Nuclear Equation of State for Hot Dense Matter

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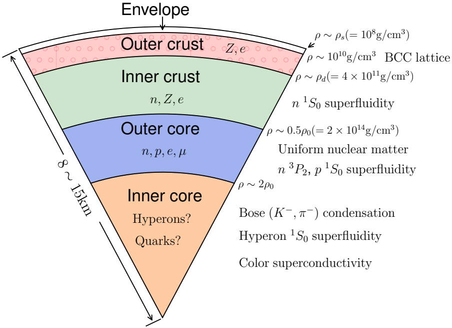
Neutron Stars

- Formed after core collapsing supernovae.
- Suggested by Walter Baade and Fritz Zwicky (1934) - Only a year after the discovery of the neutron by James Chadwick
- Jocelyn Bell Burnell and Antony Hewish observed pulsar in 1965.
- Neutron star is cold after 30s $\sim$ 60s of its birth
  - inner core, outer core, inner crust, outer crust, envelope
  - $R : \sim 10$Km, $M : 1.2 \sim 2.x\ M_{\odot}$
  - 2×10^{11} earth g $\rightarrow$ General relativity
  - B field : $10^8 \sim 10^{12}$G.
  - Central density : $3 \sim 10\rho_0 \rightarrow$ Nuclear physics!!
- TOV equations for macroscopic structure

$$\begin{aligned} \frac{dp}{dr} &= -\frac{G(M(r) + 4\pi r^3 p/c^2)(\epsilon + p)}{r(r - 2GM(r)/c^2)c^2}, \\ \frac{dM}{dr} &= 4\pi \frac{\epsilon}{c^2} r^2, \end{aligned} \tag{1}$$

- Inner structure of neutron stars



- Neutron Stars:
  - Dense nuclear matter physics

Nuclear matter properties

- What is dense matter? comparable with the nuclear saturation density $n_0 = 0.16\,\mathrm{fm}^{-3}$
- How do we know? How can we extrapolate?
  - Nuclear experiments

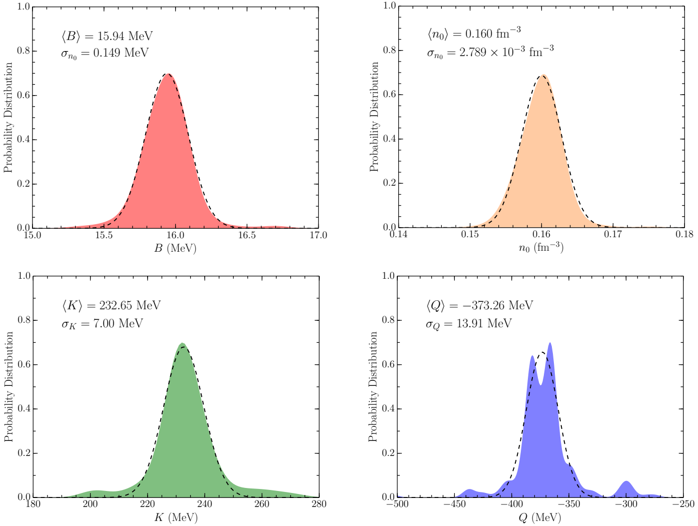
$$\frac{E}{A}(n, \delta_{np}) = A_0(n) + S_2(n)\delta_{np}^2 + \sum_{n=2}^{\infty}(S_{2n} + L_{2n}\ln|\delta_{np}|)\delta_{np}^{2n} \quad (2)$$

$$A_0(n) = -B + \frac{1}{2}K\left(\frac{n-n_0}{3n_0}\right)^2 + \frac{1}{6}Q\left(\frac{n-n_0}{3n_0}\right)^2 + \cdots \quad (3)$$

$$S_2(n) = J + L\left(\frac{n-n_0}{3n_0}\right) + \frac{1}{2}K_{\mathrm{sym}}\left(\frac{n-n_0}{3n_0}\right)^2 + \frac{1}{6}Q_{\mathrm{sym}}\left(\frac{n-n_0}{3n_0}\right)^2 + \cdots \quad (4)$$

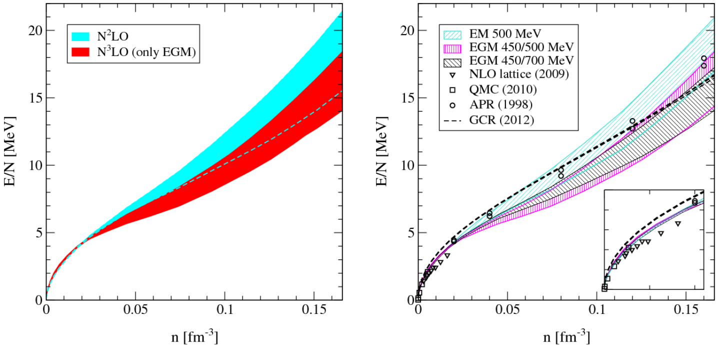
- Application?
  - Constructing nuclear equation of state for hot dense matter

- Symmetric nuclear matter(B, ρ_0, K, Q), Lim & Holt, EPJA 2019, Dutra et al. 2019)

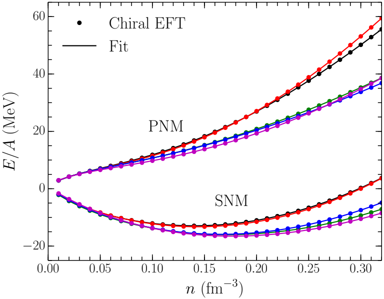


- Pure neutron matter matter
  - Chiral Effective Field Theory and Many body physics (MBPT,QMC, variational method, Green function, $\cdots$)

Krüger et al. (Left, PRC 88, 025802), Tews et al.(Right, PRC 93, 024305)



- Most neutron matter results can be fitted using the quadratic expansion.



$$\mathcal{E}(n,x) = \frac{1}{2m}\tau_n + \frac{1}{2m}\tau_p + (1-2x)^2 f_n(n) + \left[1-(1-2x)^2\right] f_s(n)\,, \tag{5}$$

$$f_s(n) = \sum_{i=0}^{3} a_i n^{(2+i/3)}\,, \quad f_n(n) = \sum_{i=0}^{3} b_i n^{(2+i/3)} \tag{6}$$

Bayesian Analysis

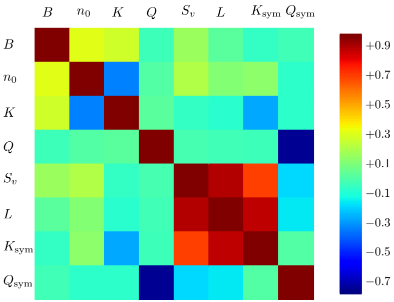
- How do we construct energy density functional (EDF) parameters $\vec{a}$?
- Baye's theorem:

$$P(\vec{a}|D) \sim P(D|\vec{a})P(\vec{a}) \tag{7}$$

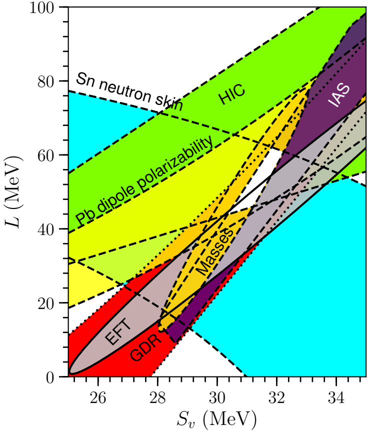
$P(\vec{a}|D)$: posterior , $P(D|\vec{a})$: likelihood, $P(\vec{a})$: prior

