TECHNISCHE UNIVERSITÄT DARMSTADT

Effective Field Theory for three-body hypernuclei

Hirschegg 2018

Fabian Hildenbrand Hans-Werner Hammer

Institute of Nuclear Physics
Technische Universität Darmstadt

GEFÖRDERT VOM

Bundesministerium
für Bildung
und Forschung

Introduction

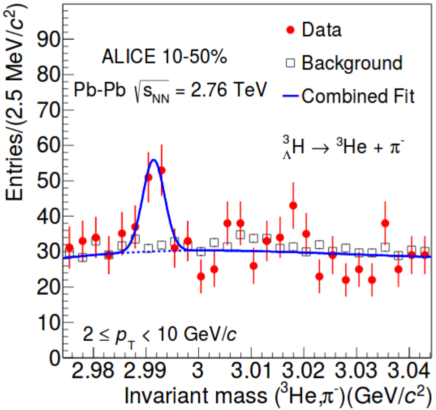
TECHNISCHE UNIVERSITÄT DARMSTADT

- Hypernuclei contain at least one hyperon (H) ($S \neq 0$)
- Consider three-body hypernuclei NNH
- Restriction to $\Lambda\,(M_\Lambda = 1115.68$ MeV$)$ particle as hyperon leads to the systems $S = -1$

$$\text{isospin}\,(I = 1) = \begin{cases} pp\Lambda \\ \frac{1}{\sqrt{2}}\,(np + pn)\,\Lambda \\ nn\Lambda \end{cases} \qquad \text{isospin}\,(I = 0) = \frac{1}{\sqrt{2}}\,(pn - np)\,\Lambda$$

- Two examples: nnΛ and hypertriton

The hypertriton



- Hypertriton is bound with a binding energy of $= 2.35 \pm 0.05$ MeV

  [M. Juric et al., Nucl. Phys. B52, 1 (1973)]

- Consists of separation energy into a deuteron ($B_d = 2.22$) MeV and a Λ $B_\Lambda = 0.13 \pm 0.05$ MeV

- Hypertriton was also produced at ALICE in Pb-Pb reaction recently

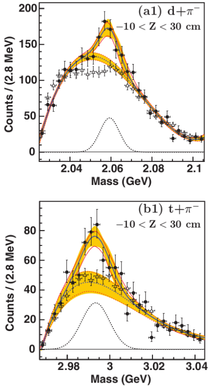
  [ALICE Collaboration Phys. Lett. B 754 (2016) 360-372]

- mean invariant mass $\mu = 2991 \pm 1 \pm 3$MeV $\Rightarrow B \approx 2$ MeV

The Λnn

- 2013 the HypHI Collaboration found evidence that the Λnn might be bound

[HypHI Collaboration C.Rappold et al. Phys.Rev.C.88(041001(R)),2013]



- d$+ \pi^-$ and $t + \pi^-$final states ^{6}Li $+$ ^{12}C reactions
- possible explanation of the observed final decay of a bound Λnn
- $^3_\Lambda n \rightarrow t + \pi^-$
  $^3_\Lambda n \rightarrow t^* + \pi^- \rightarrow d + n + \pi^-$
- mean invariant mass $\mu = 2993.7 \pm 1.3 \pm 0.6$ MeV $(t + \pi^-) \Rightarrow B \approx 1$ MeV

Theoretical framework

TECHNISCHE UNIVERSITÄT DARMSTADT

- Use pionless EFT $\rightarrow$ all interactions are contact interactions
- Exploit dibaryon formalism
- NNΛ interaction channels (only S-wave)

