Dense Matter in Supernovae and Compact Objects

C. Constantinou

IKP, FZ Jülich

19 January 2016, Hirschegg
Workshop on Gross Properties of Nuclei and Nuclear Excitations

Collaborators: M. Prakash, B. Muccioli & J.M. Lattimer

	Core-collapse supernovae	**Proto-neutron stars**	**Mergers of compact binary stars**
Baryon Density(n_0)	$10^{-8} - 10$	$10^{-8} - 10$	$10^{-8} - 10$
Temperature(MeV)	$0 - 30$	$0 - 50$	$0 - 100$
Entropy(k_B)	$0.5 - 10$	$0 - 10$	$0 - 100$
Proton Fraction	$0.35 - 0.45$	$0.01 - 0.3$	$0.01 - 0.6$

$n_0 \simeq 0.16$ fm^{-3} (equilibrium density of symmetric nuclear matter)

- Objective: Construction of EOS based on the best nuclear physics input for use in hydrodynamic simulations of core-collapse supernova explosions and binary mergers as well as the thermal evolution of proto-neutron stars.
- Status: Nearly there.

- Due to Akmal & Pandharipande (Phys. Rev. C. 56, 2261 (1997)):

$$v_{NN} = v_{18,ij} + V_{IX,ijk} + \delta v(\mathbf{P}_{ij})$$

where

$$v_{18,ij} = \sum_{p=1,18} v^p(r_{ij}) O^p_{ij} + v_{em} \qquad \text{(Argonne)}$$

$$V_{IX,ijk} = V^{2\pi}_{ijk} + V^R_{ijk} \qquad \text{(Urbana)}$$

$$\delta v(\mathbf{P}) = -\frac{P^2}{8m^2} u + \frac{1}{8m^2}[\mathbf{P.r}\ \mathbf{P}.\nabla, u] + \frac{1}{8m^2}[(\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_j) \times \mathbf{P}.\nabla, u]$$

(relativistic boost correction)

- Parametric fit by Akmal, Pandharipande, and Ravenhall (Phys. Rev. C. 58, 1804 (1998)).
- Hamiltonian density:

$$\begin{aligned}\mathcal{H}_{APR} &= \left[\frac{\hbar^2}{2m} + (p_3 + (1-x)p_5)ne^{-p_4 n}\right]\tau_n \\ &+ \left[\frac{\hbar^2}{2m} + (p_3 + (1-x)p_5)ne^{-p_4 n}\right]\tau_n \\ &+ g_1(n)(1-(1-2x)^2) + g_2(n)(1-2x)^2\end{aligned}$$

$\tau_{n(p)}$ - neutron(proton) kinetic energy density
p_i - fit parameters

- Skyrme-like ⇒ Landau effective mass:

$$m_i^* = \left(\frac{\partial \varepsilon_{k_i}}{\partial k_i}\bigg|_{k_{Fi}}\right)^{-1} k_{Fi} = \left[\frac{1}{m} + \frac{2}{\hbar^2}(p_3 + Y_i p_5)ne^{-p_4 n}\right]^{-1}$$

T=0 Thermodynamics

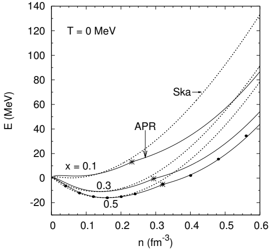
- Energy per particle $E(n,x) = \mathcal{H}/n \simeq E(n,1/2) + S_2(n)(1-2x)^2 + \ldots$
- Pressure $P(n,x) = n^2 \frac{\partial E}{\partial n} \simeq n^2 [\frac{K_0}{9n_0}(\frac{n}{n_0} - 1) + (1-2x)^2 \frac{L_0}{3n_0}] + \ldots$
- Chemical Potentials $\mu_i = \frac{\partial E}{\partial n_i}|_{n_j}$
- Incompressibility $K = 9n \frac{\partial^2 \mathcal{H}}{\partial n^2}$
- Symmetry energy $S_2 = \frac{1}{8} \left. \frac{\partial^2 E}{\partial x^2} \right|_{x=1/2}, \quad S_4 = \frac{1}{384} \left. \frac{\partial^4 E}{\partial x^4} \right|_{x=1/2}, \quad \ldots$
- Slope of S_2 $L = 3n \frac{dS_2}{dn}$
- Susceptibilities $\chi_{ij} = \left(\frac{\partial \mu_i}{\partial n_j} \right)^{-1}$
- Sound Speed $\left(\frac{c_s}{c} \right)^2 = \frac{dP}{d\epsilon}$

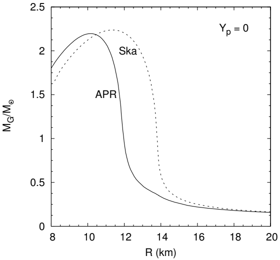
At saturation density, $n_0 = 0.16$ fm^{-3}:

- $K_0 = 266$ MeV (263)
- $E_0 = -16$ MeV (-16)
- $S_2 = 32.6$ MeV (32.9)
- $M_0^*/M = 0.7$ (0.61)
- $L_0 = 58.5$ MeV (74.6)

NS properties:

- $M_{max} = 2.19 M_\odot$ (2.22)
- $R_{max} = 10.2$ km (11.7)
- $R_{1.4} = 11.8$ km (13.8)
- $n_{dU} = 0.79/0.99$ fm^{-3} (0.38/0.54)





Finite T

- Single-particle energy spectrum:

$$\varepsilon_i = k_i^2 \frac{\partial \mathcal{H}}{\partial \tau_i} + \frac{\partial \mathcal{H}}{\partial n_i} \equiv \varepsilon_{k_i} + V_i$$

