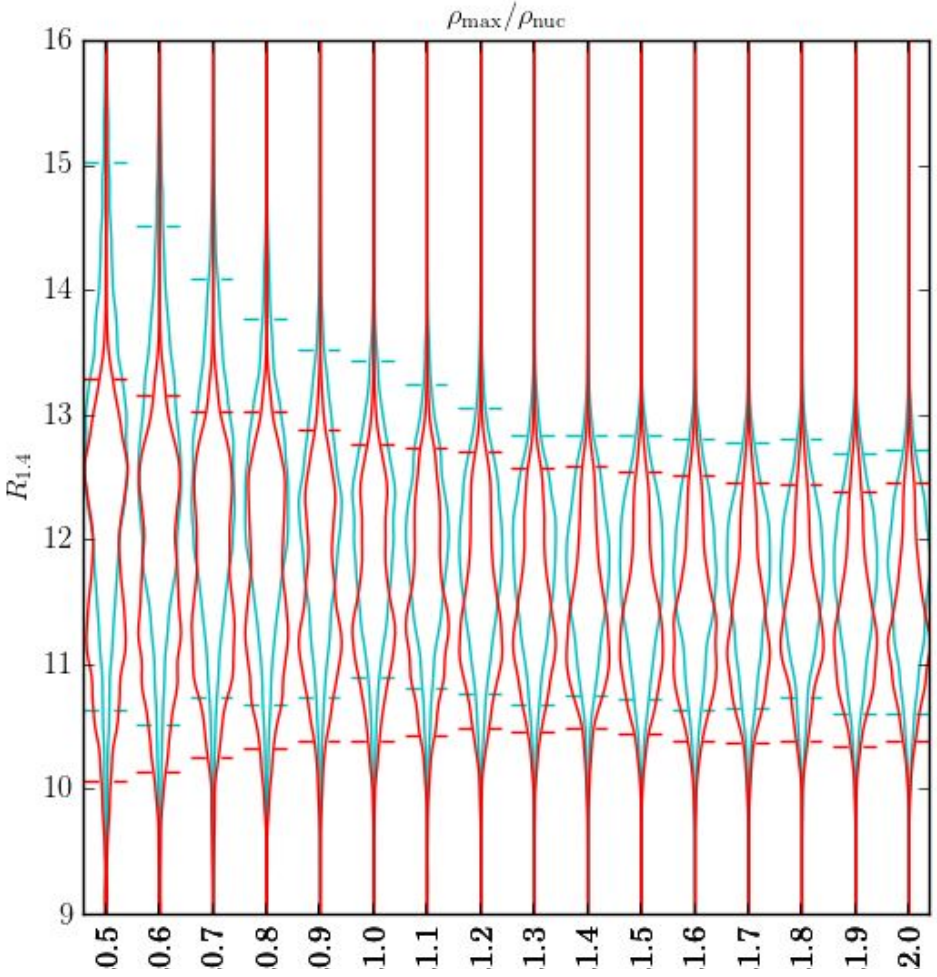


preliminary

Essick, Landry, Tews, Reddy, Holz (*in prep*)

EFT predictions are not just consistent with agnostic posterior, ***they fall near the maxima a posteriori for all densities up to ~2x saturation***!

Including more rigorous estimates theoretical uncertainty (chiral EFT)



preliminary

Essick, Landry, Tews, Reddy, Holz (*in prep*)

Trusting EFT up to

0.5x saturation → $R_{1.4}$ ~ 11.70 km (10.07, 13.29)

1.0x saturation → $R_{1.4}$ ~ 11.44 km (10.38, 12.76)

2.0x saturation → $R_{1.4}$ ~ 11.23 km (10.39, 12.45)

"stringent constraints" on **$R_{1.4}$ come primarily from strong assumption of EFT up to 2x saturation**

summary

Advantages of Nonparametric approaches

- Self-consistently incorporate information from arbitrary tabulated EOS models
  - condition prior directly on proposed EOS
  - Gaussian processes can be trained to emulate the behavior seen in an arbitrary collection of proposed EOS without knowing what that behavior is a priori
  - What's more, we can *tune* the amount we want to believe theoretical models easily

- Automatically incorporate causality constraints and thermodynamic stability
  - A modified sound-speed prescription via an auxiliary variable

$$\phi = \log\left(\frac{c^2}{c_s^2} - 1\right)$$

- Allow for large amounts of model freedom
  - Gaussian process formally supports any possible EOS, although our priors favor a subset
  - i.e., they emulate "reasonable behavior" ***but also produce behavior not seen in training set if desired***

