Coupling Relativistic Transport Theory to Decaying Color-electric Flux Tubes

Armando Puglisi

Physics and Astronomy Department, Laboratori Nazionali del Sud - INFN, Catania University

QCD matter: dense and hot

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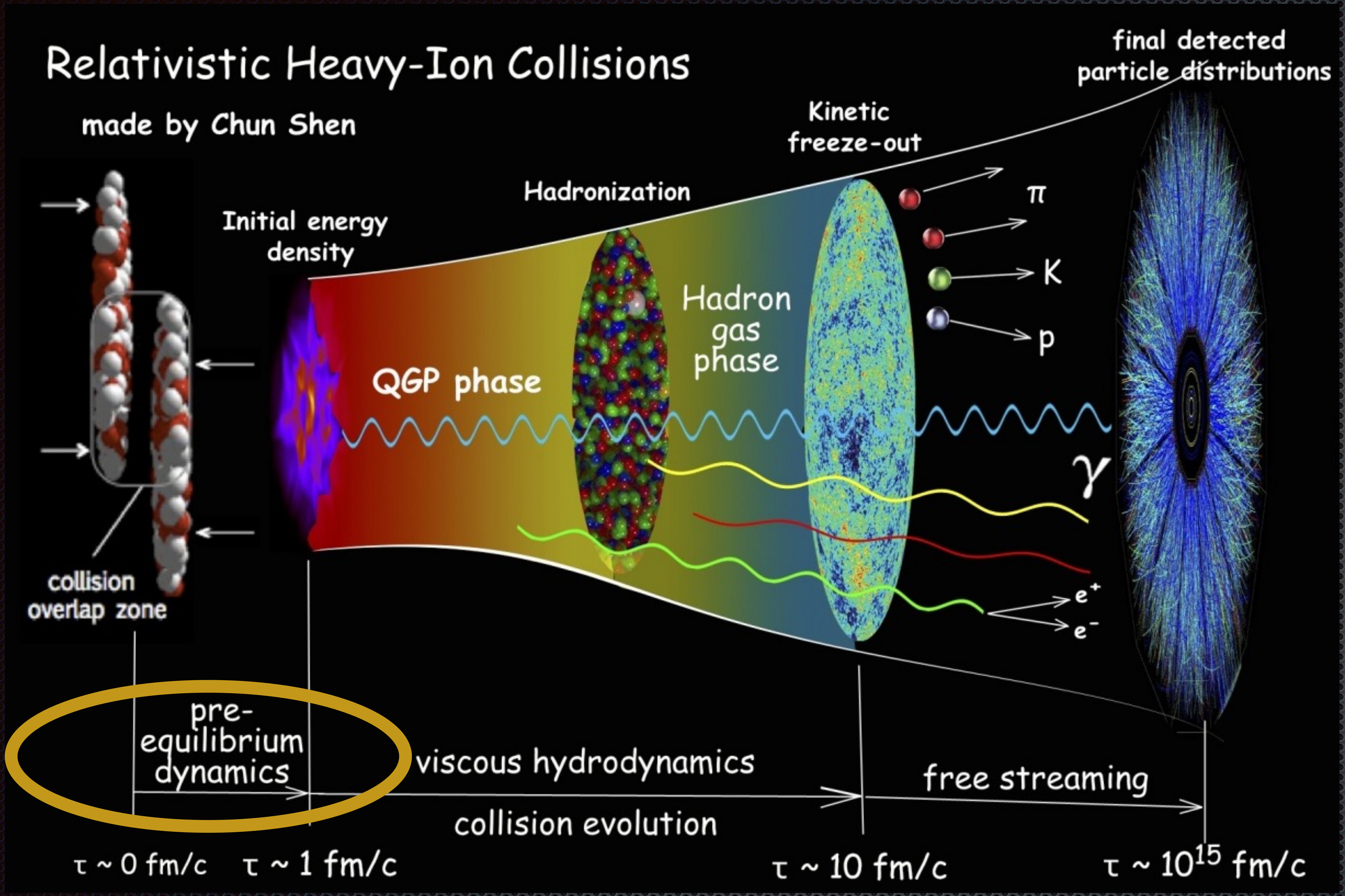
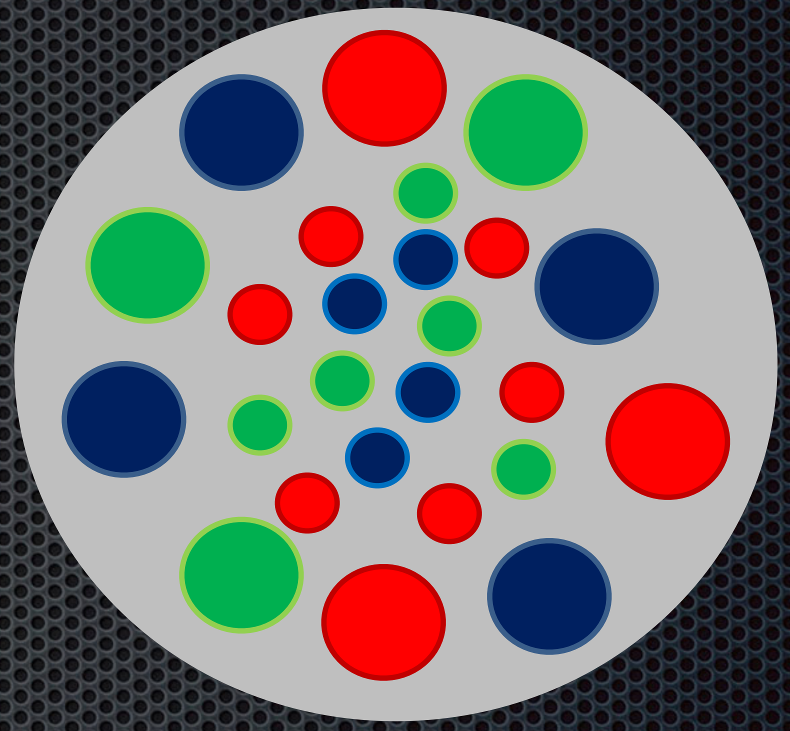
Collaborators

Vincenzo Greco
Lucia Oliva
Salvatore Plumari
Marco Ruggieri
Francesco Scardina

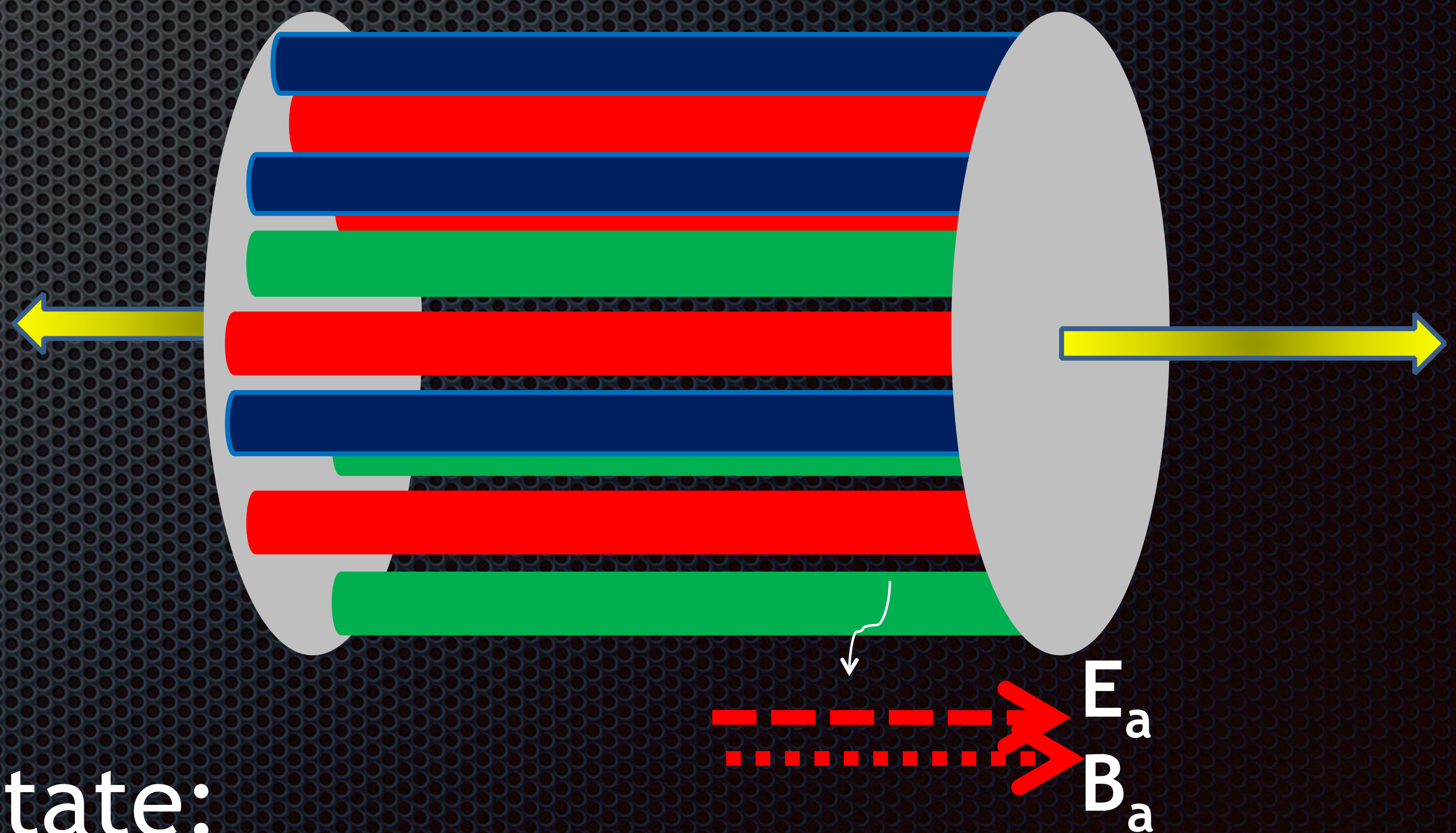
Evolution of a Relativistic Heavy Ion collision



Glasma: Transverse plane



Glasma: Longitudinal view



Initial out-of-equilibrium state:
Glasma, namely, a configuration of ***longitudinal color-electric and color-magnetic flux tubes.***

Lappi and McLerran, NPA 772 (2006)
Gelis and Venugopalan, Acta Phys. Polon. B37 (2006)
Gelis et al., NPA 828 (2009)
Fukushima and Gelis, NPA 874 (2012)
Iida et al., PRD 88 (2013)

Boltzmann Transport equation

$$(p_\mu \partial^\mu + gQF^{\mu\nu} p_\mu \partial^p_\nu) f = \mathcal{C}[f]$$

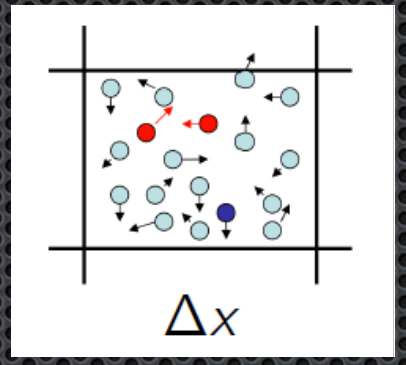
Free streaming — Field interaction — Collision integral

Field interaction: change of f due to interactions of the partonic plasma with a field (e.g. color-electric field).

Collision integral: change of f due to collision processes in the phase space volume centered at (x,p). **Responsible η/s**.

- TEST PARTICLES METHOD to map the phase space
- STOCHASTIC METHOD to simulate collisions

Z. Xu and C. Greiner, Phys.Rev. C71 (2005) 064901



$$\mathcal{C}_{22} = \frac{1}{2E_1} \int \frac{d^3p_2}{(2\pi)^3 2E_2} \frac{d^3p'_1}{(2\pi)^3 2E'_1} \frac{d^3p'_2}{(2\pi)^3 2E'_2} (f_{1'} f_{2'} - f_1 f_2) \times |\mathcal{M}|^2 \delta^4(p_1 + p_2 - p_{1'} p_{2'})$$

Test-particle method

$$f(x,p) = \sum_{i=1}^{N} \delta^4(x_i(t) - x)\delta^4(p_i(t) - p)$$

$$N = N_{real} \times N_{test} \quad , \quad \sigma \to \sigma / N_{test}$$

Stochastic method

$$P_{22} = \frac{\Delta N_{coll}^{2\to 2}}{\Delta N_1 \Delta N_2} = v_{rel} \frac{\sigma_{22}}{N_{test}} \frac{\Delta t}{\Delta^3 x}$$

Boltzmann Transport equation

$$(p_\mu \partial^\mu + gQF^{\mu\nu} p_\mu \partial^p_\nu) f = \mathcal{C}[f]$$

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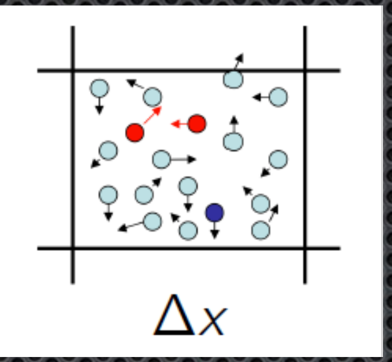

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Advantages of transport for RHICs

➢ Starting from 1-body distribution function f(x,p) and not from $T_{\mu\nu}$:

- Implement non-equilibrium initial conditions
- Include off-equilibrium at high and intermediate p_T:
- freeze-out self-consistently related with η/s(T)

➢ Good tool to compute transport coefficients

➢ one single theoretical approach to follow the entire dynamical evolution of system produced in uRHICs

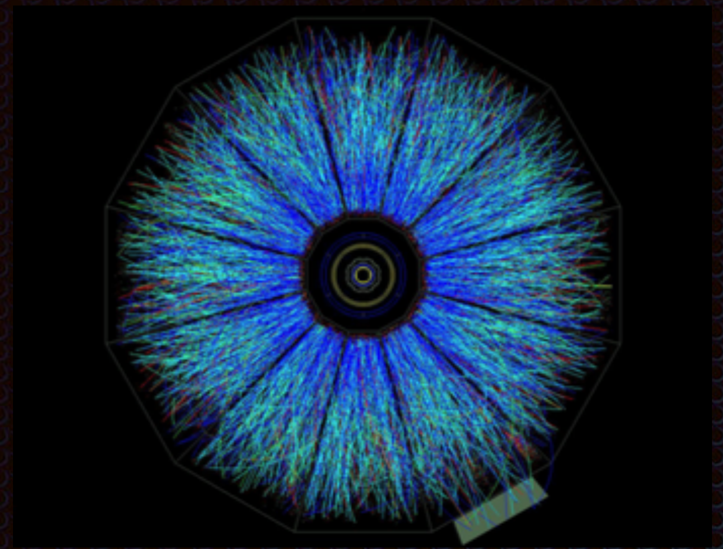
Boltzmann Transport equation at fixed η/s

What is η/s ?

Why η/s ?

What is the viscosity of QGP?

A little digression about shear viscosity

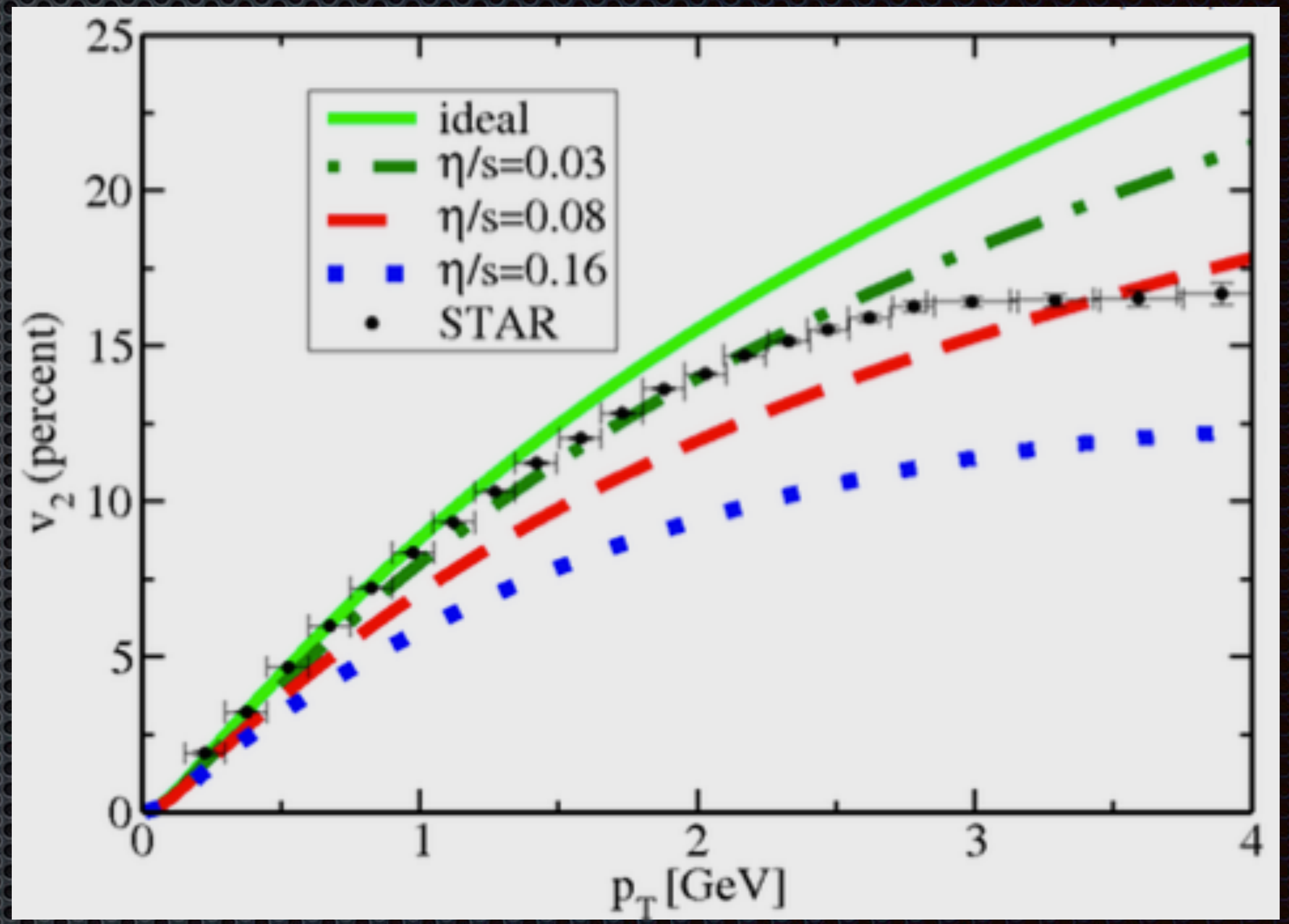


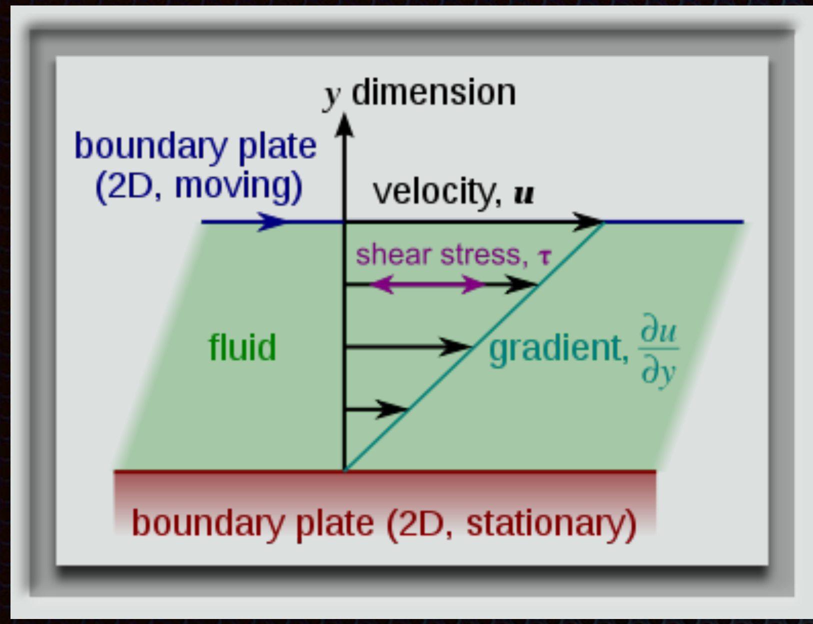
First Au-Au @ 100 GeV, RHIC, STAR

$$\frac{d^3N}{p_T dp_T dy d\phi} = \frac{1}{2\pi p_T}\frac{d^2N}{dp_T dy}\left(1 + 2\sum_{n=1}^{\infty} v_n(p_T, y)\cos[n(\phi - \psi_R)]\right)$$

