

# Experimental Constraints on the EoS of Dense Matter

“Physics in *still* an Experimental Science”

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# Nuclear Equation of State

- An equation that describes the relations among the pressure, energy, temperature, density and isospin asymmetry of nuclear systems.

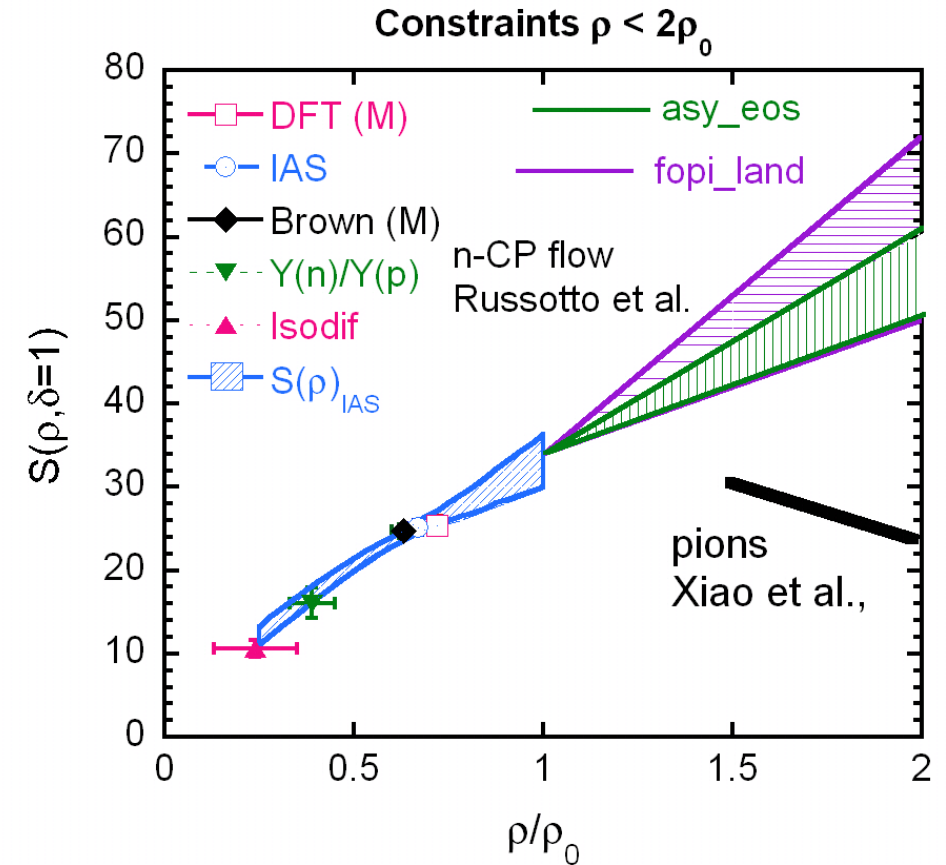
$$E / A \equiv \varepsilon(\rho, \delta, T = 0) = \underbrace{\varepsilon(\rho, 0, 0)}_{\text{EOS of symmetric nuclear matter}} + \underbrace{S(\rho, T = 0)}_{\text{Symmetry energy}} \cdot \delta^2$$

## EOS of symmetric nuclear matter

- Experimentally constrained in  $0.5\rho_0 > \rho > 4.5\rho_0$ 
  - Collective flows
  - Isoscalar collective vibrations
  - Kaon production in HIC
- Behavior at  $\rho = 2\rho_0$  influences neutron star deformability and its mass radius relationship

## Symmetry energy

- Constrained at  $0.25 < \rho/\rho_0 < 0.75$
- Constrains crust/core transition density
- Relatively unconstrained at  $\rho > \rho_0$
- Behavior at  $\rho = 2\rho_0$  influences neutron star deformability and its mass radius relationship



# What density do nuclear masses constrain?

- **Crossover technique:**

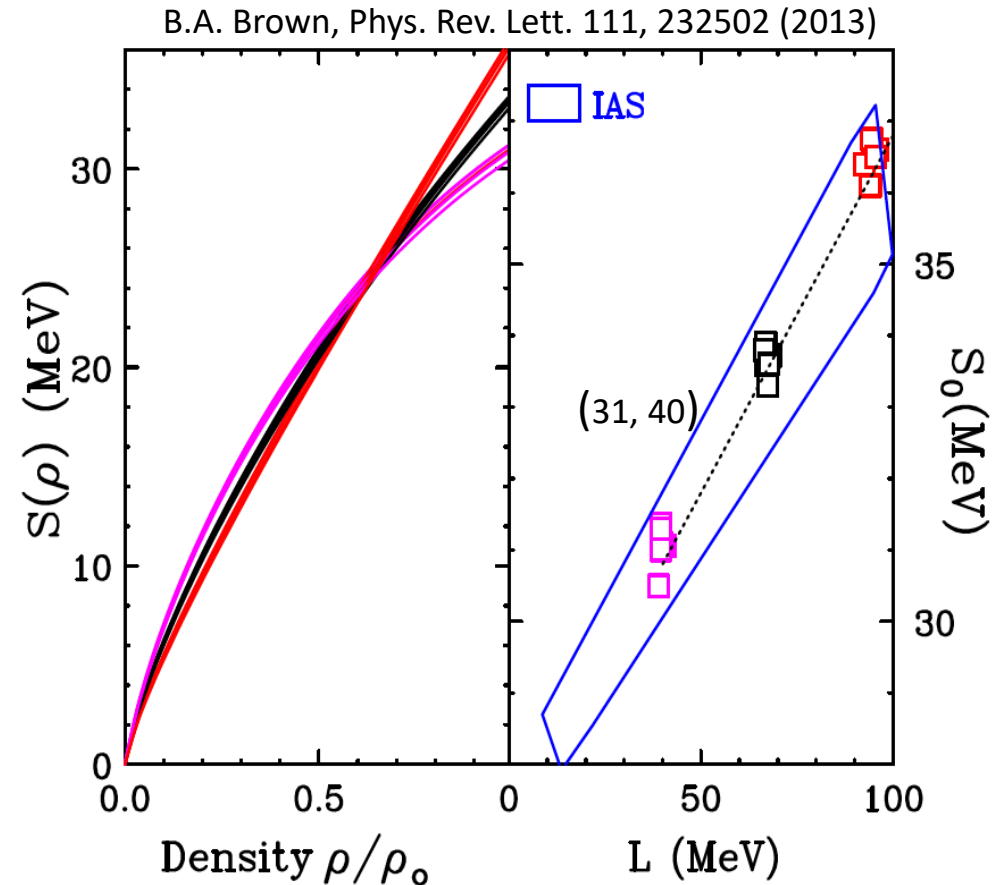
- Alex Brown fit the masses of doubly closed shell nuclei, while setting different values neutron skin thicknesses of  $\Delta R_{np}=0.16, 0.20$  and  $0.24$  fm. All Brown fits provide  $S(\rho)=24.8\pm 0.7$  MeV at  $\rho/\rho_0=0.63\pm 0.03$  as shown at left.

- **Slope technique**

- The values for  $S_0$  and  $L$  lie on a line in the  $S_0$  and  $L$  plane along which  $S(0.63\rho_0)$  remains constant. This line lies perpendicular to the gradient of  $S(0.63\rho_0)$  in the the  $S_0$  and  $L$  plane.
- The slope,  $M$ , of the line and the form of the function is sufficient to determine  $\rho/\rho_0=0.63\pm 0.03$  and  $S(\rho)=24.8 \pm 0.7$  MeV.