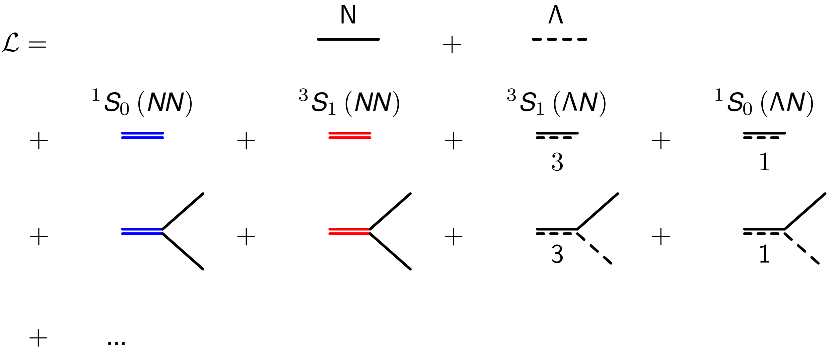
$$\mathrm{NN\Lambda} = \begin{cases} {}^1S_0\,(\mathrm{NN}) + \Lambda, & \Lambda\mathrm{nn} \\ {}^3S_1\,(\mathrm{NN}) + \Lambda, & \text{hypertriton} \\ {}^3S_1\,(\mathrm{N\Lambda}) + N, & \text{hypertriton and } \Lambda\mathrm{nn} \\ {}^1S_0\,(\mathrm{N\Lambda}) + N, & \text{hypertriton and } \Lambda\mathrm{nn} \end{cases}$$

- Explicit $\Lambda \Leftrightarrow \Sigma$ conversions are not included (implicit in the 3-body force)

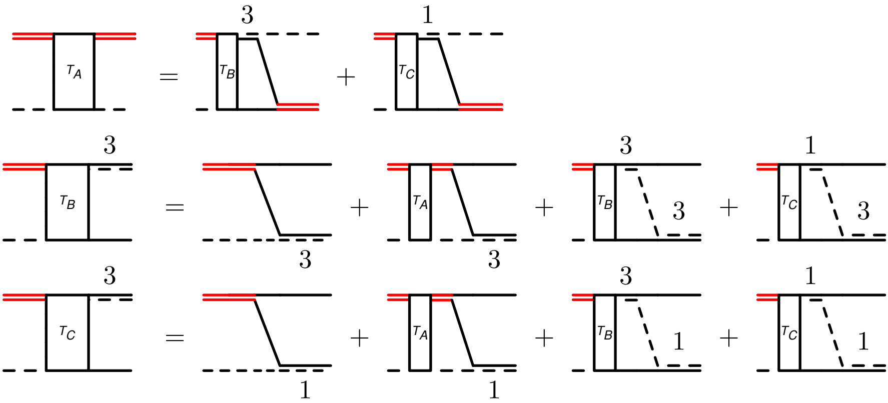
$$\gamma_3^\Lambda \sim 2\sqrt{MB_3^\Lambda/3} \approx 1.2\gamma_d \approx 54 \text{ MeV}$$
$$\ll \sqrt{M_\Lambda\,(M_\Sigma - M_\Lambda)} \approx 300 \text{ MeV}$$