elliptic flow

$$v_2(p_T, b) = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle$$





shear stress

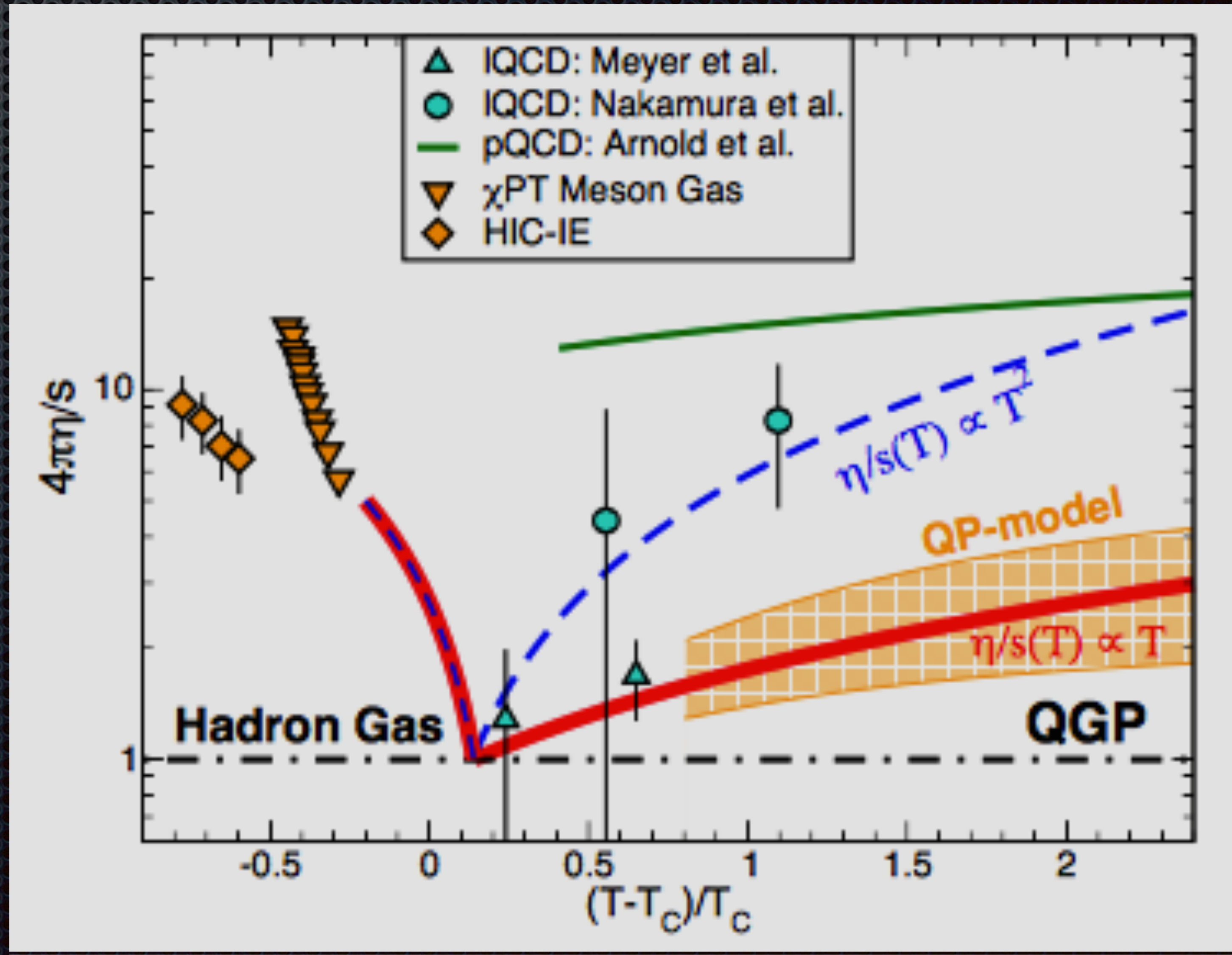
$$\tau = \frac{F_{xy}}{A_{yz}} = -\eta\frac{\partial u_x}{\partial y}$$

fluid	$P\,[Pa]$	$T\,[K]$	$\eta\,[Pa \cdot s]$	$\eta/s\,[\hbar/k_B]$
H_2O	$0.1 \cdot 10^6$	370	$2.9 \cdot 10^{-4}$	8.2
^4He	$0.1 \cdot 10^6$	2.0	$1.2 \cdot 10^{-6}$	1.9
H_2O	$22.6 \cdot 10^6$	650	$6.0 \cdot 10^{-5}$	2.0
^4He	$0.22 \cdot 10^6$	5.1	$1.7 \cdot 10^{-6}$	0.7
QGP	$88 \cdot 10^{33}$	$2 \cdot 10^{12}$	$\leq 5 \cdot 10^{11}$	≤ 0.4

T.Schäfer, D. Teaney, Rep. Prog. Phys 72 (2009) 126001

pitch drop experiment

- started in 1927
- 9th drop in April 2014
- $\eta_{pitch} = 2 \times 10^{11} \eta_{water}$
- $\eta_{pitch} \ll \eta_{QGP}$



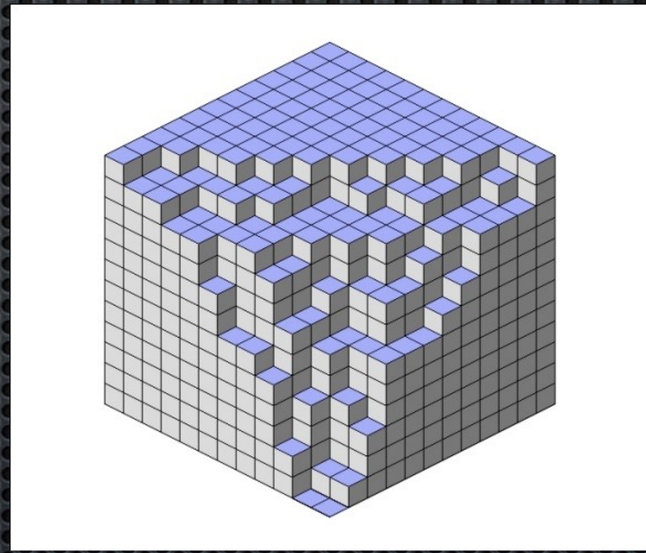
Boltzmann Transport equation at fixed η/s

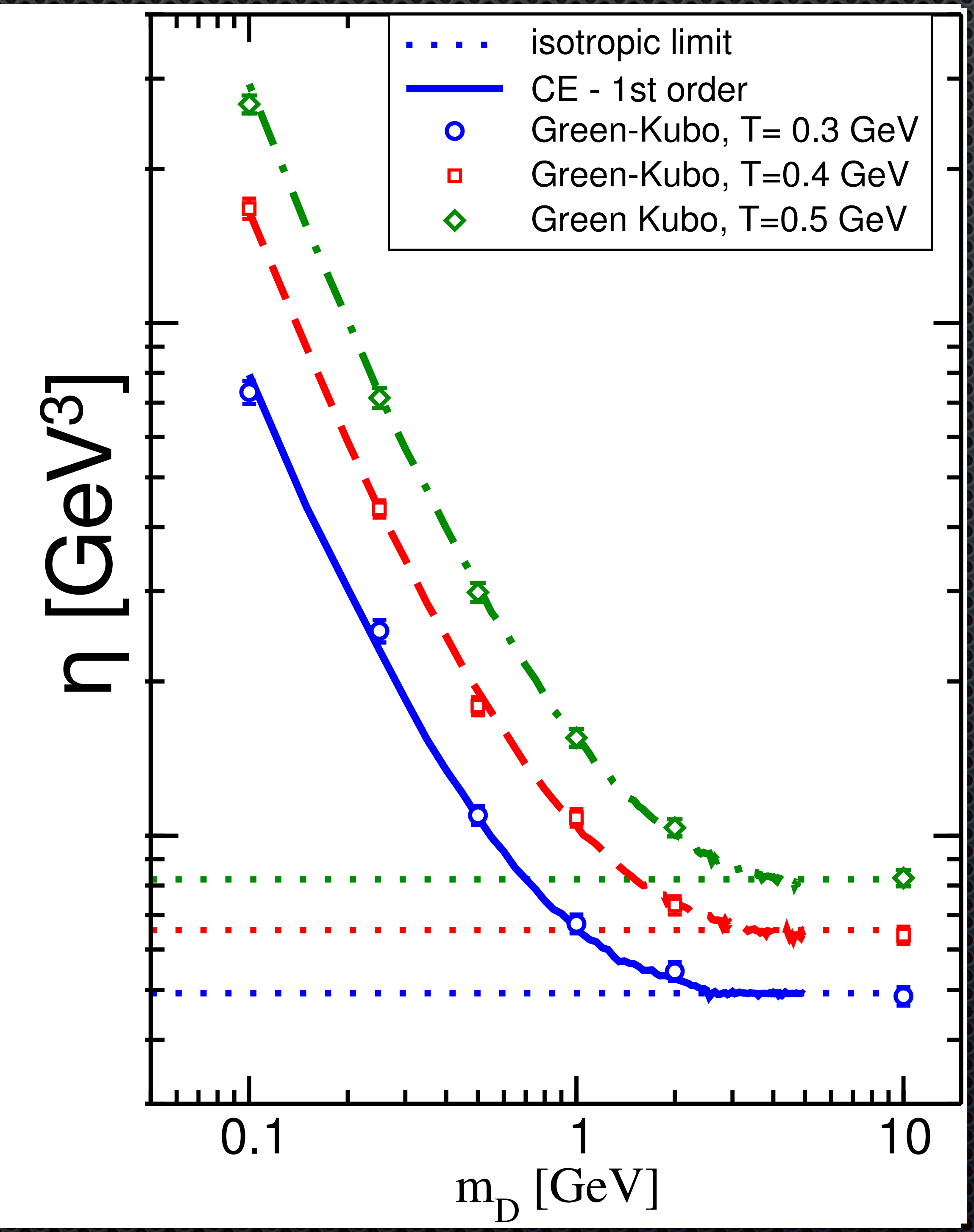
Total Cross section is computed in each configuration space cell according to Chapman-Enskog equation to give the wished value of η/s.

(.) Collision integral is gauged in each cell to assure that the fluid dissipates according to the desired value of η/s.

(.) Microscopic details are not important: the specific microscopic process producing η/s is not relevant, only macroscopic quantities are, in analogy with hydrodynamics.

$$\frac{\eta}{s} = \frac{\langle p \rangle}{g(m_D)} \frac{1}{\rho\sigma}$$





Green-Kubo correlator

$$\eta = \frac{V}{T}\langle \pi^{xy}(0)^2 \rangle$$

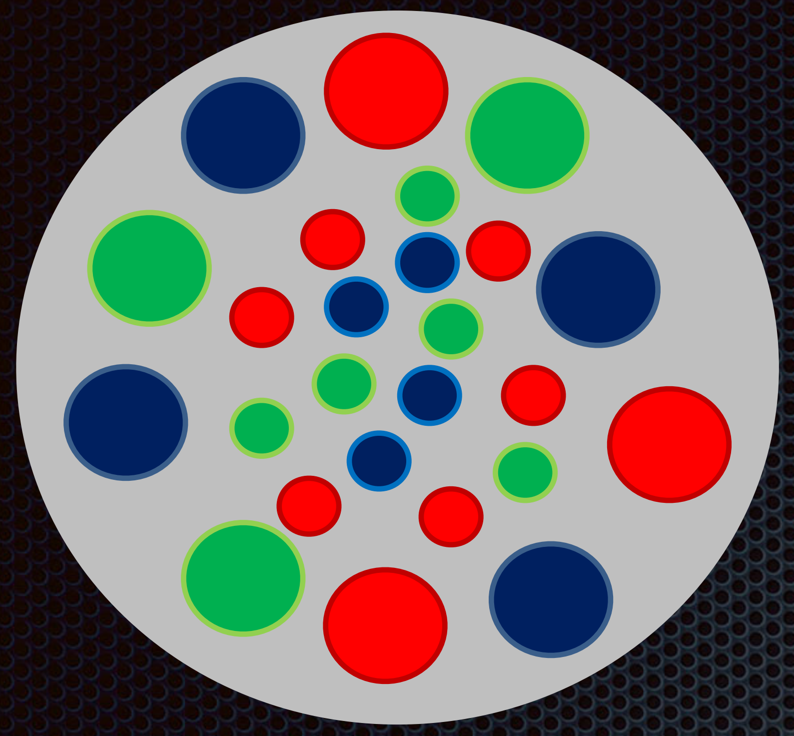
<…> correlation function of energy-momentum tensor π

C. Wesp et al, Phys.Rev. C84 (2011) 054911

A. Puglisi et al., Phys.Rev. C86 (2012) 054902

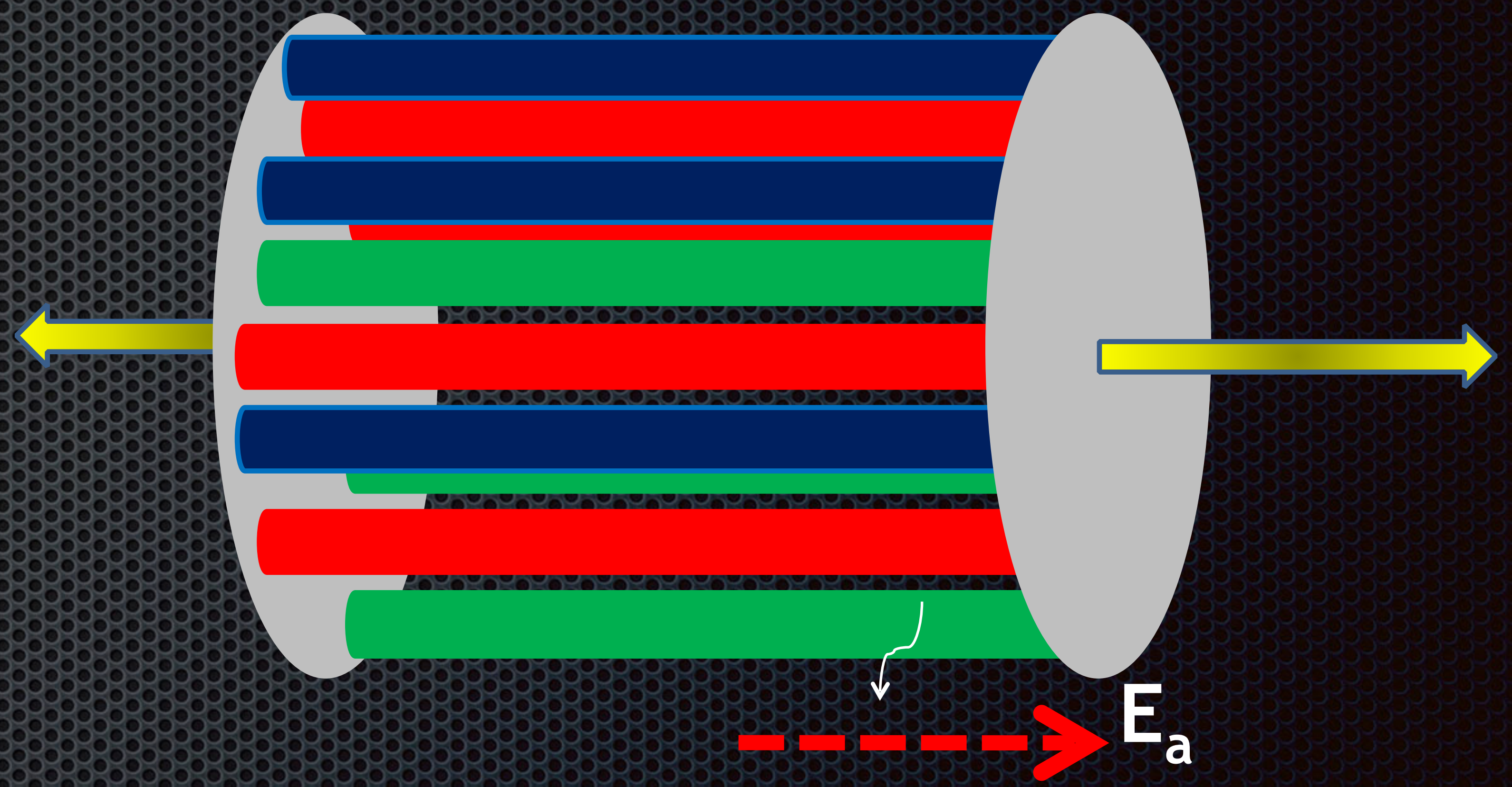
From Glasma to QGP: Schwinger effect

Glasma: Transverse plane



How does this configuration of classical color fields become a thermalized and isotropic QGP?

Glasma: Longitudinal view



Schwinger effect + collisions

Based on the assumption that classical color fields decay to a QGP via vacuum tunneling, namely via the Schwinger effect.

Schwinger Mechanism

Classical fields decay to particles pairs via tunneling due to vacuum instability

Vacuum Decay Probability

per unit of spacetime to create an electron-positron pair from the vacuum

$$\mathcal{W}(x) = \frac{e^2 E^2}{4\pi^3} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp\left(-\frac{n\pi m^2}{|eE|}\right)$$

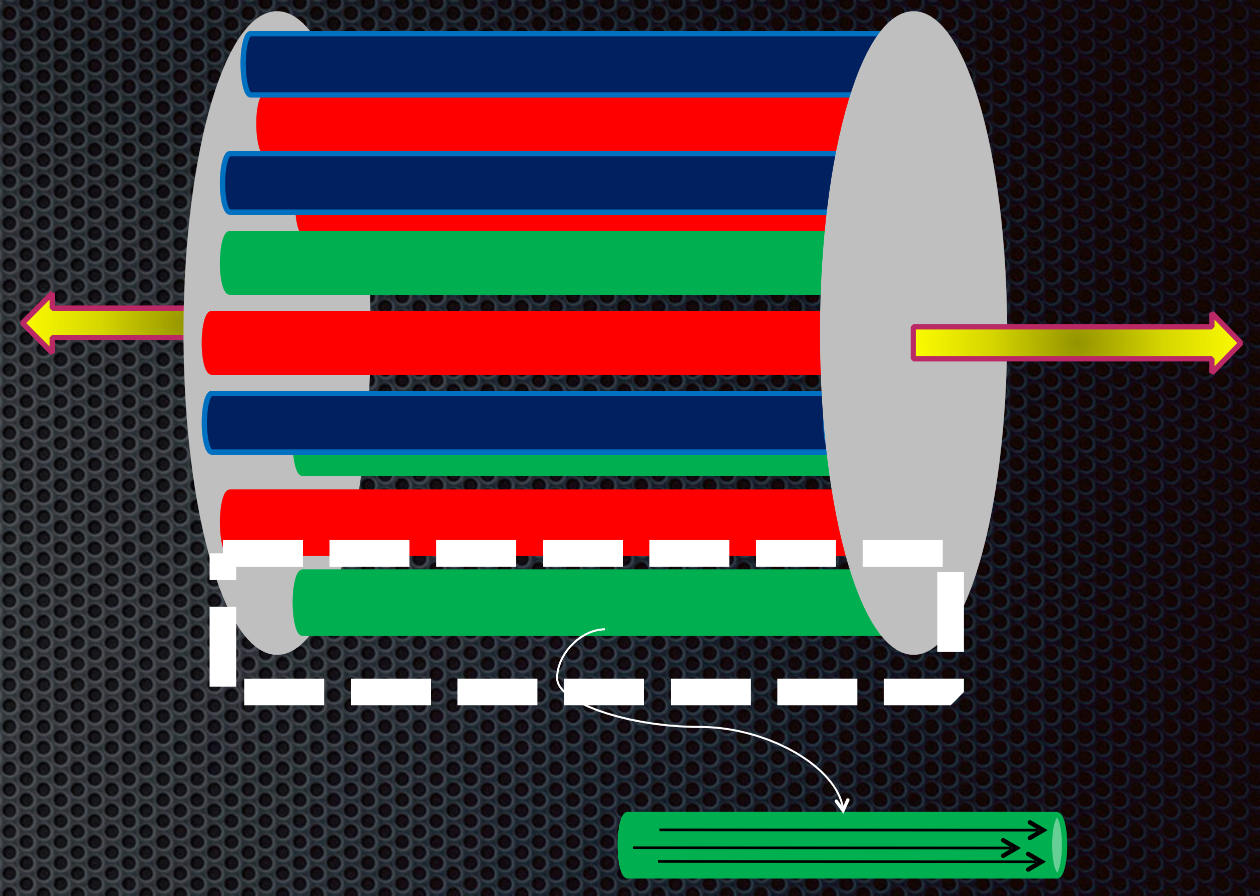
Once pairs pop up from the vacuum, charged particles propagate in real time producing electric currents:

$$J = \sigma E$$

Euler-Heisenberg (1936)
Schwinger, PR 82, 664 (1951)

