

Chiral symmetry breaking in continuum QCD

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und Forschung

fQCD collaboration:

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N. Müller, J. M. Pawłowski, S. Rechenberger, F. Rennecke, N. Strodthoff

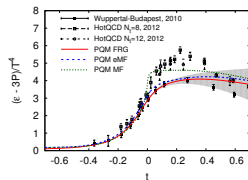
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QCD phase diagram with functional methods

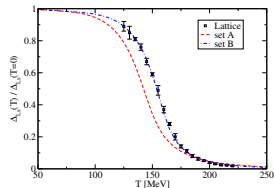
cf. talks J. M. Pawłowski, C. Fischer

- works well at $\mu = 0$: agreement with lattice



[Herbst, MM, Pawłowski, Schaefer, Stiele, '13]

[Braun, Haas, Pawłowski, unpublished]

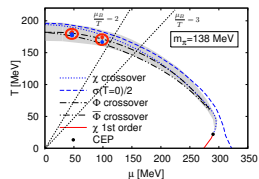


[Luecker, Fischer, Welzbacher, 2014]

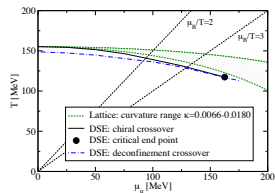
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(possibly already at small μ)



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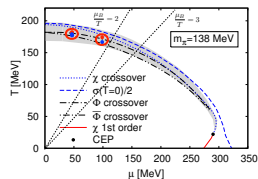


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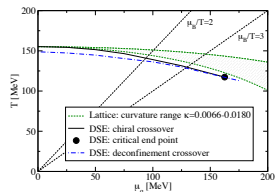
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- calculations need model input:
 - ▶ Polyakov-quark-meson model with FRG:
 - ★ initial values at $\Lambda \approx \mathcal{O}(\Lambda_{\text{QCD}})$
 - ★ input for Polyakov loop potential
 - ▶ quark propagator DSE:
 - ★ IR quark-gluon vertex



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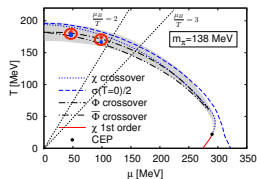
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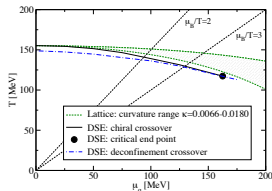
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possible explanation for disagreement:

- $\mu \neq 0$: relative importance of diagrams changes
 \Rightarrow summed contributions vs. individual contributions



[Herbst, Pawłowski, Schaefer, 2013]



[Luecker, Fischer, Fister, Pawłowski, '13]

Back to QCD in the vacuum

- use only perturbative QCD input
 - ▶ $\alpha_S(\Lambda = \mathcal{O}(10) \text{ GeV})$
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$$\partial_k \Gamma_k = \frac{1}{2} \left(\text{Diagram 1} - \text{Diagram 2} - \text{Diagram 3} \right)$$

$$\Rightarrow \text{effective action } \Gamma[\Phi] = \lim_{k \rightarrow 0} \Gamma_k[\Phi]$$

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derivatives \Rightarrow equations for 1PI n -point functions

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- approximations necessary - vertex expansion:
derivatives \Rightarrow equations for 1PI n -point functions
- $\lim_{k \rightarrow 0} \Gamma_k[\Phi]$ should not depend on approximation in the vacuum

“Quenched” Landau gauge QCD

[MM, Strodthoff, Pawłowski, 2014]

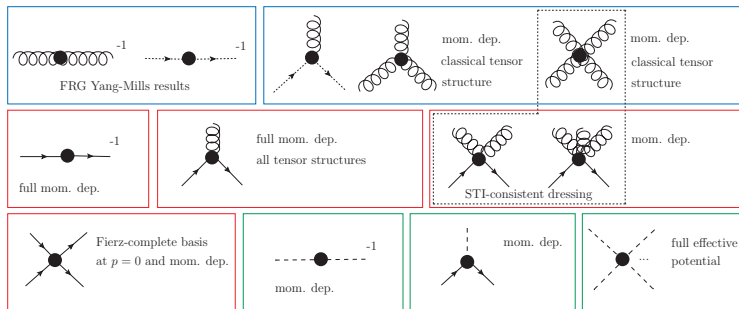
- two crucial phenomena: $S\chi$ SB and confinement
- similar scales - hard to disentangle
- quenched QCD: allows separate investigation

see e.g. [Williams, Fischer, Heupel, 2015]

