On the nature of the low-lying scalar mesons

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An effective chiral approach: the extended Linear Sigma Model (eLSM)

Chiral symmetry of QCD (classically): global $U(N_f)_r \times U(N_f)_\ell$ symmetry

$\Longrightarrow$ spontaneously broken in vacuum by nonzero quark condensate $\langle \bar{q}q \rangle \neq 0$

$\Longrightarrow$ restored at nonzero temperature T and chemical potential μ

$\Longrightarrow$ degeneracy of hadronic chiral partners in the chirally restored phase

$\Longrightarrow$ for this application: chiral symmetry must be linearly realized

$\Longrightarrow$ Linear Sigma Model, extended by (axial-)vector mesons $\Longrightarrow$ eLSM

Disclaimer: No attempt to fit precision data for hadron vacuum phenomenology!

(No attempt to compete with chiral perturbation theory)

Nevertheless: achieve reasonable description of hadron vacuum phenomenology!

Moreover: strong statement on the nature of the scalar mesons!

scalar-meson puzzle: too many scalar states to fit into a $q\bar{q}$ meson nonet

$$f_0(500),\ f_0(980),\ f_0(1370),\ f_0(1500),\ f_0(1710)$$

$\Longrightarrow$ Jaffe's conjecture: R.L. Jaffe, PRD 15 (1977) 267, 281

light scalars $f_0(500)$, $f_0(980)$ are (predominantly) $[qq][\bar{q}\bar{q}]$ tetraquark states

$\Longrightarrow$ fifth scalar meson $f_0(1710)$ could be (predominantly) glueball state

Scalar and pseudoscalar mesons

Assume mesons to be $\bar{q}q$ states: $\Phi \sim \bar{q}_r q_\ell$, $\Phi^\dagger \sim \bar{q}_\ell q_r$

$\Longrightarrow$ $\Phi \in (N_f^*, N_f)$ irrep of $U(N_f)_r \times U(N_f)_\ell$

$\Longrightarrow$ $\Phi \equiv \phi_a T_a$, T_a generators of $U(N_f)$, $\phi_a \equiv \sigma_a + i\pi_a$

$$\mathcal{L}_S = \mathrm{Tr}\left(\partial_\mu \Phi^\dagger \partial^\mu \Phi - m^2\, \Phi^\dagger \Phi\right) - \lambda_1 \left[\mathrm{Tr}\left(\Phi^\dagger \Phi\right)\right]^2 - \lambda_2 \mathrm{Tr}\left(\Phi^\dagger \Phi\right)^2 + c\left(\det\Phi - \det\Phi^\dagger\right)^2 + \mathrm{Tr}\left[H\left(\Phi + \Phi^\dagger\right)\right] + \mathrm{Tr}\left[E\, \Phi^\dagger \Phi\right]$$

$H \equiv h_a C_a$, $E \equiv \epsilon_a C_a$, $C_a \equiv T_a$, $a = 3, 8$

$\Longrightarrow$ H, E account for different non-zero quark masses

$h_a = \epsilon_a = c = 0$, $m^2 > 0$: $U(N_f)_r \times U(N_f)_\ell$ symmetry

$h_a = \epsilon_a = c = 0$, $m^2 < 0$: v.e.v. $\langle \Phi \rangle = \phi\, N_f\, T_0$, $\phi \equiv \langle \sigma_0 \rangle > 0$

Spont. symm. breaking (SSB): $U(N_f)_r \times U(N_f)_\ell \rightarrow U(N_f)_V$ $(V \equiv \ell + r)$

$h_a = \epsilon_a = 0$, $c \neq 0$: $U(1)_A$ anomaly $(A \equiv \ell - r)$

Expl. symm. breaking (ESB): $U(N_f)_r \times U(N_f)_\ell \rightarrow SU(N_f)_r \times SU(N_f)_\ell \times U(1)_V$

$m^2 < 0$: SSB: $SU(N_f)_r \times SU(N_f)_\ell \rightarrow SU(N_f)_V$

$\dim[SU(N_f)_r \times SU(N_f)_\ell / SU(N_f)_V] = N_f^2 - 1$

$\Longrightarrow$ $N_f^2 - 1$ Goldstone bosons $\Longrightarrow$ pseudoscalar mesons!

h_a, ϵ_a, $c \neq 0$, $m^2 < 0$: ESB $\Longrightarrow$ $N_f^2 - 1$ pseudo–Goldstone bosons

Vector and axial-vector mesons

Introduce left- and right-handed vector fields $\mathcal{L}_\mu \sim \bar{q}_\ell \gamma_\mu q_\ell$, $\mathcal{R}_\mu \sim \bar{q}_r \gamma_\mu q_r$,

$\Longrightarrow$ $\mathcal{L}_\mu \in (1, N_f^2)$ **irrep of** $U(N_f)_r \times U(N_f)_\ell$

$\Longrightarrow$ $\mathcal{R}_\mu \in (N_f^2, 1)$ **irrep of** $U(N_f)_r \times U(N_f)_\ell$

$\Longrightarrow$ $\mathcal{L}_\mu \equiv L_\mu^a T_a$, $\mathcal{R}_\mu \equiv R_\mu^a T_a$

$$\begin{aligned}
\mathcal{L}_V = & -\frac{1}{4} \mathrm{Tr}(\mathcal{L}^0_{\mu\nu}\mathcal{L}_0^{\mu\nu} + \mathcal{R}^0_{\mu\nu}\mathcal{R}_0^{\mu\nu}) + \mathrm{Tr}\left[\left(\frac{1}{2} m_1^2 + \Delta\right) \left(\mathcal{L}_\mu\mathcal{L}^\mu + \mathcal{R}_\mu\mathcal{R}^\mu\right)\right] \\
& + i\,\frac{g_2}{2}\,\mathrm{Tr}\left\{\mathcal{L}^0_{\mu\nu}[\mathcal{L}^\mu,\mathcal{L}^\nu] + \mathcal{R}^0_{\mu\nu}[\mathcal{R}^\mu,\mathcal{R}^\nu]\right\} \\
& + g_3\,\mathrm{Tr}\left(\mathcal{L}^\mu\mathcal{L}^\nu\mathcal{L}_\mu\mathcal{L}_\nu + \mathcal{R}^\mu\mathcal{R}^\nu\mathcal{R}_\mu\mathcal{R}_\nu\right) - g_4\,\mathrm{Tr}\left(\mathcal{L}^\mu\mathcal{L}_\mu\mathcal{L}^\nu\mathcal{L}_\nu + \mathcal{R}^\mu\mathcal{R}_\mu\mathcal{R}^\nu\mathcal{R}_\nu\right) \\
& + g_5\,\mathrm{Tr}\left(\mathcal{L}^\mu\mathcal{L}_\mu\right)\,\mathrm{Tr}\left(\mathcal{R}^\nu\mathcal{R}_\nu\right) \\
& + g_6\,\left[\mathrm{Tr}\left(\mathcal{L}^\mu\mathcal{L}_\mu\right)\,\mathrm{Tr}\left(\mathcal{L}^\nu\mathcal{L}_\nu\right) + \mathrm{Tr}\left(\mathcal{R}^\mu\mathcal{R}_\mu\right)\,\mathrm{Tr}\left(\mathcal{R}^\nu\mathcal{R}_\nu\right)\right]
\end{aligned}$$

$\mathcal{L}^0_{\mu\nu} \equiv \partial_\mu\mathcal{L}_\nu - \partial_\nu\mathcal{L}_\mu$, $\mathcal{R}^0_{\mu\nu} \equiv \partial_\mu\mathcal{R}_\nu - \partial_\nu\mathcal{R}_\mu$

vector mesons: $V_\mu^a \equiv \frac{1}{2}\left(L_a^\mu + R_\mu^a\right)$, **axial-vector mesons:** $A_\mu^a \equiv \frac{1}{2}\left(L_a^\mu - R_\mu^a\right)$

$\Delta = \delta_a C_a$ **: accounts for different quark masses (like** E**)**

g_3, g_4, g_5, g_6**: not determined by global fit to masses and decay widths**

(mild impact on $\pi\pi$ **scattering lengths,**

can be determined from LECs of QCD)

Scalar – vector interactions

$$\mathcal{L}_{SV} = i\, g_1 \,\mathrm{Tr}\left[\partial_\mu\Phi\left(\Phi^\dagger\mathcal{L}^\mu - \mathcal{R}^\mu\Phi^\dagger\right) - \partial_\mu\Phi^\dagger\left(\mathcal{L}^\mu\Phi - \Phi\mathcal{R}^\mu\right)\right]$$
$$+\frac{h_1}{2}\,\mathrm{Tr}\left(\Phi^\dagger\Phi\right)\mathrm{Tr}\left(\mathcal{L}_\mu\mathcal{L}^\mu + \mathcal{R}_\mu\mathcal{R}^\mu\right) + (g_1^2 + h_2)\,\mathrm{Tr}\left(\Phi^\dagger\Phi\mathcal{R}_\mu\mathcal{R}^\mu + \Phi\Phi^\dagger\mathcal{L}_\mu\mathcal{L}^\mu\right)$$
$$-\,2(g_1^2 - h_3)\,\mathrm{Tr}\left(\Phi^\dagger\mathcal{L}_\mu\Phi\mathcal{R}^\mu\right)$$

SSB:

- induces mass splitting, e.g. $m_{a_1}^2 - m_\rho^2 = (g_1^2 - h_3)\phi_N^2$
- induces bilinear terms, e.g. $\sim g_1\, d_{abc}\,\phi_a\, A_b^\mu\,\partial_\mu\pi_c$:

$\Longrightarrow$ eliminate by shift, e.g. $A_a^\mu \to A_a^\mu + w_{a_1}(\phi_N)\,\partial^\mu\pi_a\ ,\ \ a = 1,2,3,$

$$w_{a_1}(\phi_N) \equiv \frac{g_1\phi_N}{m_{a_1}^2}$$

$\Longrightarrow$ wave function renormalization of scalar and pseudoscalar fields, e.g.

$$\pi_a \to Z_\pi\,\pi_a\ ,\quad Z_\pi^2 \equiv \left(1 - \frac{g_1^2\phi_N^2}{m_{a_1}^2}\right)^{-1} \qquad (\ \mathrm{KSFR}: Z_\pi \equiv \sqrt{2}\ )$$

$\Longrightarrow$ v.e.v. $\phi_N \equiv Z_\pi\, f_\pi$

$\Longrightarrow$ complete meson Lagrangian $\boxed{\mathcal{L}_M = \mathcal{L}_S + \mathcal{L}_V + \mathcal{L}_{SV}}$

Vacuum phenomenology: Global fit for $N_f = 3$ (I)

$N_f = 3$ $\Longrightarrow$ two scalar-isoscalar mesons $f_0^L\,,\ f_0^H$ (combinations of $\bar{q}q$ and $\bar{s}s$)

$\Longrightarrow$ all (pseudo-)scalar masses and decay widths except those of $f_0^L\,,\ f_0^H$ determined by linear combination of $m^2\,,\ \lambda_1$ and of $m_1^2\,,\ h_1$

Since nature of scalar-isoscalar mesons (quarkonium, glueball, or four-quark state?) is unclear

$\Longrightarrow$ at first omit scalar-isoscalar mesons from the fit

$\Longrightarrow$ perform χ^2–fit of $m^2, \lambda_2\,, c\,, h_0\,, h_8\,, m_1^2\,, \delta_S\,, g_1\,,\ g_2\,,\ h_2\,,\ h_3$ (11 parameters) to 21 experimental quantities

