Constraints on the Symmetry Energy from the Nuclear Dipole Response

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Agenda

- **The Giessen EDF approach**

- **Infinite matter and nuclear binding energies**

- **Pygmy dipole (PDR) modes in Sn-isotopes**

- **Dipole response and capture cross sections for N≈50 isotones**

- **Cosmo-chronometry: PDR modes in ^{206}Pb**

- **Photoabsorption cross sections and symmetry energy**

**Review on the Giessen Nuclear Structure Approach:
H.L. and N. Tsoneva, Eur.Phys.J. A55 (2019) 238**

The Giessen EDF Approach
- GiEDF -

A Primer on Density Functional Theory

Hamiltonian Density:

$$E\left(\Psi^{+},\Psi\right)=\int\frac{d^{3}k}{(2\pi)^{3}}\frac{\hbar^{2}}{2m_{q}}\left|\nabla\Psi_{k}\right|^{2}+\frac{1}{4}\int\frac{d^{3}k}{(2\pi)^{3}}\int\frac{d^{3}k'}{(2\pi)^{3}}\Psi_{k}^{+}\Psi_{k'}^{+}V_{NN}^{*}\left(k,k'\right)\Psi_{k}\Psi_{k'}$$

Nuclear In-Medium Interaction

$$\mathcal{V}_{NN}^{*}=\left(1-V_{NN}G_{NN}(\rho)\right)^{-1}V_{NN}=\mathcal{V}_{NN}^{*}\left(\Psi^{+},\Psi\right)$$

Kohn's Lemma :

$$E\left(\Psi^{+},\Psi\right)\mapsto E\left(\rho,\tau\right)\ \ \text{where}\ \rho=\Psi^{+}\Psi\ ,\ \tau=\left|\nabla\Psi\right|^{2}$$

Dirac-Brueckner Approach to DFT:
H. Lenske, Lect.Notes Phys. 641 (2004) 147

The Giessen EDF Approach

Phys.Atom.Nucl. 79 (2016) 885, EPJ A55 (2019) 238

$$E(\rho, \kappa) \approx E(\rho_0, \kappa_0) + \sum_{q=p,n} \left(\left(T_q + U_q(\rho_0) \right) \delta\rho_q + \Delta_q \delta\kappa_q \right) + \sum_{q,q'=p,n} f_{qq'}(\rho_0) \delta\rho_q \delta\rho_{q'} + \dots$$

$$\delta\rho_q \sim \varphi_k^\dagger \varphi_n - \left\langle \varphi_k^\dagger \varphi_n \right\rangle \; ; \; \delta\kappa_q \sim \varphi_k \varphi_n - \left\langle \varphi_k \varphi_n \right\rangle \; \& \; h.c. \; ; \; q=p,n$$
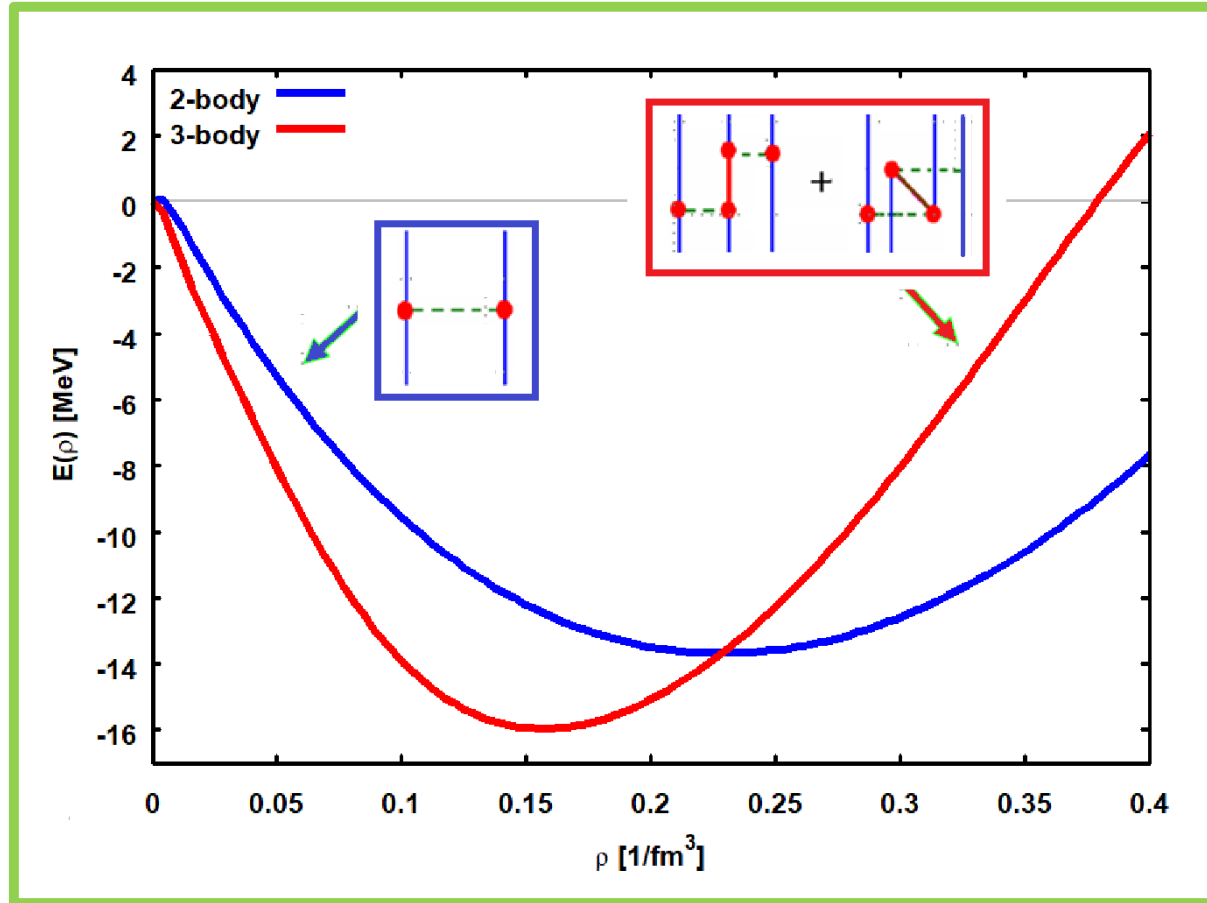
First variation - Single Particle Self-Energy:

$$U_q = \frac{\delta}{\delta\rho_q} \frac{1}{2} \langle V \rangle = \sum_{q'} V_{qq'}(\rho)\rho_{q'} + \frac{1}{2}\sum_{q'q''} \rho_{q'}\rho_{q''} \frac{\delta}{\delta\rho_q} V_{q'q''}(\rho)$$

