GOETHE UNIVERSITÄT FRANKFURT AM MAIN

Linear and nonlinear realizations of chiral symmetry

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Motivation: QCD at low energies

Chiral symmetry

$\mathrm{SU}(n_f) \times \mathrm{SU}(n_f)$

Linear realization

Linear sigma models (LSMs)
Extended LSMs
Nucleon-meson models
Quark-meson models

Nonlinear realization

Chiral perturbation theory (ChPT)

Integration

Low-energy limit same as for QCD?

Functional renormalization group (FRG)
[JE, Divotgey, Mitter, Rischke '18, '19]

Symmetry realizations: [Weinberg '68; Callan, Coleman, Wess, Zumino '69]

Synergies with the (nuclear) EOS

Validity of (hybrid) microscopic calculations

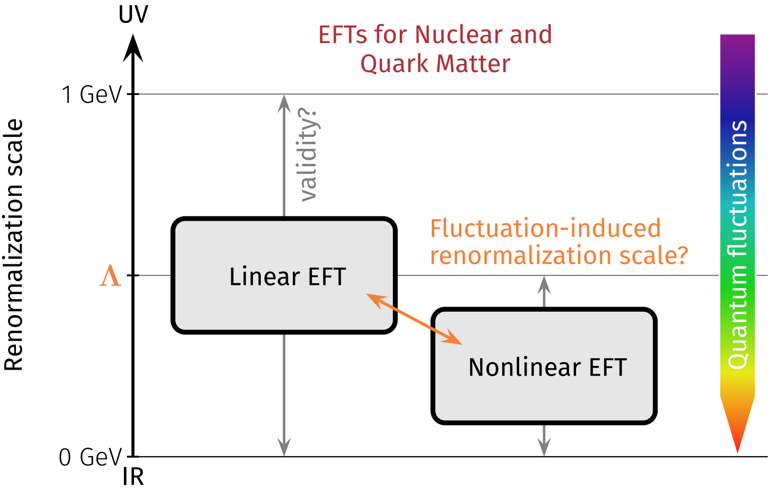


Figure 1: Hierarchy of effective field theories (EFTs); adapted from [Weise '18].

EOS scenarios: Combine symmetry realizations

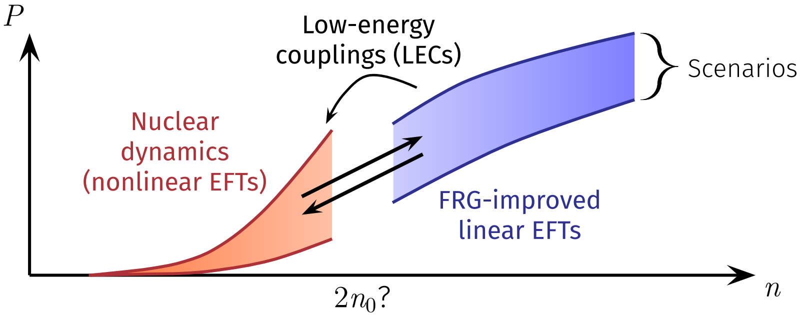


Figure 2: Sketch of the EOS; pressure P as function of the baryon density n.

FRG-improved EFTs

- **FRG:** Readily change degrees of freedom in **Yukawa interactions** (from quarks to nucleons and vice versa)
- Complement/assist (perturbative) EFT calculations with the computation of **LECs**
- (Qualitatively) contribute to unraveling the **nature of the EOS** at intermediate densities:
  (a) **Transition scales** between linear and nonlinear realizations of chiral symmetry
  (b) Compare FRG results to ChPT (chiral EFTs), e.g. [Baym, Braun, De, Drews, Drischler, Entem, Epelbaum, Hammer, Hebeler, Holt, Kaiser, Kojo, Krüger, Lattimer, Leonhardt, Lim, Lynn, Machleidt, Meißner, Reddy, Schallmo, Schwenk, Tews, Weise, Wellenhofer, ...]
  (c) Focus on **higher-derivative interactions**

Effective field theories

Chiral perturbation theory (ChPT)