- **Pearson correlation technique**

- Standard statistical analysis technique that can correlate data or assumption with conclusion



$$M(\rho_s) = -\left(\frac{\partial S(\rho_s)}{\partial L}\right) / \left(\frac{\partial S(\rho_s)}{\partial S_0}\right);$$

**M depends monotonically on  $\rho_s$**

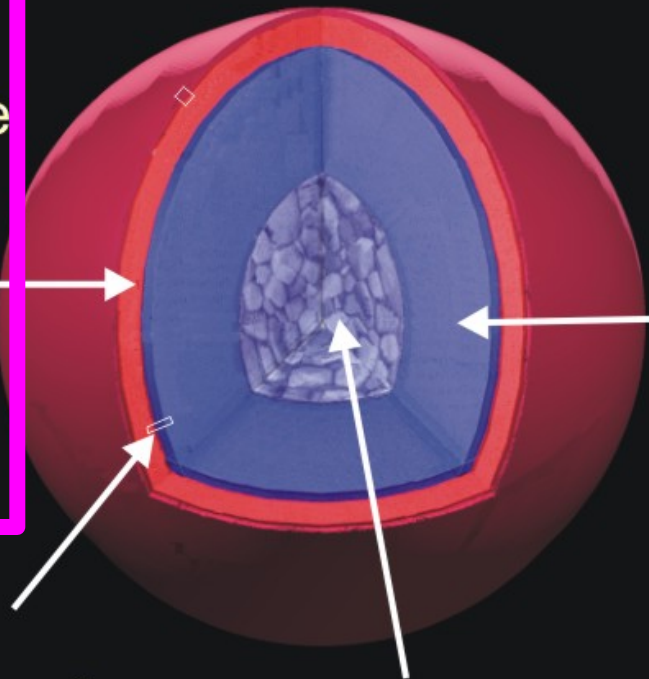
# Influence of $S(\rho)$ on a neutron star

$S(\rho)$ : = density dep. of symmetry energy

$\rho < \rho_0$

Inner crust:  
Neutron gas in coexistence with "Coulomb lattice" of nuclei.  $S(\rho)$  governs thickness of crust and the observed frequencies in star quakes.

Inner boundary of inner crust: Cylindrical and plate-like nuclear "pasta"



Inner core:

$\rho > \rho_0$

Outer core:  
Composed of neutron-rich nuclear matter.  $S(\rho)$  governs stellar radii, and moments of inertia.

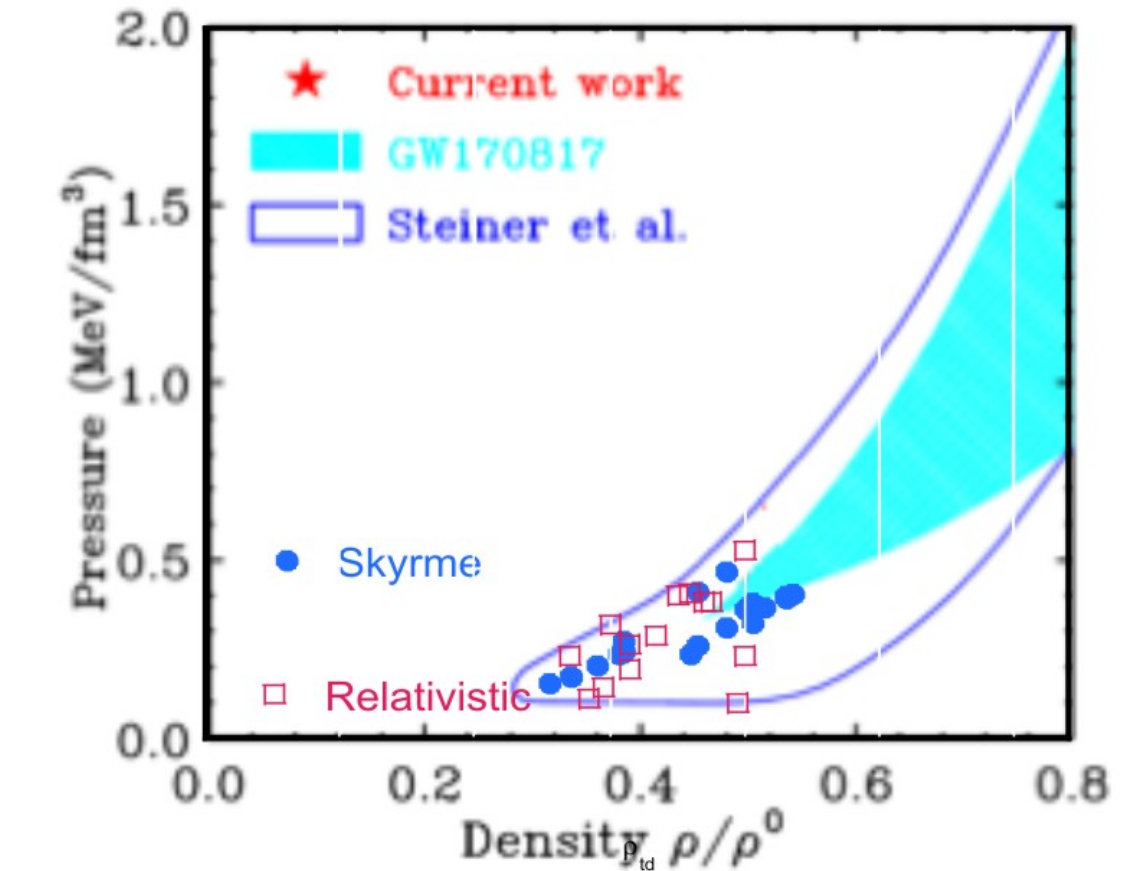
At the crust-core boundary uniform matter becomes adiabatically unstable to break into isolated liquid structures surrounded by neutron-rich gas.

# What a constraint on masses provides.

- Ducoin et al., calculated the crust-core transition density, pressure and proton fraction for 21 Skyrme and 14 relativistic models. Ducoin et al, C 83, 045810 (2011)
- Observed that the transition density, pressure and proton fraction are strongly correlated with the first three terms of a Taylor expansion of the symmetry energy functional about  $\rho_{01}=0.10 \text{ fm}^{-3}$ .

$$S(\rho) = S_{0.1} + \frac{L_{0.1}}{3} \left( \frac{\rho}{\rho_{0.1}} - 1 \right) + \frac{K_{\text{sym},0.1}}{18} \left( \frac{\rho}{\rho_{0.1}} - 1 \right)^2 + \dots$$

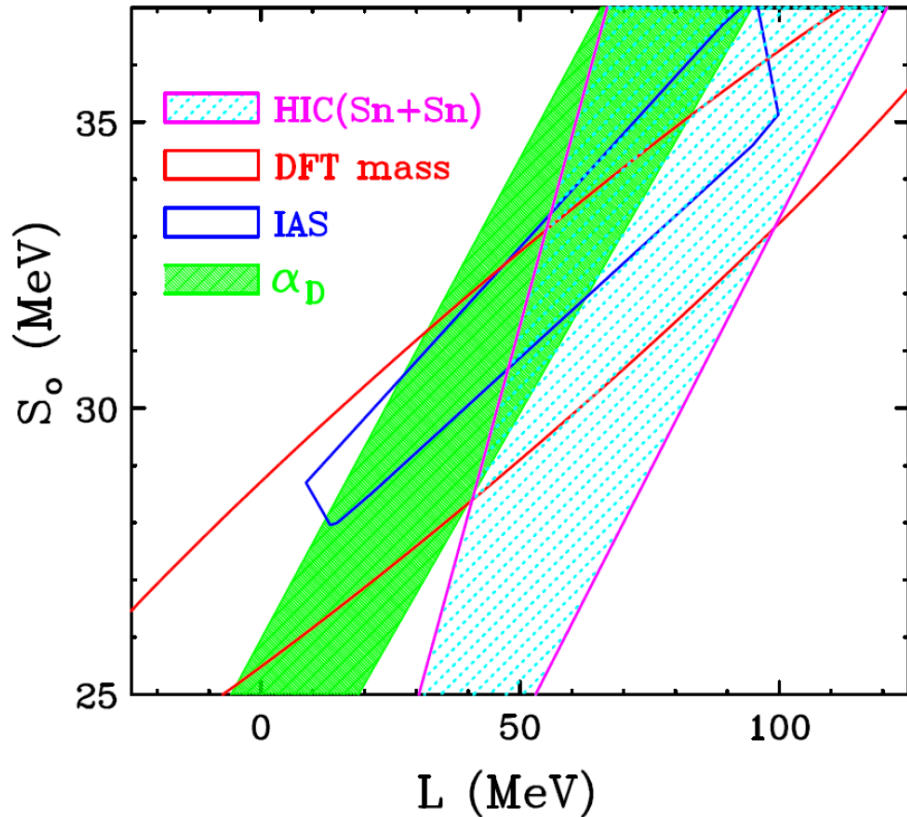
- They showed that these differences in the predictions for  $P_{\text{cc}}$  and  $\rho_{\text{cc}}$  stem from differences in  $L_{01}$  and  $K_{01}$



