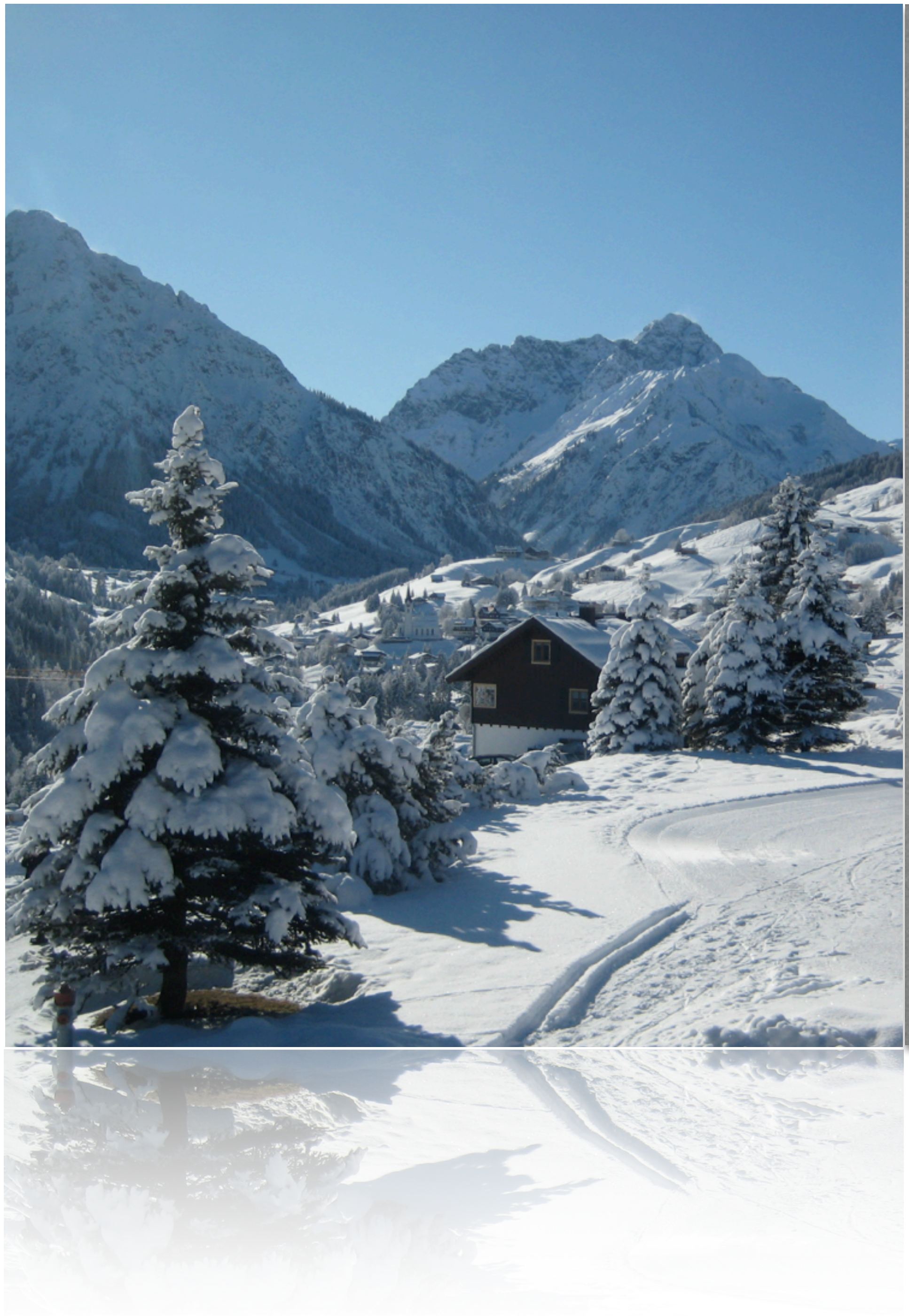

Two photon decay width of $a_0(980)$ in a dispersive approach

Igor Danilkin

Phys. Rev. D96 (2017) in coll. with
Oleksandra Deineka & Marc Vanderhaeghen

Hirschegg 2018, January 17, 2018

SFB 1044
THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

JG|U

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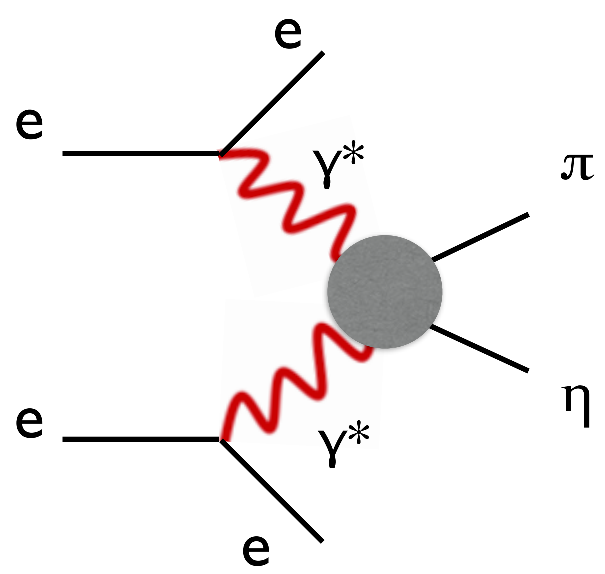
- Introduction & motivation for $a_0(980)\to\gamma\gamma$ (and related $\gamma\gamma\to\pi\eta$)
- First principle constraints (unitarity and analyticity)
- Method (Omnes formalism)
- Results and outlook

Experiment

Observables in experiment $e^+e^- \to e^-e^+\pi\eta$



$$d\sigma = \frac{\alpha^2}{16\pi^4\, Q_1^2\, Q_2^2}\, \frac{2\sqrt{X}}{s(1-4m^2/s)^{1/2}} \cdot \frac{d^3\vec{p}_1{}'}{E_1'} \cdot \frac{d^3\vec{p}_2{}'}{E_2'}$$
$$\times \left\{4\,\rho_1^{++}\,\rho_2^{++}\,\sigma_{TT} + \rho_1^{00}\,\rho_2^{00}\,\sigma_{LL} + 2\,\rho_1^{++}\,\rho_2^{00}\,\sigma_{TL} + \ldots\right\},$$

C=+1: $J^{PC}=0^{++}, 2^{++}, \ldots.$

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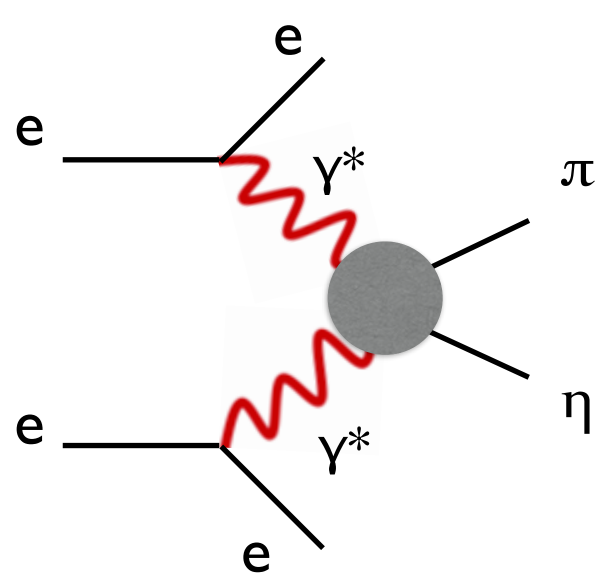

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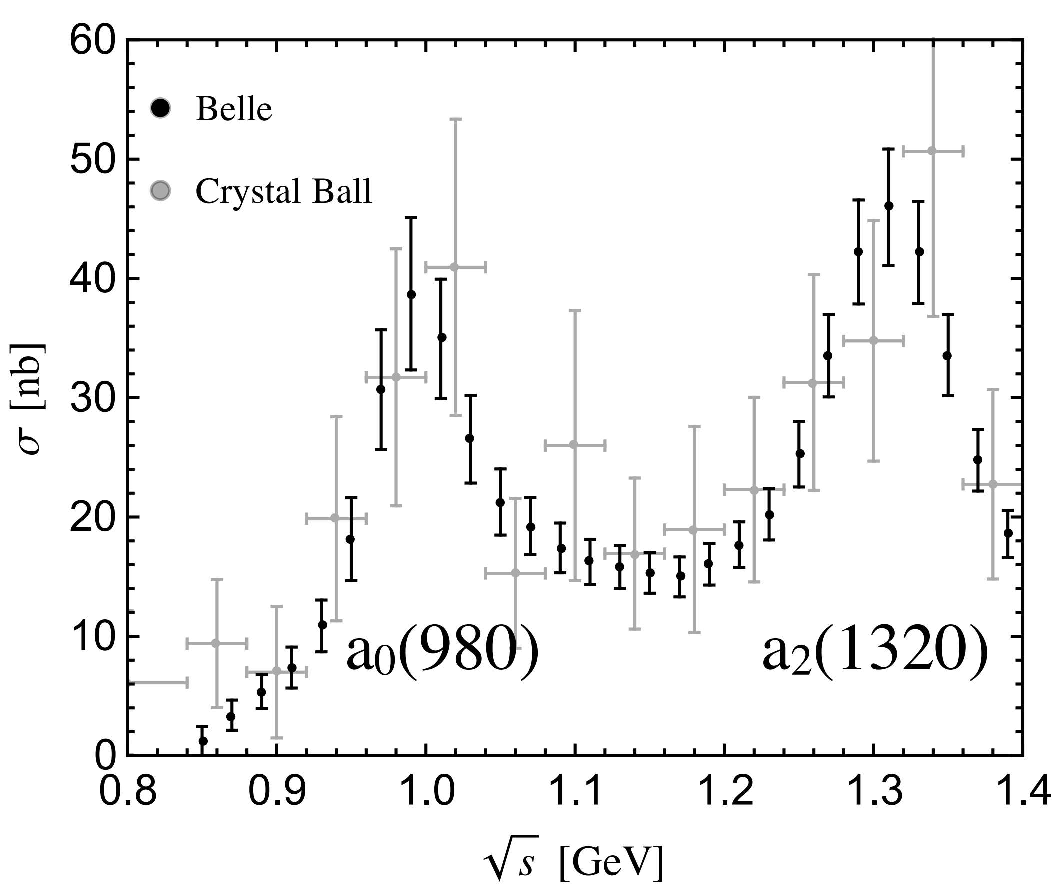

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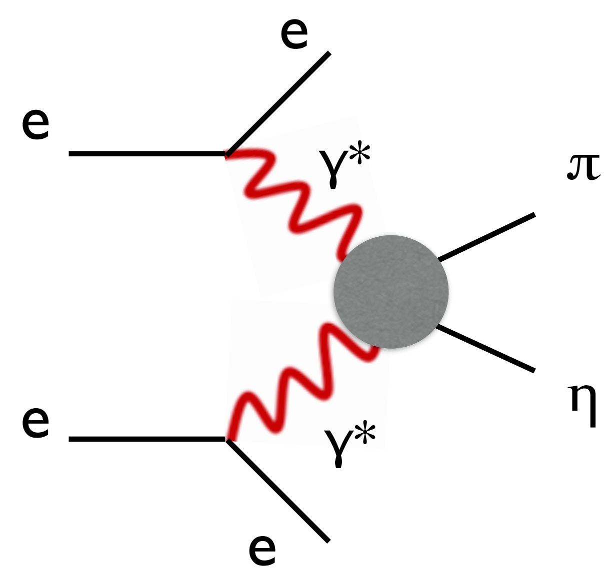

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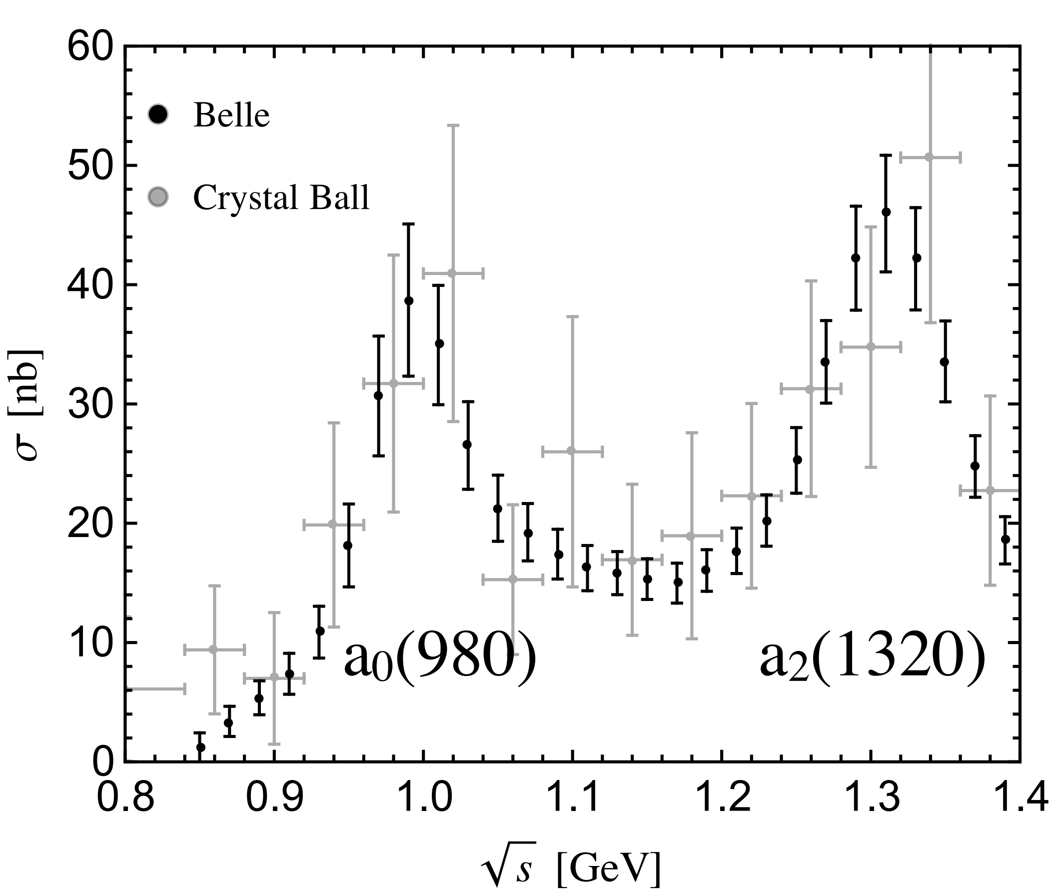
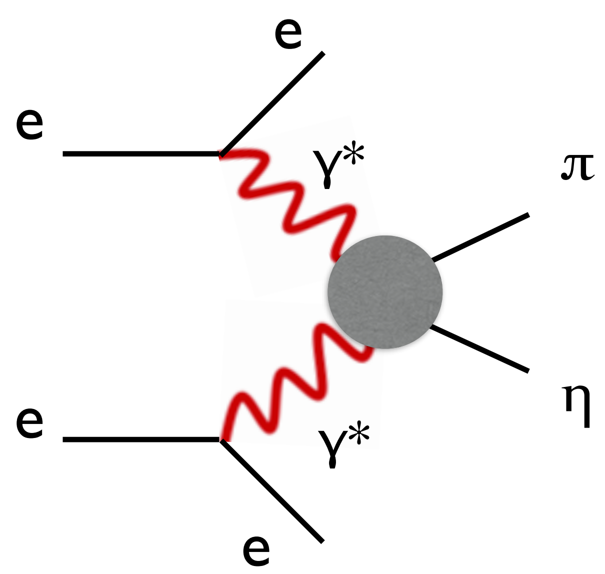


- Perform a theoretical analysis based on the S-matrix constraints

Experiment

Observables in experiment $e^+e^- \to e^-e^+\pi\eta$



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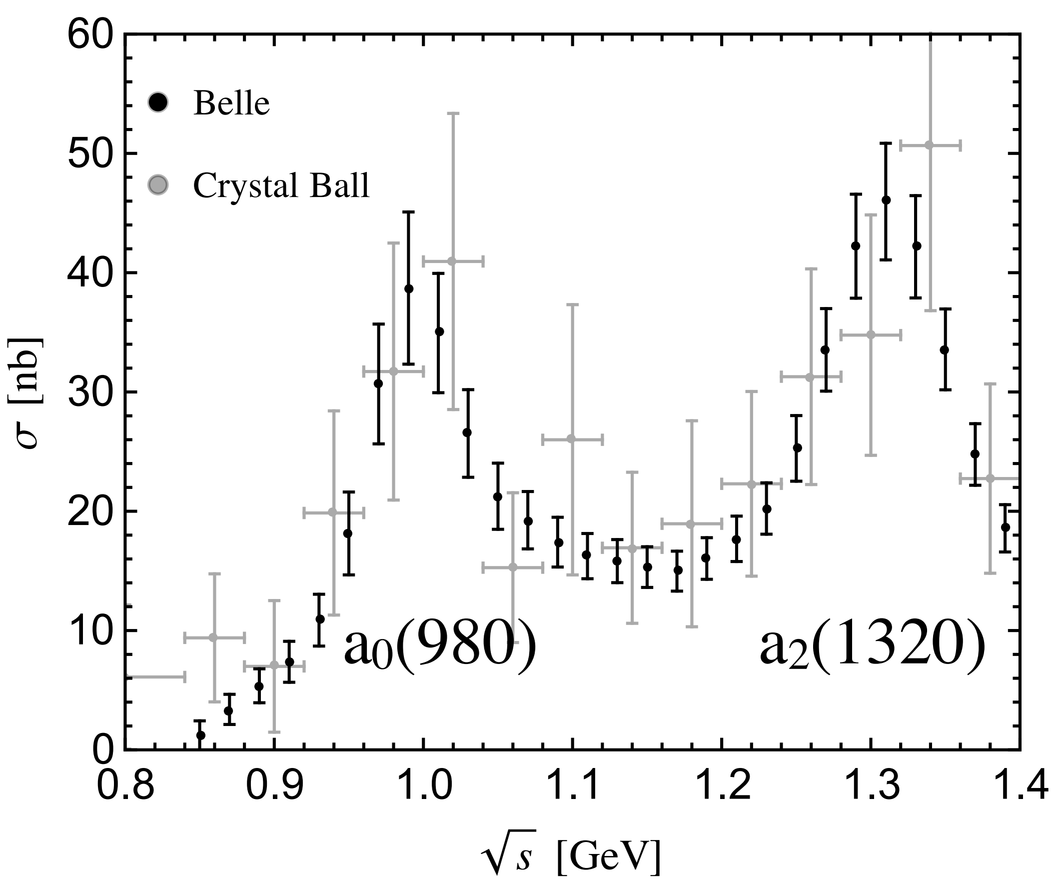
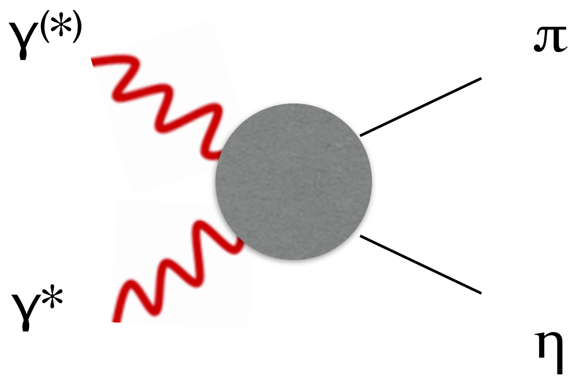


- Perform a theoretical analysis based on the S-matrix constraints
- Extract resonance properties, e,g. pole position, **two photon coupling** etc.