- **Nonlinear realization** of chiral symmetry
- Systematic analysis of **hadronic n-point functions** of QCD
  [Gasser, Leutwyler '84, '85; Leutwyler '94; ...]
- Pion interactions dominate the low-energy regime ($n_f = 2$)
- **Chiral expansion** contains the LECs of QCD,
  $\{\mathcal{C}_{i,\mathrm{ChPT}}\}, i = 1, 2, 3, \ldots,$

$$\mathcal{L}_{\mathrm{ChPT}} = \frac{1}{2}\left(\partial_\mu \vec{\pi}\right)^2 - \frac{1}{2} m_\pi^2 \vec{\pi}^{\,2} + \mathcal{C}_{1,\mathrm{ChPT}} \left(\vec{\pi}^{\,2}\right)^2 + \mathcal{C}_{2,\mathrm{ChPT}}\, \vec{\pi}^{\,2} \left(\partial_\mu \vec{\pi}\right)^2 + \mathcal{C}_{3,\mathrm{ChPT}} \left(\partial_\mu \vec{\pi}\right)^2 \left(\partial_\nu \vec{\pi}\right)^2 + \mathcal{C}_{4,\mathrm{ChPT}} \left(\partial_\mu \vec{\pi} \cdot \partial_\nu \vec{\pi}\right)^2 + \mathcal{O}\left(\pi^6, \partial^6\right),$$

$\vec{\pi}$: "stereographic pions"

SO(4) quark-meson model

- **Linear realization** of chiral symmetry
  [Schwinger '57; Gell-Mann, Lévy '60; Weinberg '67; ...]
- Two dynamical quark flavors ($n_f = 2$)
- **(Euclidean) action**: [Jungnickel, Wetterich '96]

$$S = \int_{\mathbb{E}^4} \left\{ \frac{1}{2} \left(\partial_\mu \sigma\right) \partial_\mu \sigma + \frac{1}{2} \left(\partial_\mu \vec{\pi}\right) \cdot \partial_\mu \vec{\pi} + V(\rho) - h_{\mathrm{ESB}} \sigma + \bar{\psi} \left(\gamma_\mu \partial_\mu + y\, \Phi_5\right) \psi \right\},$$

$$\Phi_5 = \sigma\, t_0 + i \gamma_5 \vec{\pi} \cdot \vec{t},$$

$$\rho = \sigma^2 + \vec{\pi}^2$$

- **Explicit** and **spontaneous** symmetry breaking implemented,

$$\mathrm{SO}(4) \longrightarrow \mathrm{SO}(3): \qquad h_{\mathrm{ESB}} \neq 0, \qquad \sigma \to \langle \sigma \rangle + \sigma$$

Outline

- **Research objective:**
  (Ab initio) generation of mesonic (and baryonic) **LECs** from quantum fluctuations **(quark-meson interactions)**
  [**JE**, Divotgey, Mitter, Rischke '18, '19]

- **Ongoing works:**
  (a) Verification of LECs via $\pi\pi$ scattering
  (b) Computation of LECs in various EFTs
  (nucleon-meson, quark-meson-diquark models)
  [Cichutek, Divotgey, **JE**, in preparation]

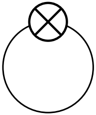
- **Planned projects:**
  (a) Exploit synergies with (hybrid) calculations of the **(nuclear) EOS**
  (at nonzero temperatures and densities)
  (b) Determine appropriate **renormalization scales** for pionic interactions as obtained from the FRG

Functional renormalization group

Functional renormalization group (FRG)

- Implementation of the **Wilsonian RG** philosophy
- Renormalization scale(k)-dependent **effective action** Γ_k
- FRG flow equation: [Wetterich '93]

$$\partial_k \Gamma_k = \frac{1}{2} \operatorname{tr} \left[\partial_k \mathcal{R}_k \left(\Gamma_k^{(2)} + \mathcal{R}_k \right)^{-1} \right] = \frac{1}{2}$$



- Regulator function $\mathcal{R}_k$ provides correct integration limits
- **Truncations** needed for solving the flow equation
- **Nonperturbative continuum method**