- $$n_i = \frac{1}{2\pi^2} \left(\frac{2m_i^* T}{\hbar^2} \right)^{3/2} F_{1/2i}$$

$$\tau_i = \frac{1}{2\pi^2} \left(\frac{2m_i^* T}{\hbar^2} \right)^{5/2} F_{3/2i}$$

$$F_{\alpha i} = \int_0^\infty \frac{x_i^\alpha}{e^{-\psi_i} e^{x_i} + 1} dx_i$$

$$x_i = \frac{\varepsilon_{ki}}{T}, \quad \psi_i = \frac{\mu_i - V_i}{T} = \frac{\nu_i}{T}$$

- Non-Relativistic Johns-Ellis-Lattimer's (JEL) Scheme (ApJ, 473 (1020),1996)

  Fermi-Dirac integrals as algebraic functions of the degeneracy parameter ψ only:

$$
\begin{aligned}
F_{3/2} &= \frac{3f(1+f)^{1/4-M}}{2\sqrt{2}} \sum_{m=0}^{M} p_m f^m \\
F_{\alpha-1} &= \frac{1}{\alpha}\frac{\partial F_\alpha}{\partial \psi} \\
\psi &= 2\left(1+\frac{f}{a}\right)^{1/2} + \ln\left[\frac{(1+f/a)^{1/2}-1}{(1+f/a)^{1/2}+1}\right]
\end{aligned}
$$

Finite T

Rest of state variables :

- Energy density $$\varepsilon = \frac{\hbar^2}{2m_n^*}\tau_n + \frac{\hbar^2}{2m_p^*}\tau_p + g_1(n)\left[1-(1-2x)^2\right] + g_2(n)(1-2x)^2$$
- Chemical potentials $$\mu_i = T\psi_i + V_i$$
- Entropy density $$s_i = \frac{1}{T}\left(\frac{5}{3}\frac{\hbar^2}{2m_i^*}\tau_+ n_i(V_i - \mu_i)\right)$$
- Pressure $$P = T(s_n + s_p) + \mu_n n_n + \mu_p n_p - \varepsilon$$
- Free energy density $$\mathcal{F} = \varepsilon - Ts$$

- To infer thermal contributions, eliminate terms that depend only on density :

$$X_{th} = X(n, x, T) - X(n, x, 0)$$

- Compare graphically the exact X_{th} with its degenerate and non-degenerate limits

Degenerate Limit

Landau Fermi Liquid Theory

- Interaction switched-on adiabatically
- Entropy density and number density maintain their free Fermi-gas forms:

$$s_i = \frac{1}{V}\sum_{k_i}[f_{k_i}\ln f_{k_i} + (1-f_{k_i})\ln(1-f_{k_i})]$$

$$n_i = \frac{1}{V}\sum_{k} f_{k_i}(T)$$

- $\int d\varepsilon \frac{\delta s}{\delta T} \quad \Rightarrow \quad s_i = 2a_i n_i T$

$$a_i = \frac{\pi^2}{2}\frac{m_i^*}{k_{Fi}^2}$$

level density parameter

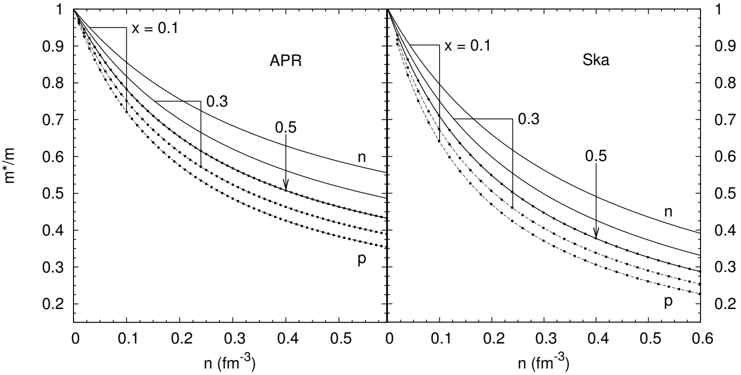
Other thermodynamics via Maxwell's relations:

- Energy density $\frac{d\varepsilon}{ds} = T$
  $\varepsilon(n,T) = \varepsilon(n,0) + \frac{T^2}{n}\sum_i a_i n_i$
- Pressure $\frac{dp}{dT} = -n^2 \frac{d(s/n)}{dn}$
  $p(n,T) = p(n,0) + \sum_i \left[a_i n_i - n\frac{d(a_i n_i)}{dn}\right] T^2$
- Chemical potentials $\frac{d\mu}{dT} = -\frac{ds}{dn}$
  $\mu(n,T) = \mu(n,0) - T^2\left[\frac{a_i}{3} + \sum_j \frac{n_j a_j}{m_j^*}\frac{dm_j^*}{dn_i}\right]$
- Free energy density $\frac{d\mathcal{F}}{dT} = -s$
  $\mathcal{F}(n,T) = \mathcal{F}(n,0) - \frac{T^2}{n}\sum_i a_i n_i$

Non-Degenerate Limit

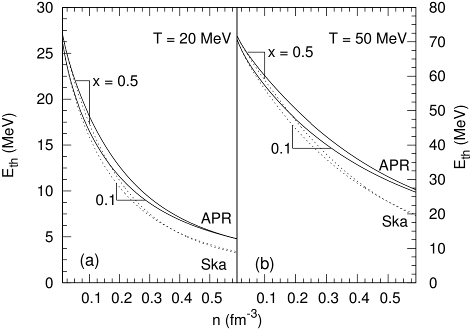
1. $F_\alpha \xrightarrow{z \ll 1} \Gamma(\alpha+1)\left(z - \frac{z^2}{2^{\alpha+1}} + \ldots\right)$
2. Invert $F_{1/2}$ to get z :
   $z = \frac{n\lambda^3}{\gamma} + \frac{1}{2^{3/2}}\left(\frac{n\lambda^3}{\gamma}\right)^2, \quad \lambda = \left(\frac{2\pi\hbar^2}{m^* T}\right)^{1/2}$ (quantum concentration)
3. Plug z in F_α's :