D. Parganlija, F. Giacosa, P. Kovacs, Gy. Wolf, DHR, PRD 87 (2013) 014011

Constraints: (i) no isospin violation

$\Longrightarrow$ experimental error $=$ max(PDG error , 5%)

(ii) $m^2 < 0$ (SSB)

(iii) $\lambda_2 > 0\,,\ \lambda_1 > -\lambda_2/2$ (boundedness of potential)

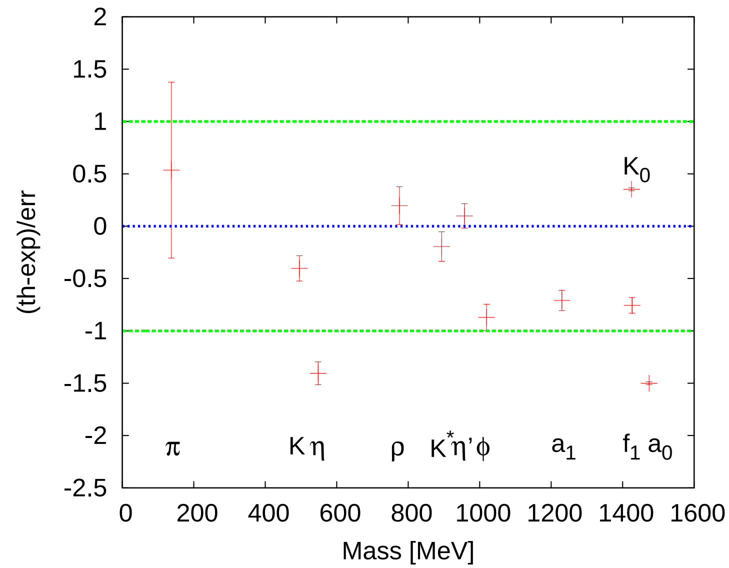
(iv) $m_1 \geq 0$ (boundedness of potential)

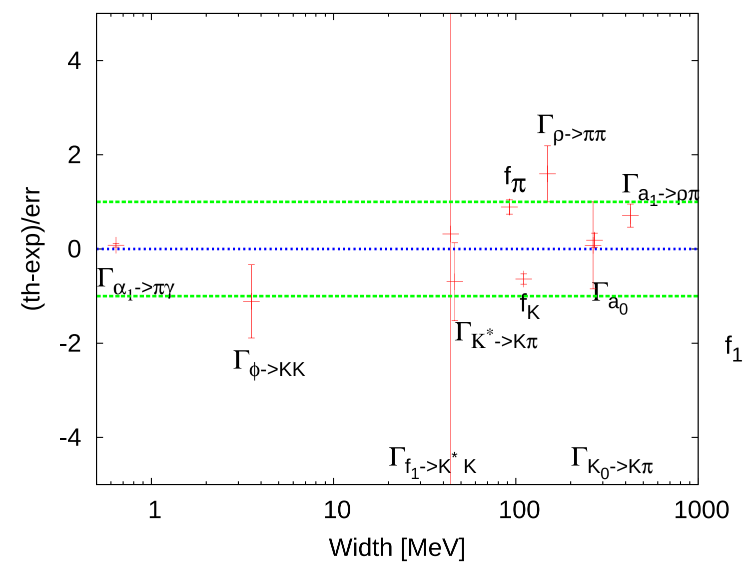
(v) $m_1 \leq m_\rho$ (SSB increases mass of vector mesons)

Vacuum phenomenology: Global fit for $N_f = 3$ (II)

Observable	Fit [MeV]	Experiment [MeV]
f_π	96.3 ± 0.7	92.2 ± 4.6
f_K	106.9 ± 0.6	110.4 ± 5.5
m_π	141.0 ± 5.8	137.3 ± 6.9
m_K	485.6 ± 3.0	495.6 ± 24.8
m_η	509.4 ± 3.0	547.9 ± 27.4
$m_{\eta'}$	962.5 ± 5.6	957.8 ± 47.9
m_ρ	783.1 ± 7.0	775.5 ± 38.8
$m_{K^\star}$	885.1 ± 6.3	893.8 ± 44.7
m_ϕ	975.1 ± 6.4	1019.5 ± 51.0
m_{a_1}	1186 ± 6	1230 ± 62
$m_{f_1(1420)}$	1372.5 ± 5.3	1426.4 ± 71.3
m_{a_0}	1363 ± 1	1474 ± 74
$m_{K_0^\star}$	1450 ± 1	1425 ± 71
$\Gamma_{\rho\to\pi\pi}$	160.9 ± 4.4	149.1 ± 7.4
$\Gamma_{K^\star\to K\pi}$	44.6 ± 1.9	46.2 ± 2.3
$\Gamma_{\phi\to\bar{K}K}$	3.34 ± 0.14	3.54 ± 0.18
$\Gamma_{a_1\to\rho\pi}$	549 ± 43	425 ± 175
$\Gamma_{a_1\to\pi\gamma}$	0.66 ± 0.01	0.64 ± 0.25
$\Gamma_{f_1(1420)\to K^\star K}$	44.6 ± 39.9	43.9 ± 2.2
Γ_{a_0}	266 ± 12	265 ± 13
$\Gamma_{K_0^\star\to K\pi}$	285 ± 12	270 ± 80

accuracy of fit: $\chi^2/\text{d.o.f.} \simeq 1.23$





Vacuum phenomenology: Global fit for $N_f = 3$ (III)

Fits with combinations $(a_0(980), K_0^\star(800))$, $(a_0(980), K_0^\star(1430))$, $(a_0(1450), K_0^\star(800))$ **have larger** χ^2/d.o.f. $\sim 2 - 24$

large–N_c suppressed parameters $\lambda_1 = h_1 \equiv 0$**:**

$\Longrightarrow$ **prediction for the masses of the isoscalar-scalar states:**

$m_{f_0^L} = 1362.7$ MeV, $m_{f_0^H} = 1531.7$ MeV

$\Longrightarrow$ **masses are in the range of the heavy scalar states:**

$m_{f_0(1370)} = (1350 \pm 150)$ MeV, $m_{f_0(1500)} = (1505 \pm 75)$ MeV,

$m_{f_0(1710)} = 1720 \pm 86$ MeV

$\Longrightarrow$ **mass of** f_0^L **close to mass of** $f_0(1370)$

$\Longrightarrow$ **mass of** f_0^H **close to** $f_0(1500)$

$\Longrightarrow$ $f_0(1370)$, $f_0(1500)$ **appear to be (predominantly)** $\bar{q}q$**–states**

$\Longrightarrow$ **chiral partners of** π, η'!

$\Longrightarrow$ **light scalar states** $f_0(500)$, $f_0(980)$ **could be (predominantly)** $[qq][\bar{q}\bar{q}]$**–states, as suggested by Jaffe** R.L. Jaffe, PRD 15 (1977) 267, 281

see, however, W. Heupel, G. Eichmann, C.S. Fischer, PLB 718 (2012) 545

$\Longrightarrow$ **light scalars have a dominant** $(\bar{q}q)(\bar{q}q)$ **component!**

Low-energy limit (I)

Does the model have the same low-energy limit as QCD?

$\Longrightarrow$ **low-energy limit of QCD: chiral perturbation theory (χPT)**

$\Longrightarrow$ **take $\mathcal{L}_{\chi PT} = \mathcal{L}_2 + \mathcal{L}_4$**

$\Longrightarrow$ **use $U = (\sigma + i\vec{\pi}\cdot\vec{\tau})/f_\pi$, $\sigma \equiv \sqrt{f_\pi^2 - \vec{\pi}^2}$, and expand $\mathcal{L}_{\chi PT}$ to order π^4, $(\partial\pi)^4$:**

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\vec{\pi})^2 - \frac{1}{2}m_\pi^2\vec{\pi}^2 + C_1(\vec{\pi}^2)^2 + C_2(\vec{\pi}\cdot\partial_\mu\vec{\pi})^2 + C_3(\partial_\mu\vec{\pi})^2(\partial_\nu\vec{\pi})^2 + C_4[(\partial_\mu\vec{\pi})\cdot\partial_\nu\vec{\pi}]^2$$

Similarly, in eLSM, integrate out all fields except pions, match coefficients:

F. Divotgey, P. Kovacs, F. Giacosa, DHR, arXiv:1605.05154 [hep-ph]

	χPT	eLSM (tree level!)
C_1	$-M^2/(8f_\pi^2) = -0.28 \pm 1.9$	-0.268 ± 0.021
C_2 [MeV]$^{-2}$	$1/(2f_\pi^2) = (5.882 \pm 0.013) \cdot 10^{-5}$	$(5.399 \pm 0.081) \cdot 10^{-5}$
C_3 [MeV]$^{-4}$	$\ell_1/f_\pi^4 = (-5.61 \pm 0.89) \cdot 10^{-11}$	$(-9.302 \pm 0.591) \cdot 10^{-11} - \frac{g_3 - g_4}{4} w_{a_1}^4 Z_\pi^4$
C_4 [MeV]$^{-4}$	$\ell_2/f_\pi^4 = (2.51 \pm 0.41) \cdot 10^{-11}$	$(9.448 \pm 0.589) \cdot 10^{-11} + \frac{g_3}{2} w_{a_1}^4 Z_\pi^4$

χPT: $m_\pi^2 = M^2(1 + 2\ell_3 M^2/f_\pi^2)$

eLSM: results for C_3 , C_4 for large-N_c suppressed $g_5 = g_6 = 0$

G. Ecker, J. Gasser, A. Pich, E. de Rafael, NPB 321 (1989) 311

$\Longrightarrow$ **resonances saturate LECs in χPT**

$\Longrightarrow$ **pion-loop corrections are small**

$\Longrightarrow$ **tree-level calculation should suffice**

Low-energy limit (II)

⟹ numerical values for C_3, C_4 do not agree between χPT and eLSM

⟹ g_5, g_6 are large–N_c suppressed ⟹ set $g_5 = g_6 = 0$

⟹ use χPT results for C_3, C_4 to determine $g_3 = -74 \pm 33$, $g_4 = 5 \pm 52$

⟹ compute other quantities to check consistency

⟹ e.g. $\pi\pi$ scattering lengths:

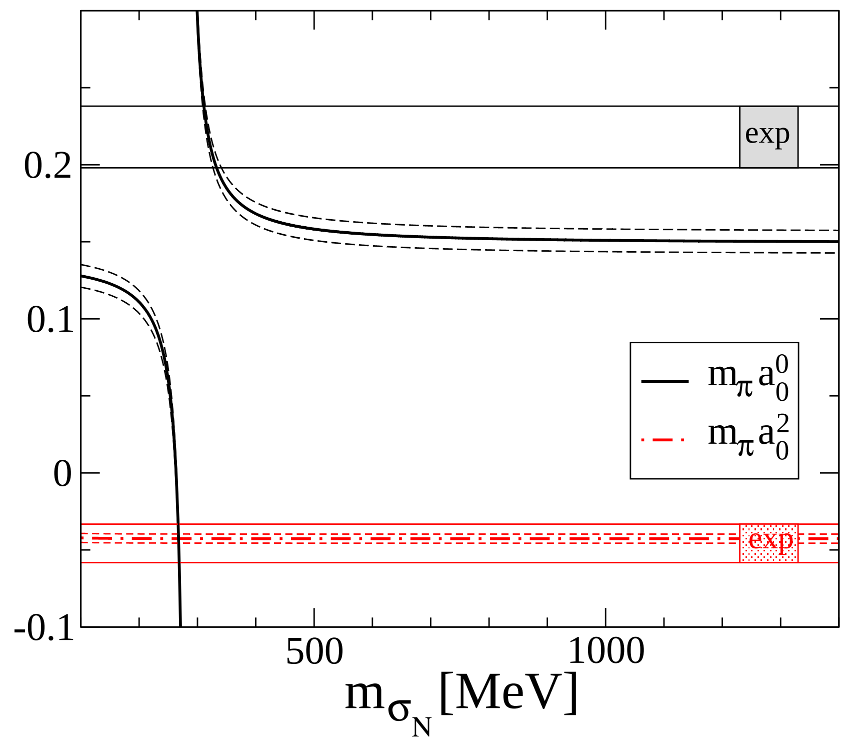
⟹ varying g_3, g_4 between ± 100 has small effect on $a_0^{0,2}$

⟹ a_0^2 agrees well with data

⟹ a_0^0 indicates influence of additional light scalar resonance

⟹ $f_0(500)$!

