FRG study of the chiral phase transition in a quark-meson model with (axial-)vector mesons

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Overview

1. Theoretical setup
   - Motivation
   - Extended linear sigma model and quark-meson model
   - Functional renormalization group (FRG)
2. Numerical results (preliminary)
   - Phase transitions
   - Summary and outlook

Phase diagram of quantum chromodynamics (QCD)

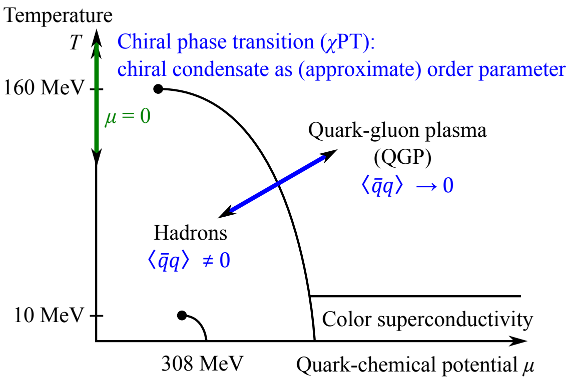

Figure 1: **Schematic QCD phase diagram and the χPT**; following [`Rischke Prog.Part.Nucl.Phys.52(2004)197`].

Accelerator facilities

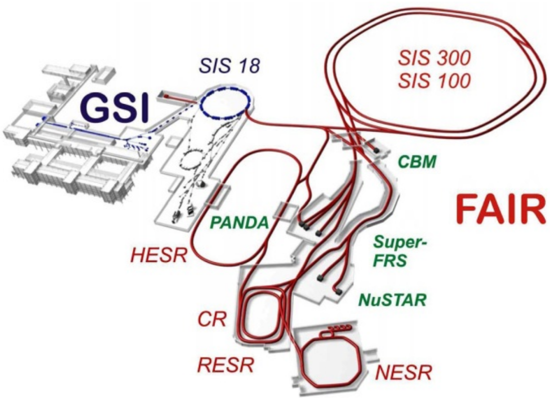

Figure 2: **Facility for Antiproton and Ion Research (FAIR)**; [Gianotti et al. arXiv:1307.4537 [physics.ins-det]].

Symmetry and symmetry breaking

Chiral symmetry of QCD

$$U(N_f)_L \times U(N_f)_R \cong SU(N_f)_V \times SU(N_f)_A \times U(1)_V \times U(1)_A$$

Symmetry breaking

1. **axial anomaly** breaks $U(1)_A$
2. **explicitly broken** for nonvanishing quark masses
3. **spontaneously broken** by the chiral condensate $\langle \bar{q}q \rangle \neq 0$

Theoretical implementation

1. extended linear sigma model (eLSM)
2. eLSM with quarks (quark-meson model)

Why is this study relevant?/What is new?

Vector mesons

Indicate the restoration of chiral symmetry in the QGP

Restoration of chiral symmetry in theoretical models

Mass degeneracy of **chiral partners**

Moreover...

- for a comparison with the **CBM** experiment
- Nonzero (T, μ) study of **(axial-)vector mesons** within the FRG missing
- alternative/complementary to lattice QCD

Models

Mesonic degrees of freedom:

- $N_f = 2$
- (Pseudo-)scalars (σ, $\vec{a}_0$, η, $\vec{\pi}$) and (axial-)vectors (ω, $\vec{\rho}$, f_1, $\vec{a}_1$)

Effective action Γ_k (UV-cutoff Λ):

$$\Gamma_\Lambda = \int_x \left[\frac{1}{2}\partial_\mu\phi\partial_\mu\phi + \frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}_{\mu\nu} + U_\Lambda(\bar{\xi}_1, \bar{\xi}_2, \bar{\xi}_3, \xi_3, \xi_4) - h\sigma \right.$$
$$\left. + \bar{\psi}\gamma_\mu\partial_\mu\psi + y\bar{\psi}\Sigma_5\psi\right],$$
$$\Sigma_5 = (\sigma_a + i\gamma_5\pi_a)t_a$$

($U(2)$-generators t_a, renormalization scale k)

Effective potential without axial anomaly

Truncation:

$$U_\Lambda(\xi_1, ..., \xi_4) = V_\Lambda(\xi_1) + W_\Lambda(\xi_1)\xi_2 + X_\Lambda(\xi_1)\xi_3 + Y_\Lambda(\xi_1)\xi_4$$

