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From spectral functions to viscosity in the Quark-Gluon Plasma

N.C., Haas, Pawlowski, Strodthoff: Phys. Rev. Lett. 115.112002, 2015

Hirschegg
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Outline

- Introduction

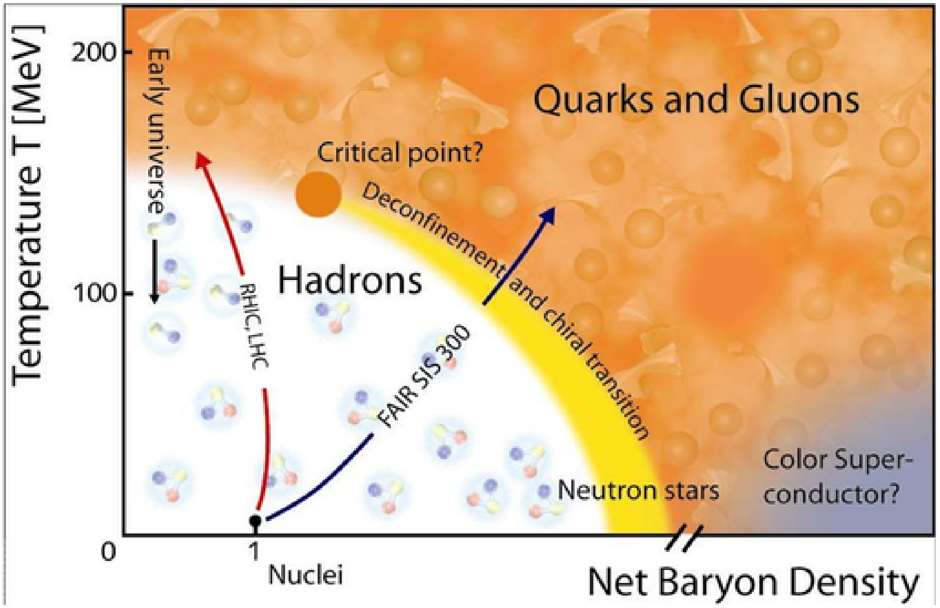
- Framework for transport coefficients in YM/QCD
  - Kubo formula
  - Correlators of Composite Operators
  - Spectral Functions in the Real-Time Formalism

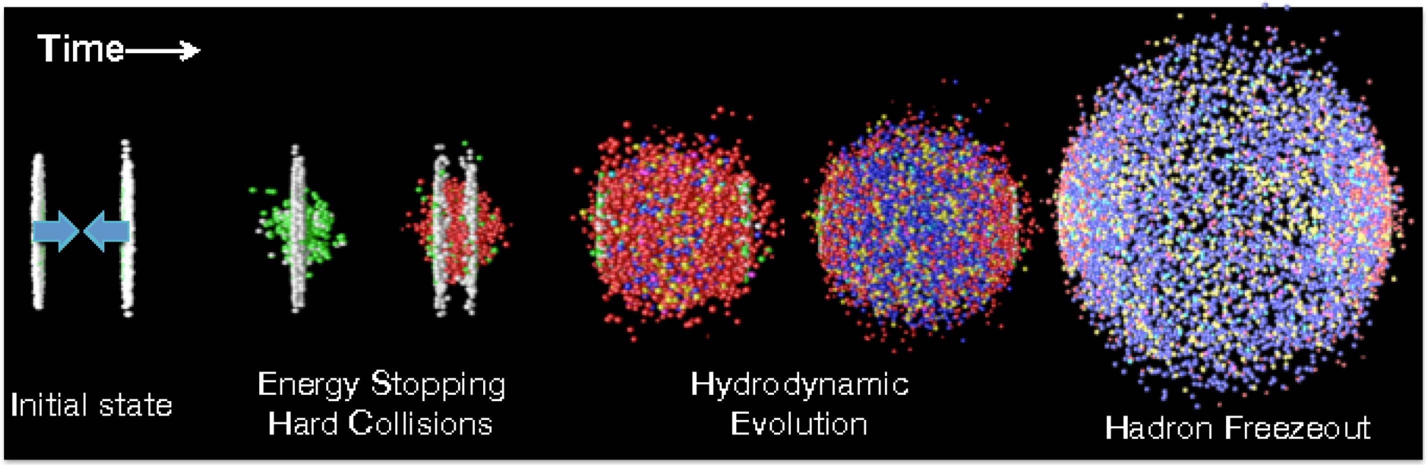
- Results for viscosity/entropy in YM/QCD