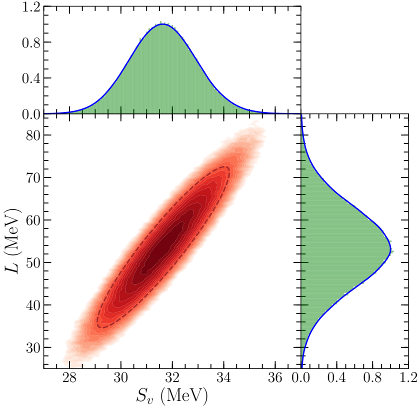
- Strategy:
  - Find useful parametrizations for the equation of state
  - Obtain priors from chiral EFT predictions
  - Use laboratory measurements of finite nuclei to obtain likelihood functions and posteriors
- Neutron & Symmteric matter from EFT (N2LO450, N2LO500, N3LO414, N3LO450, N3LO500 + N2LO 3-Body interaction, MBPT)
- Symmetric matter from Finite nuclei properties (205 Skyrme force models)
- Or Bayesian (EFT$\rightarrow$ prior; Skyrme, FLT $\rightarrow$ Likelihood)

Correlation between SNM & PNM

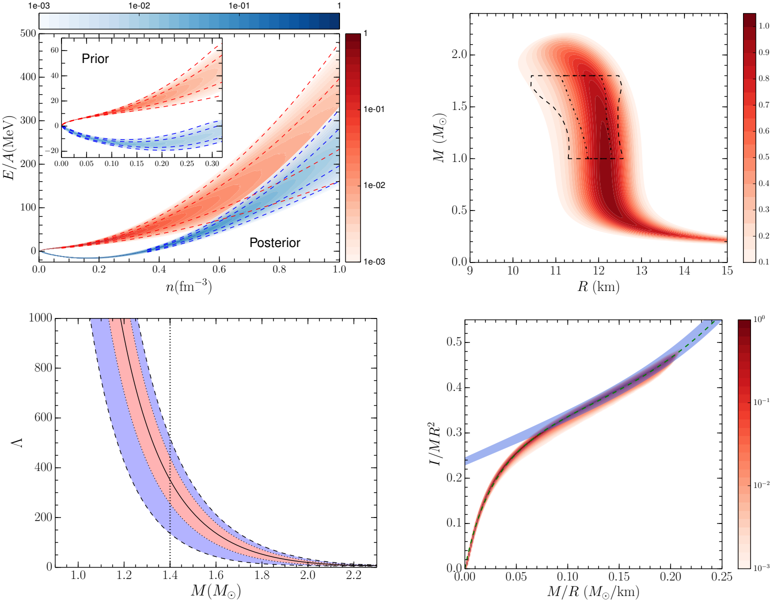


- Correlation between S_v and L (Left: Lattimer & Lim ApJ 2013, Lim & Holt, EPJA 2019)

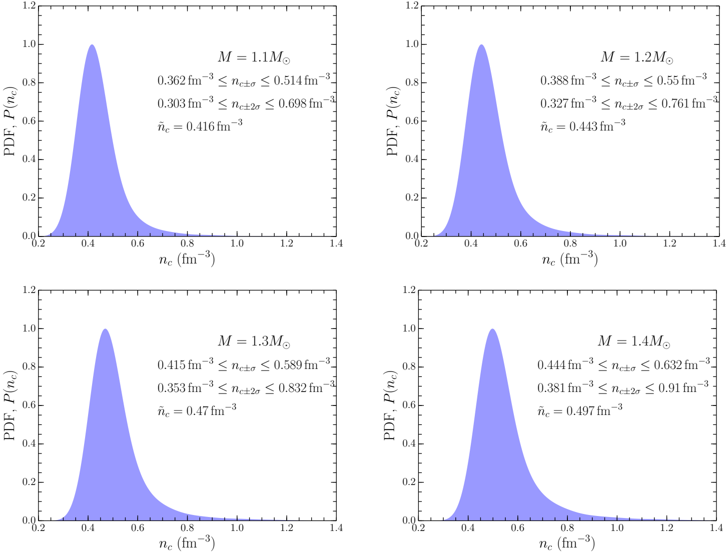




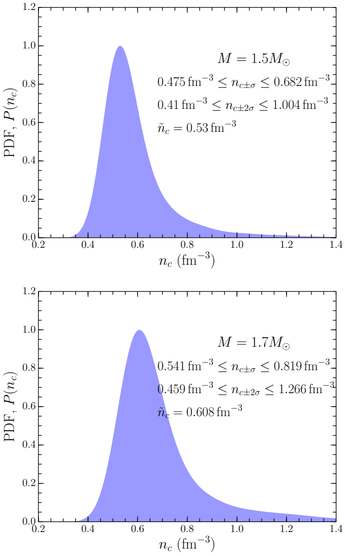
Neutron Star Phenomenology

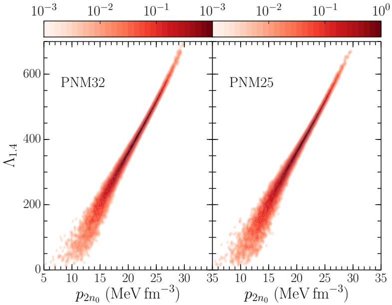


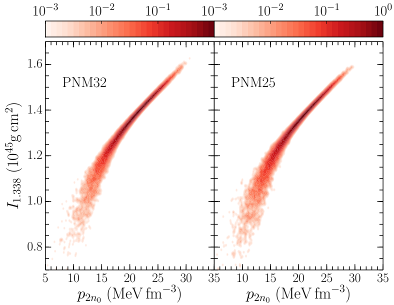
Probability distribution of central density I

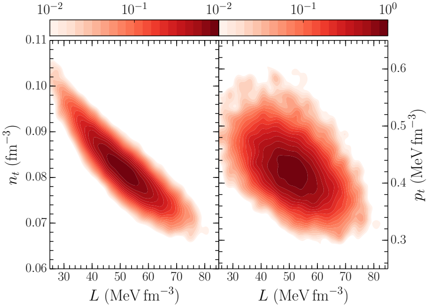


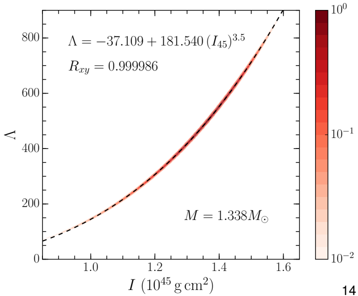
Probability distribution of central density II



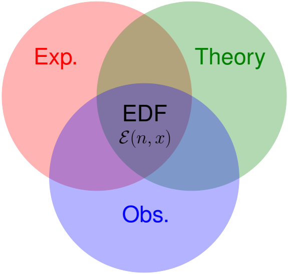

PNM32
PNM25
$\Lambda_{1.4}$
p_{2n_0} (MeV fm^{-3})