F. Divotgey, P. Kovacs, F. Giacosa, DHR, arXiv:1605.05154 [hep-ph]



Low-energy limit (III)

do loop corrections spoil nice agreement at tree level?

$\Longrightarrow$ compute loops to all orders via the Functional Renormalization Group!

J. Eser, F. Divotgey, M. Mitter, DHR, in preparation

$\Longrightarrow$ first step: consider $O(4)$ quark-meson model

$\Longrightarrow$ Ansatz for scale-dependent effective action:

$$\Gamma_k[\sigma,\vec{\pi}] = \int_X \Big\{ \frac{Z_k^\sigma}{2} (\partial_\mu \sigma)^2 + \frac{Z_k^\pi}{2} (\partial_\mu \vec{\pi})^2 + U_k(\rho) - h\sigma + C_{2,k} (\vec{\pi}\cdot\partial_\mu\vec{\pi})^2 - C_{3,k} (\partial_\mu\vec{\pi})^2 (\partial_\nu\vec{\pi})^2 + \bar{\psi}\left[Z_k^\psi \gamma_\mu \partial_\mu + y\,(\Sigma_5 + \phi t_0) \right]\psi \Big\}$$

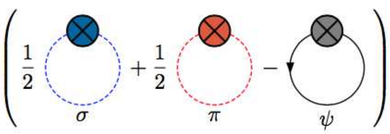
where $\rho \equiv \sigma^2 + \vec{\pi}^{\,2}$, $\phi \equiv \langle \sigma \rangle$, $\Sigma_5 \equiv \sigma t_0 + i\gamma_5 \vec{\pi}\cdot\vec{t}, t_0 \equiv \mathbb{1}/2$, $\vec{t} \equiv \vec{\tau}/2$

Note: $C_4 = 0$ at tree level without (axial-)vector mesons

Low-energy limit (IV)

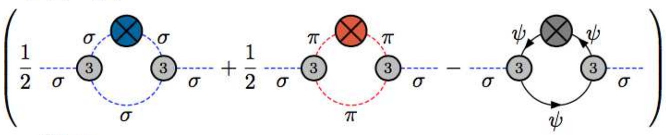
FRG flow equations for scale-dependent quantities:

$$\partial_k U_k = \mathcal{V}^{-1}\partial_k\Gamma_k = \mathcal{V}^{-1}\left(\ldots \right), \qquad (1)$$



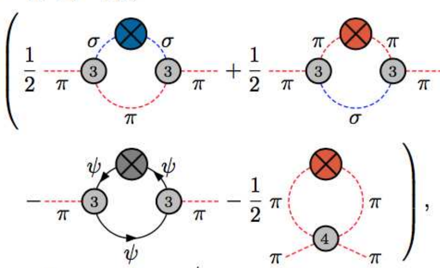
$$\partial_k Z_k^{\sigma} = \mathcal{V}^{-1}\left.\frac{\mathrm{d}}{\mathrm{d}p^2}\right|_{p^2=0}\frac{\delta^2\partial_k\Gamma_k}{\delta\sigma(-p)\delta\sigma(p)}$$

$$= \mathcal{V}^{-1}\left.\frac{\mathrm{d}}{\mathrm{d}p^2}\right|_{p^2=0}\left(\ldots \right), \qquad (2)$$



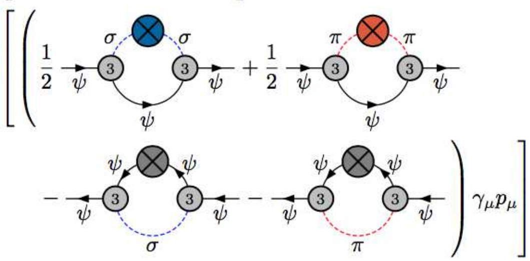
$$\partial_k Z_k^{\pi} = \mathcal{V}^{-1}\left.\frac{\mathrm{d}}{\mathrm{d}p^2}\right|_{p^2=0}\frac{\delta^2\partial_k\Gamma_k}{\delta\pi_1(-p)\delta\pi_1(p)}$$

$$= \mathcal{V}^{-1}\left.\frac{\mathrm{d}}{\mathrm{d}p^2}\right|_{p^2=0}\left(\ldots \right), \qquad (3)$$



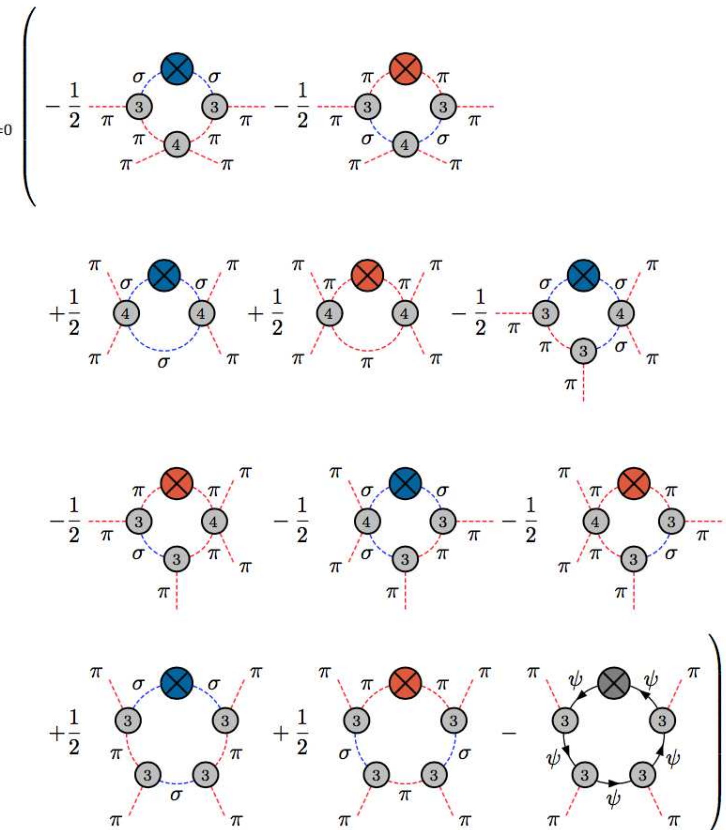
$$\partial_k Z_k^{\psi} = \frac{i}{4}\mathcal{V}^{-1}\left.\frac{\mathrm{d}}{\mathrm{d}p^2}\right|_{p^2=0}\mathrm{tr}_{\gamma}\left[\frac{\delta}{\delta\bar{\psi}(p)}\,\partial_k\Gamma_k\,\frac{\overleftarrow{\delta}}{\delta\psi(p)}\,\gamma_\mu p_\mu\right]$$

$$\frac{i}{4}\mathcal{V}^{-1}\left.\frac{\mathrm{d}}{\mathrm{d}p^2}\right|_{p^2=0}\mathrm{tr}_{\gamma}\left[\left(\ldots \right)\gamma_\mu p_\mu\right], \qquad (4)$$



$$\partial_k C_{2,k} = \frac{1}{8}\mathcal{V}^{-1}\left.\frac{\mathrm{d}}{\mathrm{d}p^2}\right|_{p^2=0}\frac{\delta^4\partial_k\Gamma_k}{\delta\pi_2(-p)\delta\pi_1(p)\delta\pi_2(-p)\delta\pi_1(p)}$$

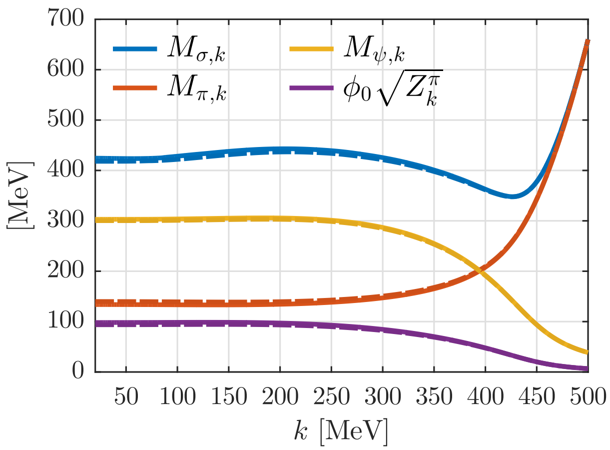
$$= \frac{1}{8}\mathcal{V}^{-1}\left.\frac{\mathrm{d}}{\mathrm{d}p^2}\right|_{p^2=0}\left(\ldots \right), \qquad (5)$$

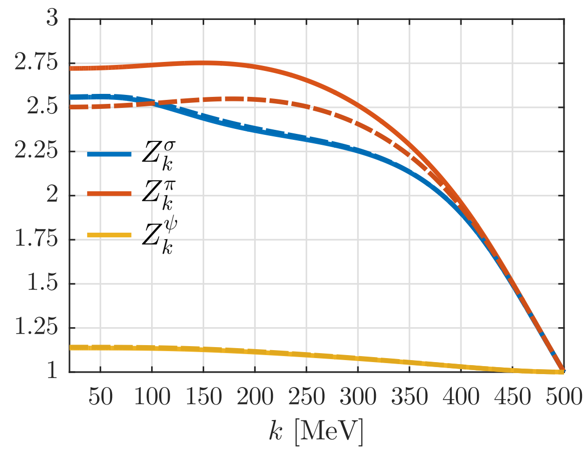


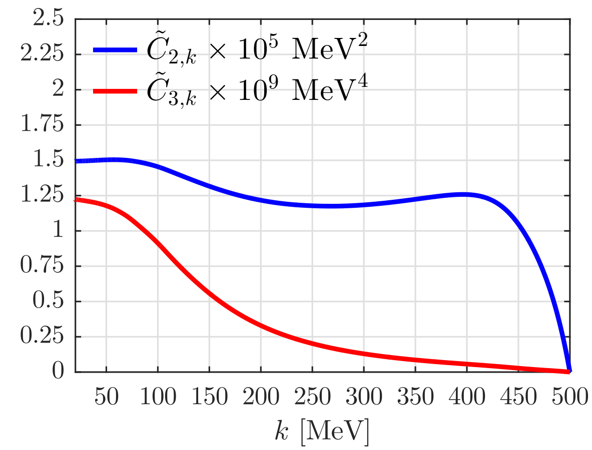
$$\partial_k C_{3,k} = -\frac{1}{16}\mathcal{V}^{-1}\left.\frac{\mathrm{d}^2}{\mathrm{d}\left(p^2\right)^2}\right|_{p^2=0}\frac{\delta^4\partial_k\Gamma_k}{\delta\pi_2(-p)\delta\pi_1(p)\delta\pi_2(-p)\delta\pi_1(p)}$$

$$= -\frac{1}{16}\mathcal{V}^{-1}\left.\frac{\mathrm{d}^2}{\mathrm{d}\left(p^2\right)^2}\right|_{p^2=0}\left(\text{same diagrams as in Eq. (5)}\right), \qquad (6)$$