Flow in theory space

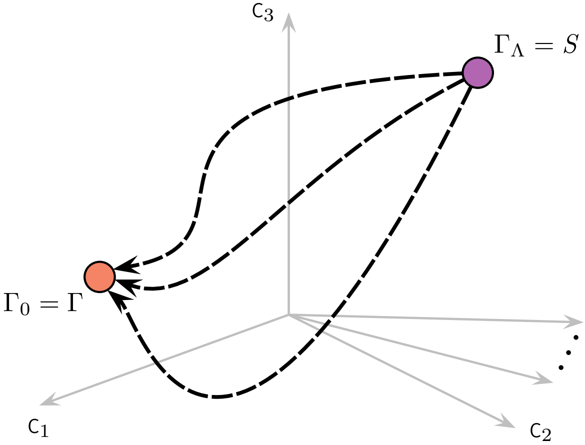


Figure 3: Theory space; spanned by the generic couplings $\{c_1, c_2, c_3, \ldots\}$.

SO(4) quark-meson model: Truncations

- **LPA (local potential approximation):** Consider scale-dependent **effective potential** V_k,

$$\Gamma_k = \int_{\mathbb{E}^4} \left\{ \frac{1}{2} (\partial_\mu \sigma) \partial_\mu \sigma + \frac{1}{2} (\partial_\mu \vec{\pi}) \cdot \partial_\mu \vec{\pi} + V_k(\rho) - h_{\mathrm{ESB}} \sigma + \bar{\psi} (\gamma_\mu \partial_\mu + y_k \Phi_5) \psi \right\}$$

- **LPA′:** Include **wave-function renormalization** Z_k,

$$\Gamma_k = \int_{\mathbb{E}^4} \left\{ \frac{Z_k^\sigma}{2} (\partial_\mu \sigma) \partial_\mu \sigma + \frac{Z_k^\pi}{2} (\partial_\mu \vec{\pi}) \cdot \partial_\mu \vec{\pi} + V_k(\rho) - h_{\mathrm{ESB}} \sigma + \bar{\psi} \left(Z_k^\psi \gamma_\mu \partial_\mu + y_k \Phi_5 \right) \psi \right\}$$

Low-energy couplings

Higher-derivative interactions

- $\mathbb{E}^4$ field-space variable $\varphi = (\vec{\pi}, \sigma)$
- Introduce complete sets of **higher-derivative couplings**,

$$\begin{aligned}\Gamma_k = \int_{\mathbb{E}^4} \Big\{ & \frac{Z_k}{2} (\partial_\mu \varphi) \cdot \partial_\mu \varphi + V_k(\rho) - h_{\mathrm{ESB}} \sigma \\ & + C_{2,k} (\varphi \cdot \partial_\mu \varphi)^2 + Z_{2,k} \varphi^2 (\partial_\mu \varphi) \cdot \partial_\mu \varphi \\ & - C_{3,k} [(\partial_\mu \varphi) \cdot \partial_\mu \varphi]^2 - C_{4,k} [(\partial_\mu \varphi) \cdot \partial_\nu \varphi]^2 \\ & - C_{5,k} \varphi \cdot (\partial_\mu \partial_\mu \varphi) (\partial_\nu \varphi) \cdot \partial_\nu \varphi \\ & - C_{6,k} \varphi^2 (\partial_\mu \partial_\nu \varphi) \cdot \partial_\mu \partial_\nu \varphi \\ & - C_{7,k} (\varphi \cdot \partial_\mu \partial_\mu \varphi)^2 - C_{8,k} \varphi^2 (\partial_\mu \partial_\mu \varphi)^2 \\ & + \bar{\psi} \left(Z_k^\psi \gamma_\mu \partial_\mu + y_k \Phi_5 \right) \psi \Big\}\end{aligned}$$