Importance for (g-2)

$\gamma\gamma^* \rightarrow \pi\eta$ (BESIII in progress)



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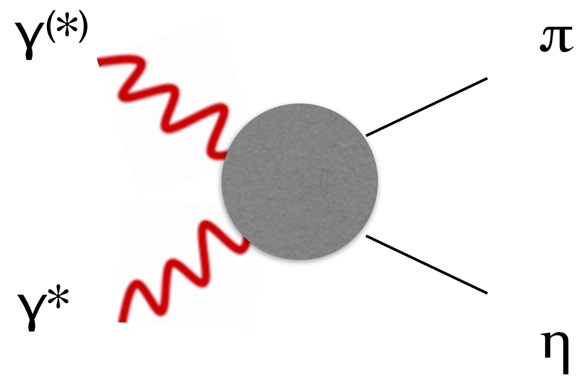
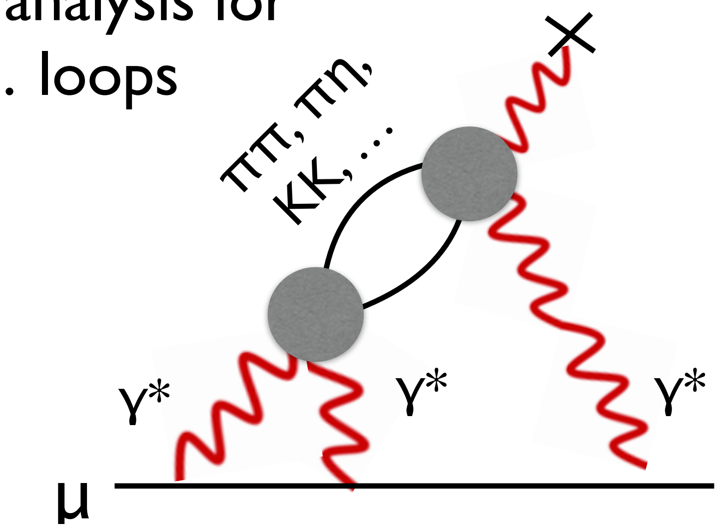


Dispersive analysis for ππ, πη, … loops



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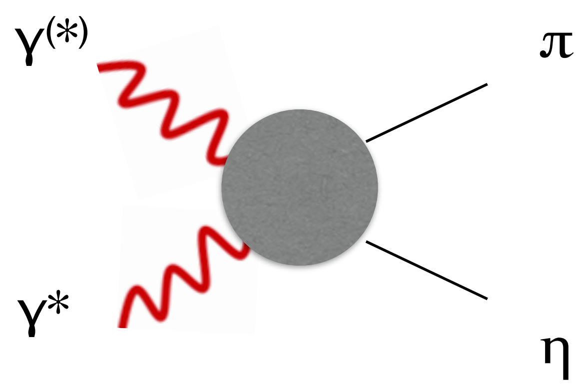
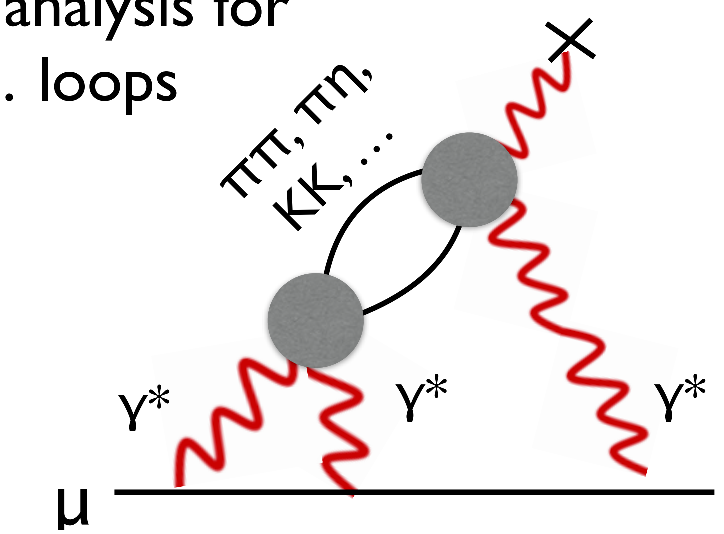




$$a_\mu^{exp} = (11\,659\,208.9 \pm 6.3) \times 10^{-10}$$
$$a_\mu^{SM} = (11\,659\,182.8 \pm 4.9) \times 10^{-10}$$

$$a_\mu^{exp} - a_\mu^{SM} =$$
$$(26.1 \pm 4.9_{th} \pm 6.3_{exp}) \times 10^{-10}$$

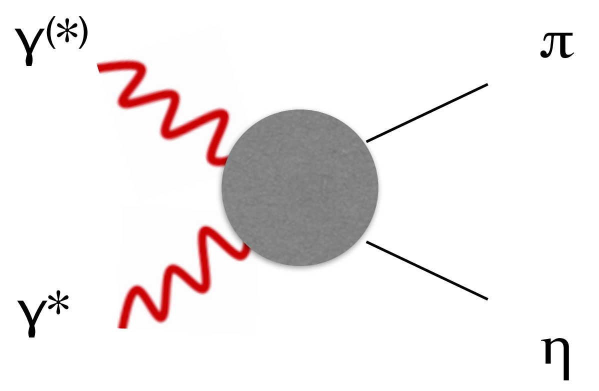
1.6_{exp}

3 - 4 σ deviation!

BNL (2006)
FNAL, J-PARC

Importance for (g-2)

γγ*→πη (BESIII in progress)



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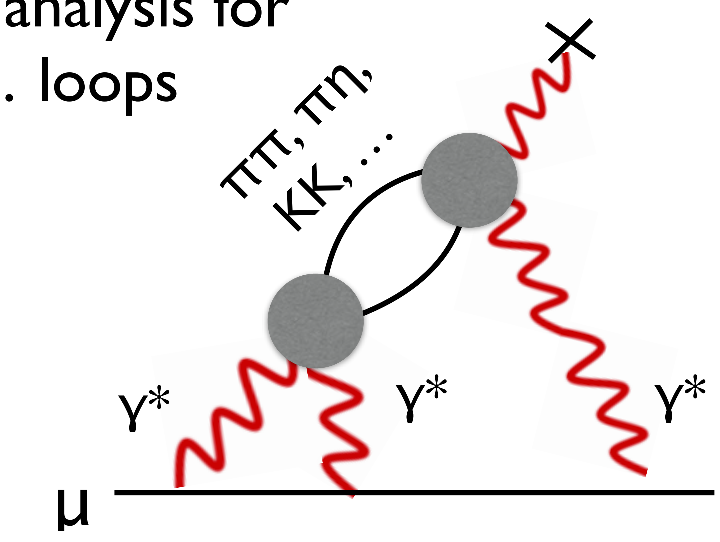

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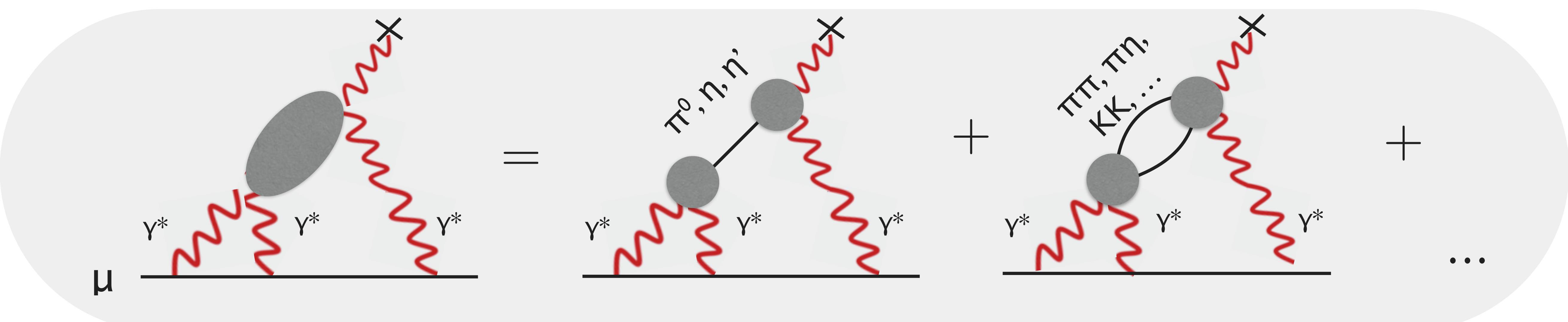
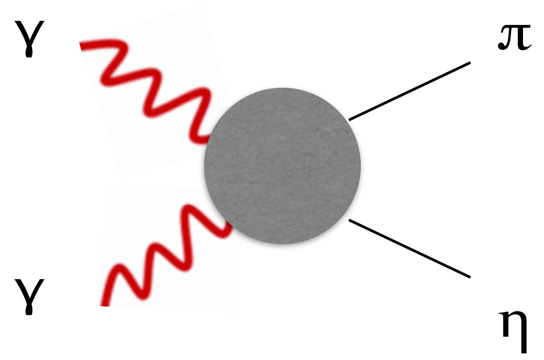
FNAL, J-PARC

Hadronic light-by-light contributions to g-2



Cross section



C=+1: $J^{PC}=0^{++}, 2^{++}, \ldots.$

Helicity amplitudes

$$\langle \pi(p_1)\eta(p_2)|T|\gamma(q_1,\lambda_1)\gamma(q_2,\lambda_2)\rangle = (2\pi)^4\,\delta^{(4)}(p_1+p_2-q_1-q_2)\,H_{\lambda_1\lambda_2}$$

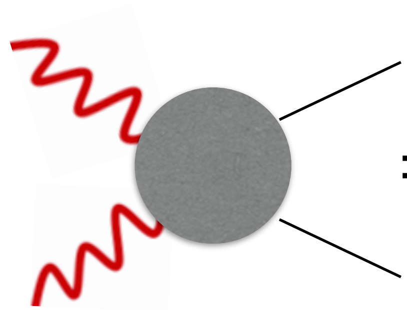
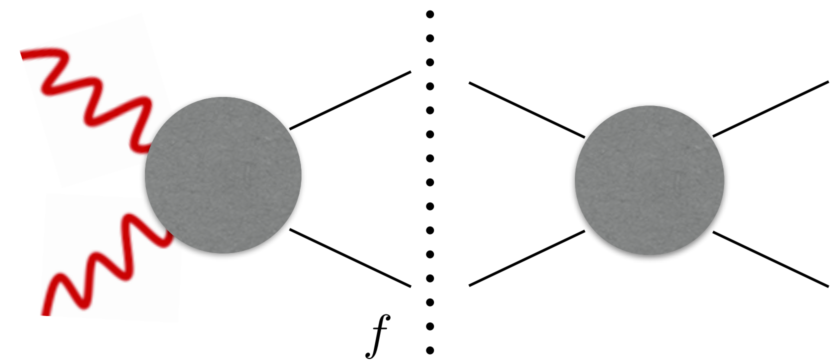
$$H_{\lambda_1\lambda_2} = H^{\mu\nu}\epsilon_\mu(\lambda_1)\,\epsilon_\nu(\lambda_2), \quad \lambda_{1,2}=\pm 1$$

P symmetry: **4** ⇒ **2** independent amplitudes H_{++}, H_{+-}

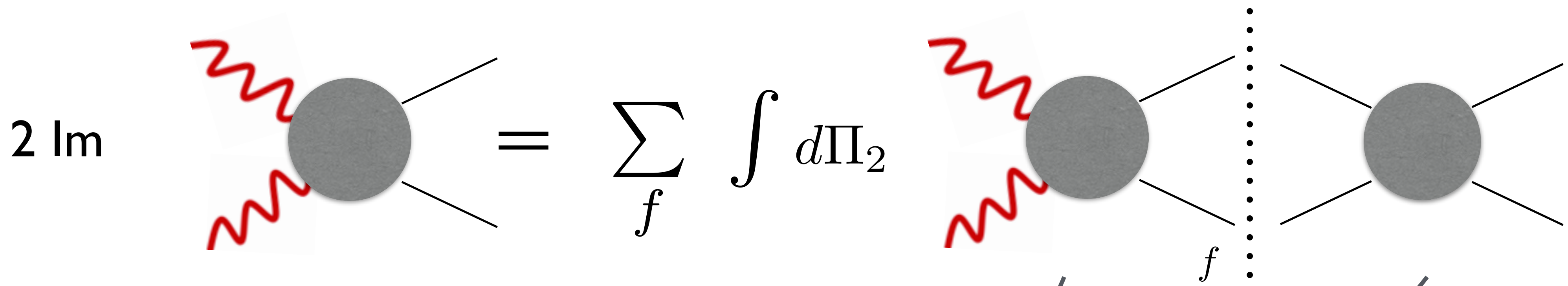
Cross section

$$\frac{d\sigma}{d\cos\theta} = \frac{\rho_{\pi\eta}(s)}{4\,s}\left(|H_{++}|^2 + |H_{+-}|^2\right)$$

Unitarity

2 Im  $= \sum_f \int d\Pi_2$ 

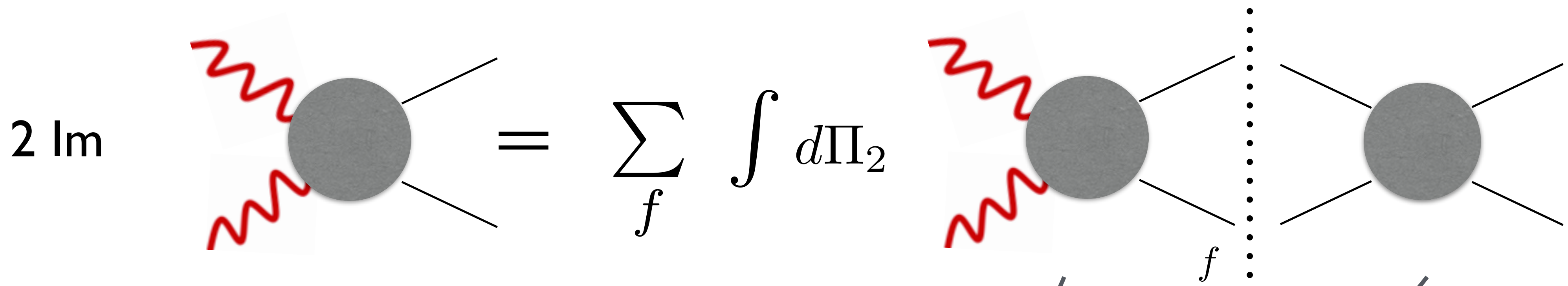
Unitarity



Partial wave expansion

$$H_{\lambda_1\lambda_2}(s,t) = \sum_{J=0}^{\infty}(2J+1)\, h_{J,\lambda_1\lambda_2}(s)\, d^J_{\lambda_1-\lambda_2,0}(\theta)$$

$$T(s,t) = \sum_{J=0}^{\infty}(2J+1)\, t_J(s)\, P_J(\theta)$$

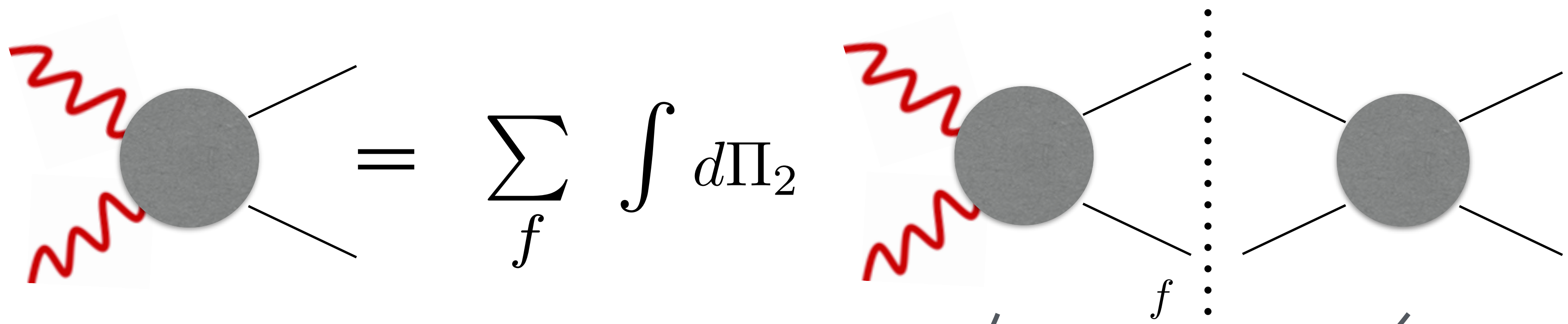


Partial wave expansion

$$H_{\lambda_1\lambda_2}(s,t) = \sum_{J=0}^{\cancel{\infty}\; J_{max}=2} (2J+1)\, h_{J,\lambda_1\lambda_2}(s)\, d^J_{\lambda_1-\lambda_2,0}(\theta)$$

$$T(s,t) = \sum_{J=0}^{\cancel{\infty}\; J_{max}=2} (2J+1)\, t_J(s)\, P_J(\theta)$$

Unitarity

2 Im 

Partial wave expansion

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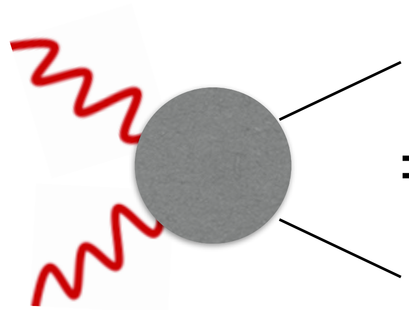
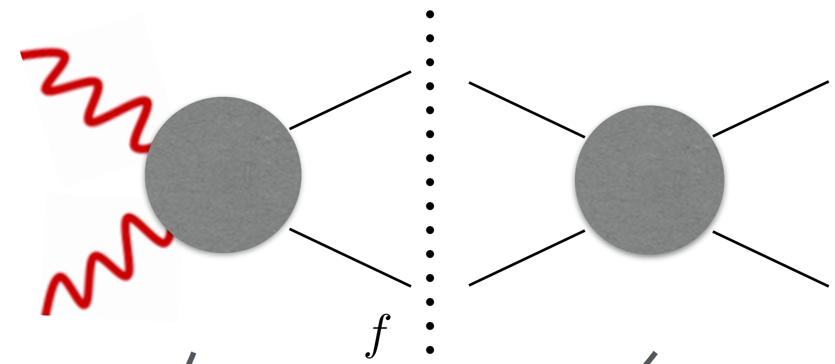
These "diagonalise unitarity" and contain resonance information

Definite: J, λ_1, λ_2

$$\mathrm{Im}\, h_{\gamma\gamma\to\pi\eta}(s) = h_{\gamma\gamma\to\pi\eta}(s)\, \rho_{\pi\eta}(s)\, t^*_{\pi\eta\to\pi\eta}(s)$$

Unitarity

2 Im  $= \sum_f \int d\Pi_2$ 

Partial wave expansion

$$H_{\lambda_1\lambda_2}(s,t) = \sum_{J=0}^{\not\infty \; J_{max}=2} (2J+1)\, h_{J,\lambda_1\lambda_2}(s)\, d^J_{\lambda_1-\lambda_2,0}(\theta)$$