PNM32
PNM25
$I_{1.338}$ (10^{45}g cm^2)
p_{2n_0} (MeV fm^{-3})


n_t (fm^{-3})
p_t (MeV fm^{-3})
L (MeV fm^{-3})


$\Lambda = -37.109 + 181.540\,(I_{45})^{3.5}$
$R_{xy} = 0.999986$
$M = 1.338 M_\odot$
Λ
I (10^{45} g cm^2)

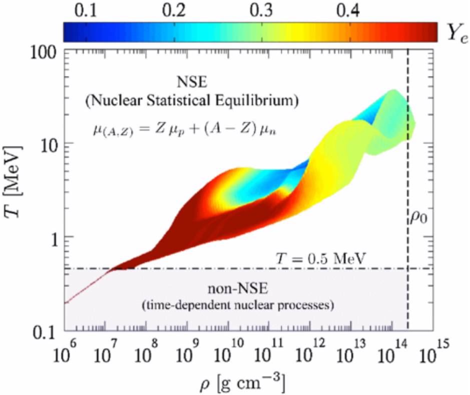
Neutron Star EOS constraints



- Experiments See William, Arnaud's talk
  - Symmetric nuclear matter properties, Neutron skin thickness, binding energies
- Theory
  - Neutron matter calculations (QMC, MBPT, ..), ; See Ingo, Corbinian's talk
- Observation
  Gravitation wave : tidal deformabilities, Moment of inertia, Nicer (mass-radius), ; Soumi, Svenja, Geerts's talk

Nuclear Equation of State

- Nuclear EOS is thermodynamic relations for given ρ, Y_e, T with wide range of varibles (Fig: Oertel et al., Rev. Mod. Phys. 89, 015007).
(ρ : $10^4 \sim 10^{14}$g/cm^3, Y_e : $0.01 \sim 0.56$, T : $0.5 \sim 200$MeV)



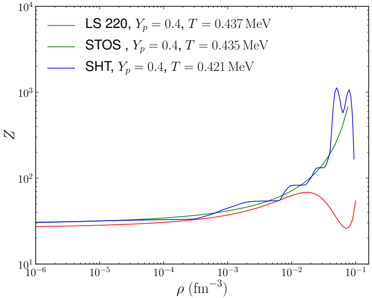
- Nuclear EOS is important to simulate core collapsing supernova explosion, proto-neturon stars, and compact binary mergers involving neutron stars.

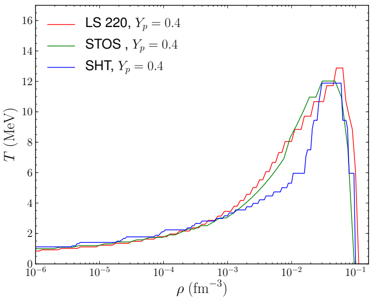
- Three representative EOSs.

- LS EOS (Lattimer Swesty 1991)
  Use Skyrme type potential with Liquid droplet approach
  - Consider phase transition, several *K*
- STOS EOS (H Shen, Toki, Oyamastu, Sumiyoshi 1998), new version (2011)
  Use RMF with TF approximation and parameterized density profile (PDP)
  - Old : awkward grid spacing
  - New : finer grid spacing, adds Hyperon(Λ, $\Sigma^{+,-,0}$)
- SHT EOS (G Shen, Horowitz, Teige 2010)
  Use RMF with Hartree approximation

	LS 220	STOS	SHT
$\rho(\mathrm{fm}^{-3})$	$10^{-6} \sim 1$ (121)	$7.58 \times 10^{-11} \sim 6.022$ (110)	$10^{-8} \sim 1.496$ (328)
Y_p	$0.01 \sim 0.5$ (50)	$0 \sim 0.65$ (66)	$0 \sim 0.56$ (57)
T (MeV)	$0.3 \sim 30$ (50)	$0.1 \sim 398.1$ (90)	$0 \sim 75.0$ (109)

- STOS & SHT tables don't provide second derivative $\left(\frac{\partial(P,S)}{\partial(T,\rho,Y_p)}\right)$.





How can we construct EOS table ?

How can we construct EOS table ?
We need nuclear force model and numerical method.

Nuclear force model	Numerical technique
Skyrme Force model (non-relativistic potential model)	Liquid Drop(let) approach (LDM)
Relativistic Mean Field model (RMF)	Thomas Fermi Approximatoin (TF)
Finite-Range Force model	Hartree-Fock Approximation (HF)

- LS EOS $\Rightarrow$ Skyrme force + LDM (without neutron skin)
- STOS $\Rightarrow$ RMF + Semi TF (parameterized density profile)
- SHT $\Rightarrow$ RMF + HARTREE

Nuclear force model should be picked up to represent both finite nuclei and neutron star observation + Neutron matter calculation.

Nuclear Statistical Equilibrium method

- Hempel et al. 2010, 2012
  - Use Relativistic mean field model (TM1, TMA, FSUgold)
  - Nuclear statistical equilibrium (alpha, deutron, triton, helion)
- Blinnikov et al. 2011
  - Used Saha equation to find fraction of multi-component of nuclei
  - Nuclear mass formula
- Furusawa, 2011
  - Used Saha equation to find fraction of multi-component of nuclei
  - Used Relativistic mean field model
  - Consider phase transtion using geometric function

- The EOS table should be thermodynamically consistent up to second order,
  (numerical derivative may not be smooth !! $\frac{\partial p}{\partial \rho}$, $\frac{\partial p}{\partial T}$ $\frac{\partial p}{\partial Y_e}$...)
- EOS should also fit astrophysical observation (M-R relation, Λ, maximum mass)

Finite Nuclei : compressible model

The total binding energy of the finite nuclei at $T = 0$ MeV,
i) Bulk, ii) Surface, iii) Coulomb,

$$F(A,Z) = \left[-B + S_v\delta^2\right] A + 4\pi r^2(\sigma_0 - \delta^2\sigma_\delta) + \frac{3}{5}\frac{Z^2e^2}{r} + \lambda_1\left[\frac{4\pi}{3}r^3 n - A\right] + \lambda_2\left[\frac{4\pi}{3}r^3 nx - Z\right] \quad (8)$$

where $\delta = 1 - 2x$ and $\sigma = \sigma_0 - (1-2x)^2\sigma_\delta$ is surface tension.