Invariants:

$$\begin{aligned}
\xi_1 &= \sigma^2 + \vec{\pi}^2 + \eta^2 + \vec{a}_0^2 \\
\xi_2 &= \left(\sigma^2 + \vec{\pi}^2\right)\left(\eta^2 + \vec{a}_0^2\right) - \left(\sigma\eta - \vec{\pi}\cdot\vec{a}_0\right)^2 \\
\xi_3 &= \left(\vec{\pi}\cdot\vec{a}_{1\mu} + \eta f_{1\mu}\right)^2 + \left(\vec{a}_0\cdot\vec{a}_{1\mu} + \sigma f_{1\mu}\right)^2 \\
&\quad + \left(\vec{\rho}_\mu \times \vec{a}_0 + \eta\vec{a}_{1\mu} + \vec{\pi} f_{1\mu}\right)^2 + \left(\vec{\pi}\times\vec{\rho}_\mu + \sigma\vec{a}_{1\mu} + \vec{a}_0 f_{1\mu}\right)^2 \\
\xi_4 &= f_{1\mu}^2 + \vec{a}_{1\mu}^2 + \omega_\mu^2 + \vec{\rho}_\mu^2
\end{aligned}$$

Effective potential with axial anomaly

Truncation:

$$U_\Lambda(\bar{\xi}_1, ..., \xi_4) = \bar{V}_\Lambda(\bar{\xi}_1) + \bar{W}_\Lambda(\bar{\xi}_1)\bar{\xi}_2 + \bar{X}_\Lambda(\bar{\xi}_1)\bar{\xi}_3 + \bar{Y}_\Lambda(\bar{\xi}_1)\xi_3 + \bar{Z}_\Lambda(\bar{\xi}_1)\xi_4$$

Invariants:

$$\begin{aligned}
\bar{\xi}_1 &= \sigma^2 + \vec{\pi}^2 \\
\bar{\xi}_2 &= \eta^2 + \vec{a}_0^2 \\
\bar{\xi}_3 &= (\sigma\eta - \vec{\pi} \cdot \vec{a}_0)^2 \\
\xi_3 &= (\vec{\pi} \cdot \vec{a}_{1\mu} + \eta f_{1\mu})^2 + (\vec{a}_0 \cdot \vec{a}_{1\mu} + \sigma f_{1\mu})^2 \\
&\quad + (\vec{\rho}_\mu \times \vec{a}_0 + \eta \vec{a}_{1\mu} + \vec{\pi} f_{1\mu})^2 + (\vec{\pi} \times \vec{\rho}_\mu + \sigma \vec{a}_{1\mu} + \vec{a}_0 f_{1\mu})^2 \\
\xi_4 &= f_{1\mu}^2 + \vec{a}_{1\mu}^2 + \omega_\mu^2 + \vec{\rho}_\mu^2
\end{aligned}$$

FRG

FRG flow equation

Integrate differential equation from $k = \Lambda$ to $k = 0$:

$$\partial_k \Gamma_k = \frac{1}{2} \mathrm{str} \left[\partial_k \mathbf{R}_k \left(\mathbf{\Gamma}_k^{(2)} + \mathbf{R}_k \right)^{-1} \right]$$

Local potential approximation: $\partial_k \Gamma_k \propto \partial_k U_k$

Grid method

- discretize effective potential U_k on the $\xi_1(\bar{\xi}_1)$-grid
- tune the UV potential such that the IR minimum lies at $\sigma_0 = \sqrt{\xi_{10}}(\sqrt{\bar{\xi}_{10}}) = f_\pi m_{a_1} / m_\rho$

χPT in the eLSM

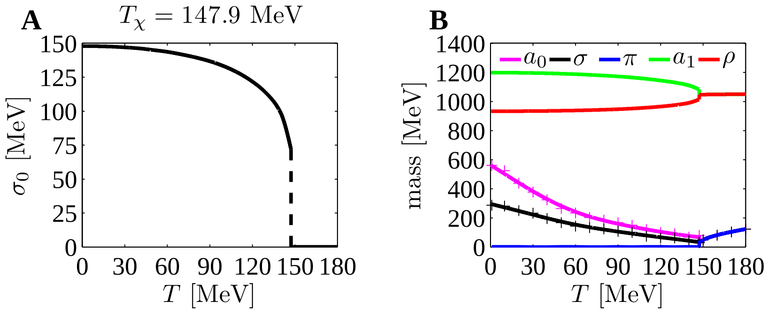

Figure 3: **1st order transition (A) and mass degeneracy (B) in the eLSM for vanishing quark masses and without axial anomaly**.
$m_{f_1} = m_{a_1}$, $m_\omega = m_\rho$, $m_\eta = m_\pi$;
[Eser, Grahl, Rischke Phys.Rev.D92(2015)096008].

χPT in the eLSM

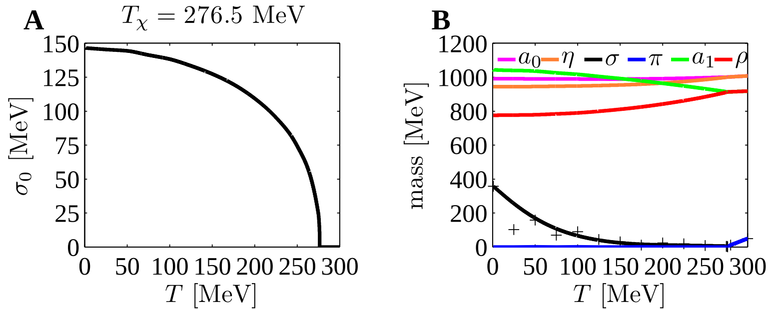

Figure 4: **2nd order transition (A) and mass degeneracy (B) in the eLSM for vanishing quark masses and with axial anomaly**.
$m_{f_1} = m_{a_1}$, $m_\omega = m_\rho$;
[Eser, Grahl, Rischke Phys.Rev.D92(2015)096008].