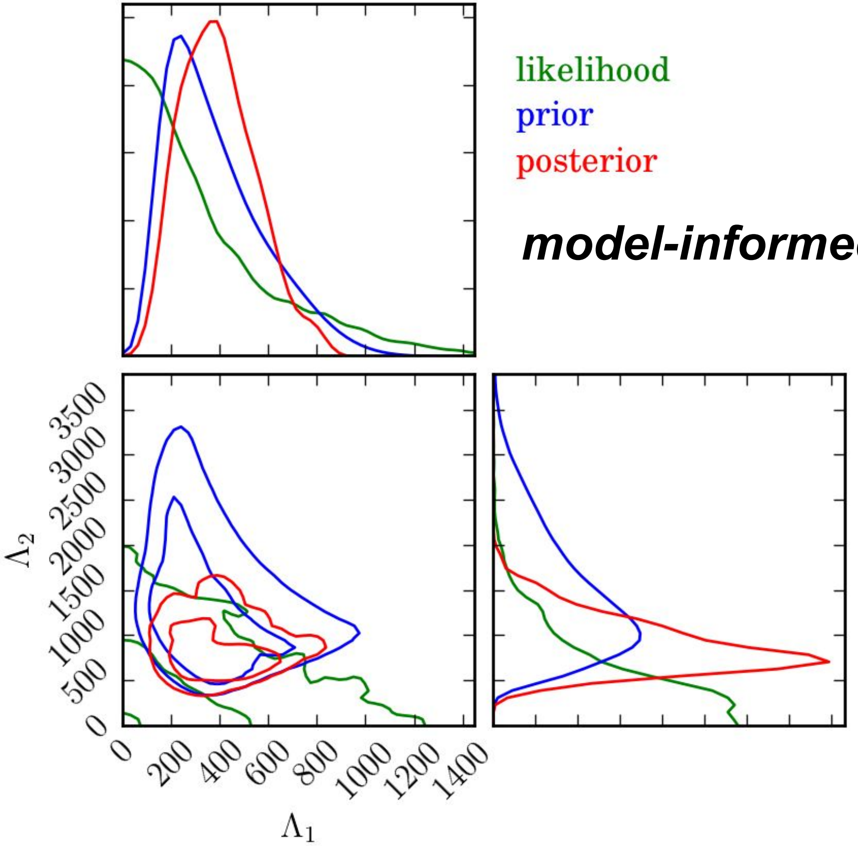
- Incorporate transparent priors
  - configurable "confidence" in tabulated EOS
  - different uncertainty at different pressures
    - small uncertainty near the crust
    - large uncertainty near the central core

} determined by the *covariance kernel* and *hyperparameters*, but come automatically from the training set

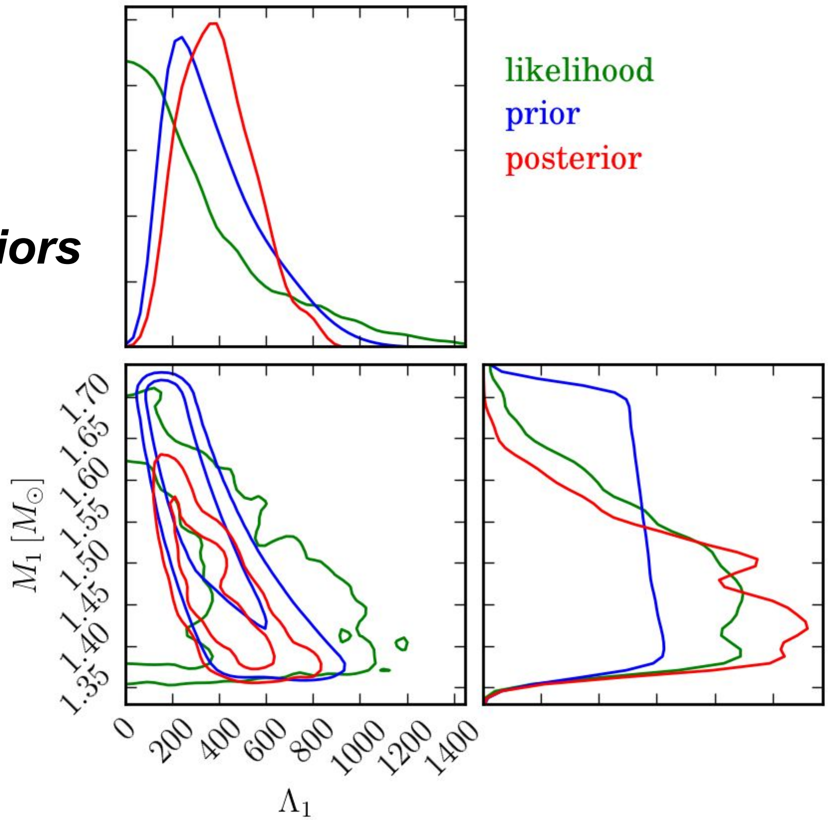
P. Landry and R. Essick. *Nonparametric inference of the neutron star equation of state from gravitational wave observations*.Phys. Rev. D **99**, 084049 (2019)