“Quenched” Landau gauge QCD

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- two crucial phenomena: S_χ SB and confinement
- similar scales - hard to disentangle see e.g. [Williams, Fischer, Heupel, 2015]
- quenched QCD: allows separate investigation
- YM propagators: FRG input [Fischer, Maas, Pawłowski, 2009], [Fister, Pawłowski, unpublished]
- matter part:



Equations

[MM, Strodthoff, Pawlowski, 2014]

$$\partial_t \text{---}^{-1} = \text{---}^{\text{---}} + \text{---}^{\text{---}} + \frac{1}{2} \text{---}^{\text{---}} + \text{---}^{\text{---}} + \text{---}^{\text{---}} - \text{---}^{\text{---}}$$

$$\partial_t \text{---}^{\text{---}} = - \text{---}^{\text{---}} - \text{---}^{\text{---}} - \text{---}^{\text{---}} - \text{---}^{\text{---}} - \frac{1}{2} \text{---}^{\text{---}} - \frac{1}{2} \text{---}^{\text{---}} + 2 \text{---}^{\text{---}} - \text{---}^{\text{---}} + \text{perm.}$$

$$\partial_t \text{---}^{\text{---}} = 2 \text{---}^{\text{---}} - \text{---}^{\text{---}} - \text{---}^{\text{---}} - \text{---}^{\text{---}} - \text{---}^{\text{---}} - \text{---}^{\text{---}} - \text{---}^{\text{---}} - \text{---}^{\text{---}} + \text{perm.}$$

Equations

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$$\partial_t \text{ (wavy line) } = - \text{ (triangle) } + 2 \text{ (triangle) } - \text{ (circle) } + \text{perm.}$$

$$\partial_t \text{ (solid line) }^{-1} = \text{ (loop) } + \text{ (loop) } + \frac{1}{2} \text{ (loop) } + \text{ (loop) } + \text{ (loop) } - \text{ (loop) }$$

$$\partial_t \text{ (dotted line) } = - \text{ (triangle) } - \text{ (triangle) } + \text{perm.}$$

$$\partial_t \text{ (wavy line) } = - \text{ (triangle) } - \text{ (triangle) } - \text{ (circle) } - \text{ (loop) } - \frac{1}{2} \text{ (loop) } + 2 \text{ (loop) } - \text{ (triangle) } + \text{perm.}$$

$$\partial_t \text{ (cross) } = 2 \text{ (loop) } - \text{ (loop) } - \text{ (loop) } - \text{ (loop) } - \text{ (loop) } - \text{ (loop) } - \text{ (loop) } + \text{perm.}$$

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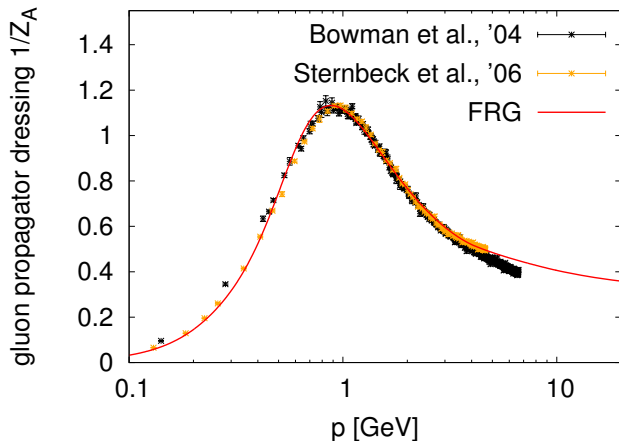
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cf. talk J. M. Pawłowski



- FRG result \Rightarrow self-consistent calculation within FRG approach
- sets the scale in comparison to lattice QCD