HF Mean-Field
G-Matrix plus 3-body interaction

Rearrangement Self-Energy
Static polarization
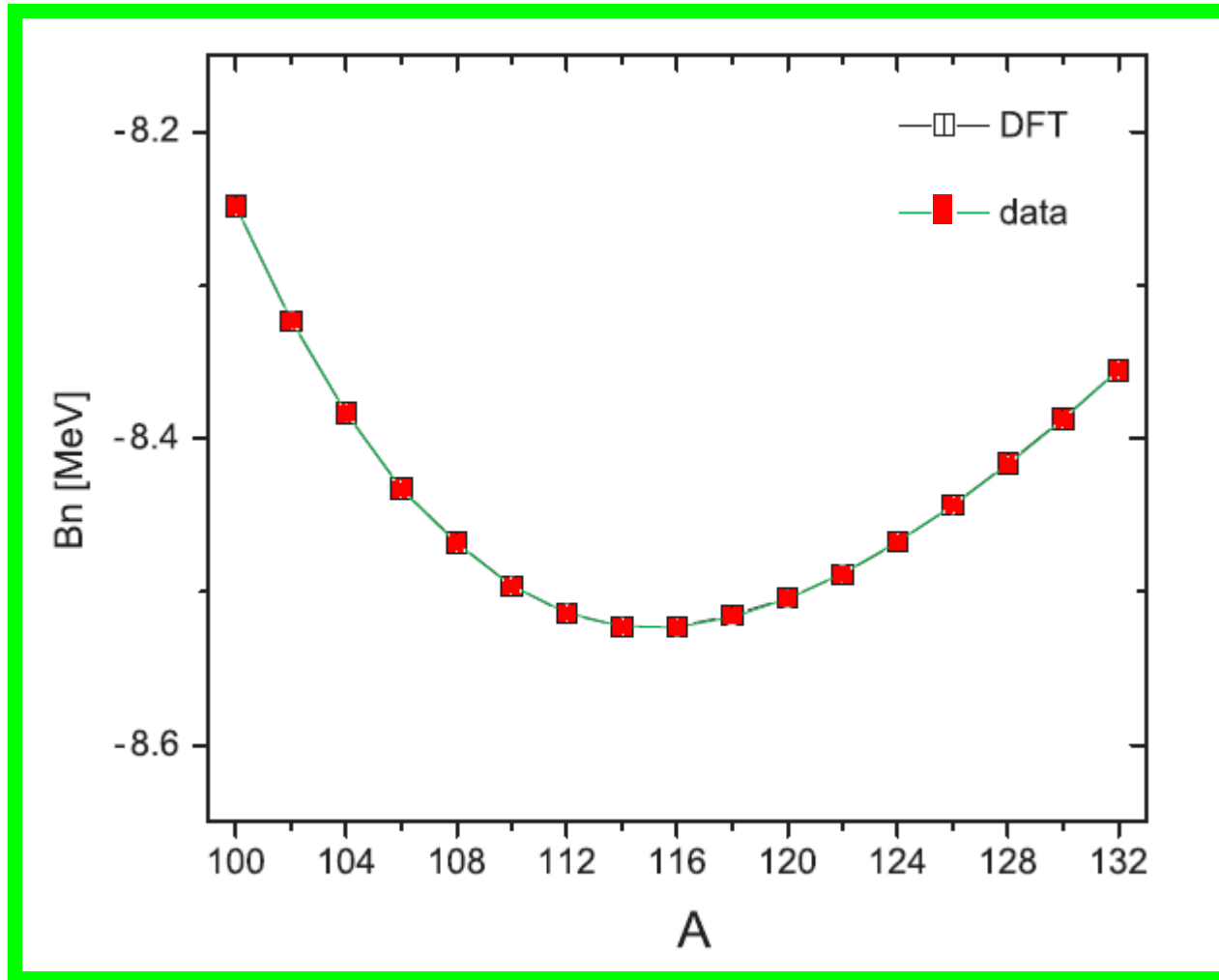
GiEDF Interaction and Nuclear Matter Binding Energy
Role of 2-body and 3-body Interactions (Urbana IX)

ρ_{sat} : 0.16 [1/fm³]
E/A : -15.8 [MeV]
K_∞ : 233.6 [MeV]

GiEDF HFB-Binding Energies of Sn-Isotopes

N. Tsoneva, H.L., Phys.Rev. C78 (2008) 064314

Nuclear Excitations

Residual Two-Quasiparticle Interactions in Fermi-Liquid Theory

Second Variation of the EDF:
(q,q'=isospin and spin)

$$f_{qq'}(\rho) = V_{qq'}(\rho) + 2\sum_{q''}\rho_{q''}\frac{\delta}{\delta\rho_q}V_{q'q''}(\rho) + \frac{1}{2}\sum_{k'k''}\rho_{k'}\rho_{k''}\frac{\delta^2}{\delta\rho_q\delta\rho_{q'}}V_{k'k''}(\rho)$$

ph-term   **polarization contributions**

$$f(\mathbf{k},\mathbf{k}') = F(\mathbf{k},\mathbf{k}') + F'(\mathbf{k},\mathbf{k}')\boldsymbol{\tau}\cdot\boldsymbol{\tau}' + [G(\mathbf{k},\mathbf{k}') + G'(\mathbf{k},\mathbf{k}')\boldsymbol{\tau}\cdot\boldsymbol{\tau}']\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}'$$

Landau-Migdal Residual Interaction
„Restoring Forces"

The Multi-Phonon QRPA Approach to Nuclear Spectroscopy

$$\Omega_a^\dagger = \sum_{\mu\nu} \left(x_{\mu\nu}^{(a)} \alpha_\mu^+ \alpha_\nu^+ - y_{\mu\nu}^{(a)*} \tilde{\alpha}_\nu \tilde{\alpha}_\mu \right)$$

- Anharmonicities due to coupling to multi-quasiparticle configurations
- Core polarization effects
- Damping width and energy shifts of ph-/2QP-states
- Quenching of interaction vertices and transition strengths („effective charge")