- Outlook

QCD phase diagram and HIC

- The QCD phase diagram and heavy-ion collisions





Heavy Ion Collisions and Viscosity

- Experimental data: **(viscous) hydrodynamics**

Elliptic flow coefficient: $\nu_2 = f\left(\frac{\eta}{s}\right)$ ← viscosity (η), entropy (s)

- Temperature dependence of $\frac{\eta}{s}$?

→ **phenomenology**

necessary input for
precision hydro-simulations!

→ **theory**

Interesting theory playground:

$\frac{\eta}{s}(T)$ in non-pert. regime

→ test your favourite method!

Kubo relation

- Application of the fluctuation dissipation theorem to (shear) viscosity:

$$\eta = \lim_{\omega\to 0} \frac{1}{20} \frac{\rho_{\pi\pi}(\omega, 0)}{\omega}$$

Kubo relation

with $\rho_{\pi\pi}(\omega, p) = \mathcal{FT}[\langle[\pi_{ij}(x), \pi_{ij}(0)]\rangle$

spectral function of the spatial, traceless energy-momentum tensor

- central object: spectral function of a composite operator $\pi(A)$ at finite T

Strategy:

Real-time !!!

→ representation of composite operators → magic formula

→ Real Time → Schwinger Keldysh, spectral representations

→ Gluon spectral functions → MEM

Composite Operators

- Expectation value of composite operator: $\Phi[\varphi]$

$$\langle \Phi_c[\varphi_a] \rangle = \Phi_c[G_{ab}\frac{\delta}{\delta\phi_b} + \phi_a]$$

magic formula!

propagator of φ

expectation $\langle \varphi \rangle = \phi$

→ „Dyson-Schwinger Equation for composite operators"

→ representation in terms of propagators and vertices of the fundamental field φ

→ RHS: finitely many diagrams with full propagators and vertices

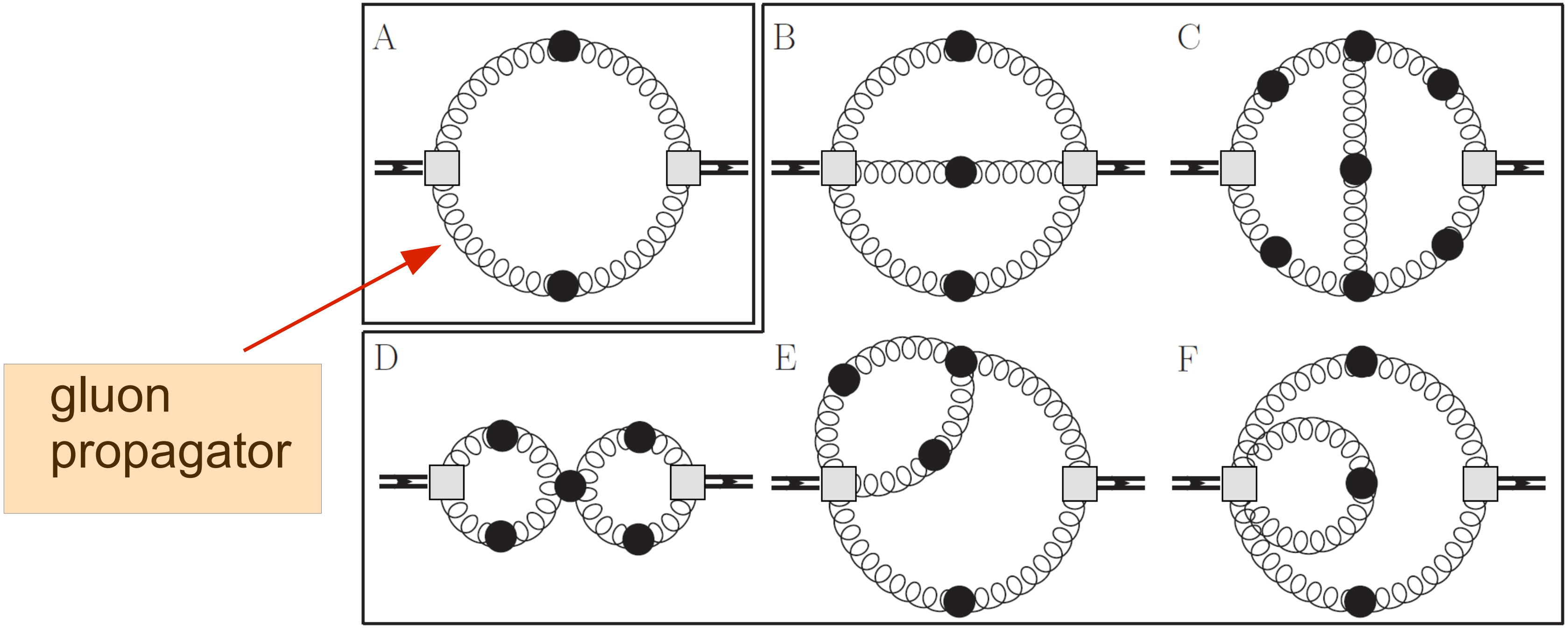
Correlator of the Energy-Momentum Tensor