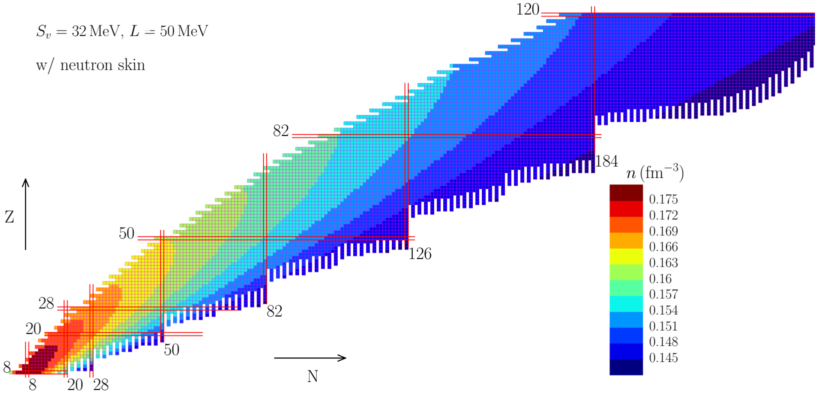
$$\frac{\partial F}{\partial n} = \frac{\partial F}{\partial x} = \frac{\partial F}{\partial r} = \frac{\partial F}{\partial \lambda_1} = \frac{\partial F}{\partial \lambda_2} = 0 \quad (9)$$

gives

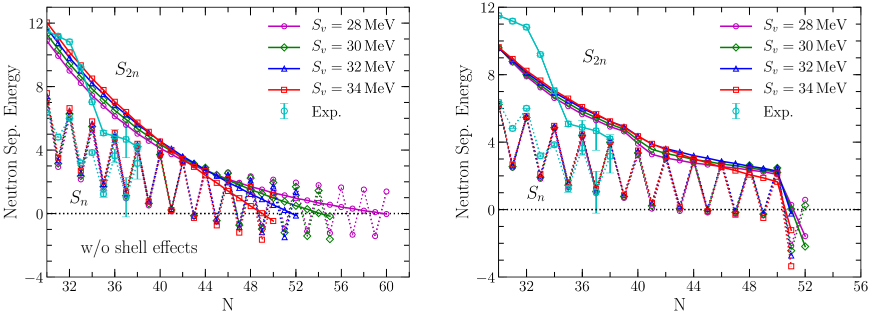
$$2\cdot 4\pi r^2\left[\sigma_0 - \sigma(1-2x)^2\right] - \frac{3}{5}\frac{Z^2e^2}{r} = 0, \quad (10)$$

$$\frac{4\pi r^3}{3} n = A, \quad \left(x = \frac{Z}{A}\right). \quad (11)$$

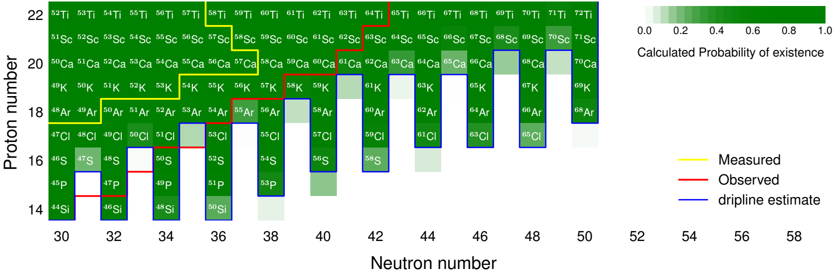
- Nuclear mass table from LDM ($S_v = 32\,\mathrm{MeV}$, $L = 50\,\mathrm{MeV}$)



- Neutron drip line study using LDM, Calcium (Z=20); without shell effect (left), with shell effect(right)



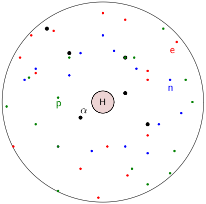
Probability of bound nuclei for $Z \leq 22$ constructed by LDM



Hot Dense Matter

For hot dense matter equation of state,

- Realistic nuclear force model
  Bulk energy $(-B + S_v\delta^2)$ should be replaced. Energy density functional (n, x, T)
- Wigner Seitz Cell method + Liquid drop approach



- Heavy nuclei at the center, n,p, α, e, exist outside + d, t, h.

$$\langle A\rangle \simeq \sum_{A,Z} A n_{A,Z}, \quad \langle Z\rangle \simeq \sum_{A,Z} Z n_{A,Z}. \tag{12}$$

(1) Bulk energy

- Nuclear Energy density functional

$$E_{\mathrm{bulk}}(n,\delta,T) = \sum_t \frac{\hbar^2 \tau_t}{2m_t^*} + \delta^2 V_{\mathcal{N}}(n) + (1-\delta^2) V_{\mathcal{S}}(n), \qquad (13)$$

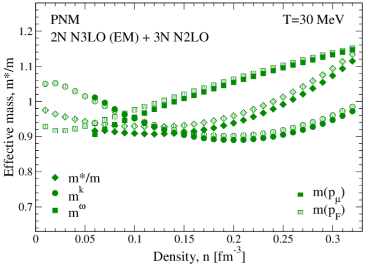
$$V_{\mathcal{N}}(n) = \sum_{i=0}^{i_{\mathrm{max}}} a_i n^{(2+i/3)}, \quad V_{\mathcal{S}}(n) = \sum_{i=0}^{i_{\mathrm{max}}} b_i n^{(2+i/3)}. \qquad (14)$$

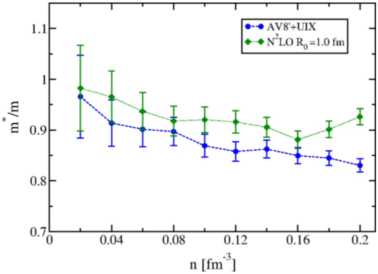
$$\frac{\hbar^2}{2m_n^*} = \frac{\hbar^2}{2m_n} + \alpha_L n_n + \alpha_U n_p, \quad \frac{\hbar^2}{2m_p^*} = \frac{\hbar^2}{2m_p} + \alpha_L n_p + \alpha_U n_n. \qquad (15)$$

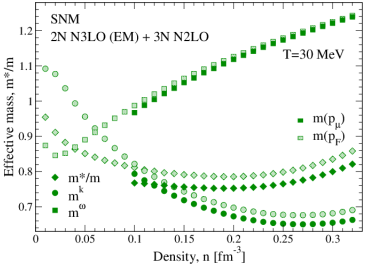
$$\langle M_{\mathcal{S}}^*/M_N \rangle = 0.774, \quad \langle M_{\mathcal{N}}^*/M_N \rangle = 0.9$$

- M^* plays important role in proton-neutron stars; See Luke, Sabrina's talk

M^* from Greens function approach (Carbone & Schwenk, PRC (2019)) and AFDMC (Buraczynski *et al.* PRL (2019))





