Nuclear matter from an effective lattice theory of heavy QCD

HIC for FAIR
Helmholtz International Center

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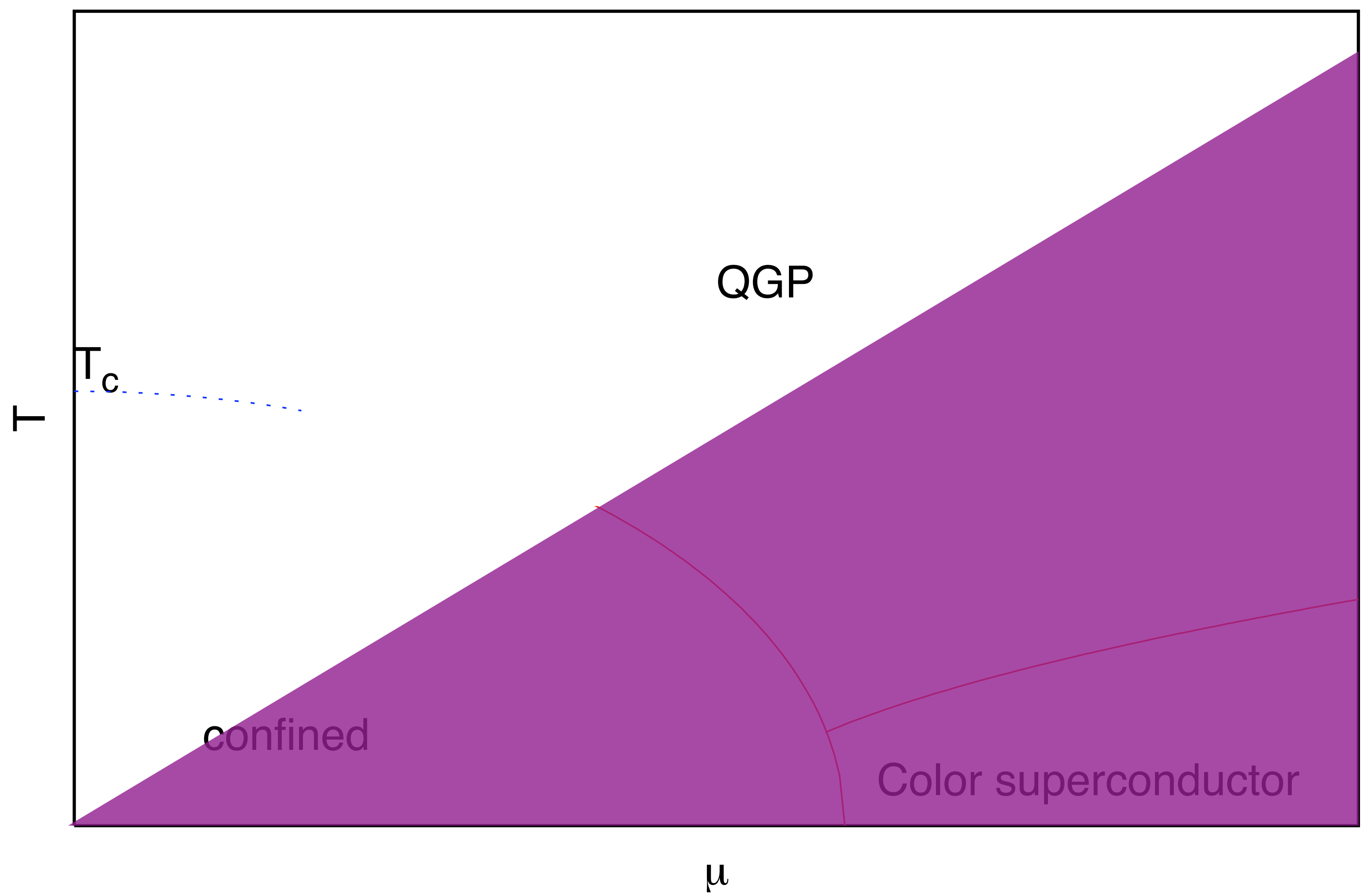
- A 3d effective theory for heavy QCD:
  Strong coupling + hopping expansions
- Simulation results
- Solution by analytic linked cluster expansion
- The nuclear equation of state in the continuum

The lattice-calculable region of the phase diagram



- Sign problem prohibits direct simulation, circumvented by approximate methods: reweigthing, Taylor expansion, imaginary chem. pot., need $\mu/T \lesssim 1 \quad (\mu = \mu_B/3)$
- No critical point in the controllable region, some signals beyond
- Complex Langevin: lots of progress, but not in all parameter space, no "guarantees"

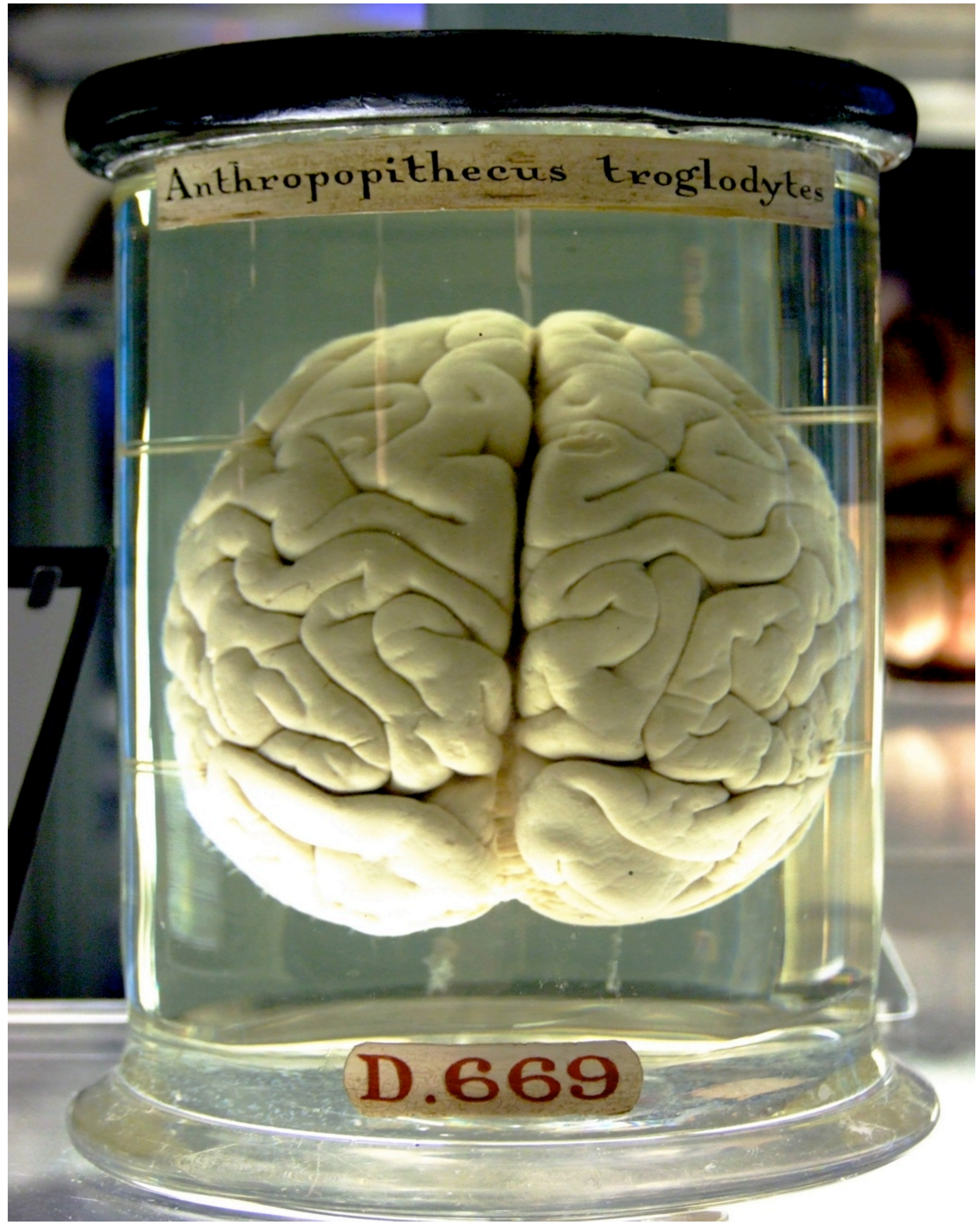
New computational avenues in LQCD:

"(Wall)Time is Money (CPU hrs)"

CPU ⟶ GPU

Here, very old-fashioned approach: BPU!

Biological Processing Unit!



Large densities? Effective theories!

Effective lattice theory for heavy and dense QCD

with M.Fromm, J.Langelage, S.Lottini, M.Neuman, J.Glesaaen

- Two-step treatment:

  I. Calculate effective theory analytically
  II. Simulate effective theory

- Step I.: split temporal and spatial link integrations:

$$Z = \int DU_0 DU_i \; \det Q \; e^{S_g[U]} \equiv \int DU_0 e^{-S_{eff}[U_0]} = \int DL \; e^{-S_{eff}[L]}$$

  Spatial integration after analytic strong coupling and hopping expansion

- Truncation valid for heavy quarks on reasonably fine lattices, a~0.1 fm

- Step II.: Mild sign problem, complex Langevin, Monte Carlo

  Check in SU(2): Scior, von Smekal 15

- New Step II.: Analytic solution by cluster expansion!

Starting point: Wilson's lattice Yang-Mills action

Partition function; link variables as degrees of freedom

$$Z = \int \prod_{x,\mu} dU(x;\mu) \exp\left(-S_g[U]\right) \equiv \int DU \exp\left(-S_g[U]\right)$$

Wilson's gauge action

$$S_g[U] = \sum_x \sum_{1\leq\mu<\nu\leq 4} \beta\left(1 - \frac{1}{3}\mathrm{ReTr}U_p\right) \equiv \sum_p S_p \qquad \beta = \frac{2N}{g^2}$$

Plaquette:

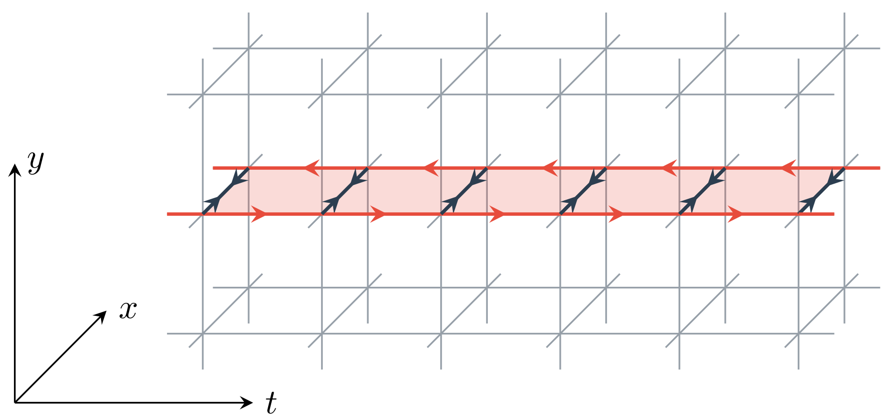
$$\square \to 1 + ia^2 g F_{\mu\nu} - \frac{a^4 g^2}{2} F_{\mu\nu}F^{\mu\nu} + O(a^6) + \ldots$$

$U_\mu(x) = \mathrm{e}^{-iagA_\mu(x)}$

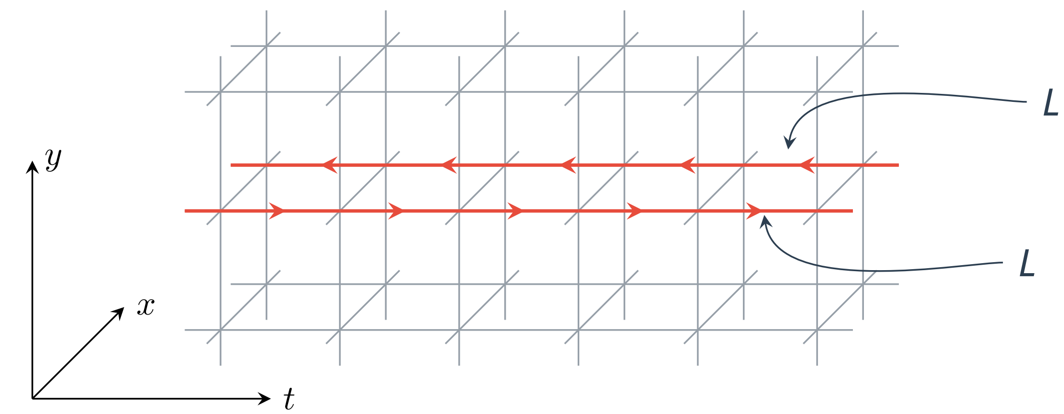
$$T = \frac{1}{aN_t} \qquad \text{continuum limit} \quad a \to 0, N_t \to \infty$$

Small $\beta(a) \Rightarrow$ small T

LO 3d effective theory for lattice YM



Integrate over all spatial gauge links



What remains is an interaction between Polyakov Loops

$$-S_1 = u^{N_\tau} \sum_{\langle ij \rangle} \mathrm{tr}\, W_i \, \mathrm{tr}\, W_j$$

Polonyi, Szachlanyi 82

Character expansion: $u = \frac{\beta}{18} + O(\beta^2) < 1$

- larger distances between loops, higher power of loops
- higher representations of loops
- decorations of LO graphs by additional plaquettes

Effective one-coupling theory for SU(3) YM

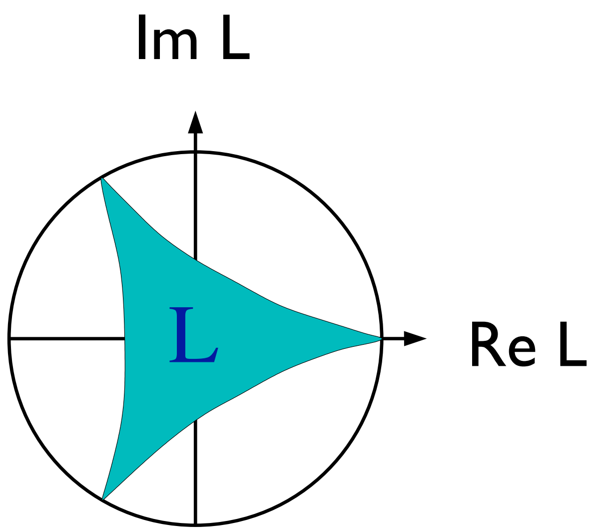
Langelage, Lottini, O.P. 10

(L= Tr W)

$$Z = \int [dL] \exp\left[-S_1 + V_{SU(3)}\right]$$

$$= \int [dL] \prod_{\langle ij \rangle} \left[1 + 2\lambda_1 \mathrm{Re}\left(L_i L_j^*\right)\right] *$$

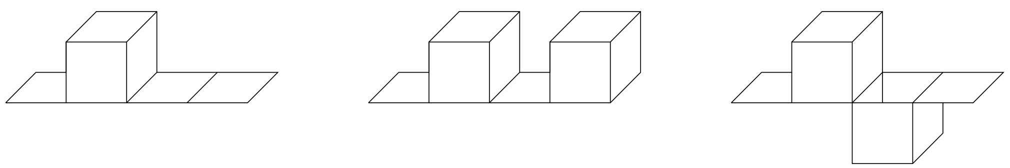
$$* \prod_i \sqrt{27 - 18|L_i|^2 + 8\mathrm{Re}L_i^3 - |L_i|^4}$$



Resummations:

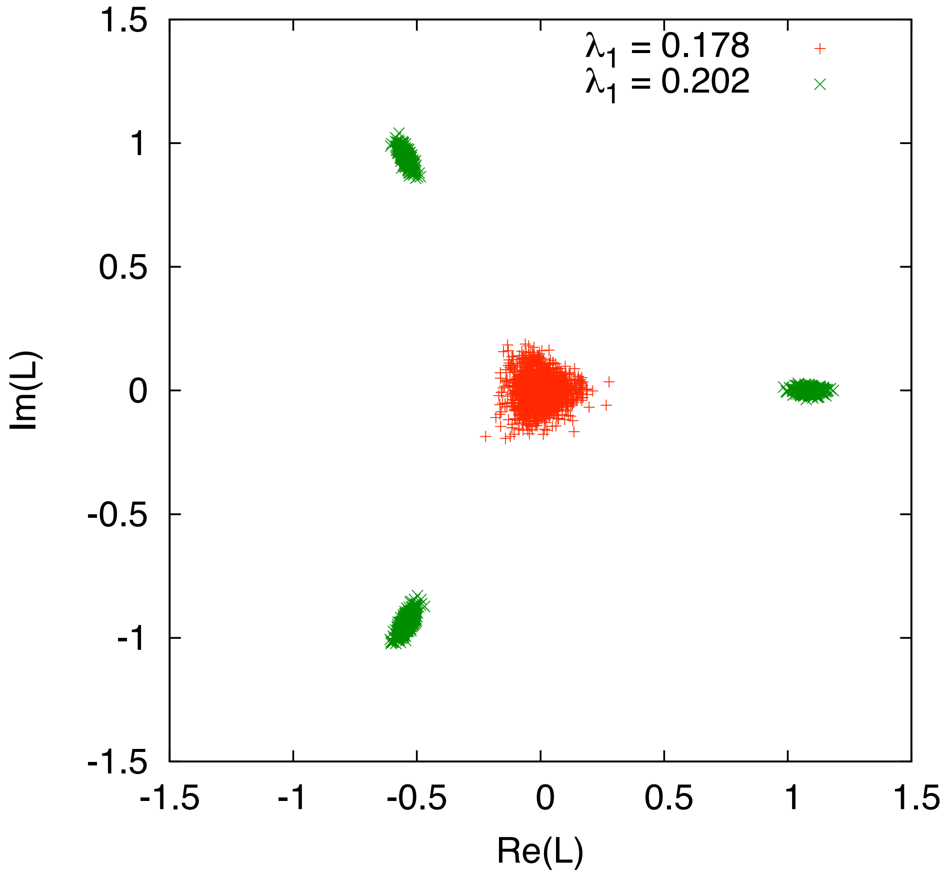
$$\sum_{<ij>} \left(\lambda_1 L_i L_j - \frac{\lambda_1^2}{2} L_i^2 L_j^2 + \frac{\lambda_1^3}{3} L_i^3 L_j^3 - \ldots\right) = \sum_{<ij>} \ln\left(1 + \lambda_1 L_i L_j\right)$$

$$\lambda(u, N_\tau \geq 5) = u^{N_\tau} \exp\left[N_\tau \left(4u^4 + 12u^5 - 14u^6 - 36u^7 + \frac{295}{2}u^8 + \frac{1851}{10}u^9 + \frac{1055797}{5120}u^{10}\right)\right]$$

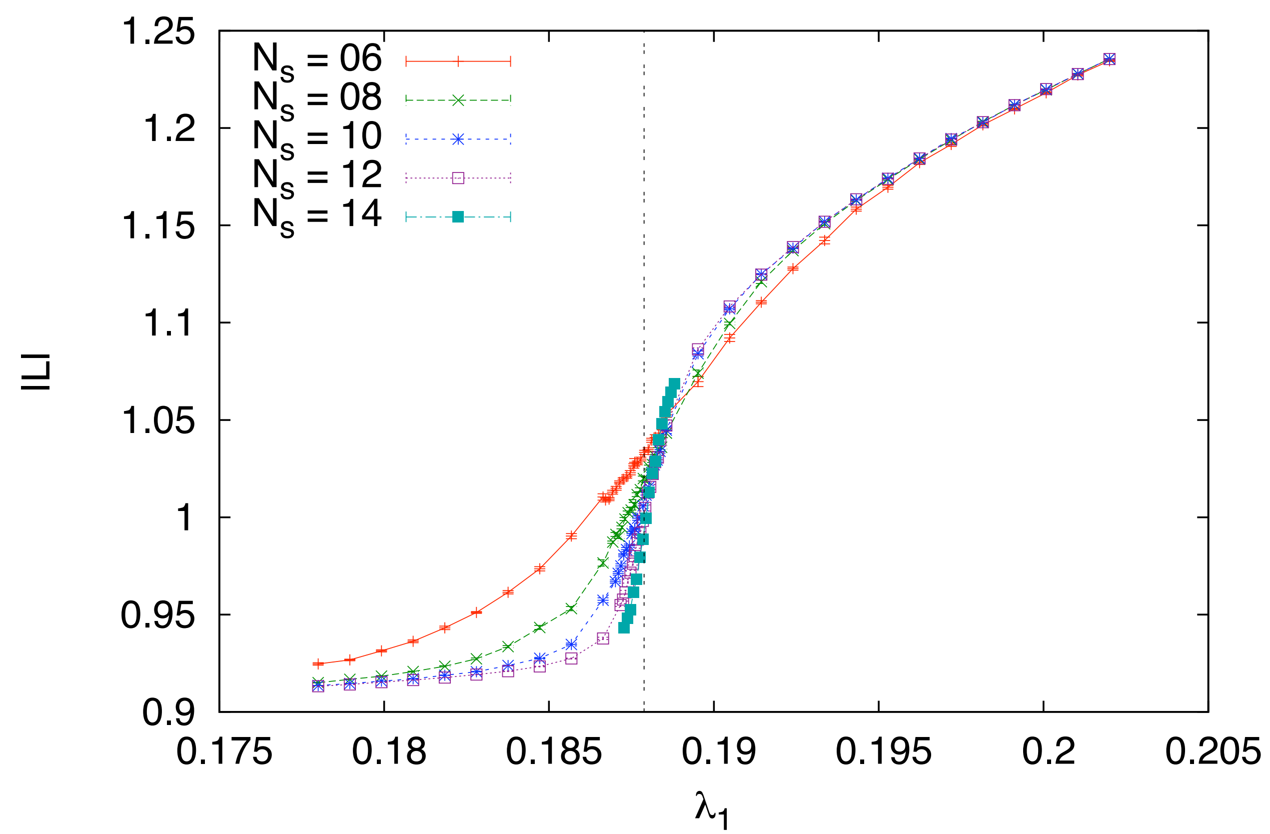


...

Numerical results for SU(3), one coupling



Order-disorder transition
=Z(3) breaking

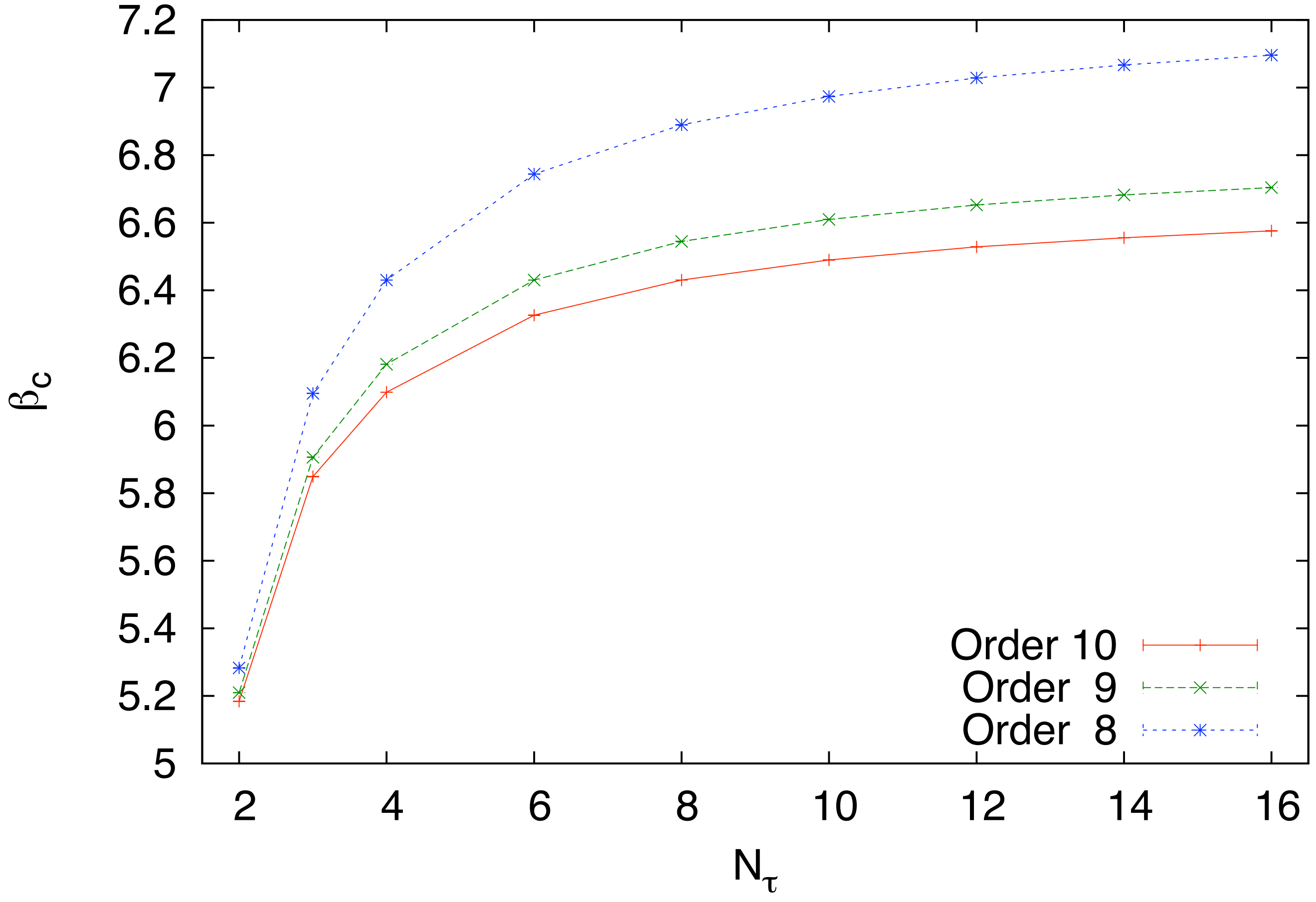


Mapping back to 4d finite T Yang-Mills

Inverting

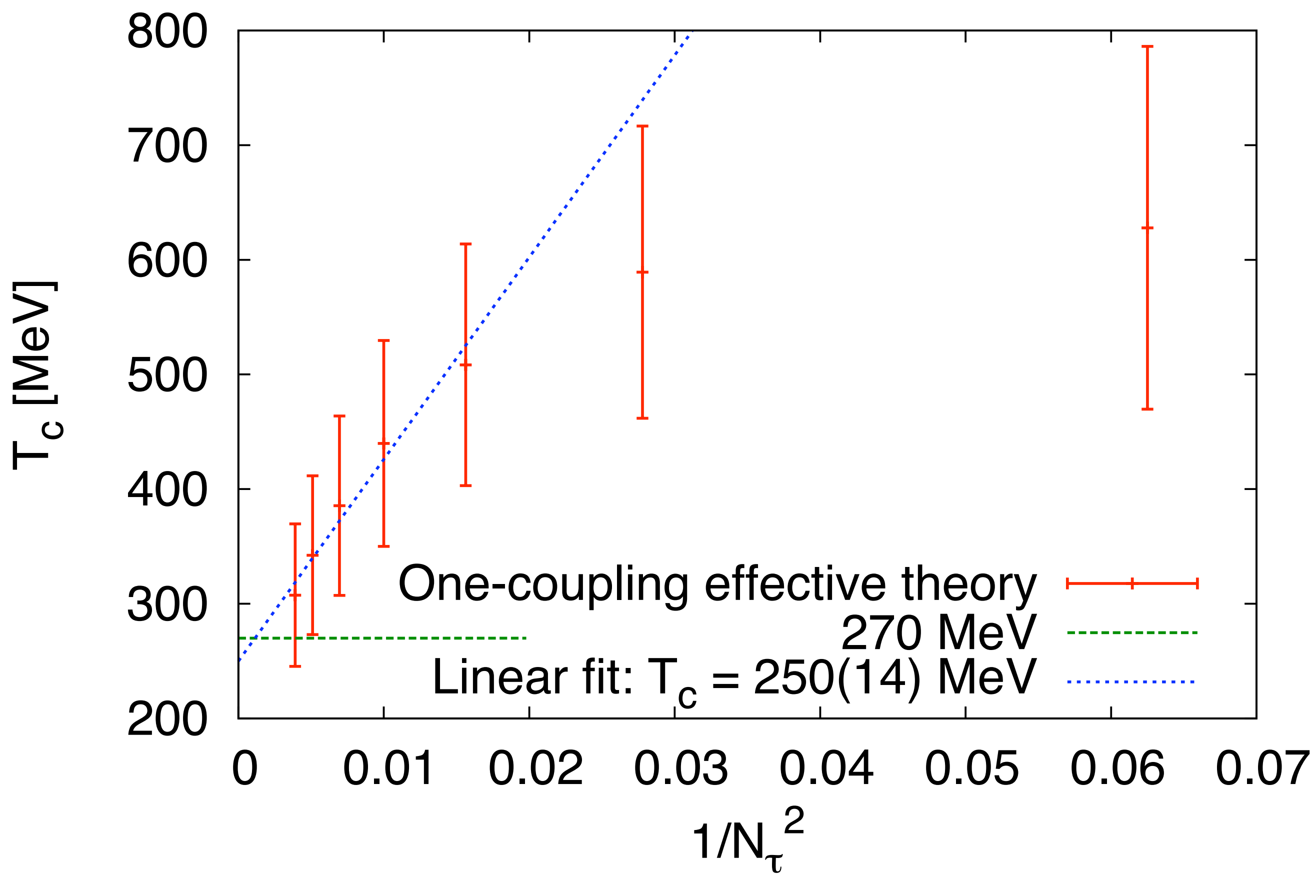
$$\lambda_1(N_\tau, \beta) \rightarrow \beta_c(\lambda_{1,c}, N_\tau)$$

...points at reasonable convergence



SU(3)

Continuum limit feasible!



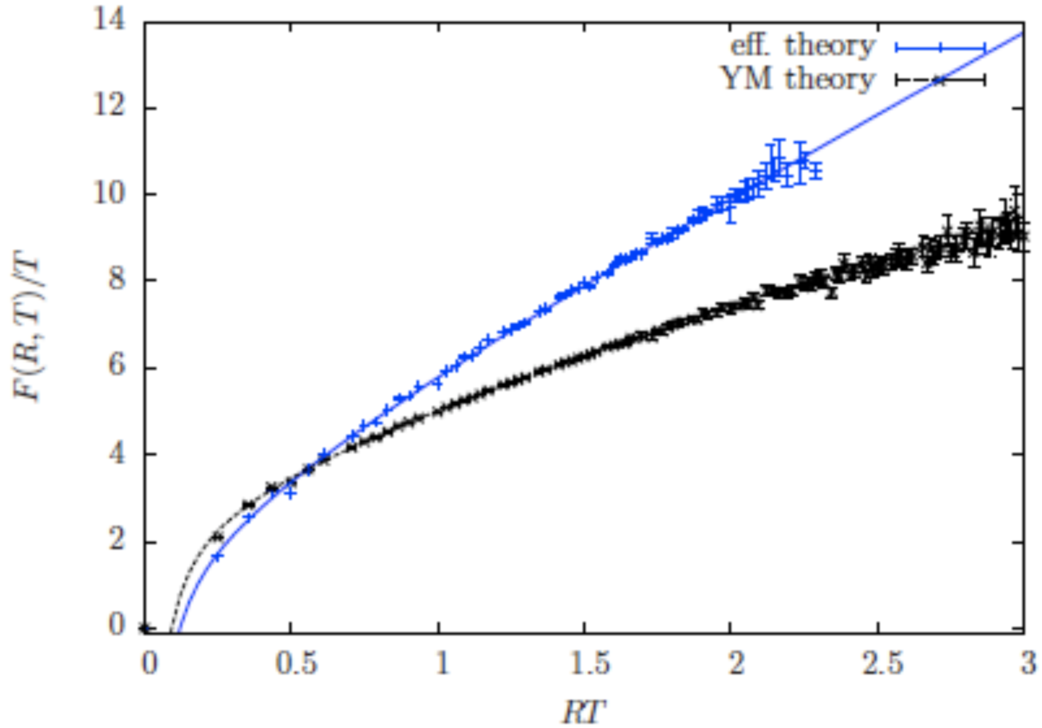
-error bars: difference between last two orders in strong coupling exp.