for solving the Kubo relation

→ apply the magic formula to

$$\Phi_{ij}[\hat{A}] = \pi_{ij}(\hat{A})\pi_{ij}(\hat{A})$$

→ diagrammatic representation of $\langle \pi_{ij}(\hat{A})\pi_{ij}(\hat{A}) \rangle$ to two-loop order:



maximal loop order: 6

↓

resummation: 3 loop

Schwinger-Keldysh Formalism I

- the expectation values are all defined at real-time!

  $\neq$ Euclidean correlation functions!

- what we need: real-time correlation functions at finite temperature!

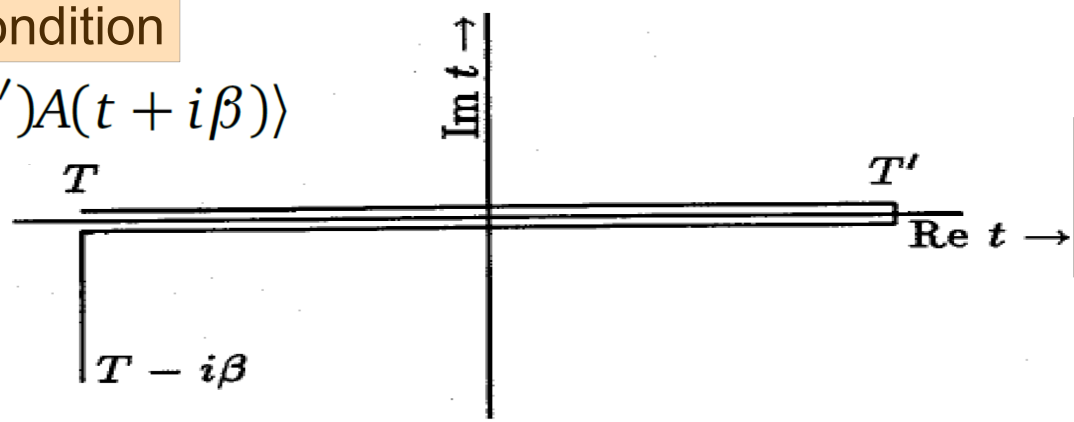
→ Schwinger-Keldysh Formalism

→ in general: non-equilibrium formulation

→ generating functional: $$Z[J_c] = \int \mathrm{d}\phi_c e^{i\int_c \{\mathcal{L}_c + J_c\phi_c\}}$$

equilibrium: KMS condition

$$\langle A(t)B(t')\rangle = \langle B(t')A(t+i\beta)\rangle$$



Schwinger-Keldysh contour C

Schwinger-Keldysh Formalism II

- doubling of degrees of freedom (doubling of Hilbert space)

→ fields ϕ_+ and ϕ_- with domains C_+ and C_-

upper contour branch (→ C_+) lower contour branch (→ C_-)