R. Essick, P. Landry, D. Holz. *Nonparametric Inference of Neutron Star Composition, Equation of State, and Maximum Mass with GW170817*. arXiv:1910.09740 (2019)

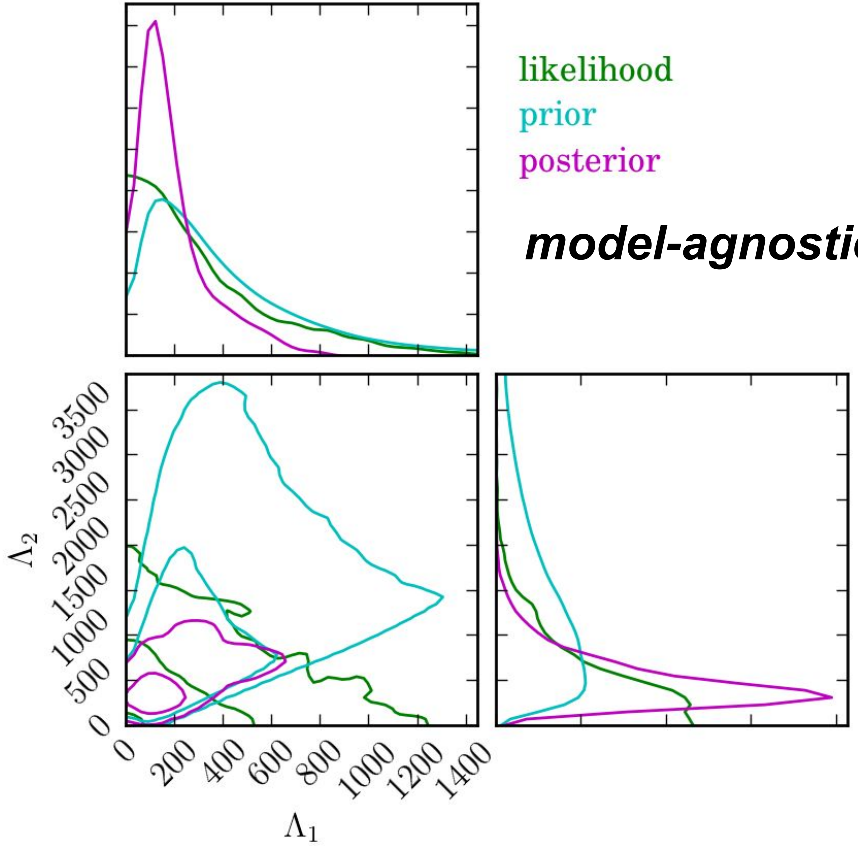
Constraints on GW170817's components



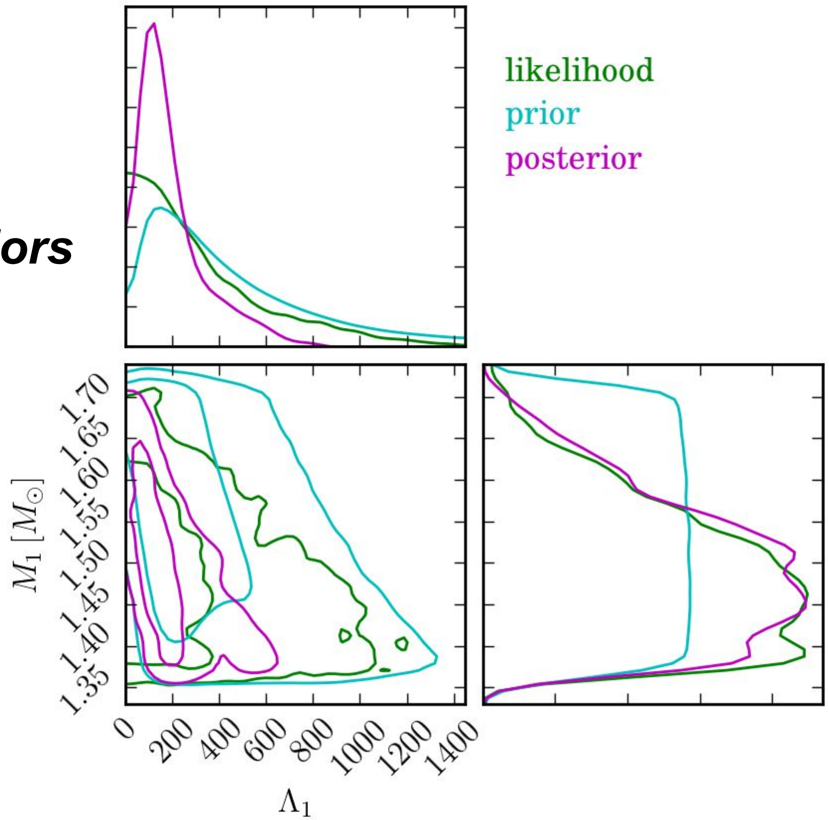