Lagrangian for the hypertriton and Λnn system

TECHNISCHE UNIVERSITÄT DARMSTADT

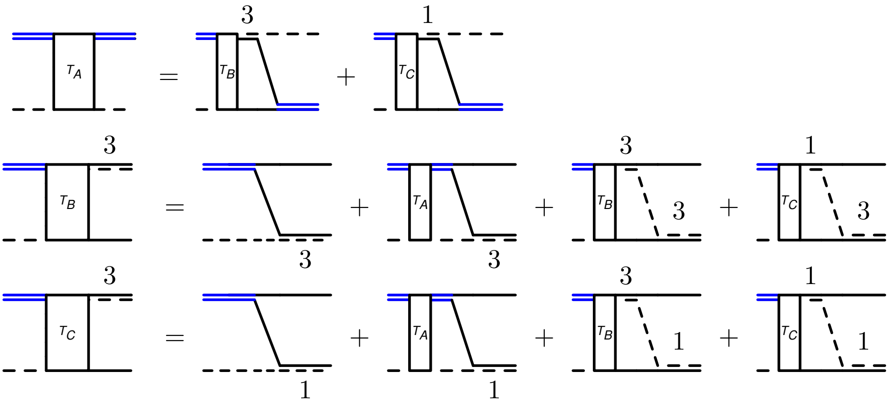


Integral equations for the hypertriton



Integral equations the Λnn system

TECHNISCHE UNIVERSITÄT DARMSTADT



Asymptotic Analysis of the hypertriton and Λnn system

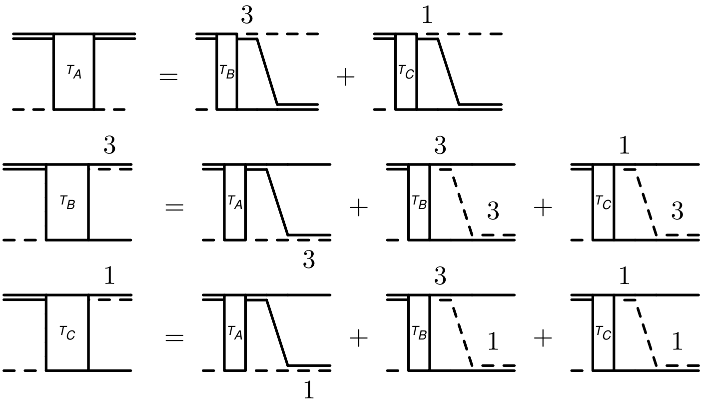
TECHNISCHE UNIVERSITÄT DARMSTADT

asymptotic limit
$\Lambda_c \gg p, q \gg 1/a, \gamma \sim k$
vanishing mass difference

→

equation becomes homogeneous
and can be decoupled
much simpler Kernel

Integral equations for the hypertriton and the Λnn system in the asymptotic limit



Asymptotic Analysis of the hypertriton and Λnn system

asymptotic limit
$\Lambda_c \gg p, q \gg 1/a, \gamma \sim k$
vanishing mass difference

equation becomes homogeneous
and can be decoupled
much simpler Kernel

$$T_i(p) = \frac{4}{\sqrt{3}\pi} \tilde{\lambda}_i \int dq \frac{1}{q} \ln\left(\frac{p^2+q^2+pq}{p^2+q^2-pq}\right) T_i(q), i \in 1, 2, 3$$

solution: power law
$T_i(p) = p^{s_i}$

$\tilde{\lambda}_1 = 1 \Rightarrow$
complex exponent
$s_1 = 1.00624i$

Efimov effect

one-parameter three-body-force

different result compared to [Hammer Nucl.Phys. A 705:173 (2002)]

Asymptotic Analysis of the hypertriton and Λnn system

asymptotic limit
$\Lambda_c \gg p, q \gg 1/a, \gamma \sim k$
vanishing mass difference

→ equation becomes homogeneous and can be decoupled
much simpler Kernel

→ $T_i\left(p\right) = \frac{4}{\sqrt{3}\pi}\tilde{\lambda}_i \int dq \frac{1}{q} \ln\left(\frac{p^2+q^2+pq}{p^2+q^2-pq}\right) T_i\left(q\right), i \in 1, 2, 3$

→ solution: power law
$T_i\left(p\right) = p^{s_i}$

→ $\tilde{\lambda}_1 = 1 \Rightarrow$
complex exponent
$s_1 = 1.00624i$

→ Efimov effect

→ one-parameter three-body-force

Efimov physics → universal relations and correlations

Asymptotic Analysis of the hypertriton and Λnn system-with different masses

TECHNISCHE UNIVERSITÄT DARMSTADT

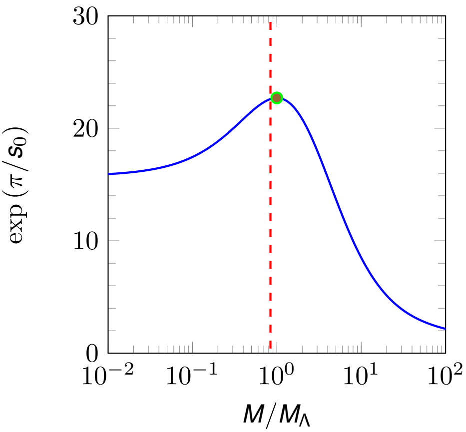
- Physical mass of Λ and nucleons are different
- Therefore asymptotic equations do not decouple
- Since the result of a power law should be reproduced for the case of $y = 0$ we choose as an ansatz

$$T_j\left(p\right) = \alpha_j p^{s_1} + \beta_j p^{s_2} + \gamma_j p^{s_3}, j \in \{A, B, C\}$$