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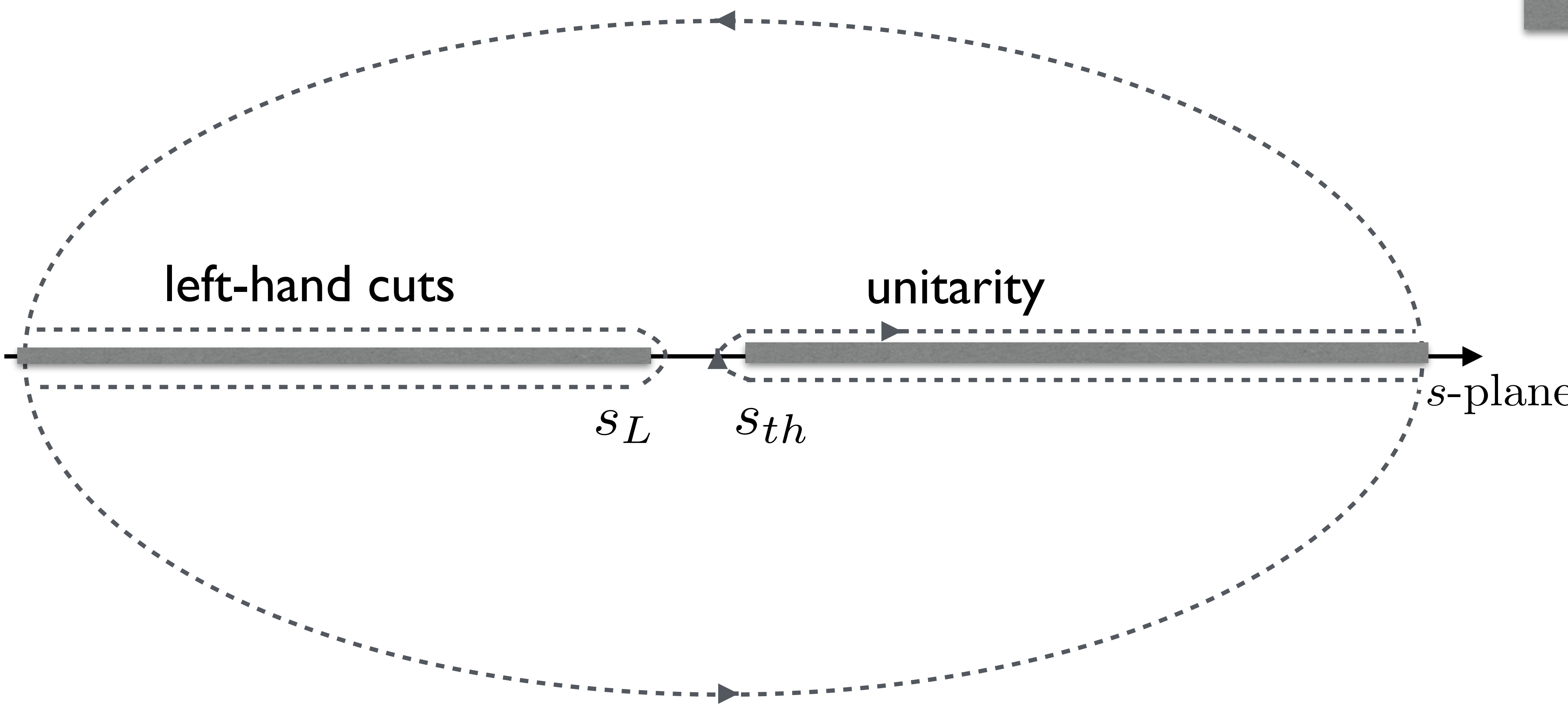
These "diagonalise unitarity" and contain resonance information (coupled-channel unitarity)

Definite: J, λ_1, λ_2

$$\mathrm{Im}\, h_{\gamma\gamma\to\pi\eta}(s) = h_{\gamma\gamma\to\pi\eta}(s)\, \rho_{\pi\eta}(s)\, t^*_{\pi\eta\to\pi\eta}(s) + h_{\gamma\gamma\to KK}(s)\, \rho_{KK}(s)\, t^*_{KK\to\pi\eta}(s)$$

Dispersion relation

Definite: J, λ_1, λ_2



$$h(s) = \frac{1}{2\pi i}\int_C ds' \frac{h(s')}{s'-s} = \int_{-\infty}^{s_L} \frac{ds'}{\pi}\frac{\mathrm{Im}\, h(s')}{s'-s} + \int_{s_{th}}^{\infty} \frac{ds'}{\pi}\frac{\mathrm{Im}\, h(s')}{s'-s}$$

analyticity relates scattering amplitude at different energies

Formalism

Morgan et. al. (1998)
Garcia-Martin et. al. (2010)
Moussallam (2013)

Write a dispersive representation for

$$\Omega^{-1}(s)(h(s) - h^{Born}(s))$$

Helicity - 0, s-wave

$$\begin{pmatrix} h_{++}(s) \\ k_{++}(s) \end{pmatrix} = \begin{pmatrix} 0 \\ k_{++}^{Born}(s) \end{pmatrix} + \Omega(s)\left[\begin{pmatrix} a \\ b \end{pmatrix} + \frac{s - s_{th}}{\pi}\int_{-\infty}^{s_L} \frac{ds'}{s' - s_{th}} \frac{\Omega(s')^{-1}}{s' - s} \begin{pmatrix} \mathrm{Im}\, h_{++}(s') \\ \mathrm{Im}\, \bar{k}_{++}(s') \end{pmatrix} \right.$$
$$\left. - \frac{s - s_{th}}{\pi}\int_{s_{th}}^{\infty} \frac{ds'}{s' - s_{th}} \frac{\mathrm{Im}\,\Omega(s')^{-1}}{s' - s} \begin{pmatrix} 0 \\ k_{++}^{Born}(s') \end{pmatrix}\right]$$

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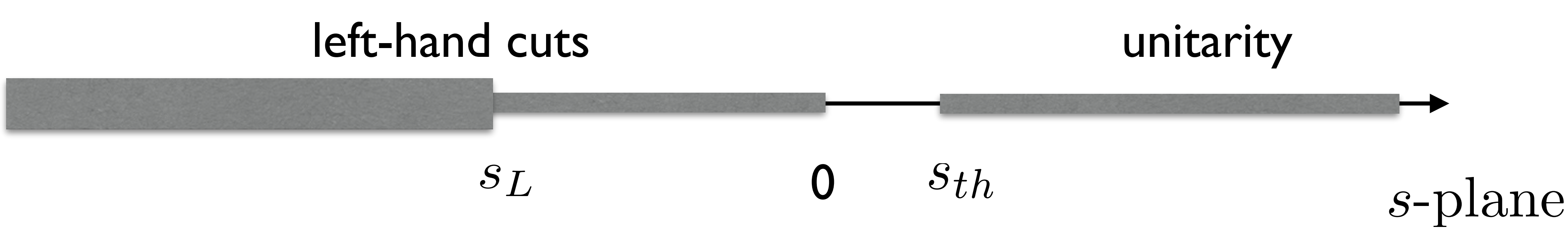

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$$- \frac{s - s_{th}}{\pi} \int_{s_{th}}^{\infty} \frac{ds'}{s' - s_{th}} \frac{\mathrm{Im}\,\Omega(s')^{-1}}{s' - s} \begin{pmatrix} 0 \\ k_{++}^{Born}(s') \end{pmatrix} \Bigg]$$

Unitarity $\quad s \geq s_{th} = (m_\pi + m_\eta)^2$

$$\mathrm{Im}\, h(s) = h(s)\, \rho(s)\, t^*(s)$$
$$\mathrm{Im}\, \Omega(s) = \Omega(s)\, \rho(s)\, t^*(s)$$

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Coupled-channel Omnes function

$$\Omega(s) = \begin{pmatrix} \Omega_{\pi\eta\to\pi\eta} & \Omega_{\pi\eta\to K\bar{K}} \\ \Omega_{K\bar{K}\to\pi\eta} & \Omega_{K\bar{K}\to K\bar{K}} \end{pmatrix}$$

Unitarity $s \geq s_{th} = (m_\pi + m_\eta)^2$

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Omnes function {πη, KK}

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Bounded p.w. amplitudes and
Omnes at large energies

$$T(s) = \Omega(s)\, N(s)$$

$$N(s) = U(s) + \frac{s - s_{th}}{\pi} \int_R \frac{ds'}{s' - s_{th}} \frac{\rho(s')N(s')(U(s) - U(s'))}{s' - s}$$

$$\Omega^{-1}(s) = 1 - \frac{s - s_{th}}{\pi} \int_R \frac{ds'}{s' - s_{th}} \frac{\rho(s')N(s')}{s' - s}$$

$$U(s) = \sum_k C_k\, \xi(s)^k$$

Chew, Mandelstam
Lutz, Gasparyan, I.D., Gill

C_k **matched** to SU(3)
ChPT at threshold

Omnes function {πη, KK}

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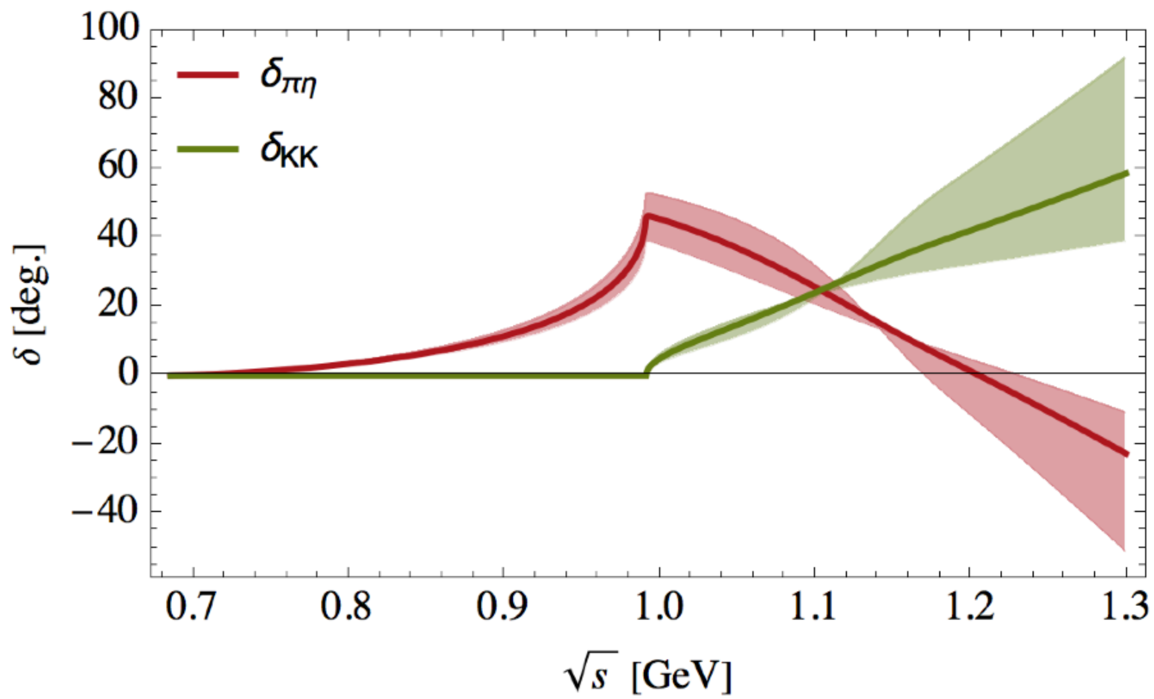
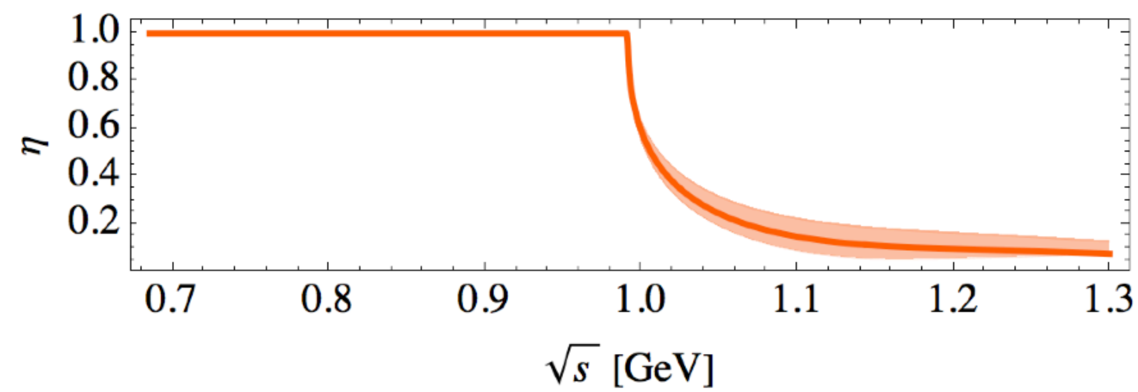
C_k **matched** to SU(3) **ChPT** at threshold





Poles in the complex plane

Single channel case (II Riemann sheet)

Poles in the complex plane

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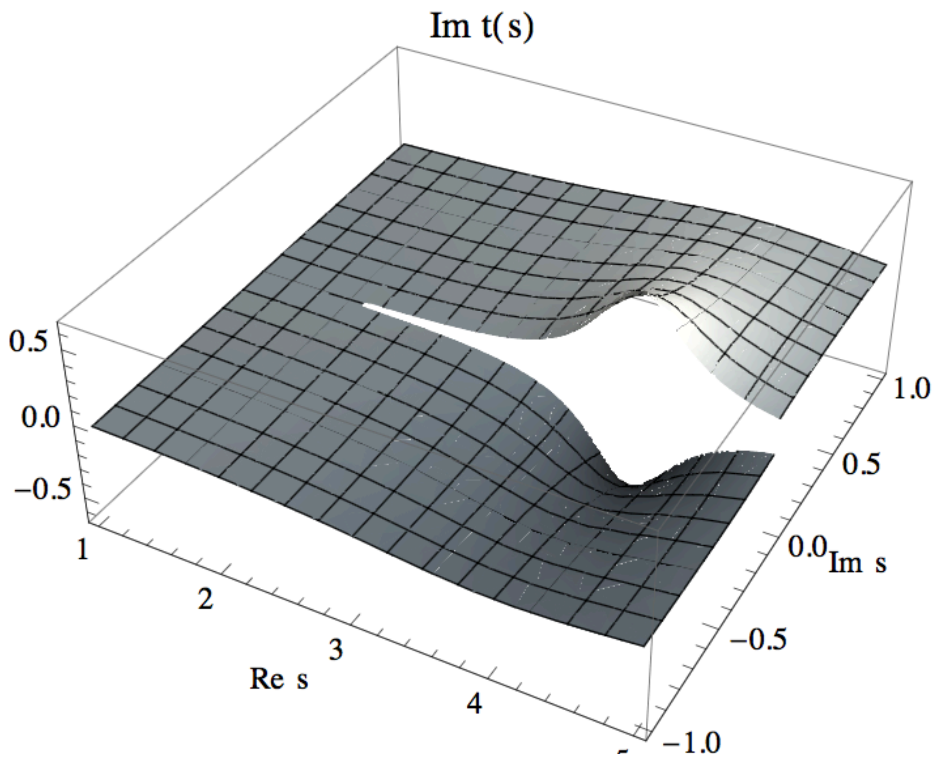

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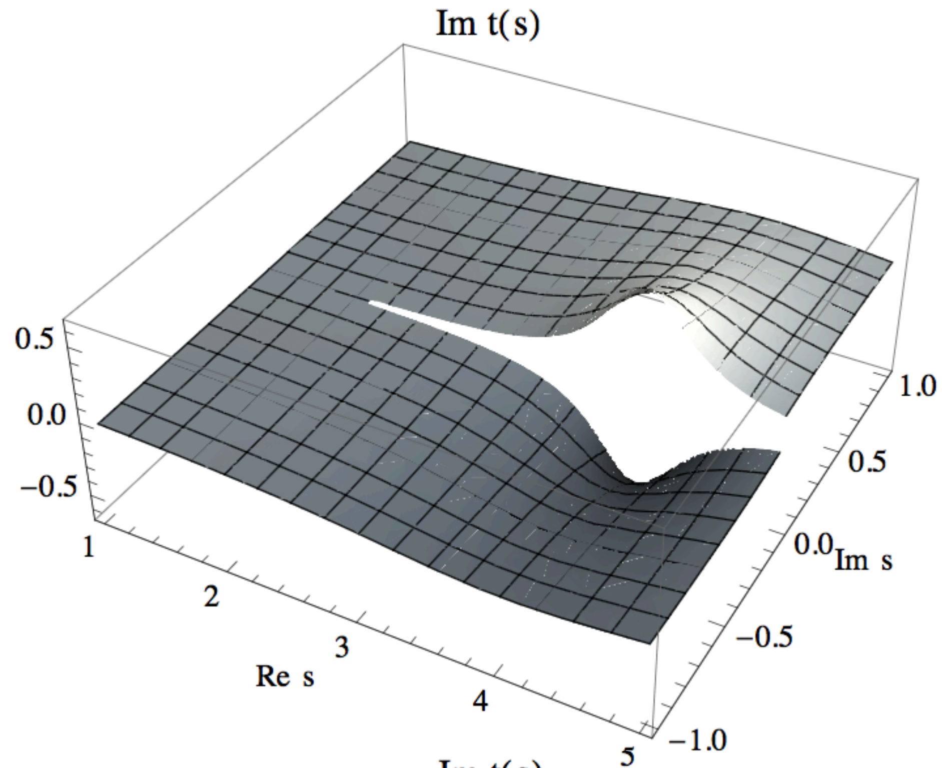
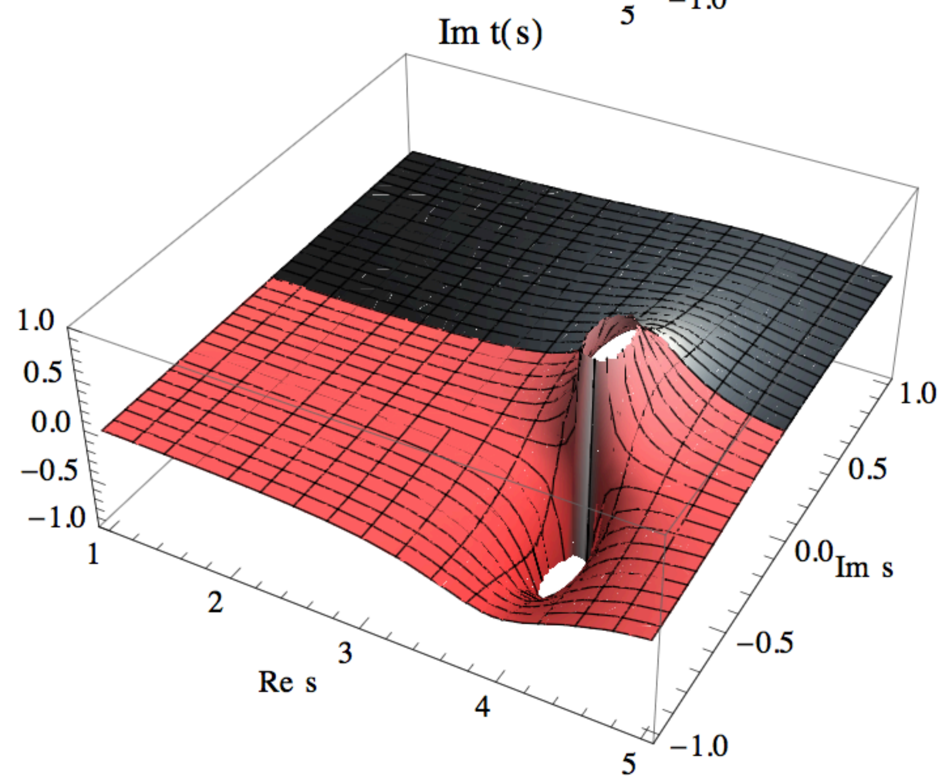
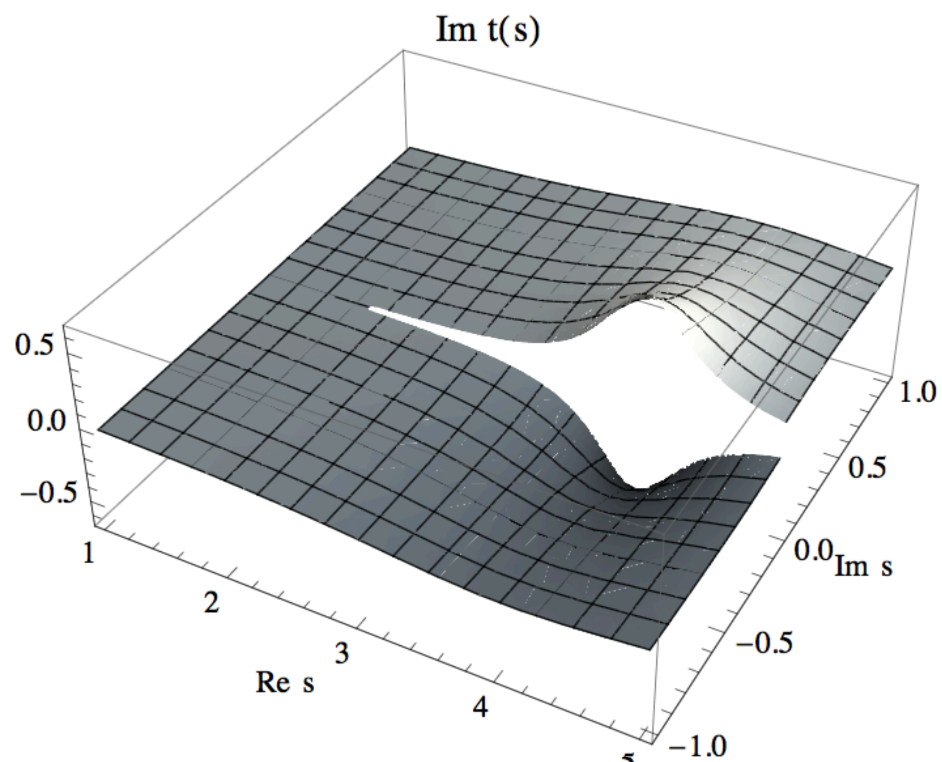
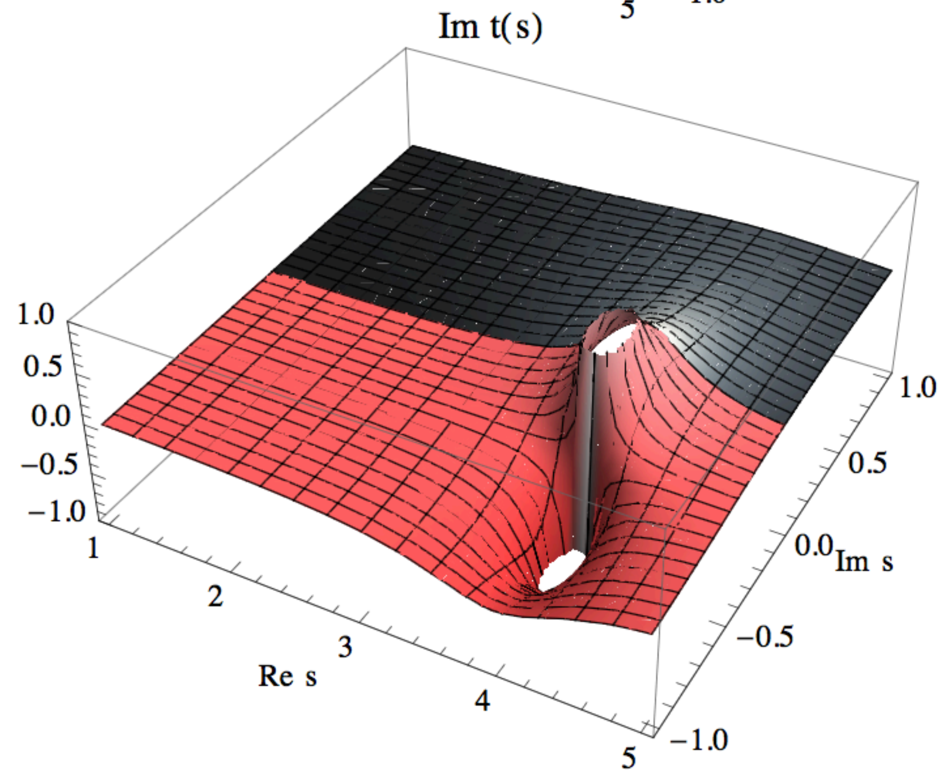