Low-energy limit (V)







FRG determines $\Gamma_k[\sigma, \vec{\pi}]$, but we require

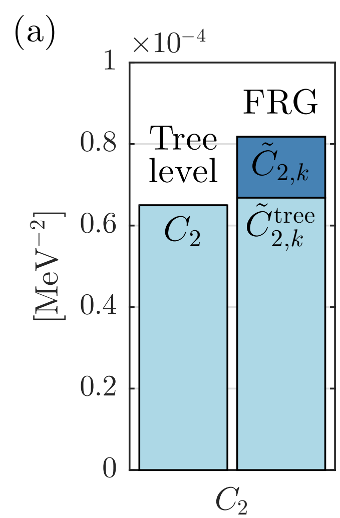
$$\Gamma_k[\tilde{\vec{\pi}}] = \int_X \left[\frac{1}{2}\left(\partial_\mu \tilde{\vec{\pi}}\right)^2 + \tilde{U}_k(\tilde{\vec{\pi}}) + \tilde{C}_{2,k}^{\text{tot}} \left(\tilde{\vec{\pi}} \cdot \partial_\mu \tilde{\vec{\pi}}\right)^2 - \tilde{C}_{3,k}^{\text{tot}} \left(\partial_\mu \tilde{\vec{\pi}}\right)^2 \left(\partial_\nu \tilde{\vec{\pi}}\right)^2 \right]$$

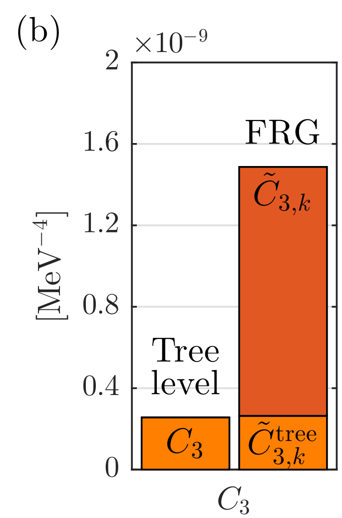
$\Longrightarrow$ **eliminate σ using eq. of motion**
$0 = \delta\Gamma_k[\sigma, \vec{\pi}]/\delta\sigma$

$\Longrightarrow$ $\tilde{C}_{i,k}^{\text{tot}} = \tilde{C}_{i,k}^{\text{tree}} + \tilde{C}_{i,k}\,,$
$\tilde{C}_{i,k} = C_{i,k}/(Z_k^\pi)^2\,,\ i = 2, 3$

$\Longrightarrow$ **loop corrections small for C_2, but large for C_3!!**

$\Longrightarrow$ **can (axial-)vector mesons (for eLSM) change this conclusion??**





Incorporating the scalar glueball (I)

Another confirmation of the (predominantly) $\bar{q}q$ assignment for the heavy scalar mesons: $\Longrightarrow$ coupling to the glueball/dilaton field!

$N_f = 2$: S. Janowski, D. Parganlija, F. Giacosa, DHR, PRD 84 (2011) 054007

$N_f = 3$: S. Janowski, F. Giacosa, DHR, PRD 90 (2014) 11, 114005

- dilatation symmetry $\Longrightarrow$ dynamical generation of tree-level meson mass parameters through glueball field G: $m^2 \to m^2 \left(\frac{G}{G_0}\right)^2$, $\quad m_1^2 \to m_1^2 \left(\frac{G}{G_0}\right)^2$
- add glueball Lagrangian:

$$\mathcal{L}_G = \frac{1}{2}\,(\partial_\mu G)^2 - \frac{1}{4}\,\frac{m_G^2}{\Lambda^2}\,G^4 \left(\ln\left|\frac{G}{\Lambda}\right| - \frac{1}{4}\right)$$

$\Lambda \sim$ gluon condensate $\langle G^a_{\mu\nu} G_a^{\mu\nu}\rangle$

$\Longrightarrow \quad \mathcal{L}_M \longrightarrow \mathcal{L}_M + \mathcal{L}_G$

- shift σ_N, σ_S, and G by their v.e.v.'s, $\sigma_{N,S} \to \sigma_{N,S} + \phi_{N,S}\,,\ G \to G + G_0$

$\Longrightarrow$ v.e.v. G_0 given by $\quad -\dfrac{m^2\,\Lambda^2}{m_G^2}\,(\phi_N^2 + \phi_S^2) = G_0^4\,\ln\left|\dfrac{G_0}{\Lambda}\right|$

$\Longrightarrow$ glueball mass given by $\quad M_G^2 = \frac{m^2}{G_0^2}\,\left(\phi_N^2 + \phi_S^2\right) + m_G^2\,\frac{G_0^2}{\Lambda^2}\,\left(1 + 3\ln\left|\frac{G_0}{\Lambda}\right|\right)$

$\Longrightarrow$ diagonalize mass matrix

$$M \equiv \begin{pmatrix} m^2_{\sigma_N} & 2\,\lambda_1\phi_N\phi_S & 2\,m^2\phi_N G_0^{-1} \\ 2\,\lambda_1\phi_N\phi_S & m^2_{\sigma_S} & 2\,m^2\phi_S G_0^{-1} \\ 2\,m^2\phi_N G_0^{-1} & 2m^2\phi_S G_0^{-1} & M_G^2 \end{pmatrix}$$

Incorporating the scalar glueball (II)

$\Longrightarrow$ χ^2–fit of Λ, λ_1, h_1, m_G, ϵ_S to the following experimental quantities:

Quantity	Our Value [MeV]	Experiment [MeV]
$M_{f_0(1370)}$	1444	1350 ± 150
$M_{f_0(1500)}$	1534	1505 ± 6
$M_{f_0(1710)}$	1750	1720 ± 6
$f_0(1370) \to \pi\pi$	423.6	325 ± 100
$f_0(1500) \to \pi\pi$	39.2	38.04 ± 4.95
$f_0(1500) \to K\bar{K}$	9.1	9.37 ± 1.69
$f_0(1710) \to \pi\pi$	28.3	29.3 ± 6.5
$f_0(1710) \to K\bar{K}$	73.4	71.4 ± 29.1

χ^2/d.o.f. $\simeq 0.35$

$\Longrightarrow$ $O(3)$–mixing matrix $O \equiv \begin{pmatrix} -0.91 & 0.24 & -0.33 \\ 0.30 & 0.94 & -0.17 \\ -0.27 & 0.26 & 0.93 \end{pmatrix}$

$\Longrightarrow$

$f_0(1370)$: $83\%\,\sigma_N$ $6\%\,\sigma_S$ $11\%\,G$

$f_0(1500)$: $9\%\,\sigma_N$ $88\%\,\sigma_S$ $3\%\,G$

$f_0(1710)$: $8\%\,\sigma_N$ $6\%\,\sigma_S$ $86\%\,G$

Note: demanding dilatation symmetry of full effective model

$\Longrightarrow$ analyticity prohibits operators with naive scaling dimension higher than 4 in Φ, $\mathcal{L}^\mu$, $\mathcal{R}^\mu$ (would require inverse powers of dilaton field)

$\Longrightarrow$ effective model is complete!

Low-lying scalars (I)

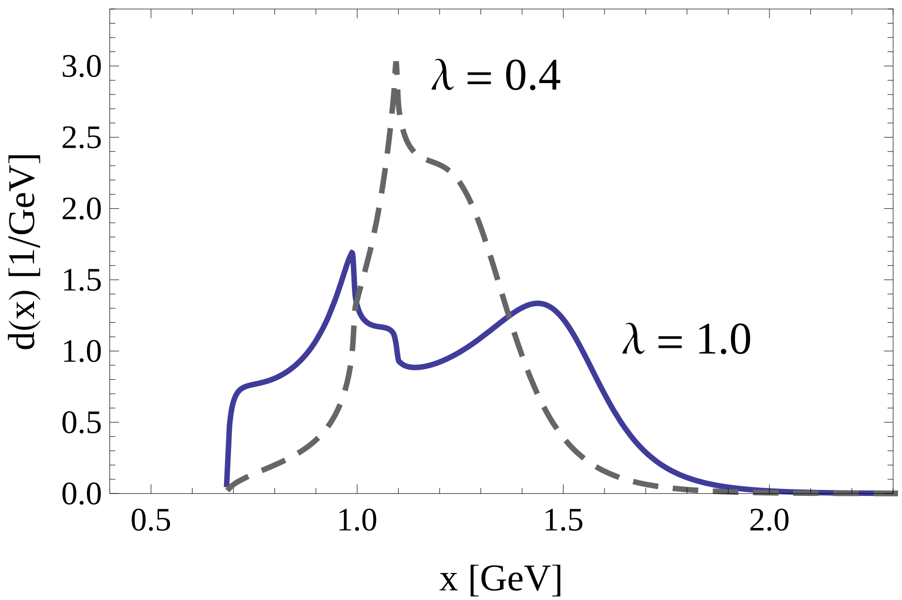
Can the low-lying scalars be "dynamically generated"?

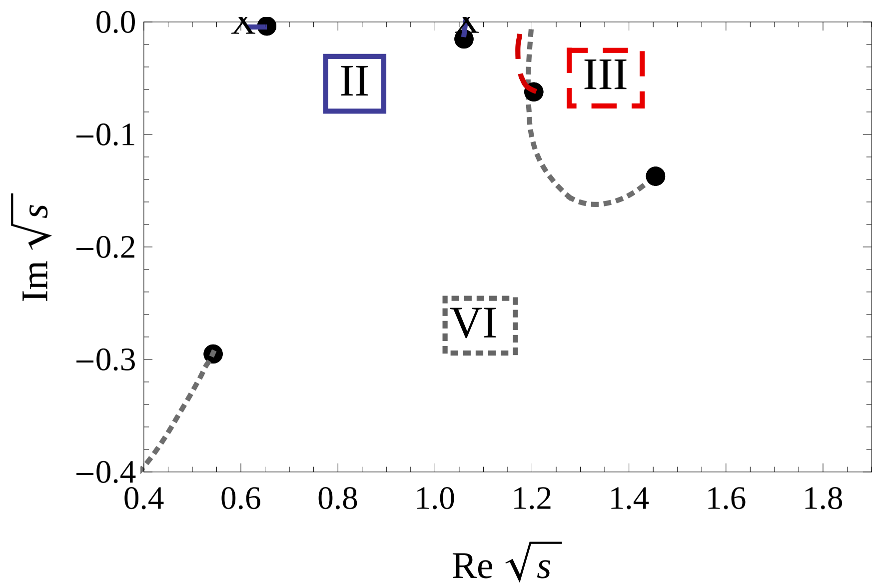
$\Longrightarrow$ **look for zeros of** $\Delta^{-1}(s) = s - m_0^2 - \Pi(s)$ **, where** $\Pi(s)$ **is 1-loop self-energy**

N.A. Törnqvist, M. Roos, PRL 76 (1996) 1575

M. Boglione, M.R. Pennington, PRD 65 (2002) 114010

$\Longrightarrow$ **study toy model inspired by eLSM**





$\Longrightarrow$ **dynamical generation of** $a_0(980)$ **,** $a_0(1450)$ **with "seed state",** $m_0 = 1.2$ **GeV**

T. Wolkanowski, F. Giacosa, DHR, PRD 93 (2016) 1, 014002

similarly: dynamical generation of $K_0^\star(800)$ **,** $K_0^\star(1430)$

T. Wolkanowski, M. Soltysiak, F. Giacosa, NPB 909 (2016) 41893

Low-lying scalars (II)

$N_f = 3$: **tetraquarks (either $[qq][\bar{q}\bar{q}]$ or $(\bar{q}q)(\bar{q}q)$ configuration) form nonet (just as $\Phi \sim \bar{q}q$)**

D. Black, A.H. Fariborz, F. Sannino, J. Schechter, PRD 59 (1999) 074026,
D. Black, A.H. Fariborz, J. Schechter, PRD 61 (2000) 074001,
A.H. Fariborz, R. Jora, J. Schechter, PRD 72 (2005) 034001,
T.K. Mukherjee, M. Huang, Q. Yan, PRD 86 (2012) 114022

$N_f = 2$: **single scalar-isoscalar state χ** $\Longrightarrow$ $f_0(500)$!!