$$\Psi_i^\dagger = \sum_a z_a^{(i)} \Omega_a^\dagger + \sum_{ab} z_{ab}^{(i)} \Omega_a^\dagger \Omega_b^\dagger + \sum_{abc} z_{abc}^{(i)} \Omega_a^\dagger \Omega_b^\dagger \Omega_c^\dagger$$
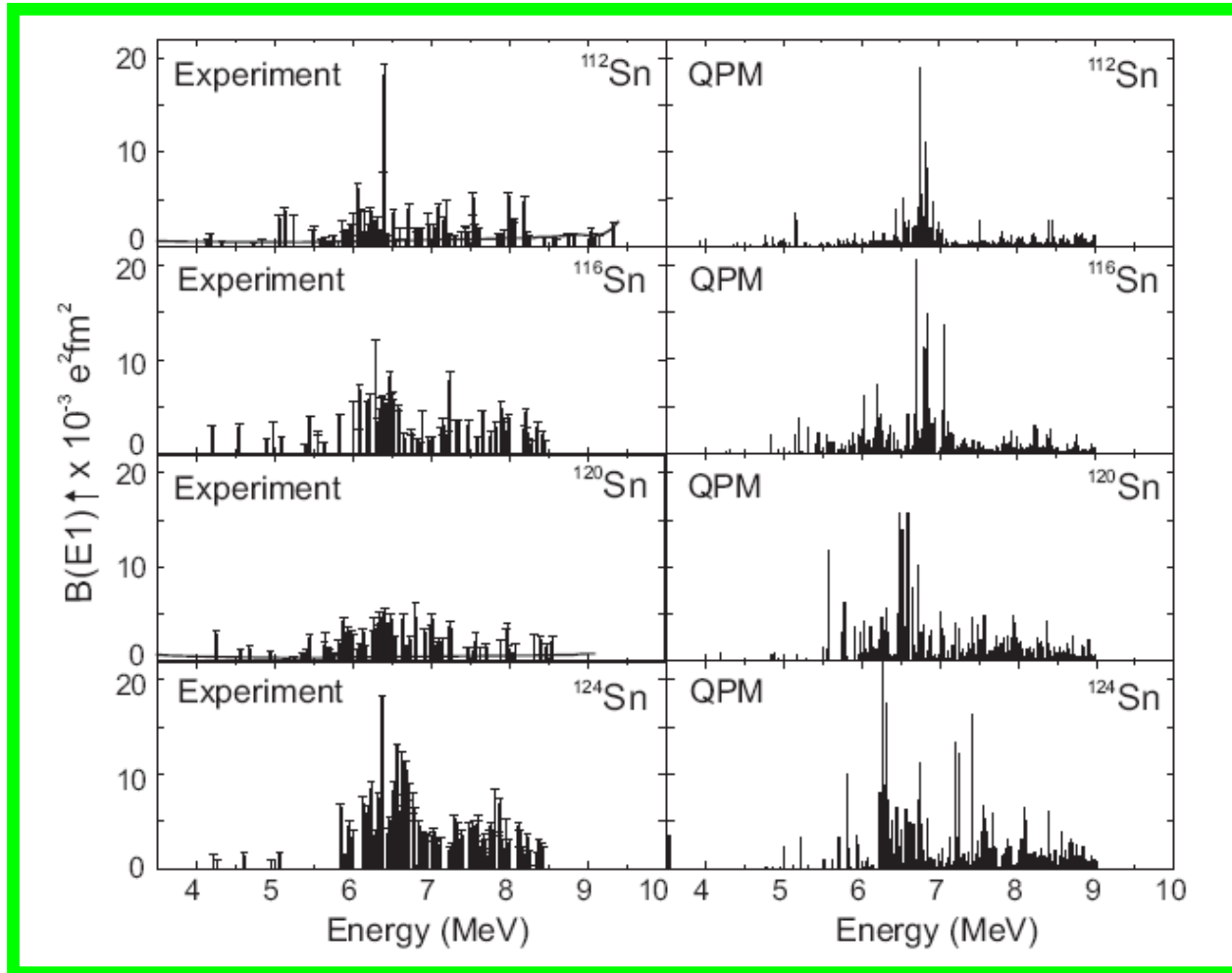
Multi-Configuration Multi-Quasiparticle Wave Function

→ Implemented for up to 6QP/3p3h configurations

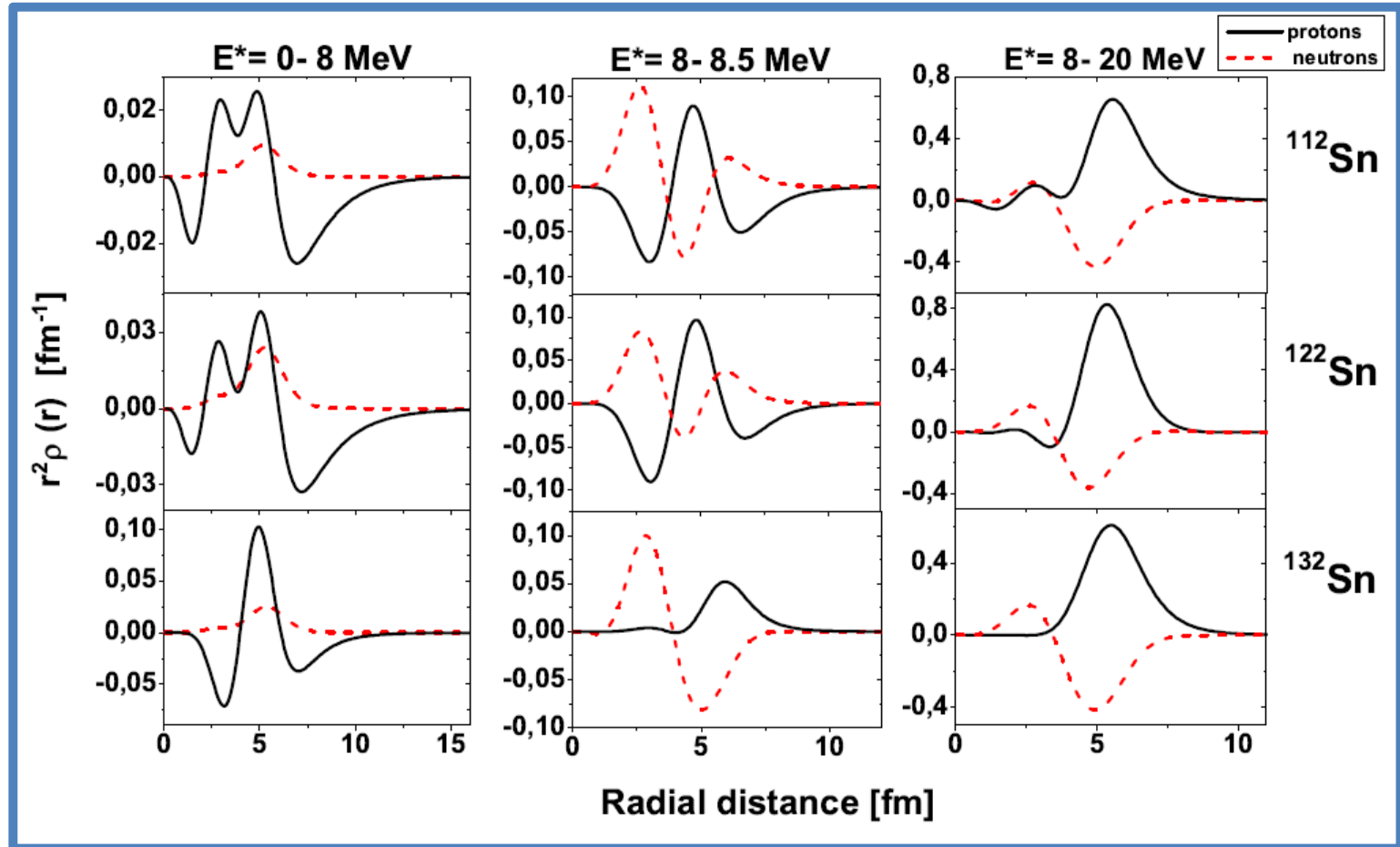Phys.Atom.Nucl. 79 (2016) no.6, 885; EPJA 55 (2019) 238