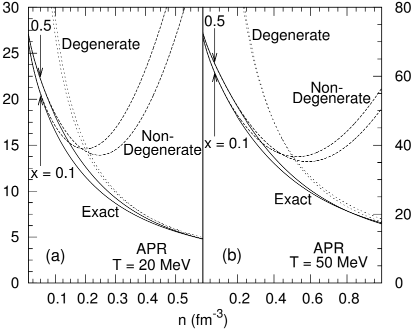
$$\begin{aligned} F_{3/2} &= \frac{3\pi^{1/2}}{4}\frac{n\lambda^3}{\gamma}\left[1 + \frac{1}{2^{5/2}}\frac{n\lambda^3}{\gamma}\right] \\ F_{1/2} &= \frac{\pi^{1/2}}{2}\frac{n\lambda^3}{\gamma} \\ F_{-1/2} &= \pi^{1/2}\frac{n\lambda^3}{\gamma}\left[1 - \frac{1}{2^{3/2}}\frac{n\lambda^3}{\gamma}\right] \end{aligned}$$



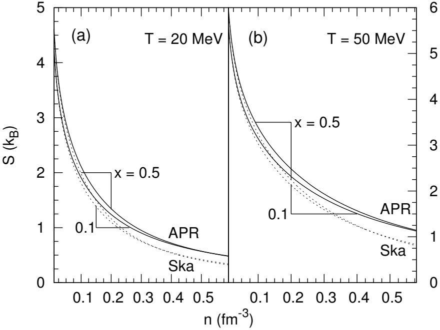
x = 0.1
APR
0.3
0.5
n
p
m*/m
n (fm-3)
x = 0.1
Ska
0.3
0.5
n
p
n (fm-3)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

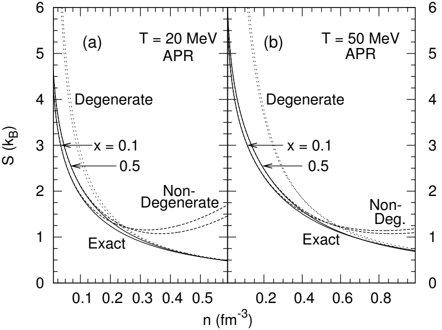
Thermal Energy



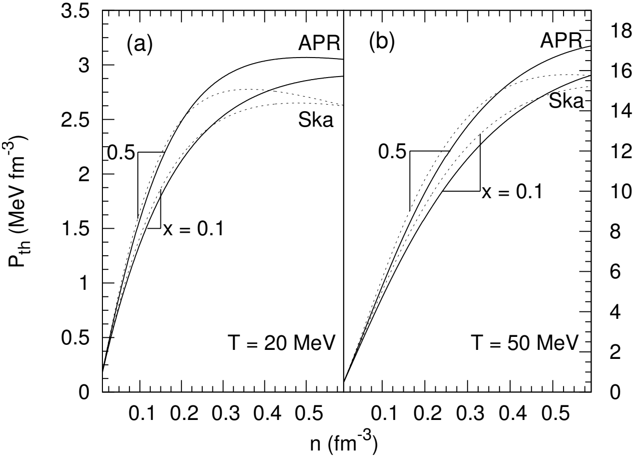


- Exact calculations agree with analytical limits where expected.
- Around nuclear saturation density exact results needed.
- Differences between models more pronounced at high densities.



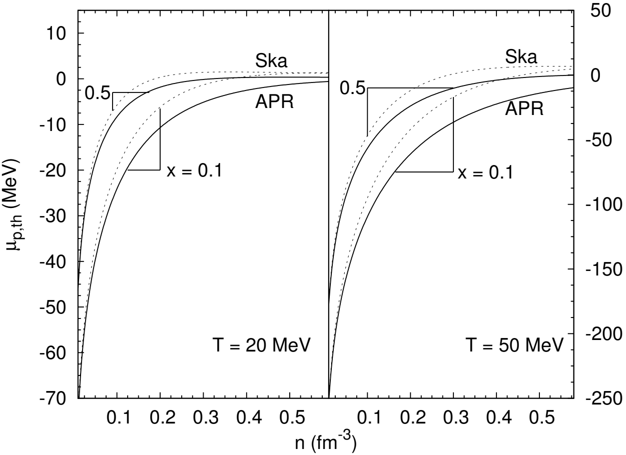


- Exact calculations agree with analytical limits where expected.
- Differences between models more pronounced at high densities.



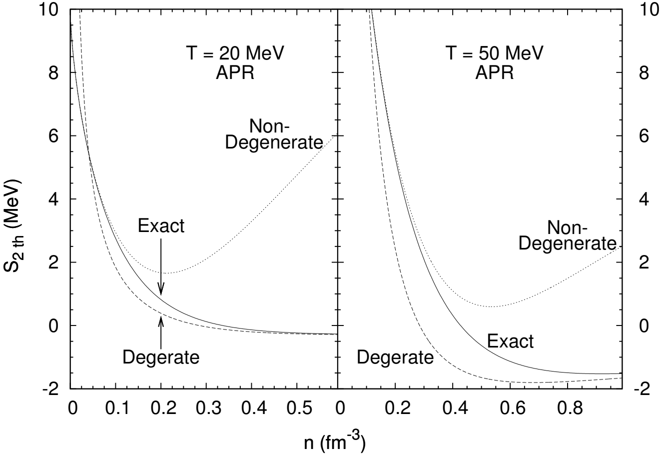
- Differences between models more pronounced at high densities.
- Note flattening at high densities.

Thermal Chemical Potential



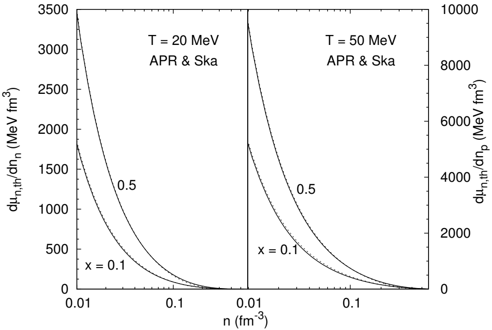
- Differences at intermediate densities.
- Not always < 0.

