Photons from partonic transport

Hirschegg workshop

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with Kai Zhou, Florian Senzel, Hendrik v. Hees, Carsten Greiner, Zhe Xu

Dynamical photon production in the QGP

Overview:

1)Photons from Heavy Ion Collisions

Physical motivation, open questions, usual approaches

2) BAMPS: $3+1d$, $N_f = 3$ transport code

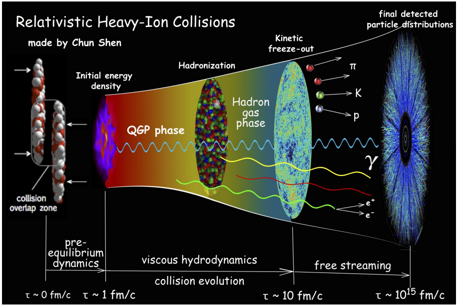
Solving of the Boltzmann equation

3) QGP Photonproduction from BAMPS

Diagrams, problems, results, future projects

If you have any ideas or questions, feel free to interrupt! :-)

Light in heavy ion collision



Direct photons come from:

1. Nucleus-Nucleus scattering
2. **QGP: nonequilibrium**
3. **QGP: equilibrium**
4. photons from hot hadron gas

Interests:

- Photon p_T-spectra from different phases
- Photon momentum anisotropies
- Contribution of weights in average anisotropy
- $\Rightarrow$ (Non)-Thermal production mechanisms?
  $\Rightarrow$ Spacetime evolution well understood?
  $\Rightarrow$ Role of chemical/kinetical nonequilibrium?

How to study dynamical direct ($\neq$ decay) photons

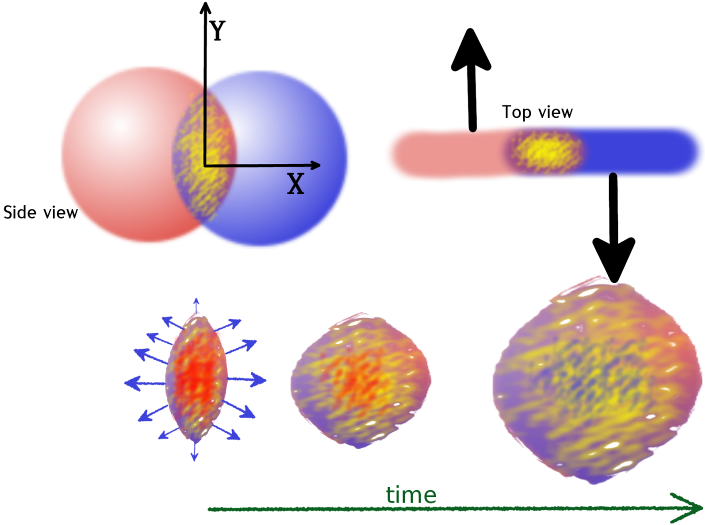
Usual approach to study photons from heavy ion collisions:

1. A **model** (e.g. hydro, fireball,...) gives temperature & velocity evolution for the full collision
2. Theory: analytic **equilibrium** γ **production rate** R
3. Integrate:
   Spectrum$= E\frac{\mathrm{d}N}{\mathrm{d}^3p} = \int_{\text{spacetime evolution}} \mathrm{d}^4x \, (R)_{\text{Local rest frame}}$

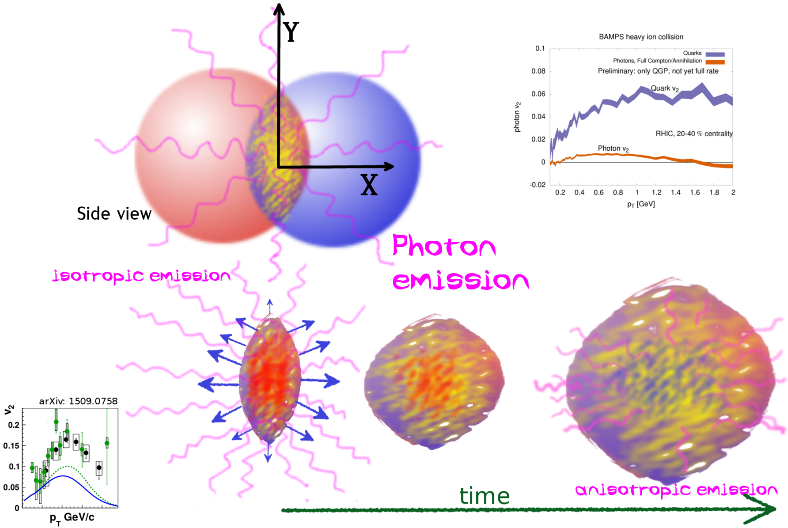
Other possibility: microscopic parton collisions make photons (our approach)

- Partonic cascade: **B**oltzmann **A**pproach to **M**ulti **P**arton **S**catterings **BAMPS**
- Spacetime evolution and γ-production in common framework
- Microscopic γ production using pQCD matrix elements
- QCD effects: LPM eff. modelled, (+ Debye masses from lattice?)

Elliptic flow for photons: Transfer of anisotropy from quarks to photons

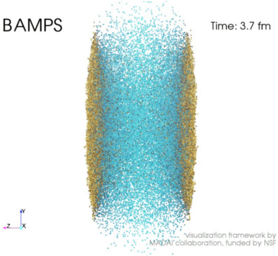


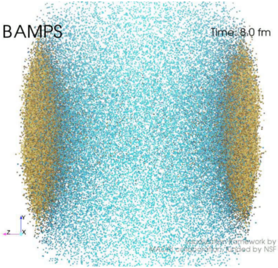
Elliptic flow for photons: Transfer of anisotropy



Our approach (in short): BAMPS

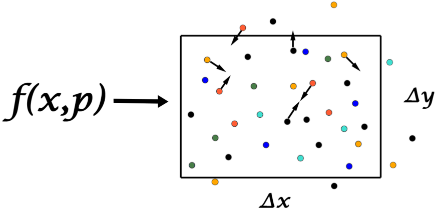
Boltzmann **A**pproach to **M**ulti **P**arton **S**catterings





$$p^{\mu}\partial_{\mu}f(x,p) = \mathcal{C}_{22}[f] + \mathcal{C}_{23}[f]$$

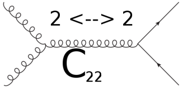
Zhe Xu & Carsten Greiner, 2005.
Phys. Rev. C 71 (2005) 064901.

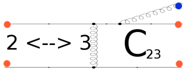


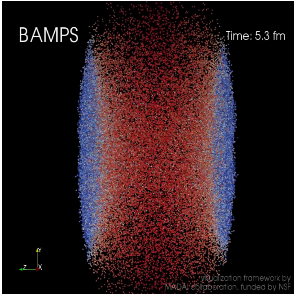
Many people have contributed to BAMPS:
...,Oliver Fochler, Jan Uphoff, Florian Senzel, MG, Kai Zhou + the Bejing group

BAMPS: Boltzmann Approach To Multi-Parton Scatterings

$$p^\mu \partial_\mu f(x,p) = \mathcal{C}_{22}[f] + \mathcal{C}_{23}[f]$$







Stochastic collision probability, Total cross sections $\sigma_{22}(s), \sigma_{23}(s)$, Fully Lorentz-invariant formulation