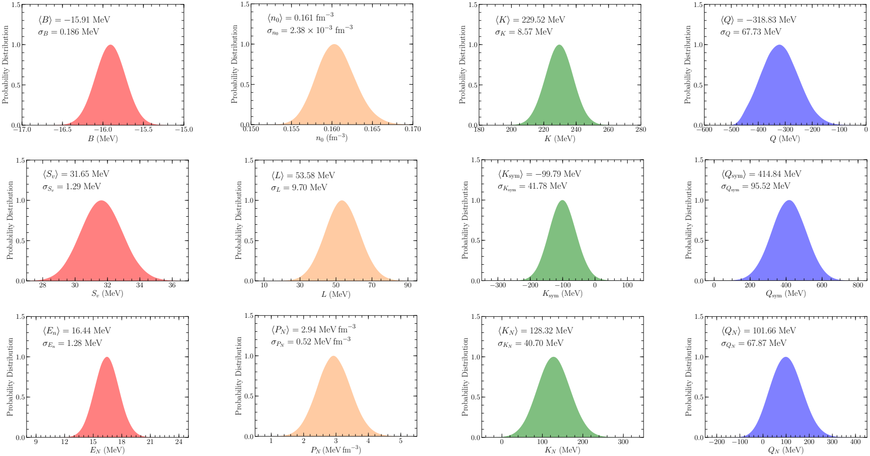


$$B = \left.\frac{E_{\text{bulk}}}{n}\right|_{n=n_0}, \qquad S_v = E_{\text{sym}}(n_0), \qquad E_n = E_{\text{pnm}}(n_0), \tag{16}$$

$$P = n^2 \frac{\partial}{\partial n}\left(\frac{E_{\text{bulk}}}{n}\right)\bigg|_{n=n_0} = 0, \quad L = 3n^2 \frac{\partial^2}{\partial n^2}\left(\frac{E_{\text{bulk}}}{n}\right) \qquad P_n = n^2 \left.\frac{\partial E_{\text{pnm}}}{\partial n}\right|_{n=n_0}, \tag{17}$$

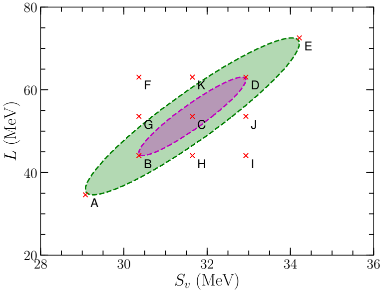
$$K = 9n^2 \frac{\partial^2}{\partial n^2}\left(\frac{E_{\text{bulk}}}{n}\right)\bigg|_{n=n_0} \qquad K_{\text{sym}} = 9n^2 \left.\frac{\partial^2 E_{\text{sym}}}{\partial n^2}\right|_{n=n_0}, \quad K_n = 9n^2 \left.\frac{\partial^2 E_{\text{pnm}}}{\partial n^2}\right|_{n=n_0}, \tag{18}$$

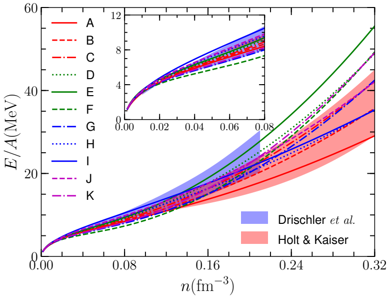
$$Q = 27n^3 \frac{\partial^3}{\partial n^3}\left(\frac{E_{\text{bulk}}}{n}\right)\bigg|_{n=n_0}, \quad Q_{\text{sym}} = 27n^3 \left.\frac{\partial E_{\text{sym}}}{\partial n^3}\right|_{n=n_0}, \quad Q_n = 27n^3 \left.\frac{\partial E_{\text{pnm}}}{\partial n^3}\right|_{n=n_0}. \tag{19}$$


$\langle B \rangle = -15.91$ MeV
$\sigma_B = 0.186$ MeV
B (MeV)
$\langle n_0 \rangle = 0.161$ fm^{-3}
$\sigma_{n_0} = 2.38 \times 10^{-3}$ fm^{-3}
n_0 (fm^{-3})
$\langle K \rangle = 229.52$ MeV
$\sigma_K = 8.57$ MeV
K (MeV)
$\langle Q \rangle = -318.83$ MeV
$\sigma_Q = 67.73$ MeV
Q (MeV)
$\langle S_v \rangle = 31.65$ MeV
$\sigma_{S_v} = 1.29$ MeV
S_v (MeV)
$\langle L \rangle = 53.58$ MeV
$\sigma_L = 9.70$ MeV
L (MeV)
$\langle K_{\rm sym} \rangle = -99.79$ MeV
$\sigma_{K_{\rm sym}} = 41.78$ MeV
$K_{\rm sym}$ (MeV)
$\langle Q_{\rm sym} \rangle = 414.84$ MeV
$\sigma_{Q_{\rm sym}} = 95.52$ MeV
$Q_{\rm sym}$ (MeV)
$\langle E_n \rangle = 16.44$ MeV
$\sigma_{E_n} = 1.28$ MeV
E_N (MeV)
$\langle P_N \rangle = 2.94$ MeV fm^{-3}
$\sigma_{P_N} = 0.52$ MeV fm^{-3}
P_N (MeV fm^{-3})
$\langle K_N \rangle = 128.32$ MeV
$\sigma_{K_N} = 40.70$ MeV
K_N (MeV)
$\langle Q_N \rangle = 101.66$ MeV
$\sigma_{Q_N} = 67.87$ MeV
Q_N (MeV)
Probability Distribution

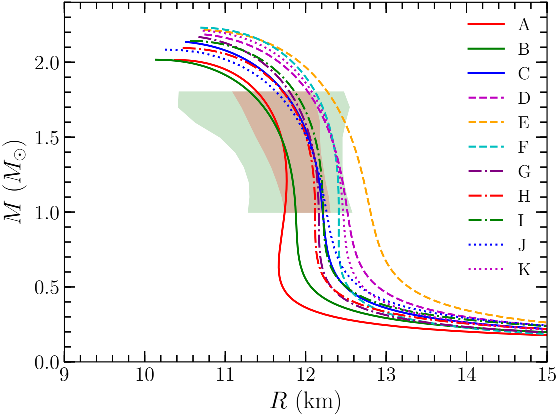
- Nuclear models for supernova EOSs

S_v and L of nuclear models for supernova EOSs and neutron matter energy per baryon in case of each model.





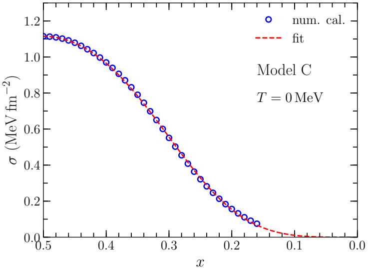
Mass and radius curves for selected models

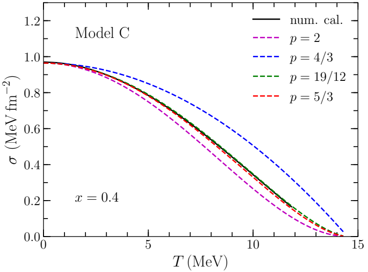


(2) Surface energy : Uniform nuclear matter with Finite Range force model

- Nuclear Surface tension for given temperature and proton fraction can be obtained from finite range force model

$$\sigma(x,T) = \sigma(x)\left[1-\left(\frac{T}{T_c}\right)^2\right]^p, \quad \sigma(x) = \sigma_0 \frac{2\cdot 2^\alpha + q}{(1-x)^{-\alpha}+q+x^{-\alpha}} \qquad (20)$$





(3) Coulomb energy

- Free energy density from Liquid Drop Model without light nuclei

$$F = un_i f_i + \frac{\sigma(x_i, T)ud}{r} + 2\pi(n_i x_i e r_N)^2 u D_d(u) + (1-u) n_{no} f_o \,, \qquad (21)$$

where

$$D_d(u) = \frac{1}{d+2}\left[\frac{2}{d-2}\left(1 - \frac{1}{2} d u^{1-2/d}\right) + u\right] , \qquad (22)$$

for each d, calculate energy density and find out lowest energy state (bubble phase $u \to (1-u)$). spherical $\to$ cylindrical $\to$ slab $\to$ cylindrical hole $\to$ spherical hole $\to$ uniform matter

- Quantum fluctuation allows the continuous dimension (Lattimer and Swesty, Nucl. Phys. A535, 331 (1991))

$F_S + F_C = \beta\mathcal{D} \to \mathcal{D} = u\left[\frac{d^2 D_d(u)}{9}\right]^{1/3}$.