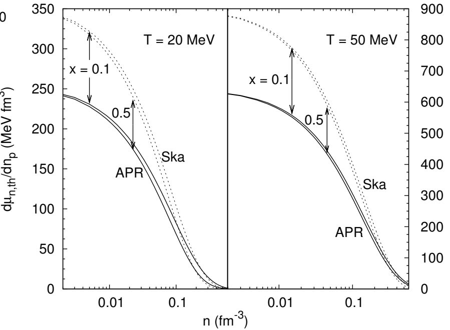
Thermal Symmetry Energy



- Significant thermal contributions at low densities.
- Negative at high densities.

Thermal Susceptibilities



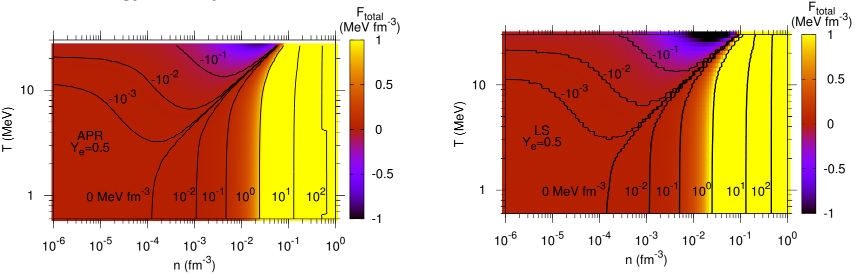


- In ND Limit, to leading order, $\chi_{ii} \sim \frac{T}{n_i}$.
  However, $\chi_{ij} \sim \frac{T}{m_i^*}\frac{\partial m_i^*}{\partial n_j}$.

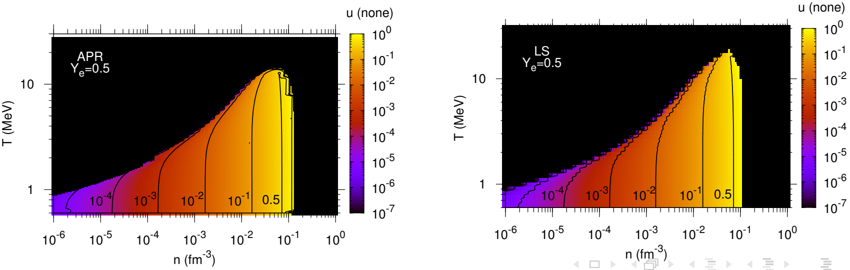
Lattimer-Swesty approach (Nucl. Phys. A **535**, 331 (1991)):

- Minimize $F = F_{bulk} + F_\alpha + F_N + F_e + F_\gamma$ with respect to nuclear radius, proton fraction, nucleon density, volume occupied by nuclei, and density of α-particles to obtain equations for chemical and mechanical equilibrium and optimization of nuclear size under baryon number conservation, $n_n + n_p = n_B = const.$ and charge conservation, $n_p = n_e$.
- Approximations:
  - α-particles represent light nuclei; treated as non-interacting Boltzmann gas.
  - Single representative species of heavy nucleus; described by the compressible liquid-drop model: $F_N = F_{bulk,in} + F_s + F_C + F_{tr}$. Ignore curvature and neutron skin for surface and screening due to bulk protons and α's for Coulomb.
  - Nucleons and α's have hard-sphere interactions with nuclei.
- Advantages:
  - Thermodynamic consistency.
  - Consistent treatment of matter inside and outside nuclei.
  - Separation of bulk and surface effects.
  - Computational simplicity.
- Possible issue: Collective motion and related thermal effects in nuclear dissociation regime.

Free energy density



Volume fraction occupied by heavy nuclei



MDI: Motivation

- Single particle potential, $U_{NR}(n,p) \propto p^2$ (contact interactions)
  $U_{RMFT}(n,\varepsilon) \propto \varepsilon$;
  inconsistent with optical model fits to nucleon-nucleus reaction data.
- Microscopic calculations (RBHF, variational calculations, etc.) show distinctly different behaviors in their momentum dependence, consistent with optical model fits.
- The above features were found necessary to account for heavy-ion data on transverse momentum and energy flow in conjunction with $K \sim 230$ MeV.

MDI: Hamiltonian Density

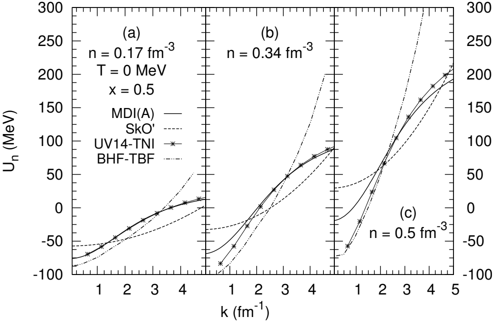
$$
\begin{aligned}
\mathcal{H} &= \frac{1}{2m}(\tau_n + \tau_p) \\
&+ \frac{A_1}{2n_0}(n_n + n_p)^2 + \frac{A_2}{2n_0}(n_n - n_p)^2 \\
&+ \frac{B}{\sigma + 1}\frac{(n_n + n_p)^{\sigma+1}}{n_0^\sigma}\left[1 - y\frac{(n_n - n_p)^2}{n_n + n_p)^2}\right] \\
&+ \frac{C_l}{n_0}\sum_i \int d^3p_i \ d^3p_i' \frac{f_i(\vec{r}_i, \vec{p}_i) f_i'(\vec{r}_i', \vec{p}_i')}{1 + \left(\frac{\vec{p}_i - \vec{p}_i'}{\Lambda}\right)^2} \\
&+ \frac{C_u}{n_0}\sum_i \int d^3p_i \ d^3p_j \frac{f_i(\vec{r}_i, \vec{p}_i) f_j(\vec{r}_j, \vec{p}_j)}{1 + \left(\frac{\vec{p}_i - \vec{p}_j}{\Lambda}\right)^2} \ ; \ i \neq j.
\end{aligned}
$$