χPT in the eLSM

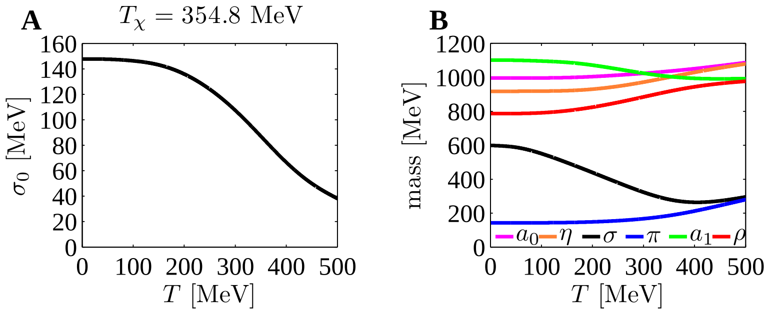

Figure 5: **Crossover transition (A) and mass degeneracy (B) in the eLSM with nonzero quark masses and axial anomaly**. $m_{f_1} = m_{a_1}$, $m_\omega = m_\rho$; [Eser, Grahl, Rischke Phys.Rev.D92(2015)096008].

$\rightarrow$ Need for improvement: unphysical transition temperature

Why to study a quark-meson model?

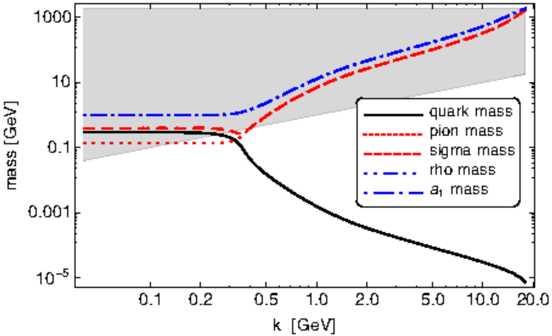

Figure 6: **Renormalized masses as a function of the RG-scale.** Reflects scale hierarchy; [Rennecke Phys.Rev.D92(2015)076012].

χPT in the eLSM with quarks (preliminary)

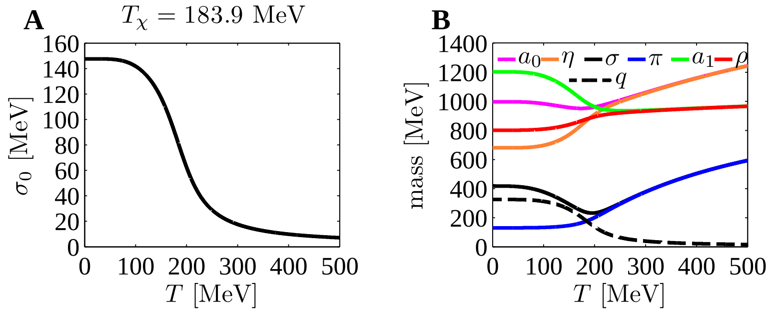

Figure 7: **Crossover transition (A) and mass degeneracy (B) in the eLSM with quarks**. $m_{f_1} = m_{a_1}$, $m_\omega = m_\rho$.

χPT in the eLSM with quarks (preliminary)

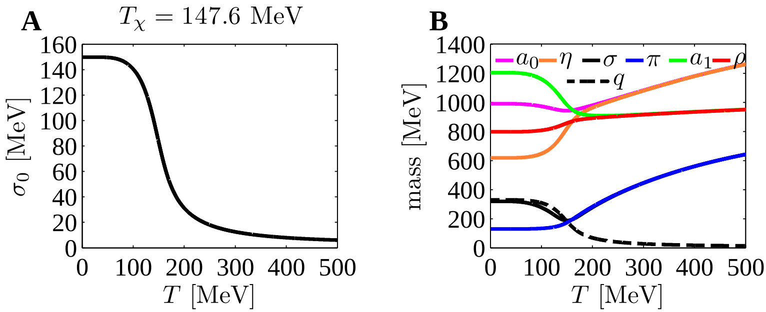

Figure 8: **Crossover transition (A) and mass degeneracy (B) in the eLSM with quarks**. $m_{f_1} = m_{a_1}$, $m_\omega = m_\rho$.

$\rightarrow$ Physical transition temperature

Summary and outlook

eLSM → eLSM with quarks

Achieved improvement

T_χ brought into **agreement** with lattice QCD

Note that...

tree level calculations in the eLSM favor the $\{f_0(1370), a_0(1450)\}$ resonances

Next steps

- calculations for $\mu \neq 0$ ($\bar{\psi}\mu\gamma_0\psi$); compute T-μ-diagram
- check quadratic order terms in the effective potential