$\Longrightarrow$ **incorporate as "interpolating field" χ in the eLSM Lagrangian**

$$\mathcal{L}_\chi = \frac{1}{2}\left(\partial_\mu \chi \partial^\mu \chi - m_\chi^2 \frac{G^2}{G_0^2} \chi^2\right) + g_\chi \frac{G}{G_0} \chi \left(\sigma^2 + \vec{\pi}^{\,2} - \eta^2 - \vec{a}_0^{\,2}\right) + g_{AV} \frac{G}{G_0} \chi \left(\vec{\rho}_\mu^{\,2} + \vec{a}_{1,\mu}^{\,2} - \omega_\mu^2 - f_{1,\mu}^2\right)$$

P. Lakaschus, J. Mauldin, F. Giacosa, DHR, in preparation

Low-lying scalars (III)

$\Longrightarrow$ set large–N_c suppressed $\lambda_1 = h_1 = 0$

$\Longrightarrow$ express m^2, c, λ_2, g_1, g_2, h_3, m_1^2 by experimental masses and decay widths

$\Longrightarrow$ perform χ^2–fit of h_2, M_G, G_0, m_χ, g_χ, g_{AV}

parameter	value
g_χ	(156 ± 215) MeV
g_{AV}	10947 ± 738 MeV
h_2	-10 ± 5
M_G	(1672 ± 120) MeV
G_0	(669 ± 738) MeV
m_χ	(539 ± 35) MeV
m^2	$-873 \cdot 10^6$ MeV^2
m_1^2	$(62 \pm 0.3) \cdot 10^3$ MeV^2
c	$(-4 \pm 11) \cdot 10^3$ MeV^2
m_σ	1401 MeV
χ_0	(13 ± 17) MeV

observable	our value	experiment
$m_{f_0(500)}$	(537 ± 34) MeV	(475 ± 75) MeV
$m_{f_0(1370)}$	(1342 ± 134) MeV	(1350 ± 150) MeV
$m_{f_0(1710)}$	(1720 ± 50) MeV	(1723 ± 5) MeV
$\Gamma_{f_0(500)\to\pi\pi}$	(505 ± 148) MeV	(550 ± 150) MeV
$\Gamma_{f_0(1370)\to\pi\pi}$	(91 ± 39) MeV	(350 ± 150) MeV
$\Gamma_{f_0(1710)\to\pi\pi}$	(29 ± 6) MeV	(29 ± 7) MeV
$m_\pi a_0^0$	0.207 ± 0.016	0.218 ± 0.02
$m_\pi a_0^2$	-0.028 ± 0.005	-0.046 ± 0.016

χ^2 test	value	
χ^2	3.1	
χ^2_{red}	1.5	

$\Longrightarrow$ reasonable description of $\pi\pi$ scattering lengths!

$\Longrightarrow$ mixing matrix:

$$
\begin{aligned}
f_0(500):&\quad 100\%\,\chi \quad 0\%\,\sigma \quad 0\%\,G \\
f_0(1370):&\quad 0\%\,\chi \quad 86\%\,\sigma \quad 14\%\,G \\
f_0(1710):&\quad 0\%\,\chi \quad 14\%\,\sigma \quad 86\%\,G
\end{aligned}
$$

Conclusions and Outlook

I. extended Linear Sigma Model (eLSM) with $U(N_f)_r \times U(N_f)_\ell$ symmetry, containing scalar and vector mesons and their chiral partners

II. Vacuum phenomenology:

1. **Excellent fit of mesonic vacuum properties for $N_f = 3$**
2. **Correct low-energy limit of QCD:**
   resonance-saturation mechanism (cf. χPT) seems to work also for eLSM
   pion-loop corrections still need to be computed via FRG
3. **Scalar-meson puzzle:**
   evidence for dominant four-quark component for the light scalar mesons
   glueball is most likely (predominantly) $f_0(1710)$
4. **Including $f_0(500)$ as an effective d.o.f. improves description of $\pi\pi$ scattering lengths**

Extension to $N_f = 4$

Fit of 3(!) additional parameters from the charm sector:

Observable	Our Value [MeV]	Exp. Value [MeV]
m_{D_0}	1981 ± 73	1864.86 ± 0.13
$m_{D_s^\pm}$	2004 ± 74	1968.50 ± 0.32
m_{η_c}	2673 ± 118	2983.7 ± 0.7
$m_{D_0^{*0}}$	2414 ± 77	2318 ± 29
$m_{D_{s0}^{*\pm}}$	2467 ± 76	2317.8 ± 0.6
$m_{\chi_{c0}}$	3144 ± 128	3414.75 ± 0.31
$m_{D^{*0}}$	2168 ± 70	2006.99 ± 0.15
$m_{D_s^*}$	2203 ± 69	2112.3 ±0.5
$m_{J/\psi}$	2947 ± 109	3096.916 ± 0.011
$m_{D_1^0}$	2429 ± 63	2421.4 ± 0.6
$m_{D_{s1}^\pm}$	2480 ± 63	2535.12 ± 0.13
$m_{\chi_{c1}}$	3239 ± 101	3510.66 ± 0.07
$\Gamma_{D_0^{*0}\to D\pi}$	139^{+243}_{-114}	$D^+\pi^-$ seen, full width 276 ± 40
$\Gamma_{D_0^{*+}\to D\pi}$	51^{+182}_{-51}	$D^+\pi^0$ seen, full width 283 ± 24 ±34
$\Gamma_{D^{*0}\to D^0\pi^0}$	0.025 ± 0.003	seen, <1.3
$\Gamma_{D^{*0}\to D^+\pi^-}$	0	not seen
$\Gamma_{D^{*+}\to D^+\pi^0}$	$0.018^{+0.002}_{-0.003}$	0.029 ± 0.008
$\Gamma_{D^{*+}\to D^0\pi^+}$	$0.038^{+0.005}_{-0.004}$	0.065 ± 0.017
$\Gamma_{D_1^0\to D^*\pi}$	65^{+51}_{-37}	$D^{*+}\pi^-$ seen, full width 27.4 ± 2.5
$\Gamma_{D_1^0\to D^0\pi\pi}$	0.59 ± 0.02	seen
$\Gamma_{D_1^0\to D^+\pi^-\pi^0}$	$0.21^{+0.01}_{-0.015}$	seen
$\Gamma_{D_1^0\to D^+\pi^-}$	0	not seen
$\Gamma_{D_1^+\to D^*\pi}$	65^{+51}_{-36}	$D^{*0}\pi^+$ seen, full width 25 ± 6
$\Gamma_{D_1^+\to D^+\pi\pi}$	0.56 ± 0.02	seen
$\Gamma_{D_1^+\to D^0\pi^0\pi^+}$	0.22 ± 0.01	seen
$\Gamma_{D_1^+\to D^0\pi^+}$	0	not seen
$\Gamma_{D_{s1}^+\to D^*K}$	25^{+22}_{-15}	seen, full width 0.92 ± 0.03 ± 0.04
$\Gamma_{D_{s1}^+\to D^+K^0}$	0	not seen
$\Gamma_{D_{s1}^+\to D^0K^+}$	0	not seen

see W.I. Eshraim, F. Giacosa, DHR, EPJA 51 (2015) 112

Electroweak interactions

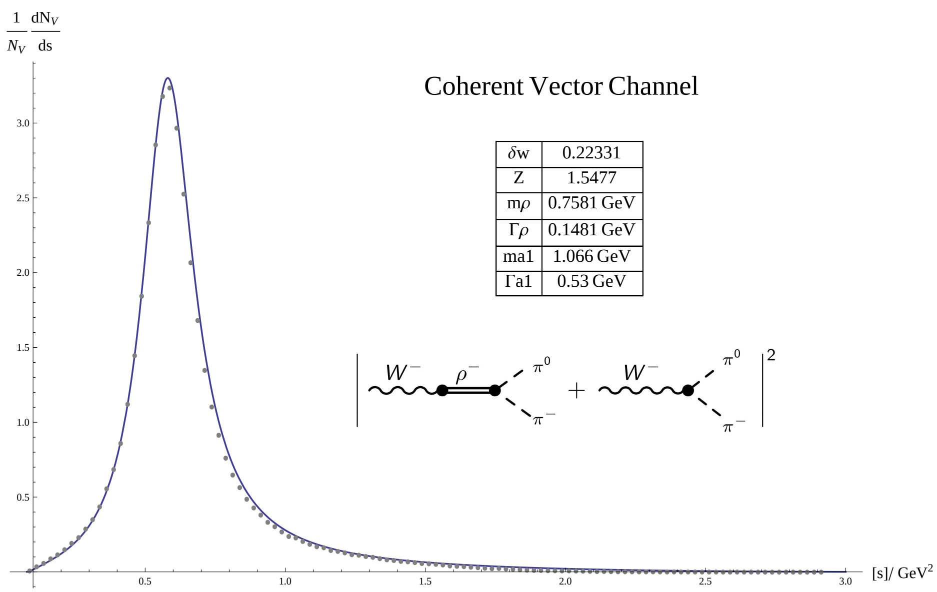
A. Habersetzer, F. Giacosa, DHR, in preparation