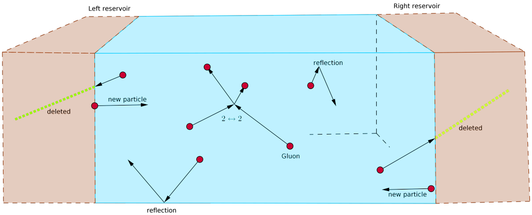
$$P_{22} = v_{rel} \frac{\sigma_{22}}{N_{test}} \frac{\Delta t}{\Delta^3 x}$$

$$P_{23} = v_{\text{rel}} \frac{\sigma_{23}}{N_{test}} \frac{\Delta t}{\Delta^3 x}, \quad P_{32} = \ldots$$

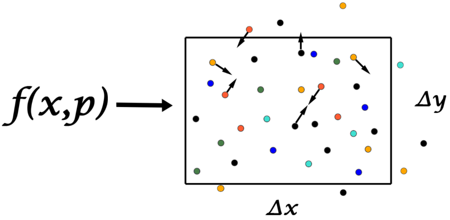
$$v_{\text{rel}} = \frac{s}{2E_1 E_2}$$

Z. Xu & C. Greiner, 2006.

BAMPS: run as a fixed box



Scattering in **cells**:



Baseline: $\frac{d\mathrm{N}}{NE^2 d\mathrm{E}} = \frac{1}{T^3} e^{-E/T}$

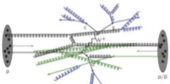
Box calculations useful:

- Test and compare effect of cross sections
- Extract Rates
- Extract transport coefficients

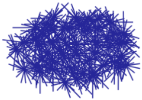
BAMPS: expanding Heavy-Ion collision

Can be used as fixed box, or expanding system

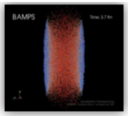
Steps:



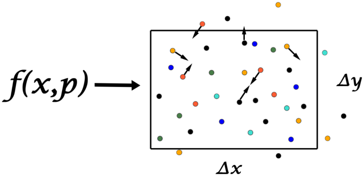
PYTHIA-Glauber



$AA = pp \times N_{\text{binary}}$



noneq. QGP



- onshell massless particles $q, \bar{q}, g$
- **New feature: Photons**
- radiative Gluons/Photons: LPM-suppression modelled
- Compton/Annihilation- γ-production
- Exact Bremsstrahlung $qq \longrightarrow qq\gamma$

BAMPS - many results in the past. Some highlights:

Many features and physical studys:

- Light quarks: up, down, strange; heavy quarks: charm, bottom, J/Ψ's, Photons
- Running coupling, Debye-screening dynamically
- Improved gluon radiation ("Gunion-Bertsch") matrix elements
- Bose-Einstein condensate (Phys.Rev.Lett. 114,182301 (2015))✓
- Jet quenching, energy loss ✓
- Elliptic flow v_2, nuclear modification factor R_{AA} ✓
- Mach cones, jet induced double-peak structure in $dN/d\phi$ ✓
- Momentum imbalance A_j of reconstructed Jets ✓
- Shear viscosity, heat conductivity, electric conductivity ✓

M.G. et al, Phys. Rev. E 87, 033019 (2013)
Heat Conductivity

C.Wesp et al., Phys. Rev. C 84, 054911 (2011)
Shear Viscosity

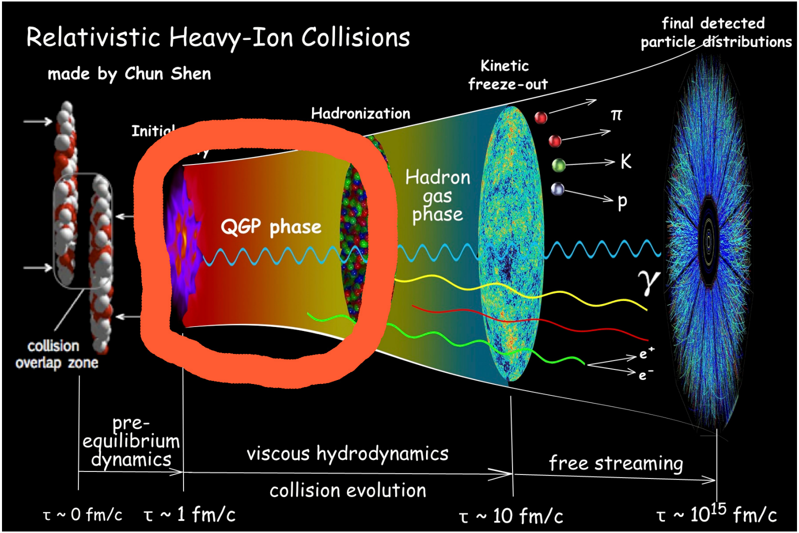
I.Bouras et al., Phys. Rev. C 90, 024904 (2014)
Mach Cones

M.G. et al, Phys.Rev. D 90, 094014 (2014)
Electric conductivity

J.Uphoff et al, Phys.Rev.Lett. 114, 112301 (2015)
$R_{AA} + v_2$

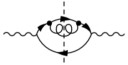
F.Senzel et al., J.Phys. G42, 115104 (2015)
Momentum imbalance of reconstructed Jets

Photons from the QGP

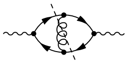


Photon production: which diagrams contribute?

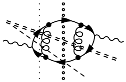
Photon rate at order $\mathcal{O}(e^2 g_s^2 T^4)$ obtained via γ-self energy.
Cutting rules give scattering matrix elements:



$2 \leftrightarrow 2$ processes, gives $|\mathcal{M}|^2$



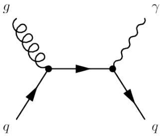
$2 \leftrightarrow 2$ processes, gives channel interference



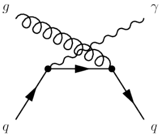
Bremsstrahlung /Inelastic pair annihilation

+ all cuts, all diagrams with infinitely more *gluon rungs*

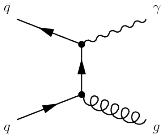
<u>$2 \leftrightarrow 2$:</u>
all in BAMPS



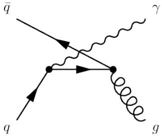
s-channel



u-channel



t-channel



u-channel

e.g. P. Arnold, G. D. Moore, and L. G. Yaffe, JHEP. 0111, 057 (2001)

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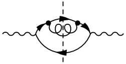
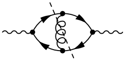
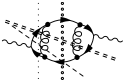


$2 \leftrightarrow 2$ processes, gives $|\mathcal{M}|^2$



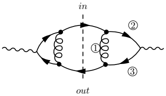
$2 \leftrightarrow 2$ processes, gives channel interference



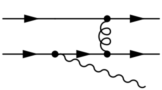
Bremsstrahlung /Inelastic pair annihilation

+ all cuts, all diagrams with infinitely more *gluon rungs*

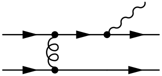
<u>$2 \rightarrow 3$:</u>



Cut for $2 \rightarrow 3$



Important contributions $2 \rightarrow 3$

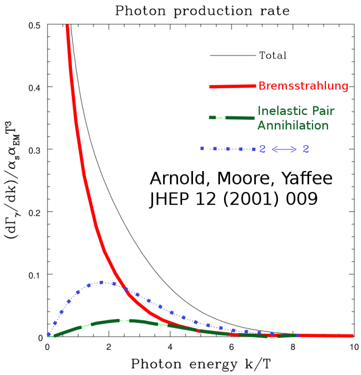


exact in BAMPS

+ $3 \rightarrow 2$, $1 \leftrightarrow 4$, ... not easily feasible in transport models

e.g. P. Arnold, G. D. Moore, and L. G. Yaffe, JHEP. 0111, 057 (2001)

How big is the influence of the diagrams?