-using non-perturbative beta-function (4d T=0 lattice)

-all data points from one single 3d MC simulation!

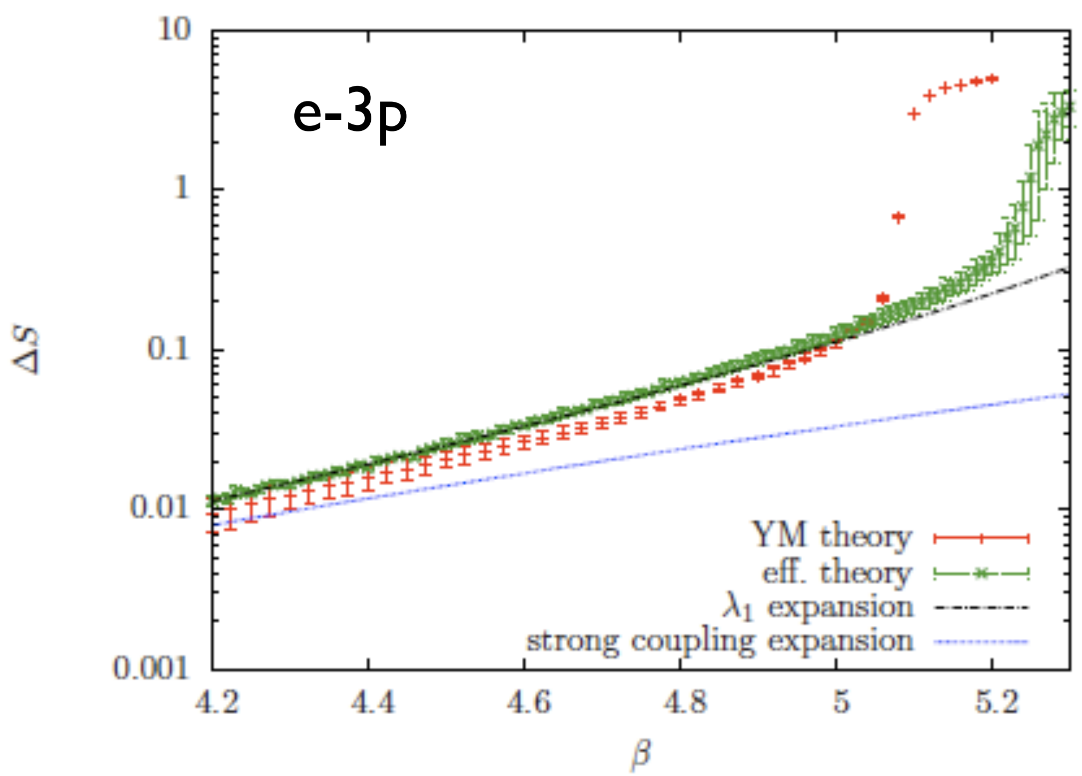
One coupling: What does and does not work?

G.Bergner, J.Langelage, O.P. 14,15



Correlation functions and spectrum:
NO

couplings over large distances needed



Thermodynamics and critical coupling:
YES

partition function needed, local!

Including dynamical Wilson fermions

Integrate the Grassmann variables $\psi, \overline{\psi}$:

$$S = S_{\rm gauge} - N_f \mathrm{Tr} \log(\mathbb{1} - \kappa H)$$

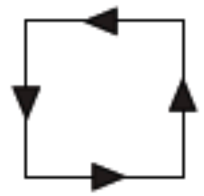
Expand in the *hopping parameter* $\kappa = 1/(2aM + 8)$

$$Z_{\rm eff}(\lambda_1, h_1, \bar{h}_1; N_\tau) = \int [\mathrm{d}L] \Big(\prod_{<ij>} [1 + 2\lambda_1 \mathrm{Re} L_i L_j^*] \Big) \Big(\prod_x \underbrace{\det[(1 + h_1 W_x)(1 + \bar{h}_1 W_x^\dagger)]^{2N_f}}_{\equiv Q(L_x, L_x^*)^{N_f}} \Big)$$

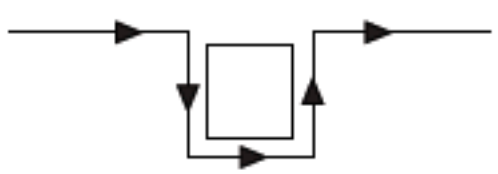
Corrections: exact expand in spatial hops

$$\begin{aligned} \det[Q] &\equiv \det[Q_{\rm stat}] \det[Q_{\rm kin}] \,, \\ \det[Q_{\rm kin}] &= \det[1 - (1 - T)^{-1}(S^+ + S^-)] \\ &\equiv \det[1 - P - M] = \exp\left[\mathrm{Tr} \ln(1 - P - M)\right] \end{aligned}$$

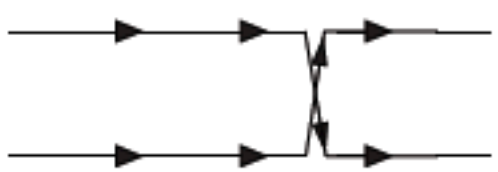
Fromm, Langelage, Lottini, Neuman, Glesaaen, O.P. 12-15

spatial:  reabsorbed in gauge part: $\begin{cases} \beta \to \beta + \mathcal{O}(\kappa^4) \\ u(\beta) \to u(\beta, \kappa) \end{cases}$

 LO Polyakov "magnetic" term $\sim \begin{cases} \underbrace{(2\kappa e^{+a\mu})^{N_\tau}}_{h_1} L \\ \underbrace{(2\kappa e^{-a\mu})^{N_\tau}}_{\bar{h}_1} L^* \end{cases}$

 higher corrections to the above:

$$h_1 = (2\kappa e^{a\mu})^{N_\tau} \left[1 + \mathcal{O}(k^2) f(u) + \ldots\right]$$

 other (suppressed) terms, such as $h_2(L_x L_{x+\hat{i}})$,

$$h_2 \sim (2\kappa e^{a\mu})^{2N_\tau} \kappa^2 N_\tau .$$

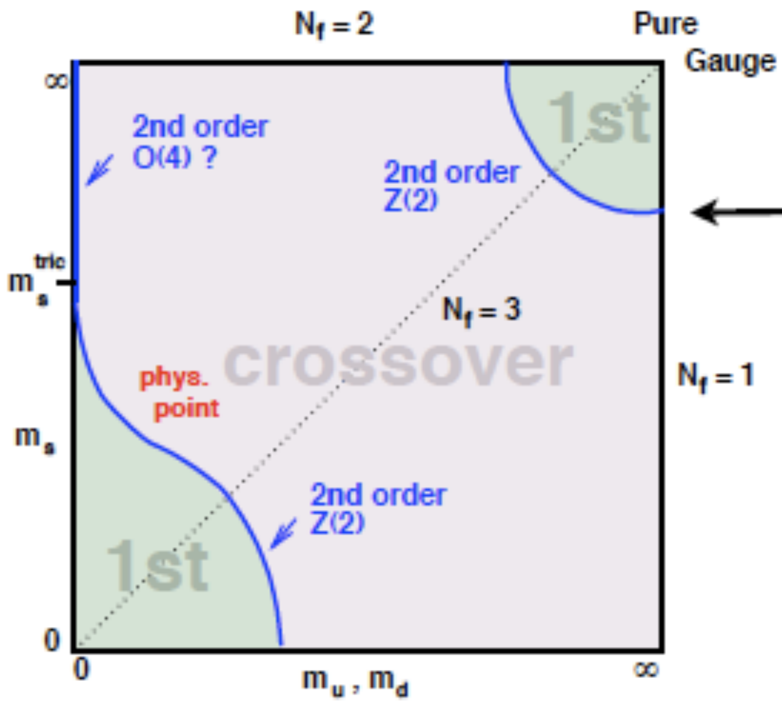
Simplification for T=0, only leading powers in Nt, fermion contribution:

$$
\begin{aligned}
-S_{\text{eff}} = &-\log \sum_{\vec{x}} (1 + h_1 \mathrm{Tr} W_{\vec{x}} + h_1^2 \mathrm{Tr} W_{\vec{x}}^\dagger + h_1^3)^2 - 2h_2 \sum_{\vec{x},i} \mathrm{Tr} \frac{h_1 W_{\vec{x}}}{1 + h_1 W_{\vec{x}}} \mathrm{Tr} \frac{h_1 W_{\vec{x}+i}}{1 + h_1 W_{\vec{x}+i}} \\
&+ 2 \frac{\kappa^4 N_\tau^2}{N_c^2} \sum_{\vec{x},i} \mathrm{Tr} \frac{h_1 W_{\vec{x}}}{(1 + h_1 W_{\vec{x}})^2} \mathrm{Tr} \frac{h_1 W_{\vec{x}+i}}{(1 + h_1 W_{\vec{x}+i})^2} \\
&+ \frac{\kappa^4 N_\tau^2}{N_c^2} \sum_{\vec{x},i,j} \mathrm{Tr} \frac{h_1 W_{\vec{x}}}{(1 + h_1 W_{\vec{x}})^2} \mathrm{Tr} \frac{h_1 W_{\vec{x}-i}}{1 + h_1 W_{\vec{x}-i}} \mathrm{Tr} \frac{h_1 W_{\vec{x}-j}}{1 + h_1 W_{\vec{x}-j}} \\
&+ 2 \frac{\kappa^4 N_\tau^2}{N_c^2} \sum_{\vec{x},i,j} \mathrm{Tr} \frac{h_1 W_{\vec{x}}}{(1 + h_1 W_{\vec{x}})^2} \mathrm{Tr} \frac{h_1 W_{\vec{x}-i}}{1 + h_1 W_{\vec{x}-i}} \mathrm{Tr} \frac{h_1 W_{\vec{x}+j}}{1 + h_1 W_{\vec{x}+j}} \\
&+ \frac{\kappa^4 N_\tau^2}{N_c^2} \sum_{\vec{x},i,j} \mathrm{Tr} \frac{h_1 W_{\vec{x}}}{(1 + h_1 W_{\vec{x}})^2} \mathrm{Tr} \frac{h_1 W_{\vec{x}+i}}{1 + h_1 W_{\vec{x}+i}} \mathrm{Tr} \frac{h_1 W_{\vec{x}+j}}{1 + h_1 W_{\vec{x}+j}} \; .
\end{aligned}
$$

Current state of the art for fermionic sector: $u^5 \kappa^8$

The deconfinement transition for heavy quarks

NLO: $\sim \kappa^2$



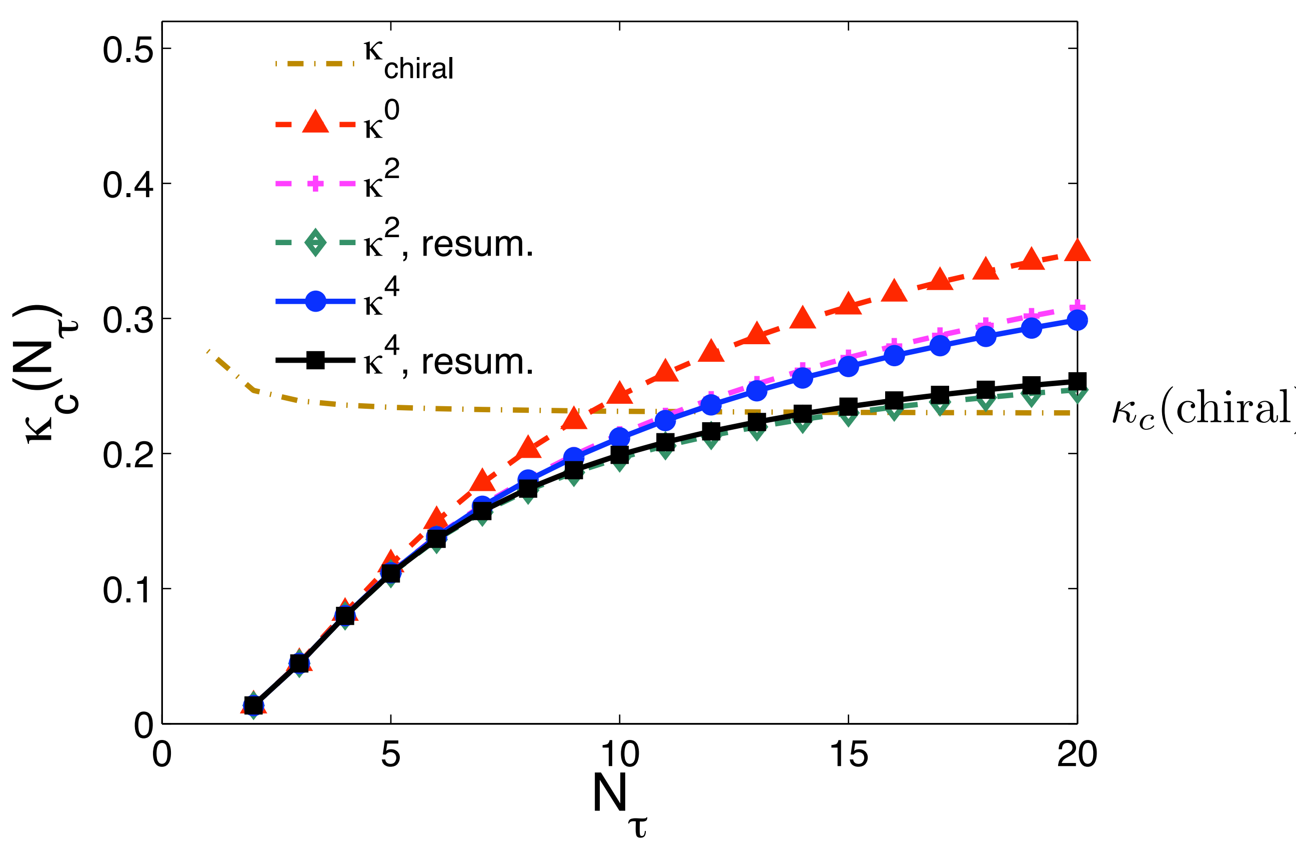
		eff. theory	4d MC, WHOT	4d MC, de Forcrand et al
N_f	M_c/T	$\kappa_c(N_\tau = 4)$	$\kappa_c(4)$, Ref. [23]	$\kappa_c(4)$, Ref. [22]
1	7.22(5)	0.0822(11)	0.0783(4)	~ 0.08
2	7.91(5)	0.0691(9)	0.0658(3)	–
3	8.32(5)	0.0625(9)	0.0595(3)	–

Accuracy ~5%, predictions for Nt=6,8,... available!