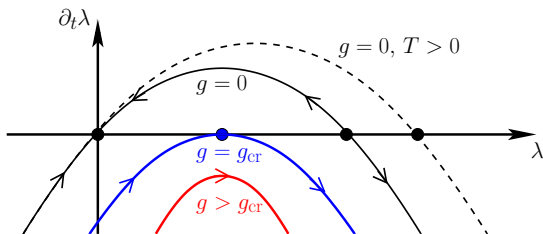
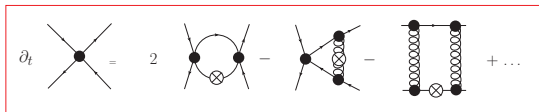
Chiral symmetry breaking

- χ SB \Leftrightarrow resonance in 4-Fermi interaction λ (pion pole):

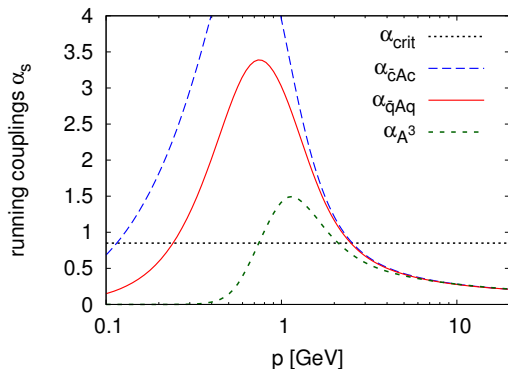
Chiral symmetry breaking

- χ SB \Leftrightarrow resonance in 4-Fermi interaction λ (pion pole):
- resonance \Rightarrow singularity without momentum dependency

$$\partial_t \lambda = a \lambda^2 + b \lambda \alpha + c \alpha^2, \quad a > 0, \quad c < 0$$



[Braun, 2011]



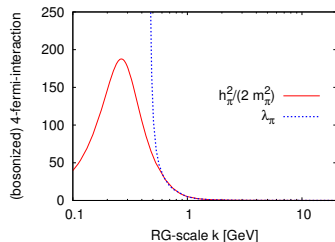
- agreement in perturbative regime required by gauge symmetry
- non-degenerate in nonperturbative regime: reflects gluon mass gap
- $\alpha_{qAq} > \alpha_{cr}$: necessary for chiral symmetry breaking
- area above α_{cr} very sensitive to errors

4-Fermi vertex via dynamical hadronization

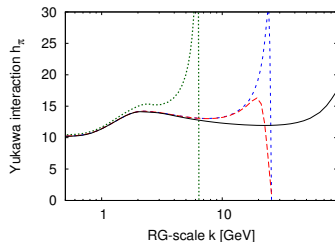
[Gies, Wetterich, 2002]

- change of variables: particular 4-Fermi channels \rightarrow meson exchange
- efficient inclusion of momentum dependence \Rightarrow no singularities
- identifies relevant effective low-energy dofs from QCD

$$\partial_k \Gamma_k = \frac{1}{2} \left(\text{ring with wavy meson} - \text{ring with dashed meson} - \text{ring with solid meson} + \frac{1}{2} \text{ring with dashed meson} \right)$$