$2 \leftrightarrow 2$ processes vs. $2 \leftrightarrow 3 + \dots$ processes

- Blue line: Included in BAMPS
- Red line: Only approximately included in BAMPS
- Bremsstrahlung in QGP as important as Compton-Scattering/Quark-Antiquark photoproduction!

see also NLO corrections: J.Ghiglieri, A. Kurkela et al. JHEP 1305 (2013) 010

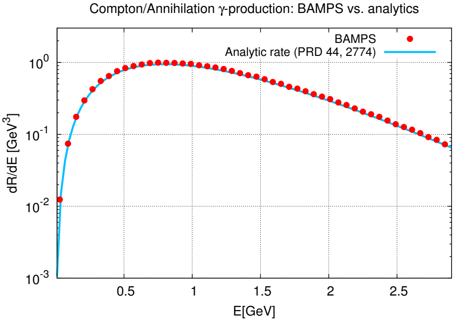
Compton/Annihilation ($2 \leftrightarrow 2$) photon production in BAMPS

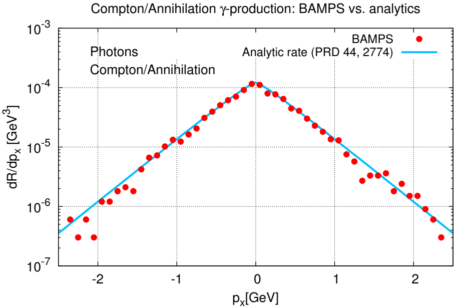
Comparison with fixed IF-cutoff

Analytic γ rate equation = BAMPS result

see Kapusta, Lichard, Seibert, Phys. Rev. D 44, 2774 (1991)

Also in boosted case perfect! (e.g. $u^\mu = (1, \beta_x, 0, 0)$)





Compton/Annihilation photon production in BAMPS

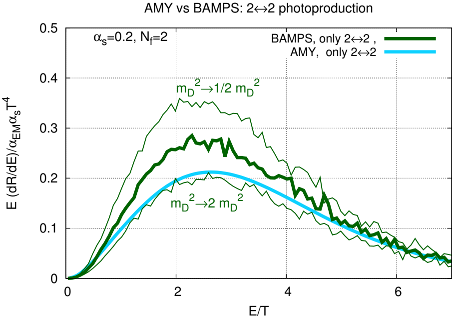
...original treatment valid for high energies (neg. Log):

Kapusta, Lichard, Seibert, Phys. Rev. D 44, 2774 (1991)

...correct treatment valid for all energies:

Arnold, Moore, Yaffe, JHEP. 0111, 057 (2001)

BAMPS: Debye-screening of quark propagators:



$$m_{D,q}^2 = 4\pi\alpha_s \frac{N_c^2 - 1}{2N_c} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \frac{1}{E} \left(f_{\text{gluon}} + f_{\text{quark}}\right)$$

$m_{D,q}^2$: Quark Debye-mass squared

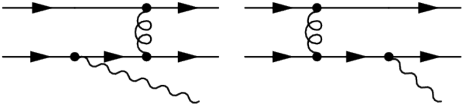
f_i: distribution of particle species i

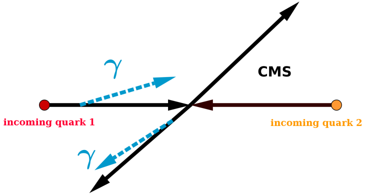
N_c: No. of colors = 3

Resummed result (AMY) within screening uncertainty ($m_{D,q}^2 \times 2^{\pm 1}$)

Possible to include lattice data for Debye-mass (equilibrium extrapolation)

Photon Bremsstrahlung processes: Some details





Useful coordinates for radiated photon:

Reference: incoming quark 1 $p_z > 0$

y: rapidity wrt incoming quark 1, $y = \frac{1}{2} \ln \frac{E+p_z}{E-p_z}$

$k_\perp$: transverse momentum of photon wrt to p_z

$q_\perp$: gluon momentum transfer

φ: $\measuredangle(\vec{q}_\perp, \vec{k}_\perp)$

- We use the **exact** pQCD computation of $|\mathcal{M}_{\mathrm{brems}}|^2$ with screened quark and gluon propagators
- Inelastic pair annihilation neglected

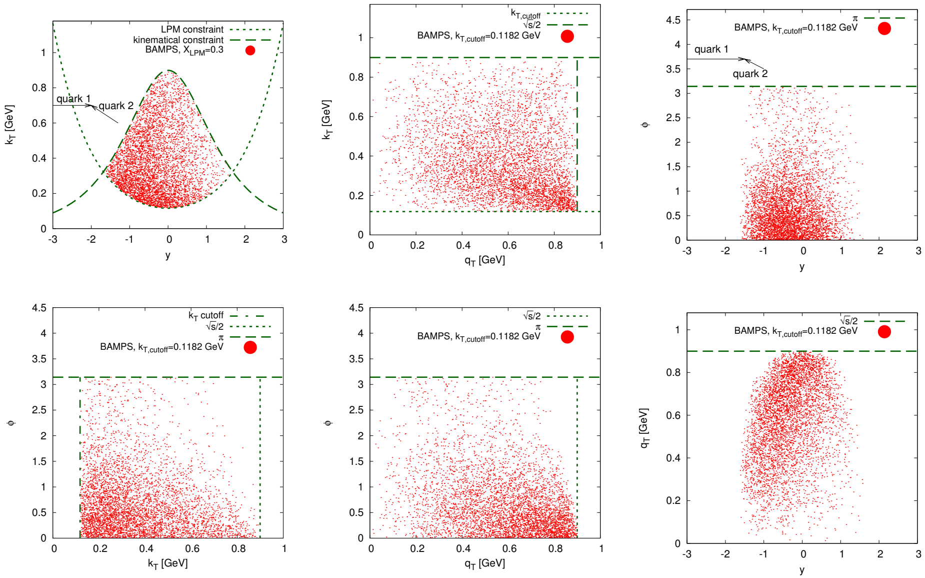
Inelastic cross section for radiated photons

Exact matrix element computed, coordinate transformation from $P_{\text{in }1}, P_{\text{in }2}, P_{\text{out }1}, P_{\text{out }2}, K \rightarrow$ integrate cross section:

$$\sigma_{23} = \frac{1}{2s} \int_{p_1'} \int_{p_2'} \int_{p_3'} \int_{p_1} \int_{p_2} \left|\mathcal{M}_{12\rightarrow 1'2'3'}\right|^2 (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_1' - p_2' - p_3')$$
$$= \frac{1}{256\pi^4 s} \int \mathrm{d}^2 q_\perp \int \mathrm{d}^2 k_\perp \int \mathrm{d}y \int \mathrm{d}\phi \left|\mathcal{M}_{12\rightarrow 1'2'3'}\right|^2 \mathcal{J}(k_\perp, q_\perp, y, \phi)$$

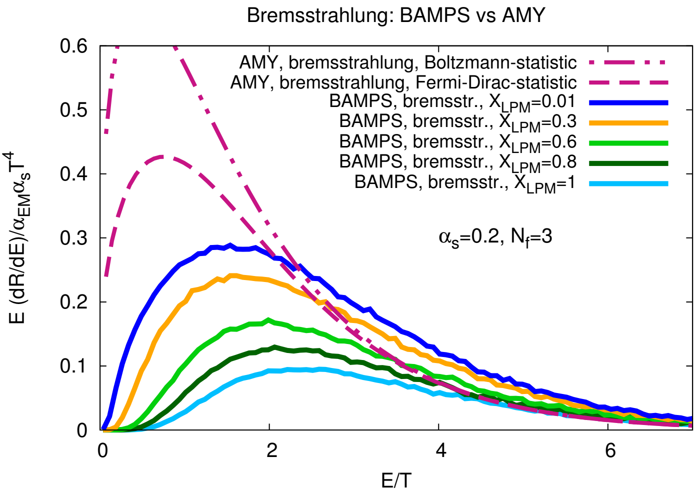
- For each particle pair in cell: compute σ_{23}
- $\left|\mathcal{M}_{12\rightarrow 1'2'3'}\right|^2 (P_{\text{in }1}, P_{\text{in }2}, k_\perp, q_\perp, y, \phi)$
- *VEGAS* integration algorithm
- If collision happens: sample outgoing momenta with *Metropolis*-algorithm according to $|\mathcal{M}|^2$
- Numerically very demanding, needs Lookup-Tables.