$$L_{01} = 2\rho_{01} \left. \frac{\partial S}{\partial \rho} \right|_{\rho_{01}} \quad K_{01} = (2\rho_{01})^2 \left. \frac{\partial^2 S}{\partial \rho^2} \right|_{\rho_{01}}$$

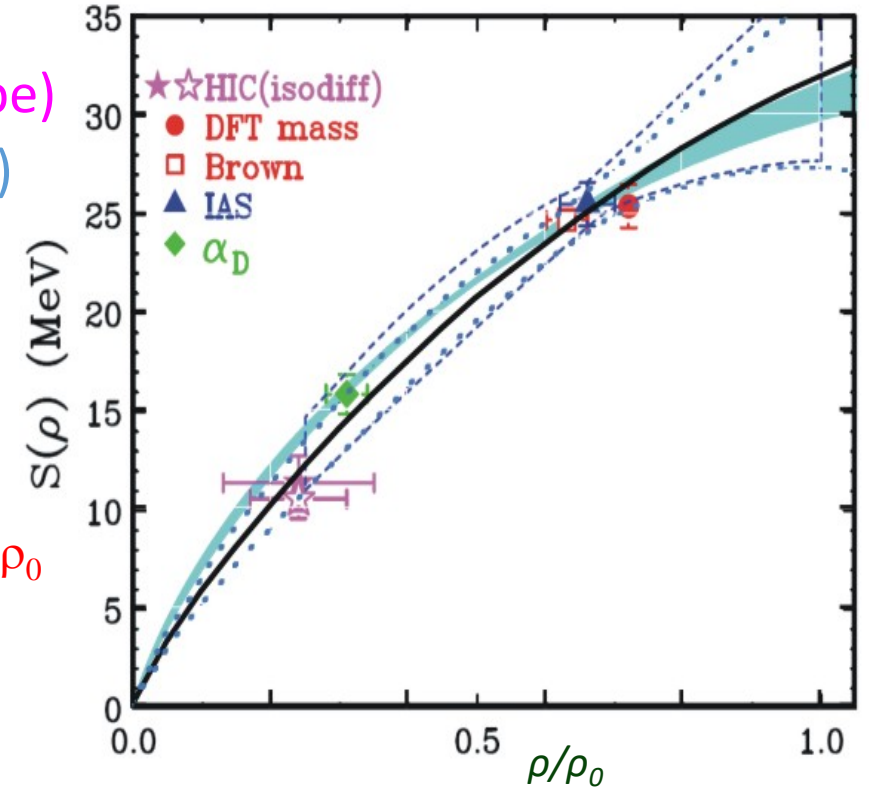
# From $S_0 - L$ correlations to $S(\rho)$

C. J. Horowitz *et al.*, J. Phys. G: Nucl. Part. Phys. **41** (2014) 093001



- HIC (cross-over, slope)
- IAS (Pearson, slope)
- $\alpha_D$  (Pearson)
- Dft (slope)\_

← The contours are the constraints extrapolated to  $\rho_0$

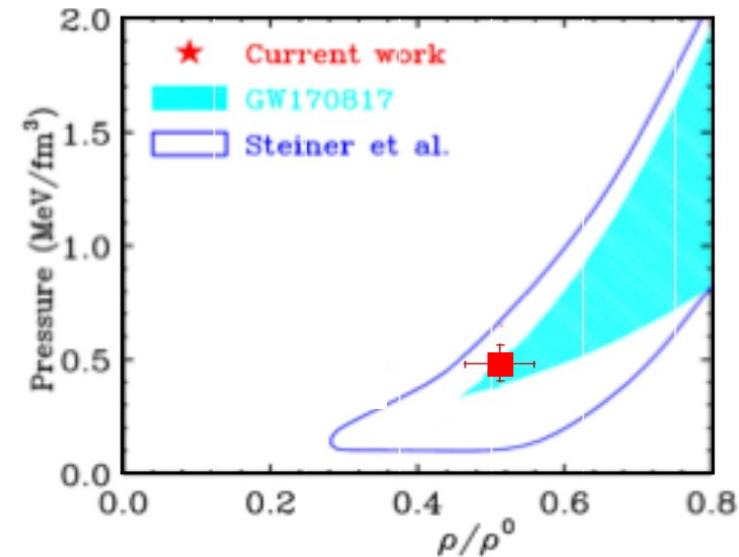
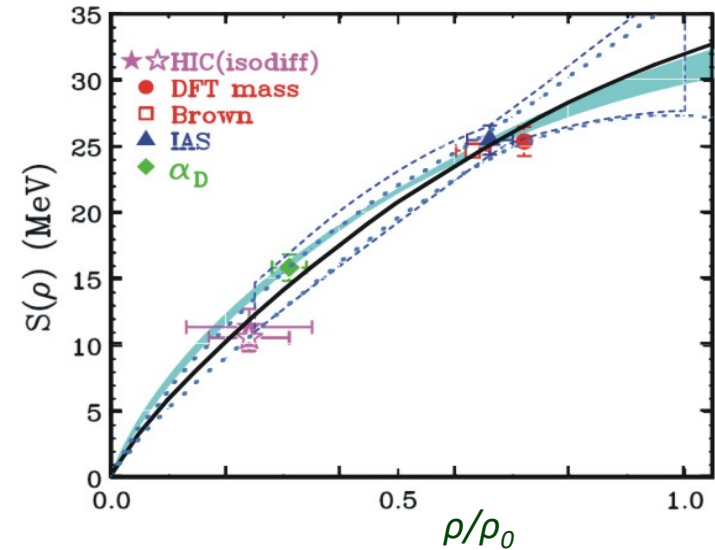


- Note the consistency of the results.
- If you want to measure  $S(\rho)$  you need a observable that probes  $\rho$  at the density you need it.