Abelian Flux Tube Model

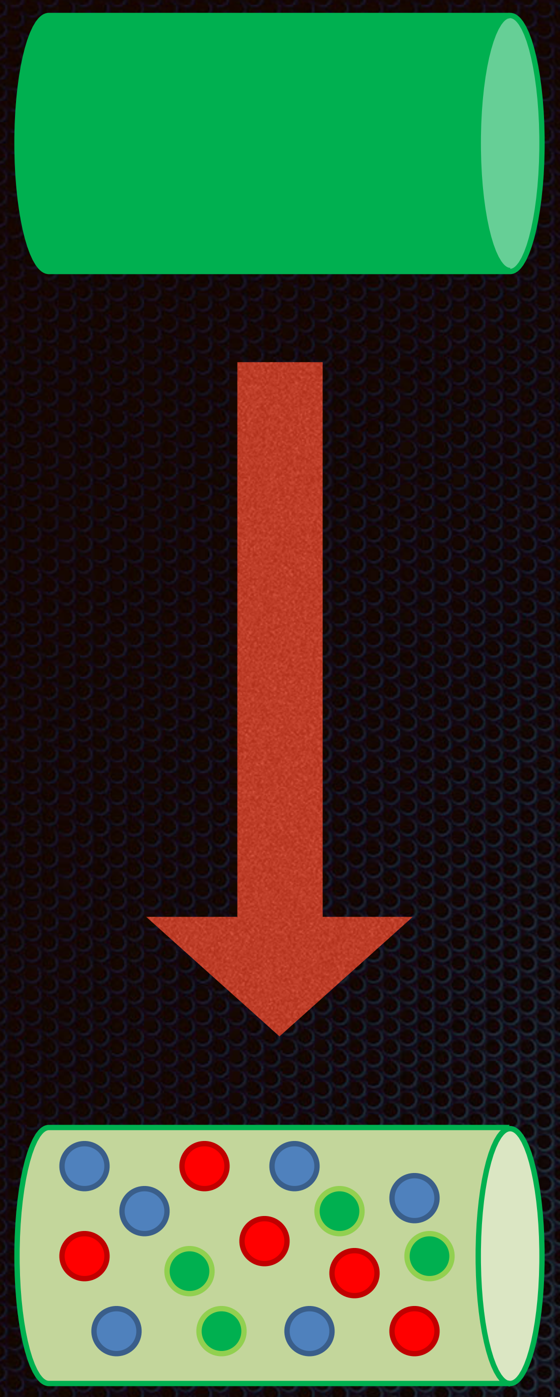
Schwinger effect in QCD



Focus on a single flux tube

Schwinger Mechanism

Classical fields decay to particles pairs via tunneling due to vacuum instability



#pairs

$$\frac{dN_{jc}}{d\Gamma} \equiv p_0 \frac{dN_{jc}}{d^4x d^2p_T dp_z} = \mathcal{R}_{jc}(p_T)\delta(p_z)p_0$$

$$\mathcal{R}_{jc}(p_T) = \frac{\mathcal{E}_{jc}}{4\pi^3}\left|\ln\left(1 \pm e^{-\pi p_T^2/\mathcal{E}_{jc}}\right)\right|$$

$$\mathcal{E}_{jc} = (g|Q_{jc}E| - \sigma_j)\,\theta\,(g|Q_{jc}E| - \sigma_j)$$

effective force

string tension

(.) Energy per unit length has to be larger than the QCD string tension
(.) Effective electric field is smaller due to string tension effect

Longitudinal Chromo-Electric fields decay to gluon pairs and quark-antiquark pairs

Casher, Neuberger and Nussinov, PRD 20, 179 (1979)
Glendenning and Matsui, PRD 28, 2890 (1983)

Abelian Flux Tube Model

- neglecting chromo-magnetic field
- abelian dynamics for the chromo-electric field
- longitudinal initial field
- Schwinger effect

Boltzmann Transport Equation

$$(p_\mu \partial^\mu + gQ_{jc}F^{\mu\nu}p_\mu \partial^p_\nu) f_{jc} = p_0 \frac{\partial}{\partial t}\frac{dN_{jc}}{d^3x d^3p} + \mathcal{C}[f]$$

Source term: change of f due to particle creation in the volume centered at (x,p).

WE SOLVE SELF-CONSISTENTLY BOLTZMANN AND MAXWELL EQUATIONS

Ryblewski and Florkowski, PRD 88 (2013)
M. Ruggieri, A.P., et al., PRC 92, 064904 (2015)

$$\frac{dE}{d\tau} = \rho \sinh\eta - j\cosh\eta$$

Field interaction + Source term
Link between parton distribution function and classical color fields evolution

$$j_M = \sum_{species} g \int \frac{d^3\mathbf{p}}{|\mathbf{p}|} p_z f(|\mathbf{p}|, t)$$

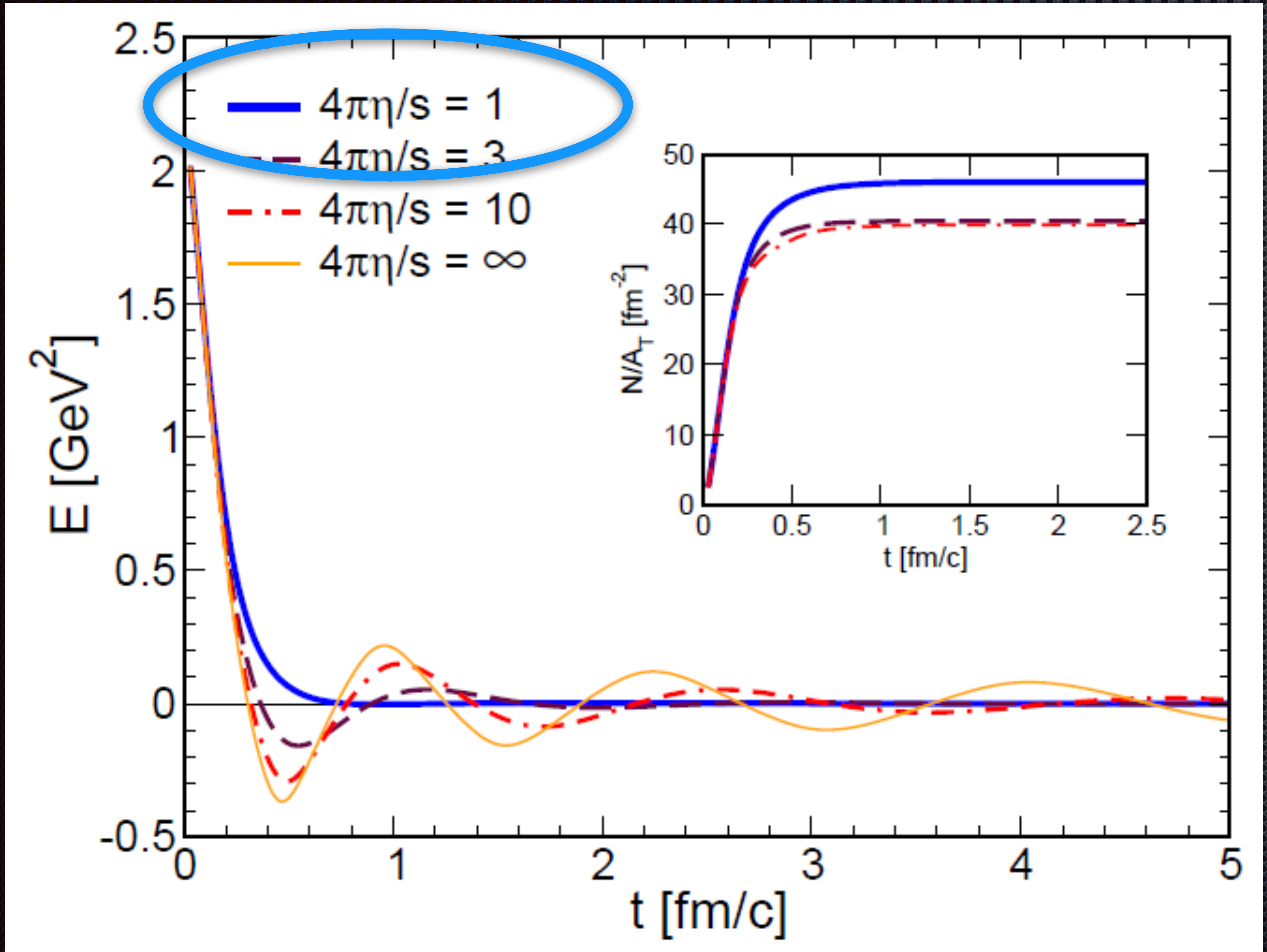
conductive current

$$j_D \equiv \frac{\partial P}{\partial t} = \int d^3p g \frac{2E_T}{gE} \times \frac{dN}{d^4x d^3p}$$

displacement current

Field decay in 1+1D expansion

M. Ruggieri, A.P., et al., PRC 92, 064904 (2015)



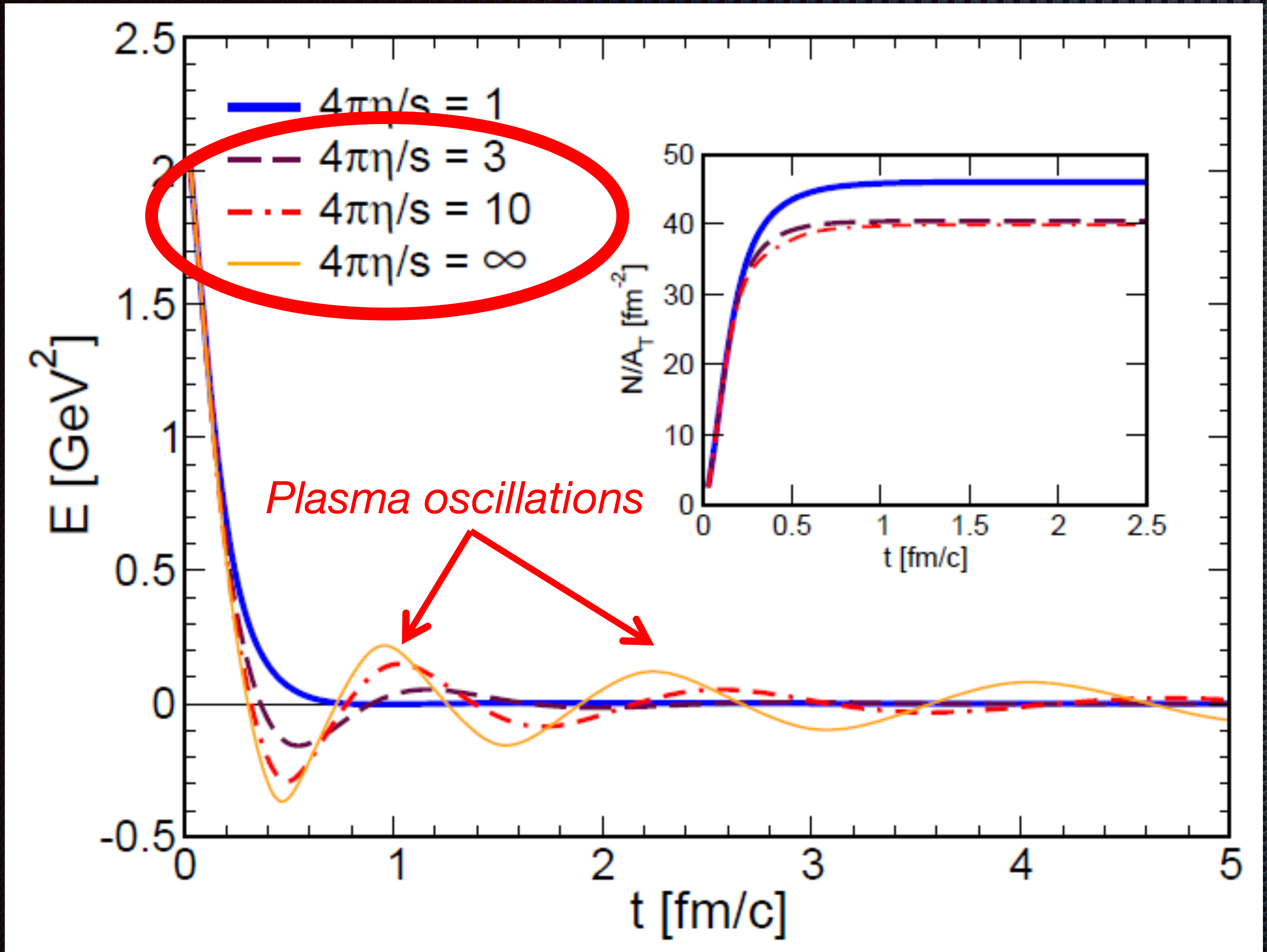
$$\frac{dE}{d\tau} = \rho \sinh\eta - j \cosh\eta$$

Small η/s implies large scattering rate, meaning efficient randomization of particles momenta
in each cell, thus damping ordered particle flow along the field direction (electric current).

Small η/s
(.) Field decays quickly (power law)

Field decay in 1+1D expansion

M. Ruggieri, A.P., et al., PRC 92, 064904 (2015)



$$\frac{dE}{d\tau} = \rho \sinh \eta - j \cosh \eta$$

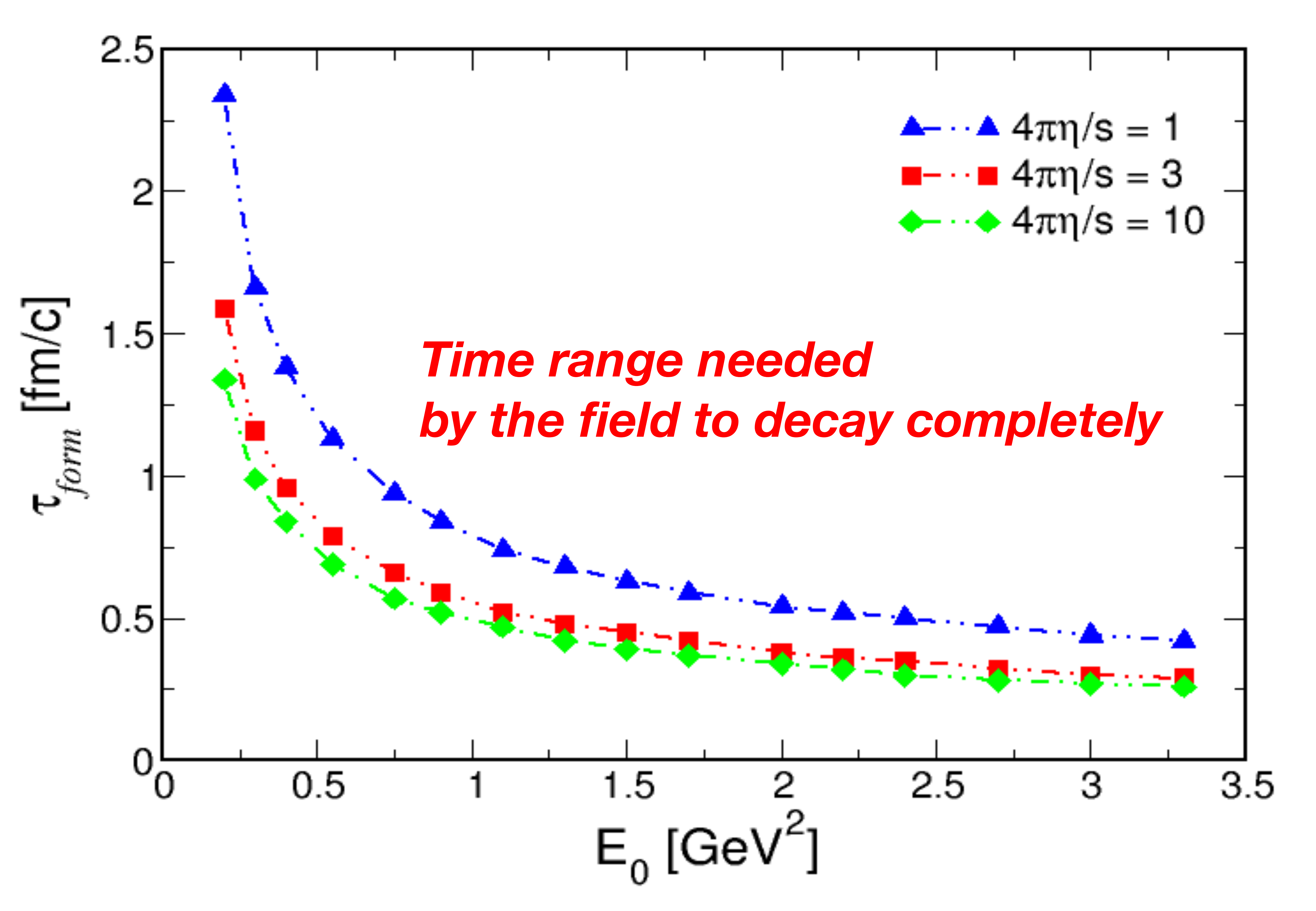
Smaller coupling (i.e. smaller isotropization efficiency) favors development of conductive electric currents: the net effect is a continuous energy exchange between particles and field.

Large η/s:
Initial times dynamics faster, due to electric current:

$$\frac{d}{dt}\left(\frac{\boldsymbol{E}^2}{2}\right) = -\boldsymbol{j} \cdot \boldsymbol{E}$$

Particles formation for 1+1D expansion

Proper time for conversion to particles



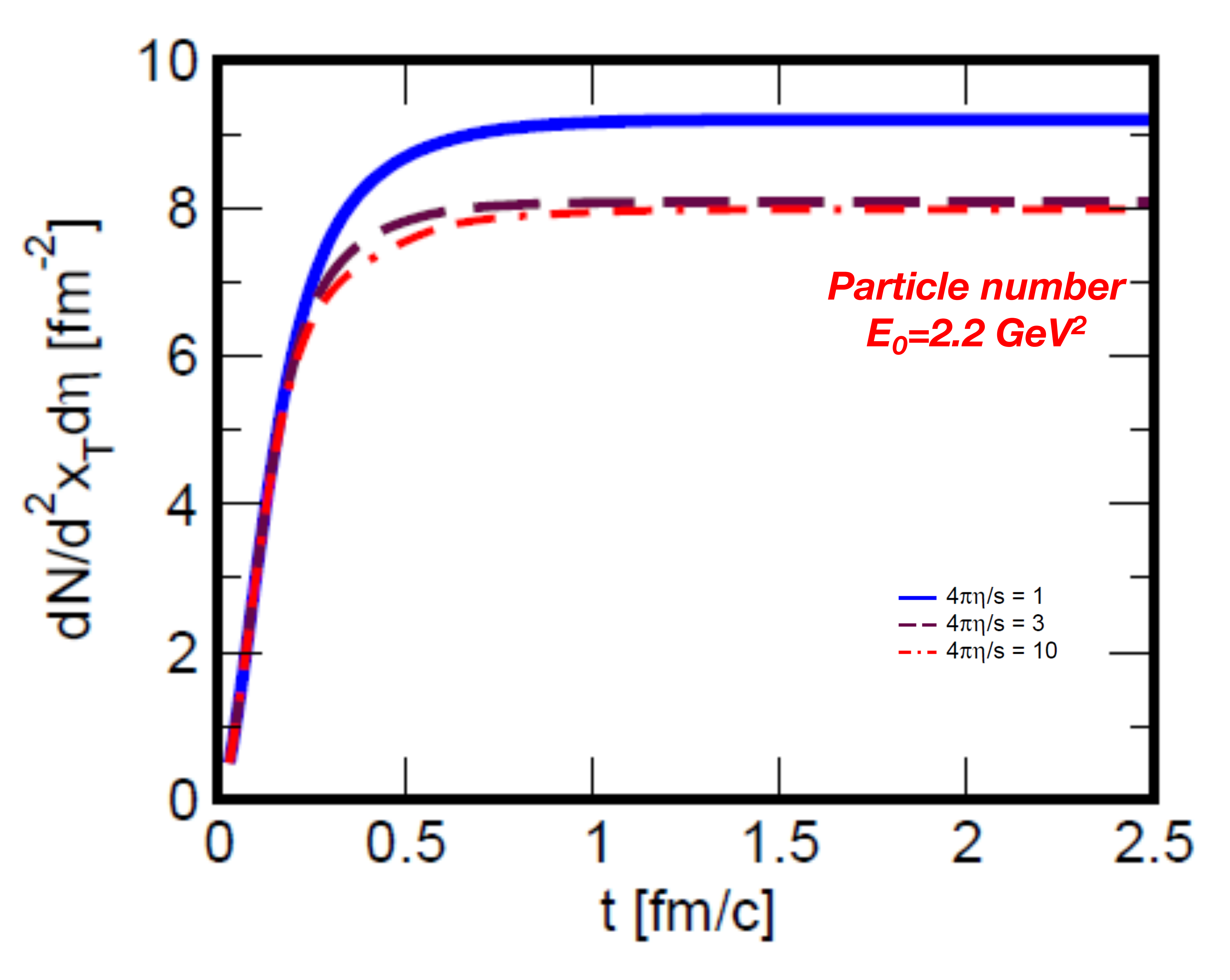
Unless initial field is very small,
formation time is less than 1 fm/c

Typical fireball lifetime: 5-10 fm/c

Field evolution satisfies:

$$\frac{d}{dt}\left(\frac{\boldsymbol{E}^2}{2}\right) = -\boldsymbol{j}\cdot\boldsymbol{E}$$

hence, smaller field implies slower decay.