- Since kernel is more complex now ⇒ integrate term by term

Scaling factor as a function of the mass ratio

TECHNISCHE UNIVERSITÄT DARMSTADT



- Ansatz reproduces the $M = M_\Lambda$ result
- $s_0 = 1.00760 (M/M_\Lambda = 0.84)$
- Results are consistent with Braaten and Hammer
  [Braaten, Hammer Phys. Rept. 428, 259–390 (2006)]
- Used chiral EFT prediction for Λ-N scattering lengths ($a_3 = -1.45 - 1.70$ fm, $a_1 = -2.90 - 2.91$ fm)
  [Haidenbauer et al Nucl. Phys. A 915 (2013) 24-58]
- considered to be large since ΛN range is given by the 2π exchange $\sim \frac{1}{2m_\pi} \approx 0.7$fm

Introduction of the 3-body force

introduce three-body force term $H(\Lambda)$ in T_1
includes $\Lambda \Leftrightarrow \Sigma$ conversions

$$T_1(p) = \frac{4}{\sqrt{3}\pi}\int^{\Lambda} dq \frac{1}{q}\left[\ln\left(\frac{p^2+q^2+pq}{p^2+q^2-pq}\right) + 2\frac{H(\Lambda)}{\Lambda^2}pq\right] T_1(q)$$

transform back to original amplitudes
T_A, T_B, T_C

solve numerical
use B as input
to fix H

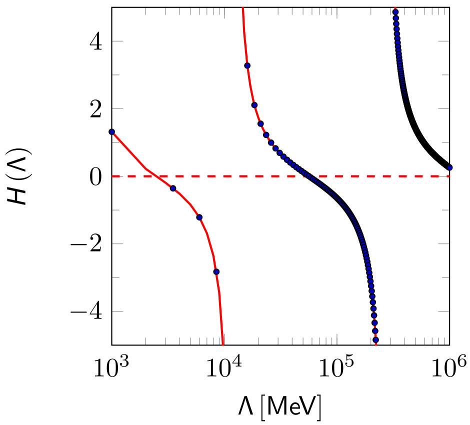
compare

theoretical expected form

$$H(\Lambda) = -\frac{\sin\left(s\log\left(\frac{\Lambda}{\Lambda_*}\right)-\arctan\left(\frac{1}{s}\right)\right)}{\sin\left(s\log\left(\frac{\Lambda}{\Lambda_*}\right)+\arctan\left(\frac{1}{s}\right)\right)}$$

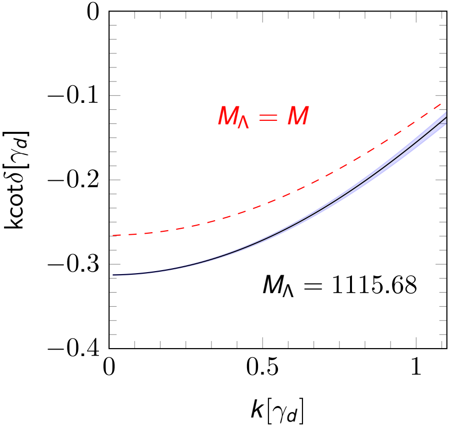
$\mathbf{H}(\Lambda)$ for the hypertriton

TECHNISCHE UNIVERSITÄT DARMSTADT



- ($a_3 = -1.64$ fm, $a_1 = -2.91$ fm)
- 3-body force shows the expected limit cycle behavior
- It is not possible to fix the Λnn 3-body force with this one due to different isospin channels ($I = 1$ and $I = 0$)
- extract the three-body parameter $\Lambda_* \approx 3.26^{+0.10}_{-0.13}\gamma_d$(hypertriton) $\Lambda_* \approx 3.54\gamma_d$ (Λnn)
- for all further calculation absorbed into the cutoff