# Constraints on the crust-core transition density

- Constraints on  $S(\rho_s)$  from structure and low energy reactions are shown by the points in the top figure. The solid line shows a constrained fit to these data.
- Following Ducoin et al., PRC, 83 (2011) 045810, this fit enables a good estimate of the NS crust-core transition density and pressure.
- The red point in the lower figure shows the cross-core transition density and pressure obtained from this fit. It implies that the crust core transition occurs at  $\approx \rho_0/2$ .
- Additional or more precise constraints on  $S(\rho_s)$  will help. A tighter constraint on  $L$  would be especially helpful.

$$S(\rho) = S_0 + \frac{L}{3} \left( \frac{\rho}{\rho_0} - 1 \right) + \frac{K_{sym}}{18} \left( \frac{\rho}{\rho_0} - 1 \right)^2 + \dots$$



Lynch and Tsang, arXiv:1805.10757 5.10757 & 2019

# What do we know to constrain EoS of symmetric nucleonic matter at $\rho > \rho_0$ ?

1. Transport Theory: e.g. BUU Time dependent Thomas Fermi
2. Density dependent momentum independent mean field potentials (Direct term HF).

$$U_n(\rho_n, \rho_p) \quad U_p(\rho_n, \rho_p)$$

- Must bind nuclei properly at correct density and nuclear radius.
- Extrapolate reasonably to high density

3. Density dependent momentum dependent mean field potentials (Exchange term. Finite range)

$$U_{n,k}(\rho_n, \rho_p, p_n^2, f_n(p_n), f_p(p_n)) \quad U_{p,k}(\rho_n, \rho_p, p_n^2, f_n(p_n), f_p(p_n))$$

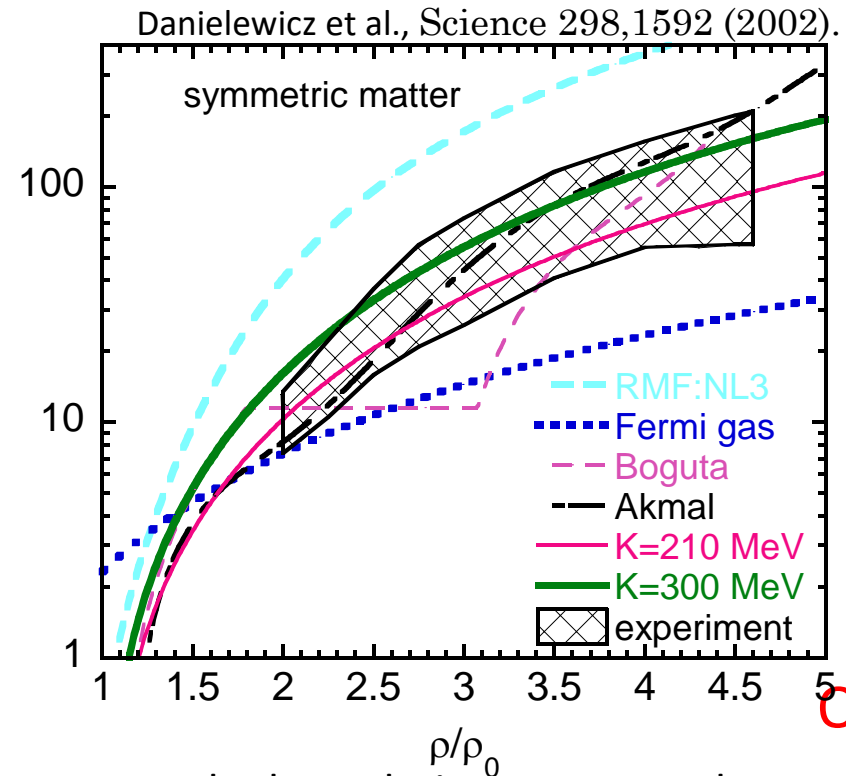
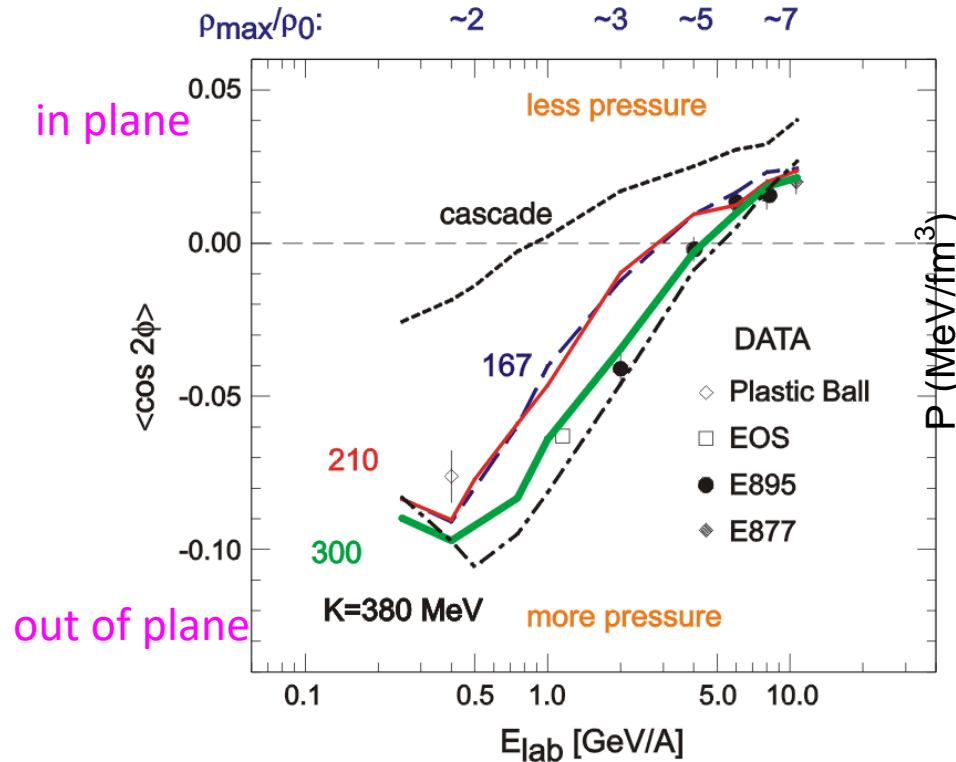
- Governs the thermal and non-equilibrium behavior and momentum dependence of optical potentials.

4. Cross sections induced by the residual interactions+ Pauli blocking

- All of these quantities must be fixed by Experimental Data!



# Determination of symmetric matter EOS from nucleus-nucleus collisions

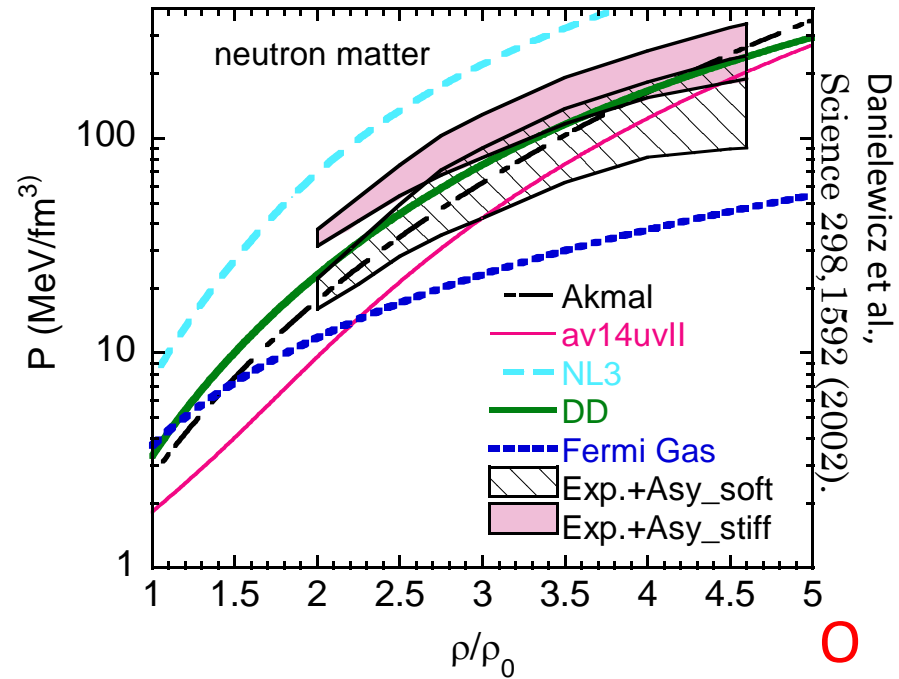
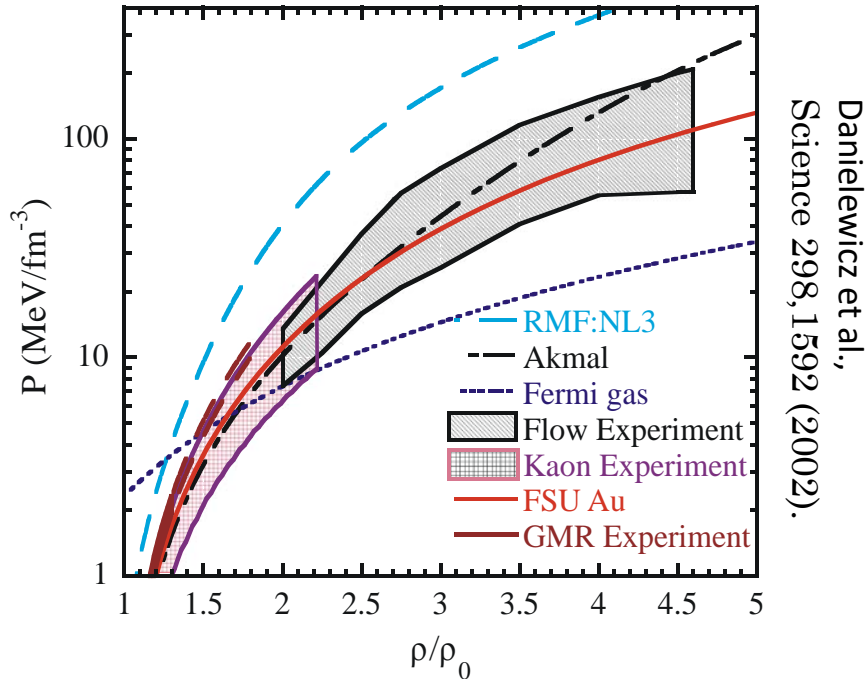