G. M. Welke, M. Prakash, T. T. S. Kuo, S. Das Gupta & C. Gale, Phys. Rev. C **38** 1545 (1990)
C. B. Das, S. Das Gupta, C. Gale & Bao-An Li, Phys. Rev. C **67**, 034611 (2003)

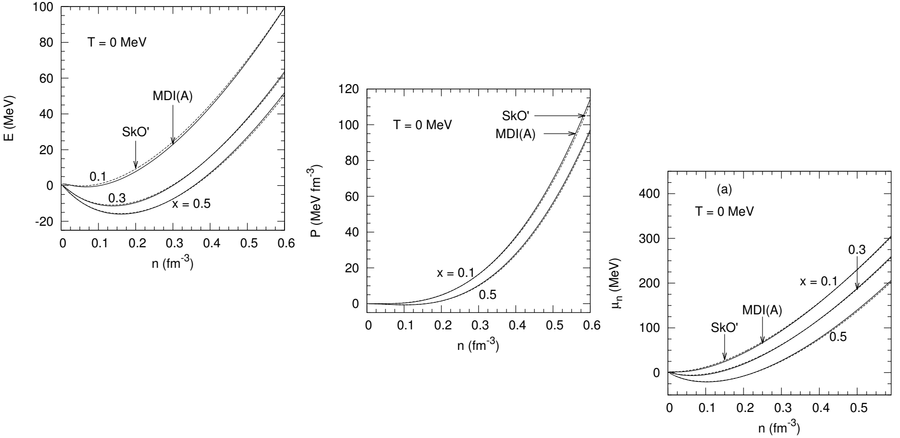
Single-Particle Potential

$$U_i(n_i, n_j, p_i) = U_i(n_i, n_j) + R_i(n_i, n_j, p_i)$$

$$R_i(n_i, n_j, p_i) = \frac{2C_l}{n_0} \frac{2}{(2\pi\hbar)^3} \int d^3p_i' \frac{f_{p_i'}}{1 + \left(\frac{\vec{p}_i - \vec{p}_i'}{\Lambda}\right)^2} + \frac{2C_u}{n_0} \frac{2}{(2\pi\hbar)^3} \int d^3p_j \frac{f_{p_j}}{1 + \left(\frac{\vec{p}_i - \vec{p}_j}{\Lambda}\right)^2}$$



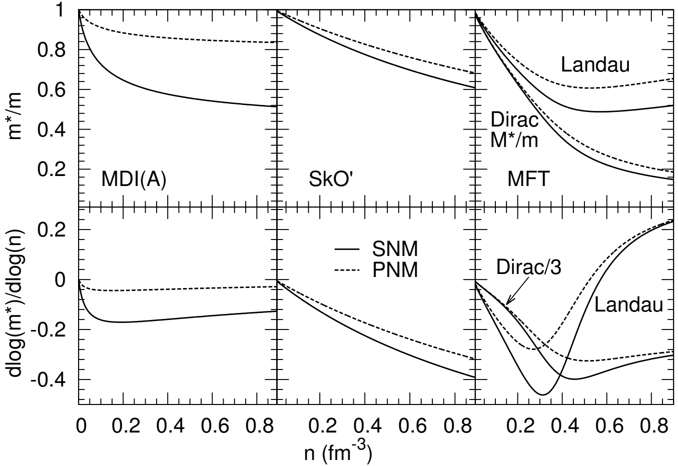
- BHF-TBF:
  W. Zuo *et al.*
  PRC **74**, 014317 (2006)
- UV14-TNI:
  R. B. Wiringa
  PRC **38**, 2967 (1988)



* P. G. Reinhard *et al.*, Phys. Rev. C **60**, 014316 (1999)

Property	MDI	SkO′	Experiment
n_0 (fm^{-3})	0.160	0.160	0.17±0.02
E_0 (MeV)	-16.00	-15.75	-16±1
K_0 (MeV)	232.0	222.3	240±20
m_0^*/m	0.67	0.90	0.8±0.1
S_v (MeV)	30.0	31.9	30-35
L_v (MeV)	65.0	68.9	40-70
K_v (MeV)	-72.0	-78.8	-100±200
$\mathrm{M}_{max}(M_\odot)$	1.9725	1.9600	2.01±0.04
R_{max}(km)	10.20	10.13	11.0± 1.0
n_c(fm^{-3})	1.2065	1.2233	
$R_{1.4}$(km)	12.21	12.17	11.5± 0.7
n_c(fm^{-3})	0.5126	0.5234	
n_{dU} (fm^{-3})	0.67(0.93)	0.59(0.92)	
M_{dU} ($M_\odot$)	1.7(1.9)	1.5(1.9)	

NS maximum masses in excess of 2 $M_\odot$ can also be obtained from a reparametrization of the MDI model, but at the expense of losing close similarity with the results of the SkO′ model.