Numerical sampling of the outgoing photon momenta

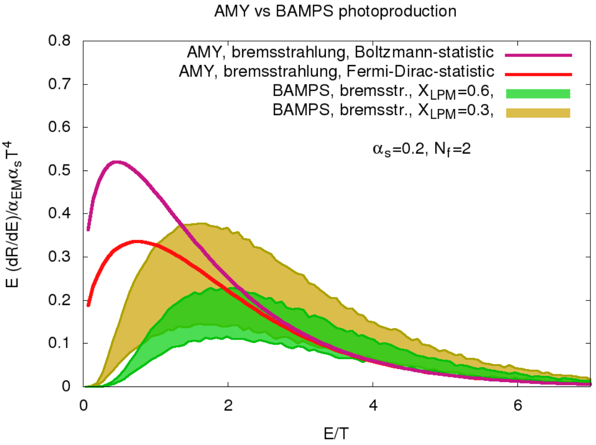


First results from BAMPS for Bremsstrahlung

Interference can only be treated phenomenologically:
$|\mathcal{M}_{23}|^2 \rightarrow |\mathcal{M}_{23}|^2\,\Theta(\lambda_{\mathrm{mfp}} - X_{\mathrm{LPM}}\tau_{\mathrm{formation}})$, we vary X_{LPM}. $\tau_{\mathrm{f}} \sim k_T^{-1}$.

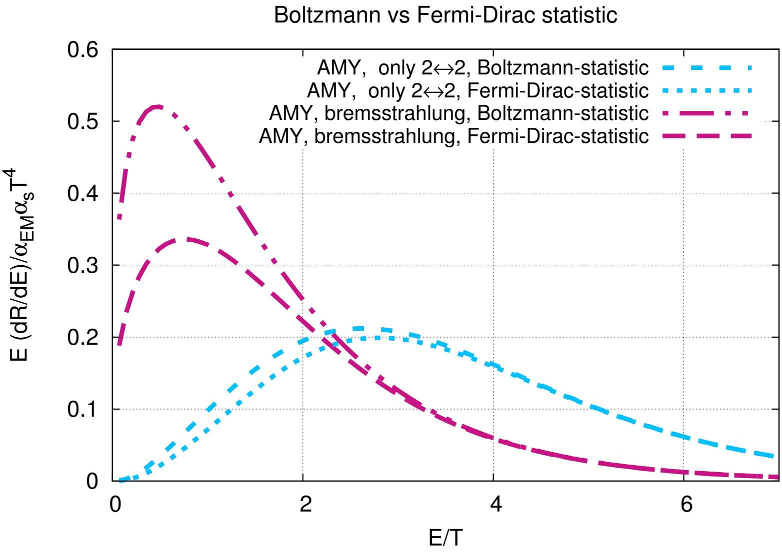


Debye-screening uncertainty for Bremsstrahlung



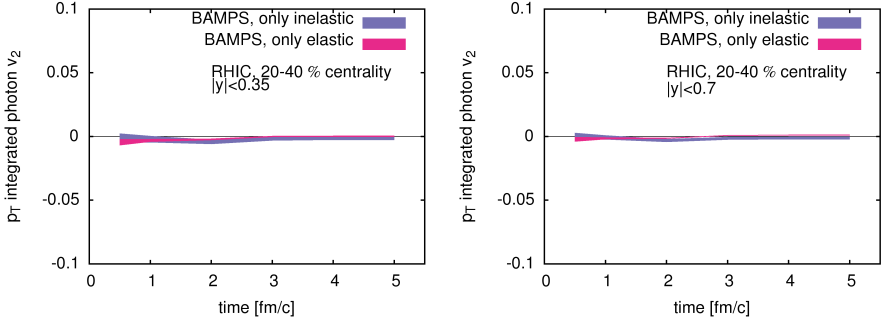
Band: Debye-mass $\times 2^{\pm 1}$. Future: use Debye-mass from lattice.

Type of statistic more important for bremsstrahlung



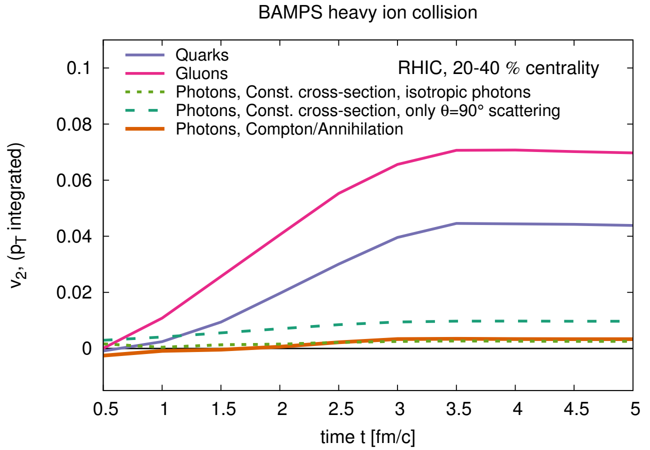
Anisotropic flow in QGP

Elastic and radiative photon production shows no $v_2^{\gamma,\mathrm{QGP}}$ ...



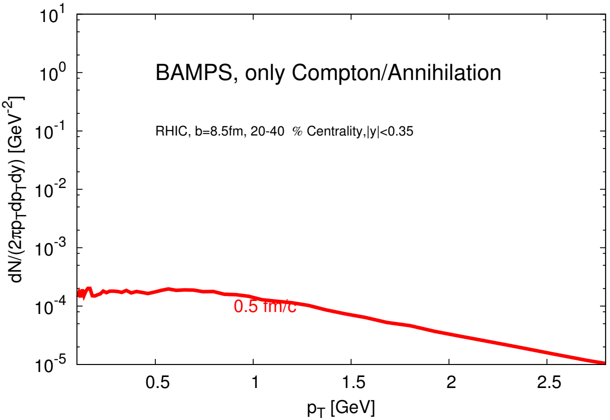
Anisotropic flow in QGP

Explanation: Quark/Gluon anisotropic flow builds up with time:



Anisotropic flow in QGP

Explanation: Quark/Gluon anisotropic flow builds up with time.
Photon production **ceases with time**:



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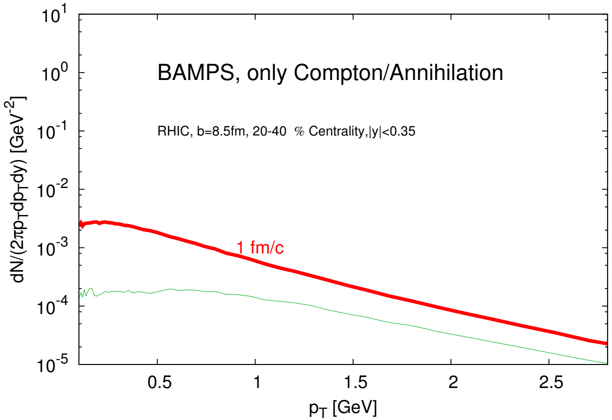