- The curves labeled by  $K_{nm}$  represent calculations with parameterized Skyrme mean fields
  - They are adjusted to find the pressure that replicates the observed transverse flow.

- The boundaries represent the range of pressures obtained for the mean fields that reproduce the data.
- They also reflect the uncertainties from the effective mass and in-medium cross sections.

# Constraints from collective flow on EOS at $\rho > 2 \rho_0$ .

$$E/A(\rho, \delta) = E/A(\rho, 0) + \delta^2 \cdot S(\rho) \quad \delta = (\rho_n - \rho_p) / (\rho_n + \rho_p) = (N-Z)/A \approx 1$$



- Note: analysis required additional constraints on  $m^*$  and  $\sigma_{NN}$ .
- Flow confirms the softening of the EOS at high density.
- Constraints from kaon production are consistent with the flow constraints and bridge gap to GMR constraints.

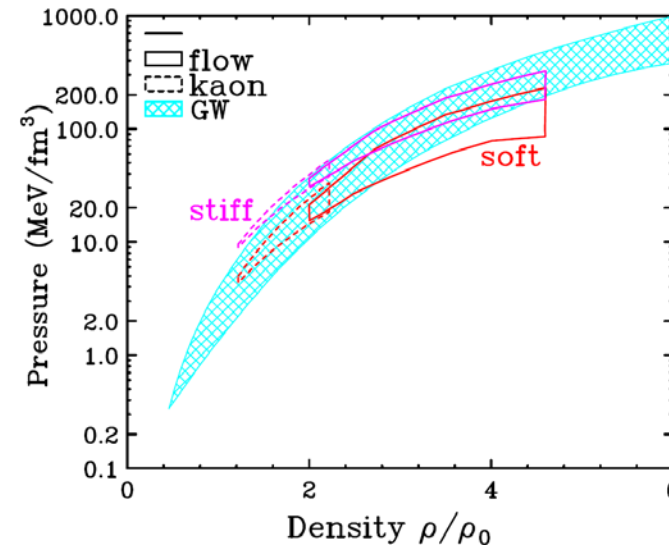
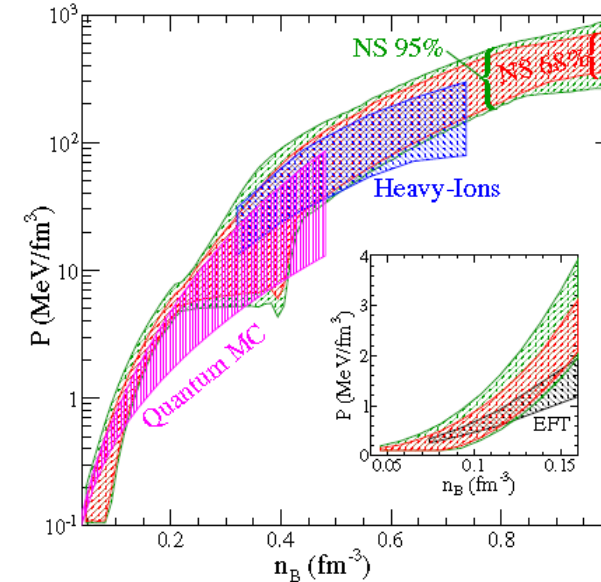
- The symmetry energy dominates the uncertainty in the n-matter EOS.
- Both laboratory and astronomical constraints on the density dependence of the symmetry energy are urgently needed.

# How does this match to various neutron star observations?

$$EoS \Leftrightarrow \varepsilon(\rho, \delta) \approx \varepsilon(\rho, 0) + S(\rho)\delta^2; \delta = (\rho_n - \rho_p)/\rho$$

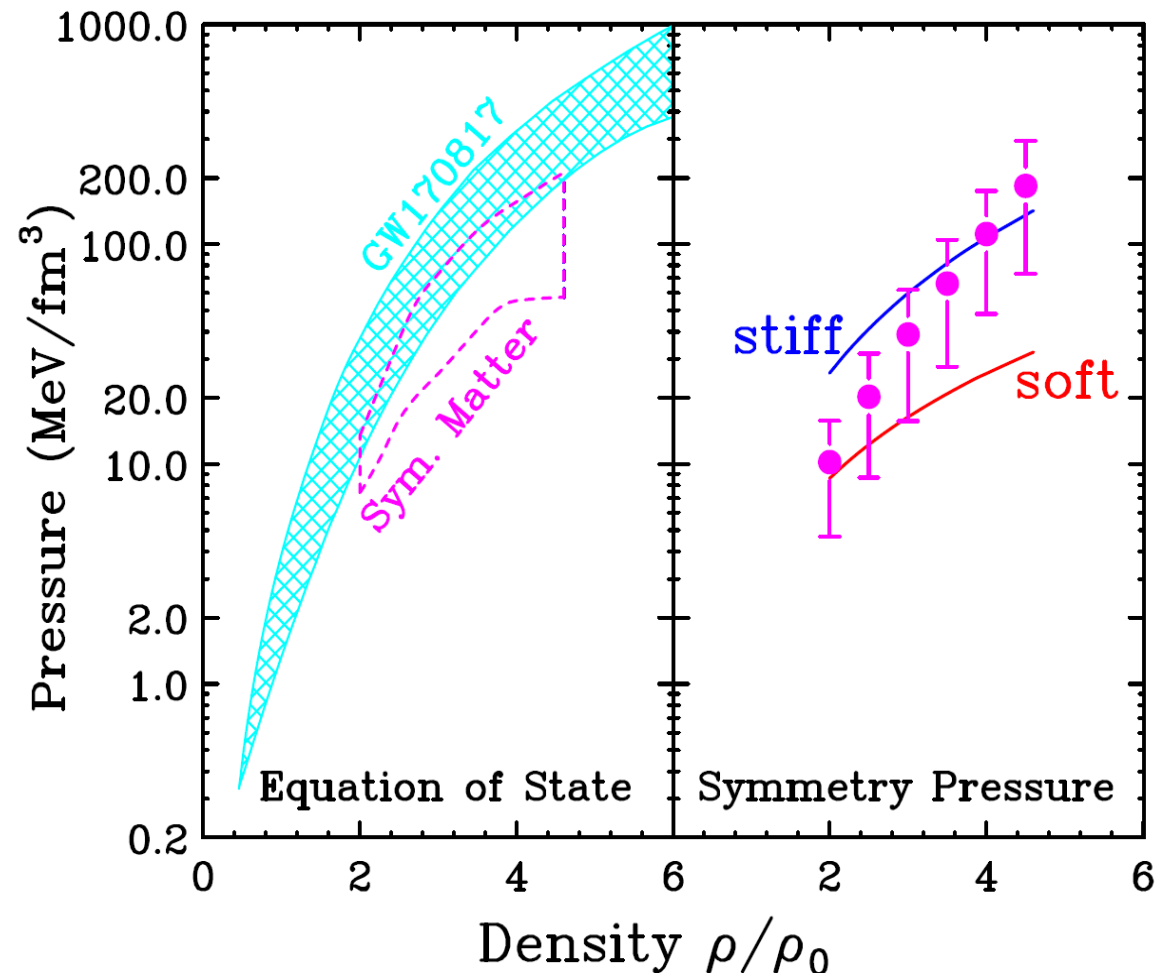
- Symmetric matter  $\varepsilon(\rho, 0)$  at  $\rho > \rho_0$ 
  - Flow with EoS TPC at LBL (2002)
  - GSI Kaon constraint (2009)
  - GSI Flow constraint (2016)
  - Essential to constrain
    - local isoscalar mean field:  $\varepsilon(\rho, 0)$
    - Non-local isoscalar mean field:  $m_s^*(\rho)$
- Extrapolation to neutron matter:
  - Using parameterization by Prakash & Latimer
 