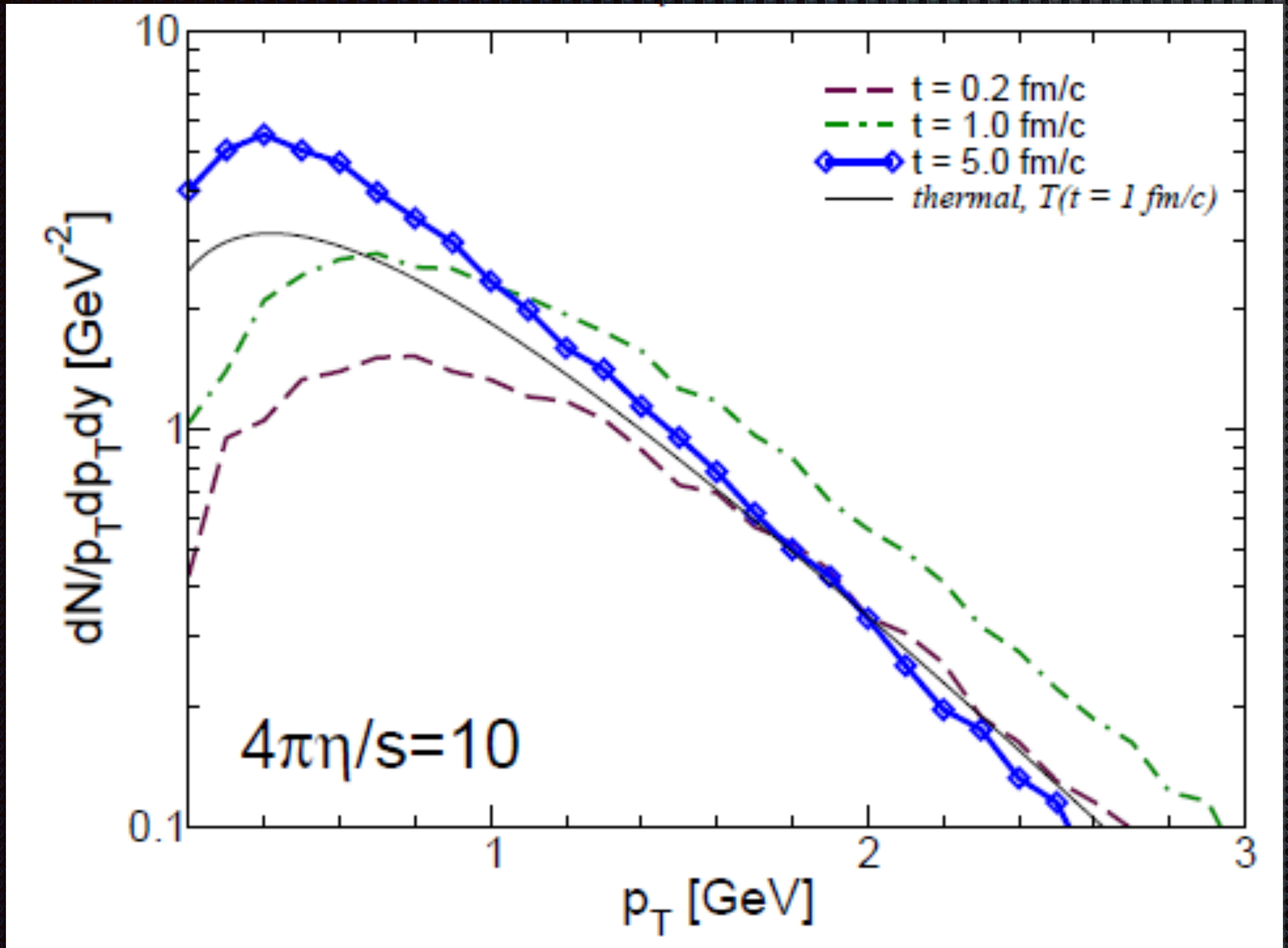


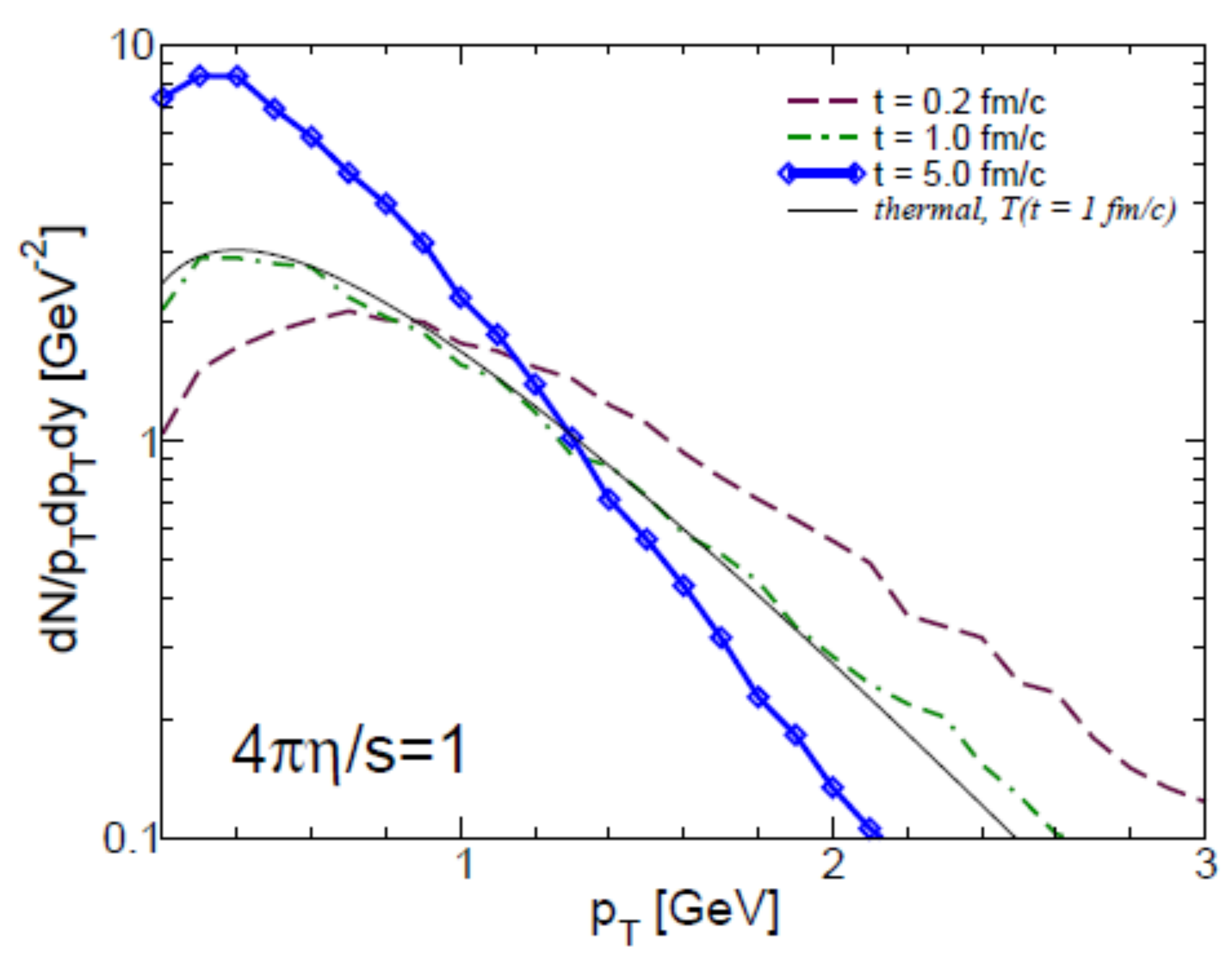
Thermalization

Comparison of *produced particles spectra* with *thermal spectra* at the *same energy density*.

Large viscosity

Particle spectra is quite different from the thermal spectrum with the same energy density





Small viscosity

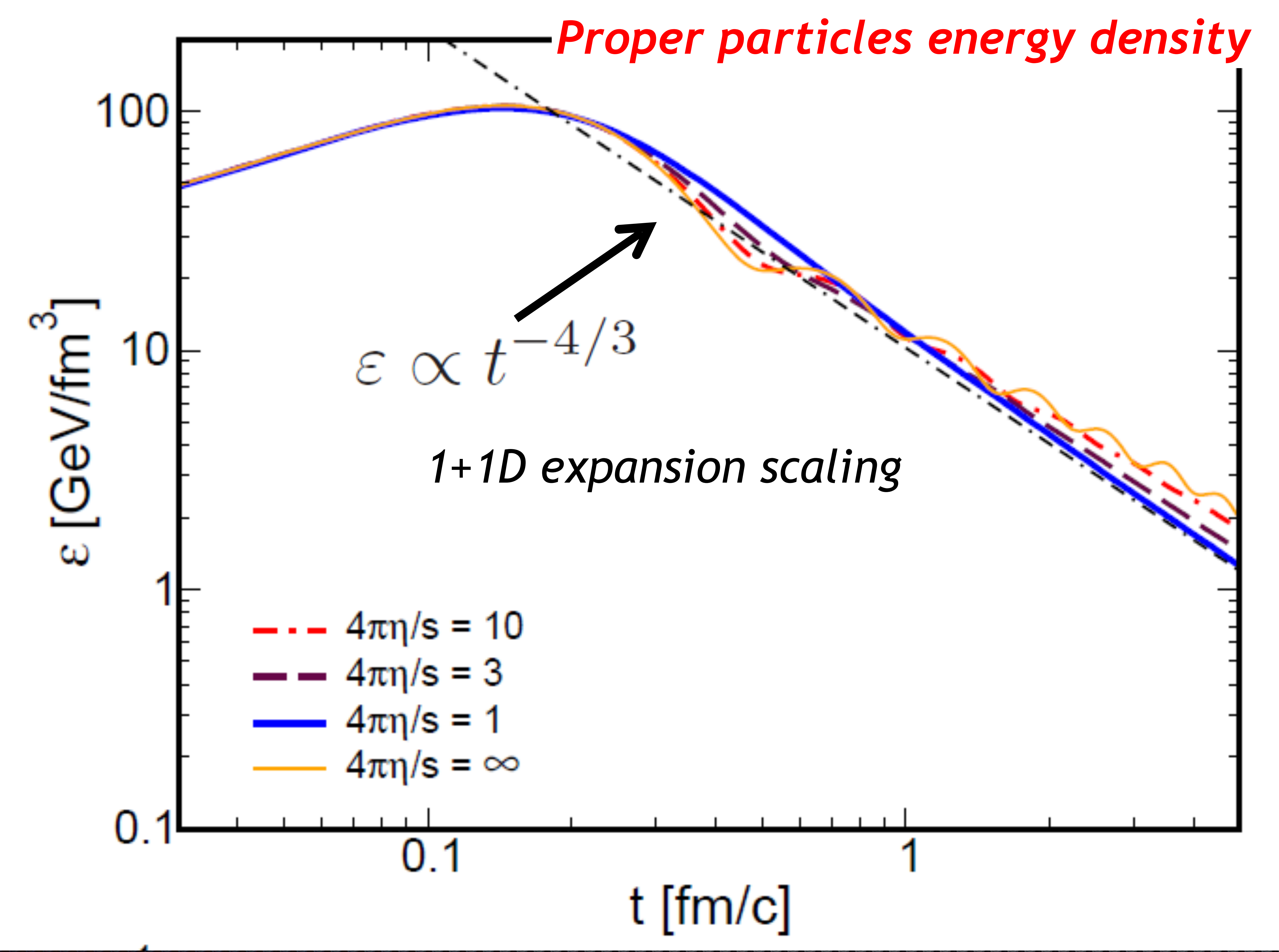
Very fast thermalization t < 1 fm/c

$$\frac{dN}{p_T dp_T dy} \propto p_T e^{-\beta p_T}$$

M. Ruggieri, A.P., et al., PRC 92, 064904 (2015)

Hydro regime

M. Ruggieri, A.P., et al., PRC 92, 064904 (2015)



Small η/s

After a short transient, the hydro regime begins:

$$\varepsilon \propto t^{-4/3}$$

Large η/s

After a short transient:

(.) dissipation keeps the system temperature higher;

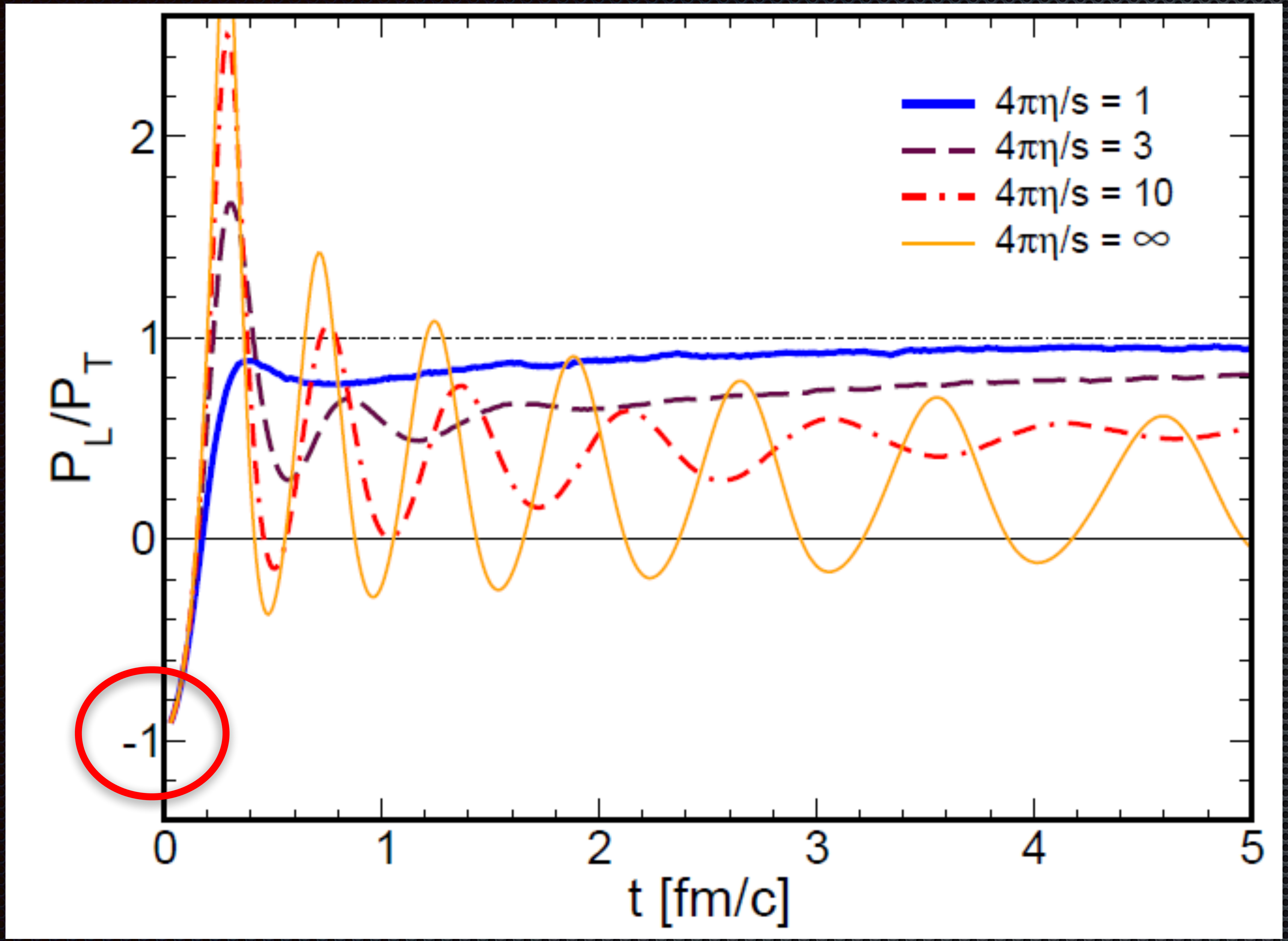
(.) oscillations arising from the field superimpose to power law decay

In agreement with *ideal hydro calculations*:
Gatoff *et al.*, PRD 36 (1987)

Transport Theory is capable to describe, even in conditions of quite strong coupling (small η/s), the evolution of physical quantities in agreement with calculations based on hydrodynamics, once the microscopic cross section is put aside in favor of fixing η/s.

Pressure isotropization

M. Ruggieri, A.P., et al., PRC 92, 064904 (2015)



$$T^{\mu\nu}_{field} = \mathrm{diag}\,(\varepsilon, P_T, P_T, P_L)$$
$$\propto \mathrm{diag}\,(\mathcal{E}^2, \mathcal{E}^2, \mathcal{E}^2, -\mathcal{E}^2)$$

$$T^{\mu\nu}_{particles} = \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \frac{p^\mu p^\nu}{E} f(\boldsymbol{x}, \boldsymbol{p})$$

$$T^{\mu\nu} = T^{\mu\nu}_{particles} + T^{\mu\nu}_{field}$$

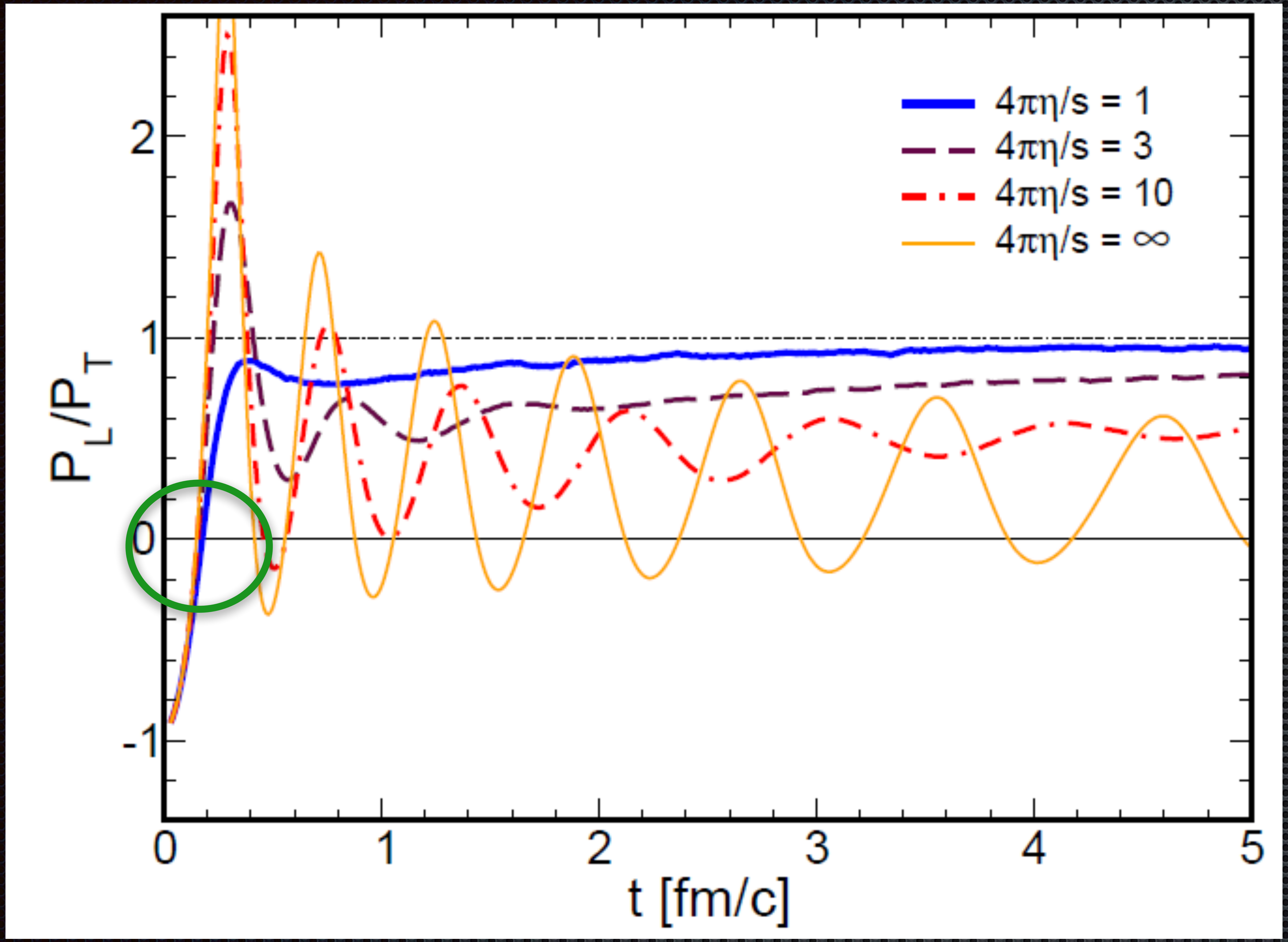
$$P_L = T_{zz}$$
$$P_T = \frac{T_{xx} + T_{yy}}{2}$$