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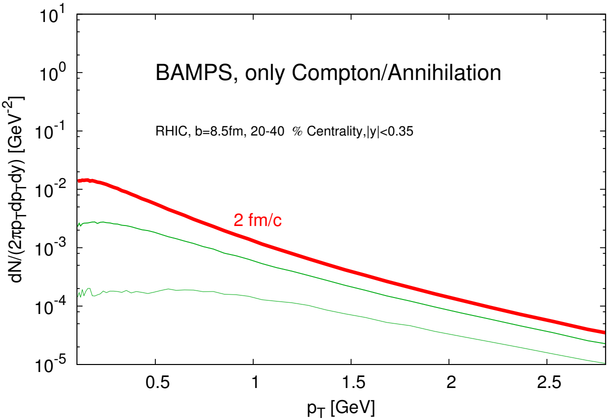

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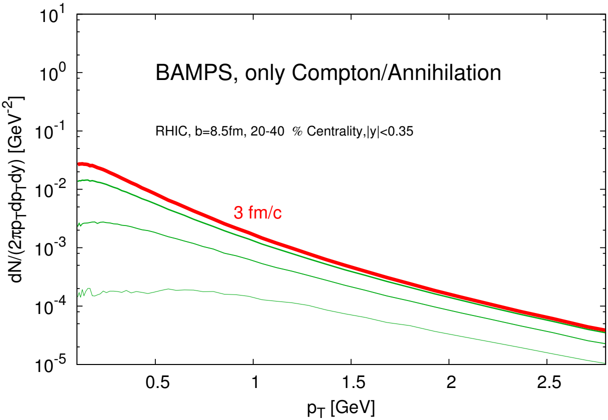

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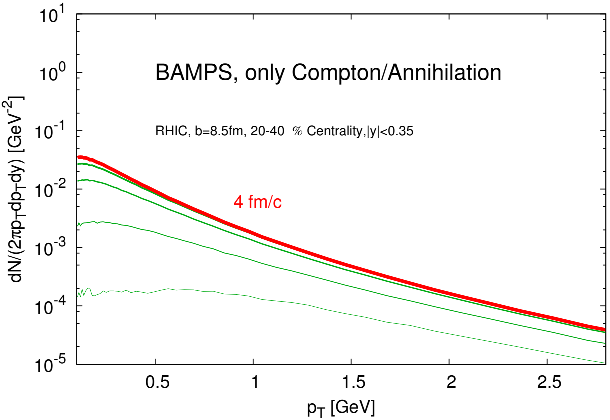

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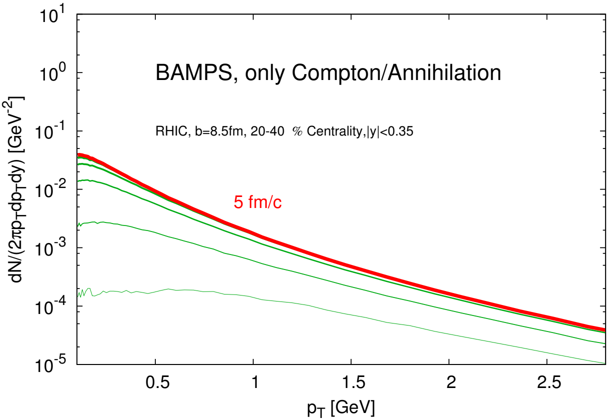
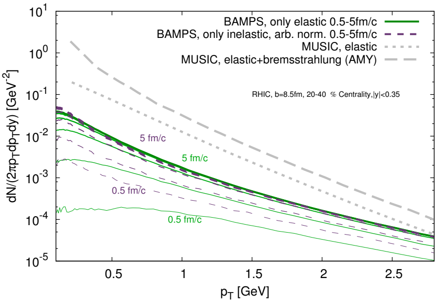
Photon production ceases with time:



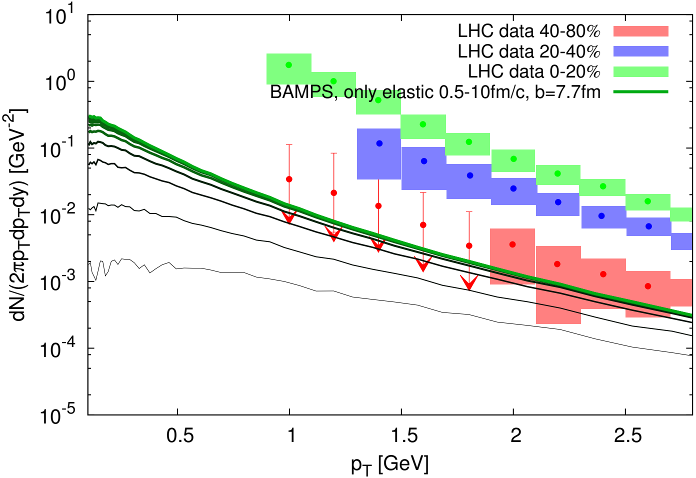
p_T-spectra from BAMPS



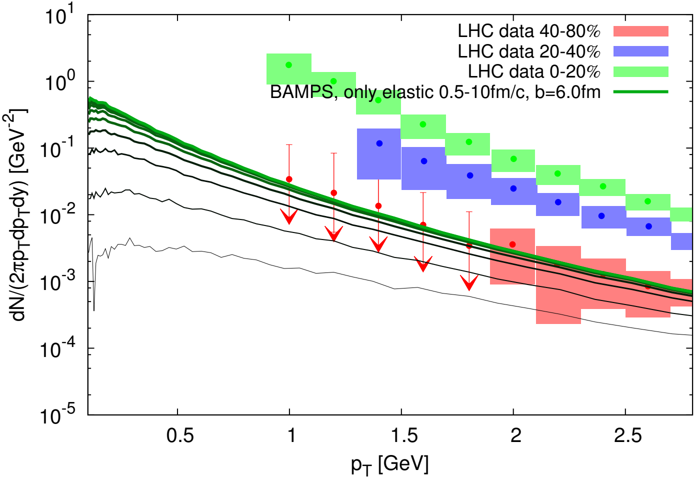
Still some problems: results not conclusive

- Radiative processes: 12 days on cluster
- Mean free path not yet correct
- No hadronic photons (Prompt photons not plotted)
- Need to include running coupling

p_T-spectra from BAMPS



p_T-spectra from BAMPS



What comes next? + Conclusions

Jet-Photon radiation

BAMPS perfectly suited for radiative $\gamma's$ from Jets + Analytic solution known (AMY-Kernel). Influence on observables?

Numerical improvements

Lookup-Tables for cross sections, Better LPM-tuning, Debye-masses

...+some other ideas

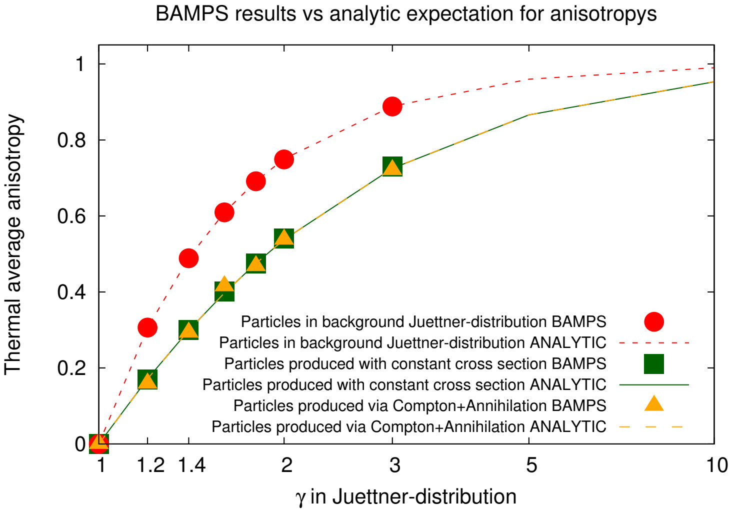
Conclusions

1. **Full leading order** γ production in microscopic transport
2. Elastic: good
3. Radiative: Only approximative right, some tuning
4. BAMPS shows no v_2^γ from QGP (independent on process!)
5. Microscopic understanding of v_2 transfer

Thank you for your attention and Hals und Beinbruch!