Poles in the complex plane

Single channel case (II Riemann sheet)





Unitarity:

$$t^I(s+i\epsilon) - t^I(s-i\epsilon) = 2i\rho(s)t^I(s+i\epsilon)t^I(s-i\epsilon)$$

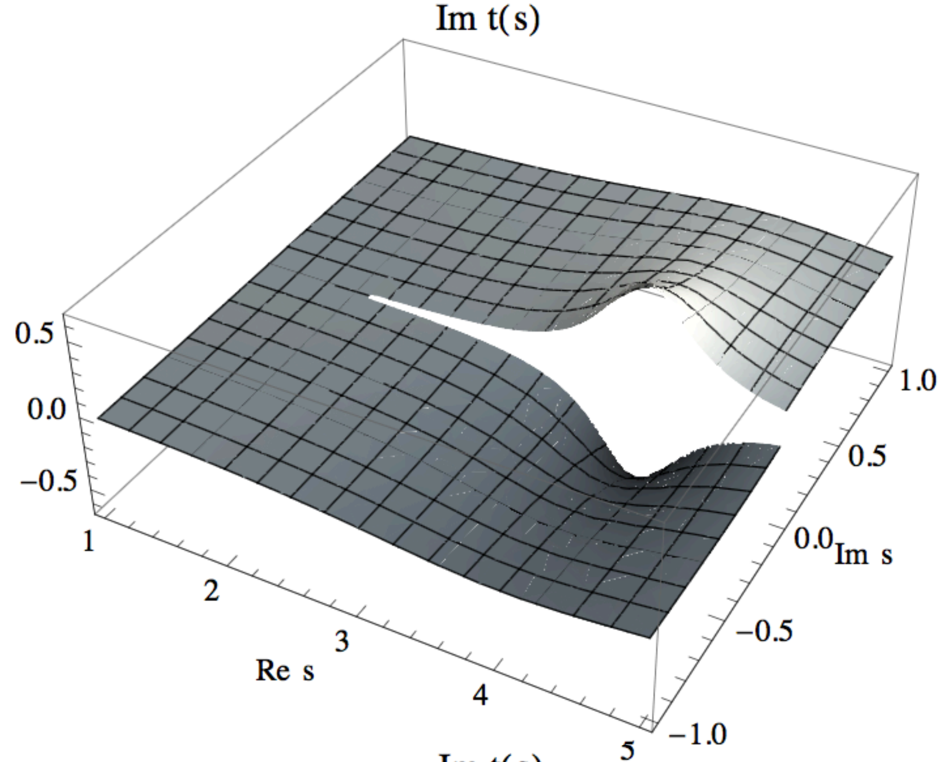
$$t^I(s+i\epsilon) = \frac{t^I(s-i\epsilon)}{1-2i\rho(s)t^I(s-i\epsilon)}$$

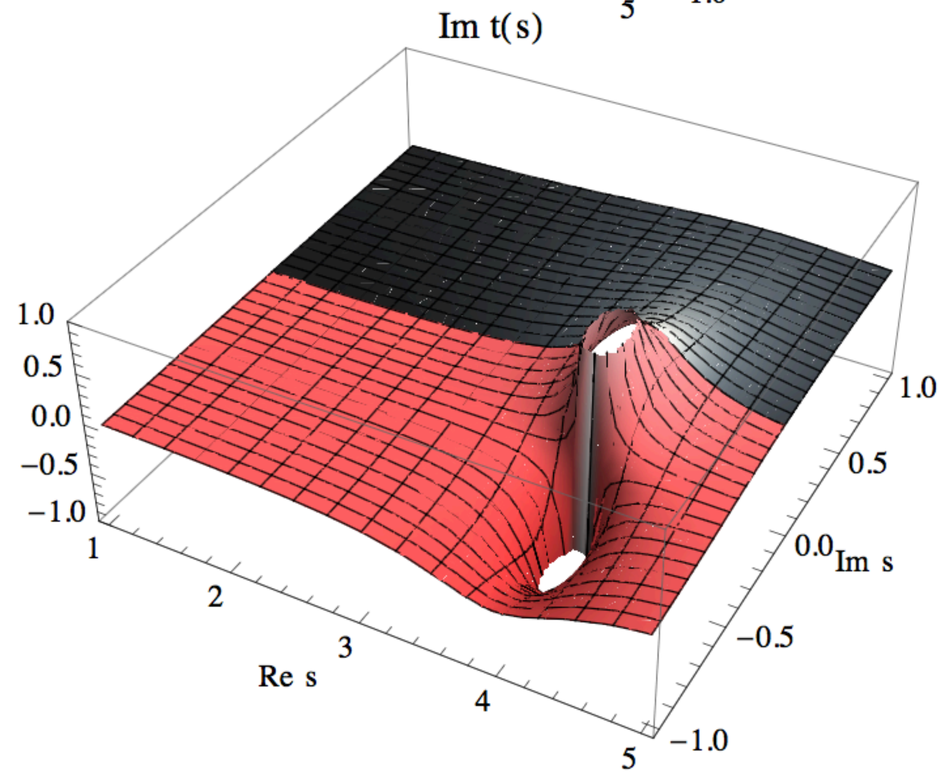
$$t^{II}(s-i\epsilon) \overset{\epsilon\to 0}{=} t^I(s+i\epsilon)$$

$$t^{II}(s) = \frac{t^I(s)}{1-2i\rho(s)t^I(s)}$$

Poles in the complex plane

Single channel case (II Riemann sheet)





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$$t^{II}(s-i\epsilon) \overset{\epsilon\to 0}{=} t^I(s+i\epsilon)$$

$$t^{II}(s) = \frac{t^I(s)}{1-2i\rho(s)t^I(s)}$$

no poles found on the II Riemann sheet!
(different from $f_0(980)$)

Poles in the complex plane

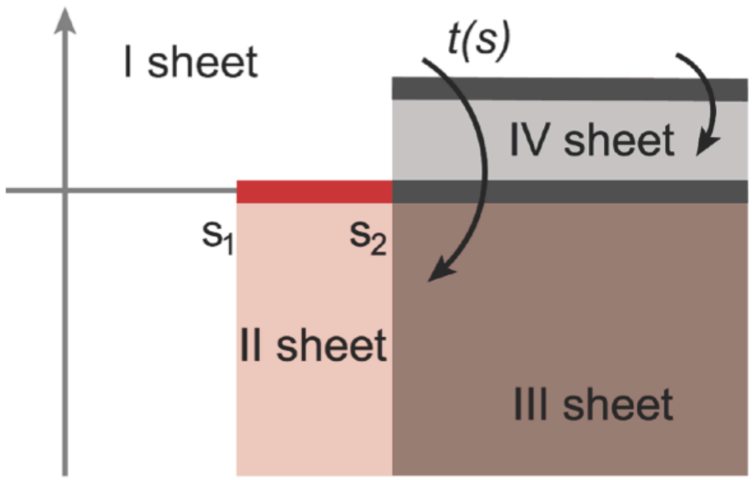
Coupled-channel case

$$t^I_{11}(s+i\epsilon) - t^I_{11}(s-i\epsilon) = 2i\rho_1(s)t^I_{11}(s+i\epsilon)t^I_{11}(s-i\epsilon) + 2i\rho_2(s)t^I_{12}(s+i\epsilon)t^I_{12}(s-i\epsilon)$$
$$t^I_{12}(s+i\epsilon) - t^I_{12}(s-i\epsilon) = 2i\rho_1(s)t^I_{11}(s+i\epsilon)t^I_{12}(s-i\epsilon) + 2i\rho_2(s)t^I_{12}(s+i\epsilon)t^I_{22}(s-i\epsilon)$$
$$t^I_{22}(s+i\epsilon) - t^I_{22}(s-i\epsilon) = 2i\rho_1(s)t^I_{12}(s+i\epsilon)t^I_{12}(s-i\epsilon) + 2i\rho_2(s)t^I_{22}(s+i\epsilon)t^I_{22}(s-i\epsilon)$$

Poles in the complex plane

Coupled-channel case

$$t_{11}^I(s+i\epsilon) - t_{11}^I(s-i\epsilon) = 2i\rho_1(s)t_{11}^I(s+i\epsilon)t_{11}^I(s-i\epsilon) + 2i\rho_2(s)t_{12}^I(s+i\epsilon)t_{12}^I(s-i\epsilon)$$
$$t_{12}^I(s+i\epsilon) - t_{12}^I(s-i\epsilon) = 2i\rho_1(s)t_{11}^I(s+i\epsilon)t_{12}^I(s-i\epsilon) + 2i\rho_2(s)t_{12}^I(s+i\epsilon)t_{22}^I(s-i\epsilon)$$
$$t_{22}^I(s+i\epsilon) - t_{22}^I(s-i\epsilon) = 2i\rho_1(s)t_{12}^I(s+i\epsilon)t_{12}^I(s-i\epsilon) + 2i\rho_2(s)t_{22}^I(s+i\epsilon)t_{22}^I(s-i\epsilon)$$



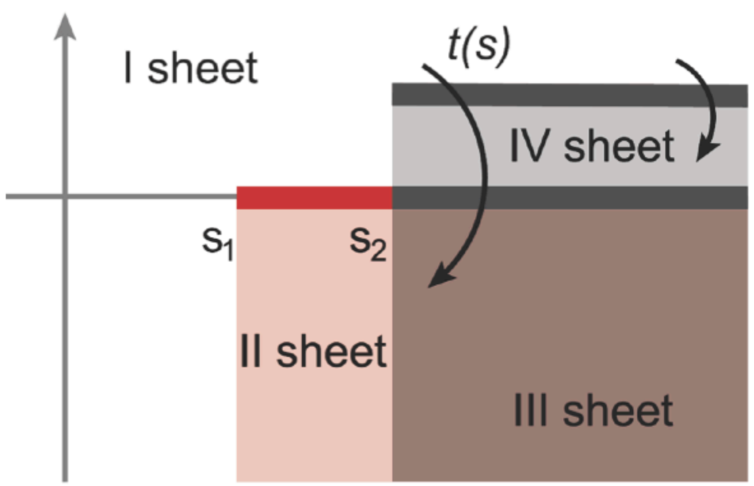
$$\rho_i(s) = 2k_i(s)/\sqrt{s}$$

Sheet	Im k_1	Im k_2
I	+	+
II	−	+
III	−	−
IV	+	−

Poles in the complex plane

Coupled-channel case

$$t_{11}^I(s+i\epsilon) - t_{11}^I(s-i\epsilon) = 2i\rho_1(s)t_{11}^I(s+i\epsilon)t_{11}^I(s-i\epsilon) + 2i\rho_2(s)t_{12}^I(s+i\epsilon)t_{12}^I(s-i\epsilon)$$
$$t_{12}^I(s+i\epsilon) - t_{12}^I(s-i\epsilon) = 2i\rho_1(s)t_{11}^I(s+i\epsilon)t_{12}^I(s-i\epsilon) + 2i\rho_2(s)t_{12}^I(s+i\epsilon)t_{22}^I(s-i\epsilon)$$
$$t_{22}^I(s+i\epsilon) - t_{22}^I(s-i\epsilon) = 2i\rho_1(s)t_{12}^I(s+i\epsilon)t_{12}^I(s-i\epsilon) + 2i\rho_2(s)t_{22}^I(s+i\epsilon)t_{22}^I(s-i\epsilon)$$



Extensions to II, III, IV Riemann sheets

$$t_{11}^{II}(s) = \frac{t_{11}^I(s)}{1 - 2\,i\,\rho_1(s)\,t_{11}^I(s)}$$

$$t_{11}^{III}(s) = t_{11}^{II}(s) + \frac{2\,i\,\rho_2(s)\,t_{12}^{II}(s)^2}{1 - 2\,i\,\rho_2(s)\,t_{22}^{II}(s)}$$

$$t_{11}^{IV}(s) = t_{11}^{I}(s) + \frac{2\,i\,\rho_2(s)\,t_{12}^{I}(s)^2}{1 - 2\,i\,\rho_2(s)\,t_{22}^{I}(s)}$$

$$\rho_i(s) = 2k_i(s)/\sqrt{s}$$

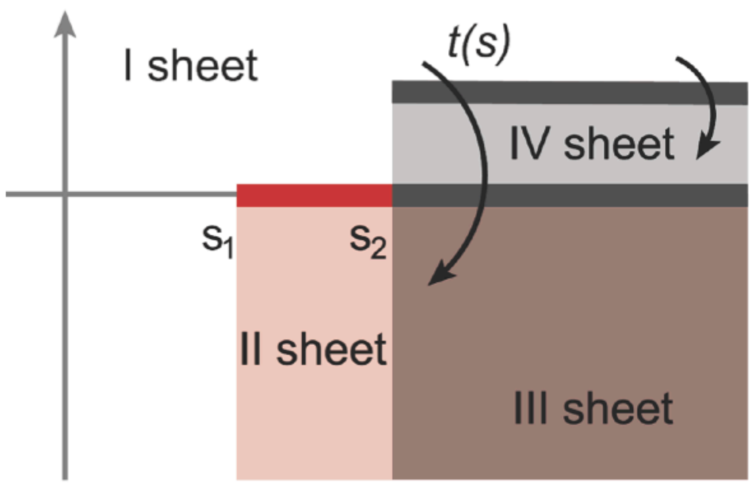
Sheet	Im k_1	Im k_2
I	+	+
II	−	+
III	−	−
IV	+	−

$$\textbf{II sheet:}\ 1 - 2i\rho_1(s)t_{11}^I(s) = 0$$
$$\textbf{III sheet:}\ 1 - 2i\rho_2(s)t_{22}^{II}(s) = 0$$
$$\textbf{IV sheet:}\ 1 - 2i\rho_2(s)t_{22}^{I}(s) = 0$$

Poles in the complex plane

Coupled-channel case

$$t^I_{11}(s+i\epsilon) - t^I_{11}(s-i\epsilon) = 2i\rho_1(s)t^I_{11}(s+i\epsilon)t^I_{11}(s-i\epsilon) + 2i\rho_2(s)t^I_{12}(s+i\epsilon)t^I_{12}(s-i\epsilon)$$
$$t^I_{12}(s+i\epsilon) - t^I_{12}(s-i\epsilon) = 2i\rho_1(s)t^I_{11}(s+i\epsilon)t^I_{12}(s-i\epsilon) + 2i\rho_2(s)t^I_{12}(s+i\epsilon)t^I_{22}(s-i\epsilon)$$
$$t^I_{22}(s+i\epsilon) - t^I_{22}(s-i\epsilon) = 2i\rho_1(s)t^I_{12}(s+i\epsilon)t^I_{12}(s-i\epsilon) + 2i\rho_2(s)t^I_{22}(s+i\epsilon)t^I_{22}(s-i\epsilon)$$