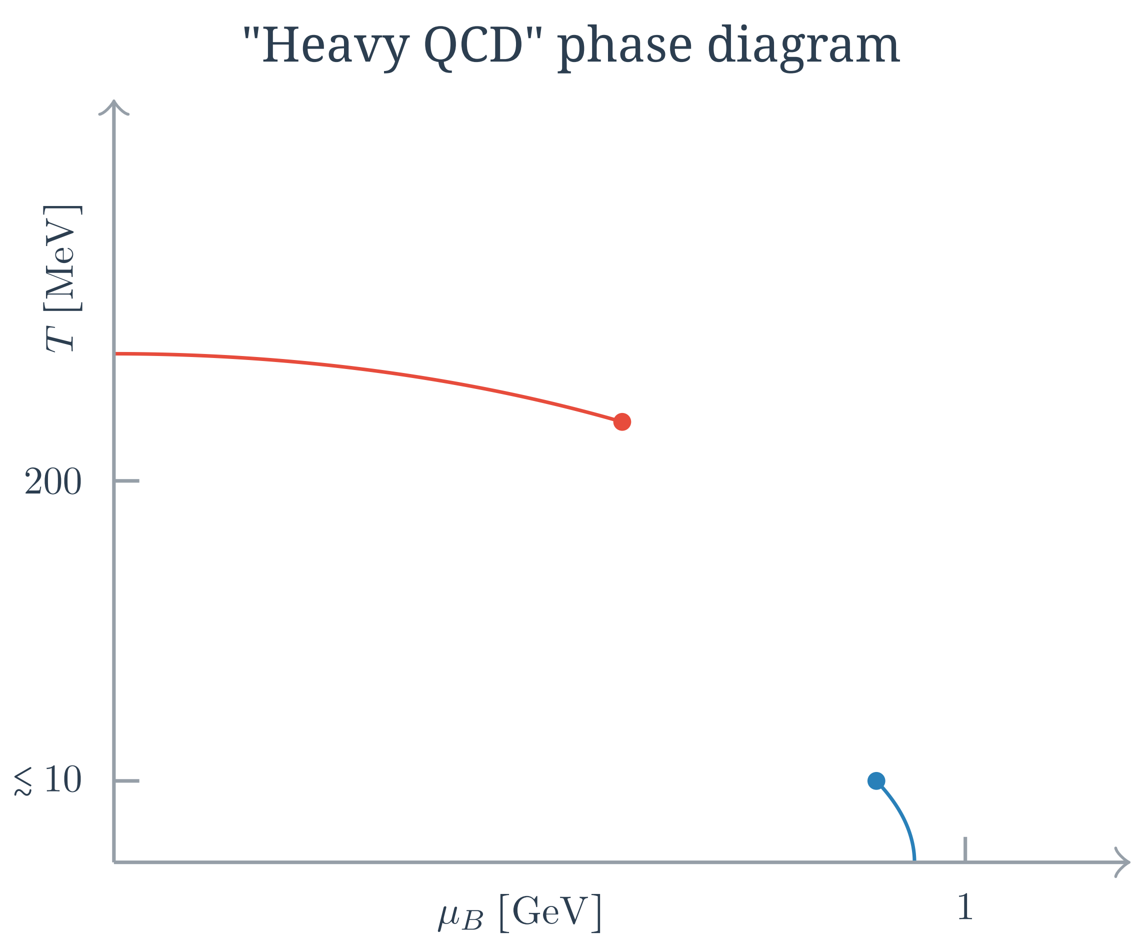
Fromm, Langelage, Lottini, O.P. 11

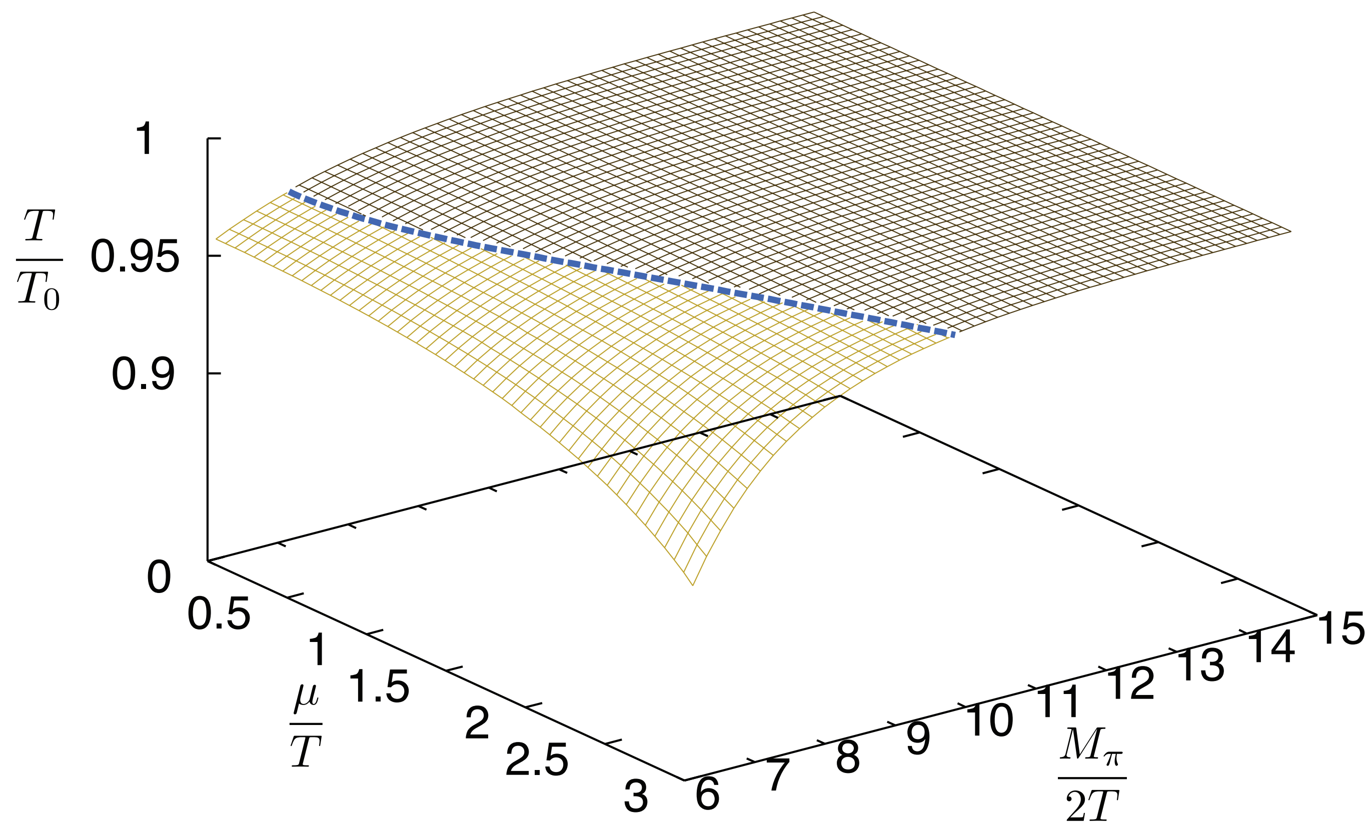
Continuum:

Friman, Lo, Redlich 14
Fischer, Lücker, Pawlowski 15



The fully calculated deconfinement transition





Fromm, Langelage, Lottini, O.P. 11

Cold and dense QCD: static strong coupling limit

Fromm, Langelage, Lottini, Neuman, O.P. , PRL 13

For T=0 (at finite density) anti-fermions decouple $N_f = 1, h_1 = C, h_2 = 0$

$$C_f \equiv (2\kappa_f e^{a\mu_f})^{N_\tau} = e^{(\mu_f - m_f)/T}, \; \bar{C}_f(\mu_f) = C_f(-\mu_f)$$

$$Z(\beta = 0) \xrightarrow{T\to 0} \left[\prod_f \int \mathrm{d}W \, (1 + C_f L + C_f^2 L^* + C_f^3)^2 \right]^{N_s^3}$$

$$= \left[1 + 4C^{N_c} + C^{2N_c}\right]^{N_s^3}$$

Free gas of baryons!

Quarkyonic?

$$n = \frac{T}{V}\frac{\partial}{\partial \mu} \ln Z = \frac{1}{a^3} \frac{4N_c C^{N_c} + 2N_c C^{2N_c}}{1 + 4C^{N_c} + C^{2N_c}}$$

$$\lim_{\mu \to \infty} (a^3 n) = 2N_c$$

Sivler blaze property + saturation!

$$\lim_{T \to 0} a^3 n = \begin{cases} 0, & \mu < m \\ 2N_c, & \mu > m \end{cases}$$

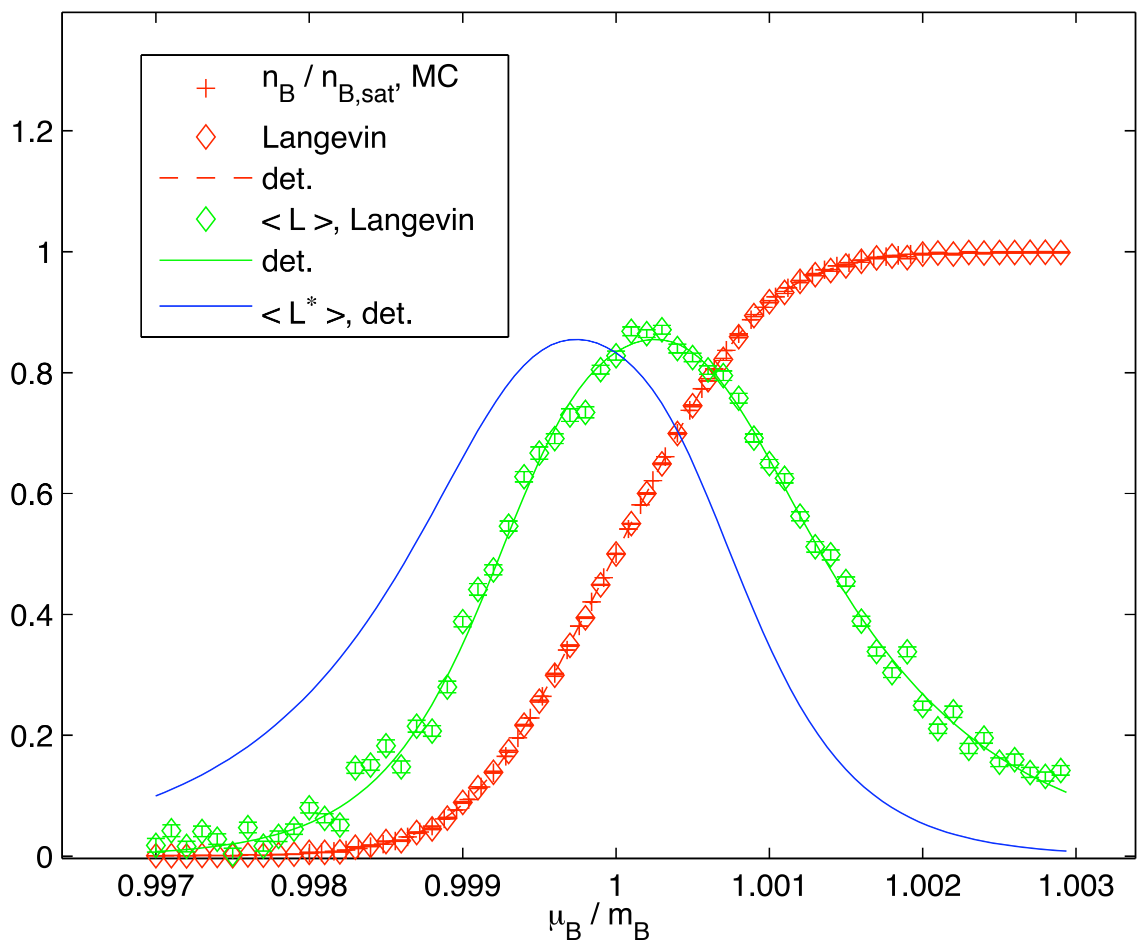
$N_f = 2$

$$z_0 = (1+4h_d^3+h_d^6)+(6h_d^2+4h_d^5)h_u+(6h_d+10h_d^4)h_u^2+(4+20h_d^3+4h_d^6)h_u^3 \\ +(10h_d^2+6h_d^5)h_u^4+(4h_d+6h_d^4)h_u^5+(1+4h_d^3+h_d^6)h_u^6\ . \quad (3.11)$$

Free gas of baryons: complete spin flavor structure of vacuum states!

Cold and dense, interacting: onset to nuclear matter

Fromm, Langelage, Lottini, Neuman, O.P. , PRL 13



$$m_\pi = 20 \text{ GeV}, T = 10 \text{ MeV}, a = 0.17 \text{ fm}$$

$$\beta = 5.7, \kappa = 0.0000887, N_\tau = 116$$

$$\lambda_1(\beta, \kappa, N_\tau) \sim 10^{-26}$$

- Silver blaze property

  no dependence on chem. pot. until onset

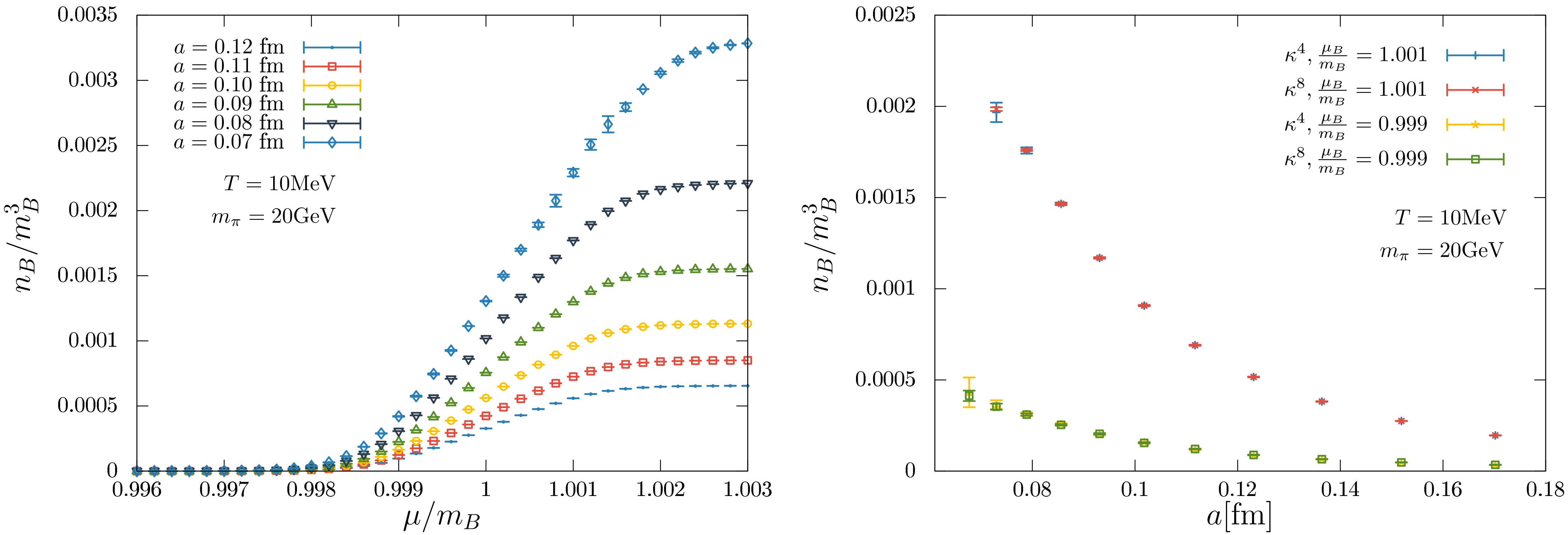
- Lattice saturation

  Pauli principle, strongly limits density!

- Screening of Polyakov loop

  But no deconfinement!