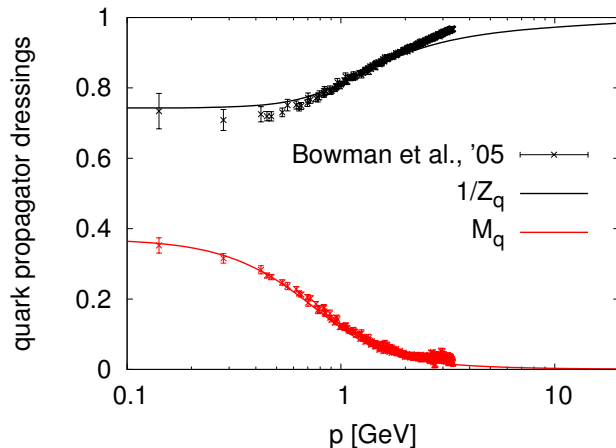


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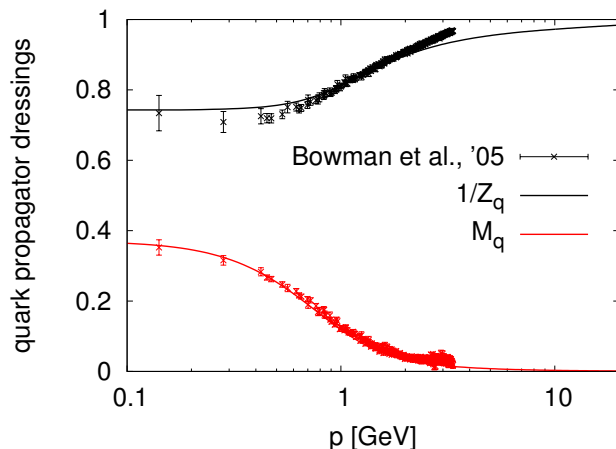


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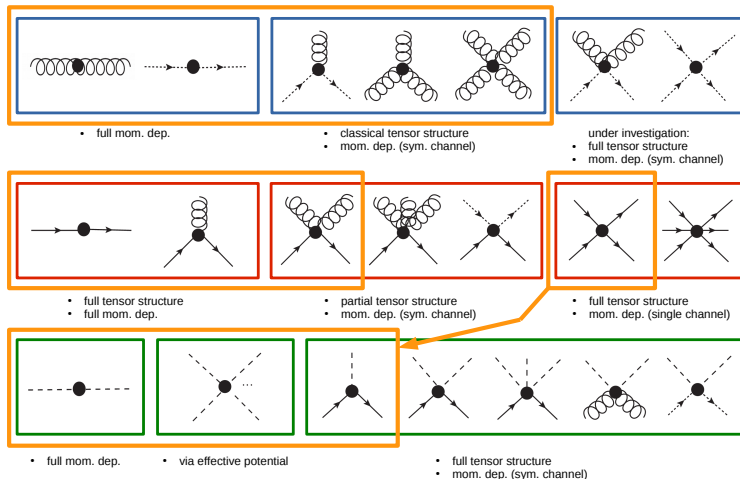
- FRG vs. lattice: bare mass, quenched, scale



- FRG vs. lattice: bare mass, quenched, scale
- agreement with lattice not sufficient for $\mu \neq 0$
 \Rightarrow need convergence in vertex expansion

Stability of truncation

Expansion of effective action in 1PI correlators



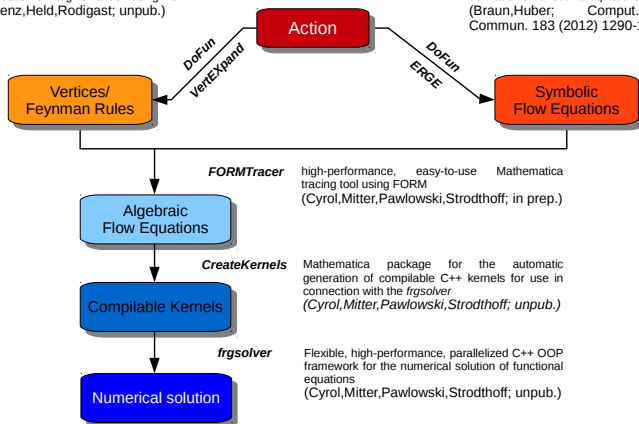
Workflow

VertEXpand

Mathematica package for the derivation of vertices from a given action using FORM (Denz,Held,Rodigast; unpub.)

DoFun

Mathematica package for the derivation of functional equations (Braun,Huber; Comput.Phys. Commun. 183 (2012) 1290-1320)



[Cyrol, MM, Pawlowski, Strodthoff, 2013-2016]

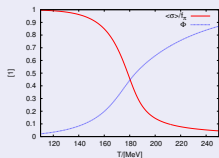
η' -meson (screening) mass at chiral crossover

- small η' -meson mass above chiral crossover? [Kapusta, Kharzeev, McLerran, 1998]
- experiment: drop in η' mass at chiral crossover [Csörgo et al., 2010]

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chiral crossover: Polyakov-Quark-Meson model (extended mean-field)