$\rho_i(s) = 2k_i(s)/\sqrt{s}$

Sheet	Im k_1	Im k_2
I	+	+
II	−	+
III	−	−
IV	+	−

Extensions to II, III, IV Riemann sheets

$$t^{II}_{11}(s) = \frac{t^I_{11}(s)}{1 - 2\,i\,\rho_1(s)\,t^I_{11}(s)}$$

$$t^{III}_{11}(s) = t^{II}_{11}(s) + \frac{2\,i\,\rho_2(s)\,t^{II}_{12}(s)^2}{1 - 2\,i\,\rho_2(s)\,t^{II}_{22}(s)}$$

$$t^{IV}_{11}(s) = t^I_{11}(s) + \frac{2\,i\,\rho_2(s)\,t^I_{12}(s)^2}{1 - 2\,i\,\rho_2(s)\,t^I_{22}(s)}$$

$$\sqrt{s^{\rm IV}_{a_0}} = (1.12^{-0.07}_{+0.02}) \pm \frac{i}{2}(0.28^{+0.08}_{-0.13})\ \mathrm{GeV}$$

$$\left|\frac{c_{K\bar K}}{c_{\pi\eta}}\right| = 0.98^{-0.07}_{+0.20}$$

II sheet: $1 - 2i\rho_1(s)t^I_{11}(s) = 0$

III sheet: $1 - 2i\rho_2(s)t^{II}_{22}(s) = 0$

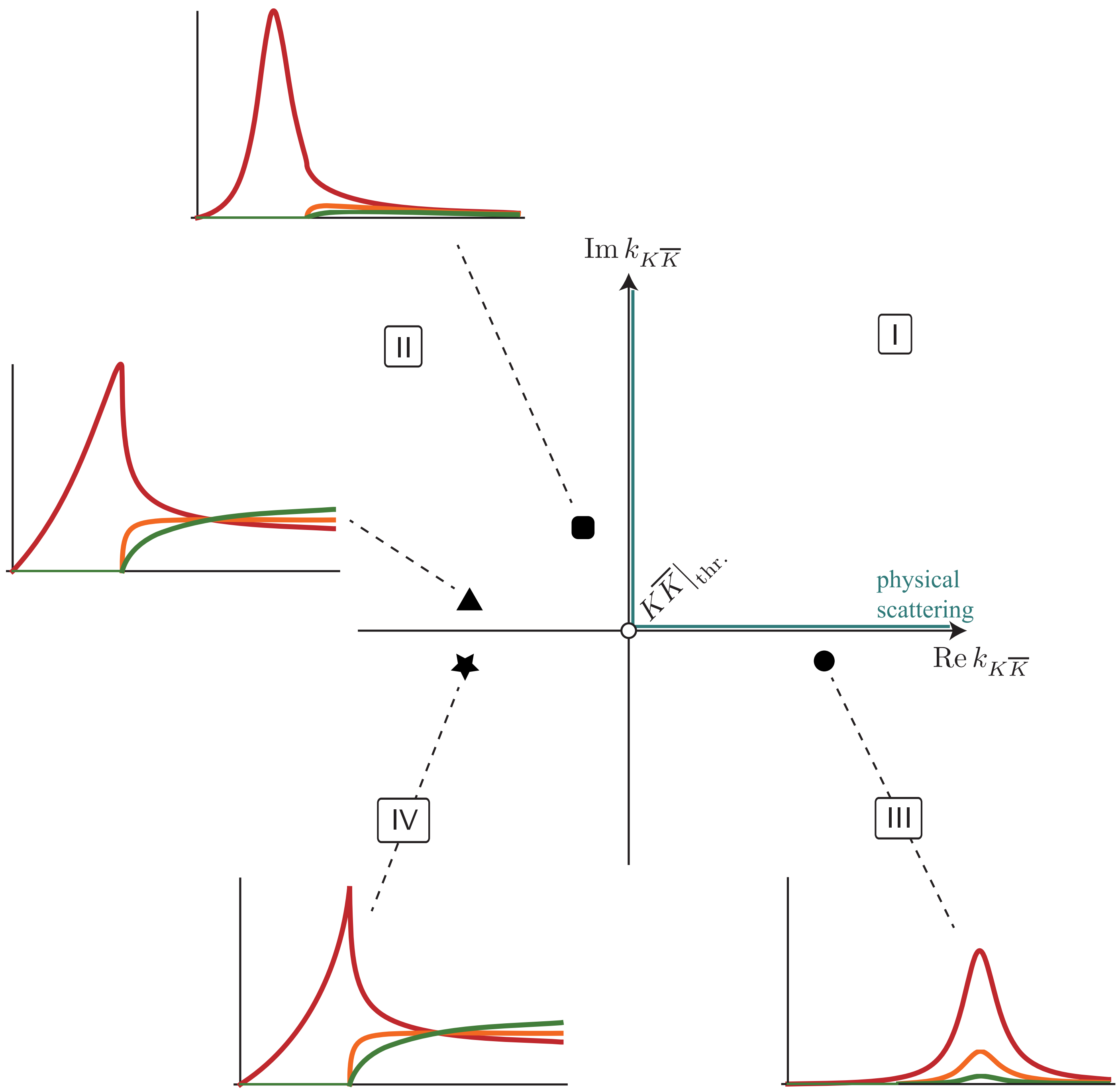
IV sheet: $1 - 2i\rho_2(s)t^I_{22}(s) = 0$

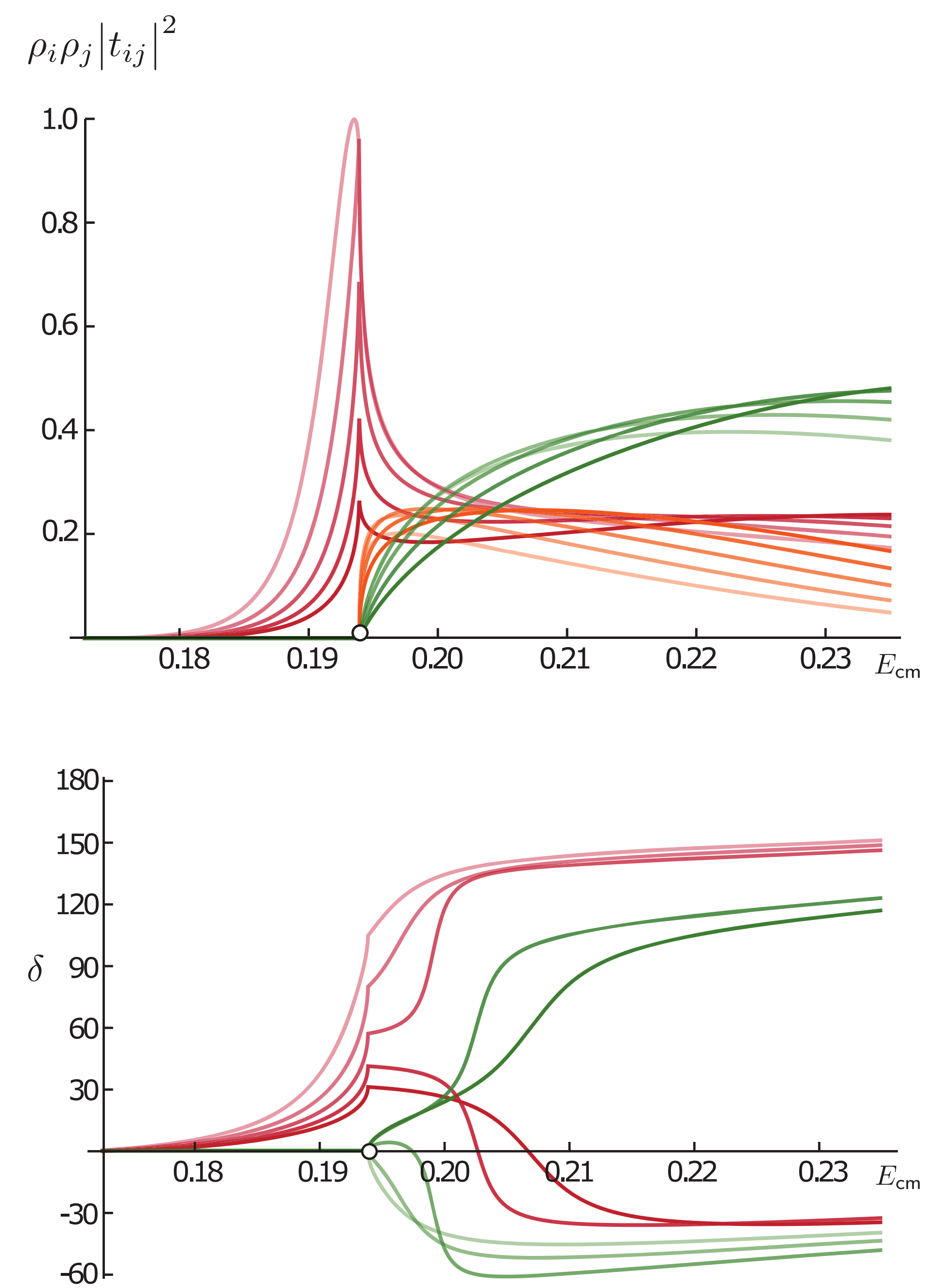
Poles in the complex plane

Simple K-matrix model [Dudek et. al. (2016)]

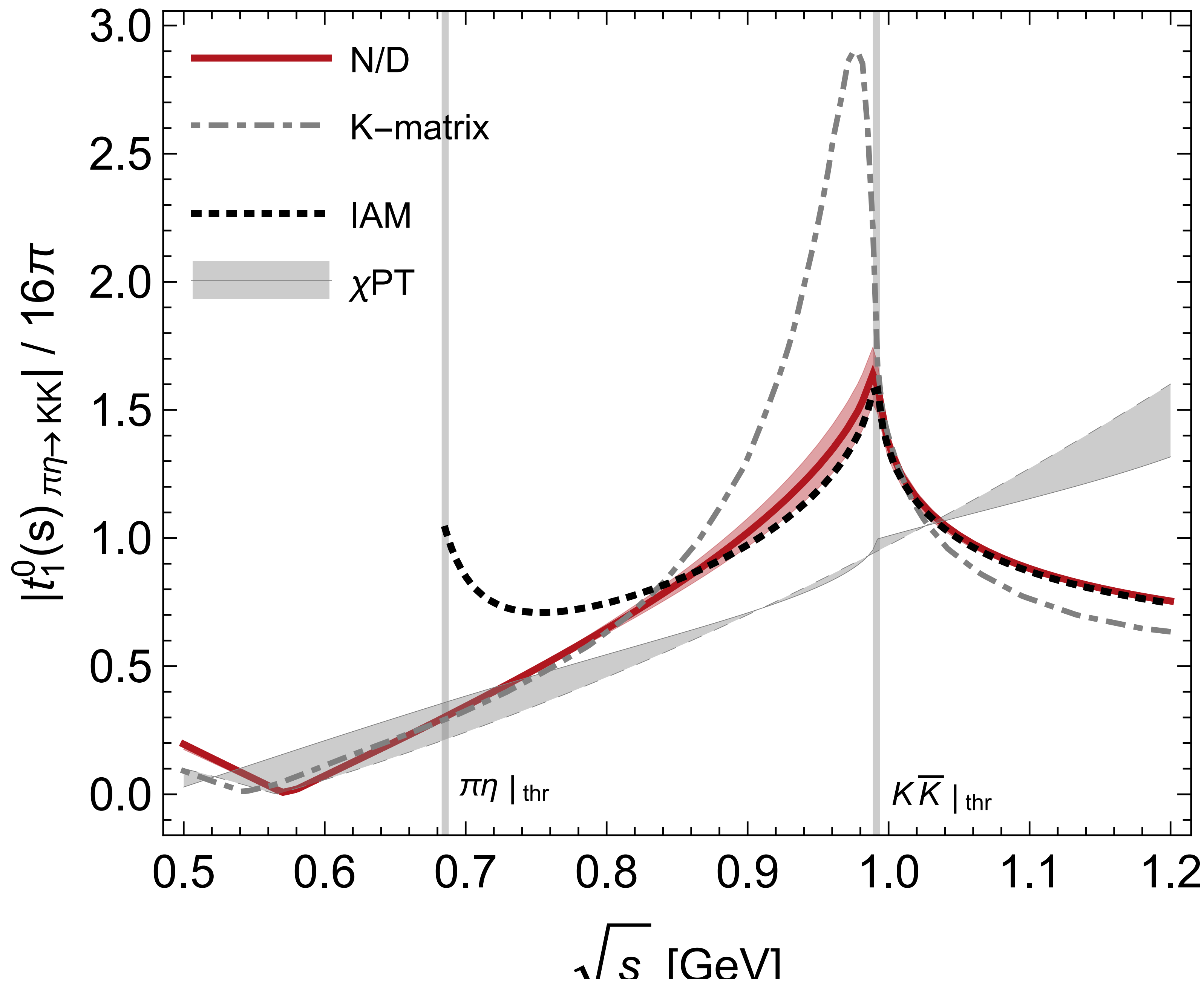
$$t^{-1}(s) = K^{-1}(s) - i\,\rho(s)$$

$$K = \frac{1}{m^2 - s}\begin{pmatrix} g_{\pi\eta}^2 & g_{\pi\eta}g_{KK} \\ g_{\pi\eta}g_{KK} & g_{KK}^2 \end{pmatrix} + \begin{pmatrix} 0 & \gamma \\ \gamma & 0 \end{pmatrix}$$





Scattering amplitude πη→KK



K-matrix:
Albaladejo et. al. (2017)

Inverse Amplitude Method (IAM)
Gomez Nicola et.al. (2002)

Chiral Perturbation Theory
Gasser et. al. (1985)

- First **lattice** analysis for m_π=391 MeV [Jozef Dudek et. al. (2016)]
- **Chiral extrapolation** of the lattice results [Zhi-Hui Guo et. al. (2017)]

Left-hand cuts

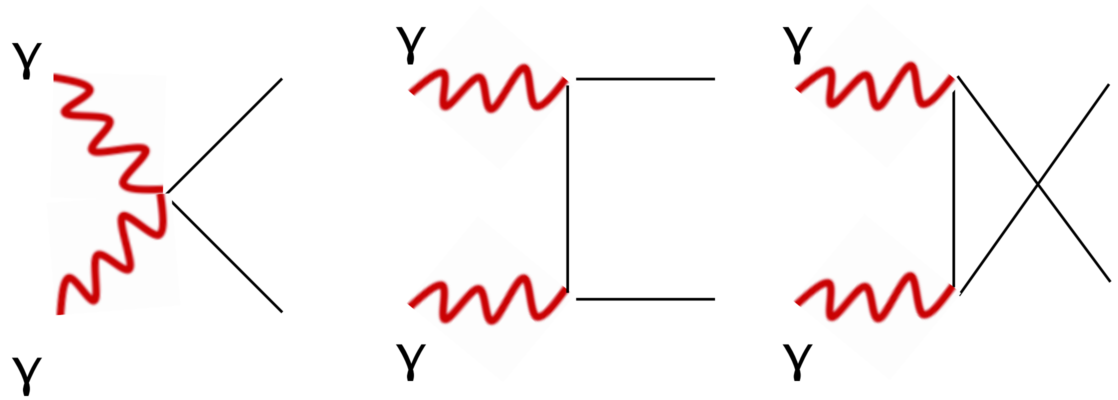
Helicity - 0, s-wave

$$\left(\begin{array}{c} h_{++}(s) \\ k_{++}(s) \end{array}\right) = \left(\begin{array}{c} 0 \\ k_{++}^{Born}(s) \end{array}\right) + \Omega(s)\left[\left(\begin{array}{c} a \\ b \end{array}\right) + \frac{s-s_{th}}{\pi}\int_{-\infty}^{s_L}\frac{ds'}{s'-s_{th}}\frac{\Omega(s')^{-1}}{s'-s}\left(\begin{array}{c} \mathrm{Im}\, h_{++}(s') \\ \mathrm{Im}\, \bar{k}_{++}(s') \end{array}\right)\right.$$
$$\left. - \frac{s-s_{th}}{\pi}\int_{s_{th}}^{\infty}\frac{ds'}{s'-s_{th}}\frac{\mathrm{Im}\,\Omega(s')^{-1}}{s'-s}\left(\begin{array}{c} 0 \\ k_{++}^{Born}(s') \end{array}\right)\right]$$

Helicity - 0, s-wave

$$\begin{pmatrix} h_{++}(s) \\ k_{++}(s) \end{pmatrix} = \begin{pmatrix} 0 \\ k_{++}^{Born}(s) \end{pmatrix} + \Omega(s)\left[\begin{pmatrix} a \\ b \end{pmatrix} + \frac{s-s_{th}}{\pi}\int_{-\infty}^{s_L}\frac{ds'}{s'-s_{th}}\frac{\Omega(s')^{-1}}{s'-s}\begin{pmatrix} \mathrm{Im}\, h_{++}(s') \\ \mathrm{Im}\, \bar{k}_{++}(s') \end{pmatrix} - \frac{s-s_{th}}{\pi}\int_{s_{th}}^{\infty}\frac{ds'}{s'-s_{th}}\frac{\mathrm{Im}\,\Omega(s')^{-1}}{s'-s}\begin{pmatrix} 0 \\ k_{++}^{Born}(s') \end{pmatrix}\right]$$

Scalar QED



Left-hand cuts

Helicity - 0, s-wave

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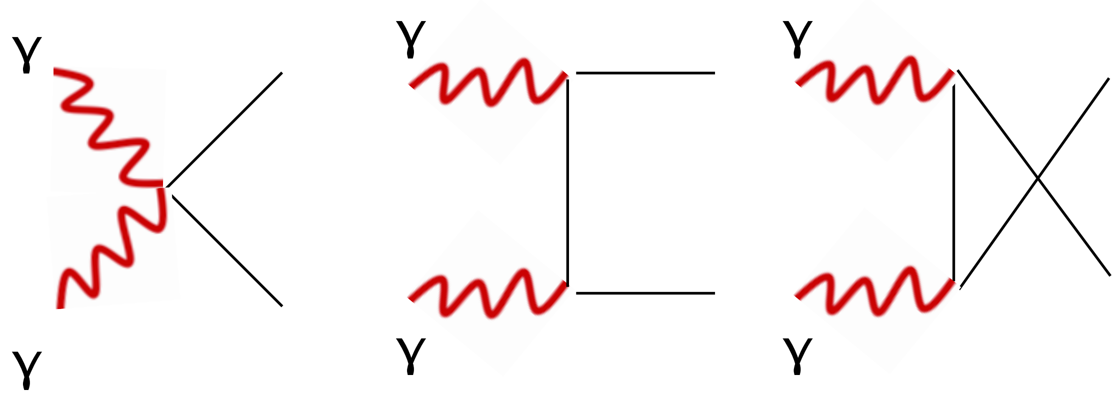
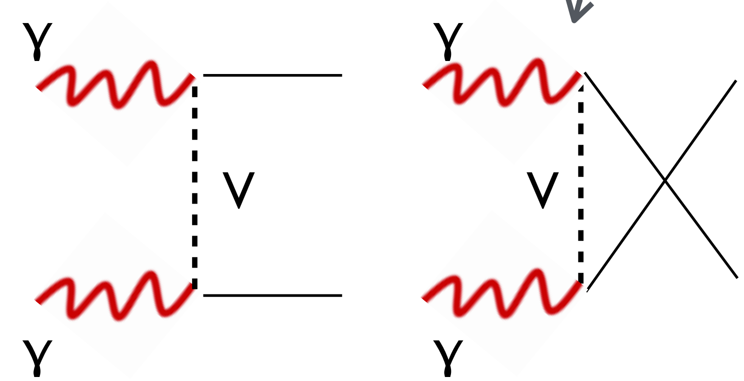
Scalar QED