- **Goal:** Compute IR values of all scale-dependent quantities

Dynamical chiral symmetry breaking

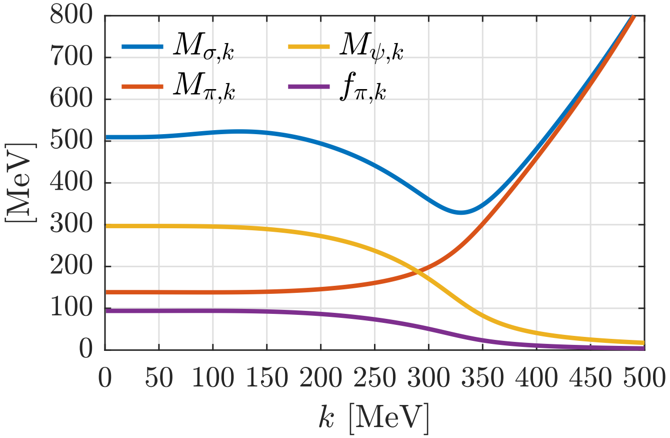


Figure 4: Scale evolution of the (renormalized) meson and quark masses as well as the pion decay constant; [Divotgey, **JE**, Mitter '19].

Dynamical generation of LECs of $\mathcal{O}(\partial^2)$

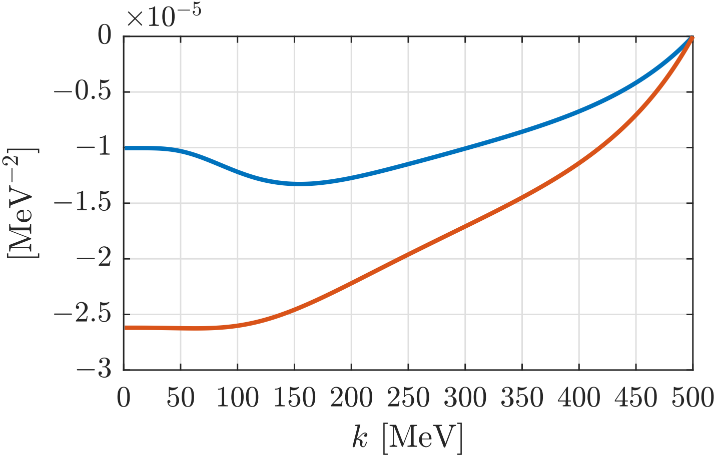


Figure 5: Scale evolution of the (renormalized) higher-derivative couplings of $\mathcal{O}(\partial^2)$; [Divotgey, **JE**, Mitter '19].

Dynamical generation of LECs of $\mathcal{O}(\partial^4)$

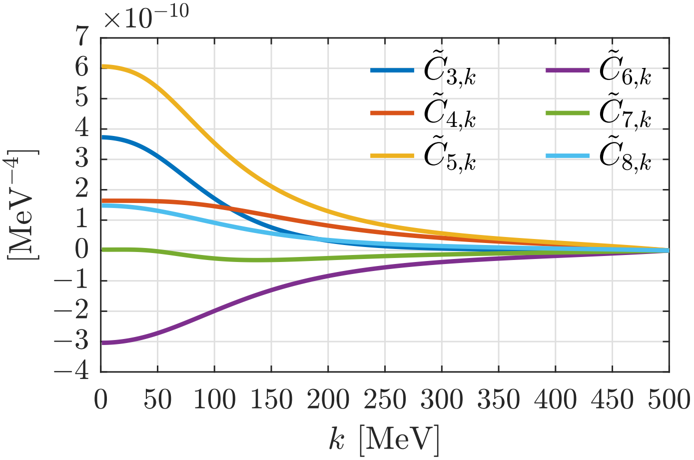


Figure 6: Scale evolution of the (renormalized) higher-derivative couplings of $\mathcal{O}(\partial^4)$; [Divotgey, JE, Mitter '19].