Giessen DFT & Multi-Phonon Approach
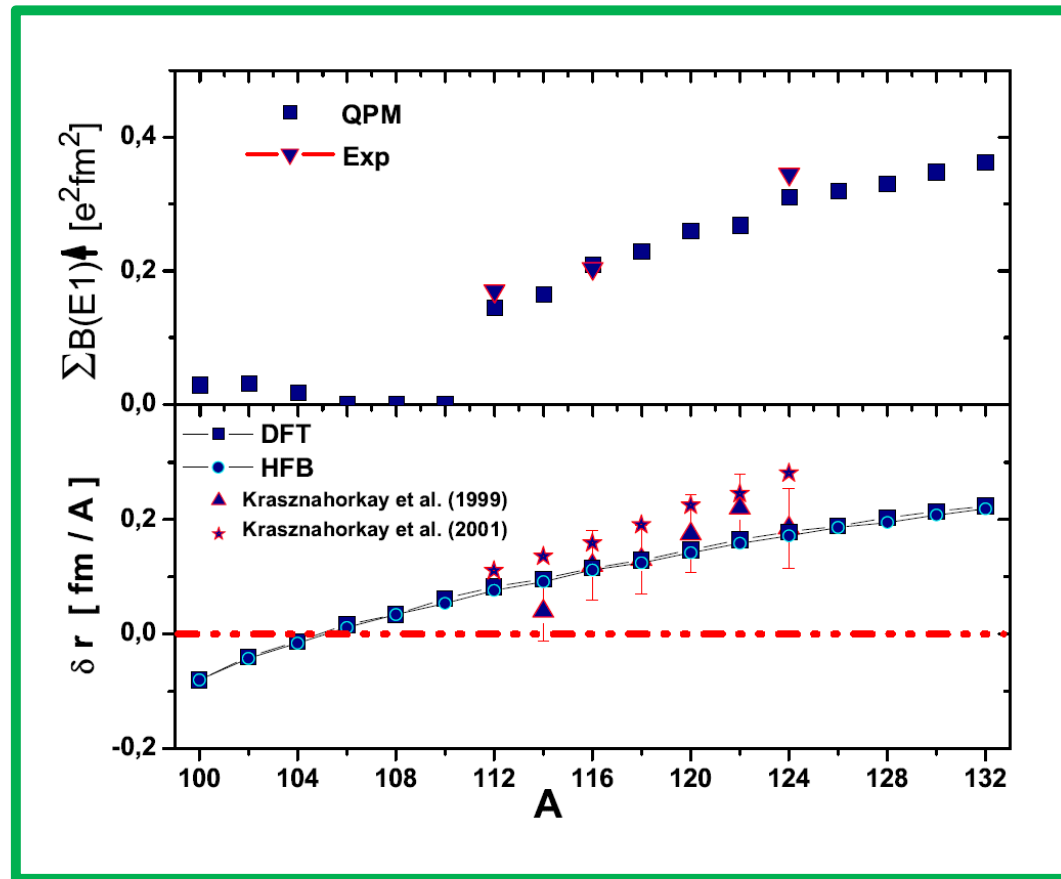
Pygmy Dipole Modes in Sn-Isotopes

PhysRevC.90.024304(2014)

Pattern of Pygmy Dipole Modes:
Compressional Dipole Surface Excitations

Pygmy Dipole Modes and Nuclear Skins

N. Tsoneva, H.L., Phys.Rev. C78 (2008) 064314

Probing nuclear skins in (n,p)- and (p,n)-type spin-dipole charge exchange reaction:

$$S_{\mathrm{SDR}}^{-} - S_{\mathrm{SDR}}^{+} = \frac{9}{2\pi}\left(N\langle r^2\rangle_n - Z\langle r^2\rangle_p\right)$$

The N=50 Isotones

**Pygmy dipole strength in ^{86}Kr and systematics of $N = 50$ isotones**

R. Schwengner,[1] R. Massarczyk,[1,2] G. Rusev,[3,4,*] N. Tsoneva,[5,6] D. Bemmerer,[1] R. Beyer,[1] R. Hannaske,[1,2] A. R. Junghans,[1] J. H. Kelley,[4,7] E. Kwan,[3,4,†] H. Lenske,[5] M. Marta,[1,‡] R. Raut,[3,4,§] K. D. Schilling,[1] A. Tonchev,[3,4,†] W. Tornow,[3,4] and A. Wagner[1]

Photo-Absorption Cross Sections
(Data: ELBE@Rossendorf and HIγS@Duke)
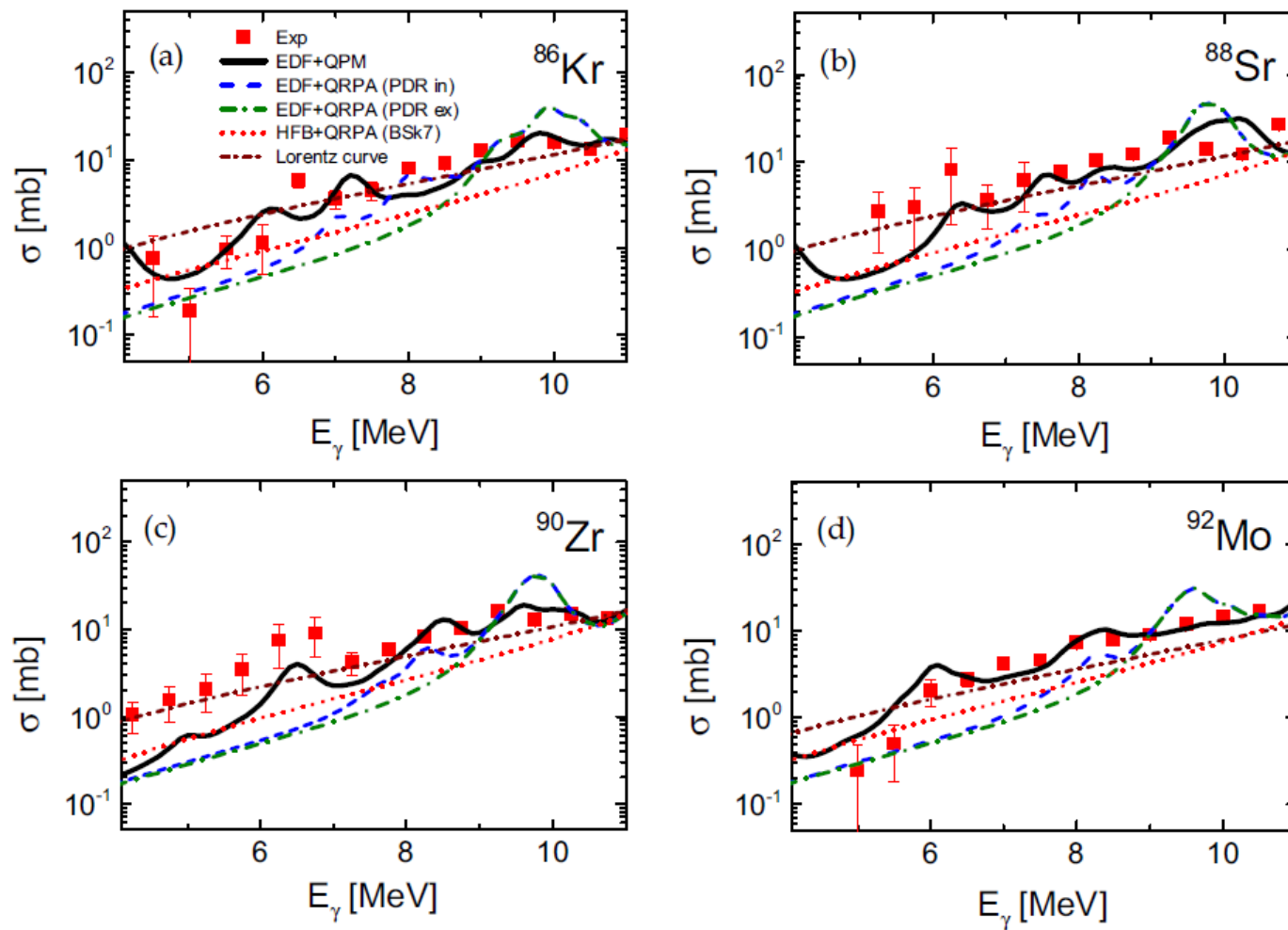
PhysRevC.91.044318 (2015)