Considering bubble phase,

$$\mathcal{D}(u) = u(1-u)\frac{(1-u)D_3^{1/3}(u) + uD_3^{1/3}(1-u)}{u^2 + (1-u)^2 + 0.6u^2(1-u)^2}. \qquad (23)$$

Free energy

Total free energy density consists of

$$F = F_N + F_o + F_\alpha + F_d + F_t + F_h + F_e + F_\gamma \quad (24)$$

where F_N, F_o, F_α, F_e, and F_γ are the free energy density of heavy nuclei, nucleons out the nuclei, alpha particles, electrons, and photons.

- $F_N = F_{bulk,i} + F_{coul} + F_{surf} + F_{trans}$
- $F_o = F_{bulk,o}$
- α, d, t, h particles : Non-interacting Boltzman gas
- e, γ : treat separately

For $F_{bulk,i}$, $F_{bulk,o}$, and F_{surf}, we use the same force model.
F_{surf} from the semi infinite nuclear matter calculation

The is the modification of LPRL (1985), LS (1991, No skin)

- Consistent calculation of surface tension
- Deuteron, triton, helion
- The most recent parameter set

Free energy minimization

For fixed independent variables (ρ, Y_p, T), we have the 11 dependent variables ($\rho_i, x_i, r_N, z_i, u, \rho_o, x_o, \rho_\alpha, \rho_d, \rho_t, \rho_h$).

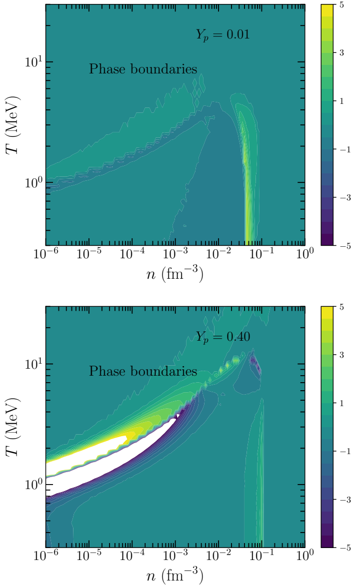
where i heavy nuclei, o nucleons outside, x proton fraction, u filling factor, and ν_n neutron skin density.

From baryon and charge conservation, we can eliminate x_o and ρ_o.

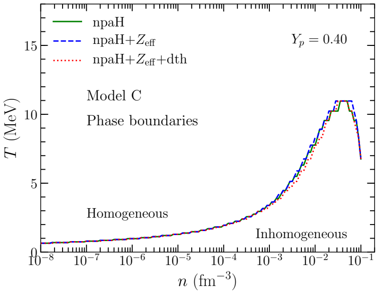
Free energy minimization,
$$\frac{\partial F}{\partial \rho_i} = \frac{\partial F}{\partial x_i} = \frac{\partial F}{\partial r_N} = \frac{\partial F}{\partial z_i} = \frac{\partial F}{\partial u} = \frac{\partial F}{\partial \rho_\alpha} = \frac{\partial F}{\partial \rho_d} = \frac{\partial F}{\partial \rho_t} = \frac{\partial F}{\partial \rho_h} = 0.$$

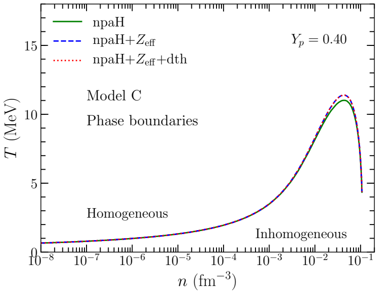
- Finally, we have 6 equations to solve and 6 unknowns. $z = (\rho_i, \ln(\rho_{no}), \ln(\rho_{po}), x_i, \ln(u), z_i)$.
- The code (f90) is fast. $165(\rho) \times 101(T) \times 65(Y_p)$ zones < 20 mins in single machine. ($1.0 \times 10^{-8} \le \rho \le 1.6\,\mathrm{fm}^{-3}$, $0.1 \le T \le 100\,\mathrm{MeV}$, $0.01 \le Y_p \le 0.65$) - more grid points (maybe) needed
- Any Skyrme functionals (or any symmetric or neutron matter property EOSs) can be compared,

Results : Relative pressure difference bewteen the current code & LS SkM*

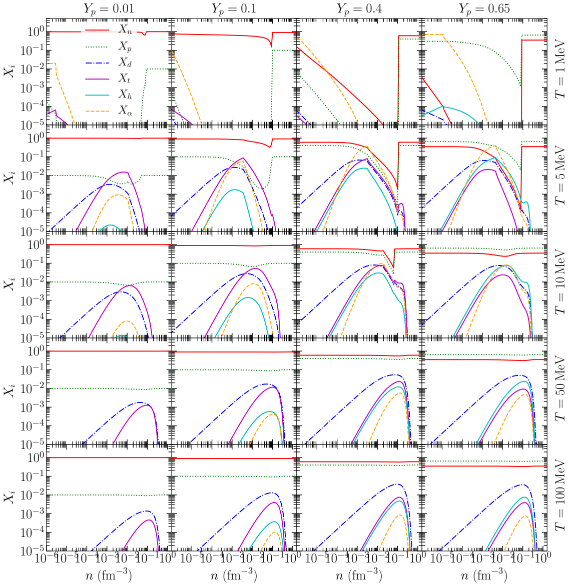


Phase Boundary





Particle fraction



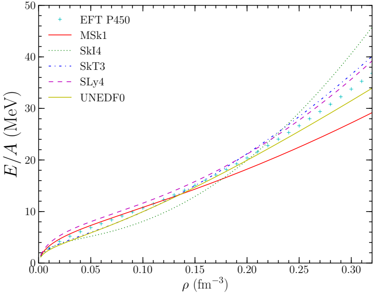
Summary

Nuclear Equation of State

- Nuclear Model : Energy density functional (Exp. + Theory + Obs.)
  - should be consistent with finite nuclei,
  - neutron matter calculation
  - maximum mass of neutron stars, *MR* (NICER) , moment of inertia in the future
- Numerical Method : Liquid Drop Model
  - surface tension, critical temperature, effective charge (Screening from outside particles)
  - deuteron, triton, helion
- The plan for improving
  - Effective mass constraints (χEFT, perturbation, Green Function, Fermi Liquid Theory)
  - Speed of Sound in dense matter; (See Sabrina's talk)

Backup : Bulk matter from Mean Field Models

- Bulk nuclear matter calculation by EFT has been done up to N3LO with two and three body interactions.
- Current potential model (non-relativistic) and relativistic mean field model