Vacuum manifold: Transition to nonlinear realization

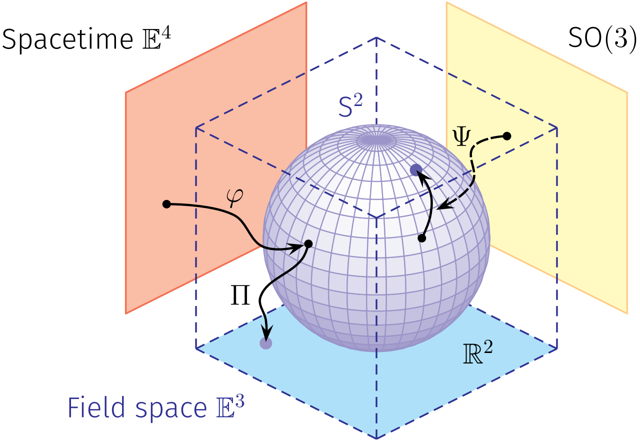


Figure 7: Spheres as vacuum manifolds of the breaking SO$(n+1) \longrightarrow$ SO(n); the two-sphere $(n = 2)$ is shown; [JE, in preparation].

Effective pion action

- (Momentum-dependent) pion self-interactions (cf. ChPT)
- Exploit "circle condition," $\varphi^2 = f_\pi^2$
- (Nonlinear, stereographic) pion action ($\tilde{\Pi}^a$, $a = 1, 2, 3$):

$$\Gamma_k = \int_{\mathbb{E}^4} \Big\{ \frac{1}{2} \left(\partial_\mu \tilde{\Pi}_a\right) \partial_\mu \tilde{\Pi}^a + \frac{1}{2} \tilde{\mathcal{M}}^2_{\Pi,k} \tilde{\Pi}_a \tilde{\Pi}^a - \tilde{\mathcal{C}}_{1,k} \left(\tilde{\Pi}_a \tilde{\Pi}^a\right)^2 \\
+ \tilde{\mathcal{Z}}_{2,k} \tilde{\Pi}_a \tilde{\Pi}^a \left(\partial_\mu \tilde{\Pi}_b\right) \partial_\mu \tilde{\Pi}^b \\
- \tilde{\mathcal{C}}_{3,k} \left[\left(\partial_\mu \tilde{\Pi}_a\right) \partial_\mu \tilde{\Pi}^a\right]^2 - \tilde{\mathcal{C}}_{4,k} \left[\left(\partial_\mu \tilde{\Pi}_a\right) \partial_\nu \tilde{\Pi}^a\right]^2 \\
- \tilde{\mathcal{C}}_{5,k} \tilde{\Pi}_a \left(\partial_\mu \partial_\mu \tilde{\Pi}^a\right) \left(\partial_\nu \tilde{\Pi}_b\right) \partial_\nu \tilde{\Pi}^b \\
- \tilde{\mathcal{C}}_{6,k} \tilde{\Pi}_a \tilde{\Pi}^a \left(\partial_\mu \partial_\nu \tilde{\Pi}_b\right) \partial_\mu \partial_\nu \tilde{\Pi}^b \\
- \tilde{\mathcal{C}}_{8,k} \tilde{\Pi}_a \tilde{\Pi}^a \left(\partial_\mu \partial_\mu \tilde{\Pi}_b\right) \partial_\nu \partial_\nu \tilde{\Pi}^b \Big\}$$

Nonlinear LECs

- Renormalized pion mass:

$$\tilde{\mathcal{M}}^2_{\Pi,k} = \frac{\tilde{h}_{\mathrm{ESB}}}{f_\pi}, \qquad \tilde{h}_{\mathrm{ESB}} = \frac{h_{\mathrm{ESB}}}{\sqrt{Z^\pi_k}}$$

- Nonlinear LECs as linear combinations of linear LECs,

$$\tilde{\mathcal{C}}_{1,k} = \frac{\tilde{\mathcal{M}}^2_{\Pi,k}}{8f_\pi^2}, \qquad \tilde{\mathcal{Z}}_{2,k} = -\frac{1}{4f_\pi^2},$$

$$\tilde{\mathcal{C}}_{3,k} = \tilde{C}_{3,k} - \tilde{C}_{5,k} + \tilde{C}_{7,k} + 2\left(\tilde{C}_{6,k} + \tilde{C}_{8,k}\right),$$

$$\tilde{\mathcal{C}}_{4,k} = \tilde{C}_{4,k}, \qquad \tilde{\mathcal{C}}_{5,k} = 2\left(\tilde{C}_{6,k} + \tilde{C}_{8,k}\right),$$

$$\tilde{\mathcal{C}}_{6,k} = -\tilde{C}_{6,k} - \tilde{C}_{8,k}, \qquad \tilde{\mathcal{C}}_{8,k} = \frac{1}{2}\left(\tilde{C}_{6,k} + \tilde{C}_{8,k}\right)$$