Neutron Capture Cross Sections
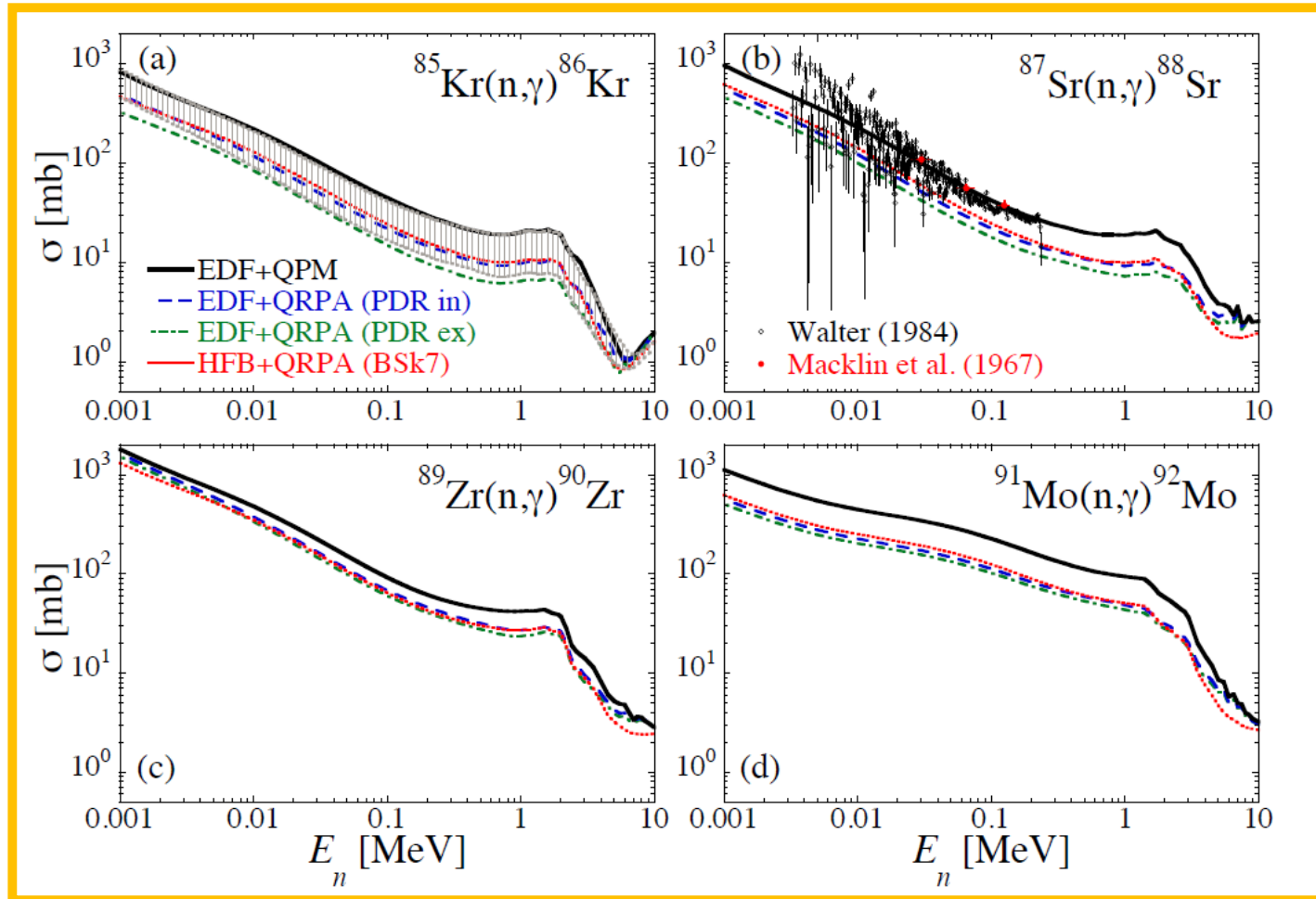
PhysRevC.91.044318 (2015)

Note:

The (n,γ) reactions are NOT part of nucleo-synthesis reactions but the inverse (γ,n) reactions act to destroy the N=50 isotones

Maxwellian –Averaged Neutron Capture Cross Sections at kT= 30 keV N=50 Nuclides

	Exp.	EDF + QPM	EDF + QRPA (PDR in)	EDF + QRPA (PDR ex)	HFB + QRPA (BSk7)
$^{85}Kr(n,\gamma)^{86}Kr$	83^{+23}_{-38} [5]	104.0	53.1	37.3	59.0
$^{87}Sr(n,\gamma)^{88}Sr$	92 ± 4 [7]	103.1	54.2	44.4	62.1
$^{89}Zr(n,\gamma)^{90}Zr$	–	224.3	170.4	152.1	159.8
$^{91}Mo(n,\gamma)^{92}Mo$	–	349.8	174.8	158.4	195.0

[5] R. Raut, A. P. Tonchev, G. Rusev, W. Tornow, C. Iliadis, M. Lugaro, J. Buntain, S. Goriely, J. H. Kelley, R. Schwengner, A. Banu, and N. Tsoneva, Phys. Rev. Lett. **111**, 112501 (2013).

[7] Z. Y. Bao, H. Beer, F. Käppeler, F. Voss, K. Wisshak, and T. Rauscher, At. Data Nucl. Data Tables **76**, 70 (2000).

Cosmo-Chronometry: ^{206}Pb

Explosive r- and AGB s-process production:
^{238}U($t_{1/2}$=4.5$\cdot$10^{+9}y)$\rightarrow$...$\rightarrow$ ^{206}Pb and ^{205}Pb + n $\rightarrow$ ^{206}Pb
$\rightarrow$ **Geo-chemical r-process indicator**
$\rightarrow$ **Tracer for the now extinct ^{205}Pb($t_{1/2}$=1.73$\cdot$10^{+7} y)**
$\rightarrow$ **Solidification of the solar nebula**

$(\vec{\gamma},\gamma')$ experiment at the HI$\vec{\gamma}$S facility

Pygmy and core polarization dipole modes in ^{206}Pb: Connecting nuclear structure to stellar nucleosynthesis