Initial time

High anisotropy: pure field with negative longitudinal pressure

Pressure isotropization

M. Ruggieri, A.P., et al., PRC 92, 064904 (2015)



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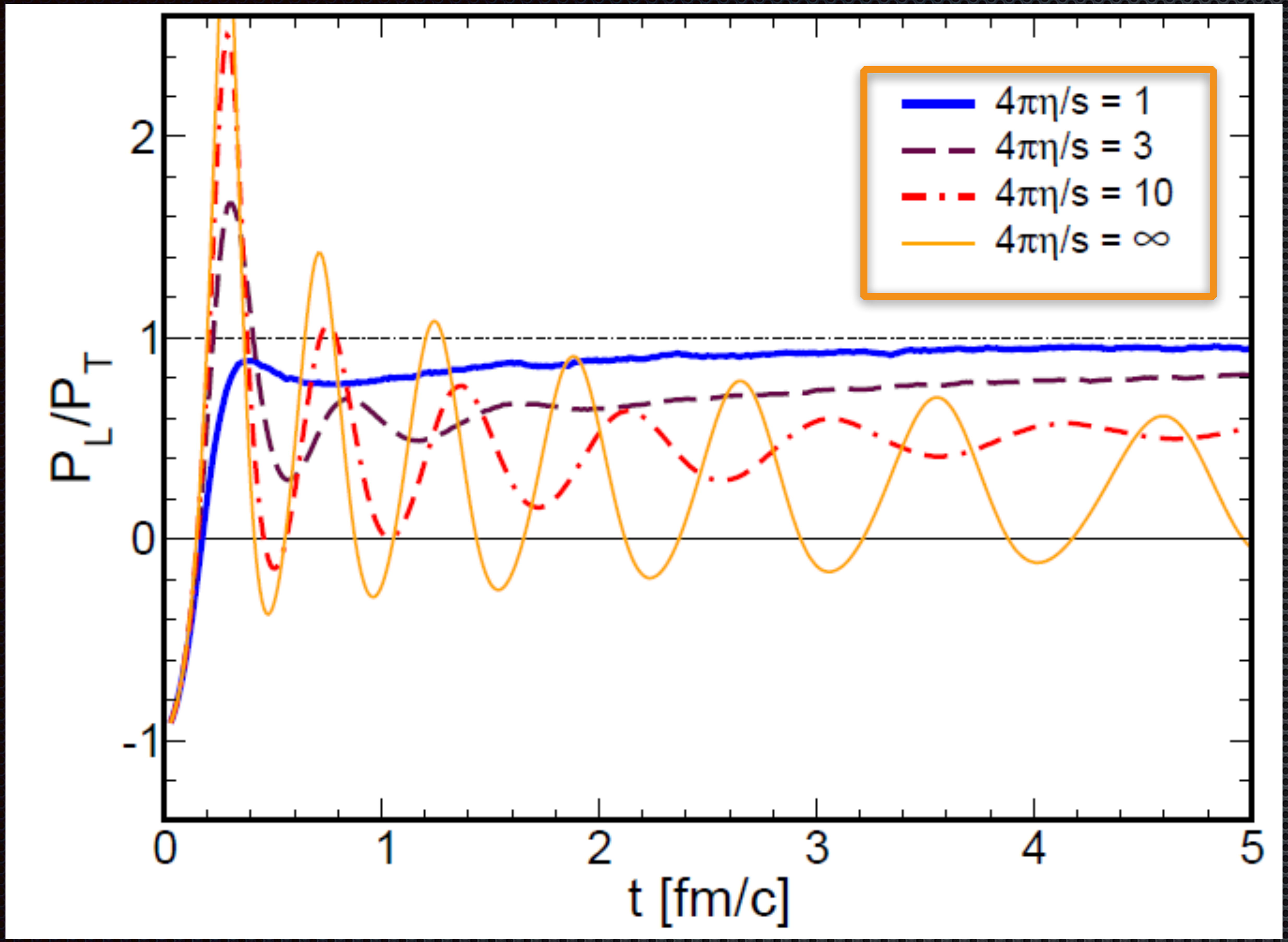
Early time evolution

Longitudinal pressure turns to be positive due to particles pop-up

Pressure isotropization

M. Ruggieri, A.P., et al., PRC 92, 064904 (2015)



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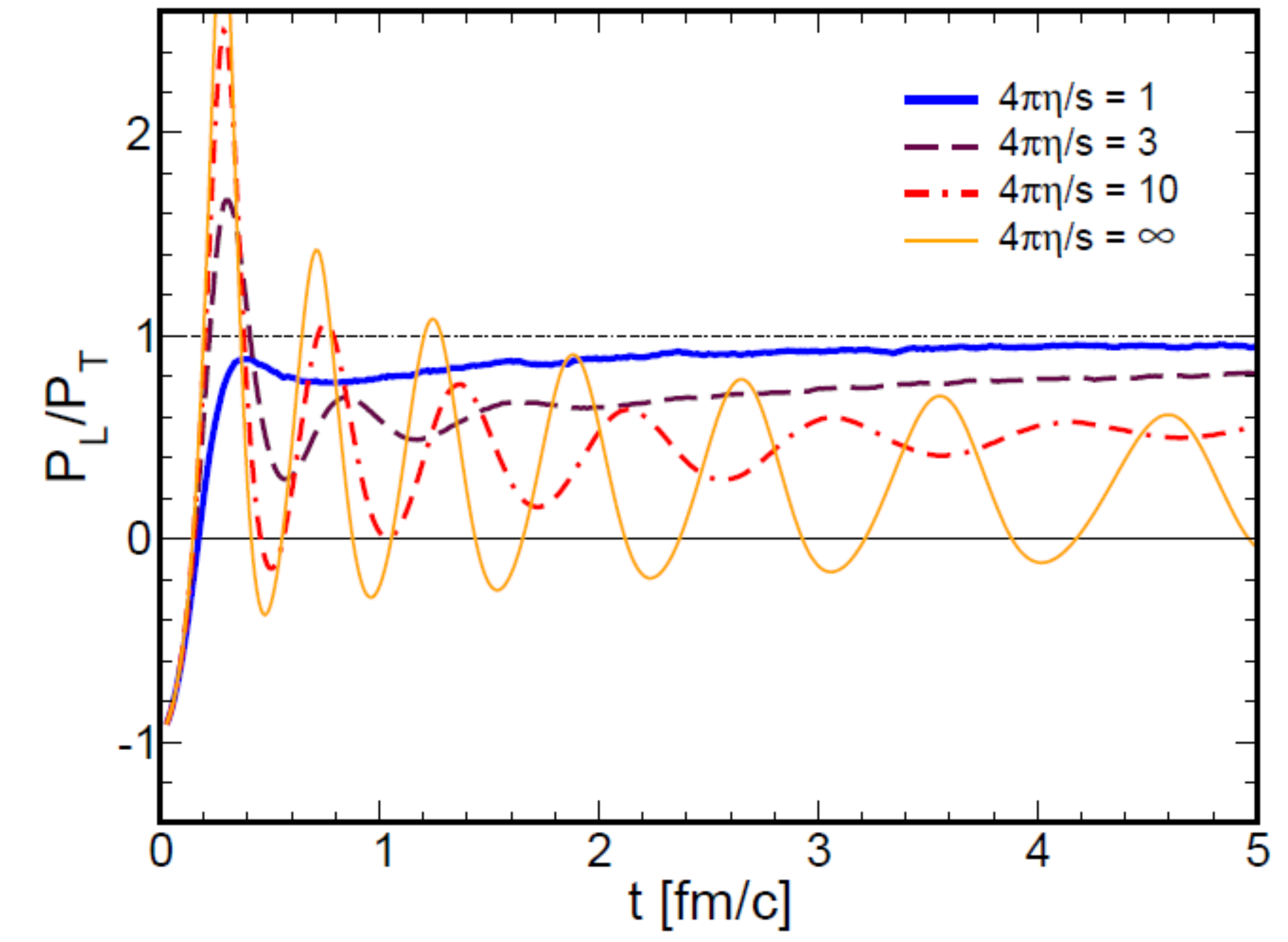
$$T^{\mu\nu} = T^{\mu\nu}_{particles} + T^{\mu\nu}_{field}$$

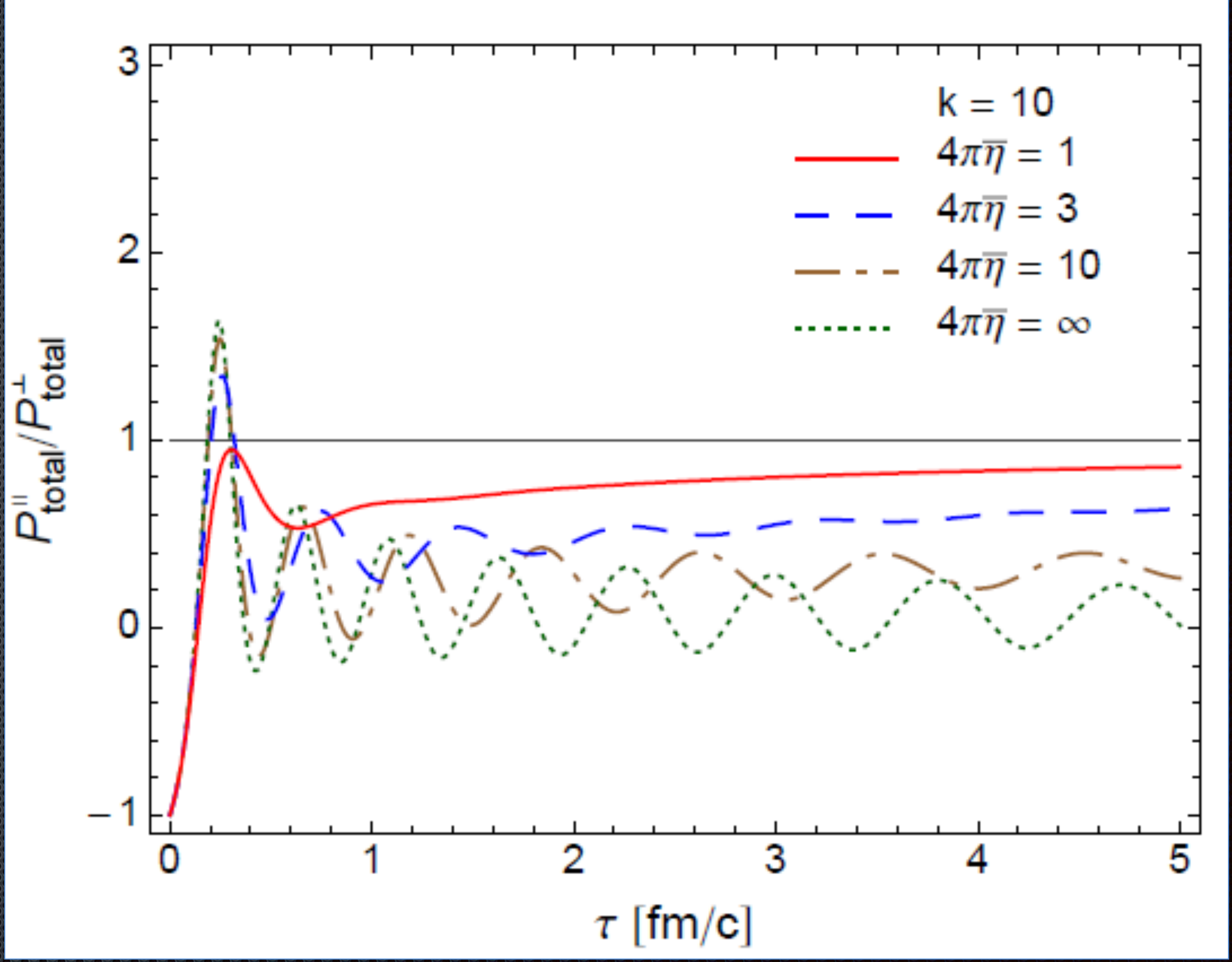
$$P_L = T_{zz}$$
$$P_T = \frac{T_{xx} + T_{yy}}{2}$$

Strong coupling: isotropization time of about 1 fm/c
Weak coupling: less efficient isotropization

Pressure isotropization

Ryblewski and Florkowski, PRD 88 (2013)
(.) RTA calculation
(.) Quarks and gluons





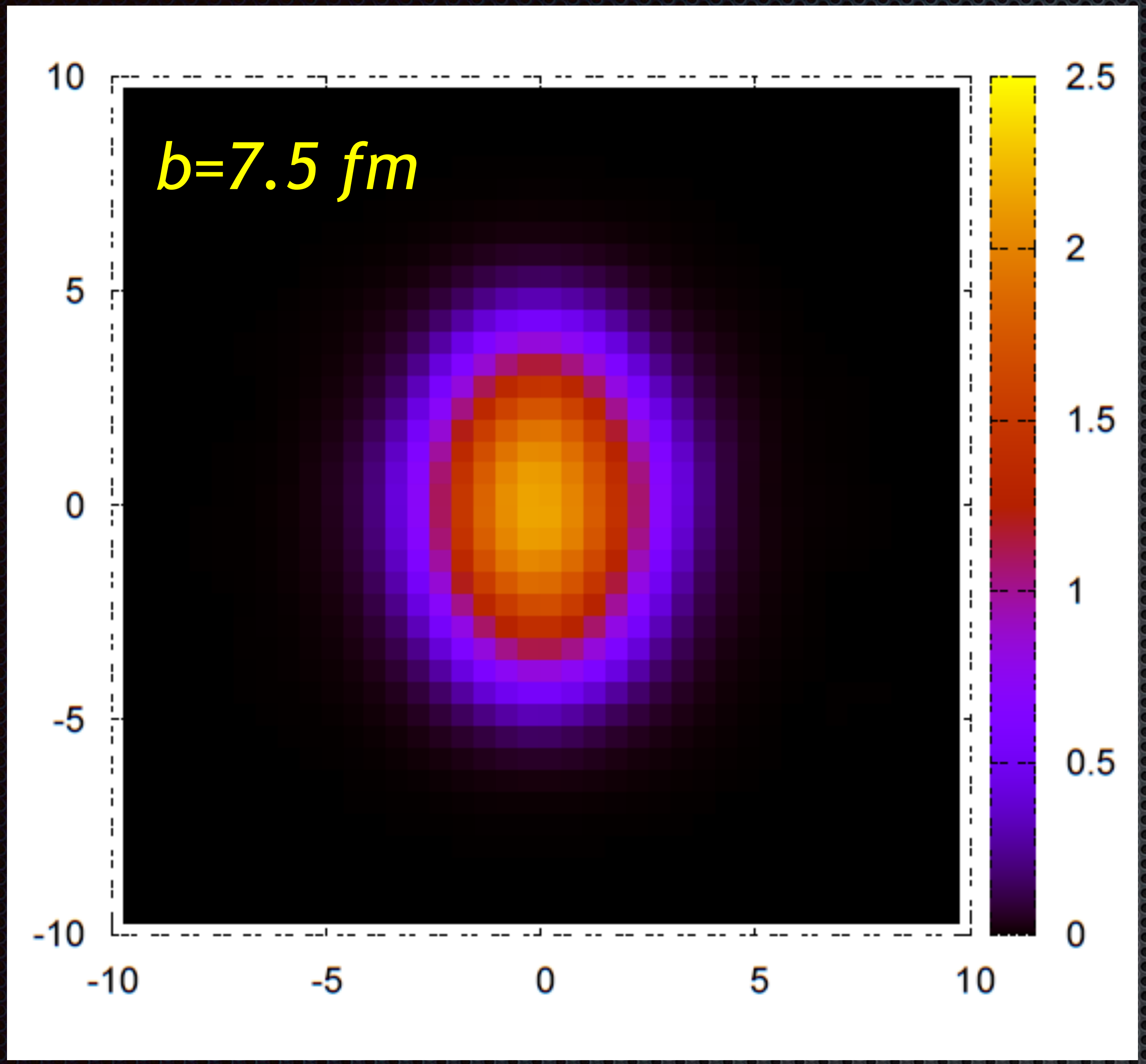
From the qualitative point of view the agreement is excellent.
Quantitatively, some difference arises because of different calculation scheme.

3+1D expansion

A. Puglisi. et al., *in preparation*

Initial field is *longitudinal*, but a realistic 3D expansion is allowed which leads to *transverse fields*.

Initial longitudinal field



A model for a realistic initial condition:
Electric field with an eccentricity

Anisotropic pressure gradients

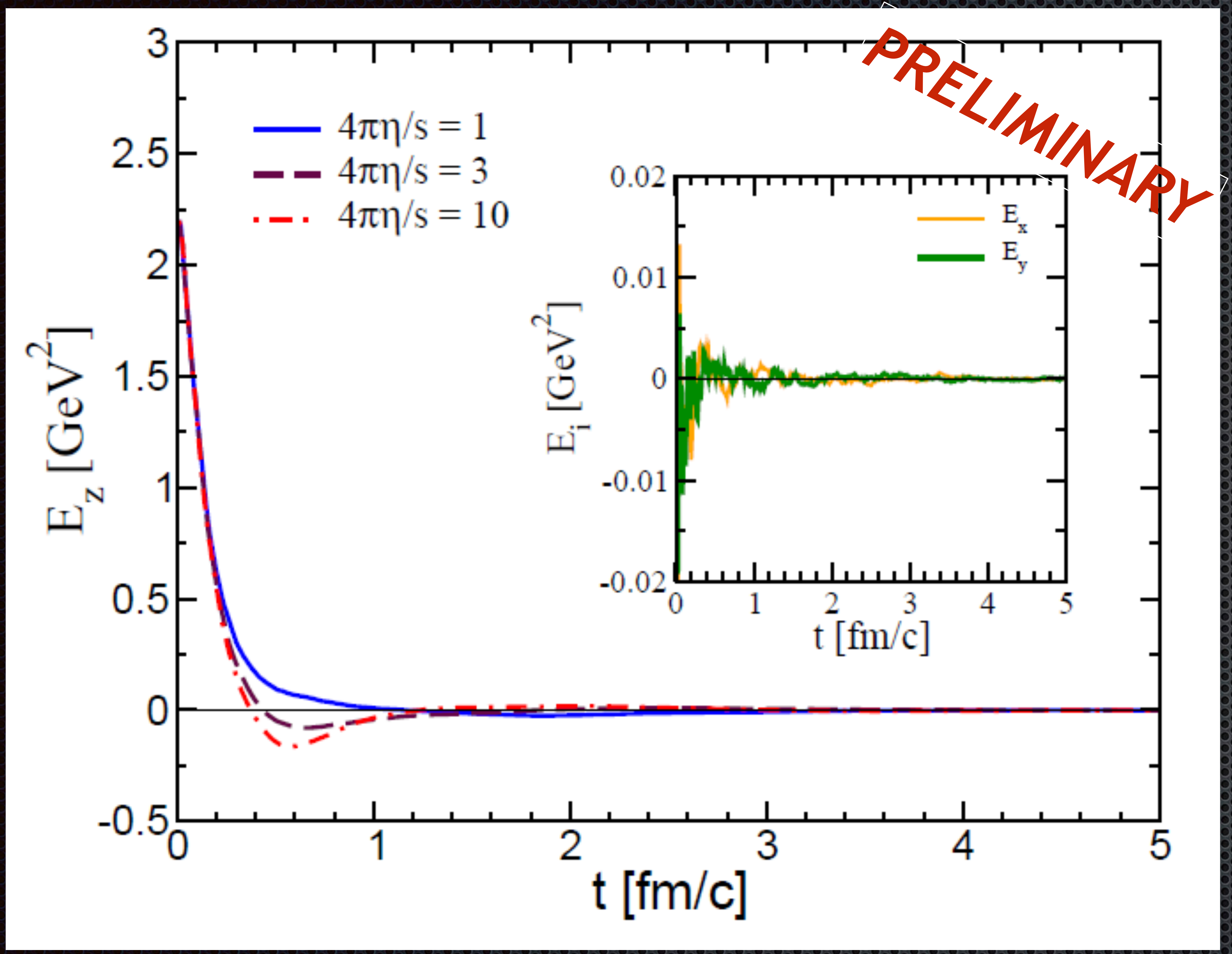
Elliptic flow production

Toy model calculation: *ignores initial state fluctuations*.
Nevertheless interesting calculation: by means of ***one theoretical framework*** we are able to describe the dynamics ***from initial state*** (classical fields) up to ***final stage*** (flows production).