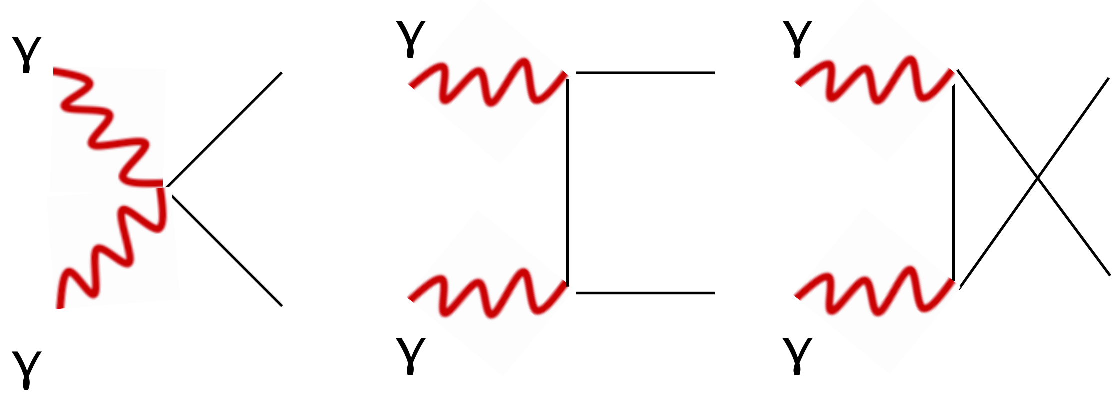
V-meson exchange



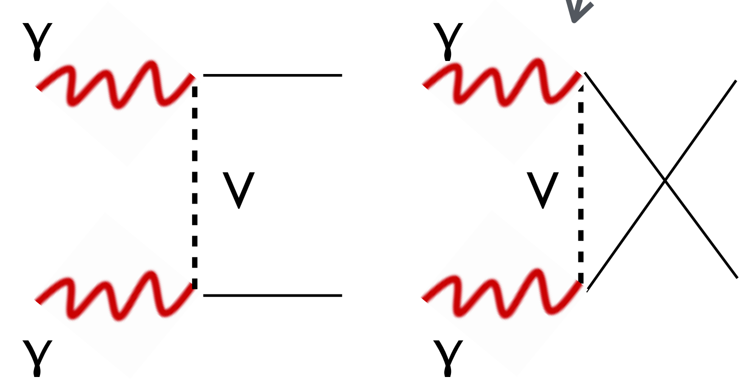
Helicity - 0, s-wave

$$\begin{pmatrix} h_{++}(s) \\ k_{++}(s) \end{pmatrix} = \begin{pmatrix} 0 \\ k_{++}^{Born}(s) \end{pmatrix} + \Omega(s)\left[\begin{pmatrix} a \\ b \end{pmatrix} + \frac{s-s_{th}}{\pi}\int_{-\infty}^{s_L} \frac{ds'}{s'-s_{th}} \frac{\Omega(s')^{-1}}{s'-s}\begin{pmatrix} \mathrm{Im}\, h_{++}(s') \\ \mathrm{Im}\, \bar{k}_{++}(s') \end{pmatrix} - \frac{s-s_{th}}{\pi}\int_{s_{th}}^{\infty} \frac{ds'}{s'-s_{th}} \frac{\mathrm{Im}\,\Omega(s')^{-1}}{s'-s}\begin{pmatrix} 0 \\ k_{++}^{Born}(s') \end{pmatrix}\right]$$

Scalar QED



V-meson exchange



Subtraction constants:

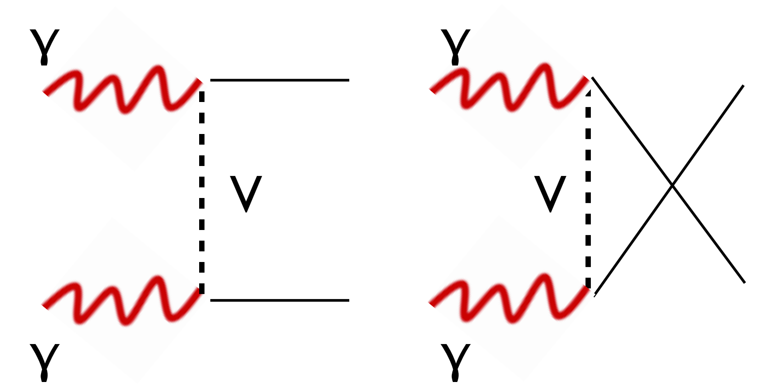
$$a = h^0_{1,++}(s_{th}) \simeq h^{0,Vexch}_{1,++}(s_{th})$$

$$b = \bar{k}^0_{1,++}(s_{th}) \simeq k^{0,Vexch}_{1,++}(s_{th})$$

Left-hand cuts

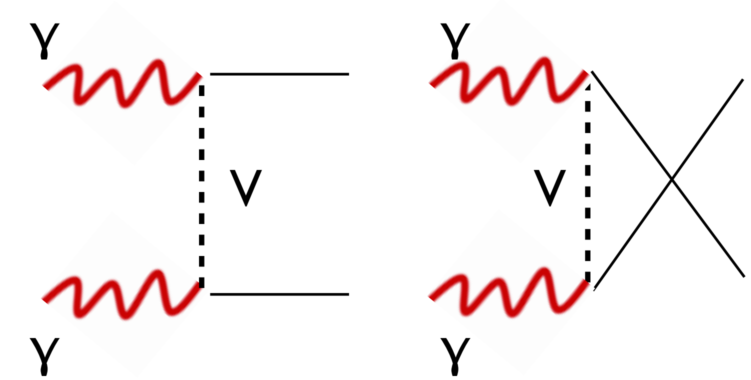
Approximate left-hand cuts by vector meson exchanges

$$\mathcal{L} = e\, C_V\, \epsilon_{\mu\nu\alpha\beta}\, F^{\mu\nu}\, \partial^\alpha \phi\, V^\beta$$

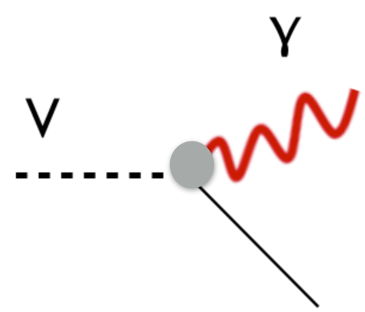


Left-hand cuts

Approximate left-hand cuts by vector meson exchanges

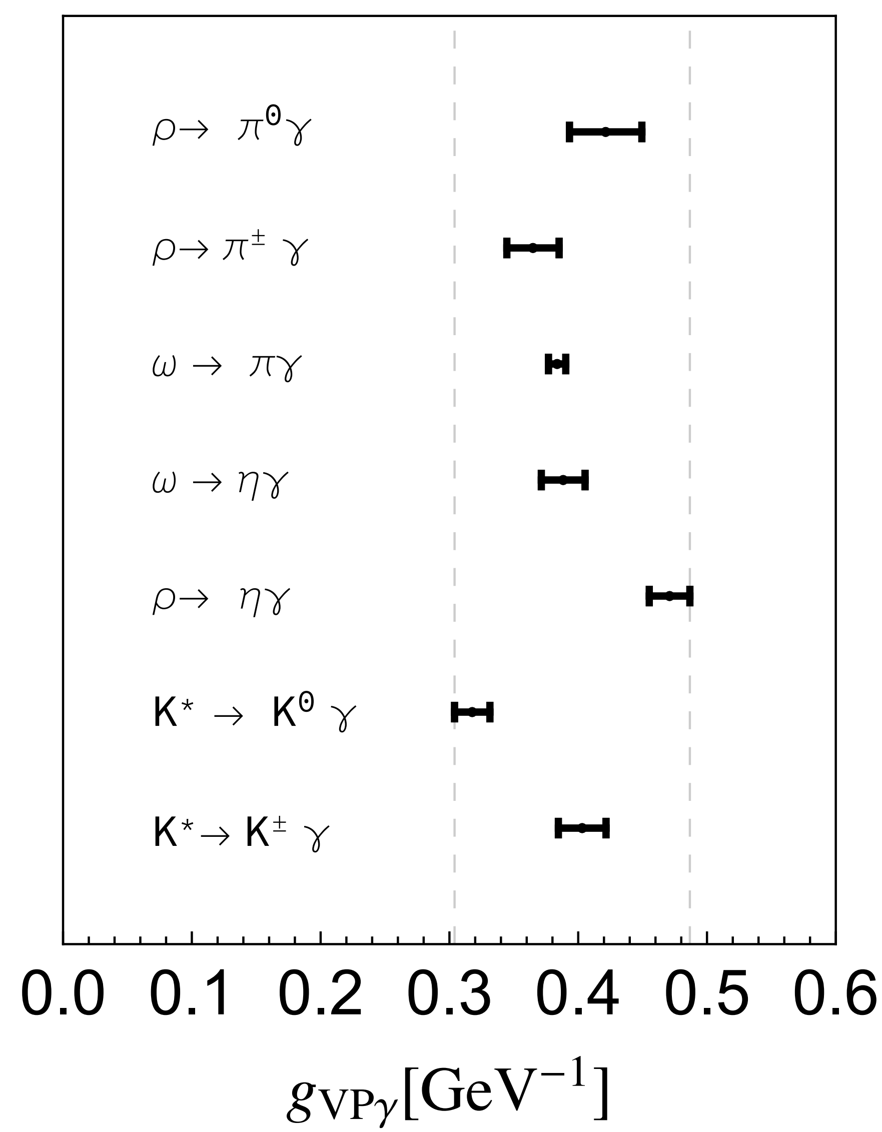


$$\mathcal{L} = e\, C_V\, \epsilon_{\mu\nu\alpha\beta}\, F^{\mu\nu}\, \partial^\alpha \phi\, V^\beta$$



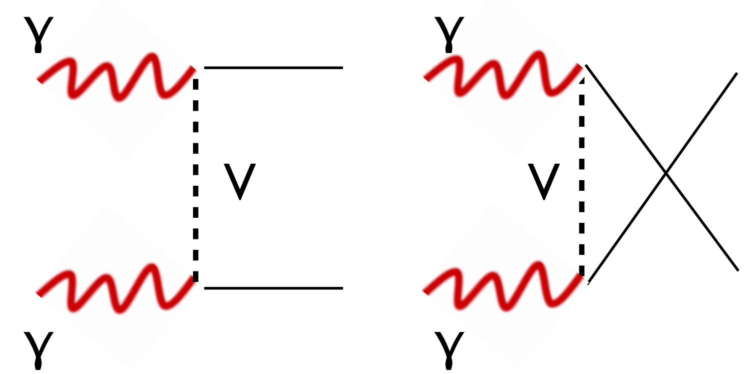
Physical couplings (rescaled using isospin symmetry)

✔ $C_{\rho\to\pi^{\pm,0}\gamma},\ \frac{1}{3} C_{\omega\to\pi^0\gamma},\ \sqrt{3}\, C_{\omega\to\eta\gamma},\ \ldots$



Left-hand cuts

Approximate left-hand cuts by vector meson exchanges



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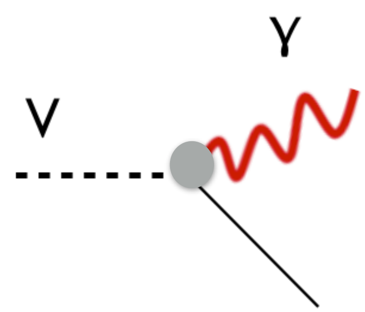


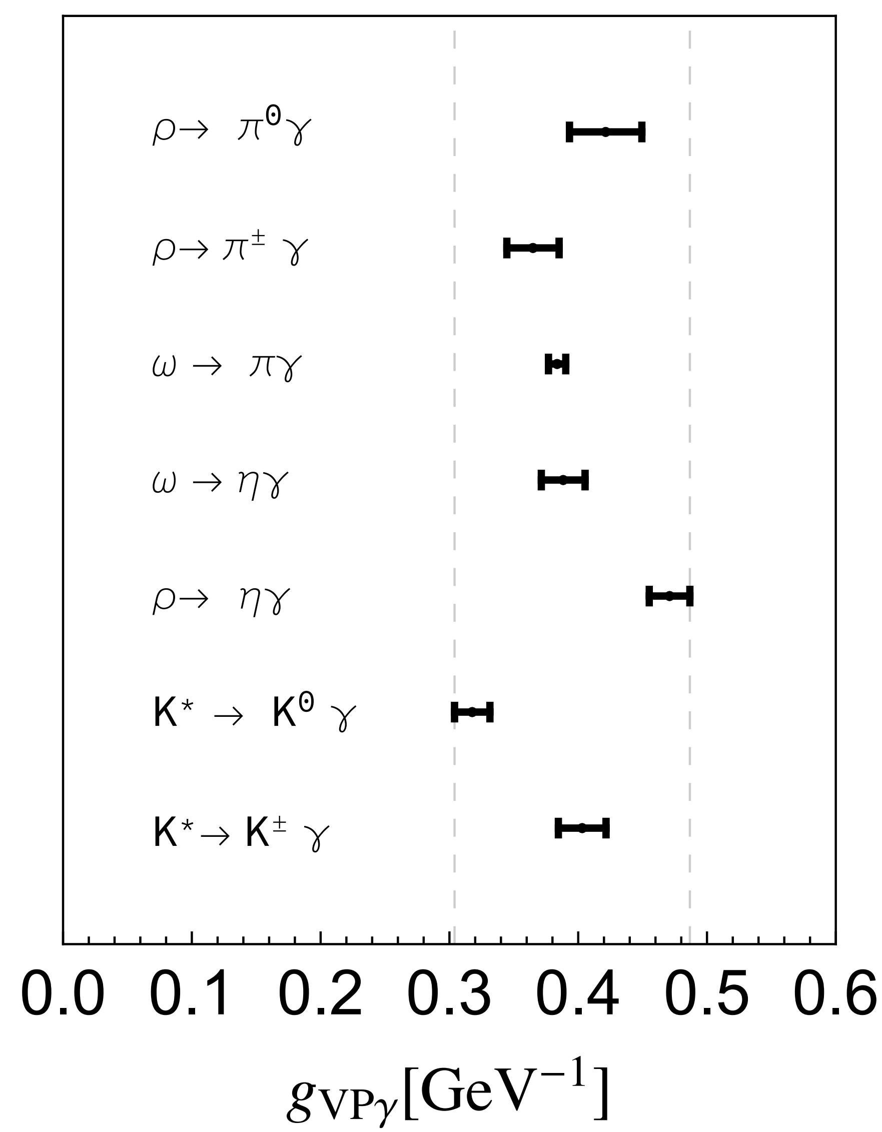
Physical couplings (rescaled using isospin symmetry)

✔ $C_{\rho\to\pi^{\pm,0}\gamma},\, \frac{1}{3}C_{\omega\to\pi^0\gamma},\, \sqrt{3}\, C_{\omega\to\eta\gamma}, \ldots$

Universal (effective) coupling

✔ $g_{VP\gamma} = C_{\rho\to\pi^{\pm,0}\gamma} = \frac{1}{3}C_{\omega\to\pi^0\gamma} = \sqrt{3}\, C_{\omega\to\eta\gamma}\ldots$

$$= 0.4 \pm 0.1\ \text{GeV}^{-1}$$



$\eta \to \pi^0\gamma\gamma$

$\eta \to \pi^0\gamma\gamma$ is linked to $\gamma\gamma \to \pi^0\eta$ by crossing symmetry

$$\frac{d^2\Gamma}{ds\,dt} = \frac{1}{(2\pi)^3}\frac{1}{32\,m_\eta^3}\sum_{\lambda_1,\lambda_2}|H_{\lambda_1\lambda_2}|^2$$

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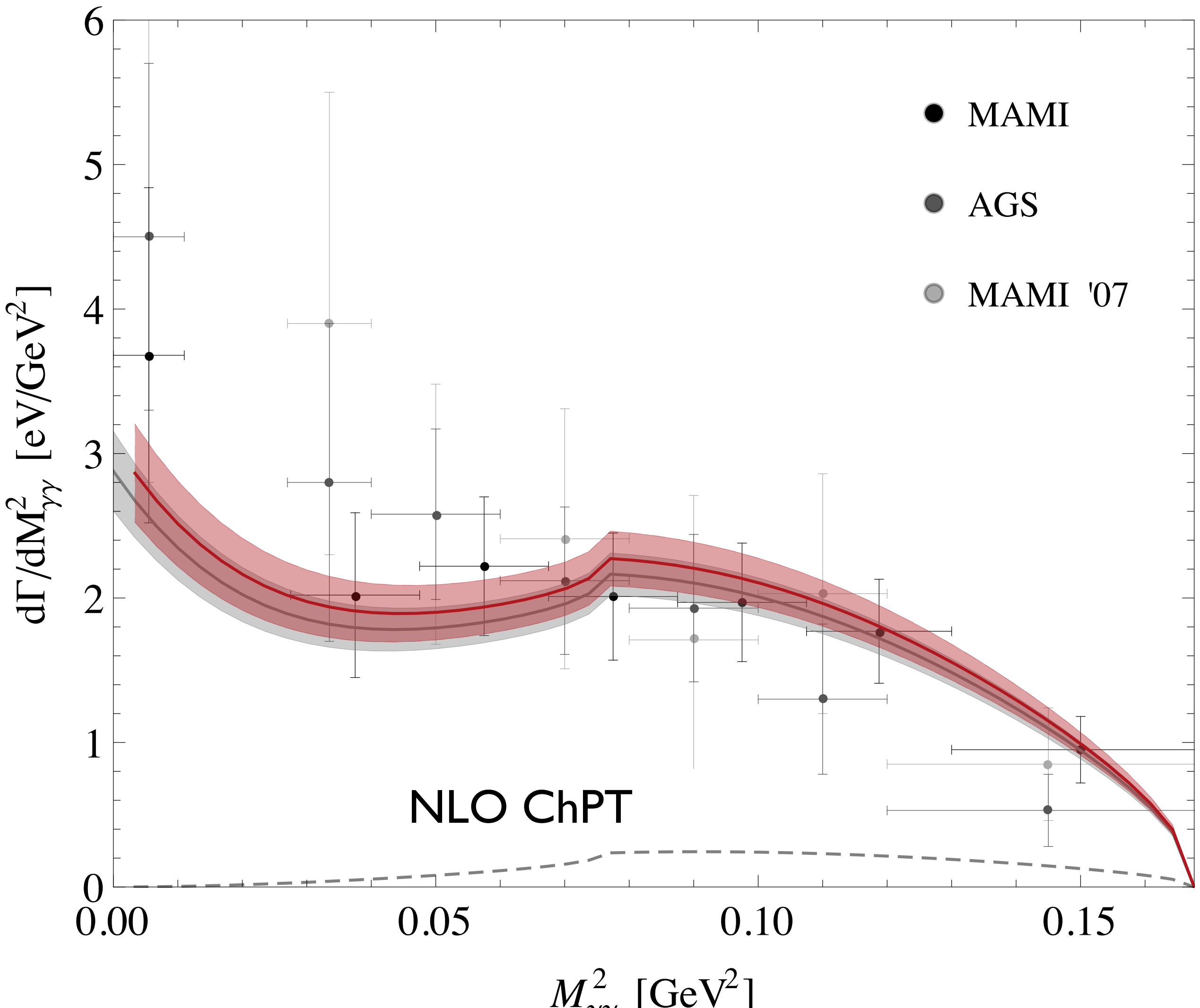