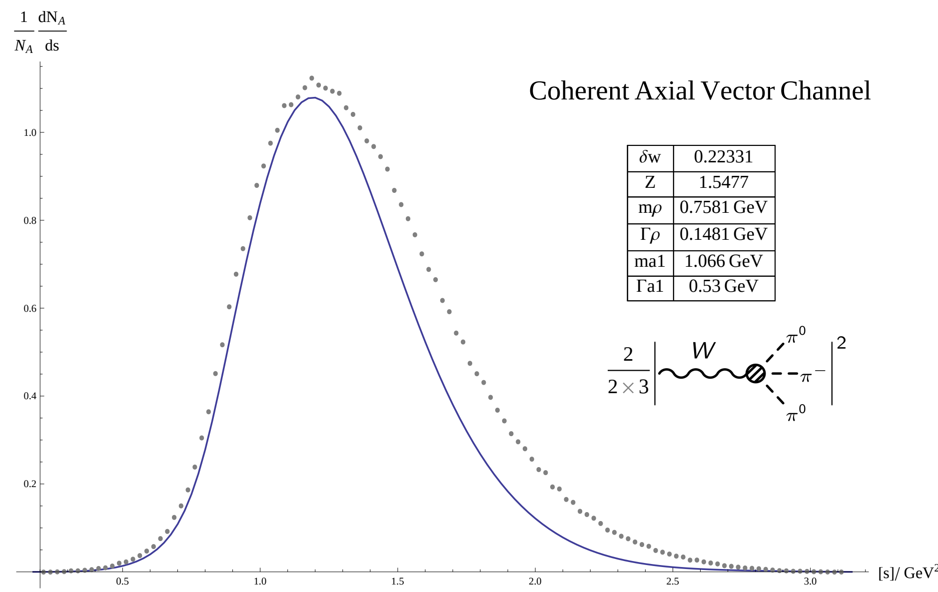
$$\partial^\mu \Phi \longrightarrow D^\mu \Phi \equiv \partial^\mu \Phi - i\, e\, A^\mu\, [T_3, \Phi] - i\, g \cos\theta_C\, (W_1^\mu T_1 + W_2^\mu T_2)\Phi$$
$$- i\, g \cos\theta_W\, (Z^\mu T_3 \Phi + \tan^2\theta_W \Phi T_3 Z^\mu)$$

$$\mathcal{L}_0^{\mu\nu} \longrightarrow \mathcal{L}^{\mu\nu} \equiv \partial^\mu \mathcal{L}^\nu - i\, e\, A^\mu [T_3, \mathcal{L}^\nu] - i\, g [W_1^\mu T_1 + W_2^\mu T_2, \mathcal{L}^\nu] - ig \cos\theta_W Z^\mu [T_3, \mathcal{L}^\nu]$$
$$- \partial^\nu \mathcal{L}^\mu + i\, e\, A^\nu [T_3, \mathcal{L}^\mu] + i\, g [W_1^\nu T_1 + W_2^\nu T_2, \mathcal{L}^\mu] + ig \cos\theta_W Z^\nu [T_3, \mathcal{L}^\mu]$$

$$\mathcal{R}_0^{\mu\nu} \longrightarrow \mathcal{R}^{\mu\nu} \equiv \partial^\mu \mathcal{R}^\nu - i\, e\, A^\mu [T_3, \mathcal{R}^\nu] - ig \sin\theta_W Z^\mu [T_3, \mathcal{R}^\nu]$$
$$- \partial^\nu \mathcal{R}^\mu + i\, e\, A^\nu [T_3, \mathcal{R}^\mu] + ig \sin\theta_W Z^\nu [T_3, \mathcal{R}^\mu]$$

$$\mathcal{L}_M \longrightarrow \mathcal{L}_M + \frac{\delta_W}{2} g\, \cos\theta_C\, \mathrm{Tr}[W_{\mu\nu} \mathcal{L}^{\mu\nu}] + \frac{\delta_Y}{2} e\, \mathrm{Tr}[B_{\mu\nu} \mathcal{R}^{\mu\nu}] + \frac{1}{4} \mathrm{Tr}[(W^{\mu\nu})^2 + (B^{\mu\nu})^2]$$





cf. **M. Urban, M. Buballa, J. Wambach, NPA 697 (2002) 338**

Baryons and their chiral partners

Inclusion of baryons and their chiral partners ($N_f = 2$):

$\Longrightarrow$ **Mirror assignment:** C. DeTar and T. Kunihiro, PRD 39 (1989) 2805

$$\Psi_{1,r} \to U_r \, \Psi_{1,r} \ , \quad \Psi_{1,\ell} \to U_\ell \, \Psi_{1,\ell} \ , \quad \text{but: } \Psi_{2,r} \to U_\ell \, \Psi_{2,r} \ , \quad \Psi_{2,\ell} \to U_r \, \Psi_{2,\ell}$$

$\Longrightarrow$ **new, chirally invariant mass term:**

$$\mathcal{L}_B = \bar{\Psi}_{1,\ell} \, i\partial\!\!\!/ \, \Psi_{1,\ell} + \bar{\Psi}_{1,r} \, i\partial\!\!\!/ \, \Psi_{1,r} + \bar{\Psi}_{2,\ell} \, i\partial\!\!\!/ \, \Psi_{2,\ell} + \bar{\Psi}_{2,r} \, i\partial\!\!\!/ \, \Psi_{2,r} + m_0 \left(\bar{\Psi}_{2,\ell} \, \Psi_{1,r} - \bar{\Psi}_{2,r} \, \Psi_{1,\ell} - \bar{\Psi}_{1,\ell} \, \Psi_{2,r} + \bar{\Psi}_{1,r} \, \Psi_{2,\ell} \right)$$

Note: chiral symmetry restoration:

chiral partners become degenerate, but not necessarily massless!

$\Longrightarrow$ **m_0 models contribution from gluon condensate to baryon mass**

$\Longrightarrow$ **allows for stable nuclear matter ground state!**

Vector – baryon interactions

$$\mathcal{L}_{VB} = c_1 \left(\bar{\Psi}_{1,\ell} \not{\mathcal{L}} \Psi_{1,\ell} + \bar{\Psi}_{1,r} \not{\mathcal{R}} \Psi_{1,r}\right) + c_2 \left(\bar{\Psi}_{2,\ell} \not{\mathcal{R}} \Psi_{2,\ell} + \bar{\Psi}_{2,r} \not{\mathcal{L}} \Psi_{2,r}\right)$$

Note: in general $c_1 \neq c_2$

$\Longrightarrow$ allows to fit axial coupling constants (see below)!

Scalar – baryon interactions

Yukawa interaction:

$$\mathcal{L}_{SB} = -\hat{g}_1 \left(\bar{\Psi}_{1,\ell}\,\Phi\,\Psi_{1,r} + \bar{\Psi}_{1,r}\,\Phi^\dagger\,\Psi_{1,\ell}\right) - \hat{g}_2 \left(\bar{\Psi}_{2,r}\,\Phi\,\Psi_{2,\ell} + \bar{\Psi}_{2,\ell}\,\Phi^\dagger\,\Psi_{2,r}\right)$$

$N_f = 2$ **mass eigenstates:**

$$\begin{pmatrix} N \\ N^\star \end{pmatrix} \equiv \begin{pmatrix} N^+ \\ N^- \end{pmatrix} = \frac{1}{\sqrt{2\cosh\delta}} \begin{pmatrix} e^{\delta/2} & \gamma_5\, e^{-\delta/2} \\ \gamma_5\, e^{-\delta/2} & -e^{\delta/2} \end{pmatrix} \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix}\,, \quad \sinh\delta = \frac{\phi}{4\,m_0}(\hat{g}_1 + \hat{g}_2)$$

$$m_\pm = \sqrt{m_0^2 + \frac{\phi^2}{16}(\hat{g}_1 + \hat{g}_2)^2} \pm \frac{\phi}{4}(\hat{g}_1 - \hat{g}_2) \longrightarrow m_0 \qquad (\phi \to 0)$$

axial coupling constant:

$$g_A = +\tanh\delta \left[1 - \frac{c_1 + c_2}{2\,g_1}\left(1 - \frac{1}{Z^2}\right)\right] - \frac{c_1 - c_2}{2\,g_1}\left(1 - \frac{1}{Z^2}\right)$$

$$g_A^\star = -\tanh\delta \left[1 - \frac{c_1 + c_2}{2\,g_1}\left(1 - \frac{1}{Z^2}\right)\right] - \frac{c_1 - c_2}{2\,g_1}\left(1 - \frac{1}{Z^2}\right) \neq -g_A\,!$$

$\Longrightarrow$ for $c_1 \neq c_2$ compatible with $g_A \simeq 1.26\,,\ g_A^\star \simeq 0\,!$

T.T. Takahashi, T. Kunihiro, PRD 78 (2008) 011503

T. Maurer, T. Burch, L.Ya. Glozman, C.B. Lang, D. Mohler, A. Schäfer, arXiv:1202.2834[hep-lat]

Vacuum phenomenology: The chiral partner of the nucleon (I)

Baryon sector ($N_f = 2$): S. Gallas, F. Giacosa, DHR, PRD 82 (2010) 014004

Determine m_0 , c_1 , c_2 , $\hat{g}_1$, $\hat{g}_2$ **through χ^2–fit to**

$$M_N\,,\ M_{N^\star}\,,\ g_A = 1.267 \pm 0.004\,,\ g_A^\star\,,\ \Gamma(N^\star \to N\pi)$$

(i) Scenario A: $N = N(940),\ N^\star = N(1535)$

$\Longrightarrow$ $g_A^\star = 0.2 \pm 0.3$ T.T. Takahashi, T. Kunihiro, PRD 78 (2008) 011503

$\Gamma(N^\star \to N\pi) = (67.5 \pm 23.6)$ MeV

(ii) Scenario B: $N = N(940),\ N^\star = N(1650)$

$\Longrightarrow$ $g_A^\star = 0.55 \pm 0.2$ T.T. Takahashi, T. Kunihiro, PRD 78 (2008) 011503

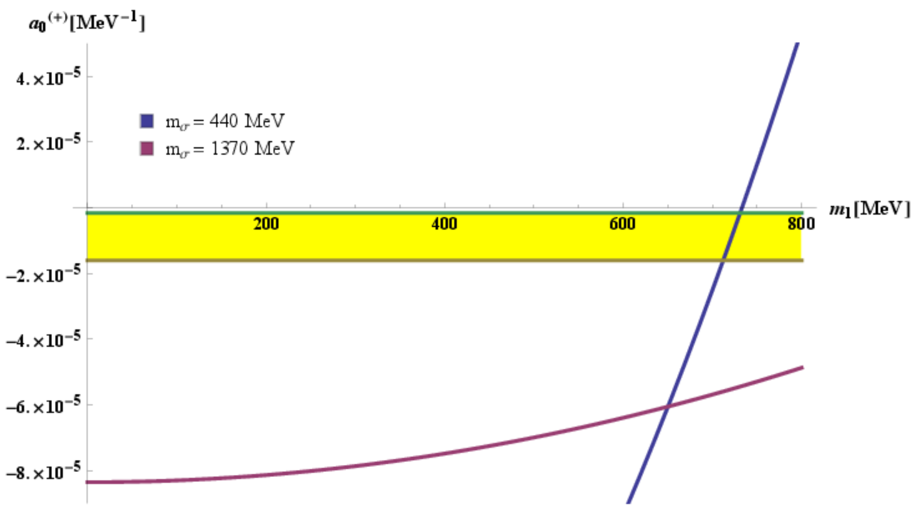
$\Gamma(N^\star \to N\pi) = (128 \pm 44)$ MeV

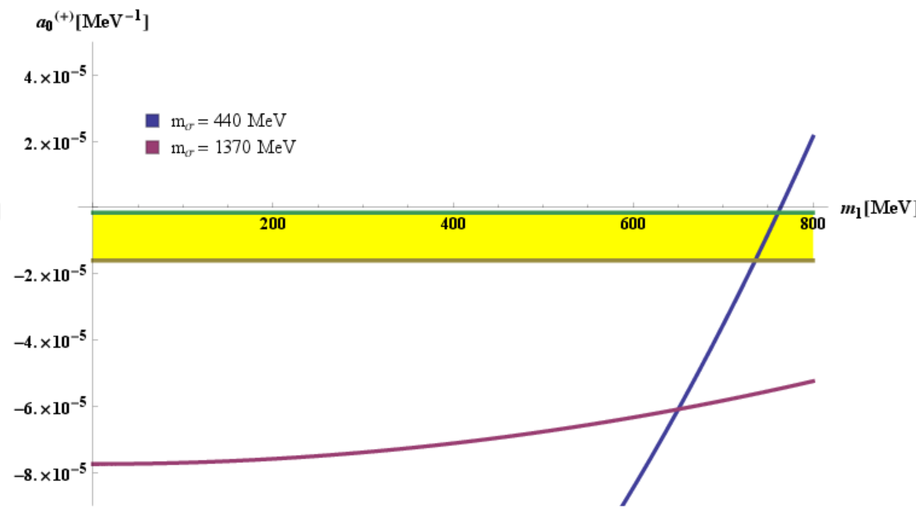
Test validity of the two scenarios through comparison to:

- πN **scattering lengths**
- **decay width** $\Gamma(N^\star \to N\eta)$

Vacuum phenomenology: The chiral partner of the nucleon (II)

πN scattering lengths $a_0^{(\pm)}$:





$\Longrightarrow$ $a_0^{(+)}$ requires a light σ!