Field decay in 3+1D expansion

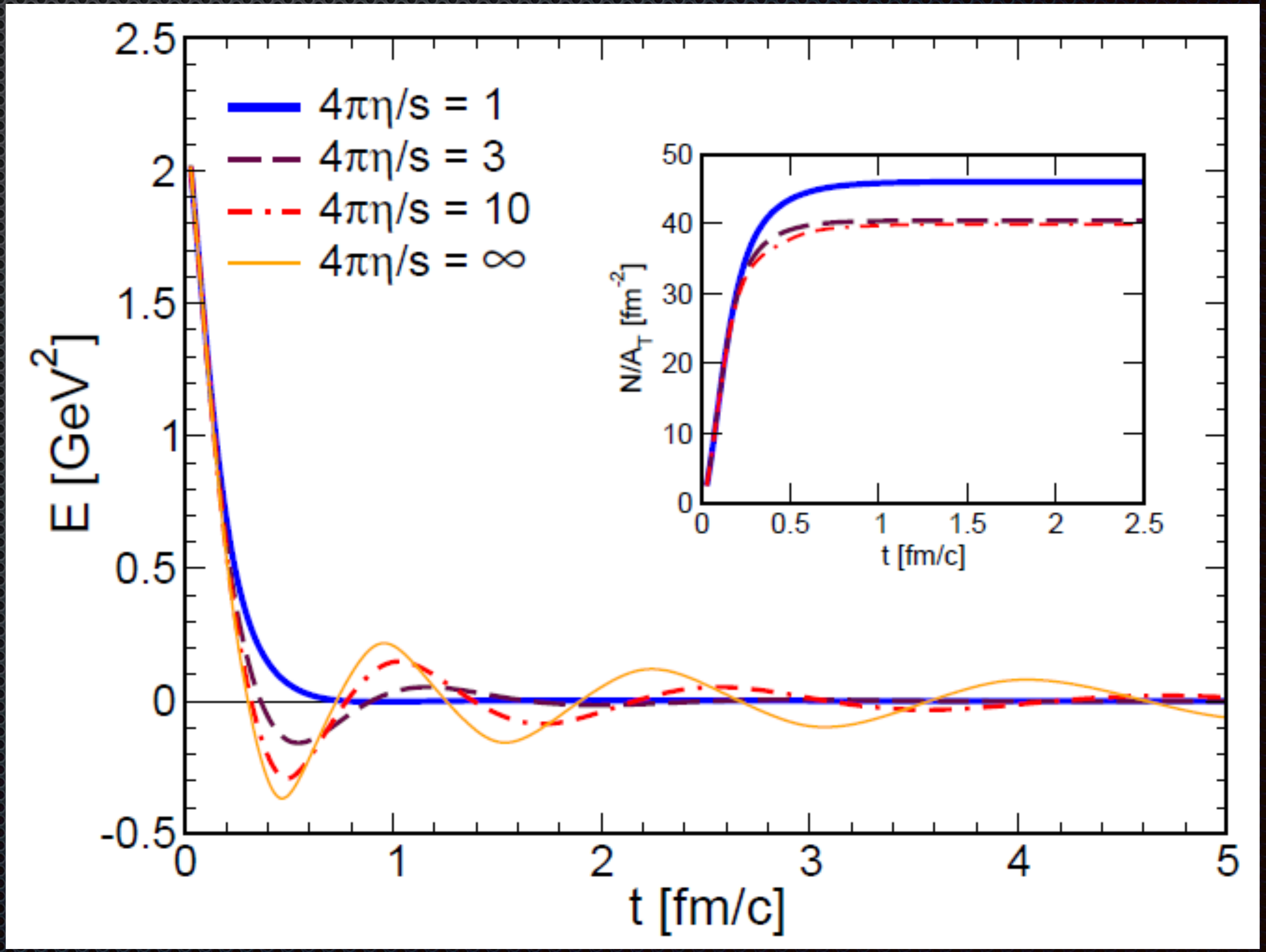
A. Puglisi. et al., in preparation

3+1D



Fields at midrapidity
averaged on the transverse plane

1+1D

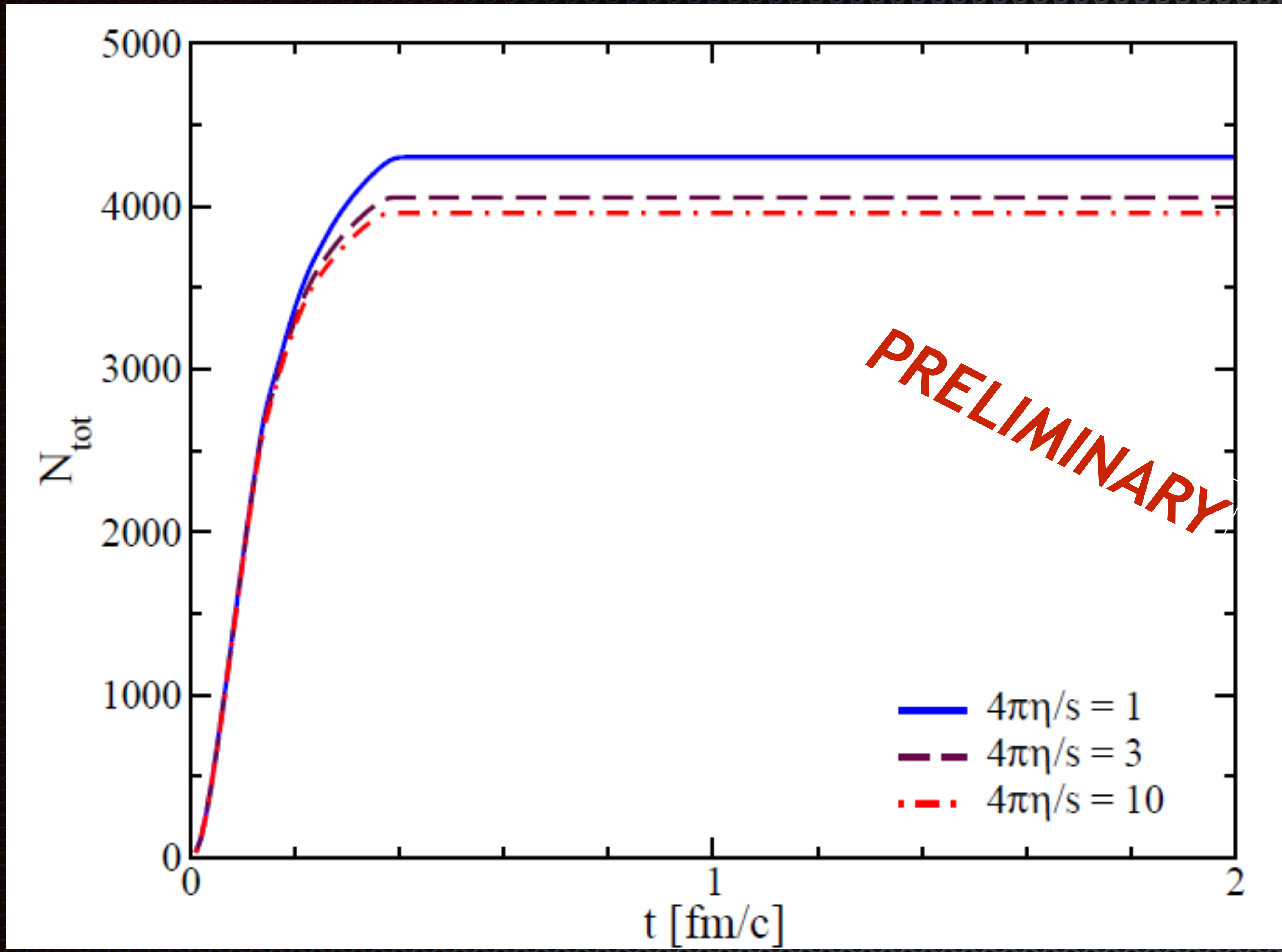


Nice agreement with the 1+1D calculation

Plasma production for 3+1D expansion

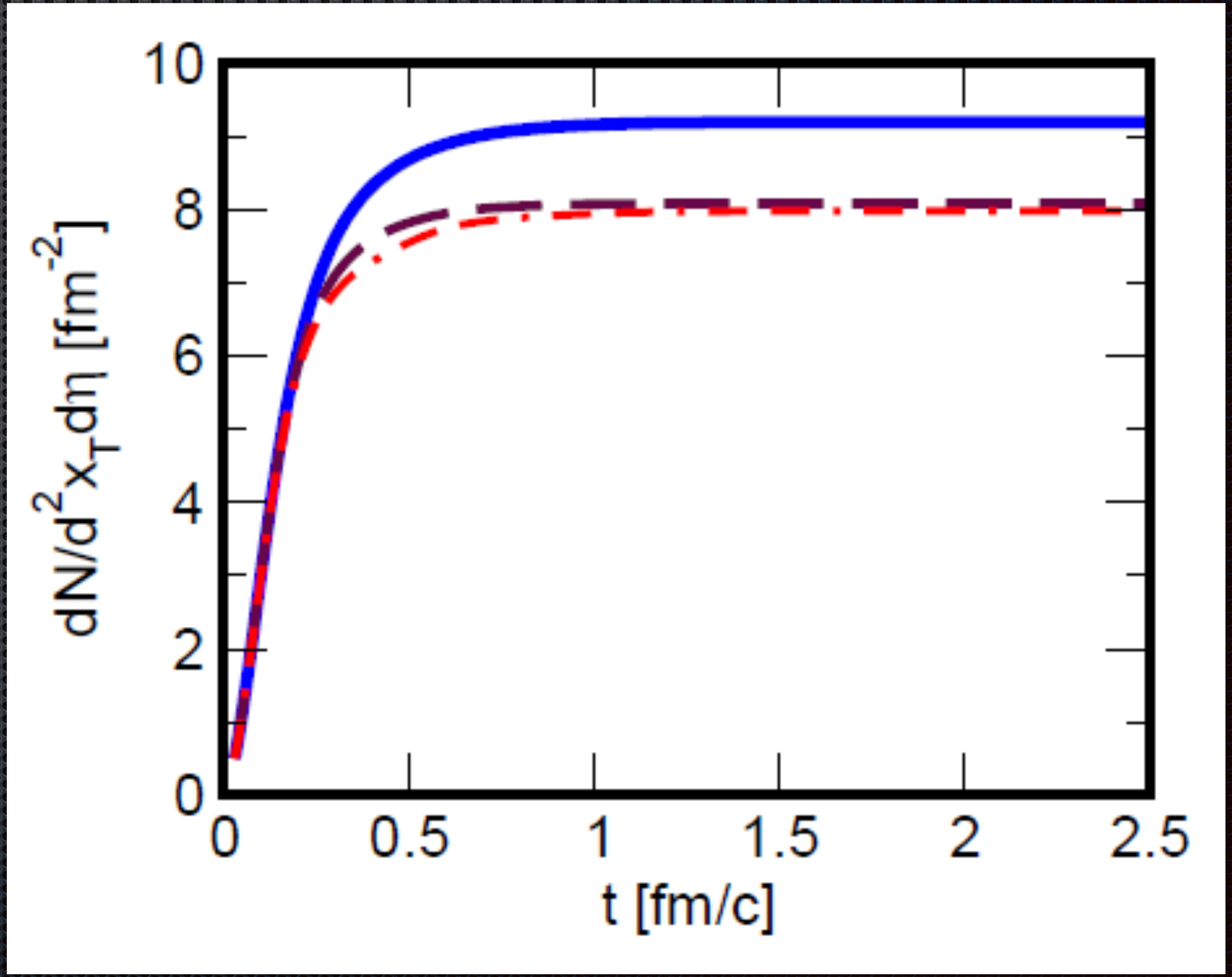
A. Puglisi. et al., in preparation

3+1D



Plasma production occurs within 0.3-1 fm/c

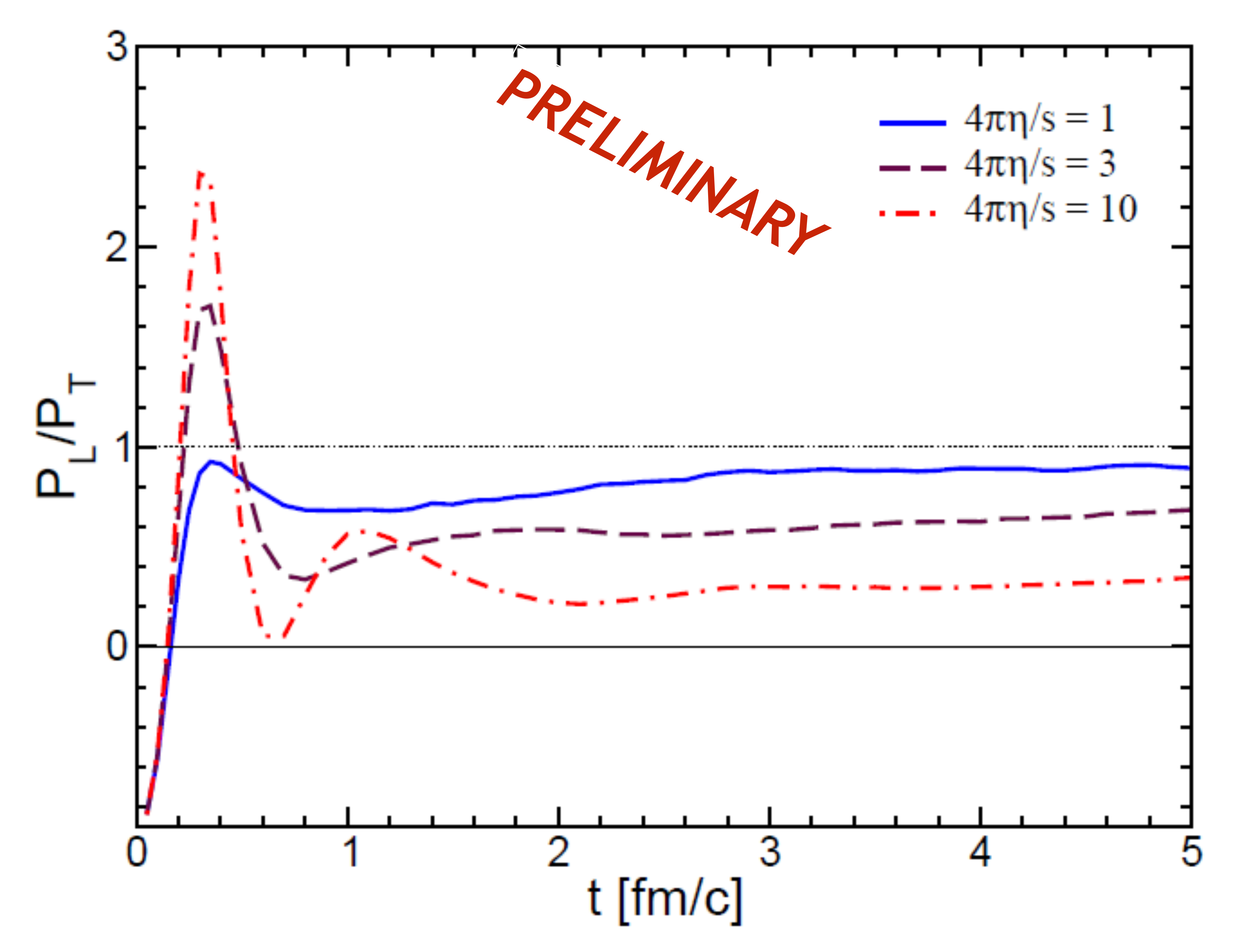
1+1D



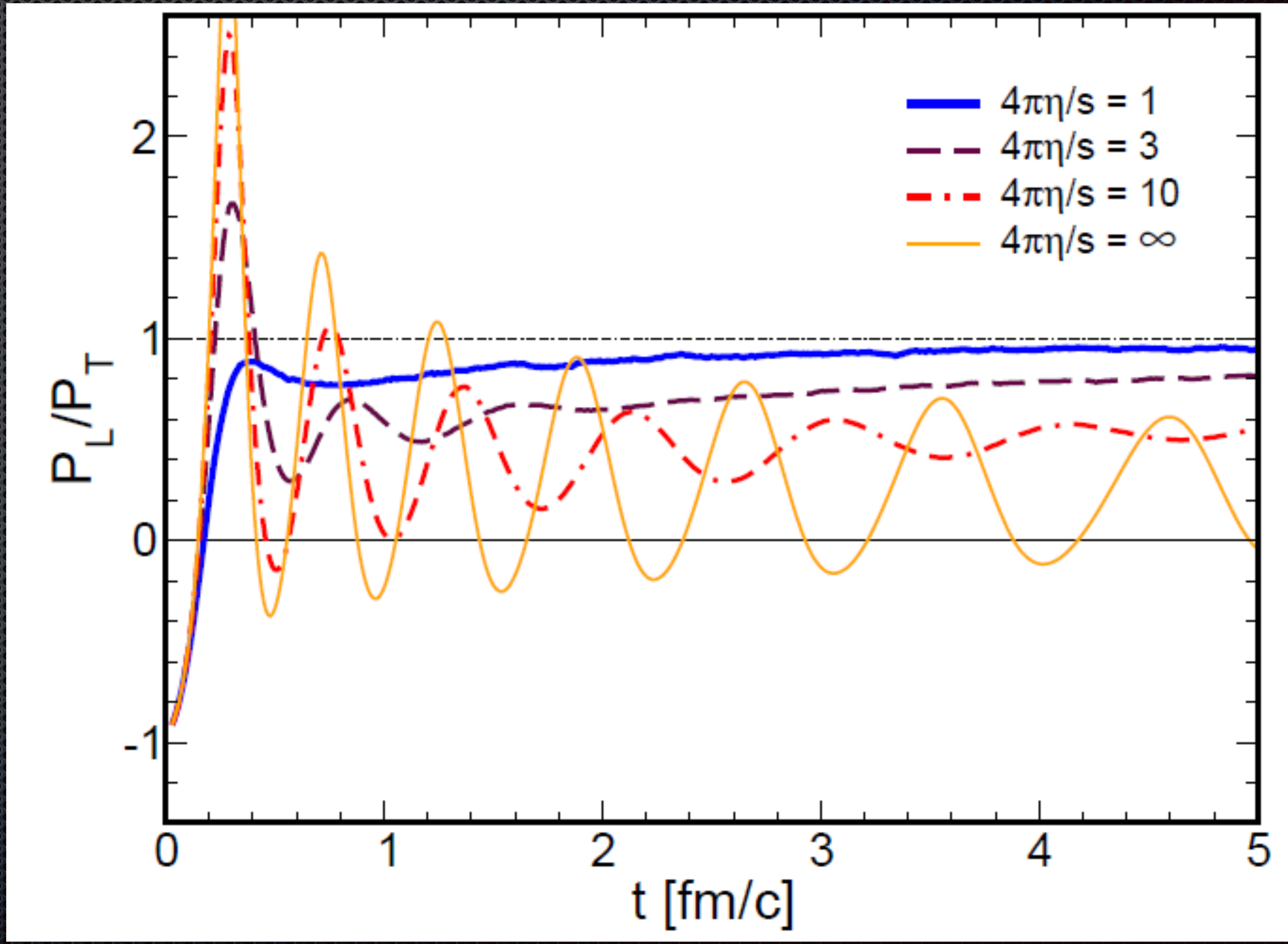
Isotropization for 3+1D expansion

A. Puglisi. et al., in preparation

3+1D



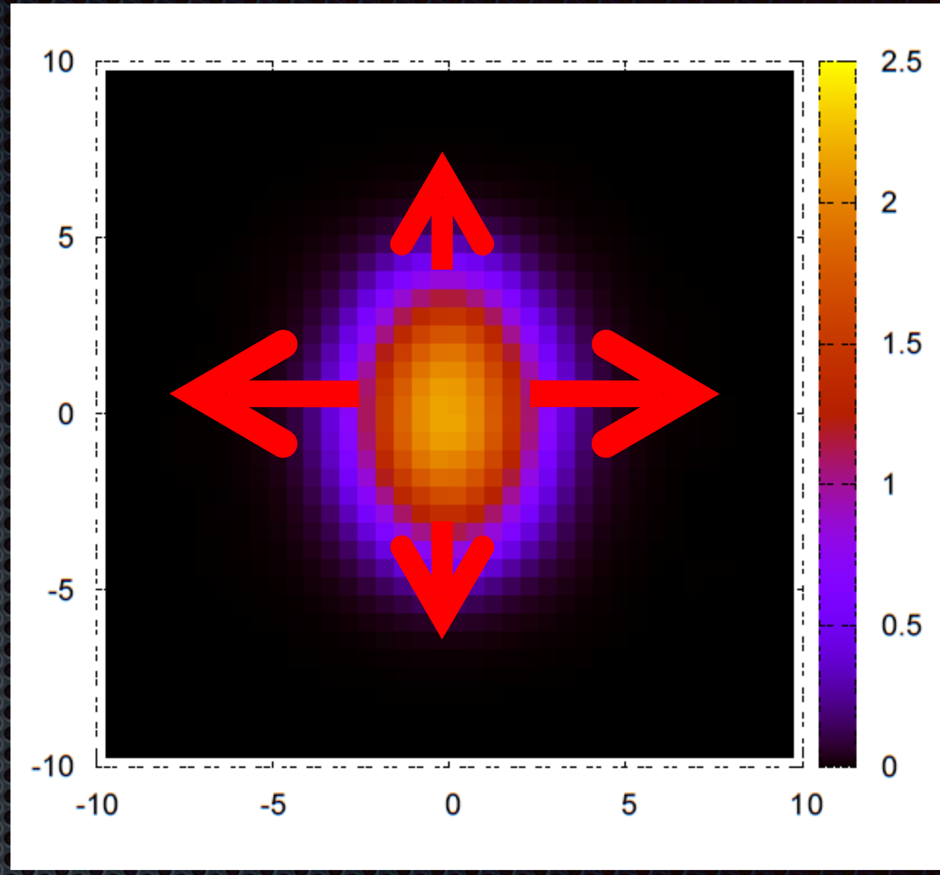
1+1D



Nice agreement with the 1+1D calculation about timescales and isotropization rate