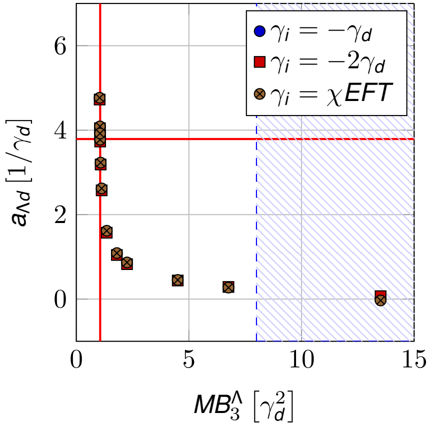
Scattering phase shift for deuteron Λ scattering



- Variation on the chiral EFT prediction by 15 %
- Scattering phase shift is insensitive towards exact values of the ΛN scattering lengths

Universal relation for the hypertriton

TECHNISCHE UNIVERSITÄT DARMSTADT

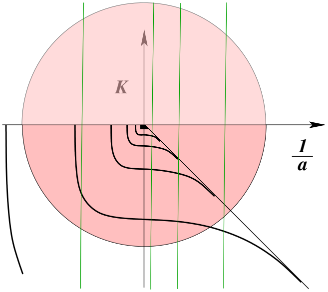


- Phillips line for the hypertriton
- Independent of the ΛN pole-position
- Differs for unphysical regions, defined by the pion mass

Λnn theory exspectations

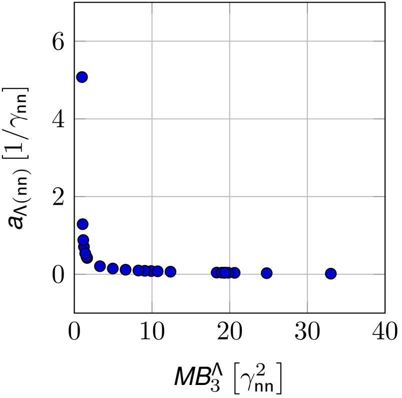
TECHNISCHE UNIVERSITÄT DARMSTADT

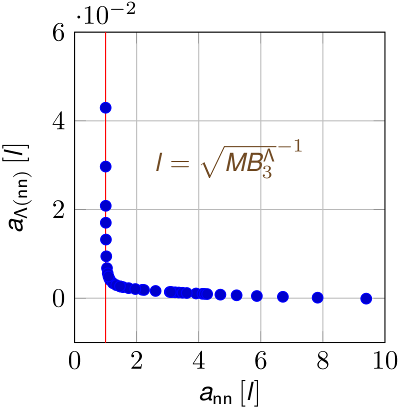
- Λnn is always bound in this theory by construction system shows the effimov effect
- BUT! Λnn may not be within range of applicability



Universal relations for the Λnn system

Use hypothetical positive scattering lengths for the n-n scattering lengths





Conclusion and Outlook

TECHNISCHE
UNIVERSITÄT
DARMSTADT

- Presented an EFT approach to strangeness $S = -1$ three-body hypernuclei
- Showed you universal relations for the Λnn system and the hypertriton
- Study Λnn system dependence on input parameters
- Include explicit $\Lambda \Leftrightarrow \Sigma$ Conversions to check estimate

$$\gamma_3^\Lambda \sim 2\sqrt{MB_3^\Lambda/3} \approx 1.2\gamma_d \approx 54 \text{ MeV}$$
$$\ll \sqrt{M_\Lambda \left(M_\Sigma - M_\Lambda\right)} \approx 300 \text{ MeV}$$

Conclusion and Outlook

TECHNISCHE
UNIVERSITÄT
DARMSTADT

- Calculate wave-function of the hypertriton
- Use this to calculate observable like matter-radius
- Test Sensitivity on $B_{\mathrm{np}\Lambda} - B_{\mathrm{d}}$