Continuum approach

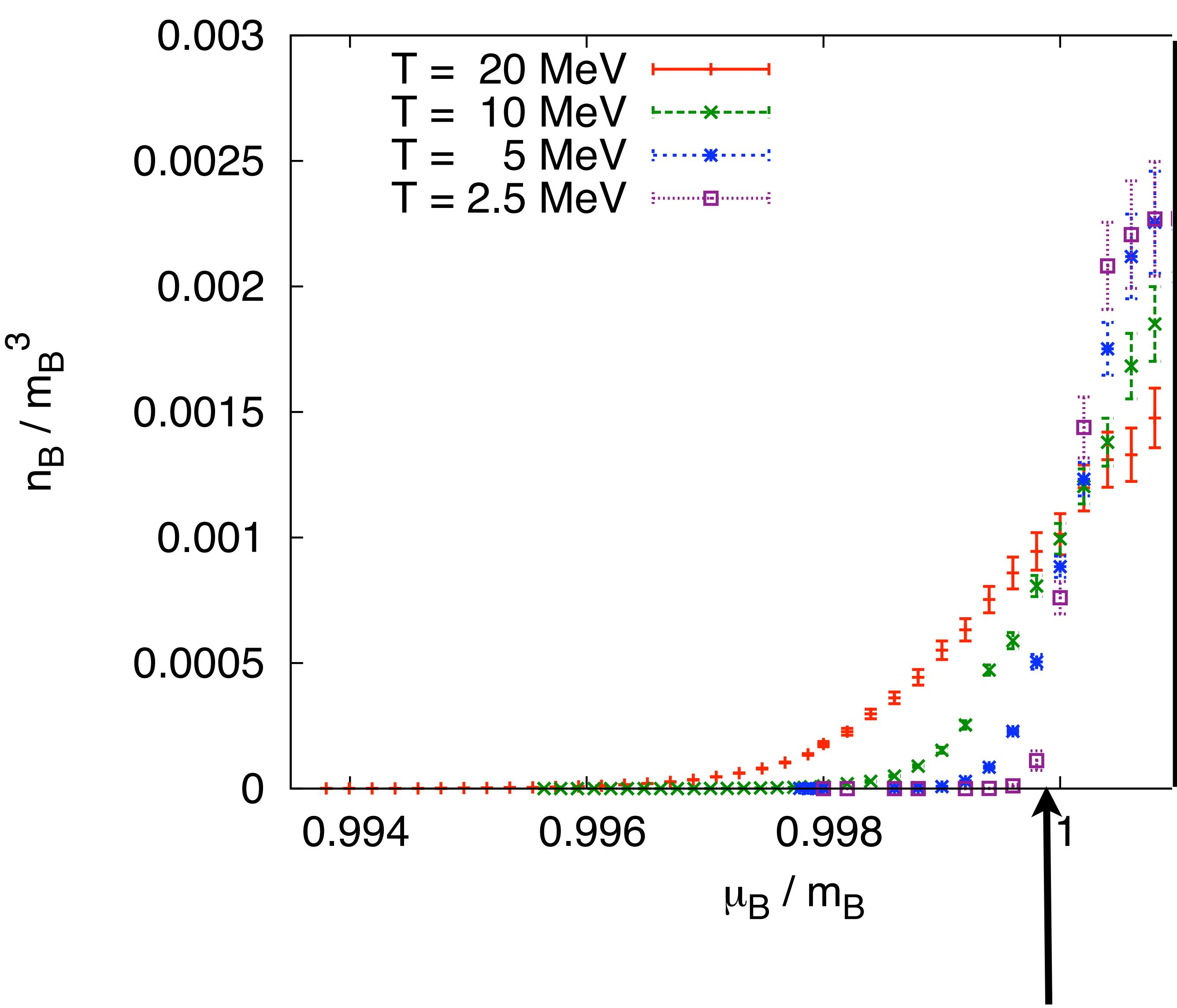


- Continuum approach ~a as expected for Wilson fermions
- Cut-off effects grow rapidly beyond onset transition
- Finer lattice necessary for larger density to avoid saturation

Cold and dense, interacting: onset to nuclear matter

continuum extrapolated

$m_\pi = 20\ \text{GeV}$



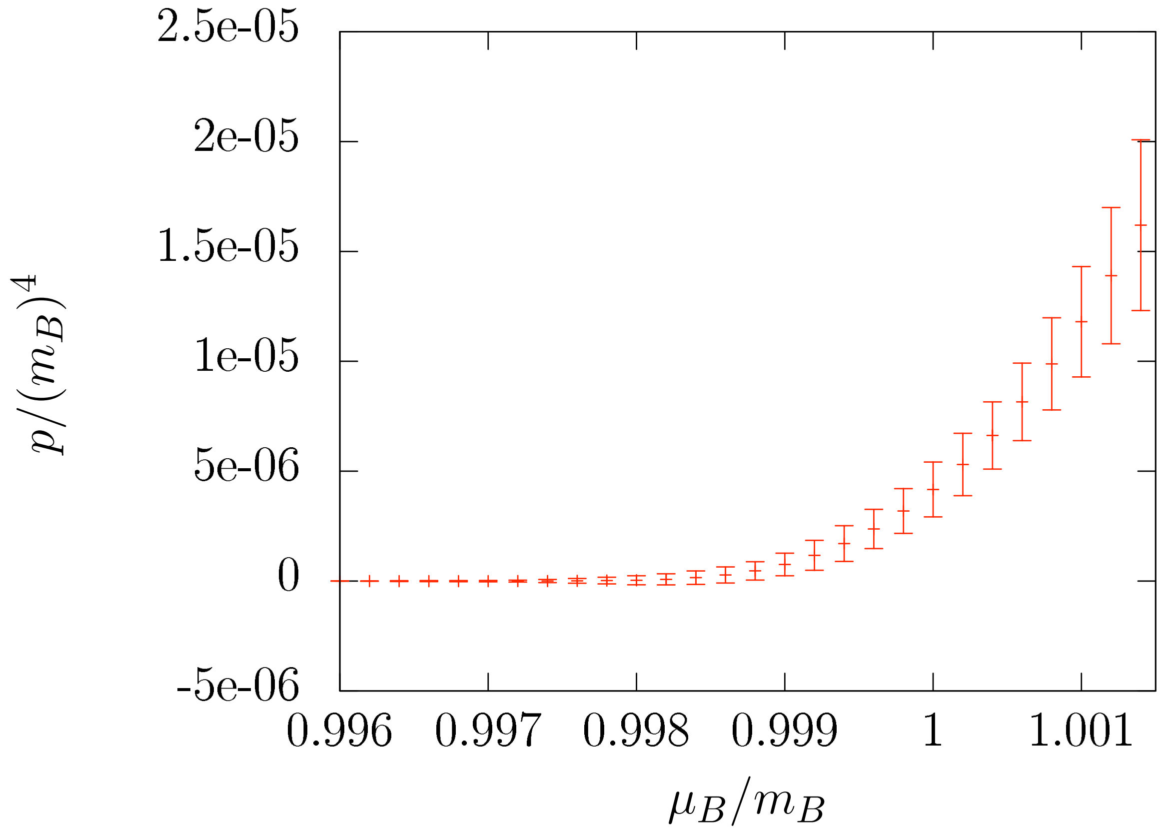
$\frac{\mu}{T} \sim 4000$

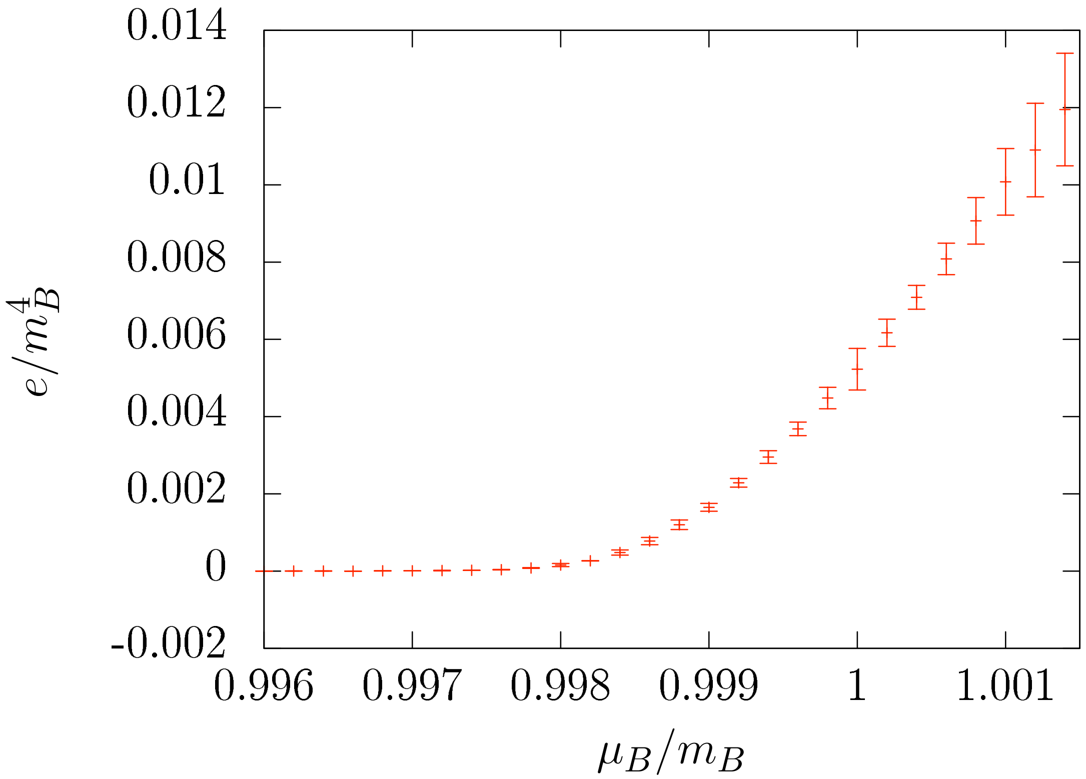
Effect of binding between baryons: $\mu_c < m_B$

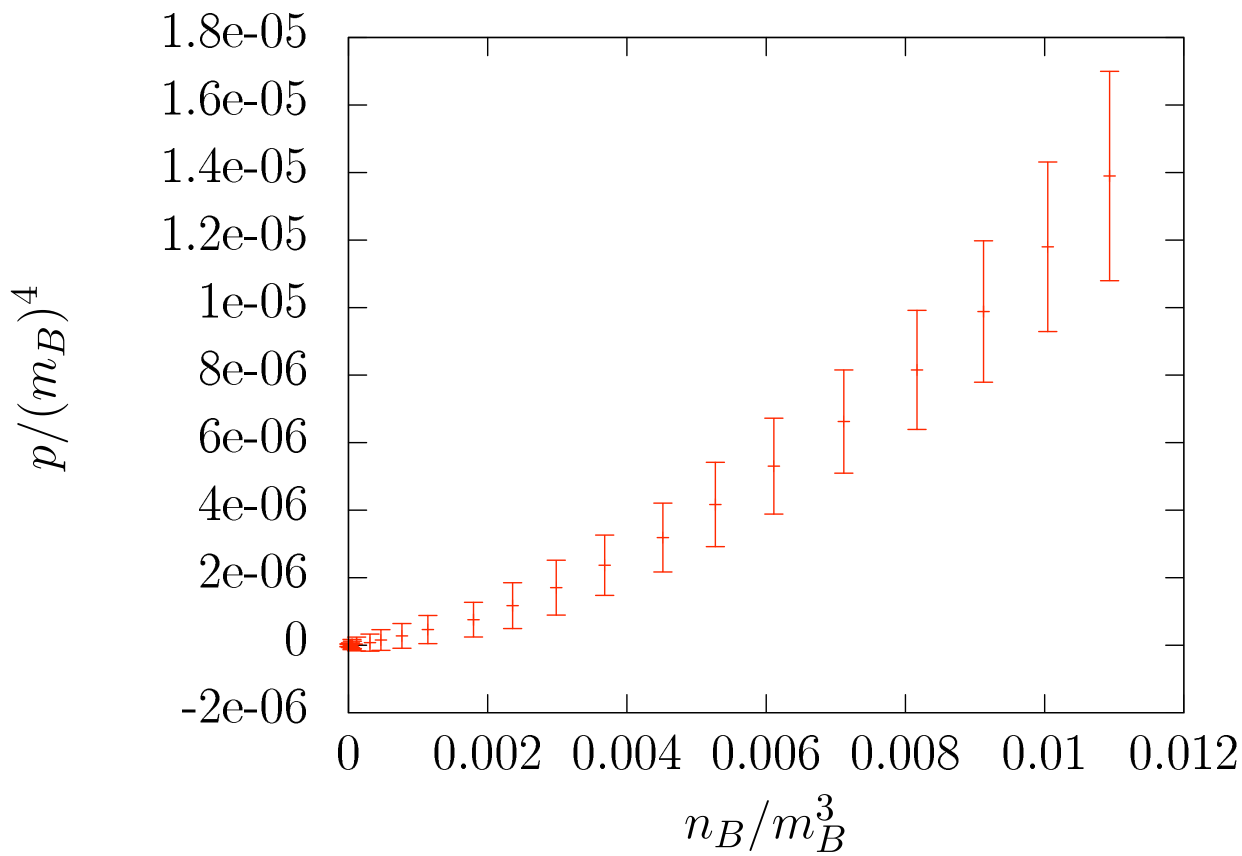
Binding energy per nucleon: $\epsilon = \frac{\mu_c - m_B}{m_B} \sim 10^{-3}$

Transition is smooth crossover: $T > T_c \sim \epsilon\, m_B$

The equation of state for nuclear matter, Nf=2

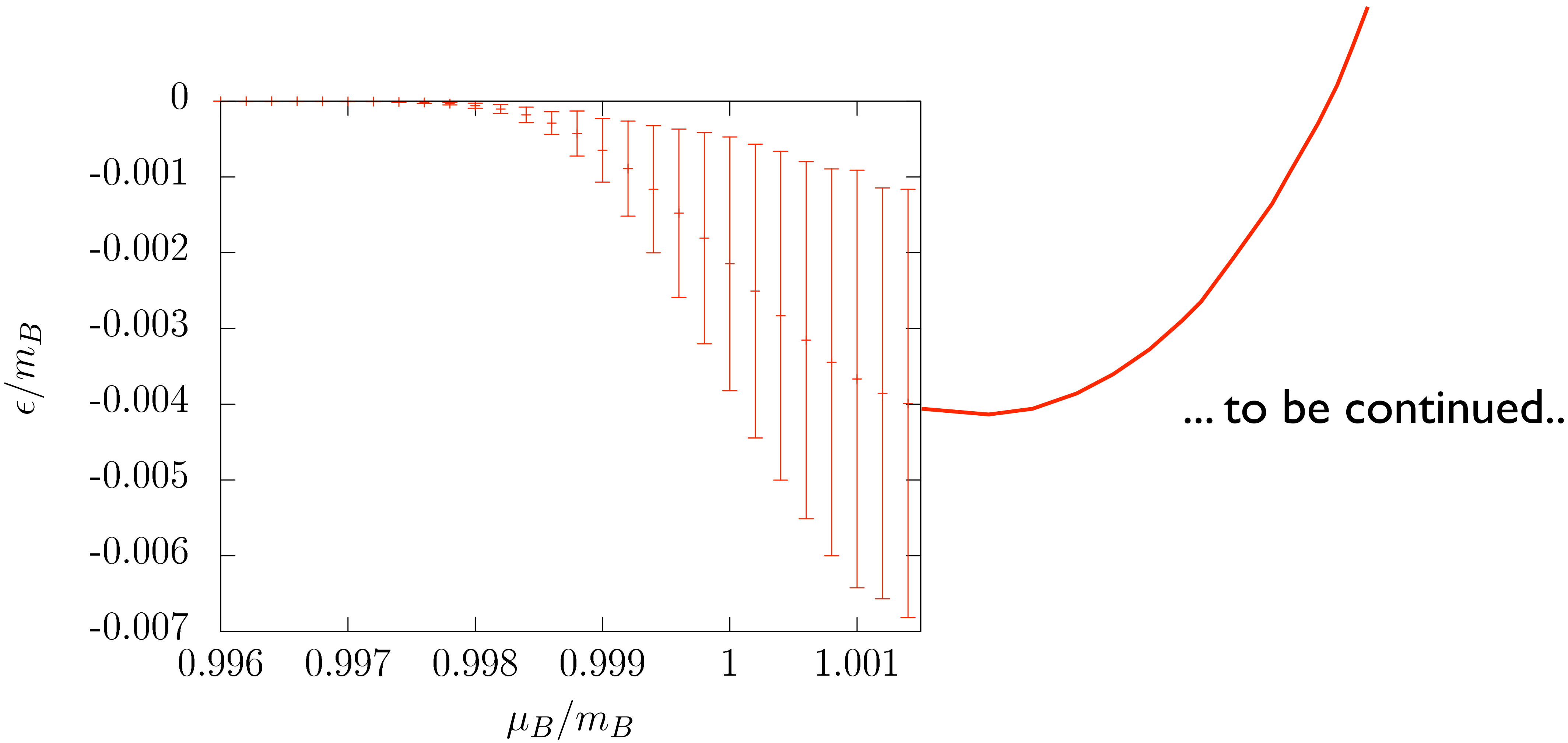






Binding energy per nucleon

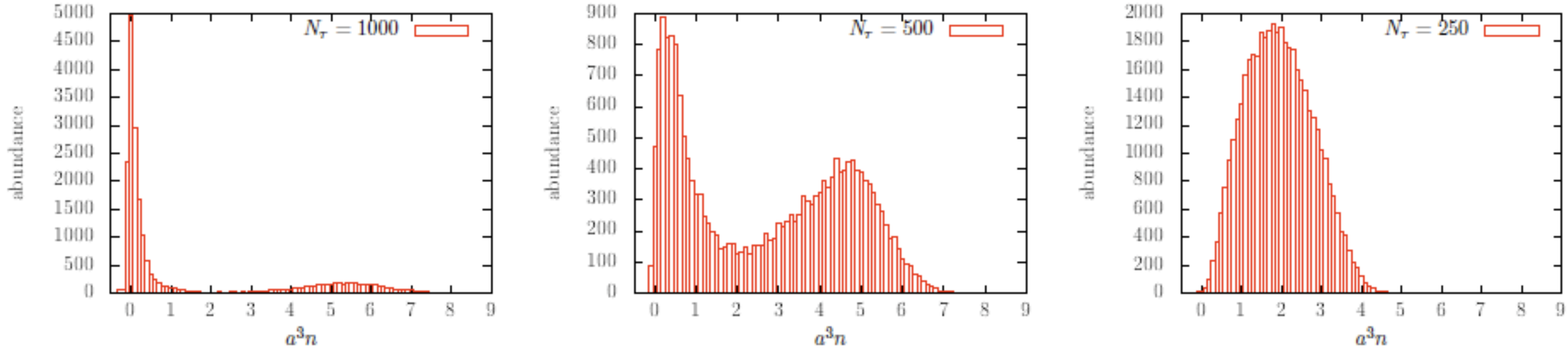
$$\epsilon = \frac{e - n_B m_B}{n_B m_B} = \frac{e}{n_B m_B} - 1$$



Minimum: access to nucl. binding energy, nucl. saturation density!