$a_0^{(-)} = (6.04 \pm 0.63) \cdot 10^{-4}$ MeV^{-1} $\qquad$ $a_0^{(-)} = (5.90 \pm 0.46) \cdot 10^{-4}$ MeV^{-1}

for comparison: $a_{0,\mathrm{exp}}^{(-)} = (6.4 \pm 0.1) \cdot 10^{-4}$ MeV^{-1}

However:

$\Gamma(N^\star \to N\eta) = (10.9 \pm 3.8)$ MeV $\qquad$ $\Gamma(N^\star \to N\eta) = (18.3 \pm 8.5)$ MeV

$\Gamma_{\mathrm{exp}}(N^\star \to N\eta) = (78.7 \pm 24.3)$ MeV! $\qquad$ $\Gamma_{\mathrm{exp}}(N^\star \to N\eta) = (10.7 \pm 6.7)$ MeV

Vacuum phenomenology: The chiral partner of the nucleon (III)

Inclusion of $f_0(500)$ and $f_0(1710)$:

P. Lakaschus, J. Mauldin, F. Giacosa, DHR, in preparation

$\Longrightarrow$ **mass parameter m_0 generated by interaction Lagrangian:**

$$\mathcal{L}_{\chi GN} = -\left[a\chi + bG + c_N\left(\det\Phi + \det\Phi^\dagger\right)\right]\left(\bar{\Psi}_{2,\ell}\,\Psi_{1,r} - \bar{\Psi}_{2,r}\,\Psi_{1,\ell} - \bar{\Psi}_{1,\ell}\,\Psi_{2,r} + \bar{\Psi}_{1,r}\,\Psi_{2,\ell}\right)$$

and condensation $\chi \to \chi_0$, $G \to G_0$, $\sigma \to \phi$: $m_0 = a\chi_0 + bG_0 + \frac{c_N}{2}\phi^2$

$\Longrightarrow$ **new contributions of χ and G to πN scattering lengths:**

parameter	our value	experiment	χ^2
$m_\pi\, a_0^{(+)}$	-0.0016	-0.0012 ± 0.0010	0.1
$m_\pi^3\, a_{1+}^{(+)}$	0.045	0.133 ± 0.004	484.2
$m_\pi^3\, a_{1-}^{(+)}$	-0.097	-0.056 ± 0.010	16.6
$m_\pi^3\, r_0^{(+)}$	0.02	-0.06 ± 0.02	15.9

$\Longrightarrow$ **description of $a_0^{(+)}$ considerably improved!**

Extension to $N_f = 3$ and four baryon multiplets (I)

L. Olbrich, M. Zetenyi, F. Giacosa, DHR, PRD 93 (2016) 3, 034021

Assume baryons to be $q[qq]$ composites $\Longrightarrow$ $B \in (N_f, N_f^*)$:

$$B = \begin{pmatrix} \frac{\Lambda}{\sqrt{6}} + \frac{\Sigma^0}{\sqrt{2}} & \Sigma^+ & p \\ \Sigma^- & \frac{\Lambda}{\sqrt{6}} - \frac{\Sigma^0}{\sqrt{2}} & n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}$$

Scalar diquark fields with definite parity:

$$J^P = 0^+ : \qquad \mathcal{D}_{ij} = \frac{1}{\sqrt{2}} \left(q_j^T C \gamma^5 q_i - q_i^T C \gamma^5 q_j \right) \equiv \sum_{k=1}^{3} D_k \epsilon_{kij}$$

$$J^P = 0^- : \qquad \tilde{\mathcal{D}}_{ij} = \frac{1}{\sqrt{2}} \left(q_j^T C q_i - q_i^T C q_j \right) \equiv \sum_{k=1}^{3} \tilde{D}_k \epsilon_{kij}$$

$\Longrightarrow$ **Right- and left-handed diquark fields:**

$$D_k^{r(\ell)} \equiv \frac{1}{\sqrt{2}} \left(\tilde{D}_k \pm D_k \right)$$

$\Longrightarrow$ **Under chiral transformations:**

$$D_k^{r(\ell)} \longrightarrow D_k^{r(\ell)} U_{r(\ell)}^{\dagger}$$

Extension to $N_f = 3$ and four baryon multiplets (II)

⟹ Right-and left-handed matrix-valued baryon fields $N_{1r(\ell)}$, $N_{2r(\ell)}$:

$$\left(N_{1r(\ell)}\right)_{ij} \equiv D_j^r q_{ir(\ell)} \;, \qquad \left(N_{2r(\ell)}\right)_{ij} \equiv D_j^\ell q_{ir(\ell)}$$

⟹ Under chiral transformations:

$$N_{1r} \longrightarrow U_r N_{1r} U_r^\dagger \,, \quad N_{1\ell} \longrightarrow U_\ell N_{1\ell} U_r^\dagger \,, \quad N_{2r} \longrightarrow U_r N_{2r} U_\ell^\dagger \,, \quad N_{2\ell} \longrightarrow U_\ell N_{2\ell} U_\ell^\dagger$$

⟹ Right-and left-handed "mirror" baryon fields $M_{1r(\ell)}$, $M_{2r(\ell)}$:

$$\left(M_{1r(\ell)}\right)_{ij} \equiv D_j^r \,\partial\!\!\!/\, q_{ir(\ell)} \;, \qquad \left(M_{2r(\ell)}\right)_{ij} \equiv D_j^\ell \,\partial\!\!\!/\, q_{ir(\ell)}$$

⟹ Under chiral transformations:

$$M_{1r} \longrightarrow U_\ell M_{1r} U_r^\dagger \,, \quad M_{1\ell} \longrightarrow U_r M_{1\ell} U_r^\dagger \,, \quad M_{2r} \longrightarrow U_\ell M_{2r} U_\ell^\dagger \,, \quad M_{2\ell} \longrightarrow U_r M_{2\ell} U_\ell^\dagger$$

⟹ Form linear combinations with definite positive/negative parity:

$$B_N = \frac{1}{\sqrt{2}}(N_1 - N_2) \;, \quad B_{N\star} = \frac{1}{\sqrt{2}}(N_1 + N_2) \,, \quad B_M = \frac{1}{\sqrt{2}}(M_1 - M_2) \;, \quad B_{M\star} = \frac{1}{\sqrt{2}}(M_1 + M_2)$$

Assignment to physical particles (zero-mixing limit):

B_N : $\{N(939),\, \Lambda(1116),\, \Sigma(1193),\, \Xi(1338)\}$, B_M : $\{N(1440), \Lambda(1600), \Sigma(1620), \Xi(1690)\}$,
$B_{N\star}$: $\{N(1535),\, \Lambda(1670), \Sigma(1620), \Xi(?)\}$, $B_{M\star}$: $\{N(1650), \Lambda(1800), \Sigma(1750), \Xi(?)\}$.

Extension to $N_f = 3$ and four baryon multiplets (III)

Lagrangian:

$$
\begin{aligned}
\mathcal{L} \; = \; & \mathrm{Tr}\left\{\bar{N}_{1r} i \not{D}_{1r} N_{1r} + \bar{N}_{1\ell} i \not{D}_{2\ell} N_{1\ell} + \bar{N}_{2r} i \not{D}_{2r} N_{2r} + \bar{N}_{2\ell} i \not{D}_{1\ell} N_{2\ell}\right\} \\
+ \; & \mathrm{Tr}\left\{\bar{M}_{1r} i \not{D}_{3\ell} M_{1r} + \bar{M}_{1\ell} i \not{D}_{4r} M_{1\ell} + \bar{M}_{2r} i \not{D}_{4\ell} M_{2r} + \bar{M}_{2\ell} i \not{D}_{3r} M_{2\ell}\right\} \\
- \; & g_N \mathrm{Tr}\left\{\bar{N}_{1\ell} \Phi N_{1r} + \bar{N}_{1r} \Phi^\dagger N_{1\ell} + \bar{N}_{2\ell} \Phi N_{2r} + \bar{N}_{2r} \Phi^\dagger N_{2\ell}\right\} \\
- \; & g_M \mathrm{Tr}\left\{\bar{M}_{1\ell} \Phi^\dagger M_{1r} + \bar{M}_{1r} \Phi M_{1\ell} + \bar{M}_{2\ell} \Phi^\dagger M_{2r} + \bar{M}_{2r} \Phi M_{2\ell}\right\} \\
- \; & m_{0,1} \mathrm{Tr}\left\{\bar{M}_{1r} N_{1\ell} + \bar{M}_{2\ell} N_{2r} + \bar{N}_{1\ell} M_{1r} + \bar{N}_{2r} M_{2\ell}\right\} \\
- \; & m_{0,2} \mathrm{Tr}\left\{\bar{M}_{1\ell} N_{1r} + \bar{M}_{2r} N_{2\ell} + \bar{N}_{1r} M_{1\ell} + \bar{N}_{2\ell} M_{2r}\right\} \\
- \; & \kappa_1 \mathrm{Tr}\left\{\bar{N}_{2\ell} \Phi N_{1r} \Phi^\dagger + \bar{N}_{1r} \Phi^\dagger N_{2\ell} \Phi\right\} - \kappa_1' \mathrm{Tr}\left\{\bar{N}_{2r} \Phi^\dagger N_{1\ell} \Phi^\dagger + \bar{N}_{1\ell} \Phi N_{2r} \Phi\right\} \\
- \; & \kappa_2 \mathrm{Tr}\left\{\bar{M}_{2\ell} \Phi^\dagger M_{1r} \Phi^\dagger + \bar{M}_{1r} \Phi M_{2\ell} \Phi\right\} - \kappa_2' \mathrm{Tr}\left\{\bar{M}_{2r} \Phi M_{1\ell} \Phi^\dagger + \bar{M}_{1\ell} \Phi^\dagger M_{2r} \Phi\right\} \\
- \; & \epsilon_1 \left(\mathrm{Tr}\left\{\bar{N}_{2r} \Phi^\dagger\right\} \mathrm{Tr}\left\{N_{1\ell} \Phi^\dagger\right\} + \mathrm{Tr}\left\{\bar{N}_{1\ell} \Phi\right\} \mathrm{Tr}\left\{N_{2r} \Phi\right\}\right) \\
- \; & \epsilon_2 \left(\mathrm{Tr}\left\{\bar{M}_{2\ell} \Phi^\dagger\right\} \mathrm{Tr}\left\{M_{1r} \Phi^\dagger\right\} + \mathrm{Tr}\left\{\bar{M}_{1r} \Phi\right\} \mathrm{Tr}\left\{M_{2\ell} \Phi\right\}\right) \\
- \; & \epsilon_3 \left(\mathrm{Tr}\left\{\Phi^\dagger \Phi\right\} \mathrm{Tr}\left\{\bar{M}_{1r} N_{1\ell} + \bar{M}_{2\ell} N_{2r} + \bar{N}_{1\ell} M_{1r} + \bar{N}_{2r} M_{2\ell}\right\}\right) \\
- \; & \epsilon_4 \left(\mathrm{Tr}\left\{\Phi^\dagger \Phi\right\} \mathrm{Tr}\left\{\bar{M}_{1\ell} N_{1r} + \bar{M}_{2r} N_{2\ell} + \bar{N}_{1r} M_{1\ell} + \bar{N}_{2\ell} M_{2r}\right\}\right)
\end{aligned}
$$

where $D^\mu_{kr} = \partial^\mu - i c_k \mathcal{R}^\mu \; , \;\; D^\mu_{k\ell} = \partial^\mu - i c_k \mathcal{L}^\mu$