$$S(\rho)_{stiff} = 12.7 \times (\rho/\rho_0)^{2/3} + 38 \times (\rho/\rho_0)^2 / (1 + \rho/\rho_0)$$

$$S(\rho)_{soft} = 12.7 \times (\rho/\rho_0)^{2/3} + 19 \times (\rho/\rho_0)^{1/2}$$
  - This a reasonable bracketing assumption.
  - Matches NS radii from x-ray bursters
  - Matches GW17081 constraints amazingly well.



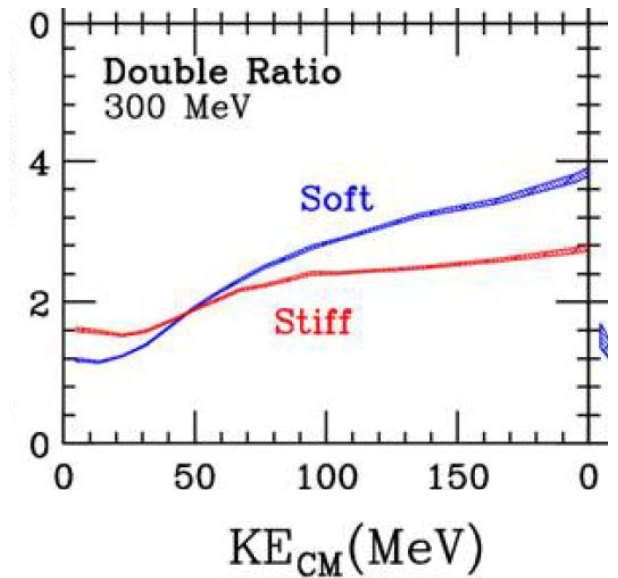
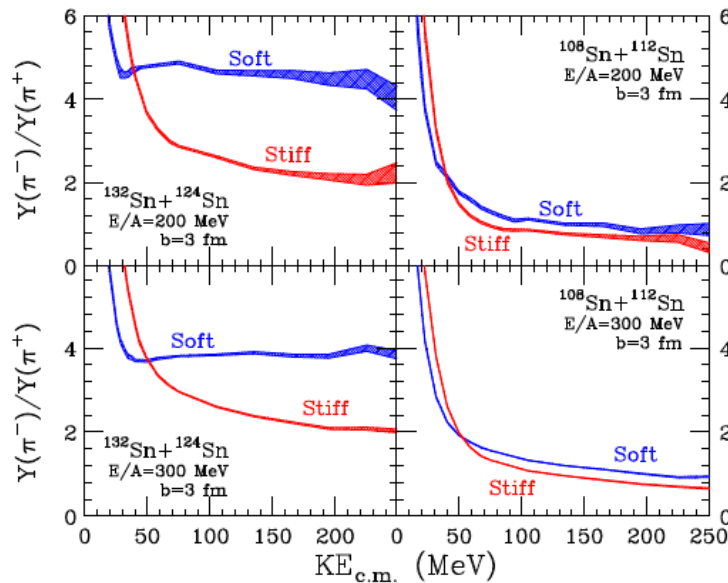
# What have we learned about the symmetry energy or symmetry pressure?

- GW neutron matter pressure exceeds symmetric matter pressure.
- Most probable symmetry pressure can be obtained by subtraction.
- “Error bars” are 50% confidence limits.
- At 90% confidence level a negative symmetry pressure, corresponding to decreasing symmetry energy cannot be precluded.
- Need laboratory constraint on symmetry energy



The Symmetry energy is key to determining where the phase transitions occur within neutron stars (crust-core, delta, strange or quark matter).

- No astronomical observation *currently constrains the symmetry energy in the NS core.*
- Goal of experiments is to probe *Symmetry energy in HIC at energies where densities approach  $2\rho_0$* 
  - For  $^{132}\text{Sn} + ^{124}\text{Sn}$ , symmetry energy repels *n's* from and *p's* to dense neutron rich region.
  - For  $^{108}\text{Xe} + ^{112}\text{Sn}$  symmetry plays almost no role  $\rightarrow$  both reactions are important.
  - 1<sup>st</sup> observable:  $Y(n, KE)/Y(p, KE)$  : (radial, transverse, elliptical flows): sensitive to  $S(\rho)$ ,  $m_v^*(\rho)$
  - Nucleonic probes improve with incident energy.
  - Gave reasonable positive pressure in Au+Au collisions
  - See talk by A. Le Fevre
- 2<sup>nd</sup> obs.:  $Y(\pi^-, KE)/Y(\pi^+, KE)$ 
  - Probe more sensitive at lower incident energy.
  - Potentially large effects
  - No published data



Proposed pion measurements of ratios and double ratios  
 Double ratio depends only on symmetry energy