Mapping onto the nonlinear effective action

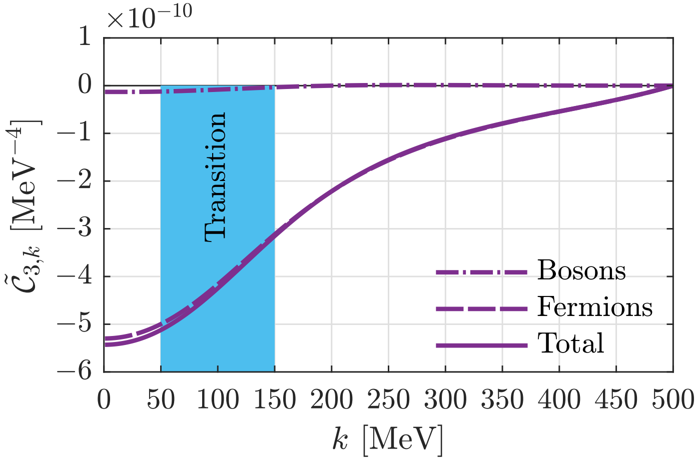


Figure 8: Scale evolution of the nonlinear LEC $\tilde{\mathcal{C}}_{3,k}$; dominance of fermion fluctuations, which decouple below 50-150 MeV; [Divotgey, **JE**, Mitter '19].

Summary

Summary

Conclusions:

- Fluctuation dynamics strongly **dominated** by fermionic loops
- **Renormalization scales** of 50-150 MeV for the purely pionic effective action as obtained from the **FRG**

Outlook:

- Compute $\pi\pi$ scattering lengths
- Determine LECs from quark-gluon fluctuations
- Confront the FRG calculation with ChPT (chiral EFTs); compatibility analysis of **renormalization schemes**

Backup

Summary of numerical results

- LECs evaluated at $k_{\mathrm{IR}} = 1$ MeV: [Divotgey, **JE**, Mitter '19]

Linear model		Nonlinear model	
$\tilde{C}_2\ [1/f_\pi^2] \times 10$	-0.88	$\ldots$	
$\tilde{Z}_2\ [1/f_\pi^2] \times 10$	-2.30	$\tilde{\mathcal{Z}}_2\ [1/f_\pi^2] \times 10$	-2.50
$\tilde{C}_3\ [1/f_\pi^4] \times 10^2$	2.88	$\tilde{\mathcal{C}}_3\ [1/f_\pi^4] \times 10^2$	-4.20
$\tilde{C}_4\ [1/f_\pi^4] \times 10^2$	1.27	$\tilde{\mathcal{C}}_4\ [1/f_\pi^4] \times 10^2$	1.27
$\tilde{C}_5\ [1/f_\pi^4] \times 10^2$	4.69	$\tilde{\mathcal{C}}_5\ [1/f_\pi^4] \times 10^2$	-2.41
$\tilde{C}_6\ [1/f_\pi^4] \times 10^2$	-2.35	$\tilde{\mathcal{C}}_6\ [1/f_\pi^4] \times 10^2$	1.21
$\tilde{C}_7\ [1/f_\pi^4] \times 10^2$	0.02	$\ldots$	
$\tilde{C}_8\ [1/f_\pi^4] \times 10^2$	1.14	$\tilde{\mathcal{C}}_8\ [1/f_\pi^4] \times 10^2$	-0.60

- $\tilde{\mathcal{C}}_{1,k_{\mathrm{IR}}} = 0.27$, $\tilde{\mathcal{M}}_{\Pi,k_{\mathrm{IR}}} = 138.5$ MeV