- N-point functions carry branch indices

→ tensors of rank N with respect to $\{+,-\}$

- e.g.: two-point function

$$G = \begin{pmatrix} G_{++} & G_{+-} \\ G_{-+} & G_{--} \end{pmatrix}$$

$$G_{-+}(x,y) := -i\,\langle \varphi_-(x)\varphi_+(y)\rangle =: G_>(x,y)$$
$$G_{+-}(x,y) := -i\,\langle \varphi_-(y)\varphi_+(x)\rangle =: G_<(x,y)$$
$$G_{++}(x,y) := -i\,\langle T\varphi_+(y)\varphi_+(x)\rangle =: G_F(x,y)$$
$$G_{--}(x,y) := -i\,\langle \tilde{T}\varphi_-(y)\varphi_-(x)\rangle =: G_{\tilde{F}}(x,y)$$

Spectral Functions

- Spectral function: Representation in the +/- formalism

$$\rho = G_{-+} - G_{+-}$$

spectral representations: some principal value integral...

$$\begin{aligned} G^{\pm\pm}(\omega,\vec{p}) &= F(\omega,\vec{p}) \pm i\left(n(\omega)+\tfrac{1}{2}\right)\rho(\omega,\vec{p}), \\ G^{+-}(\omega,\vec{p}) &= -i\,n(\omega)\rho(\omega,\vec{p}), \\ G^{-+}(\omega,\vec{p}) &= -i\,(n(\omega)+1)\,\rho(\omega,\vec{p}), \end{aligned}$$

Bose-Einstein distribution

- using the KMS relation:

$$\rho = (1-e^{-\beta\omega})G_{-+}$$

Putting the pieces together...

- Spectral function of the energy momentum tensor:

reminder: Kubo relation $\eta = \frac{1}{20} \frac{\mathrm{d}}{\mathrm{d}\omega} \rho_{\pi\pi}(\omega, 0) \Big|_{\omega=0}$

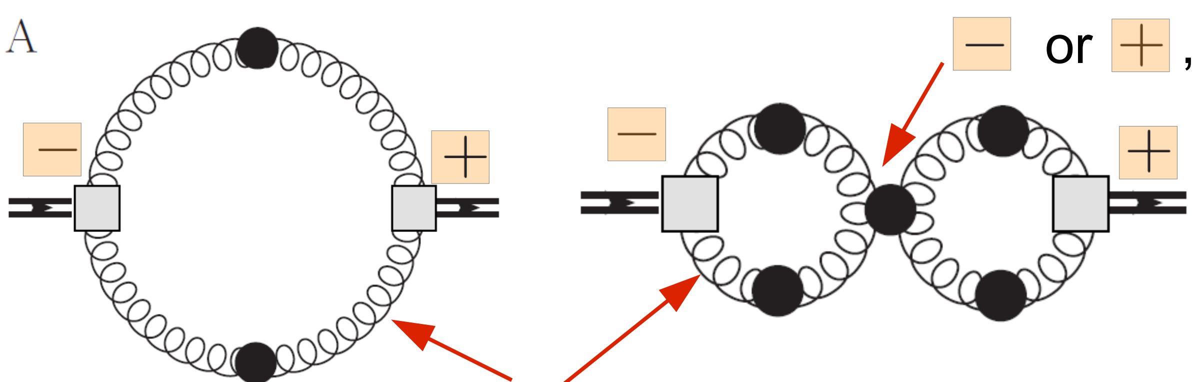
and with $\rho = (1 - e^{-\beta\omega}) G^{-+}$

→ derivative hits the exponential only!

→ $$\eta = \frac{\beta}{20} G^{-+}_{\pi\pi}(0,0)$$

Diagrams:

$-$ or $+$, sum over branch indices



and so on...

gluon propagator G_{AA}

Gluon Spectral Functions and MEM

- Gluons run in the loops

  → need the non-perturbative gluon spectral function!