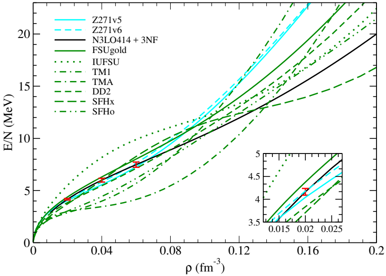


Figure: Left:energy per baryon of pure neutron matter in case of Skyrme force model. Right : the same plot but for the case of relativistic mean field model (figure from Rrapaj et al., PhysRevC.93.065801 (2015))

Backup SkT

- SkT

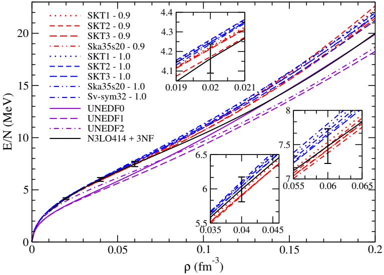


Figure: Modification of SkT3 models to fit EFT calculation by Brown and Schwenk (PRC.89.011307). Figure from Rrapaj et al., PhysRevC.93.065801 (2015) .

- It might need to find new parameter sets to fit χEFT bulk matter calculation.

Backup : Quadratic Expansion

- Nuclear matter?
  - Uniform or Non-uniform (inhomogeneous)
  - Nuclear matter properties : B, ρ_0, K, Q, S_v, L, $K_{\rm sym}$, $Q_{\rm sym}$

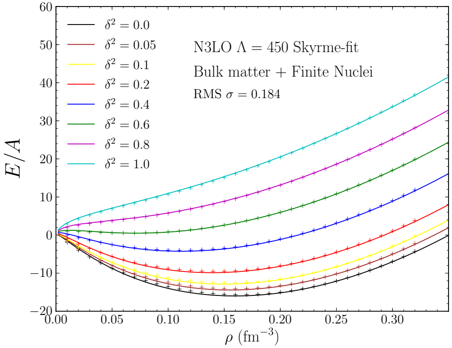
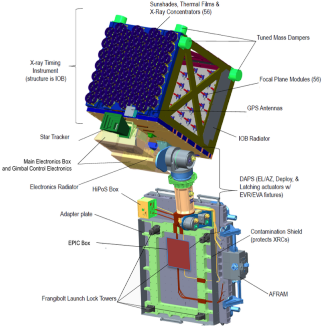


Figure: Nuclear matter energy per baryon for each δ^2 ($\delta = (n_n - n_p)/(n_n + n_p)$)

Backup : NICER

- Neutron star radii : $\pm 5\%$
- Neutron star masses : $\pm 10\%$
- First dedicated targets : PSR J0437-4715, PSR J0030+0451



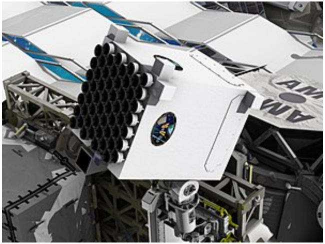


Figure: NICER instrument

Backup :*L*

- Neutron Skin : *L*

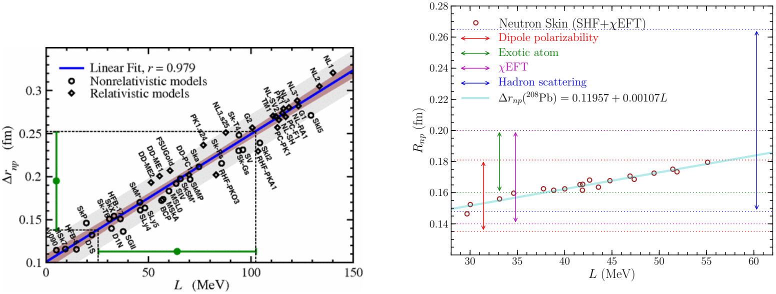


Figure: neutron skin correlation with density derivative of symmetry energy

Backup : correlation

- Correlation between S_v and L, E_N and P_N

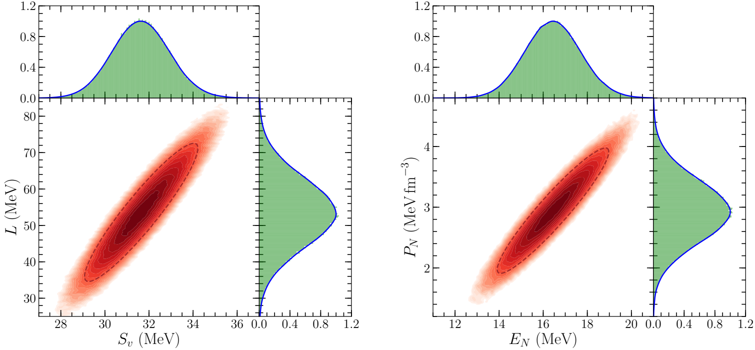


Figure: Two dimensional distribution plots for S_v and L, E_N and P_N

Backup : moment of inertia

- J0737-3039(A) :
  Radio timing → measurement of periastron advance → the effects of relativistic spin-orbit coupling → second order post-Newtonian expansion → moment of inertia

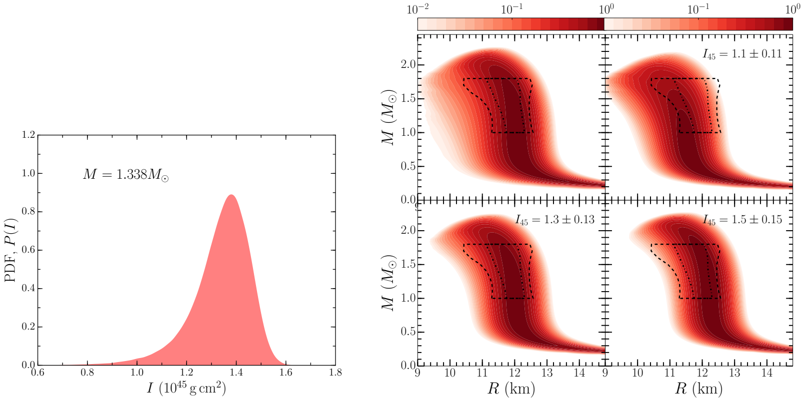


Figure: Mass and radii bands contrained from the moment of inertia of neutron star $M = 1.338 M_\odot$.
Lim & Holt, PRC 100, 035802 (2019)

Backup:pasta phase

- The transition to core happens when the uniform nuclear matter has the lowest energy state
- Nuclear pasta phase
  - The ground state of nuclear shape is determined by the competition between Coulomb interaction and surface tension

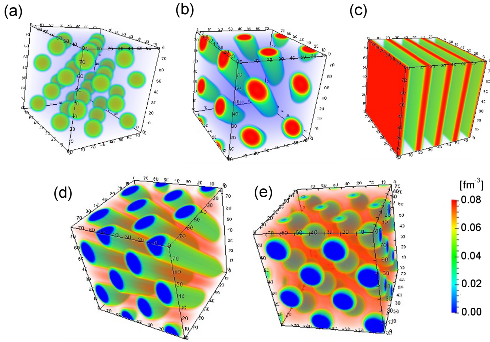


Figure: Numerical calculation of nuclear pasta phase. Figure from the work of Okamoto et al., Phys. Rev. C 88, 025801 (2013).