- For n large, $\frac{dm^*}{dn} \simeq 0$
- $m^* = \frac{m}{1+\beta(x)n}$
- $m^* = E_F^* = (p_F^2 + M^{*2})^{1/2}$
- Minimum at n s.t. $\frac{p_F}{M^*} + \frac{dM^*}{dp_F} = 0$

Degenerate limit implications:

- $\eta = \frac{\mu - \epsilon(p=0)}{T} \gg 1 \Rightarrow$ Sommerfeld
- $\epsilon = \frac{p^2}{2m} + U(n,p;T) \to \frac{p^2}{2m} + U(n,p;0)$

For a general $U(n,p)$, define an effective mass function

$$\mathcal{M}(n,p) = m\left[1 + \frac{m}{p}\left.\frac{\partial U(n,p)}{\partial p}\right|_n\right]^{-1} \quad .$$

Relation to Landau m^* : $\mathcal{M}(n, p = p_F) = m^*$
Applying the Sommerfeld expansion to the integral of the entropy density gives

$$s = 2anT - \frac{16}{5\pi^2}a^3 nT^3(1 - L_F)$$

$$L_F = \frac{7}{12}p_F^2\frac{\mathcal{M}_F'^2}{m^{*2}} + \frac{7}{12}p_F^2\frac{\mathcal{M}_F''}{m^*} + \frac{3}{4}p_F\frac{\mathcal{M}_F'}{m^*} \quad ; \quad \mathcal{M}_F' \equiv \left.\frac{\partial \mathcal{M}(n,p)}{\partial p}\right|_{p=p_F}$$

Degenerate Limit Thermodynamics Beyond Leading Order

- Thermal Energy:

$$E_{th} = aT^2 + \frac{12}{5\pi^2} a^3 T^4 (1 - L_F)$$

- Thermal Pressure:

$$P_{th} = \frac{2}{3} anQT^2 - \frac{8}{5\pi^2} a^3 nQT^4 \left(1 - L_F + \frac{n}{2Q}\frac{dL_F}{dn}\right)$$

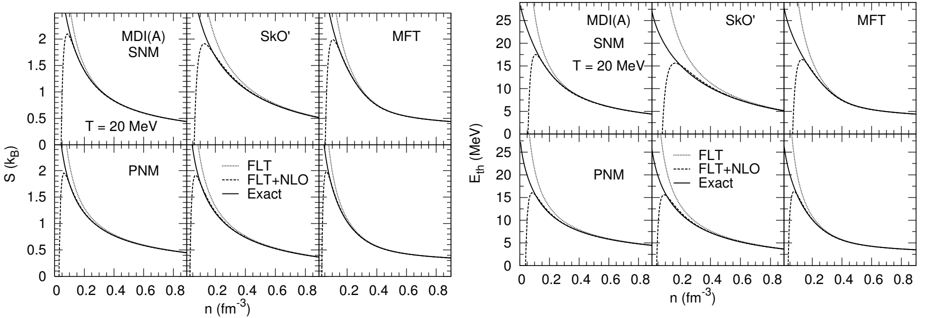
- Thermal Chemical Potential:

$$\mu_{th} = -a\left(1 - \frac{2Q}{3}\right) T^2 + \frac{4}{5\pi^2} a^3 T^4 \left[(1 - L_F)(1 - 2Q) - n\frac{dL_F}{dn}\right]$$

- Specific Heat at constant volume:

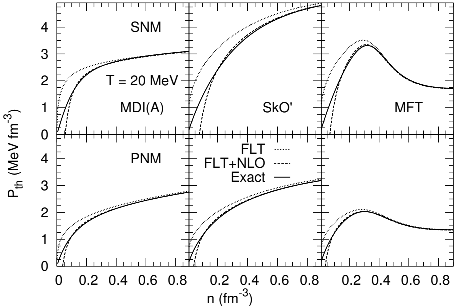
$$C_V = 2aT + \frac{48}{5\pi^2} a^3 T^3 (1 - L_F)$$

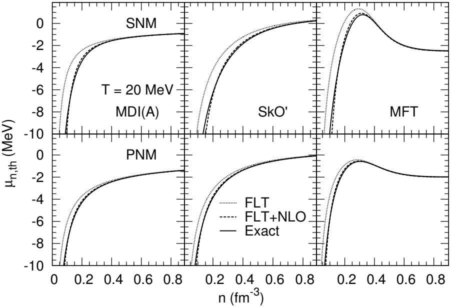
- Specific Heat at constant pressure: $C_P = C_V + \frac{T}{n^2} \frac{\left(\left.\frac{\partial P_{th}}{\partial T}\right|_n\right)^2}{\left.\frac{\partial P}{\partial n}\right|_T}$



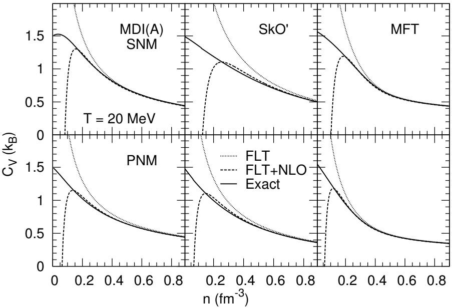
- The three models produce quantitatively similar results.
- Agreement with exact results extended down to $n \simeq 0.1$ fm^{-3}.
- Better agreement for PNM than for SNM.

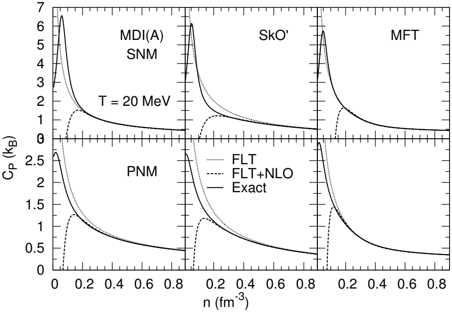
Results: P_{th} and μ_{th}





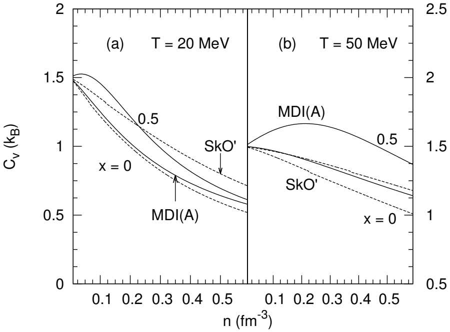
- Model dependence is evident- due to $\frac{dm^*}{dn}$.
- Agreement with exact results extended down to $n \simeq 0.1$ fm^{-3}.
- Better agreement for PNM than for SNM.

