



Effective Field Theory for three-body hypernuclei

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GEFÖRDERT VOM



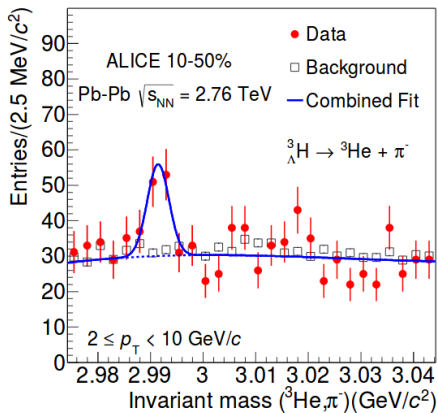
Bundesministerium
für Bildung
und Forschung

- ▶ Hypernuclei contain at least one hyperon (H) ($S \neq 0$)
- ▶ Consider three-body hypernuclei NNH
- ▶ Restriction to Λ ($M_\Lambda = 1115.68$ MeV) particle as hyperon leads to the systems $S = -1$

$$\text{isospin } (I = 1) = \begin{cases} pp\Lambda \\ \frac{1}{\sqrt{2}} (np + pn) \Lambda \\ nn\Lambda \end{cases} \quad \text{isospin } (I = 0) = \frac{1}{\sqrt{2}} (pn - np) \Lambda$$

- ▶ Two examples: $nn\Lambda$ and hypertriton

The hypertriton



- ▶ Hypertriton is bound with a binding energy of $= 2.35 \pm 0.05$ MeV

[M. Juric et al., Nucl. Phys. B52, 1 (1973)]

- ▶ Consists of separation energy into a deuteron ($B_d = 2.22$) MeV and a Λ $B_{\Lambda} = 0.13 \pm 0.05$ MeV

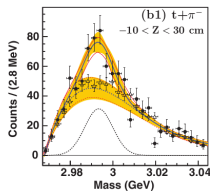
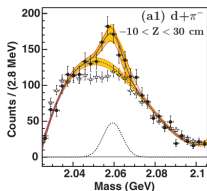
- ▶ Hypertriton was also produced at ALICE in Pb-Pb reaction recently

[ALICE Collaboration Phys. Lett. B 754 (2016) 360-372]

- ▶ mean invariant mass
 $\mu = 2991 \pm 1 \pm 3$ MeV
 $\Rightarrow B \approx 2$ MeV

- ▶ 2013 the HypHI Collaboration found evidence that the Λnn might be bound

[HypHI Collaboration C.Rappold et al. Phys.Rev.C.88(041001(R)),2013]



- ▶ $d + \pi^-$ and $t + \pi^-$ final states ${}^6\text{Li} + {}^{12}\text{C}$ reactions
- ▶ possible explanation of the observed final decay of a bound Λnn
- ▶ ${}^3_{\Lambda}n \rightarrow t + \pi^-$
- ▶ ${}^3_{\Lambda}n \rightarrow t^* + \pi^- \rightarrow d + n + \pi^-$
- ▶ mean invariant mass $\mu = 2993.7 \pm 1.3 \pm 0.6$ MeV
($t + \pi^-$) $\Rightarrow B \approx 1$ MeV

- ▶ Use pionless EFT \rightarrow all interactions are contact interactions
- ▶ Exploit dibaryon formalism
- ▶ $NN\Lambda$ interaction channels (only S-wave)

$$NN\Lambda = \begin{cases} {}^1S_0(NN) + \Lambda, & \Lambda nn \\ {}^3S_1(NN) + \Lambda, & \text{hypertriton} \\ {}^3S_1(N\Lambda) + N, & \text{hypertriton and } \Lambda nn \\ {}^1S_0(N\Lambda) + N, & \text{hypertriton and } \Lambda nn \end{cases}$$