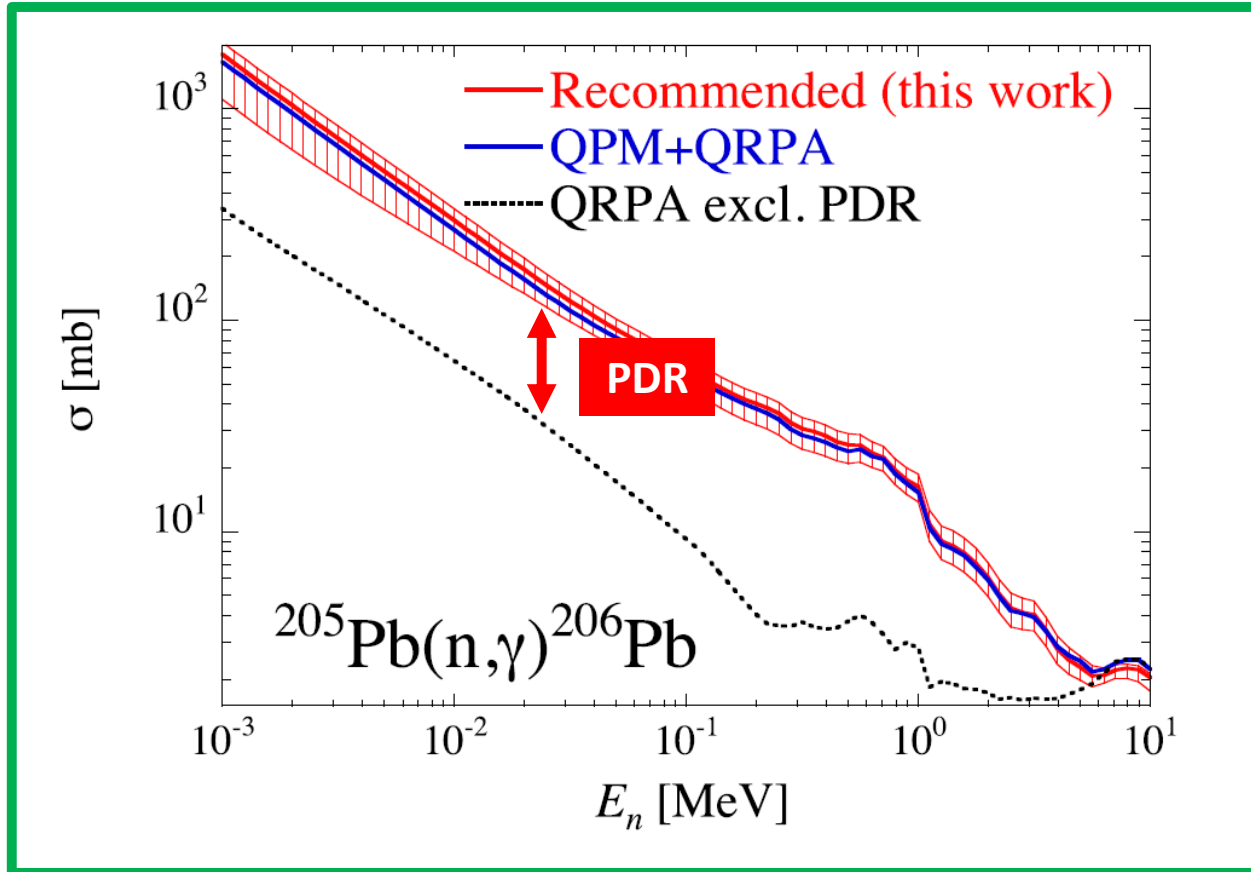A.P. Tonchev [a,b,*], N. Tsoneva [c], C. Bhatia [d,b,e], C.W. Arnold [f,g], S. Goriely [h], S.L. Hammond [i,e], J.H. Kelley [i,e], E. Kwan [j], H. Lenske [c], J. Piekarewicz [k], R. Raut [l], G. Rusev [g], T. Shizuma [m], W. Tornow [b,e]

Cumulative Electric and Magnetic Dipole Strength Functions HIγS@Duke
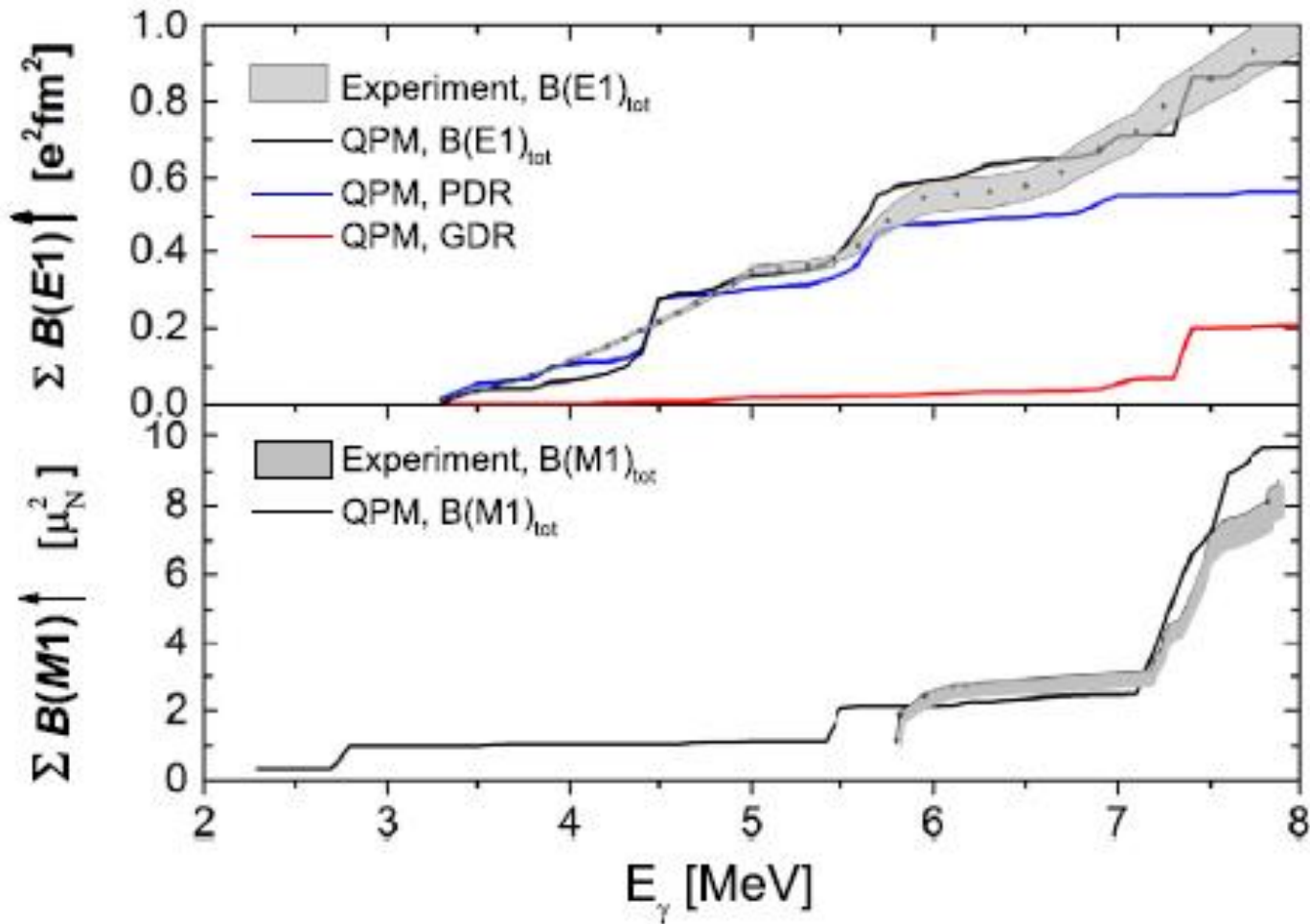
Neutron Capture Cross Section

Phys.Lett. B773 (2017) 20

Photoabsorption Cross Section and Sum Rules

Moments of the Photo-Absorption Cross Section

$$\sigma_{-n} = \int_0^\infty dE \frac{\sigma_\gamma(E)}{E^n} \quad \leftrightarrow \quad S_{-(n-1)} = \sum_c \frac{\left|M_{\gamma c}\right|^2}{E_c^{n-1}}$$

n = 0 : Energy Weighted Sum Rule (EWSR)

n = 1 : Non-Energy Weighted Sum Rule (NEWSR) - total transition strength

n = 2 : Polarizability sum rule

$$\alpha_D = \frac{1}{2\pi^2\alpha} \int_0^\infty \frac{\sigma_\gamma(E)}{E^2} dE = \frac{\sigma_{-2}}{2\pi^2\alpha} = 6.942\,\sigma_{-2}$$