# SπRIT 2016 Campaign

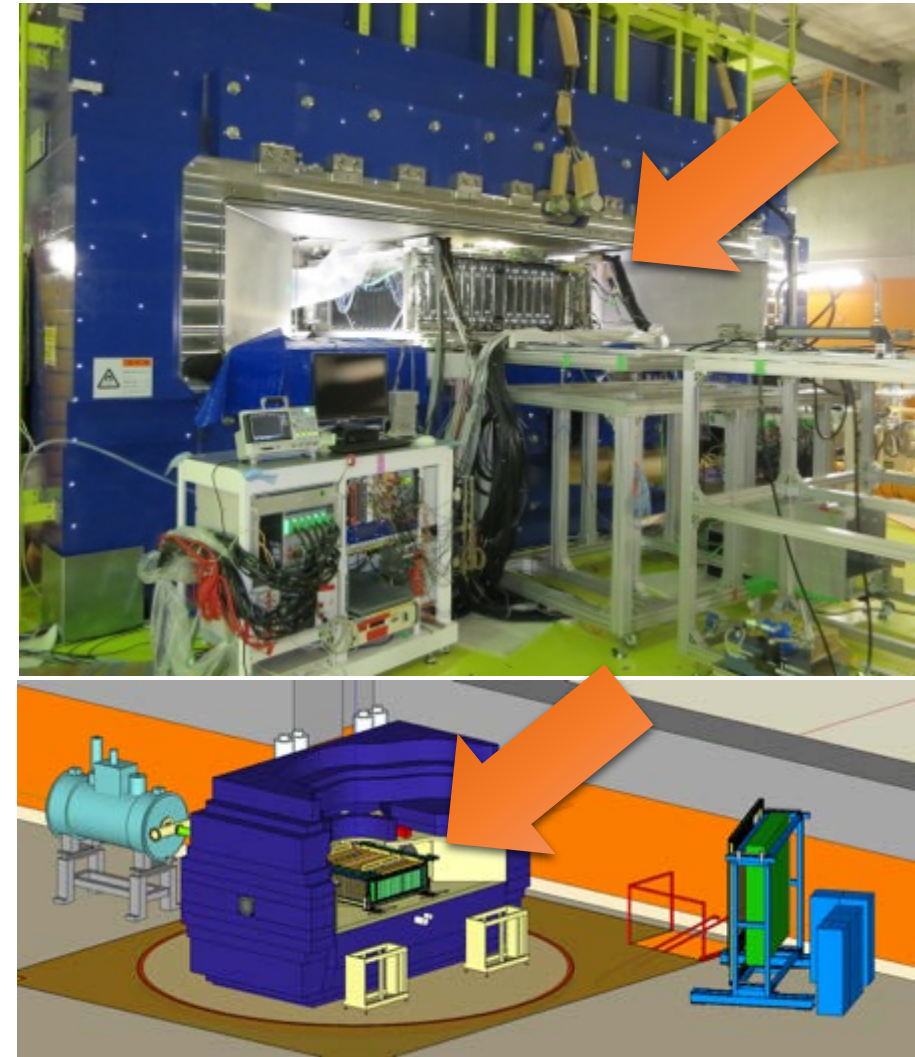


- SAMURAI Pion Reconstruction and Ion Tracker Time Projection Chamber was installed within the SAMURAI superconducting magnet at RIBF/RIKEN.
- NueLAND was located at  $\sim 30^\circ$

▪  $E_{\text{beam}} = 270 \text{ A MeV}$

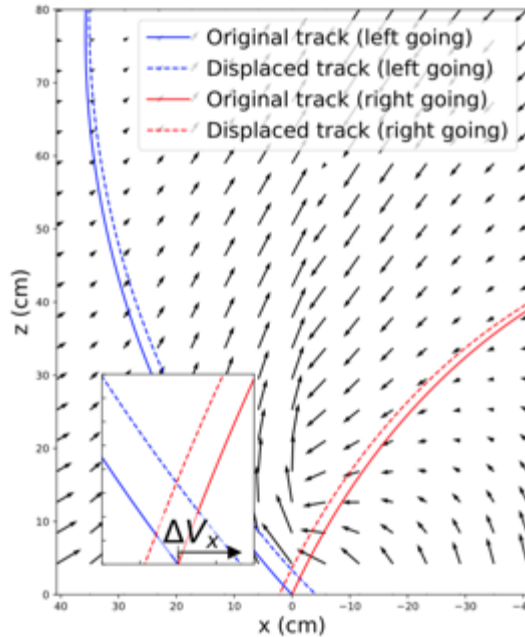
System	$\delta = (N-Z)/A$	#events
$^{132}\text{Sn} + ^{124}\text{Sn}$	0.22	$3.8 \times 10^6$
$^{108}\text{Sn} + ^{112}\text{Sn}$	0.09	$2.4 \times 10^6$
$^{112}\text{Sn} + ^{124}\text{Sn}$	0.15	$1.8 \times 10^6$
$^{124}\text{Sn} + ^{112}\text{Sn}$	0.15	$2.5 \times 10^5$

Cocktail ( $Z=1\sim 3$ ) with  $E=100, 300 \text{ MeV}$



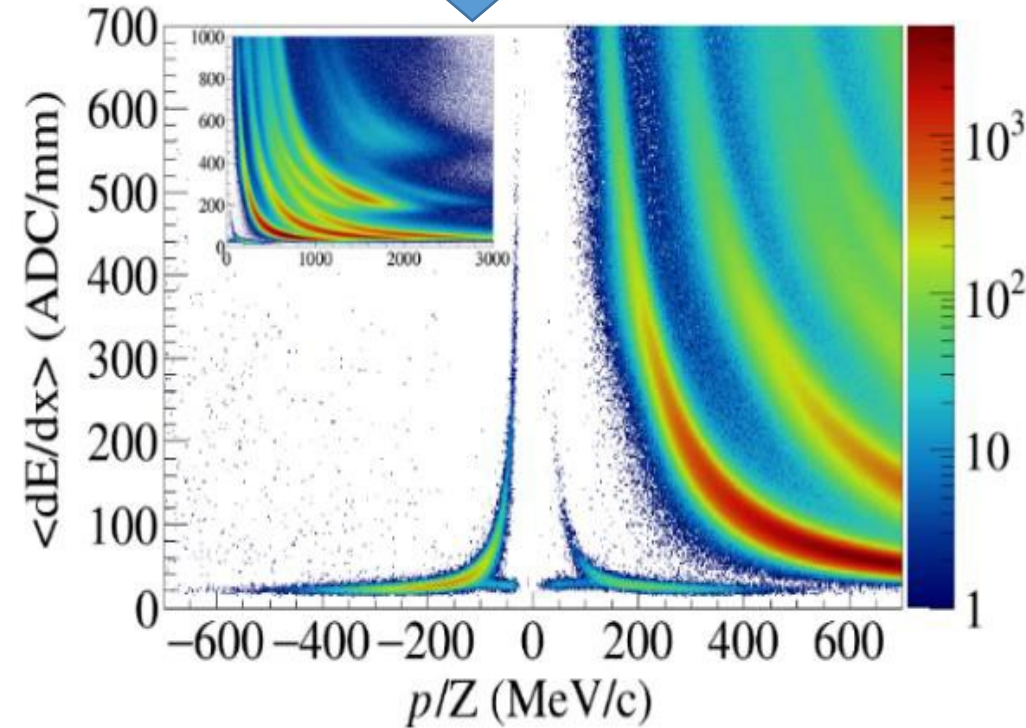
# Technical results from previous experiment

- Eight technical papers published. One patent.
- Two more under review and two more in preparation.
- Illustrative examples:
  1. Algorithm for space charge corrections in a TPC.
  2. Dynamic range extended by factor 5, effectively 4000 to 1 maximum signal to noise.
  3. Efficiency by embedding test tracks in real data.



Uncorrected track misses vertex

PID from pions beyond Lithium



# Scientific Results: 4 papers in preparation



- Seven groups of transport theorists have working together for three years to make transport theory more quantitative.
- Published 3 theoretical articles on Transport model evaluation project. One focus was on transport model comparisons with analytically known solutions.
- First focus was on the total pion ratio  $Y(\pi^-)/Y(\pi^+)$ .
- Error in data is size of points and mostly systematic: 4% for multiplicity and 2% for ratios.
- Theory underpredicts data for many models.
- Normalizing pion production mechanism on Au+Au at  $E/A=400$  MeV may have been problematic. → Normalize on Xe+Sn reactions and check the energy dependence carefully.

1<sup>st</sup> paper

G. Jhang et al

Unpublished work.  
Please do not distribute!