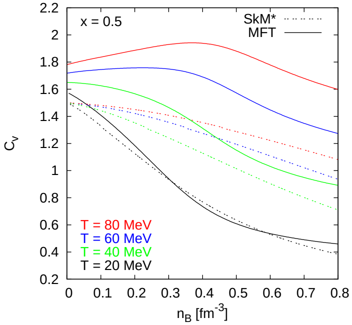




- The MDI and MFT C_V exceed the classical value of 1.5 in the nondegenerate limit. In this regime the T-dependence of the spectrum becomes important.
- The peaks in C_P are due to the proximity to the nuclear liquid-gas phase transition.

Results: Specific Heats

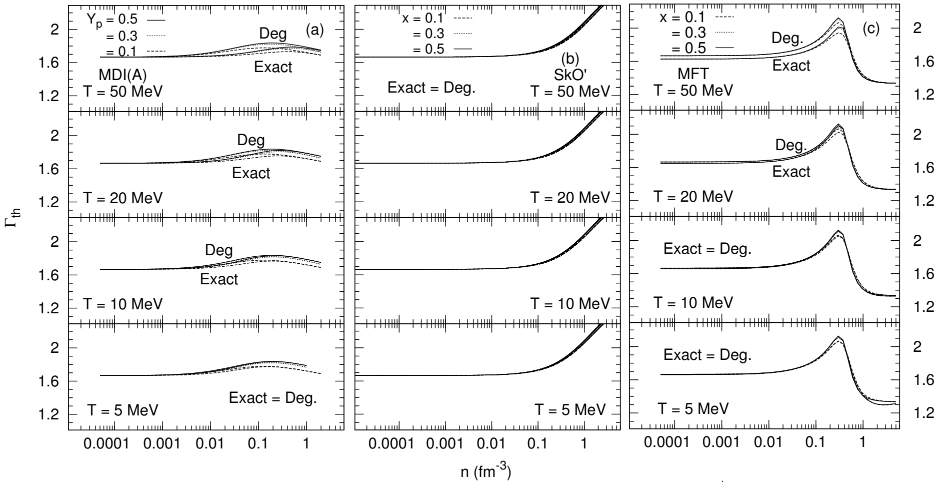




- The MDI and MFT C_V exceed the classical value of 1.5 in the nondegenerate limit. In this regime the T-dependence of the spectrum becomes important.

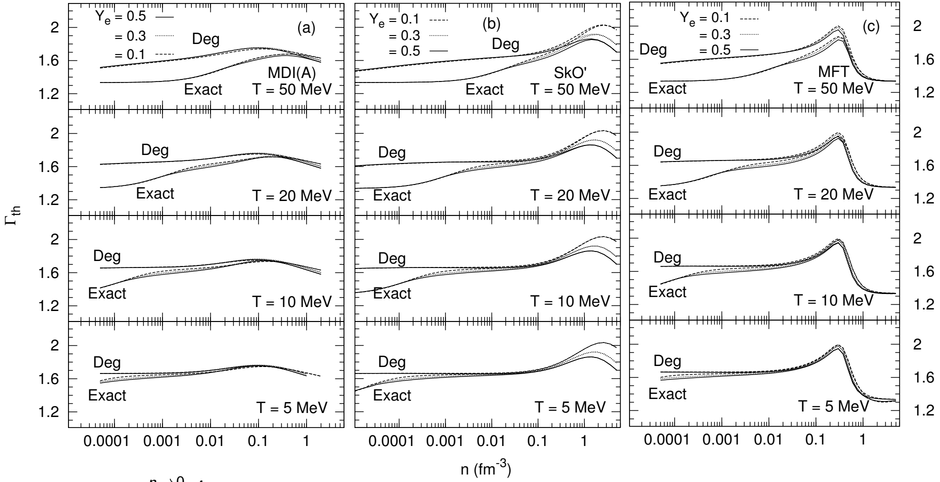
Γ_{th}-General Considerations

- The thermal properties of the EOS are relevant in a merger at the onset of matter transfer between the two compact objects which leads to shock heating. These can be characterized by the thermal index, Γ_{th}.
- $\Gamma_{th} = 1 + \frac{P_{th}}{\varepsilon_{th}}$
- Degenerate Limit
  - Nonrelativistic
    $\Gamma_{th} = 1 + \frac{2}{3}Q - \frac{4}{5\pi^2} a^2 n T^2 \frac{dL_F}{dn} \overset{n\to 0}{\longrightarrow} \frac{5}{3}$
    $Q = 1 + \frac{3}{2}\frac{n}{m^*}\frac{dm^*}{dn}$
  - Relativistic
    $\Gamma_{th} = 1 + \frac{Q}{3} + \frac{8}{15\pi^2} a^2 T^2 (1-Q) \left(L_F - \frac{5}{3}\frac{p_F^4}{E_F^{*4}} \right) \overset{n\to\infty}{\longrightarrow} \frac{4}{3}$
    $Q = 1 + \left(\frac{M^*}{E_F^*}\right)^2 \left(1 - \frac{3n}{M^*}\frac{dM^*}{dn}\right)$
- CC, B. Muccioli, M. Prakash & J.M. Lattimer, arXiv:1504.03982



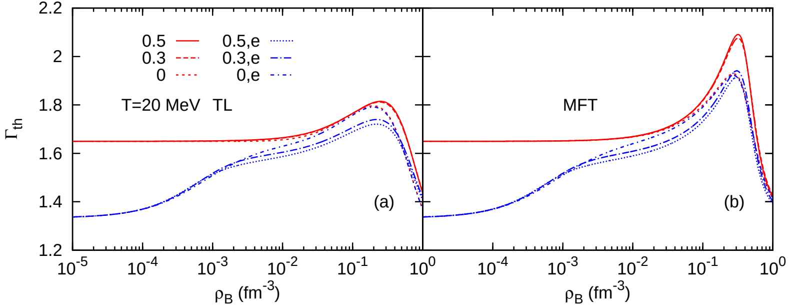
- Weak x and T dependence (none for Skyrme: $\Gamma_{th} = \frac{8}{3} - \frac{m^*}{m}$).
- Considerable n dependence in the homogeneous phase.