- ▶ Explicit $\Lambda \Leftrightarrow \Sigma$ conversions are not included (implicit in the 3-body force)

$$\gamma_3^\Lambda \sim 2\sqrt{MB_3^\Lambda/3} \approx 1.2\gamma_d \approx 54 \text{ MeV}$$

$$\ll \sqrt{M_\Lambda(M_\Sigma - M_\Lambda)} \approx 300 \text{ MeV}$$

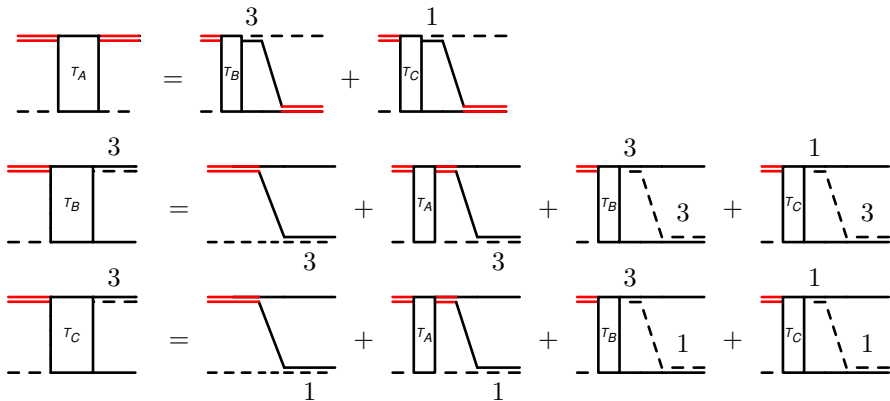
Lagrangian for the hypertriton and Λnn system

$$\begin{aligned}
 \mathcal{L} = & \quad \text{N} \quad + \quad \Lambda \\
 & \quad \text{---} \quad + \quad \text{---} \\
 & + \quad {}^1S_0(NN) \quad + \quad {}^3S_1(NN) \quad + \quad {}^3S_1(\Lambda N) \quad + \quad {}^1S_0(\Lambda N) \\
 & + \quad \text{---} \quad + \quad \text{---} \quad + \quad \frac{\text{---}}{3} \quad + \quad \frac{\text{---}}{1} \\
 & + \quad \text{---} \quad + \quad \text{---} \quad + \quad \frac{\text{---}}{3} \quad + \quad \frac{\text{---}}{1} \\
 & + \quad \dots
 \end{aligned}$$

The diagram shows the Lagrangian \mathcal{L} for the hypertriton and Λnn system. It is composed of several terms:

- A term for the nucleon (N) represented by a solid horizontal line.
- A term for the lambda baryon (Λ) represented by a dashed horizontal line.
- Four terms representing different partial waves:
 - ${}^1S_0(NN)$: A blue double horizontal line.
 - ${}^3S_1(NN)$: A red double horizontal line.
 - ${}^3S_1(\Lambda N)$: A dashed double horizontal line with a denominator of 3.
 - ${}^1S_0(\Lambda N)$: A dashed double horizontal line with a denominator of 1.
- Four corresponding vertex diagrams, each showing a double line (solid or dashed) on the left and two lines (solid or dashed) on the right, representing the interaction between the double-line particle and the two single-line particles.
- A final term with an ellipsis (\dots) indicating additional terms in the Lagrangian.

Integral equations for the hypertriton



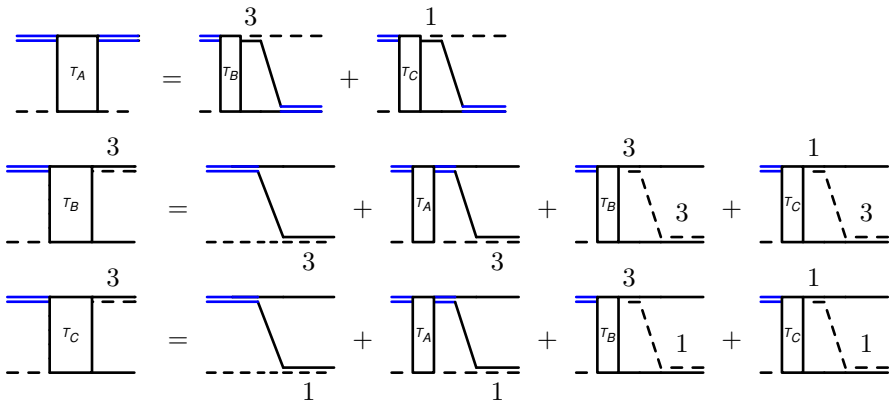
The diagrammatic equations are as follows:

$$T_A = T_B + T_C$$

$$T_B = \text{direct} + T_A + T_B \text{ (propagator 3)} + T_C \text{ (propagator 3)}$$

$$T_C = \text{direct} + T_A + T_B \text{ (propagator 1)} + T_C \text{ (propagator 1)}$$

Integral equations the Λ nn system



The diagrammatic equations are as follows:

$$\begin{aligned}
 & \tau_A = \tau_B + \tau_C \\
 & \tau_B = \tau_B^{(3)} + \tau_A^{(3)} + \tau_B^{(3,d)} + \tau_C^{(3,d)} \\
 & \tau_C = \tau_C^{(1)} + \tau_A^{(1)} + \tau_B^{(1,d)} + \tau_C^{(1,d)}
 \end{aligned}$$

Where the superscripts (3) and (1) indicate the number of nucleons in the final state, and (d) indicates a dashed line representing a nucleon in the final state.

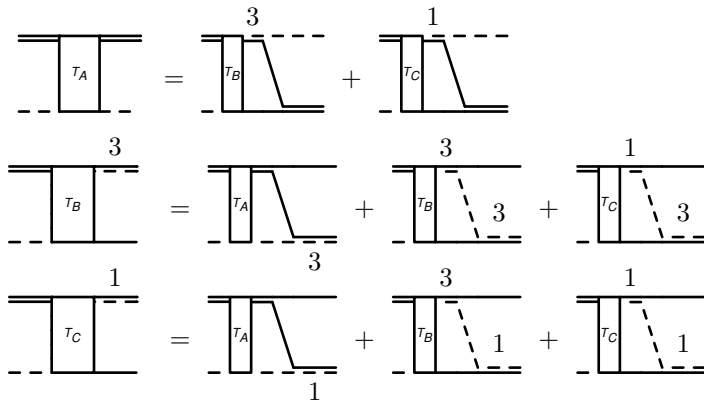
Asymptotic Analysis of the hypertriton and Λnn system

asymptotic limit
 $\Lambda_c \gg p, q \gg 1/a, \gamma \sim k$
vanishing mass difference



equation becomes
homogeneous
and can be decoupled
much simpler Kernel

Integral equations for the hypertriton and the Λnn system in the asymptotic limit



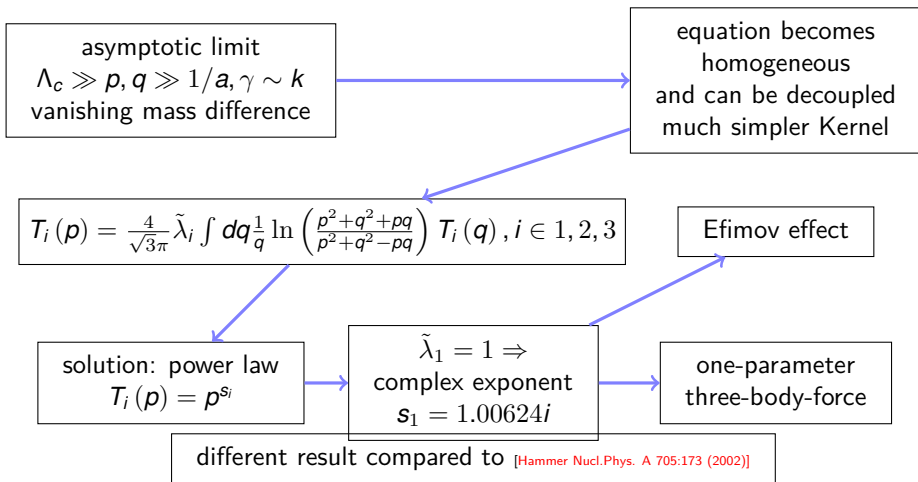
The diagrams illustrate the decomposition of transition amplitudes T_A , T_B , and T_C into sums of diagrams with dashed lines and labels 1 and 3. The diagrams are arranged in three rows, each representing an equation. The left side of each equation shows a transition amplitude T_A , T_B , or T_C between two states. The right side shows the sum of diagrams representing the asymptotic limit. The diagrams are labeled with 1 and 3, indicating the order of the expansion.