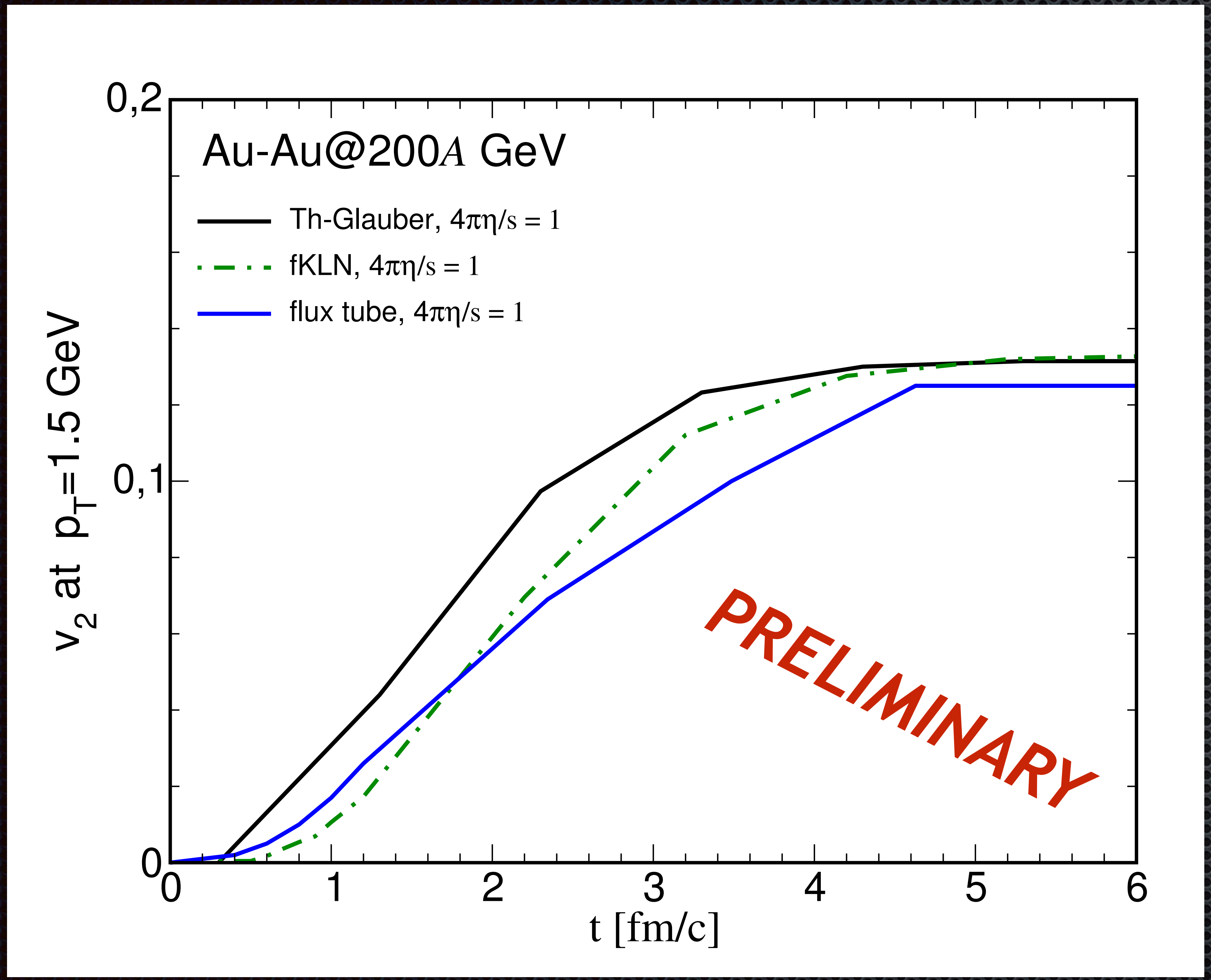
Elliptic flow

$$\frac{d^3N}{dyp_T dp_T d\phi} = \frac{1}{2\pi}\frac{d^2N}{dyp_T dp_T}\left[1 + 2v_2(y, p_T)\cos 2\phi\right]$$

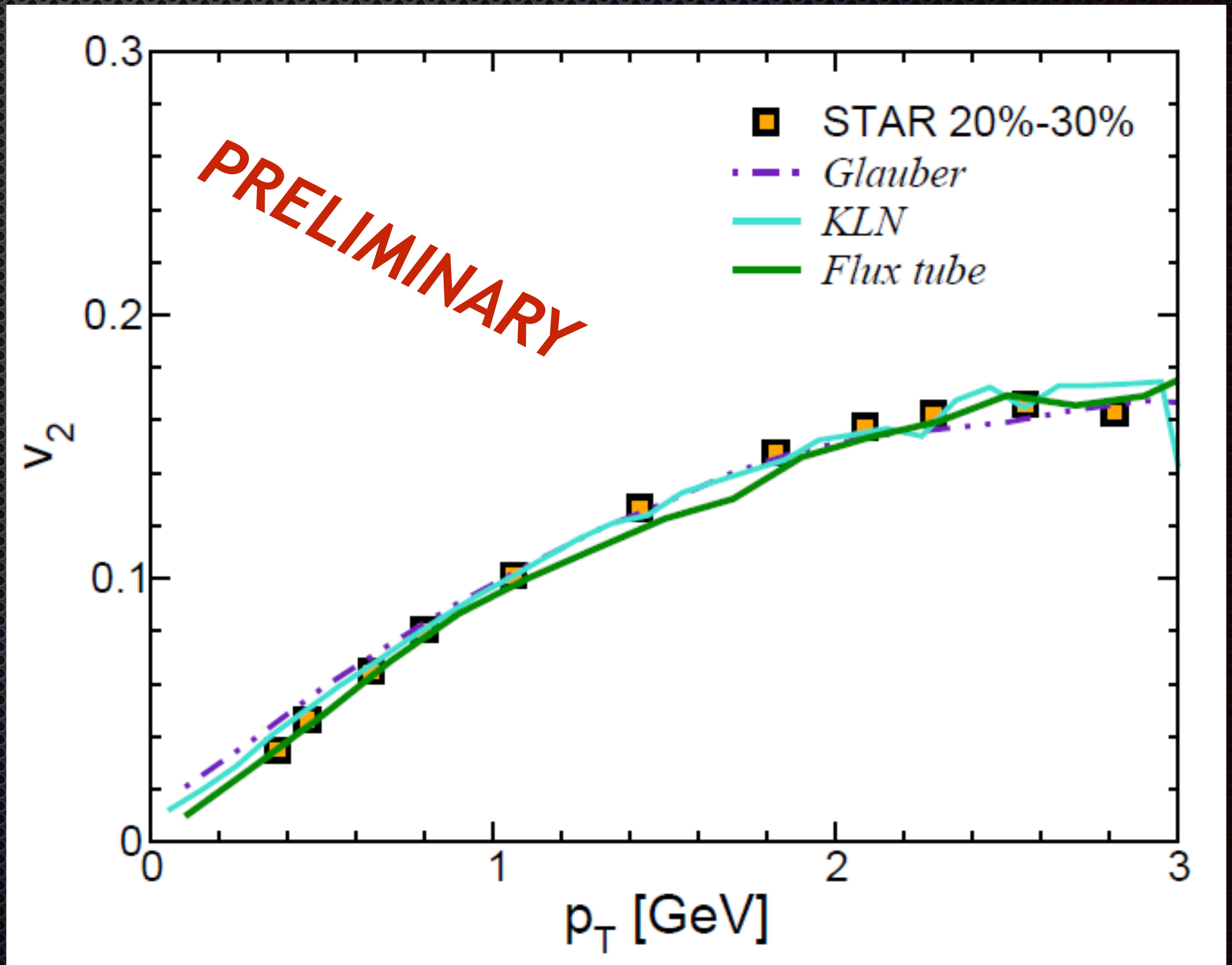


Self-consistent computation of elliptic flow

$$v_2 = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle = \left\langle \frac{p_x^2 - p_y^2}{p_T^2} \right\rangle$$



3+1D expansion
RHIC, b=7.5 fm
4πη/s=1



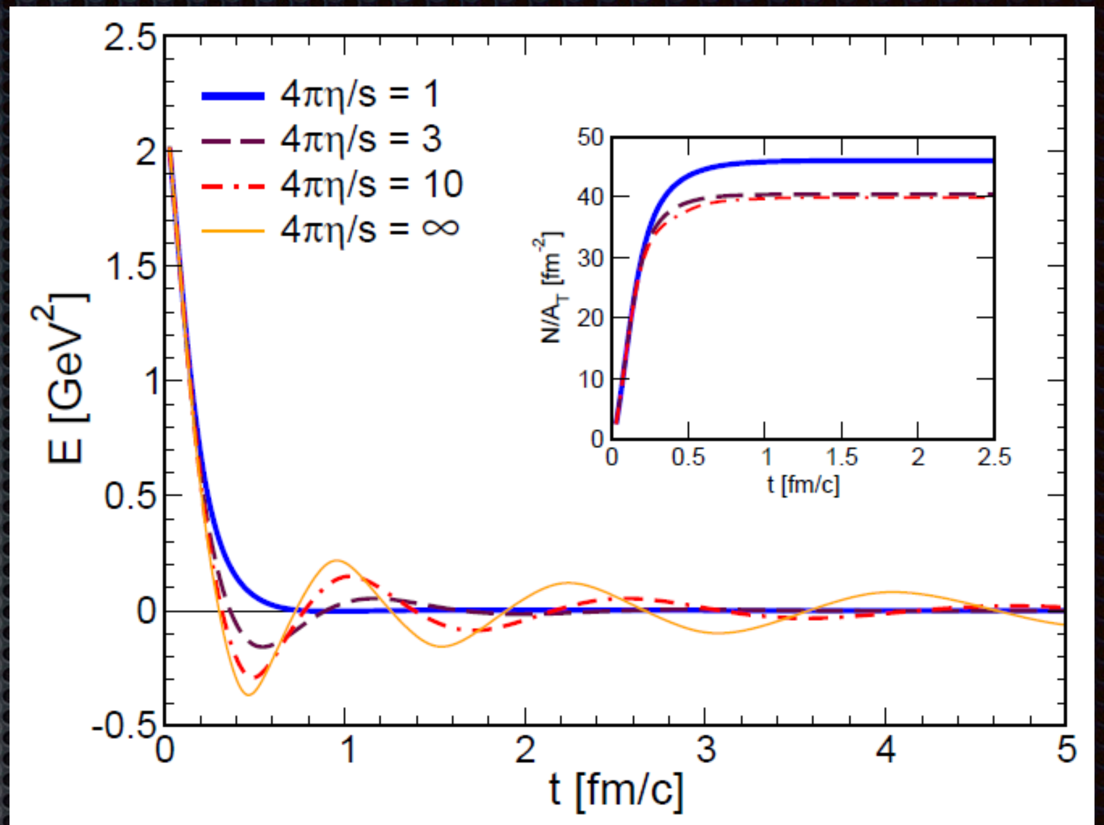
Elliptic flow in agreement with previous model calculations (hydro and/or transport)

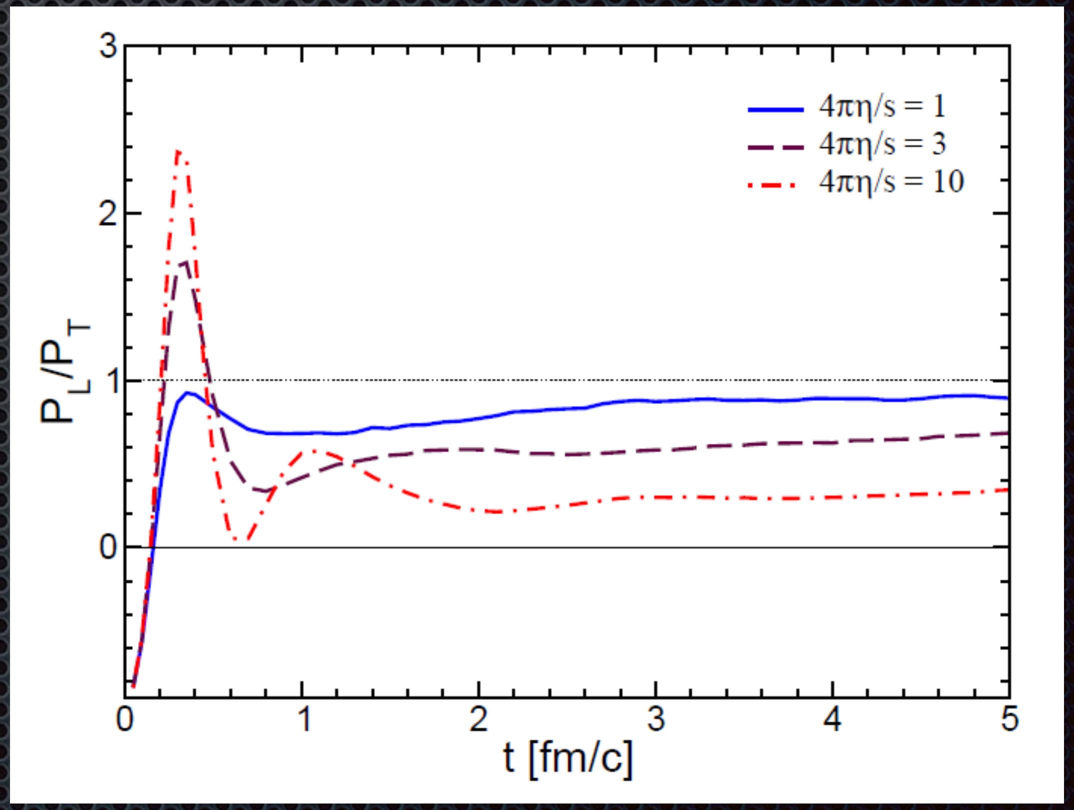
First calculation which using one single scheme, follows the system from $t=0^+$ up to the final stages

Conclusions

- Relativistic Transport Theory permits to study early times dynamics of heavy ion collisions.
- Initial color-electric field decays in ≈0.5 fm/c
- Schwinger tunneling allows a fast particle production ≈0.3-1 fm/c
- Strongly coupled plasma (small η/s) reaches a hydro regime in a very short time
- Isotropization time is less than 1 fm/c
- One single framework: from t=0 to collective flows

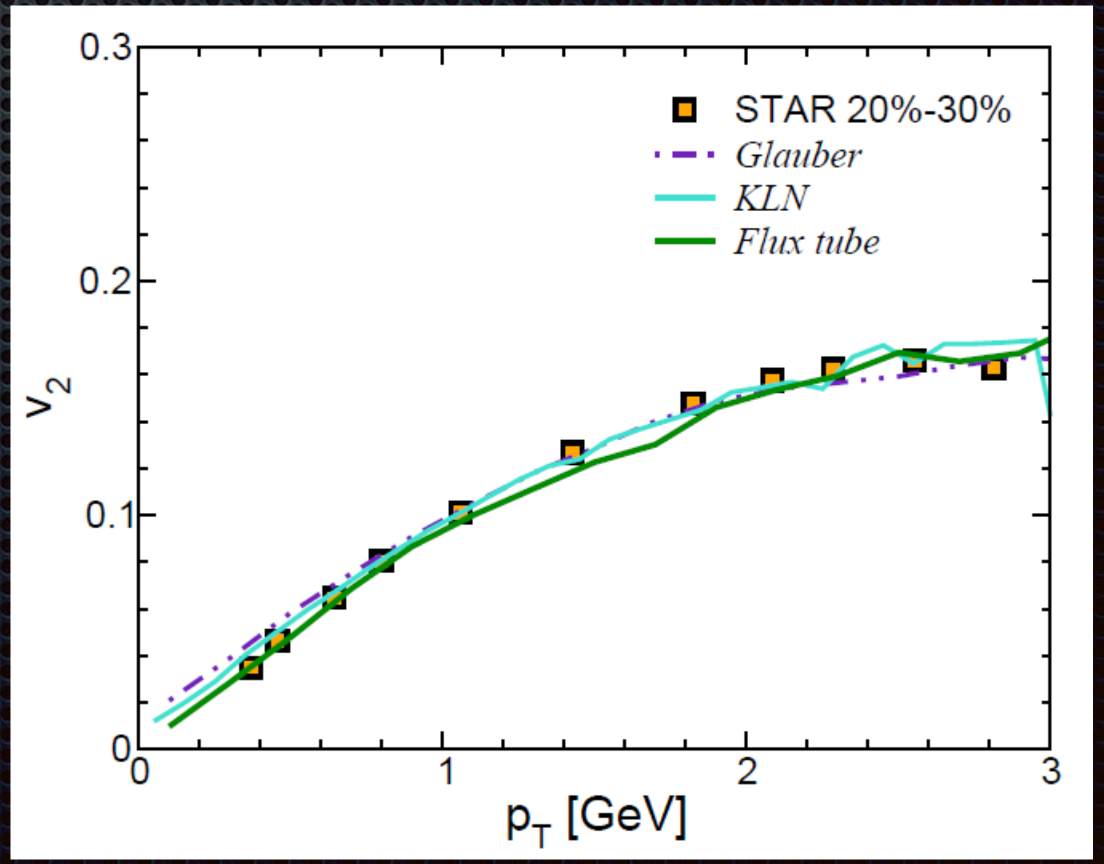
Does the plasma oscillations leave any observable fingerprint?





Outlooks

- initial condition
- field fluctuations in rapidity and transverse plane
- more flux tubes -> pA,AA
- Color-Magnetic field and its decay
- Photons and dileptons in the pre-equilibrium phase



Thank you for your attention!