$\epsilon \sim 10^{-3}$ consistent with the location of the onset transition

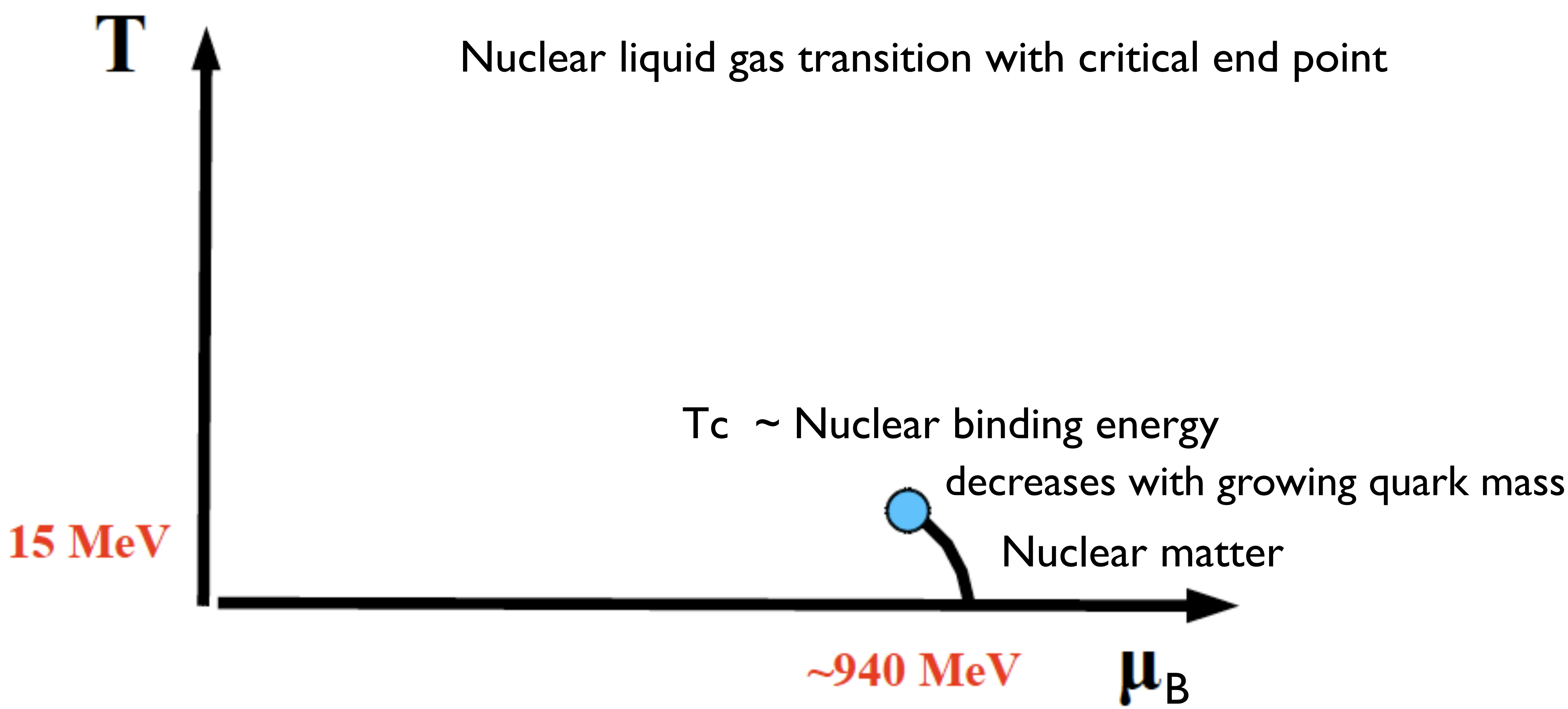
Liquid gas transition: first order + endpoint



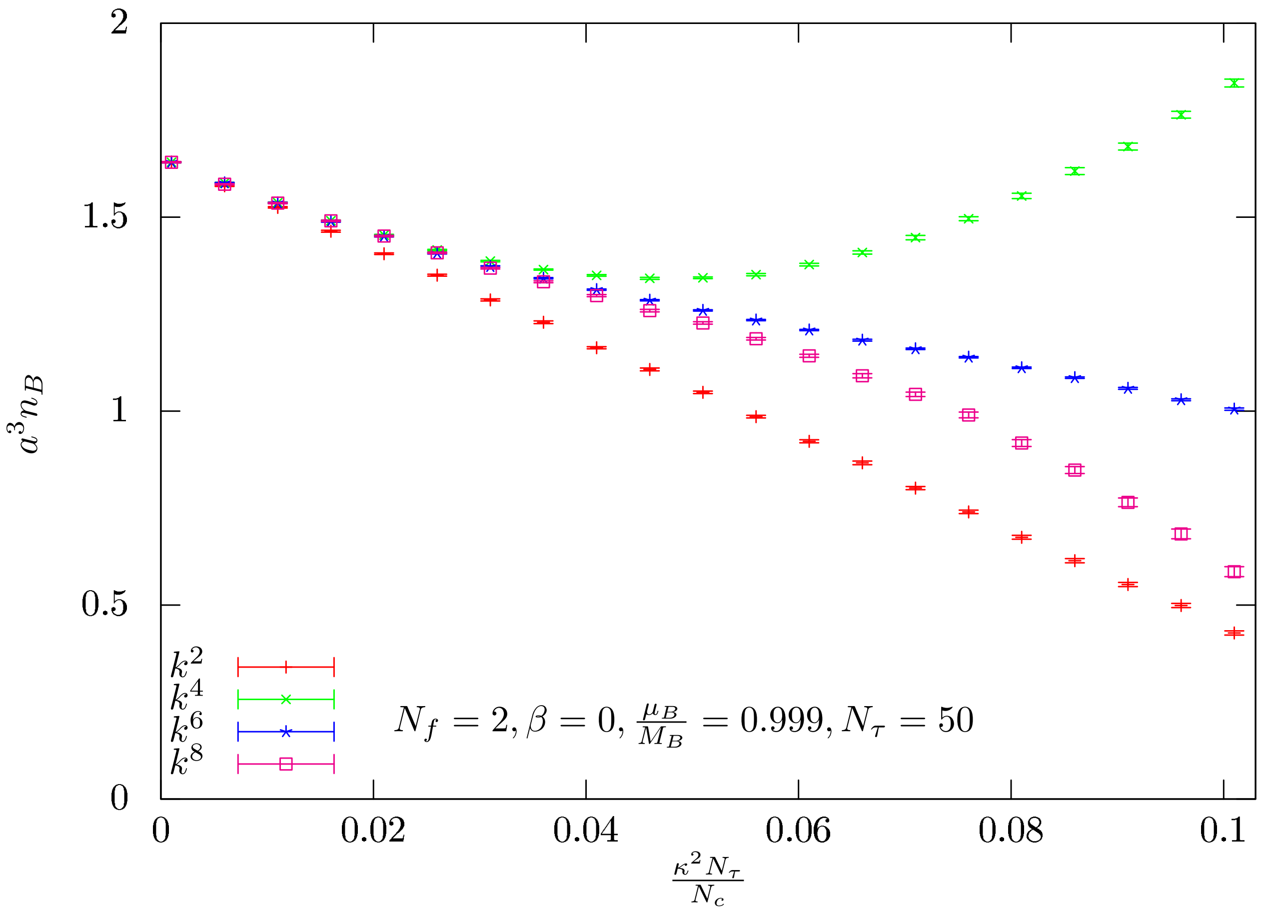
For sufficiently light quarks: $\kappa \sim 0.1$

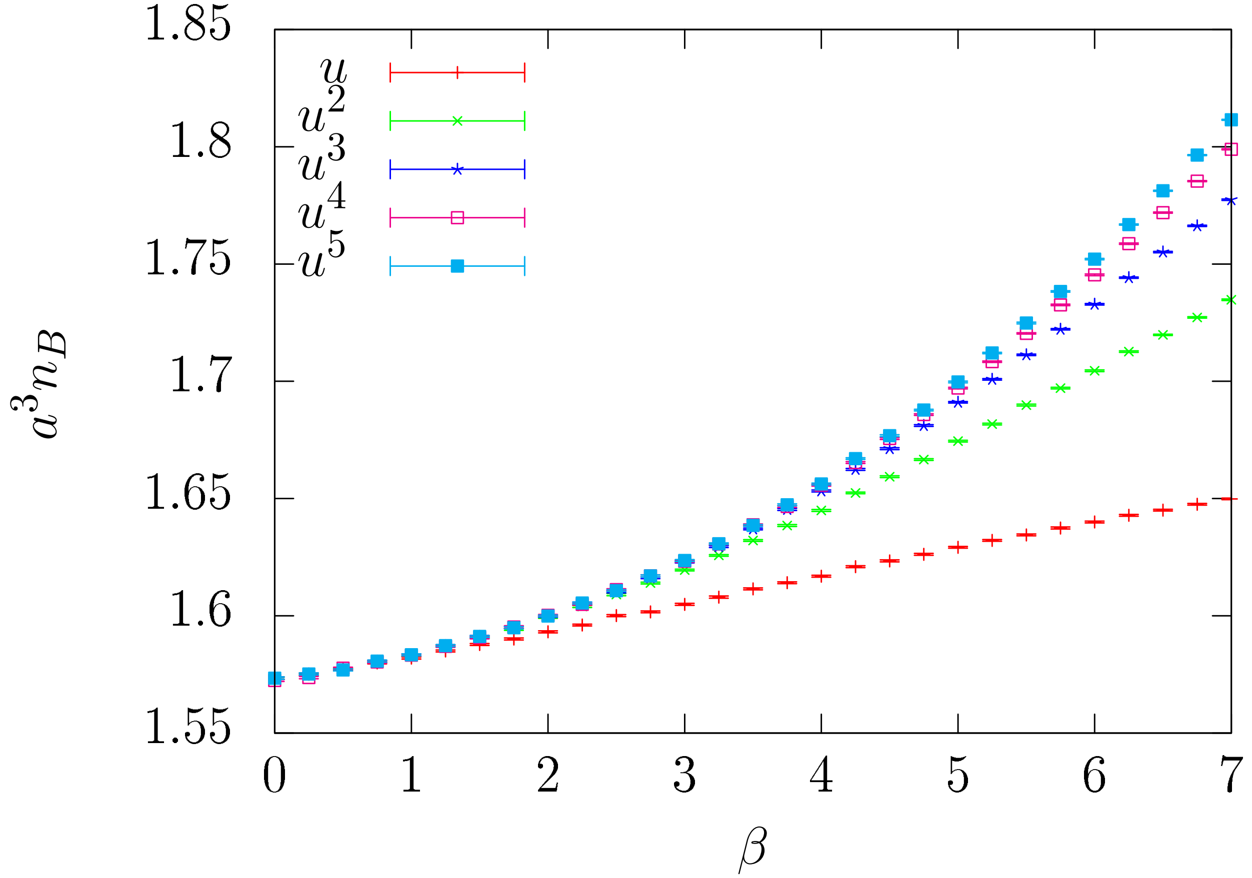
- Coexistence of vacuum and finite density phase: 1st order
- If the temperature $T = \frac{1}{aN_\tau}$ or the quark mass is raised this changes to a crossover

nuclear liquid gas transition!!!


T
Nuclear liquid gas transition with critical end point
Tc ~ Nuclear binding energy
decreases with growing quark mass
15 MeV
Nuclear matter
~940 MeV
μ_B

Convergence of the effective theory





hopping expansion

strong coupling expansion κ^8

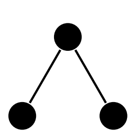
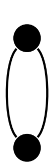
Linked cluster expansion of effective theory

Consider spin model with 2-point interactions

$$\mathcal{Z} = \int \mathcal{D}\phi \, e^{-S_0[\phi] + \frac{1}{2}\sum_{x,y}\sum_{i,j}\phi_i(x) v_{ij}(x,y)\phi_j(y)} \qquad\qquad W = -\ln \mathcal{Z}$$

Linked cluster expansion of "free energy":

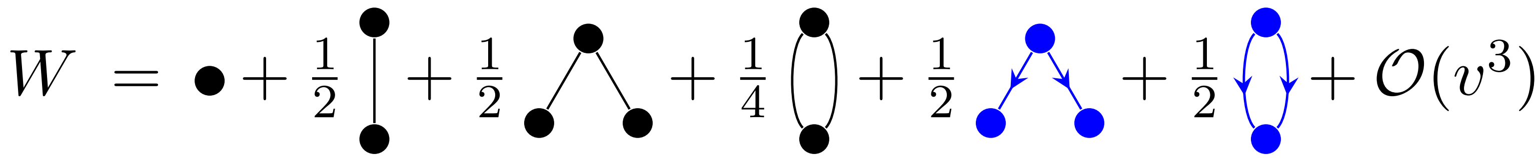
$$\begin{aligned} W = W_0 &+ \frac{1}{2}\sum_{x,y}\sum_{i,j} M_i(x) v_{ij}(x,y) M_j(y) \\ &+ \frac{1}{2}\sum_{i,j,k}\sum_{x,y,z} M_i(x) v_{ij}(x,y) M_{jk}(y) v_{kl}(y,z) M_l(z) \\ &+ \frac{1}{4}\sum_{i,j}\sum_{x,y} M_{ij}(x) v_{ik}(x,y) v_{jl}(x,y) M_{kl}(y) + \mathcal{O}(v^3) \end{aligned}$$

$= \bullet + \frac{1}{2}$  $+ \frac{1}{2}$  $+ \frac{1}{4}$  $+ \mathcal{O}(v^3)$