Row 1: T_A is equal to the sum of two diagrams. The first diagram has a solid line labeled T_B and a dashed line labeled 3. The second diagram has a solid line labeled T_C and a dashed line labeled 1.

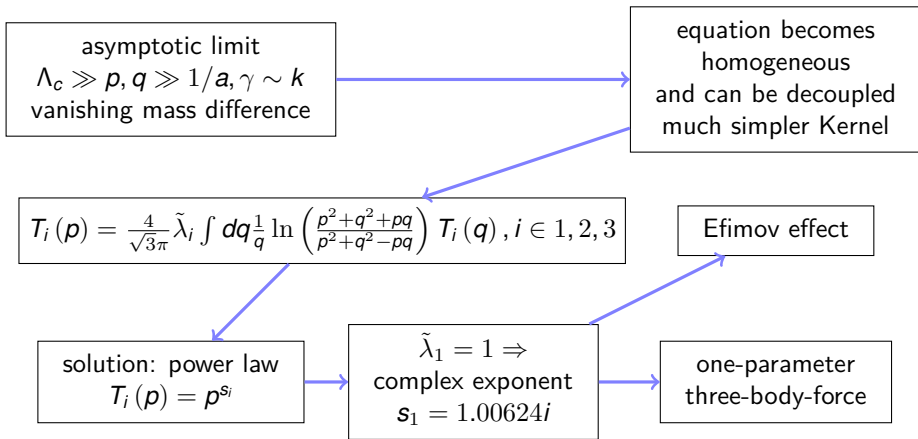
Row 2: T_B is equal to the sum of three diagrams. The first diagram has a solid line labeled T_A and a dashed line labeled 3. The second diagram has a solid line labeled T_B and a dashed line labeled 3. The third diagram has a solid line labeled T_C and a dashed line labeled 3.

Row 3: T_C is equal to the sum of three diagrams. The first diagram has a solid line labeled T_A and a dashed line labeled 1. The second diagram has a solid line labeled T_B and a dashed line labeled 1. The third diagram has a solid line labeled T_C and a dashed line labeled 1.

Asymptotic Analysis of the hypertriton and Λnn system



Asymptotic Analysis of the hypertriton and Λnn system



Efimov physics \rightarrow universal relations and correlations

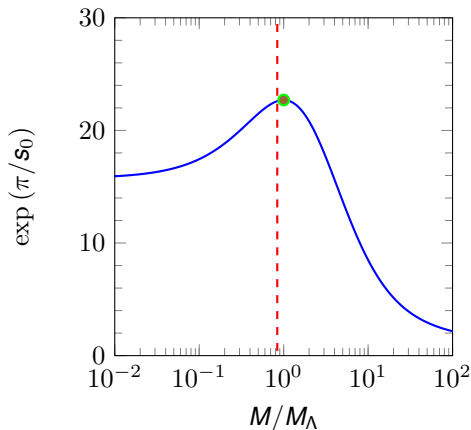
Asymptotic Analysis of the hypertriton and Λnn system-with different masses

- ▶ Physical mass of Λ and nucleons are different
- ▶ Therefore asymptotic equations do not decouple
- ▶ Since the result of a power law should be reproduced for the case of $y = 0$ we choose as an ansatz

$$T_j(p) = \alpha_j p^{s_1} + \beta_j p^{s_2} + \gamma_j p^{s_3}, j \in \{A, B, C\}$$