model-informed priors



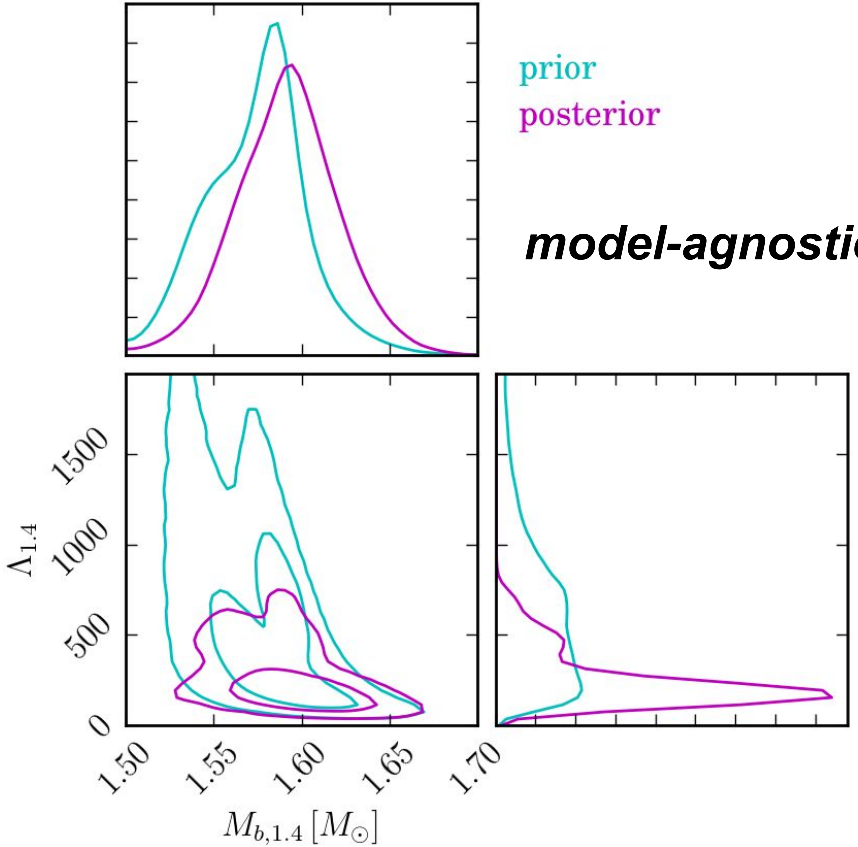
Constraints on GW170817's components



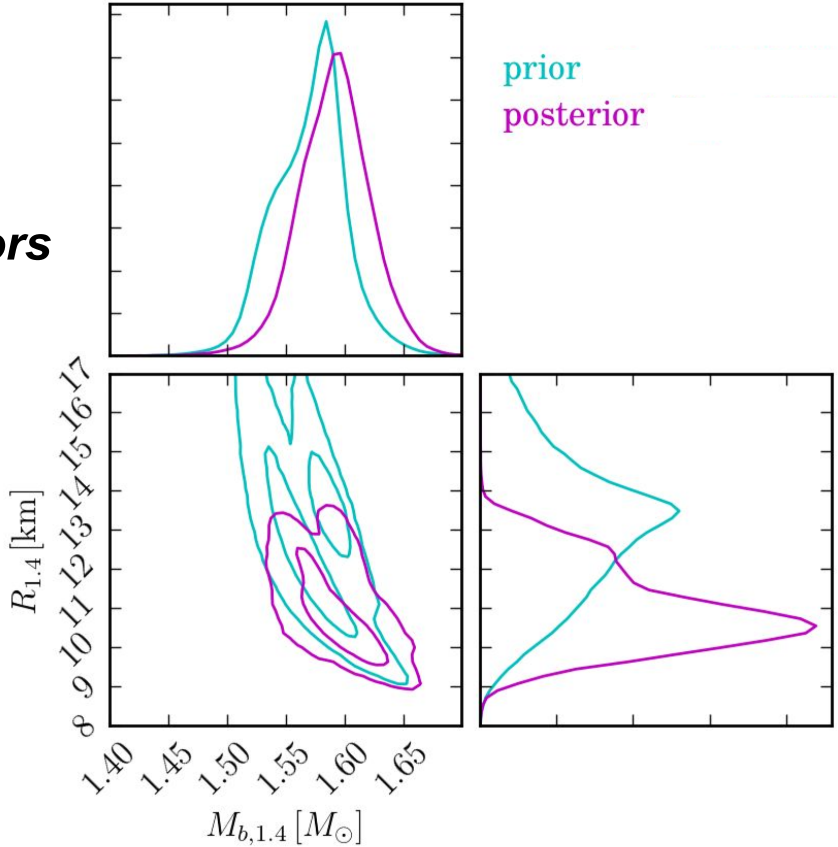
model-agnostic priors



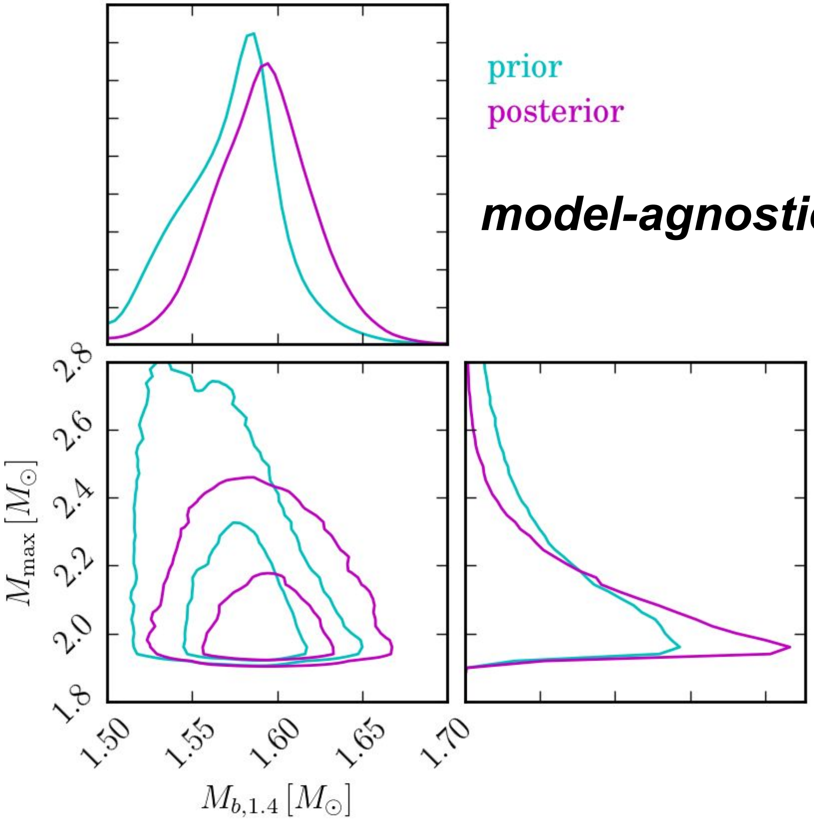
Constraints on canonical macroscopic observables



model-agnostic priors



Constraints on canonical macroscopic observables



model-agnostic priors

agnostic $M_{\max} \leq 2.32\, M_\odot$

informed $M_{\max} \leq 2.26\, M_\odot$

compare to Cromartie+(2019)

$$M = 2.14^{+0.10}_{-0.09}\, M_\odot$$