Γ_{th}-Leptons and photons included



- $\Gamma_{th} \xrightarrow{n \to 0} \frac{4}{3}$
- Maximum even for Skyrme

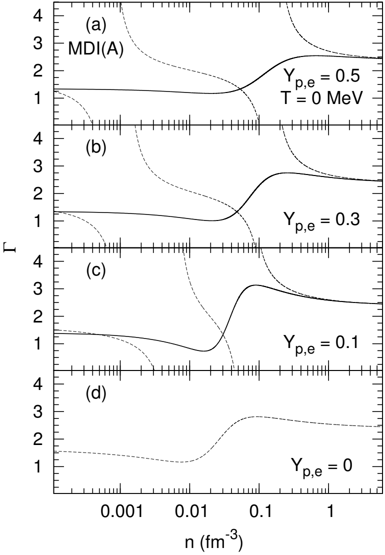
Γ_{th}-Beyond MFT



- X. Zhang & M. Prakash, work in progress
- Finite range effects via 2-loop calculation
- Lower peak relative to a similarly-calibrated MFT

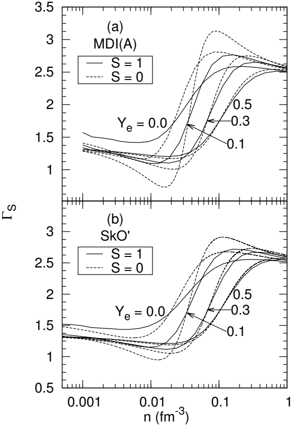
Γ_S-General Comments

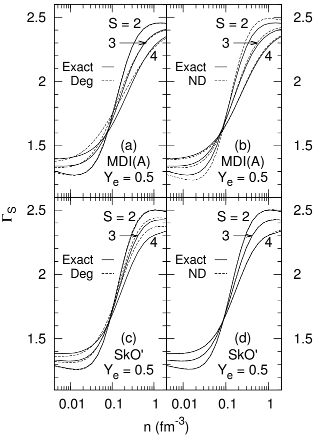
- Relevant during the early inspiralling phase of a merger when the two objects interact only gravitationally.
- $\Gamma_S(n,S) = \frac{\partial \ln P}{\partial \ln n}\Big|_S = \frac{n}{P}\ \frac{\partial P}{\partial n}\Big|_S$
- $\Gamma_S(n,T) = \frac{C_P}{C_V}\frac{n}{P}\ \frac{\partial P}{\partial n}\Big|_T$
- Relation to sound speed, c_s:
  $\left(\frac{c_s}{c}\right)^2 = \Gamma_S\ \frac{P}{h+mn}$
- Degenerate Limit:
  $\Gamma_S(n,S) = \frac{n}{P_0+\frac{nQS^2}{6a}}\left[\frac{dP_0}{dn} + \frac{QS^2}{6a}\left(1+\frac{2}{3}Q+\frac{Q}{n}\frac{dQ}{dn}\right)\right]$
  $\Gamma_S(n,T) = \frac{n}{P_0+P_{th}}\left[\frac{K}{9} + \frac{\partial P_{th}}{\partial n}\Big|_T + \frac{T}{n^2 C_V}\left(\frac{\partial P_{th}}{\partial T}\Big|_n\right)^2\right]$
  (using $C_P = C_V + \frac{T}{n^2}\frac{\left(\frac{\partial P}{\partial T}\big|_n\right)^2}{\frac{\partial P}{\partial n}\big|_T}$)



- Only nucleons ⇒ mechanical instability
- With leptons, nuclear matter is stable
- At $S = 0$, $P_l \sim n^{4/3}$
  $\Rightarrow \; \Gamma_{S=0} = \left(\frac{4}{3} + \frac{n}{P_l}\frac{dP_b}{dn}\right)\left(1 + \frac{P_b}{P_l}\right)^{-1}$
  For $n \simeq n_0$,
  $P_b(n,\alpha) \simeq \frac{n^2(n-n_0)}{9n_0^2}\left\{K_0 + \alpha^2\left[\frac{3n_0 L_v}{(n-n_0)} + K_v\right]\right\}$
- SNM ($\alpha = 0,\ P_b = 0$)
  $\Gamma_{S=0}(n = n_0) = \frac{4}{3} + \frac{K_0}{9n_0 P_l} \sim 2.1, \quad n^{(sp)} = \frac{2n_0}{3}$
- PNM ($\alpha = 1,\ P_l = 0$)
  $\Gamma_{S=0}(n = n_0) = 2 + \frac{K_0+K_v}{3L_v} \sim 2.8$
  $n^{(sp)} = n_0 \; \frac{3(K_0+K_v)}{2(K_0+K_v)-6L_v} < 0$

$\Gamma_{S\neq0}$





- Low n, $\Gamma_S > \Gamma_0$
  High n, $\Gamma_S < \Gamma_0$

- n_X such that
  $\frac{1}{P_0}\frac{dP_0}{dn} = \frac{1}{P_{th}}\left.\frac{\partial P_{th}}{\partial n}\right|_S$
  (indep. of S in Deg. Limit)

- Full APR (inclusive of subnuclear regime) coming soon.
  Improvements: Incorporation of virial EOS(done) and better treatment of nuclear properties.
- m^* is crucial in the determination of thermal effects.
- The EOS used in heavy-ion collisions can support a 2 $M_\odot$ neutron star.
- Both Γ_{th} and Γ_S depend weakly on T but their density dependence cannot be ignored.
- Finite-range effects lead to higher C_v and suppress the density dependence of Γ_{th}.
- Leptons stabilize the spinodally unstable nucleonic matter at subnuclear densities.