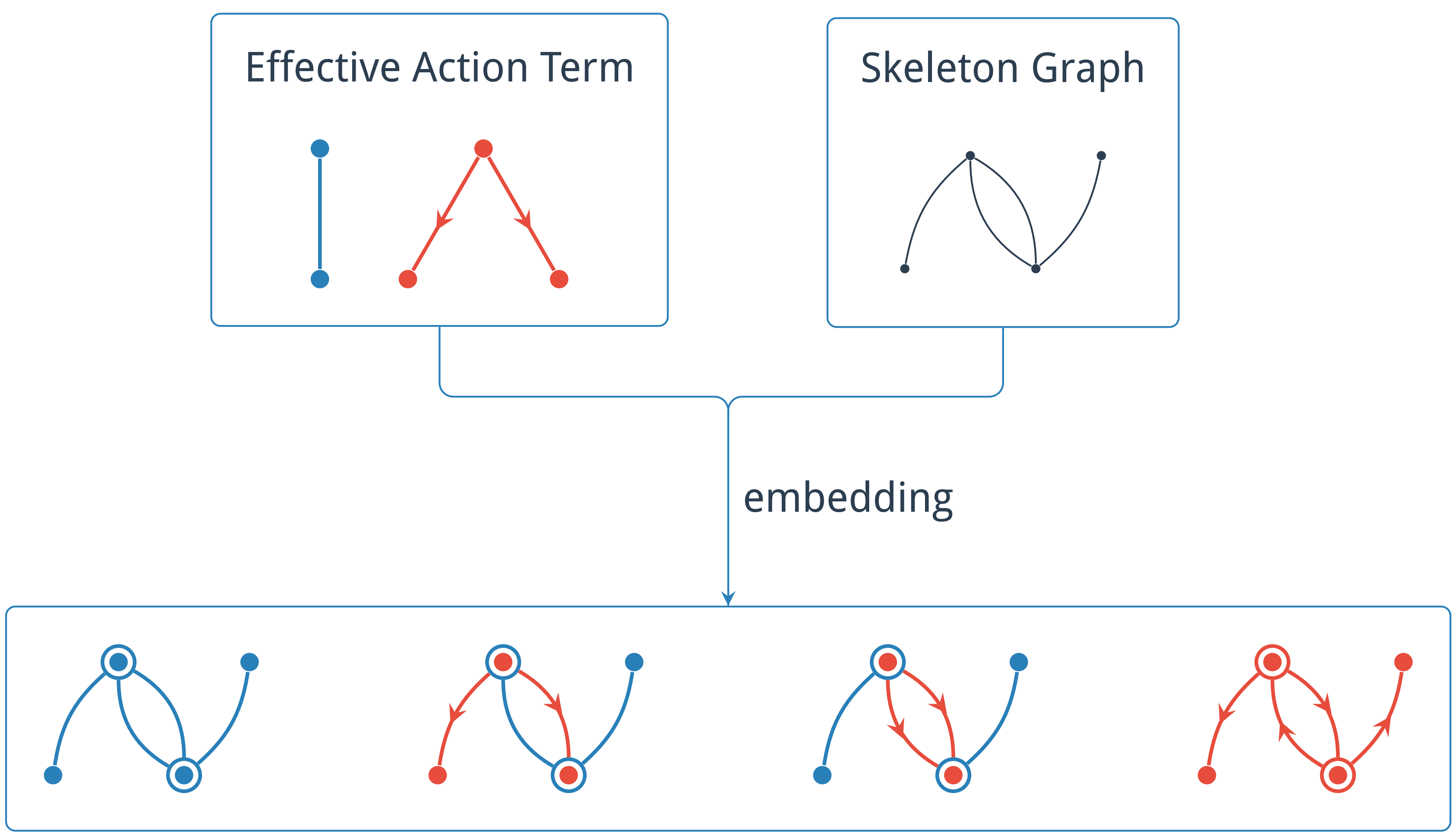
Required generalization: n-point interactions

Glesaaen, Neuman, O.P. 15

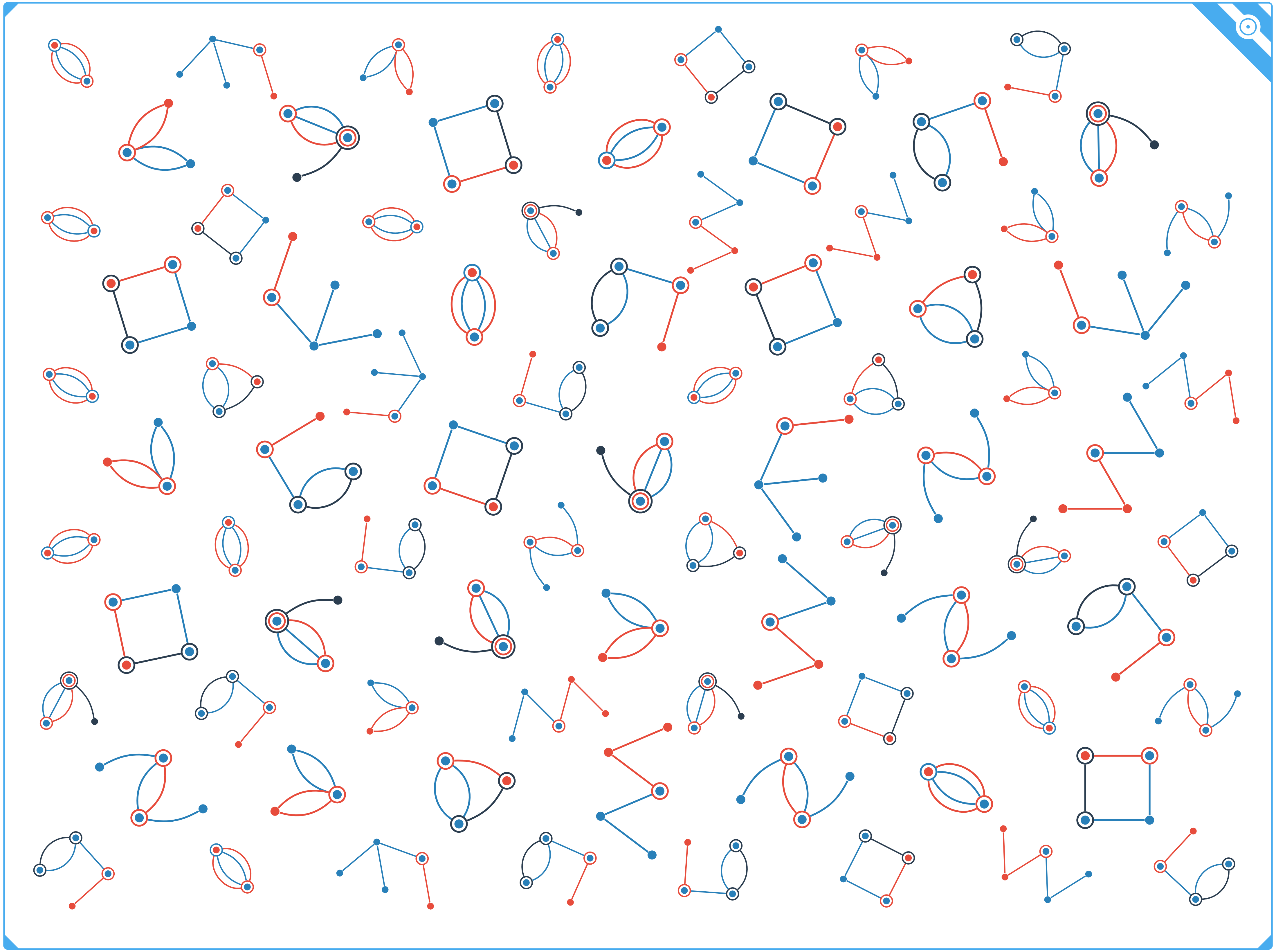
$$\mathcal{Z} = \int \mathcal{D}\phi \, e^{-S_0[\phi] + \frac{1}{2}\sum v_{ij}(x,y)\phi_i(x)\phi_j(y) + \frac{1}{3!}\sum u_{ijk}(x,y,z)\phi_i(x)\phi_j(y)\phi_k(z) + \dots}$$



Mapping of the effective theory by embedding:

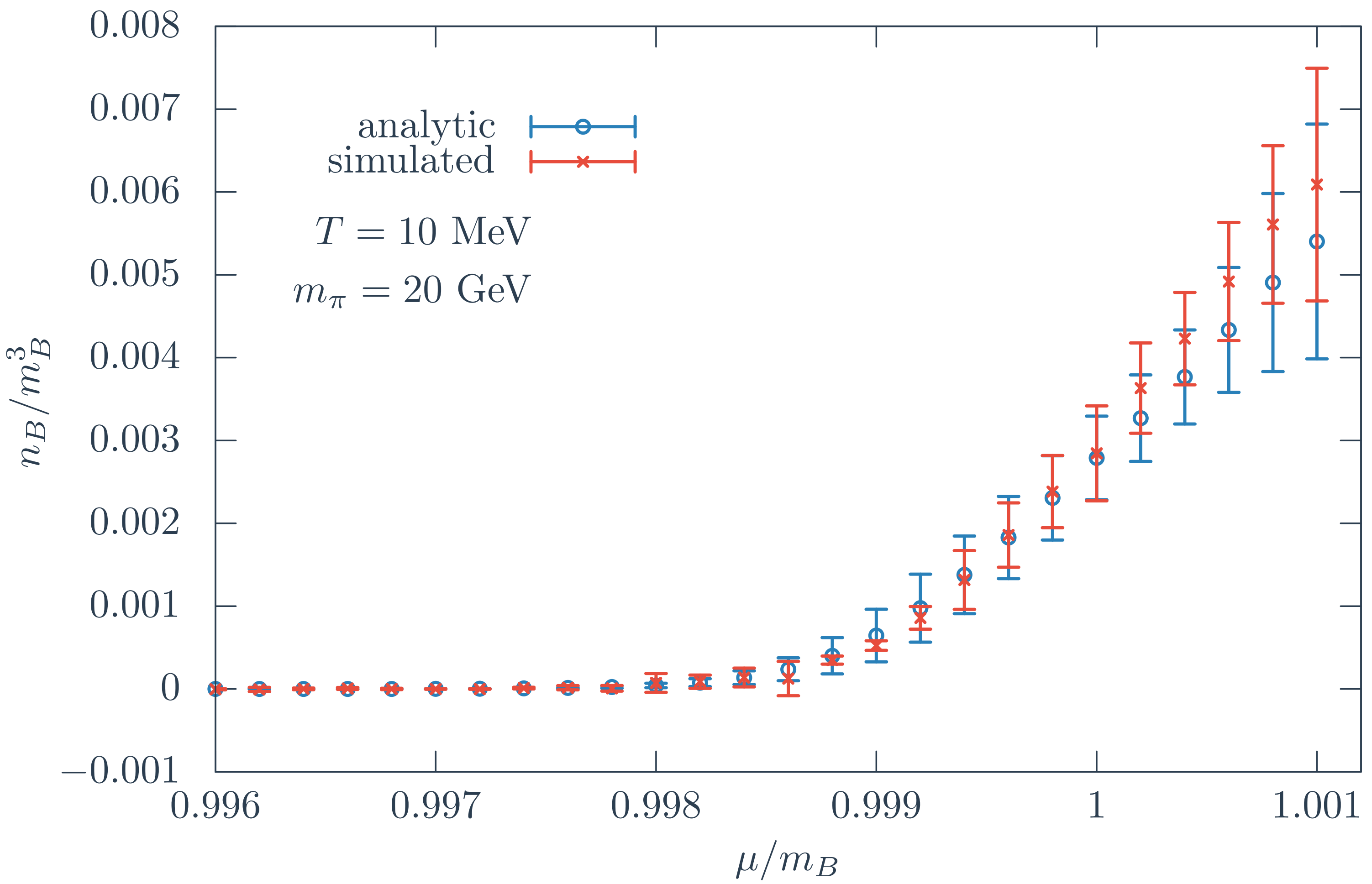


Fun with diagrams....