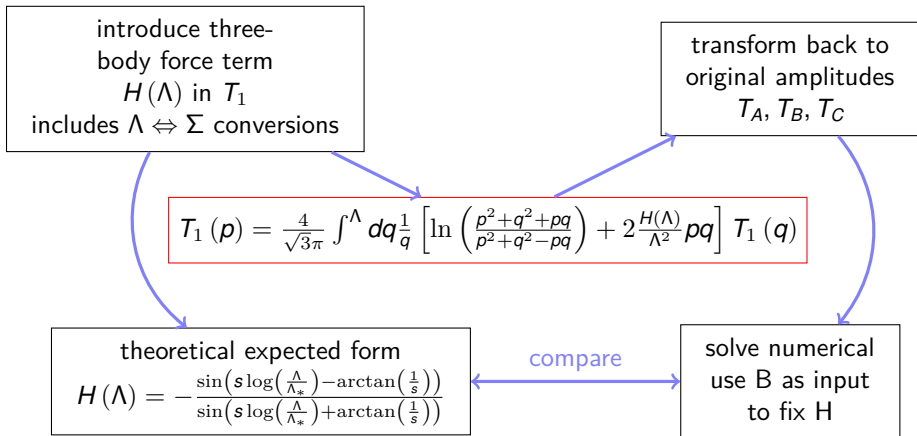
- ▶ Since kernel is more complex now \Rightarrow integrate term by term

Scaling factor as a function of the mass ratio

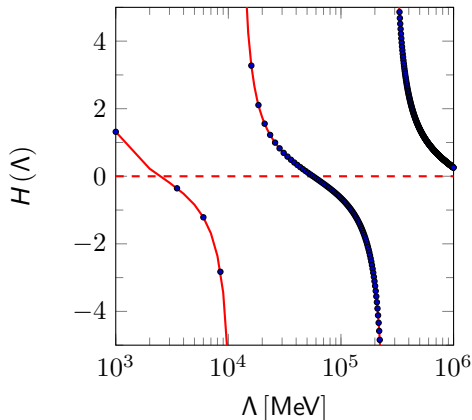


- ▶ Ansatz reproduces the $M = M_\Lambda$ result
- ▶ $s_0 = 1.00760(M/M_\Lambda = 0.84)$
- ▶ Results are consistent with Braaten and Hammer
[Braaten, Hammer Phys. Rept. 428, 259–390 (2006)]
- ▶ Used chiral EFT prediction for Λ -N scattering lengths
($a_3 = -1.45 - 1.70$ fm,
 $a_1 = -2.90 - 2.91$ fm)
[Haidenbauer et al Nucl. Phys. A 915 (2013) 24–58]
- ▶ considered to be large since ΛN range is given by the 2π exchange
 $\sim \frac{1}{2m_\pi} \approx 0.7$ fm