Backup : coexistence

- Coexistence
  - The coexistence curve shows the phase equilibrium bewteen dense and dilute phase.
  - This is the simplified case of heavy nuclei with nucleon fluids.

$$p_I = p_{II}, \quad \mu_{nI} = \mu_{nII}, \quad \mu_{pI} = \mu_{pII} \tag{25}$$

  - Under critical temperature coexistence is possible.

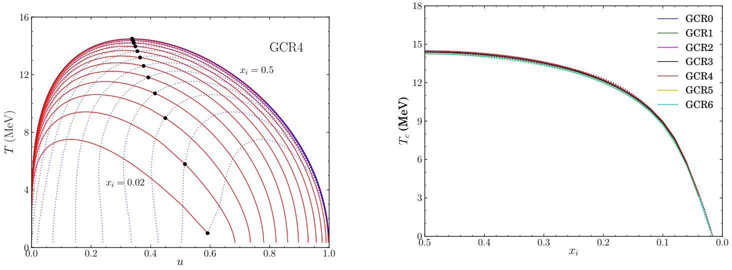


Figure: The left panel shows the coexistence curve for given Y_p and the right panel show the critical temperature as a function of Y_p.

Semi-infinite nuclear matter

The semi-infinite nuclear matter is important to provide the surface tension formula for given proton fraction and temperature.
Liquid droplet approach need surface tension information.
In general $\omega = \omega(x, T)$, and ω is given by

$$\omega = \int_{-\infty}^{\infty} [\mathcal{E} - T(S_n + S_p) - \mu_n\rho_n - \mu_p\rho_p + p_o]\ dz = -\int_{-\infty}^{\infty}[p(z) - p_o]dz. \quad (26)$$

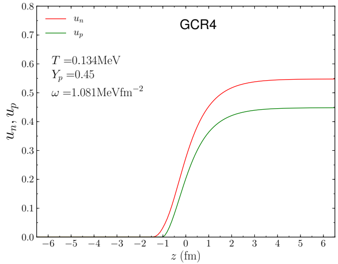


Figure: Semi-infinite nuclear matter density profile

- Liquid Drop Model provide analytic description of pasta phase

$$F = un_i f_i + \frac{\sigma(x_i) u d}{r} + 2\pi (n_i x_i e r_N)^2 u D_d(u) + (1-u) n_{no} f_o\,, \tag{27}$$

where

$$\sigma(x) = \sigma_0 \frac{2^{\alpha+1}+q}{(1-x)^{-\alpha}+q+x^{-\alpha}} \tag{28}$$

$$D_d(u) = \frac{1}{d+2}\left[\frac{2}{d-2}\left(1-\frac{1}{2} d u^{1-2/d}\right)+u\right]\,, \tag{29}$$

for each d, calculate energy density and find out lowest energy state (bubble phase $u \to (1-u)$).

- Quantum fluctuation allows the continuous dimension (Lattimer and Swesty, Nucl. Phys. A535, 331 (1991))

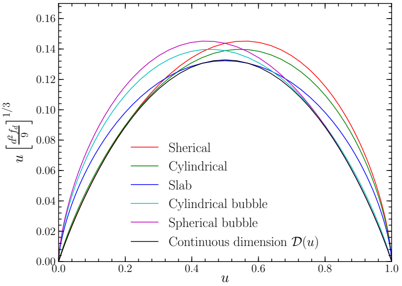
$F_S + F_C = \beta \mathcal{D} \to \mathcal{D} = u\left[\frac{d^2 D_d(u)}{9}\right]^{1/3}$.

Considering bubble phase,

$$\mathcal{D}(u) = u(1-u)\frac{(1-u) D_3^{1/3}(u) + u D_3^{1/3}(1-u)}{u^2+(1-u)^2+0.6u^2(1-u)^2}. \tag{30}$$

Pasta function as a function of u

- The continuous dimension gives the lowest energy state among all discrete dimension.



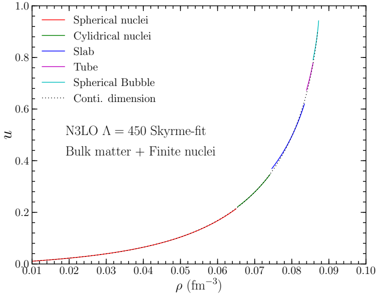


Figure: Shape functions and liquid drop model calculation with discrete dimension and shape function, Lim & Holt Physical Review C 95 (6), 065805

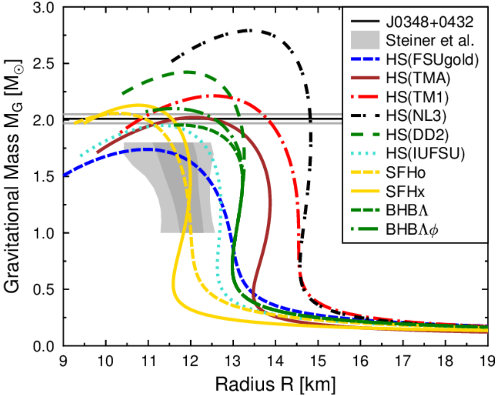


Figure: Mass and radius relation from various E.O.S. LS220, SFHo, and SFHx satisfies maximum mass of neutron star and allowed region for give neutron star's mass. (From Hempel's homepage)

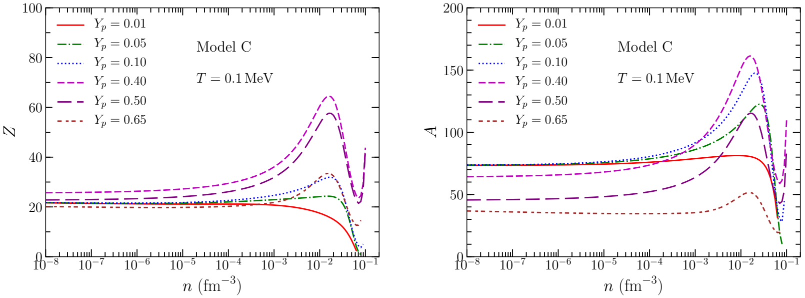


Figure: Atomic number and mass number of heavy nuclei for given conditions using model C.

Finite Range Force model

The interaction energy in finite range force is obtained by integration all over the phase space.

$$W = -\frac{8\pi^3}{\hbar^3}\int d^3r_1 \int d^3r_2\, f(r_{12}/a) \tag{31}$$
$$\times \sum_t \left[\int\int C_L f_{t_1} f_{t_2}\, d^3p_{t_1}\, d^3p_{t_2} + \int\int C_U f_{t1} f_{t_2'}\, d^3p_{t_1}\, d^3p_{t_2'}\right] ,$$

C_L and C_U are the density dependent and momentum dependent functionals between like particles and unlike particles.
ex)

$$C_{L,U} = \alpha_{L,U} + \beta_{L,U} p_{12} + \gamma_{L,U}(n_1^{1/3} + n_2^{1/3}) \tag{32}$$