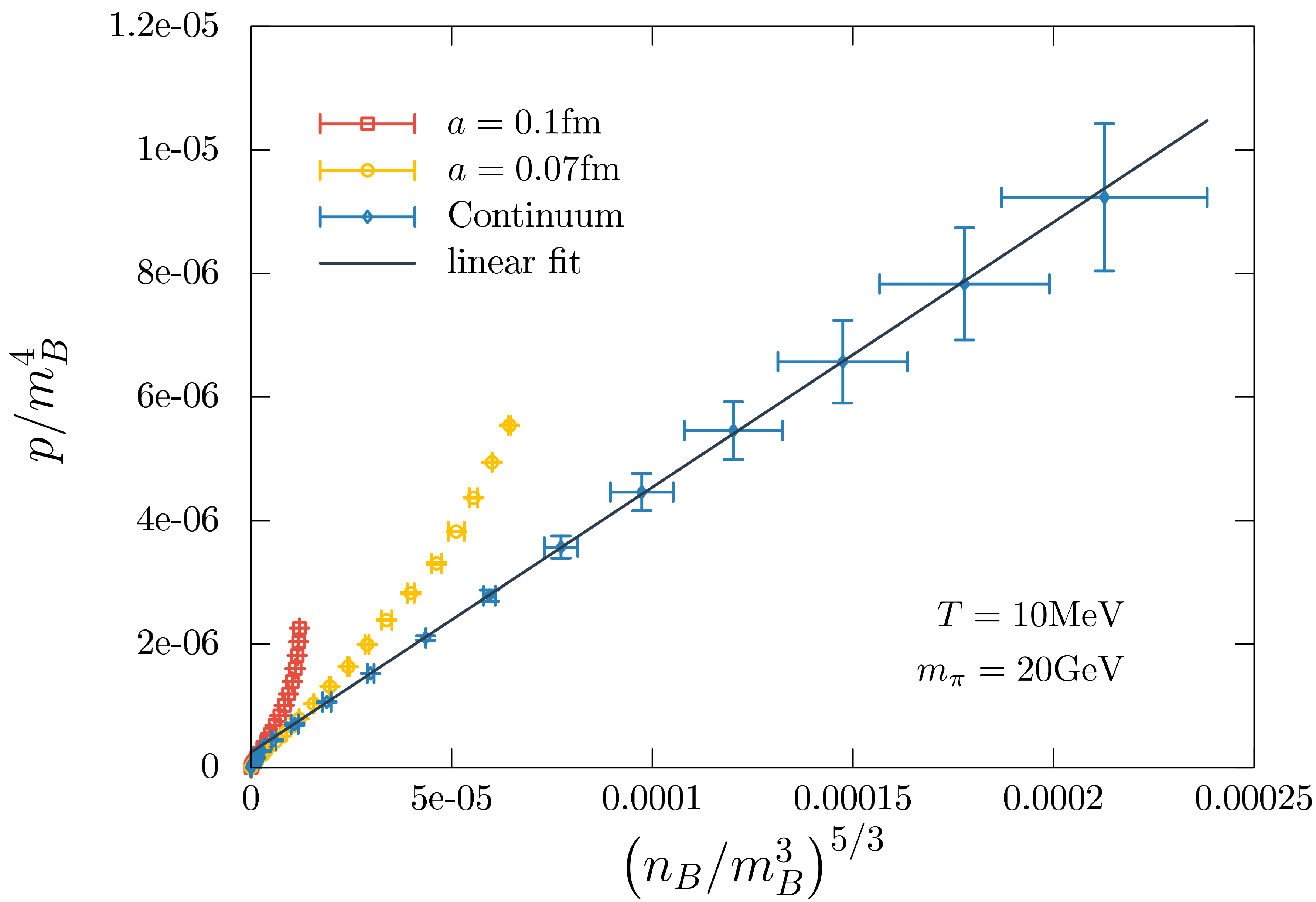


Compare continuum extrapolated results

through $u^5\kappa^8$

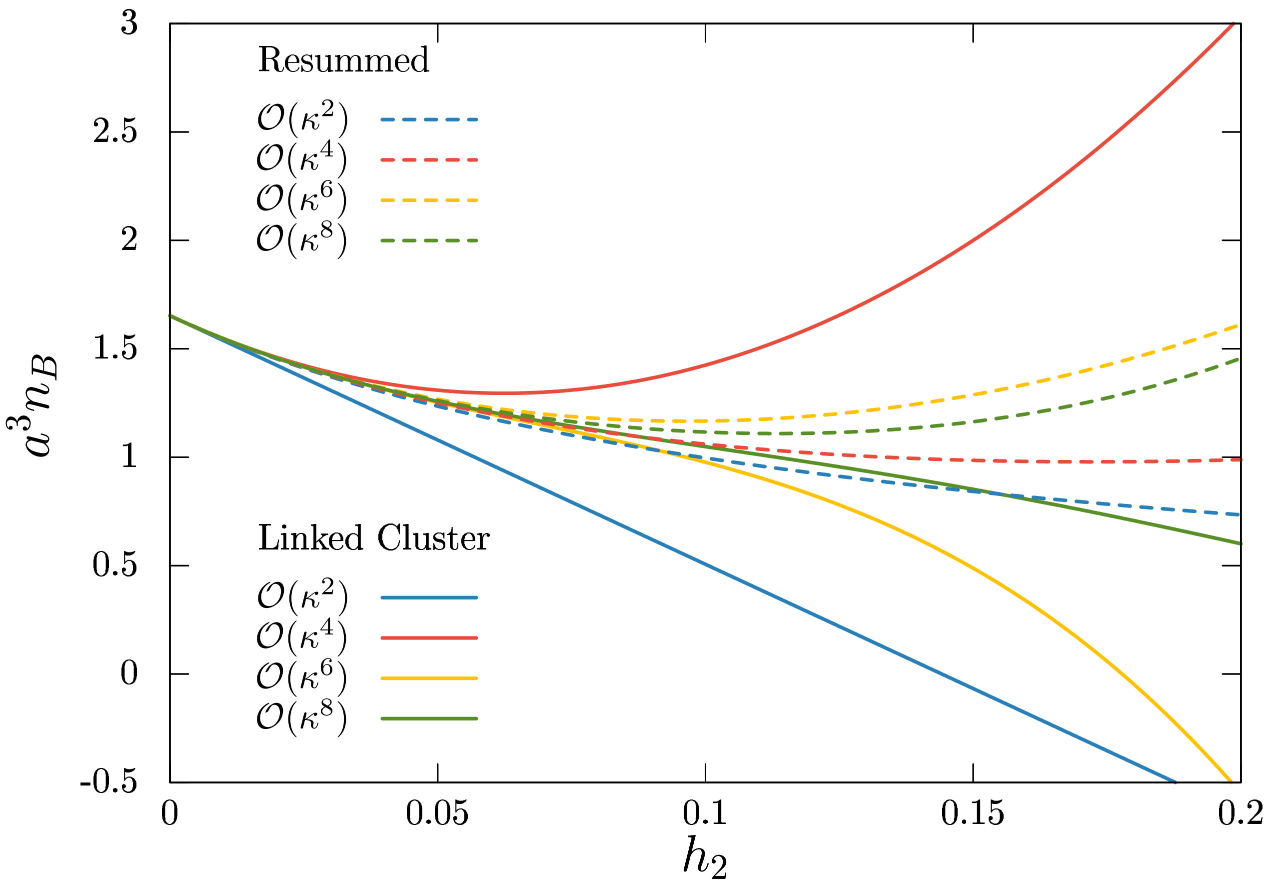


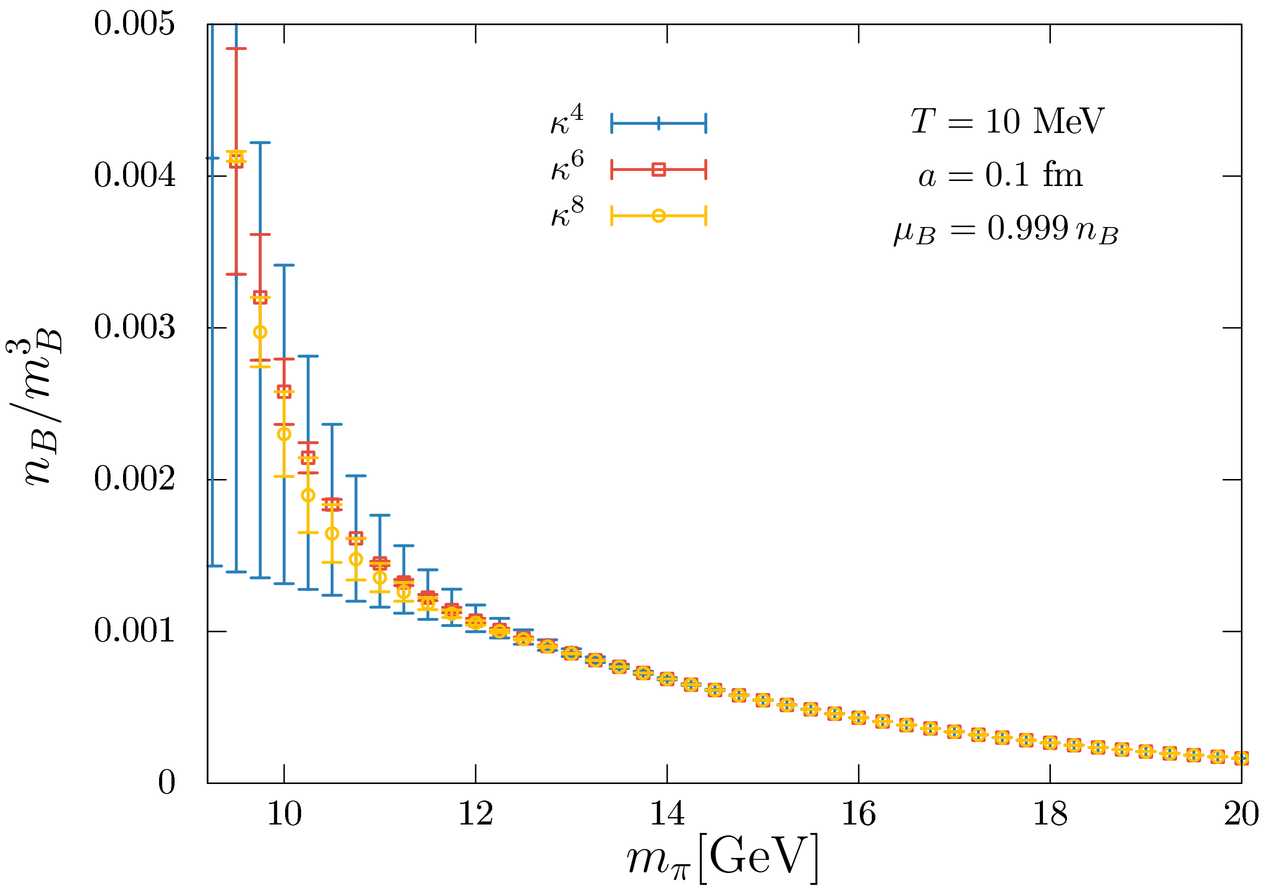
Equation of state of heavy nuclear matter, continuum



- EoS fitted by polytrope, non-relativistic fermions!
- Can we understand the pre-factor? Interactions, mass-dependence…

Resummations + reach in mass range





Resumming long range non-overlapping chains, gain in mass range “sobering”

Conclusions

- Nuclear matter directly from QCD in "one-parameter distortions":
- Heavy dense QCD near continuum with fully analytic methods
- Chiral dense QCD on coarse lattices (not shown here)
- Larger than nuclear densities out of reach because of lattice saturation

Backup slides

Strong coupling expansion (pure gauge)

Wilson action: $S_g[U] = \sum_x \sum_{1\leq\mu<\nu\leq 4} \beta \left(1 - \frac{1}{3}\mathrm{Re}\mathrm{Tr}U_p\right) \equiv \sum_p S_p$ Plaquette action

Character of rep. r: $\chi_r(U) = \mathrm{Tr}D_r(U)$

group element — representation matrix of group element

Character expansion: $\exp -S_p = c_0(\beta)[1 + \sum_{r\neq 0} d_r c_r(\beta)\chi_r(U_p)]$, convergent inside radius of c.

dimension of rep. matrix

Expansion coefficients: combinations of modified Bessel fcns. for SU(N)

$c_f \equiv u = \frac{\beta}{18} + O(\beta^2) < 1$, all others can be expressed by fundamental one

Wilson 74: static potential, string tension Münster, Seo 80-82: glueball masses,
Polonyi, Szachlanyi 82: strong coupling limit of free energy, effective action, Green 83: finite T string
Langelage, Münster, O.P. 08: strong coupling series for finite T

Subleading couplings

Subleading contributions for next-to-nearest neighbours:

$$\lambda_2 S_2 \quad \propto \quad u^{2N_\tau+2} \sum_{[kl]}{}' 2\mathrm{Re}(L_k L_l^*) \quad \text{distance} \; = \sqrt{2}$$

$$\lambda_3 S_3 \quad \propto \quad u^{2N_\tau+6} \sum_{\{mn\}}{}'' 2\mathrm{Re}(L_m L_n^*) \quad \text{distance} \; = 2$$

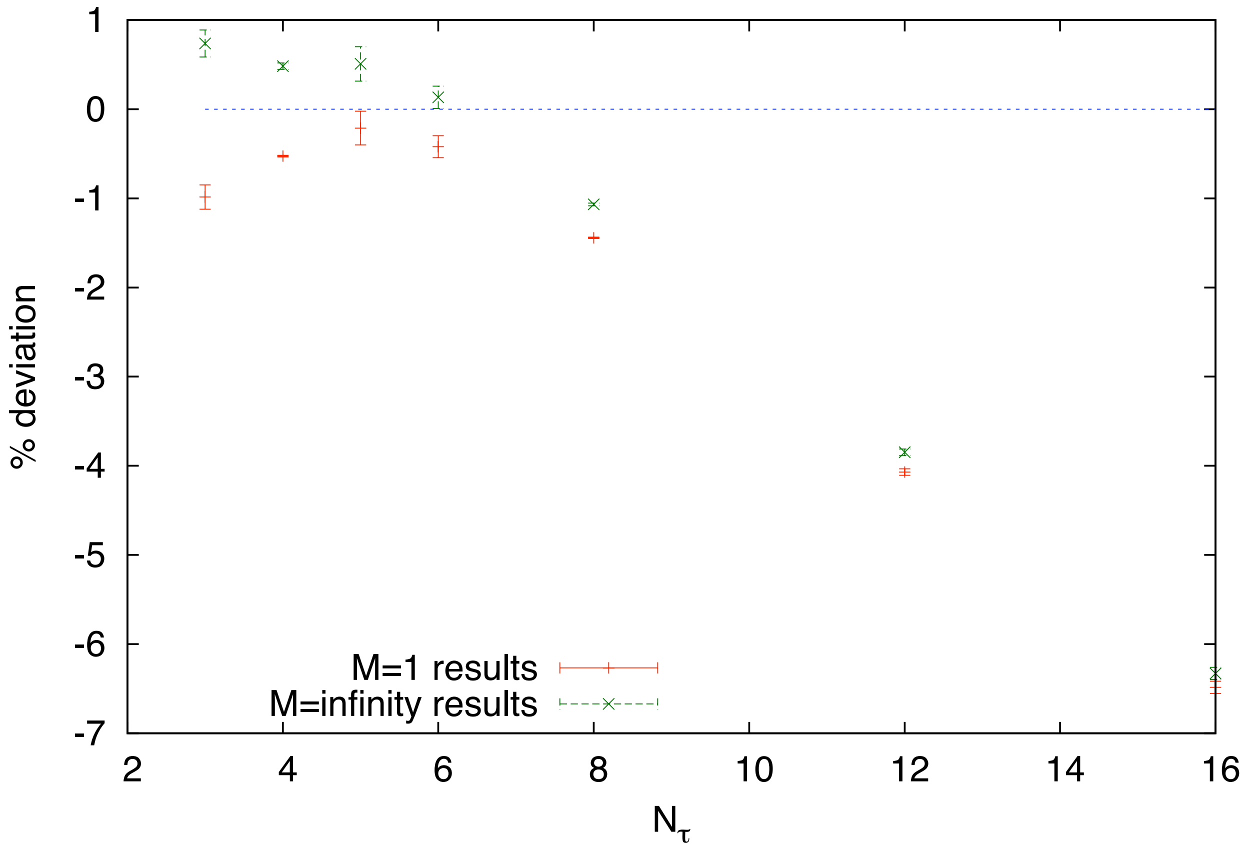
as well as terms from loops in the *adjoint* representation:

$$\lambda_a S_a \propto u^{2N_\tau} \sum_{<ij>} \mathrm{Tr}^{(a)} W_i \mathrm{Tr}^{(a)} W_j \quad ; \quad \mathrm{Tr}^{(a)} W = |L|^2 - 1$$

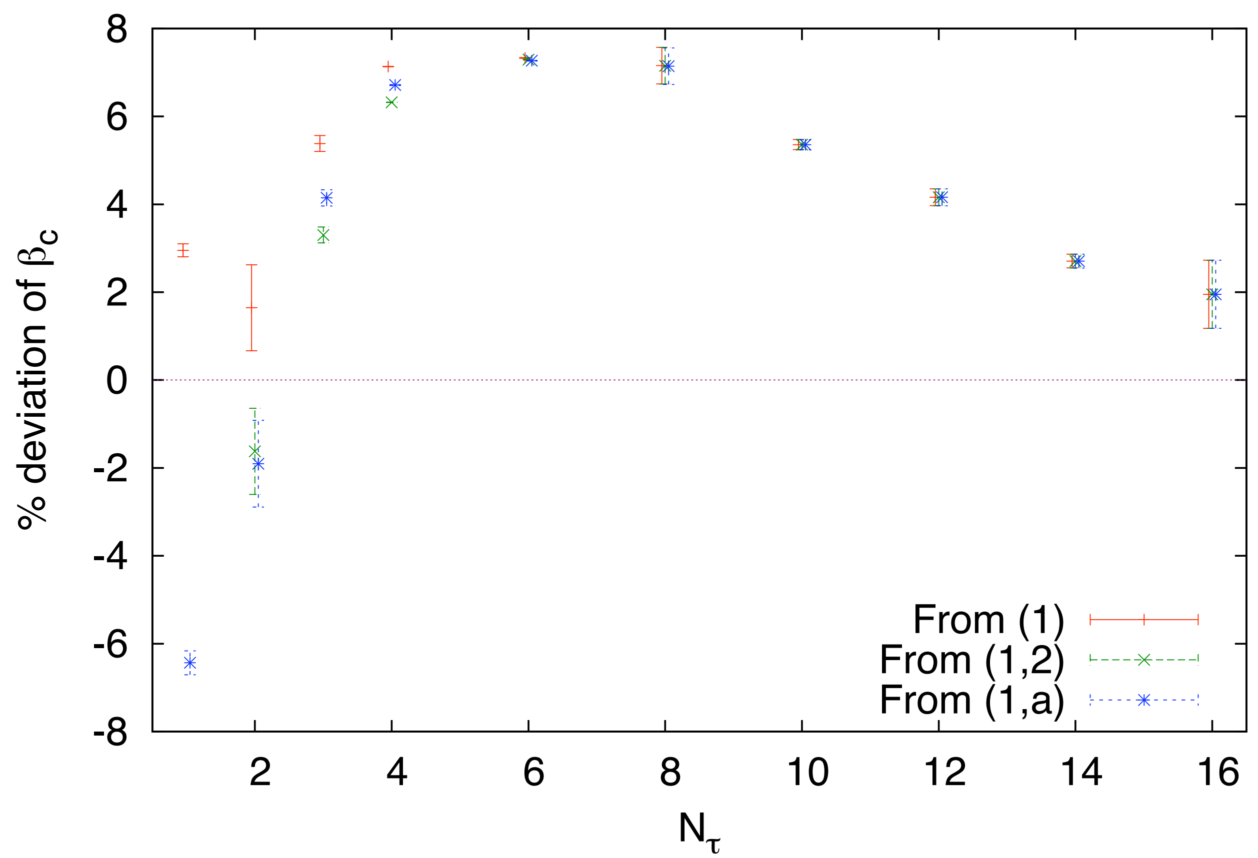
Comparison with 4d Monte Carlo

Relative accuracy for β_c compared to the full theory

SU(2)



SU(3)



Note: influence of additional couplings checked explicitly!