Effect of stiff of  $S(\rho)$   
Is known to be small.  
Our uncertainties are  
small enough, but our  
focus is on spectra





# Analysis of mirror systems

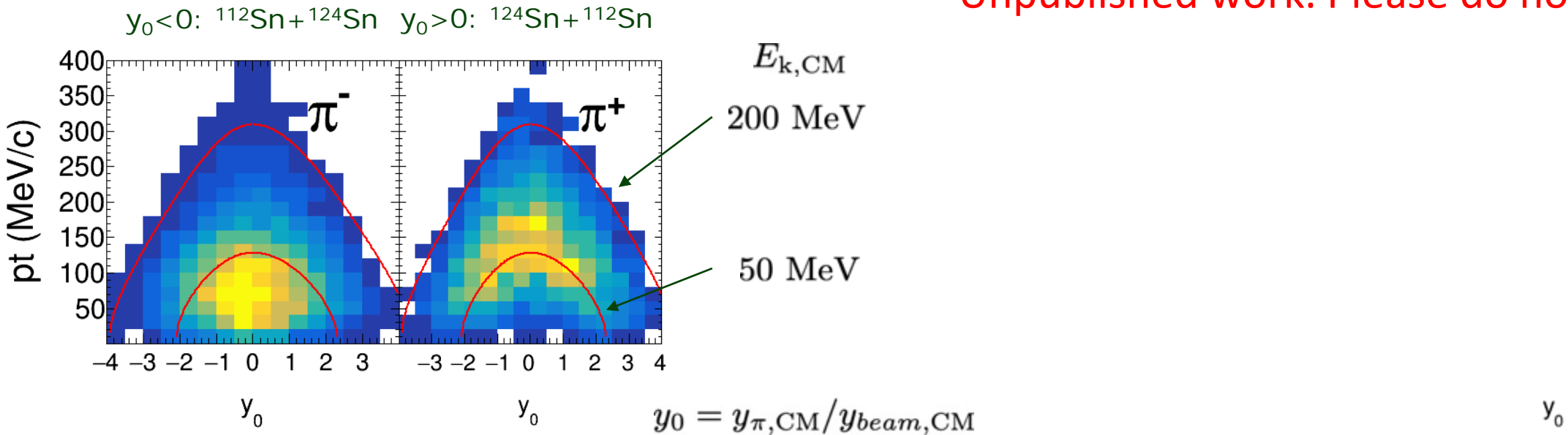
from J. Barney



2<sup>nd</sup> paper

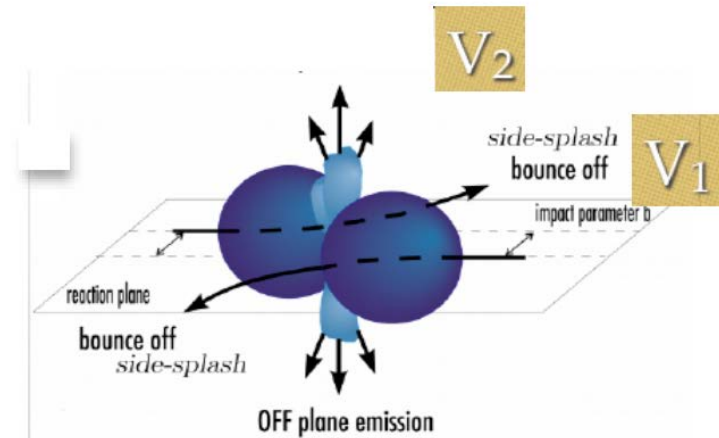
- $p_T$ - $y_0$  plot for  $\pi^-$  shows Coulomb attraction towards the collision center resulting in low emission energy. Opposite phenomenon is observed for  $\pi^+$  by Coulomb repulsion.
- $\pi^-/\pi^+$  ratio shows symmetric in forward and backward rapidity regions as expected.

Unpublished work. Please do not distribute!



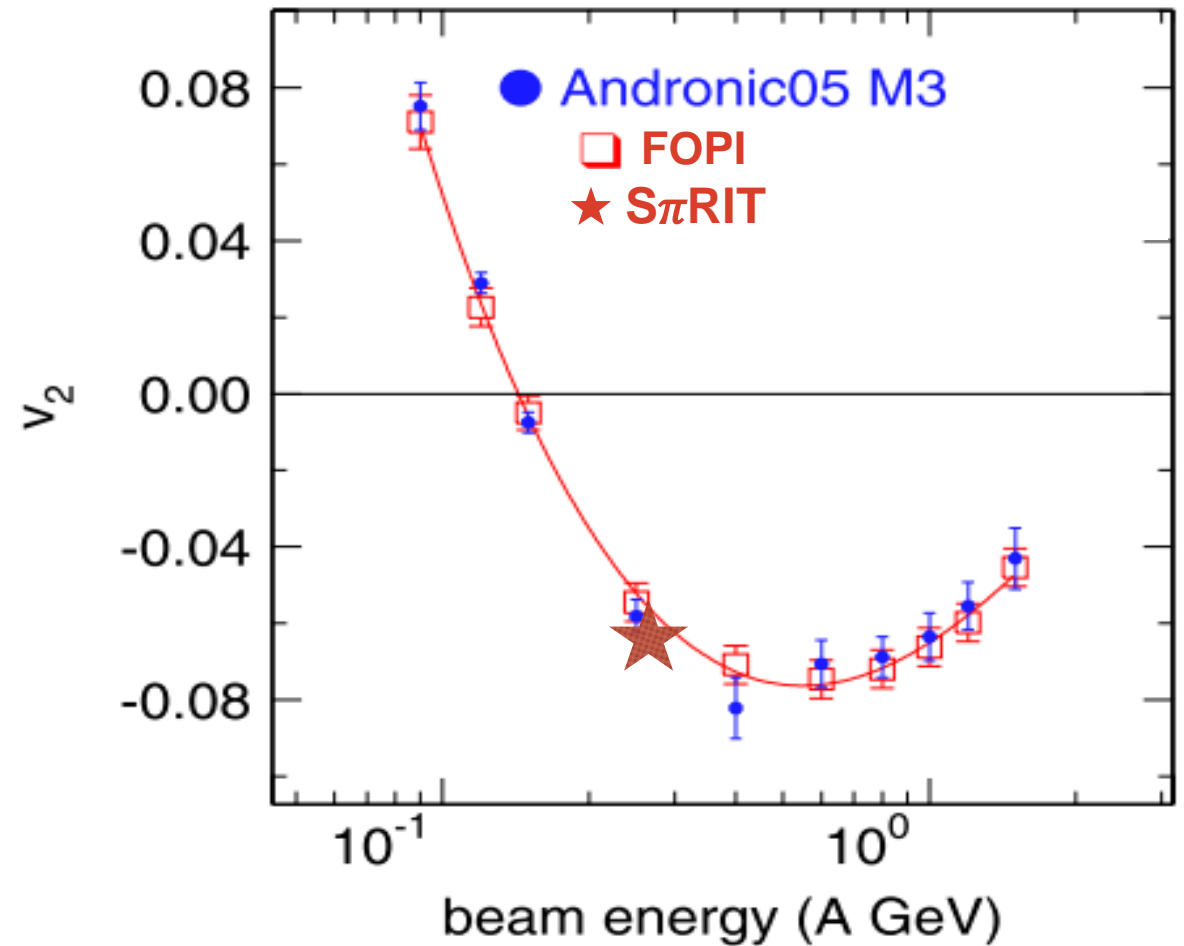
# Isospin dependence of Elliptical Flow

from M. Kurata-Nishimura



- Comparison of  $^{132}\text{Sn}+^{124}\text{Sn}$  and  $^{108}\text{Sn}+^{112}\text{Sn}$  collisions

- Comparison of  $^{132}\text{Sn}+^{124}\text{Sn}$  to systematics



# Summary

- Experiment can be useful if the observables are carefully chosen so that the unknowns are constrained.
  - Example: crust-core transition density, you need to go beyond masses!!
- You need to know what density your observable probes!
  - Please do quantitative sensitivity tests.
- For HIC you need to constrain all unknowns in your theory.
  - Density dependent, momentum independent mean field potential
  - Density dependent momentum dependent mean field potential
  - In medium cross sections.
- Until that is done, all constraints will be preliminary.
- Right now the pressures in neutron stars are known to about factor of four.
- If you can constraint the pressure to less than a factor of two, it is progress,