APPENDIX

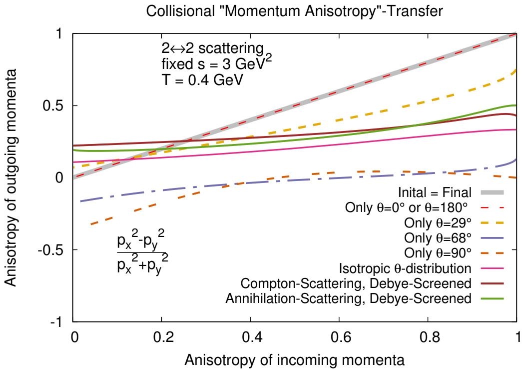
How anisotropy for photons gets lost



Change of momentum anisotropy in individual collisions

$$\langle \text{anisotropy}_{\mathbf{in}} \rangle \xrightarrow{\textbf{Boost}} \text{CM scattering } \frac{d\sigma}{d\Theta_{\text{CM}}} \xrightarrow{\textbf{Boost}} \langle \text{anisotropy}_{\mathbf{out}} \rangle \, (\Theta)$$

As an example, one **particular** momentum configuration:



Transfer of momentum anisotropy in equilibrium

Anisotropy of quark/gluon distribution

Fix $\frac{\mathrm{d}N}{\mathrm{d}^3\vec{p}} \sim \exp(-p^\mu u_\mu / T)$ equilibrium with boost: $\gamma > 1$

Analytic: $v_2 = \int \mathrm{d}^3\vec{p}\, \frac{\mathrm{d}N}{\mathrm{d}^3\vec{p}} \left(\frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right) / \int \mathrm{d}^3\vec{p}\, \frac{\mathrm{d}N}{\mathrm{d}^3\vec{p}}$

BAMPS: Average $\frac{p_x^2 - p_y^2}{p_x^2 + p_y^2}$ over all partons

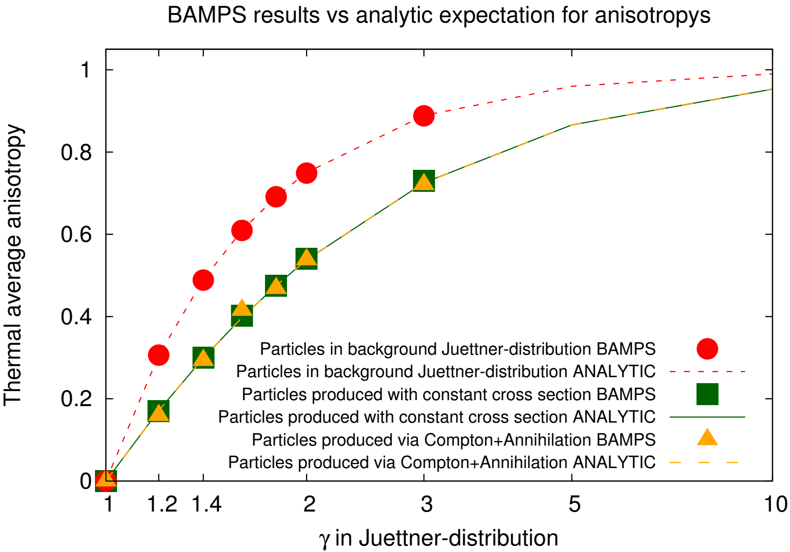
Anisotropy of produced photons

Photon production rate: $E \frac{\mathrm{d}R}{\mathrm{d}^3\vec{p}} = \mathrm{function}(p^\mu u_\mu, T)$ same $\gamma > 1$

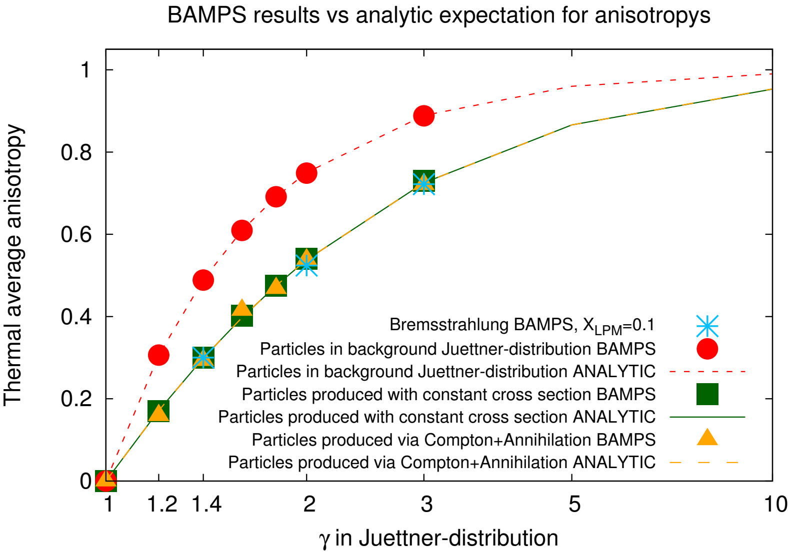
Analytic: $v_2 = \int \mathrm{d}^3\vec{p}\, \frac{\mathrm{d}R}{\mathrm{d}^3\vec{p}} \left(\frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right) / \int \mathrm{d}^3\vec{p}\, \frac{\mathrm{d}R}{\mathrm{d}^3\vec{p}}$

BAMPS: Average $\frac{p_x^2 - p_y^2}{p_x^2 + p_y^2}$ over all photons

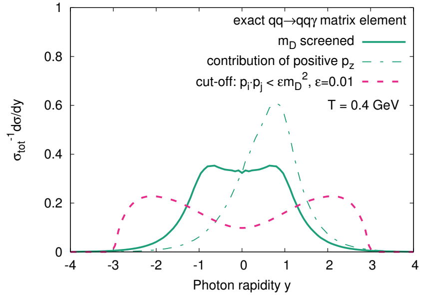
Transfer of momentum anisotropy in equilibrium