$\Longrightarrow$ reduction to $N_f = 2$: $N(939)$, $N(1440)$, $N(1535)$, $N(1650)$

Extension to $N_f = 3$ and four baryon multiplets (IV)

$\Longrightarrow$ **χ^2–fit of 12 parameters to 13 experimental quantities:**

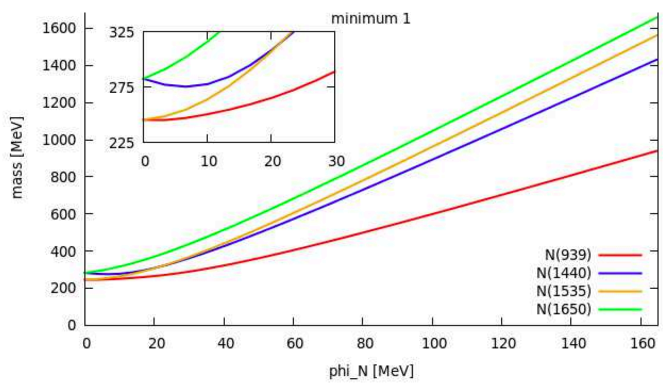
	our results [GeV]	experiment [GeV]
m_N	0.9389 ± 0.001	0.9389 ± 0.001
$m_{N(1440)}$	1.430 ± 0.0713	1.43 ± 0.0715
$m_{N(1535)}$	1.561 ± 0.0668	1.53 ± 0.0765
$m_{N(1650)}$	1.657 ± 0.0721	1.65 ± 0.087
$\Gamma_{N(1440)\to N\pi}$	0.1948 ± 0.0870	0.195 ± 0.087
$\Gamma_{N(1535)\to N\pi}$	0.0722 ± 0.0188	0.0675 ± 0.0183
$\Gamma_{N(1535)\to N\eta}$	0.0055 ± 0.0026	0.063 ± 0.0183
$\Gamma_{N(1650)\to N\pi}$	0.1121 ± 0.0331	0.105 ± 0.0366
$\Gamma_{N(1650)\to N\eta}$	0.0117 ± 0.0038	0.015 ± 0.008

	our results	experiment/lattice
g_A^N	1.267 ± 0.0025	1.267 ± 0.0025
$g_A^{N(1440)}$	1.2 ± 0.2	1.2 ± 0.2
$g_A^{N(1535)}$	0.2 ± 0.3	0.2 ± 0.3
$g_A^{N(1650)}$	0.5494 ± 0.2	0.55 ± 0.2

Mixing matrix:

$$\begin{pmatrix} N(939) \\ \gamma^5 N(1535) \\ N(1440) \\ \gamma^5 N(1650) \end{pmatrix} = \begin{pmatrix} -0.994 & -0.012 & -0.019 & 0.104 \\ 0.098 & -0.487 & -0.041 & 0.867 \\ -0.009 & 0.085 & 0.992 & 0.096 \\ -0.042 & -0.869 & 0.121 & -0.478 \end{pmatrix} \begin{pmatrix} \Psi_N \\ \gamma^5 \Psi_{N\star} \\ \Psi_M \\ \gamma^5 \Psi_{M\star} \end{pmatrix}$$

Masses as function of φ_N:



$\Longrightarrow$ **Mixing matrix:** $N(939) \to N$, $N(1535) \to M^\star$, $N(1440) \to M$, $N(1650) \to N^\star$

$\Longrightarrow$ **Chiral partners:** $N(939) \longleftrightarrow N(1535)$, $N(1440) \longleftrightarrow N(1650)$

$U(1)_A$ anomaly and $N(1535) \to N\eta$ decay

L. Olbrich, M. Zetenyi, F. Giacosa, DHR, arXiv:1708.01061 [hep-ph]

$N_f = 2$: $\det\Phi - \det\Phi^\dagger = -i(\sigma_N \eta_N - \vec{a}_0 \cdot \vec{\pi})$ is parity-odd, $U(1)_A$ violating

$\bar{\Psi}_{2,\ell}\,\Psi_{1,r} + \bar{\Psi}_{2,r}\,\Psi_{1,\ell} - \bar{\Psi}_{1,\ell}\,\Psi_{2,r} - \bar{\Psi}_{1,r}\,\Psi_{2,\ell}$ is parity-odd

$\Longrightarrow$ $$\mathcal{L}_A = \lambda_A \left(\det\Phi - \det\Phi^\dagger\right)\left(\bar{\Psi}_{2,\ell}\,\Psi_{1,r} + \bar{\Psi}_{2,r}\,\Psi_{1,\ell} - \bar{\Psi}_{1,\ell}\,\Psi_{2,r} - \bar{\Psi}_{1,r}\,\Psi_{2,\ell}\right)$$

is parity-even, $U(1)_A$ violating

$\Longrightarrow$ SSB: direct coupling $N(1535)N\eta$

$\Longrightarrow$ adjust λ_A to reproduce $\Gamma(N(1535) \to N\eta)$!

$N_f = 3$: $$\mathcal{L}_A = \lambda_A \left(\det\Phi - \det\Phi^\dagger\right) \mathrm{Tr}\left(\bar{B}_{M\star} B_N - \bar{B}_N B_{M\star} - \bar{B}_{N\star} B_M + \bar{B}_M B_{N\star}\right)$$

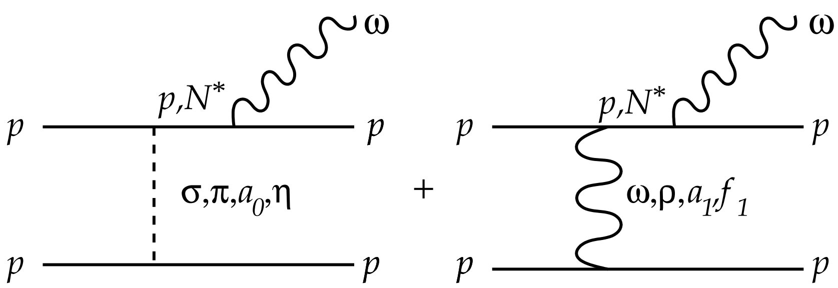
$\Longrightarrow$ adjust λ_A to reproduce $\Gamma(N(1535) \to N\eta)$

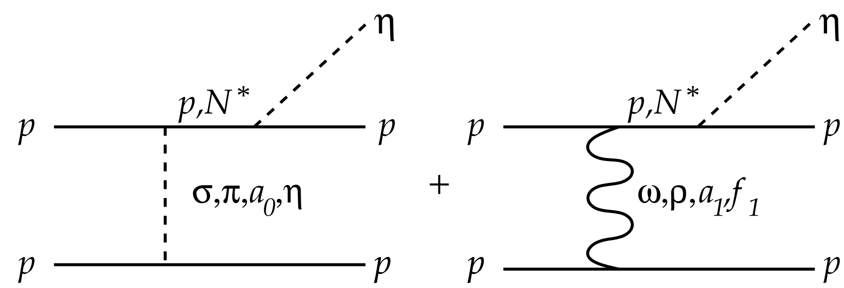
$\Longrightarrow$ predict $\Gamma(\Lambda(1670) \to \Lambda\eta) = 5.1^{+2.7}_{-2.1}$ MeV

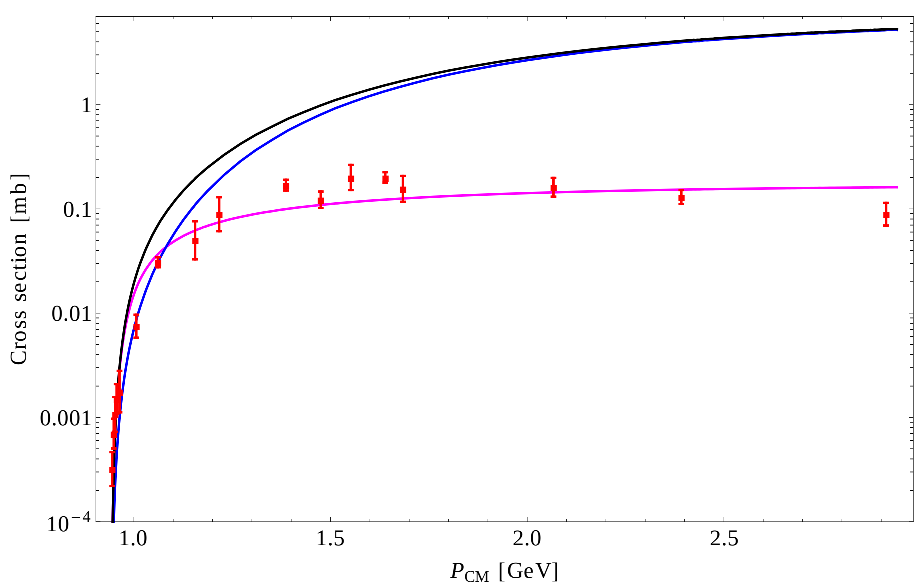
(cf. $\Gamma_{\rm exp}(\Lambda(1670) \to \Lambda\eta) = (7.5 \pm 5)$ MeV)

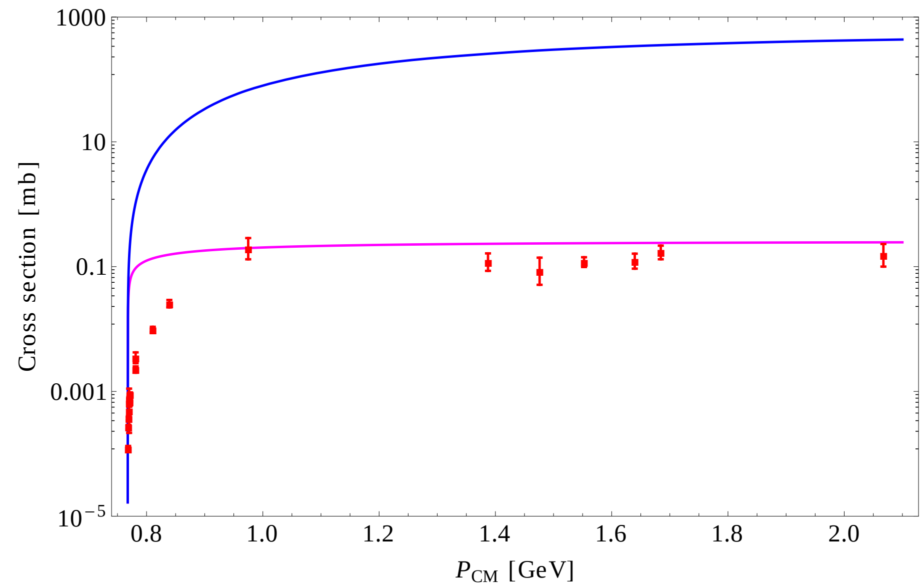
Exclusive hadron production in pp

K. Teilab, F. Giacosa, DHR, in preparation **preliminary!**









Born: p only, **Born: incl. N*,** **K-matrix unitarized,**

data: SPES III, PINOT, COSY-TOF, COSY-11