Introduction of the 3-body force

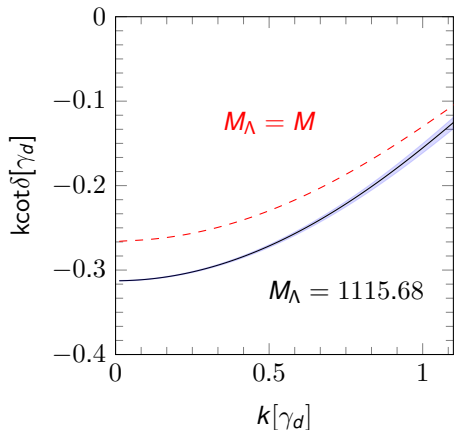


H(Λ) for the hypertriton



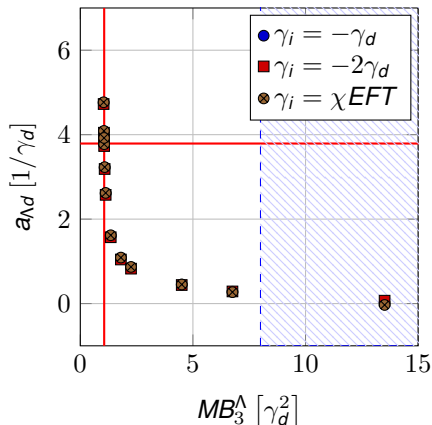
- ▶ ($a_3 = -1.64$ fm, $a_1 = -2.91$ fm)
- ▶ 3-body force shows the expected limit cycle behavior
- ▶ It is not possible to fix the Λ_{nn} 3-body force with this one due to different isospin channels ($I = 1$ and $I = 0$)
- ▶ extract the three-body parameter
 $\Lambda_* \approx 3.26^{+0.10}_{-0.13} \gamma_d$ (hypertriton)
 $\Lambda_* \approx 3.54 \gamma_d$ (Λ_{nn})
- ▶ for all further calculation absorbed into the cutoff

Scattering phase shift for deuteron Λ scattering



- ▶ Variation on the chiral EFT prediction by 15 %
- ▶ Scattering phase shift is insensitive towards exact values of the ΛN scattering lengths

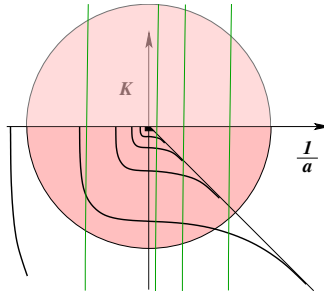
Universal relation for the hypertriton



- ▶ Phillips line for the hypertriton
- ▶ Independent of the ΛN pole-position
- ▶ Differs for unphysical regions, defined by the pion mass

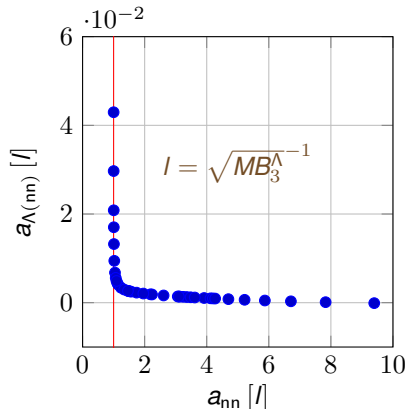
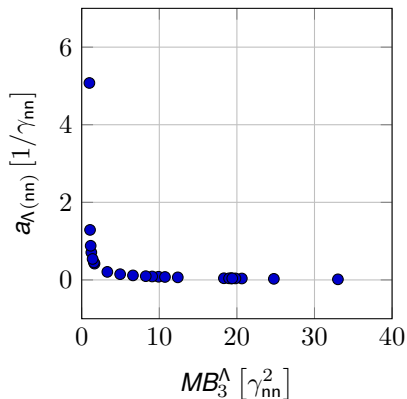
Λ nn theory expectations

- ▶ Λ nn is always bound in this theory by construction system shows the effimov effect
- ▶ BUT! Λ nn may not be within range of applicability



Universal relations for the Λ nn system

Use hypothetical positive scattering lengths for the n-n scattering lengths



- ▶ Presented an EFT approach to strangeness $S = -1$ three-body hypernuclei
- ▶ Showed you universal relations for the Λ nn system and the hypertriton
- ▶ Study Λ nn system dependence on input parameters
- ▶ Include explicit $\Lambda \Leftrightarrow \Sigma$ Conversions to check estimate

$$\begin{aligned}\gamma_3^\Lambda &\sim 2\sqrt{MB_3^\Lambda/3} \approx 1.2\gamma_d \approx 54 \text{ MeV} \\ &\ll \sqrt{M_\Lambda(M_\Sigma - M_\Lambda)} \approx 300 \text{ MeV}\end{aligned}$$

- ▶ Calculate wave-function of the hypertriton
- ▶ Use this to calculate observable like matter-radius
- ▶ Test Sensitivity on $B_{np\Lambda} - B_d$