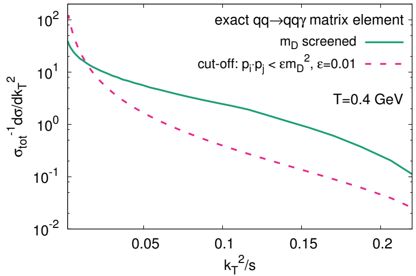


Transfer of momentum anisotropy in equilibrium

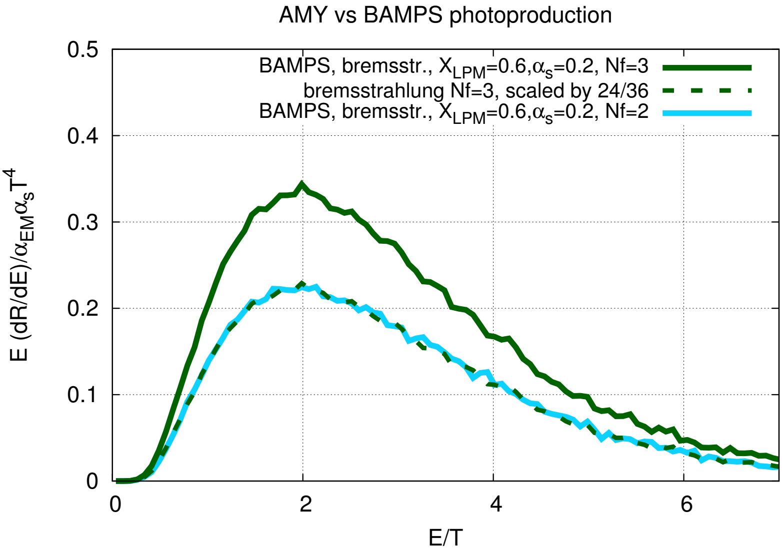


Differential cross sections for radiative photon emission:





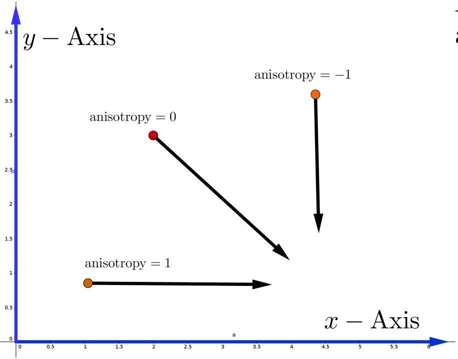
Trivial dependence on number of flavours...



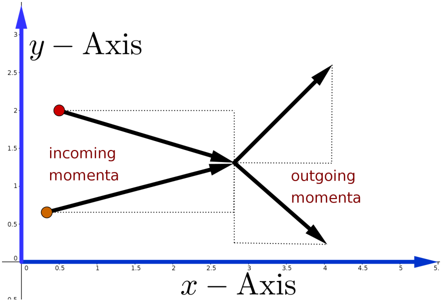
Elliptic flow in partonic transport simulations

"v_2" (rather: momentum anisotropy) can be studied per particle:

$$\text{"}v_2\text{"} = \frac{1}{N} \sum_{i=1}^{\text{all particles } N} \frac{p_{i,x}^2 - p_{i,y}^2}{p_{i,x}^2 + p_{i,y}^2} = \left\langle \frac{p_{i,x}^2 - p_{i,y}^2}{p_{i,x}^2 + p_{i,y}^2} \right\rangle_{\text{all particles}}$$



Example: anisotropy can be *lost* after collision:



Elliptic flow in partonic transport simulations

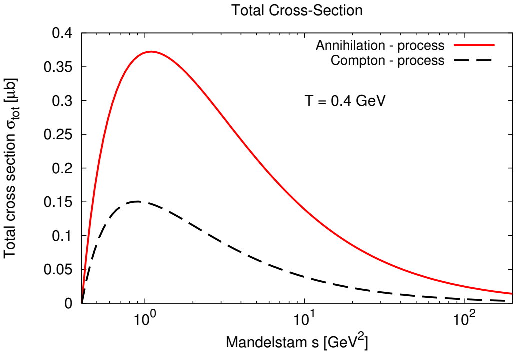
$$"v_2" = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle_{\text{all particles}}$$

Probability for collision:
s-dependent!

$$P_{22} = \sigma_{22} \frac{s}{2E_1 E_2} \frac{\Delta t}{\Delta V N_{\text{test}}}$$

<u>Two effects:</u>

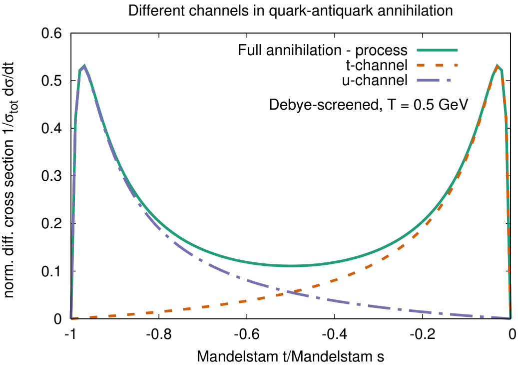
- *Total* cross section: which particles *do* collide?
- *Differential* cross section: preferred scattering angle Θ

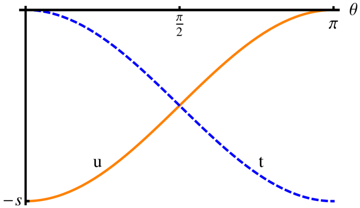


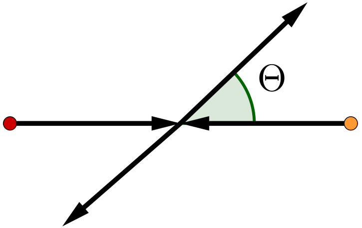
Elliptic flow in partonic transport simulations

Specific process: **distribution** of scattering angles

Mandestam $t = -\frac{s}{2}\left(1 - \cos\Theta_{\rm CM}\right)$





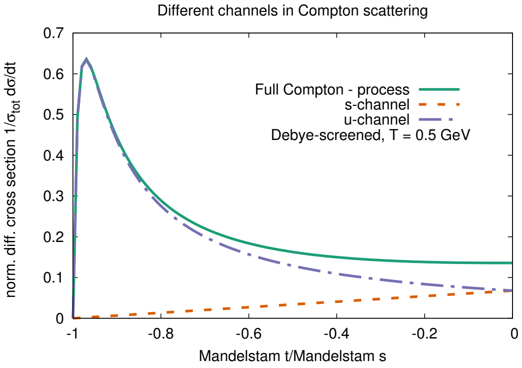


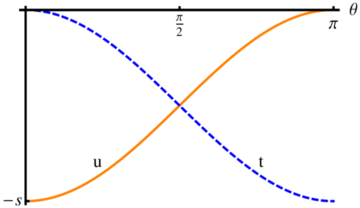
- Total cross section and $v_{\rm rel}$ "select" collision partners
- v_2 of final particles depends on $d\sigma/dt = |\mathcal{M}|^2/16\pi s^2$

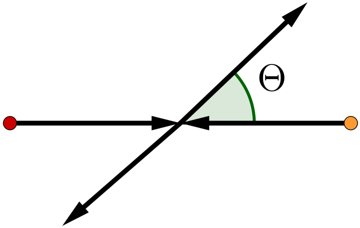
Elliptic flow in partonic transport simulations

Specific process: **distribution** of scattering angles

Mandestam $t = -\frac{s}{2}\left(1 - \cos\Theta_{\mathrm{CM}}\right)$







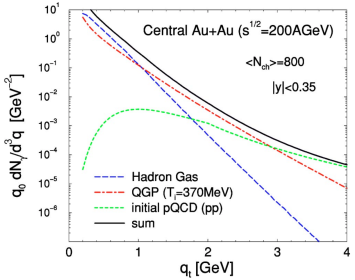
- Total cross section and v_{rel} "select" collision partners
- v_2 of final particles depends on $d\sigma/dt = |\mathcal{M}|^2/16\pi s^2$

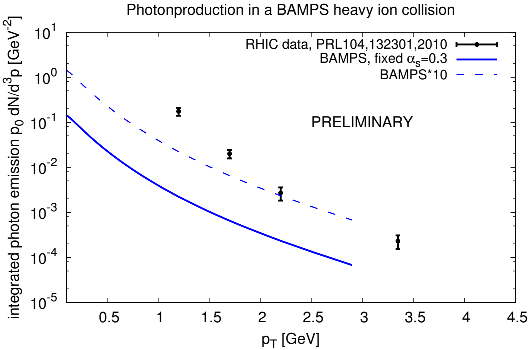
Photon Yield:

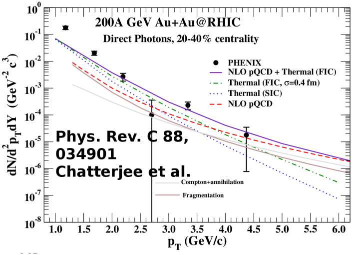
Explain difference to data:

- Only elastic leading order processes
- Only QGP contribution
- 1.5 – 2.5 GeV QGP window?