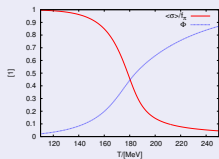


- $N_f = 2$ quark and meson degrees of freedom
- describes chiral crossover
- (de-)confinement via Polyakov loop potential
- $U(1)_A$ -anomaly: mesonic 't Hooft determinant

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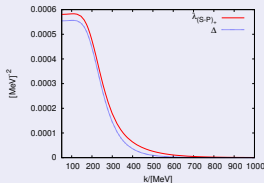
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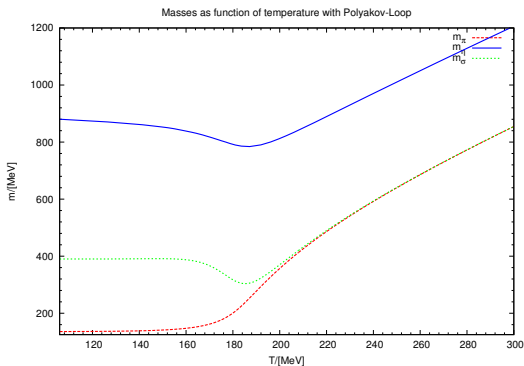
't Hooft determinant

[Heller, MM, 2015]



- RG-scale dependence from fQCD
- temperature dependence $k(T)$:
 - ▶ $\lambda_{(S-P)_+,fQCD}(k) \equiv \lambda_{(S-P)_+,PQM}(T)$

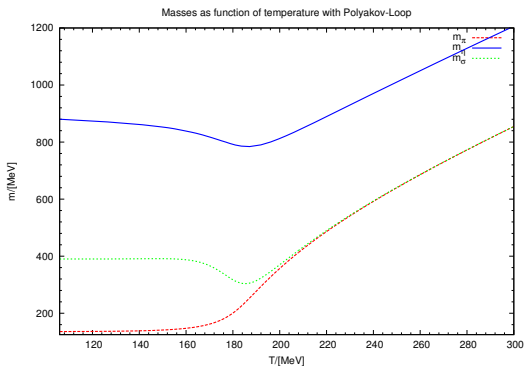
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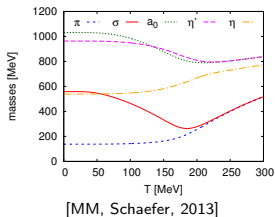
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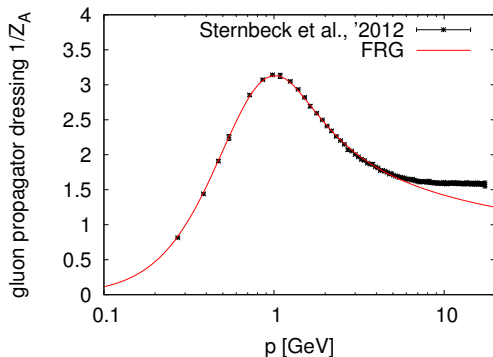
[Heller, MM, 2015]

- screening masses!
- QM-Model $N_f = 2 + 1$:



- chiral symmetry restoration:
 \Rightarrow drop in $m_{\eta'}$

Outlook: unquenched gluon propagator



[Cyrol, MM, Pawłowski, Strodthoff, in preparation]

- self-consistent solution of classical propagators and vertices
- massless quarks: quark mass gap seems unimportant

Summary and Outlook

(quenched) QCD with functional RG

- QCD phase diagram: need for quantitative control
- vacuum:
 - ▶ sole input $\alpha_S(\Lambda = \mathcal{O}(10) \text{ GeV})$ and $m_q(\Lambda = \mathcal{O}(10) \text{ GeV})$
 - ▶ good agreement with lattice simulations (sufficient?)
 - ▶ (non-perturbative) results:
 - ★ quark-propagator
 - ★ (quark-gluon vertex)
 - ★ (4-Fermi interaction channels)
 - ▶ phenomenology:
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- unquenching (partially done)
- finite temperature/chemical potential
- more checks on convergence of vertex expansion
- bound-state properties (form factor, PDA. . .)