Back-up

Schwinger effect in Chromodynamics

Numerical estimates

eE=1 GeV^2 corresponds to 5×10^{24} Volt/m

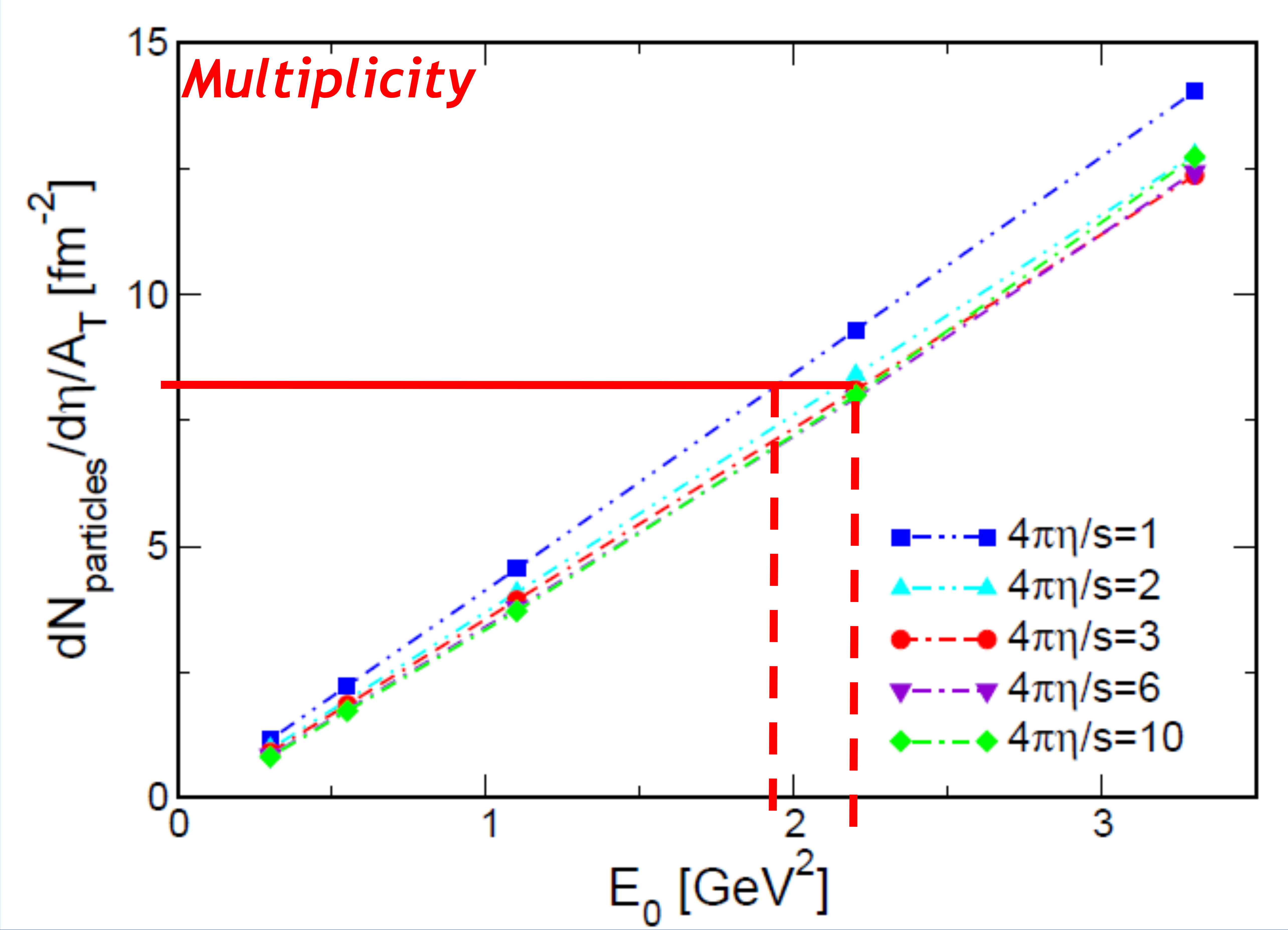
QED critical field: 2.6×10^{-7} GeV^2

In QCD the critical field is given by the string tension:
The energy per unit length carried by the field has to be larger of that required to produce a deconfined pair

QCD critical field: 0.2-0.6 GeV^2

Initial color-electric field in HICs:
gE: 1-10 GeV^2

A rough initial field estimate



$$E_0 \approx 1.9 \div 2.2 \text{ GeV}^2$$

Naïve calculation
(rough estimate)

Multiplicity for a RHIC collision, b=2.5 fm:

$$\frac{dN}{dy} = \frac{dN}{d\eta} \approx 1040$$

Transverse area:

$$A_T \approx \pi R^2 \approx \pi (6.5)^2 \approx 137 \text{ fm}^2$$

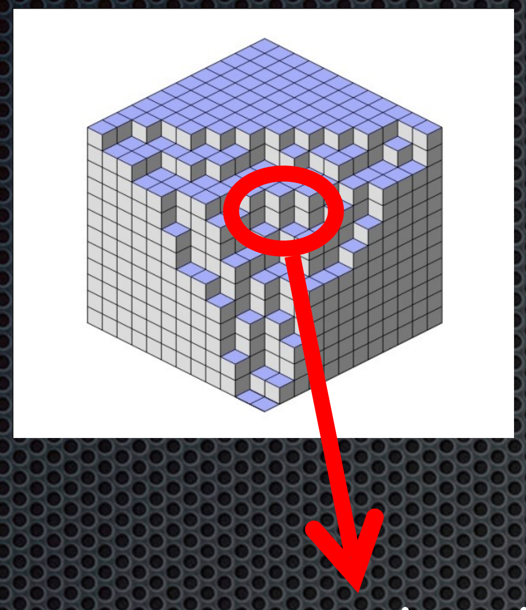
Multiplicity per transverse area:

$$\frac{dN}{A_T d\eta} \approx 8 \text{ fm}^{-2}$$

Very rough estimate:
it gives the proper order of magnitude, leaving the exact number determination to a more realistic model of the initial tubes distribution

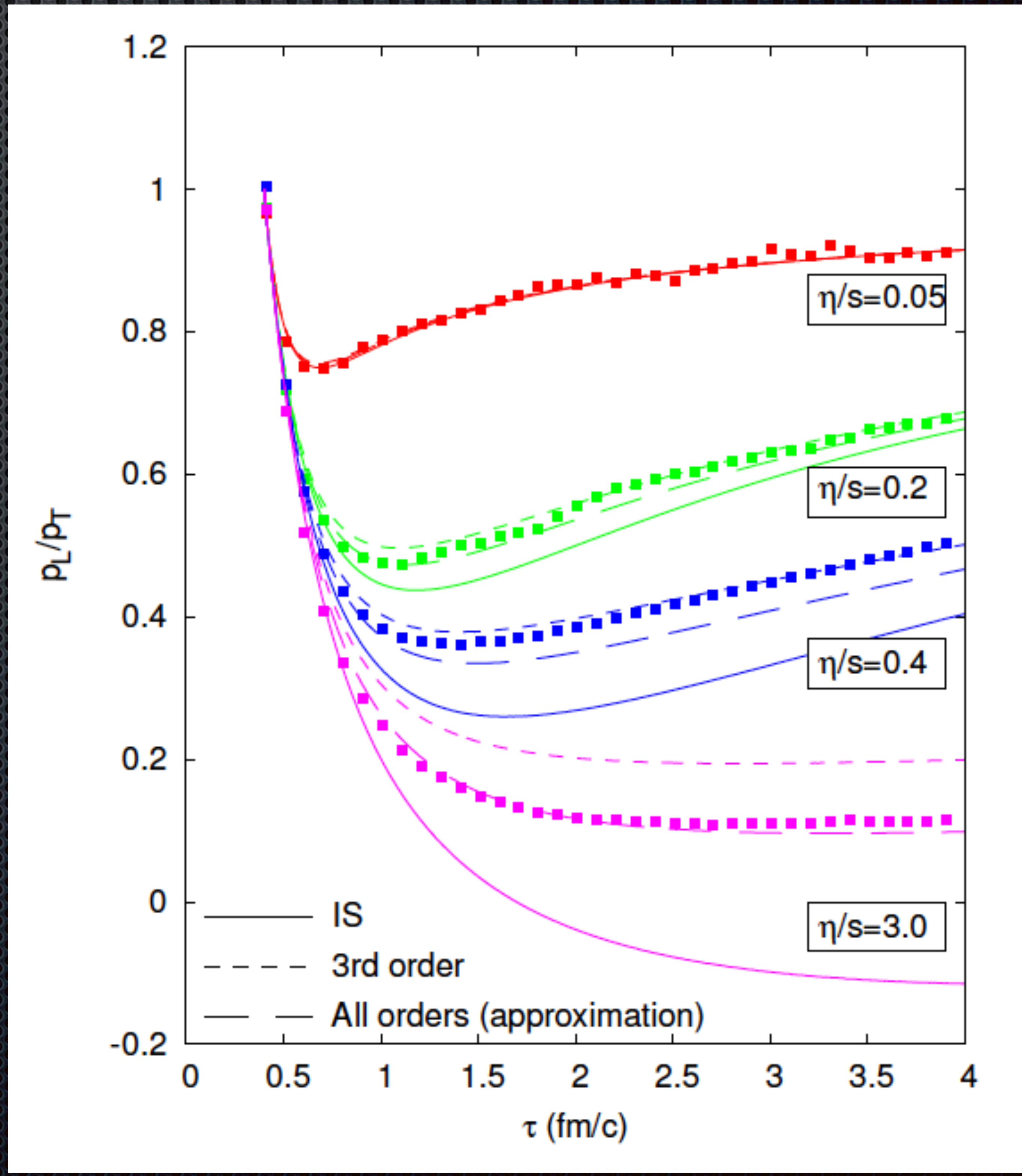
Transport *gauged* to hydro

We use ***Boltzmann equation*** to simulate a fluid at ***fixed eta/s*** rather than fixing a set of microscopic processes.
Total Cross section is ***computed*** in ***each configuration space cell*** according to ***Chapman-Enskog equation*** to give the ***wished value of eta/s***.



$$\frac{\eta}{s} = \frac{\langle p \rangle}{g(m_D)} \frac{1}{\rho\sigma}$$

There is agreement of hydro with transport also in the non dilute limit



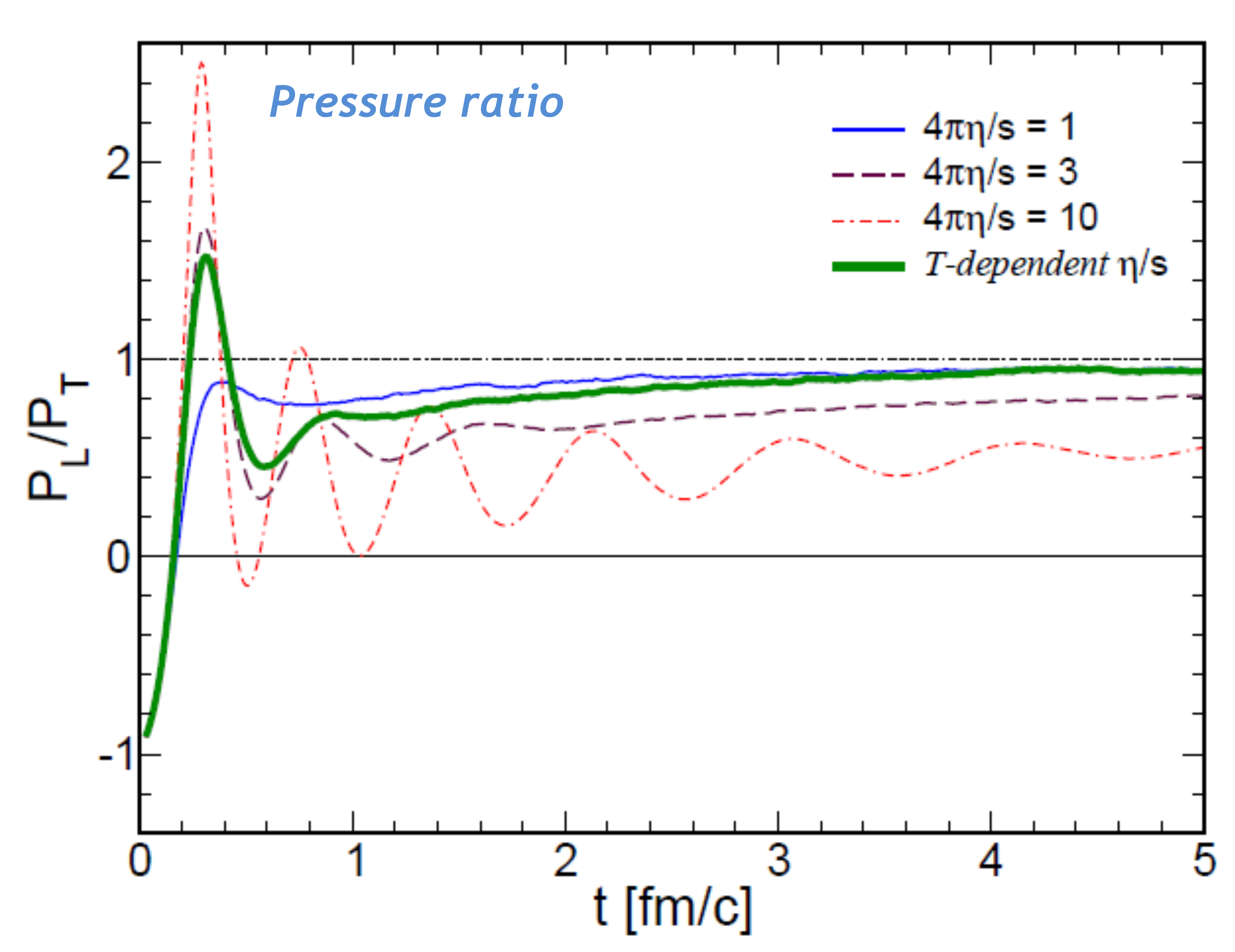
El, Xu, Greiner, Phys.Rev. C81 (2010) 041901

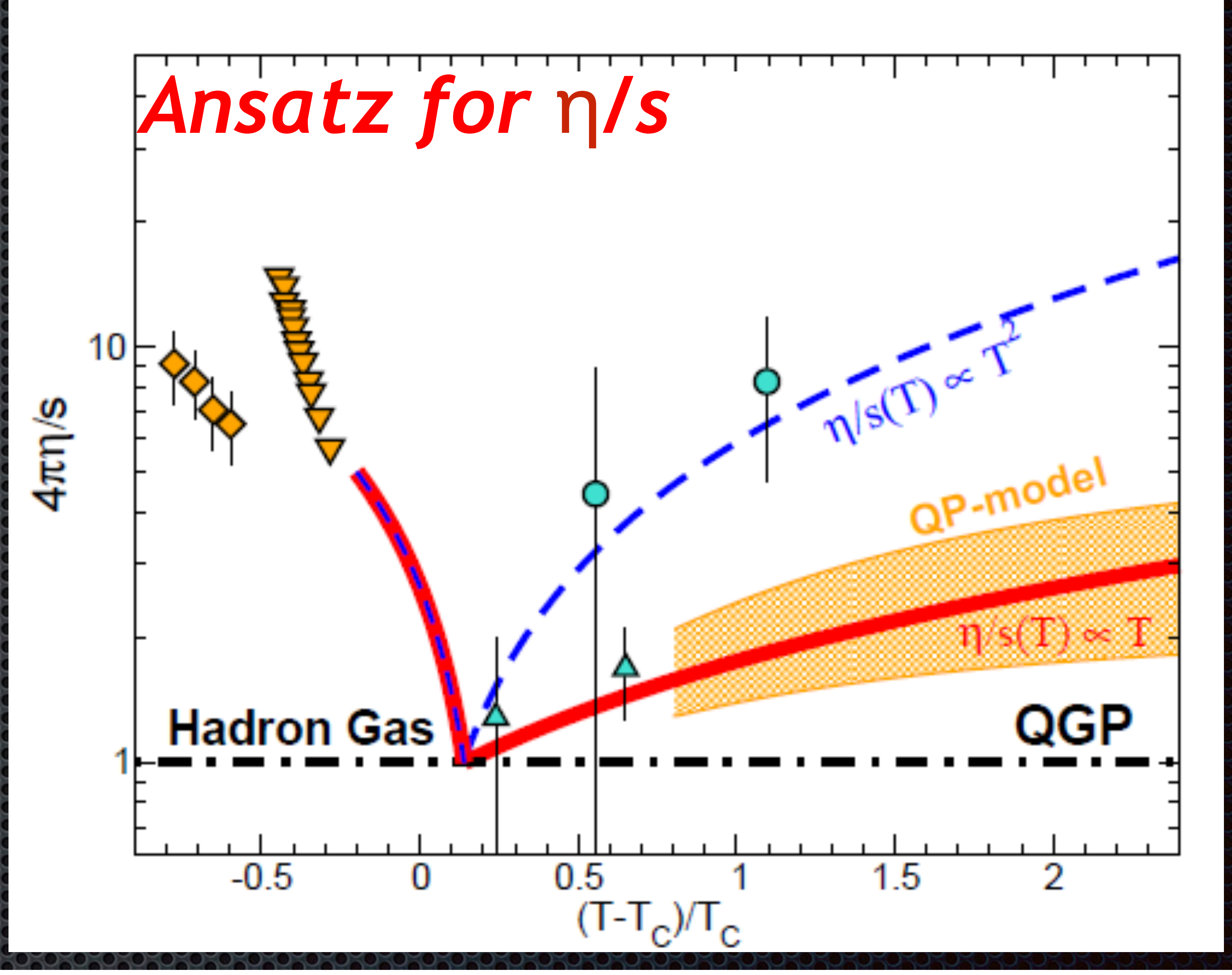
Isotropization for T-dependent η/s

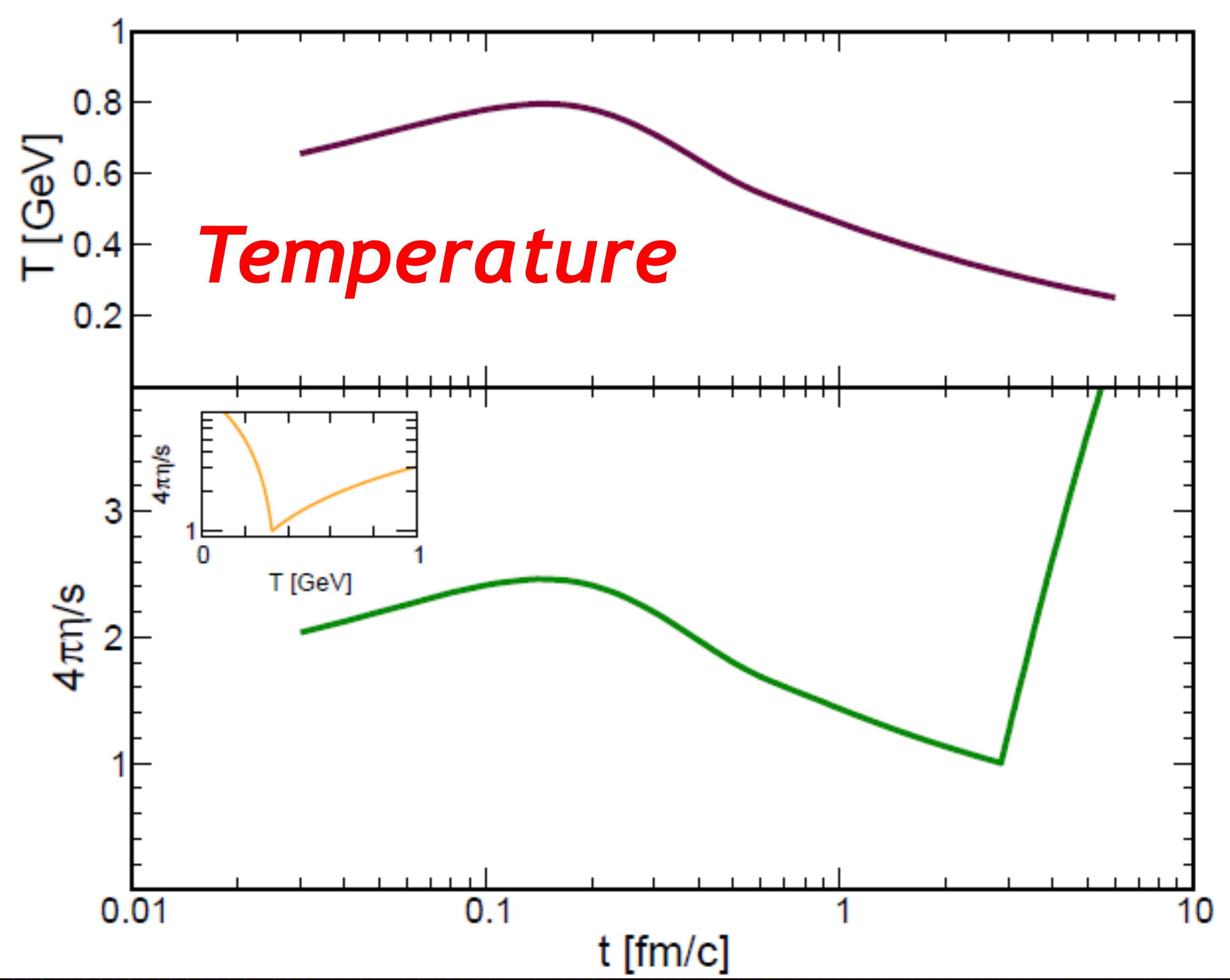
Plumari *et al.*, arXiv:1304.6566

Local temperature in realistic collisions evolves in time:

η/s should be time-dependent







Schwinger effect in Chromodynamics

Abelian *Flux Tube Model*

(.) assume ***color-electric fields*** evolve as ***classical abelian fields***

We did this choice just for sake of simplicity:
This is the *easiest way* to implement an initial classical field configuration and couple it to the kinetic equations.

There is, however, an argument (Florkowski, 2010)
Assume there is a gauge in which the *large classical initial field* is along directions 3 and 8:

$$
\begin{aligned}
F^{\mu\nu} \rightarrow \tilde{F}^{\mu\nu} &= \tilde{F}[A_3, A_8] \\
&= (\partial^\mu A_3^\nu - \partial^\nu A_3^\mu) + (\partial^\mu A_8^\nu - \partial^\nu A_8^\mu)
\end{aligned}
$$

The *initial quantum fluctuations* can develop in the full color space, but they are neglected since they are small compared to the classical part of the initial gluon field.
Then, the Schwinger effect:

(.) it produces gluon quanta in the "full" adjoint color space.
(.) gluons play the role of quantum fluctuations on the top of the classical field, with dynamics governed by kinetic equations.
(.) gluon interactions with classical field occur via *currents in the MEs*.