PHYSICAL REVIEW C **69**, 014903 (2004)
SIMON TURBIDE, RALF RAPP, AND CHARLES GALE







So far in the microscopic partonic transport model BAMPS:

$2 \leftrightarrow 2$

$gg \to gg$	
$gg \to q\bar{q}$	
$q\bar{q} \to gg$	$q\bar{q} \to q'\bar{q}'$
$qg \to qg$	$\bar{q}g \to \bar{q}g$
$q\bar{q} \to q\bar{q}$	
$qq \to qq$	$\bar{q}\bar{q} \to \bar{q}\bar{q}$
$qq' \to qq'$	$q\bar{q}' \to q\bar{q}'$

$2 \leftrightarrow 3$

$gg \leftrightarrow ggg$	
$qg \to qgg$	$\bar{q}g \to \bar{q}gg$
$q\bar{q} \to q\bar{q}g$	
$qq \to qqg$	$\bar{q}\bar{q} \to \bar{q}\bar{q}g$
$qq' \to qq'g$	
$qq' \to qq'g$	$q\bar{q}' \to q\bar{q}'g$

3 Flavours of quarks,antiquarks,gluons

New feature: Photons

- Compton/Annihilation - Debye screened
- Exact $qq \longrightarrow qq\gamma$ in near future
- LPM-suppression needs to be modelled

QGP in thermal equilibrium: Rates are known

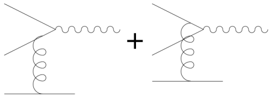
These 4 diagrams are used by

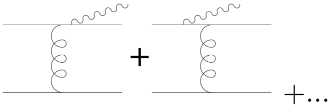
Kapusta, Lichard, Seibert, Phys. Rev. D 44, 2774 (1991)

$$E\,\frac{dR}{d^3p}\bigg|_{\text{Compton}} = \left(\sum_i q_i^2\right)\frac{2\alpha_{\text{EM}}\alpha_{\text{strong}}}{\pi^4}T^2 e^{-E/T}\ln\left(\frac{4ET}{k_c^2}+0.5-C_{\text{Euler}}\right)$$

$$E\,\frac{dR}{d^3p}\bigg|_{\text{Annih.}} = \left(\sum_i q_i^2\right)\frac{2\alpha_{\text{EM}}\alpha_{\text{strong}}}{\pi^4}T^2 e^{-E/T}\ln\left(\frac{4ET}{k_c^2}-1-C_{\text{Euler}}\right)$$

Not complete leading order! Missing contribution of collinear singularities of

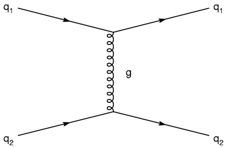




Inelastic Pair Annihilation and **Bremsstrahlung**

Leading Order pQCD Scattering in BAMPS

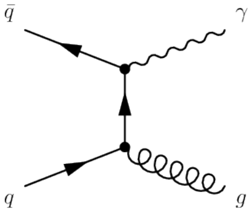
$1 + 2 \rightarrow 3 + 4$ scattering:



$$\frac{d\sigma}{dt} = \frac{|\mathcal{M}|^2}{16\pi s^2} \tag{1}$$

For example: $q + q' \rightarrow q + q'$

$$|\mathcal{M}|^2 = \frac{64\pi^2\alpha_s^2}{9}\left[\frac{u^2+s^2}{t^2}\right] \tag{2}$$



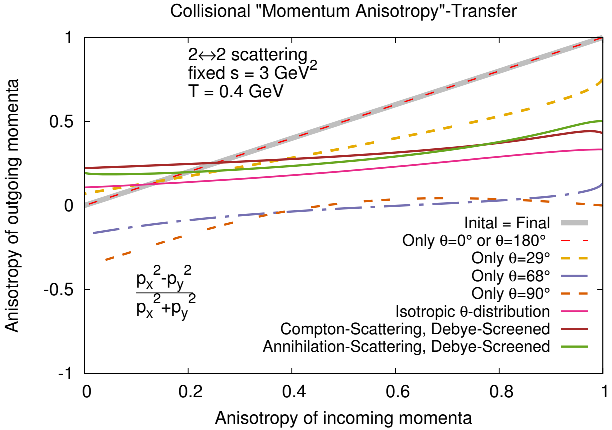
Photon prod. channel: $q + \bar{q} \rightarrow g + \gamma$

$$|\mathcal{M}|^2 = \frac{128}{9}\alpha_{\text{EM}}\alpha_s\pi^2\left[\frac{ut}{t^2}\right] \tag{3}$$

Debye-mass to screen infrared divergencies: $t^2 \rightarrow (t - m_D^2)^2$

Analytic study: momentum anisotropy in collisions

$$\langle \text{anisotropy}_{\mathbf{in}} \rangle \xrightarrow{\textbf{Boost}} \text{CM scattering } \frac{d\sigma}{d\Theta_{\text{CM}}} \xrightarrow{\textbf{Boost}} \langle \text{anisotropy}_{\mathbf{out}} \rangle \, (\Theta)$$



Leading Order pQCD Scattering in BAMPS

How the Debye-mass $m_{D,\mathbf{gluon}}/m_{D,\mathbf{quark}}$ can be calculated:

1. Boltzmann stat., thermal system $m_{\mathrm{D}}^2 = (3 + N_f)\frac{8}{\pi}\alpha_s T^2$
2. Implementation in BAMPS: Dynamically from the distribution

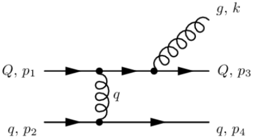
reconstruct the momentum distribution from all particles at each timestep

$$m^2_{\mathrm{D,gluons}} = 16\pi\alpha_s \int \frac{\mathrm{d}^3p}{p(2\pi)^3} \left(N_{\mathrm{color}} f_{\mathrm{gluon}} + N_{\mathrm{flavor}} f_{\mathrm{quark}}\right)$$

$$m^2_{\mathrm{D,quarks}} = 4\pi\alpha_s \frac{8}{6} \int \frac{\mathrm{d}^3p}{p(2\pi)^3} \left(f_{\mathrm{gluon}} + f_{\mathrm{quark}}\right)$$

Inelastic Scattering

elastic scattering with additional gluon radiation:



$$|\mathcal{M}_{23}|^2 = |\mathcal{M}_{22}|^2 \cdot 48\pi\alpha_s(k_\perp^2)(1-\bar{x})^2 \left[\frac{\vec{k}_\perp}{k_\perp^2} + \frac{\vec{q}_\perp - \vec{k}_\perp}{(\vec{q}_\perp - \vec{k}_\perp)^2 + m_\mathrm{D}^2(\alpha_s(k_\perp^2))}\right]$$

O.Fochler et al., Phys. Rev. D 88, 014018 (2013)
Radiative parton processes in perturbative QCD: An improved version of the Gunion and Bertsch cross section from comparisons to the exact result

LPM (Landau Pomeranchuk Migdal) effect is effectively modeled