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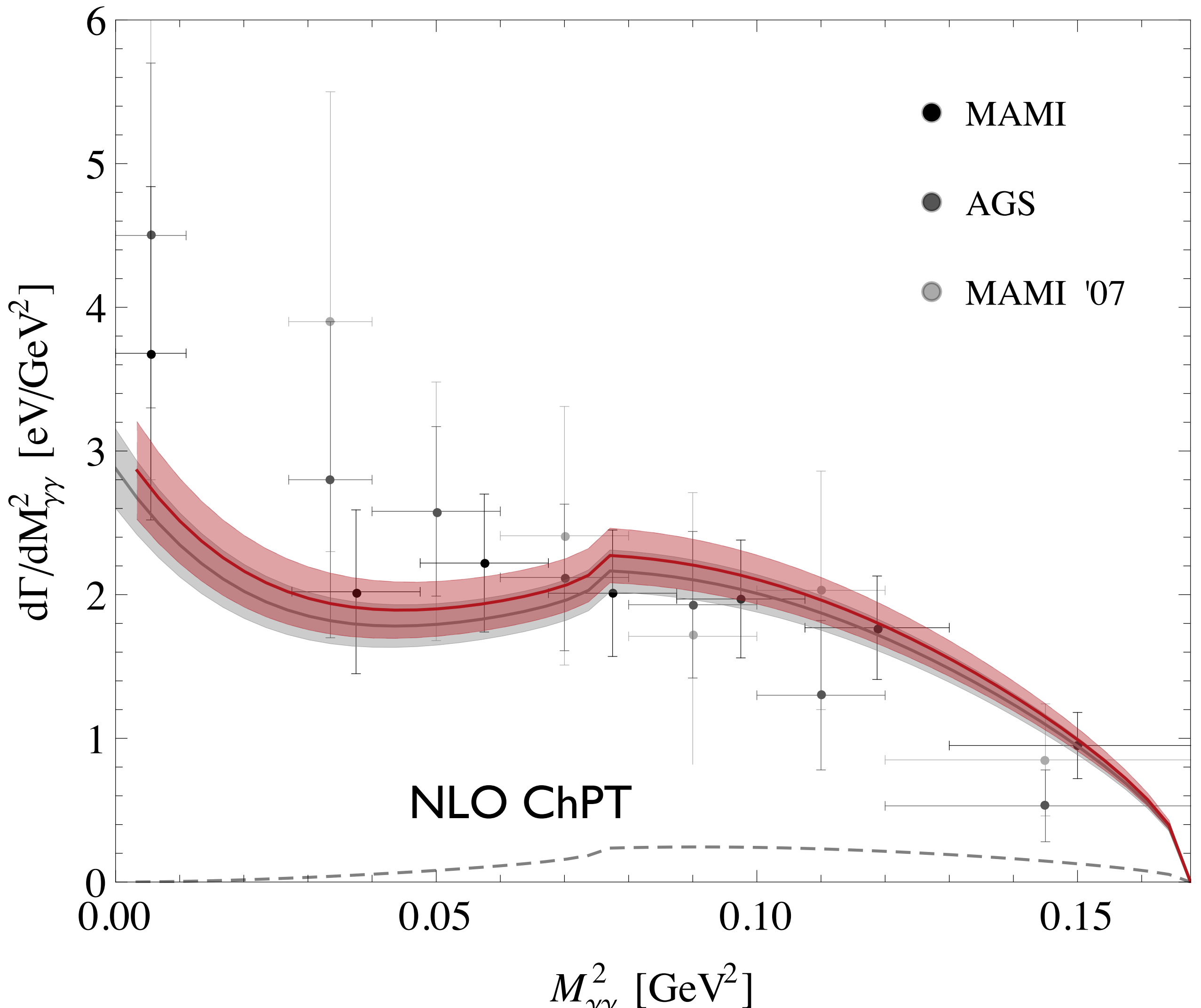
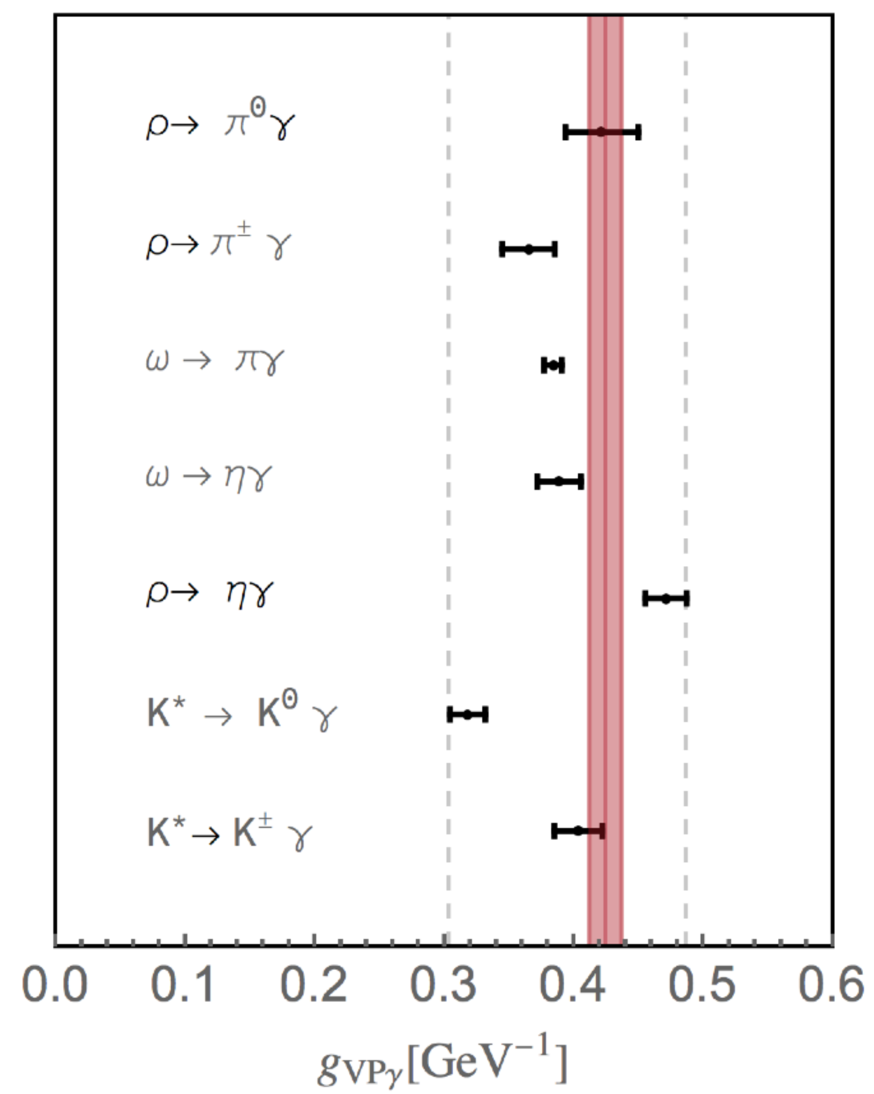


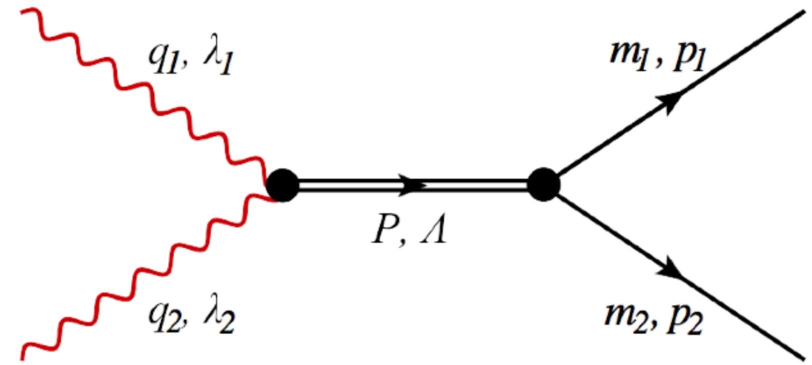
We find

$$\Gamma_{\eta\to\pi^0\gamma\gamma} = 0.291 \pm 0.022 \text{ eV} \quad \text{[Phys. coupl.]}$$
$$\Gamma_{\eta\to\pi^0\gamma\gamma} = 0.303 \pm 0.029 \text{ eV} \quad \text{[Fit } g_{VP\gamma}\text{]}$$
$$\Gamma^{\rm PDG}_{\eta\to\pi^0\gamma\gamma} = 0.334 \pm 0.028 \text{ eV}$$



$a_2(1320)$ contribution

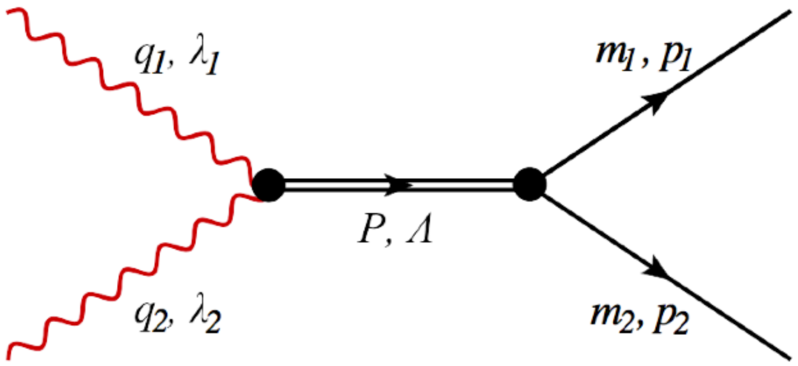
$a_2(1320)$ resonance implemented as explicit d.o.f



$$\mathcal{L}_{T\to PP} = e^2\, C_{T\to PP}\, T_{\mu\nu}\, F^{\mu\lambda} F^{\nu}_{\lambda}$$
$$\mathcal{L}_{T\to\gamma\gamma} = C_{T\to\gamma\gamma}\, T^{\mu\nu}\, \partial_\mu P \partial_\nu P$$

$a_2(1320)$ contribution

$a_2(1320)$ resonance implemented as explicit d.o.f



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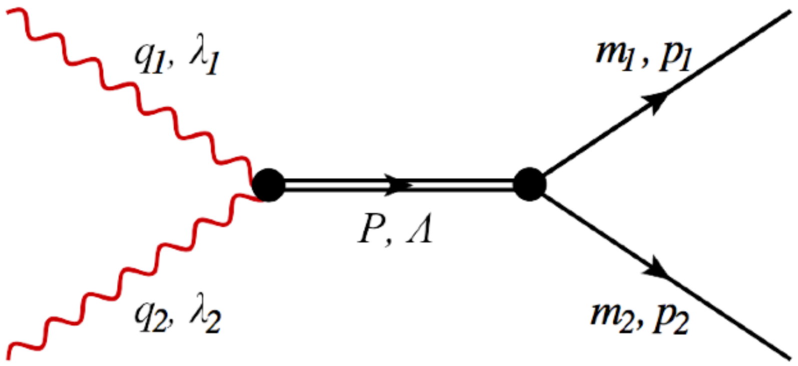
$$\mathcal{L}_{T\to\gamma\gamma} = C_{T\to\gamma\gamma}\, T^{\mu\nu}\, \partial_\mu P \partial_\nu P$$

Helicity - 2, d-wave

$$h_{+-}(s) = \frac{e^2\, C_{a_2\to\pi\eta}\, C_{a_2\to\gamma\gamma}\, s^2\, \beta^2_{\pi\eta}(s)}{10\,\sqrt{6}\,(s - M^2_{a_2} + i\, M_{a_2}\, \Gamma_{a_2}(s))}$$

$a_2(1320)$ contribution

$a_2(1320)$ resonance implemented as explicit d.o.f



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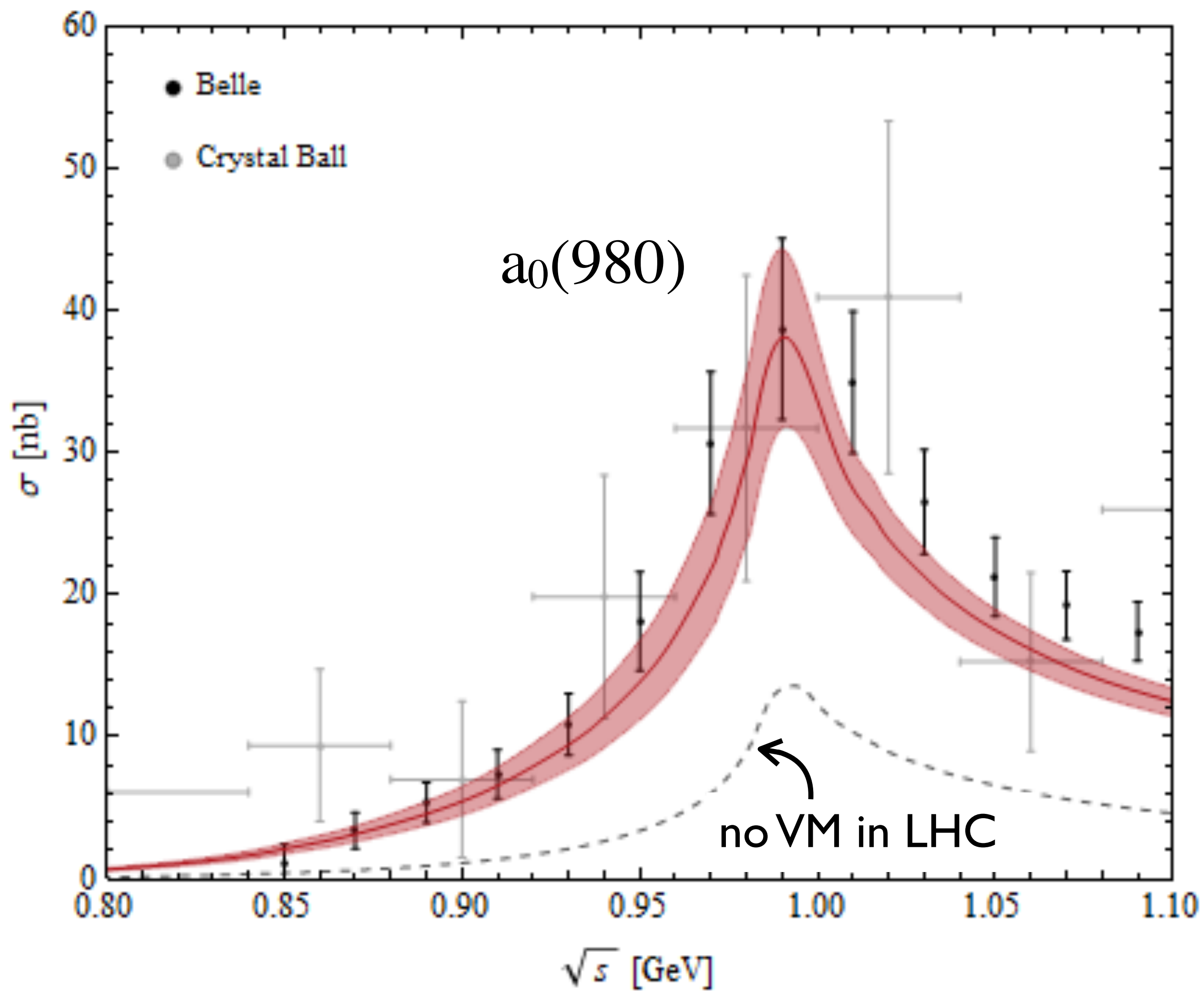
Helicity - 2, d-wave

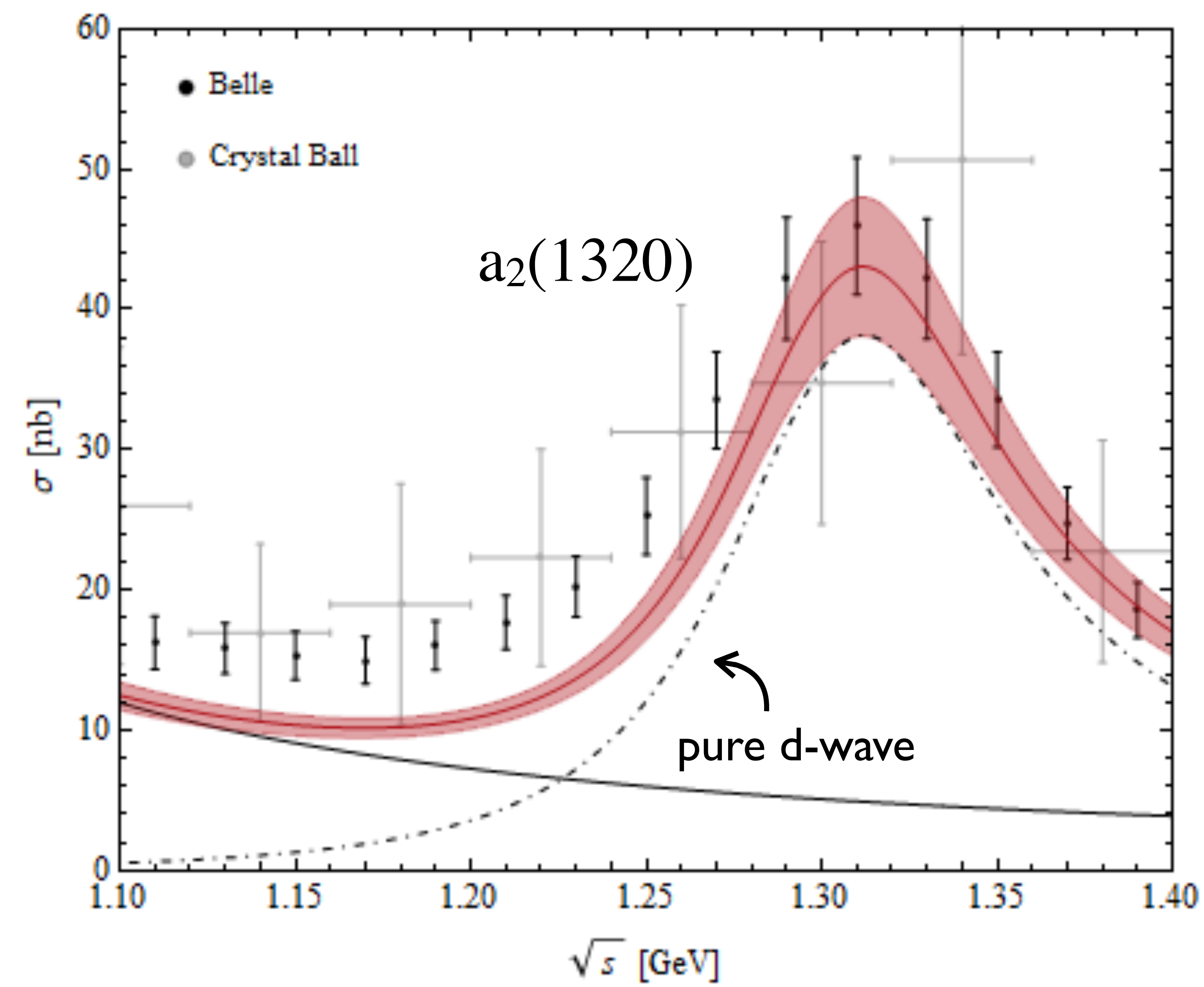
$$h_{+-}(s) = \frac{e^2\, C_{a_2\to\pi\eta}\, C_{a_2\to\gamma\gamma}\, s^2\, \beta^2_{\pi\eta}(s)}{10\sqrt{6}\,(s - M^2_{a_2} + i\, M_{a_2}\, \Gamma_{a_2}(s))}$$