Dipole Polarizability

Photo-Cross Section Moments for ^{206}Pb
Experimental, Covariant EDF and GiEDF Results

Nucleus	E_{max} (MeV)	σ_0 (mb MeV)	σ_{-1} (mb)	σ_{-2} (mb/MeV)	Ref.
^{206}Pb	26	3544±294	241±17	18±1	Present+[46, 49]
		3437	240	18	[ENDF]
^{208}Pb	25	3981±331	287±18	20±1	[50]
		3404	239	18	[ENDF]

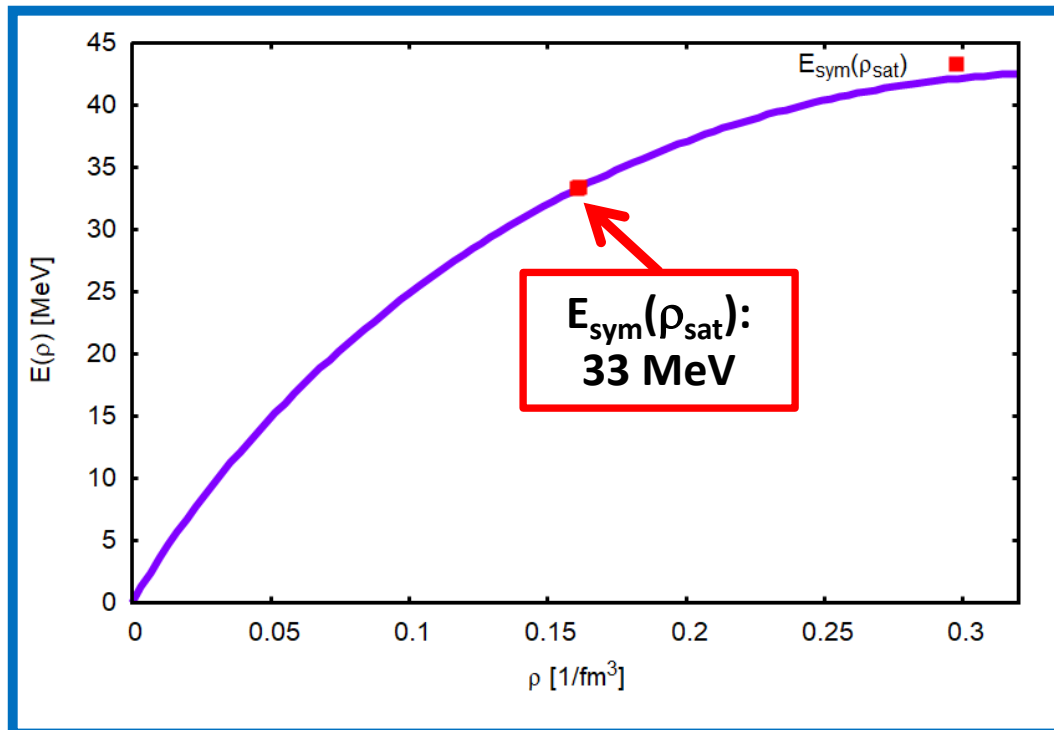| Model	σ_0 (mb MeV)	σ_{-1} (mb)	σ_{-2} (mb/MeV)	R_{skin} (fm)
RMF012	3653	237	17	0.12 [0.13]
FSUGarnet	3689	243	18	0.15 [0.16]
FSUGold	3638	251	19	0.19 [0.21]
RMF028	3711	265	21	0.26 [0.29]
RMF032	3812	262	21	0.30 [0.32]
GiEDF	3060	230	18	0.15 [0.16]

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Symmetry Energy

Symmetry Energy and the Nuclear EoS

$$S(\rho) \equiv \frac{1}{2}\left(\frac{\partial^2 \mathcal{E}(\rho,\delta)}{\partial \delta^2}\right)_{\delta=0} \approx \mathcal{E}(\rho, \delta=1) - \mathcal{E}(\rho, \delta=0).$$

$(\delta=3x)$

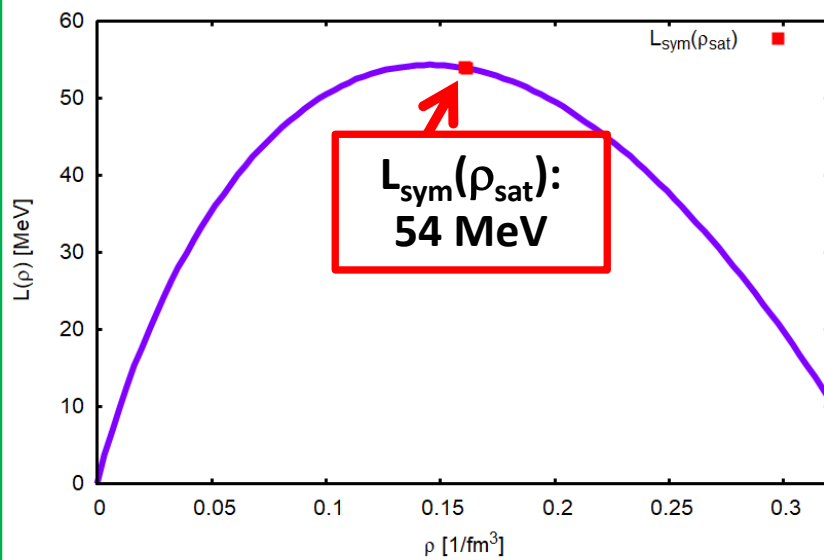$$S(\rho) = J + Lx + \frac{1}{2}K_{\mathrm{sym}}x^2 + \ldots \quad \text{with} \quad x \equiv \frac{\rho - \rho_0}{3\rho_0}.$$
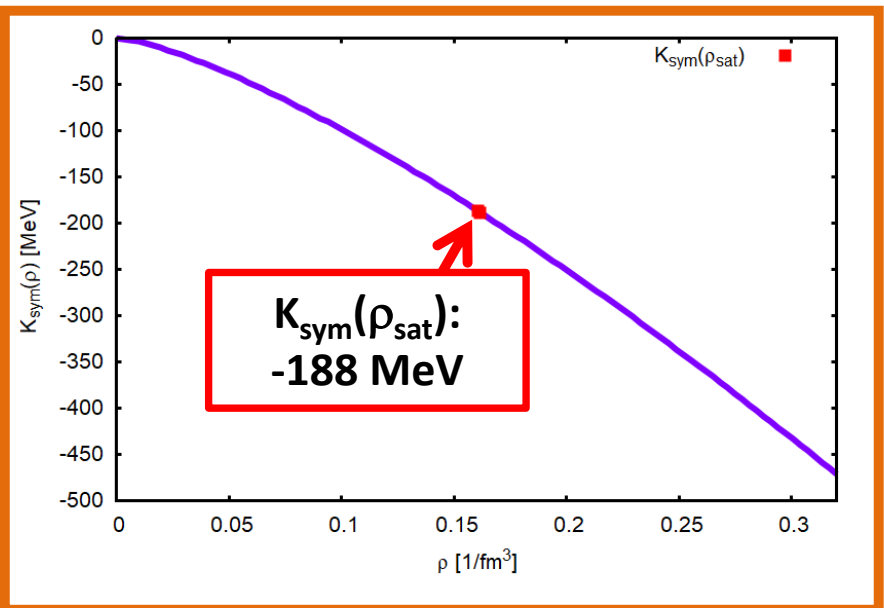