Couplings

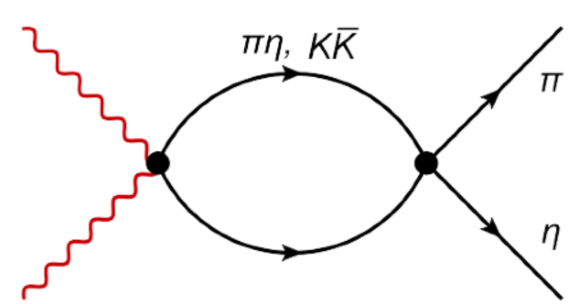
$$\Gamma_{a_2\to\pi\eta} = \frac{\beta^5_{\pi\eta}(M^2_{a_2})}{1920\,\pi}\, C^2_{a_2\to\pi\eta}\, M^3_{a_2} \overset{\text{exp}}{=} 15.5(1.5)\,\text{MeV}$$
$$\Gamma_{a_2\to\gamma\gamma} = \frac{\pi\alpha^2}{5}\, C^2_{a_2\to\gamma\gamma}\, M^3_{a_2} \overset{\text{exp}}{=} 1.0(1)\,\text{keV}$$

Results: γγ→πη

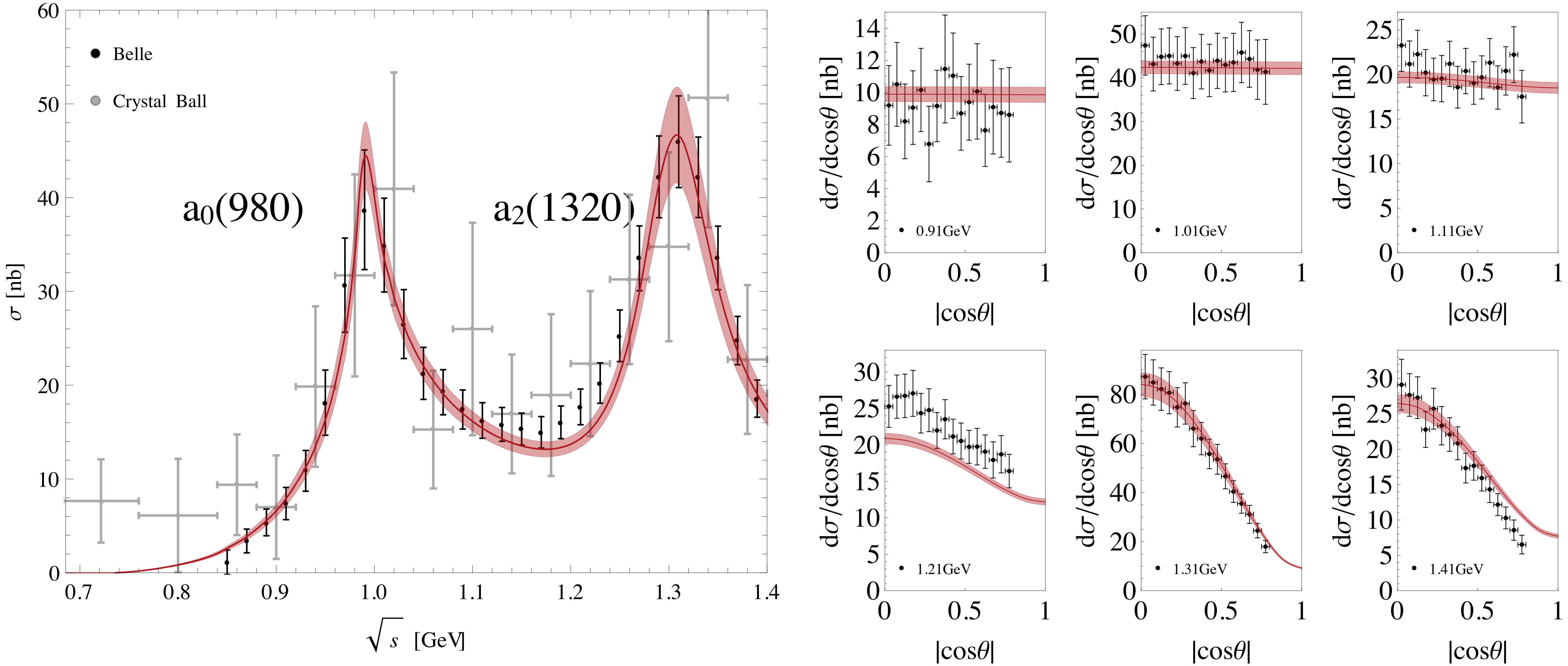




- ✔ **Parameter free** post-diction using physical couplings: $C_{\rho\to\pi^0\gamma,\eta\gamma,\ldots}$ and $C_{a_2\to\pi\eta,\gamma\gamma}$
- ✔ Coupled-channel dispersive treatment of $a_0(980)$ is **crucial**
- ✔ $a_2(1320)$ described as explicit d.o.f



Results: $\gamma\gamma \to \pi\eta$



- Scrutinise the uncertainties of our **hadronic input** using $g_{VP\gamma}$ and fitting $\gamma\gamma \to \pi\eta$ data
- Total and differential cross sections are in good agreement with the data

Two-photon coupling

For the pole on the IV Riemann sheet unitarity implies

$$h^{IV}_{\gamma\gamma\to\pi\eta}(s) - h^{I}_{\gamma\gamma\to\pi\eta}(s) = 2\,i\,\rho_{K\bar{K}}(s)\,h^{I}_{\gamma\gamma\to K\bar{K}}(s)\,t^{IV}_{K\bar{K}\to\pi\eta}(s)$$

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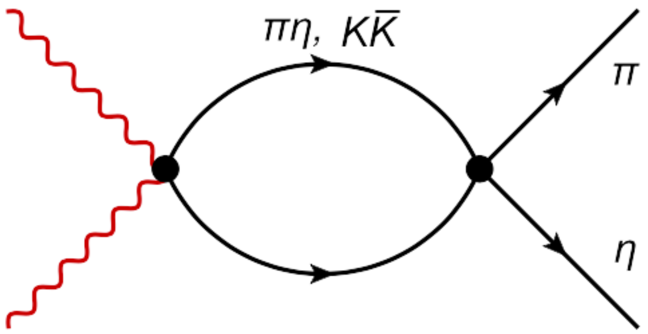
In the vicinity of the pole one can write

$$h^{IV}_{\gamma\gamma\to\pi\eta}(s) \simeq \frac{C_{\gamma\gamma}\,C_{\pi\eta}}{s^{\rm IV}_{a_0} - s}\,, \quad t^{IV}_{K\bar{K}\to\pi\eta}(s) \simeq \frac{C_{K\bar{K}}C_{\pi\eta}}{s^{\rm IV}_{a_0} - s}$$

$$\left(\frac{C_{\gamma\gamma}}{C_{K\bar{K}}}\right)^2 = -(2\rho_{K\bar{K}}(s^{IV}_{a_0}))^2(t^{I}_{\gamma\gamma\to K\bar{K}}(s^{IV}_{a_0}))^2$$



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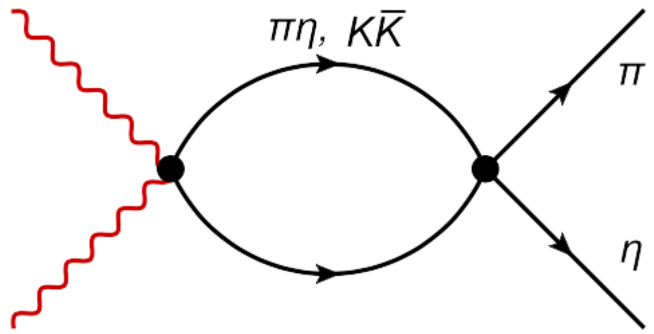


The two photon decay width

$$\Gamma_{\gamma\gamma} = \frac{|C_{\gamma\gamma}|^2}{16\pi m_0} = 0.27(4)\,\text{keV}$$

Experimental value

$$\Gamma_{a_0\to\gamma\gamma}\mathcal{B}(\pi^0\eta) = 0.21^{+0.08}_{-0.04}\ \text{keV}$$

Summary and Outlook

- We have presented a theoretical study of the **$\gamma\gamma \to \pi\eta$ reaction** from the threshold up to 1.4 GeV
- We used a coupled-channel **dispersive approach** in order to properly describe the scalar a(980) resonance, which has a dynamical {πη, KK} origin
- We analytically continued the amplitude into the unphysical regions. We found the pole on the **IV Riemann sheet**, which produces a strong **cusp-like behaviour** of the cross section exactly at the KK threshold
- At the pole position, we calculated the **two-photon coupling** which corresponds to **radiative width**: $\Gamma_{\gamma\gamma}$=0.27(4) keV
- Our analysis is a necessary starting point for a further study where one of the initial photons has a **finite virtuality ($\gamma^*\gamma^* \to \pi\eta$)**: input to (g-2) & ongoing BESIII analysis

Thank you!

Extra slides

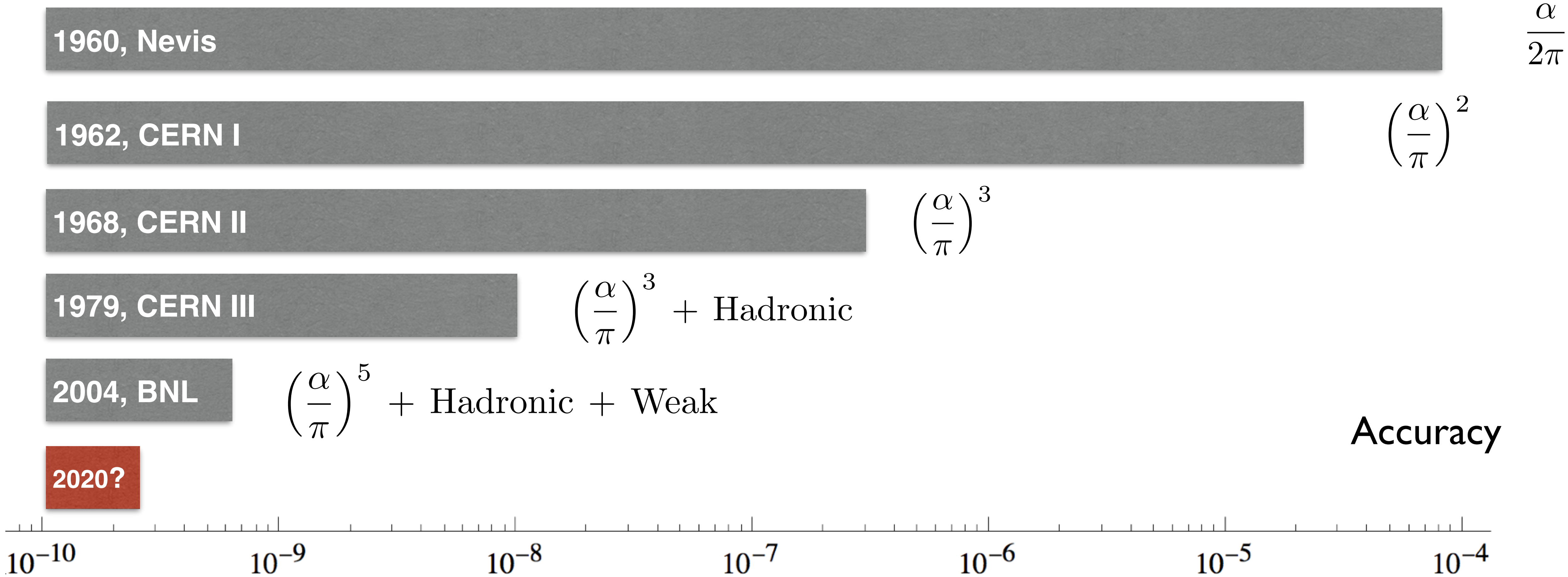
History of achieved accuracy

- Magnetic moment of the muon

$$\vec{\mu} = \frac{Q}{2m}\, g\, \vec{S}$$

- anomalous part

$$a_\mu = \frac{(g-2)_\mu}{2}$$



$$a_\mu^{exp} = (11\,659\,208.9 \pm 6.3) \times 10^{-10}$$

$$a_\mu^{SM} = (11\,659\,182.8 \pm 4.9) \times 10^{-10}$$

$$a_\mu^{exp} - a_\mu^{SM} =$$

$$(26.1 \pm 4.9_{th} \pm 6.3_{exp}) \times 10^{-10}$$

? 1.6_{exp}

3 - 4 σ deviation!

FNAL, J-PARC

QCD contribution to (g-2)

$$a_\mu^{QCD} = (695.6 \pm 4.9) \times 10^{-10}$$

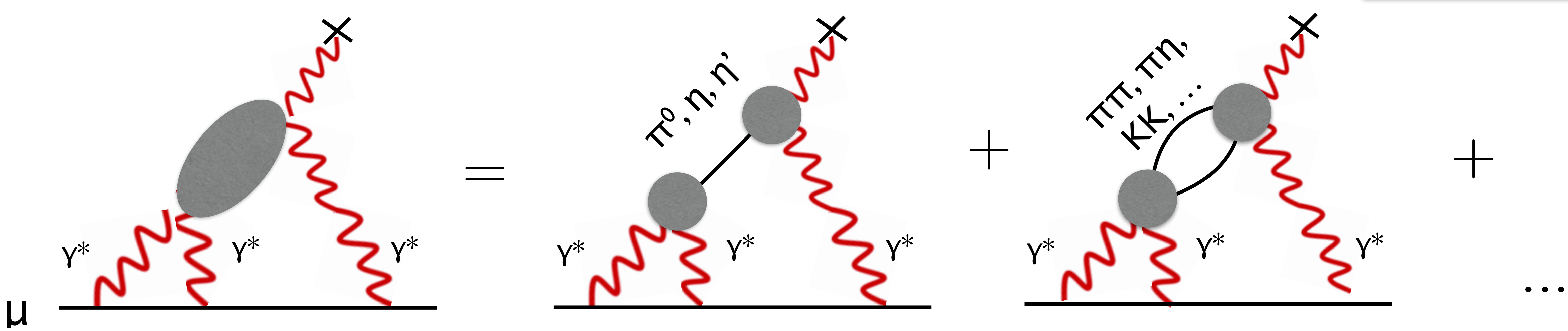
Hadronic vacuum polarization

$$a_\mu^{QCD,VP} = (685.1 \pm 4.3) \times 10^{-10}$$

Hadronic light-by-light scattering

$$a_\mu^{QCD,\,LbL} = (10.2 \pm 3.9) \times 10^{-10}$$

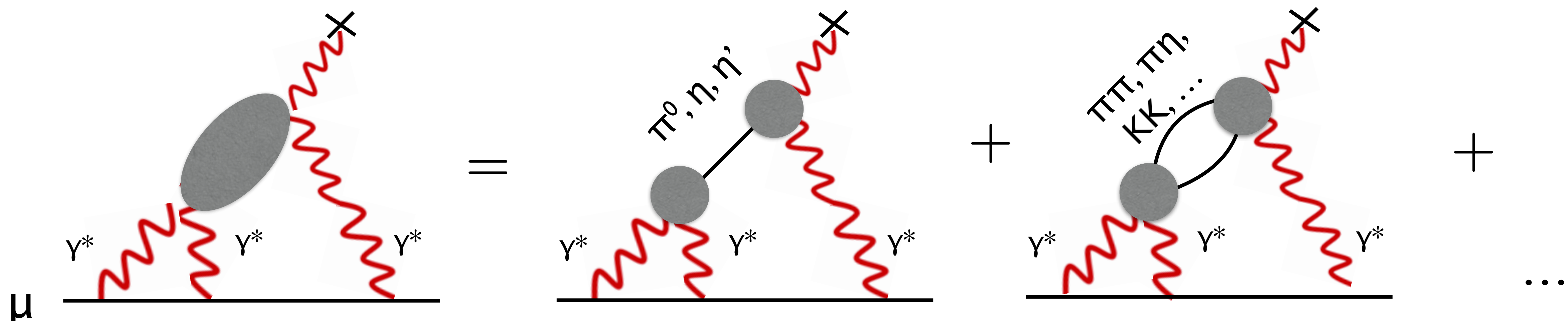
Hagivara (2011)
Jegerlehnner (2015)



Relies on measurements of **TFF** $\pi^0\gamma^*\gamma^{(*)}, \eta\gamma^*\gamma^{(*)}, \ldots$ to reduce model dependence

Dispersive analysis for ππ, πη, … loops is needed

HLbL contributions to (g-2) in units 10^{-10}



Authors	π^0, η, η'	$\pi\pi$, KK	scalars	axial vectors	quark loops	Total
BPaP(96)	8.5(1.3)	-1.9(1.3)	-0.68(0.20)	0.25(0.10)	2.1(03)	8.3(3.2)
HKS(96)	8.3(0.6)	-0.5(0.8)	—	0.17(0.17)	1.0(1.1)	9.0(1.5)
KnN(02)	8.3(1.2)	—	—	—	—	8.0(4.0)
MV(04)	11.4(1.0)	—	—	2.2(0.5)	—	13.6(2.5)
PdRV(09)	11.4(1.3)	-1.9(1.9)	-0.7(0.7)	1.5(1.0)	0.23	**10.5(2.6)**
N/JN(09)	9.9(1.6)	-1.9(1.3)	-0.7(0.2)	2.2(0.5)	2.1(0.3)	11.6(3.9)
J(15)	9.9(1.6)	-1.9(1.3)	-0.7(0.2)	**0.75(0.27)**	2.1(0.3)	**10.2(3.9)**

B=Bjnens, Pa=Pallante, P=Prades, H=Hayakawa, K=Kinoshita, S=Sanda, Kn=Knecht, N=Nyffeler, M=Melnikov, V=Vainshtein, dR=de Rafael, J=Jegerlehner

N/D technique

Solve p.w. dispersion relation using N/D technique using model-independent form for the left-hand cuts

$$T(s) = U(s) + \frac{s-s_{th}}{\pi}\int_R \frac{ds'}{s'-s_{th}}\frac{\rho(s')|T(s')|^2}{s'-s}$$

$$\downarrow$$

$$\sum_k C_k\,\xi(s)^k$$

conformal mapping expansion C_k
fitted to exp data or **matched** to ChPT

Chew, Mandelstam
Lutz, I.D, Gasparyan

$$T(s) = \frac{N(s)}{D(s)} = \Omega(s)\,N(s)$$

$$N(s) = U(s) + \frac{s-s_{th}}{\pi}\int_R \frac{ds'}{s'-s_{th}}\frac{\rho(s')N(s')(U(s)-U(s'))}{s'-s}$$

$$D(s) = 1 - \frac{s-s_{th}}{\pi}\int_R \frac{ds'}{s'-s_{th}}\frac{\rho(s')N(s')}{s'-s}$$

Conformal mapping

Solve p.w. dispersion relation using N/D technique using model-independent form for the left-hand cuts

$$T(s) = U(s) + \frac{s - s_{th}}{\pi} \int_R \frac{ds'}{s' - s_{th}} \frac{\rho(s')|T(s')|^2}{s' - s}$$

$$\downarrow$$

$$\sum_k C_k\, \xi(s)^k$$

conformal mapping expansion C_k
fitted to exp data or **matched** to ChPT

Chew, Mandelstam
Lutz, I.D, Gasparyan

$$\xi(s) = \frac{a\,(\Lambda_S^2 - s)^2 - 1}{(a - 2\,b)(\Lambda_S^2 - s)^2 + 1}$$

$$a = \frac{1}{(\Lambda_S^2 - \mu_E^2)^2}, \quad b = \frac{1}{(\Lambda_S^2 - \Lambda_0^2)^2}$$