$E_{\mathrm{sym}}(\rho_{\mathrm{sat}})$

$E_{\mathrm{sym}}(\rho_{\mathrm{sat}})$:
33 MeV

Symmetry Pressure and Incompressibility

$$S(\rho) = J + Lx + \frac{1}{2}K_{sym}x^2 \cdot \qquad \left[\; x \equiv \frac{\rho - \rho_0}{3\rho_0}. \; \right]$$

GiEDF:
Isospin Pressure $L_{sym}(\rho)$

$L_{sym}(\rho)$: 54 MeV

$K_{sym}(\rho_{sat})$: -188 MeV

GiEDF:
Isospin Incompressibility $K_{sym}(\rho)$

Symmetry Energy and Isospin Dynamics

$$E_{sym}(\rho) = \frac{1}{3}\frac{\hbar^2 k_F^2(\rho)}{2m^*(\rho)}\left(1 + f_0'(\rho)\right)$$

Relation to residual isovector Interactions

$$f_0'(\rho) = f_0^{'(\pi)}(\rho) + f_0^{'(\rho)}(\rho) + \ldots = C_0 V_{01}^*(\rho)$$

The pionic part…

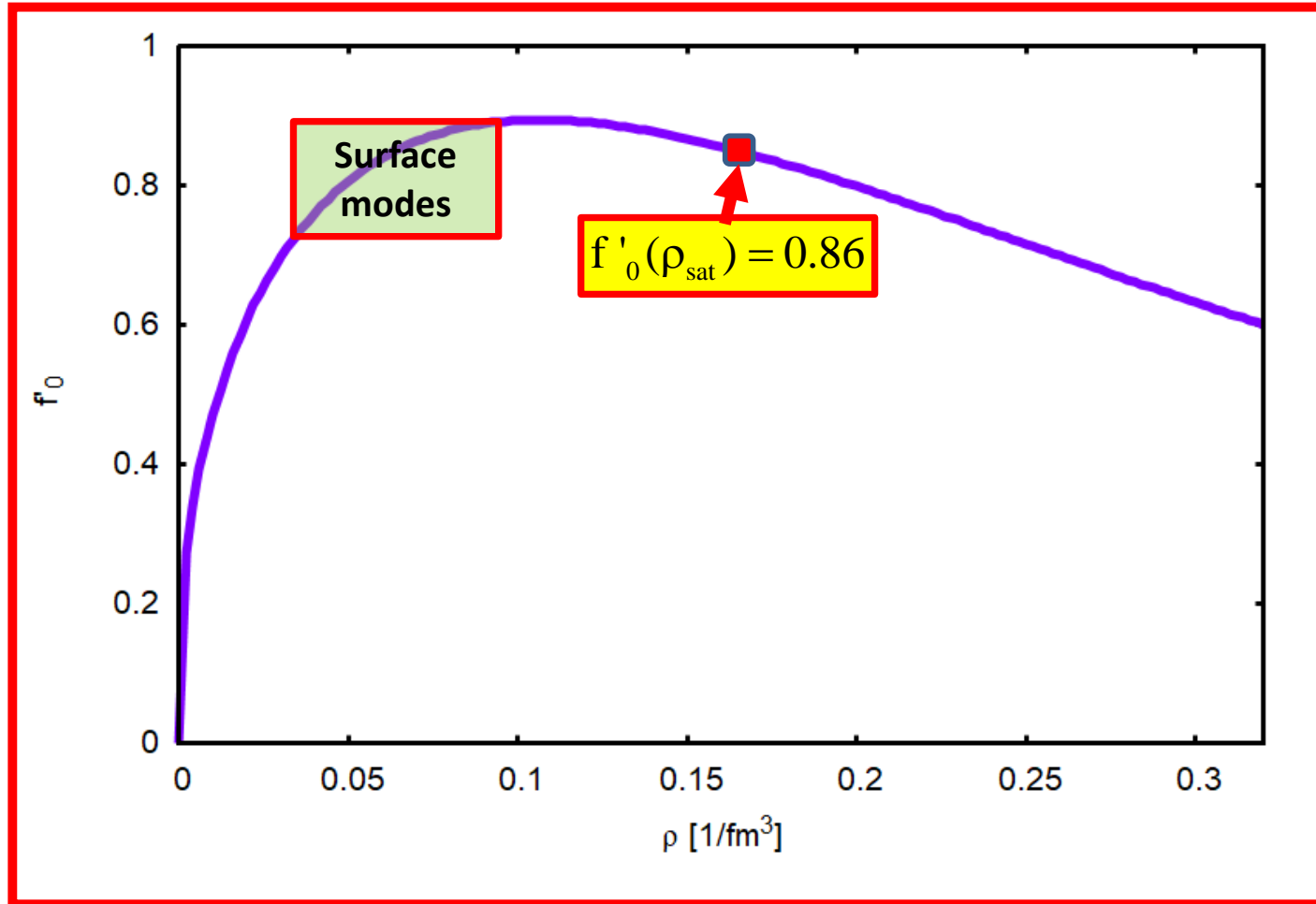$$f_0^{'(\pi)}(\rho) = +4\pi\frac{3}{4}N_0(2k_F)\frac{f_\pi^2}{2k_F^2}Q_0\left(1 + \frac{m_\pi^2}{k_F^2}\right) \qquad \left(Q_0(x) = \frac{1}{2}\log\left(\frac{1+x}{1-x}\right)\right)$$

**…and correspondingly for the rho-meson
(and supplemented by polarization interactions)**

The Landau-Migdal parameter f'$_0$
Connecting E$_{sym}$ to Nuclear Isovector Dynamics

Surface modes

$$f\,'_0(\rho_{sat}) = 0.86$$

Photo-Cross Section Moments for [206]Pb
Comparison of Covariant EDF and GiEDF Results

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Model	σ_0 (mb MeV)	σ_{-1} (mb)	σ_{-2} (mb/MeV)	R_{skin} (fm)	J (MeV)	L (MeV)	K_{sym} (MeV)
RMF012	3653	237	17	0.12 [0.13]	29.8	48.3	98.7
FSUGarnet	3689	243	18	0.15 [0.16]	30.9	51.0	59.5
FSUGold	3638	251	19	0.19 [0.21]	32.6	60.5	−51.3
RMF028	3711	265	21	0.26 [0.29]	37.5	112.6	26.2
RMF032	3812	262	21	0.30 [0.32]	41.3	125.6	28.6
GiEDF	3060	230	18	0.15 [0.16]	33.4	53.9	−188.4

- **Symmetry Energy ⟷ Isovector Restoring Force**
- **Nuclear g.s. Property ⟷ Dipole Dynamics**

Summary and Outlook

- **EDF approach to nuclear ground and excited states**

- **Nuclei by HFB and multi-phonon QRPA theory**

- **Pygmy modes as signatures for skin dynamics**

- **Low-energy PDR modes and capture cross sections**

- **Skin dynamics and symmetry energy**

- ***Low-energy quadrupole excitations: the PQR modes***
  (Phys.Lett. B695 (2011) 174;...; Nucl.Phys. A990 (2019) 183)

...in collaboration with N. Tsoneva, S. Goriely, J. Piekarewicz experimental groups at Cologne, Darmstadt, Duke, and Rossendorf