  → Analytical continuation of Euclidean propagators

  Haas,Fister,Pawlowski (2014)

  functional renormalization, lattice

- inversion problem

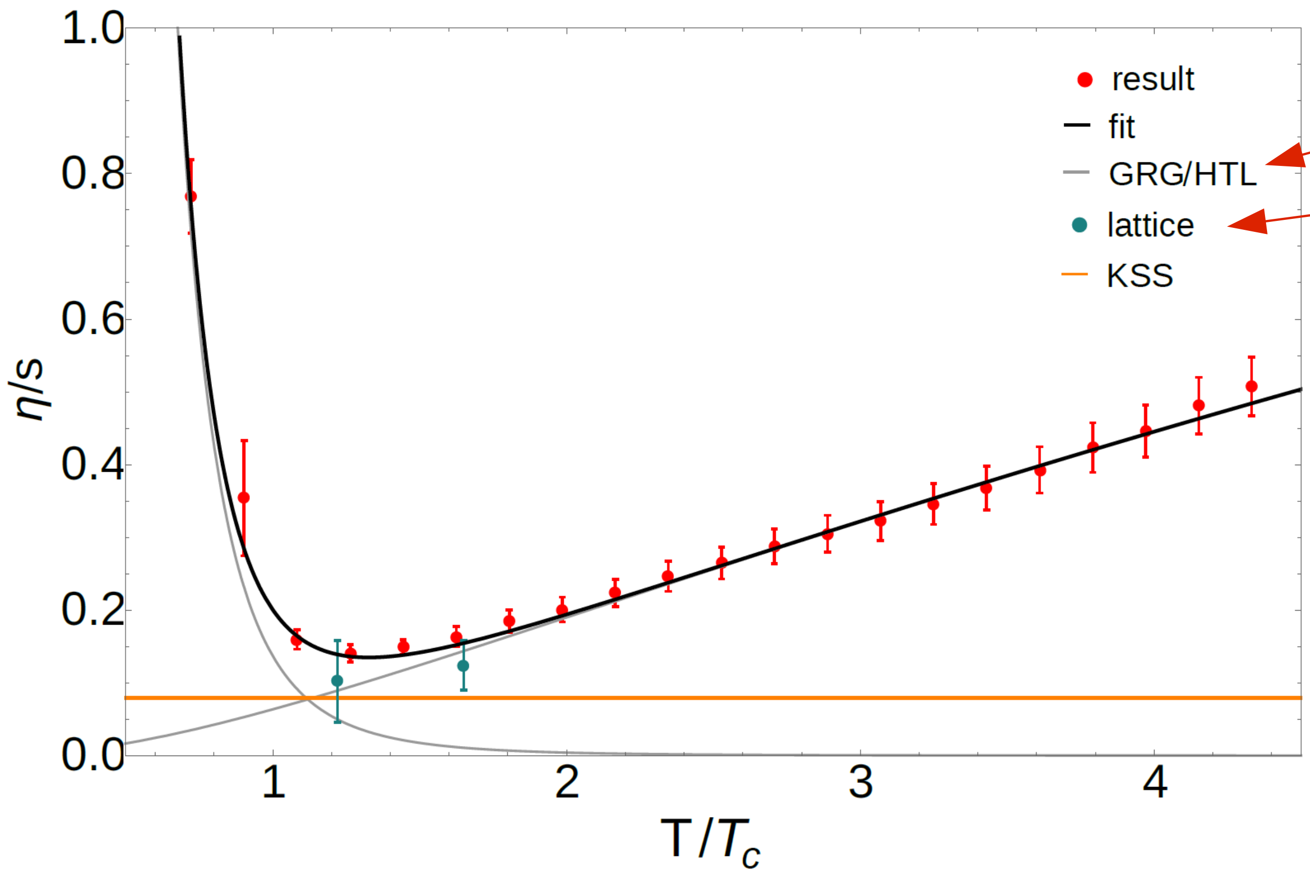
$$G(\tau = it) = \int \frac{dp^0}{2\pi} K(\tau, p^0) \rho(p^0)$$

Euclidean propagator — spectral function

  → use constraints for inversion → Maximum Entropy Method (MEM)

  → soon: direct real-time calculations a la Pawlowski,Strodthoff (2015)

Viscosity over Entropy in SU(3) YM



Arnold,Moore,Yaffe (2003)

H.B. Meyer (2007)

„glueball resonance gas
+
HTL perturbation theory“

$$\frac{\eta}{s}(T) = \frac{a}{\alpha^{\gamma}_{s,\mathrm{HQ}}(c\,T/T_c)} + \frac{b}{(T/T_c)^{\delta}}$$

$a \approx 0.15$

$b \approx 0.14$

$c \approx 0.66$

$\delta \approx 5.1$

$\gamma \approx 1.6$

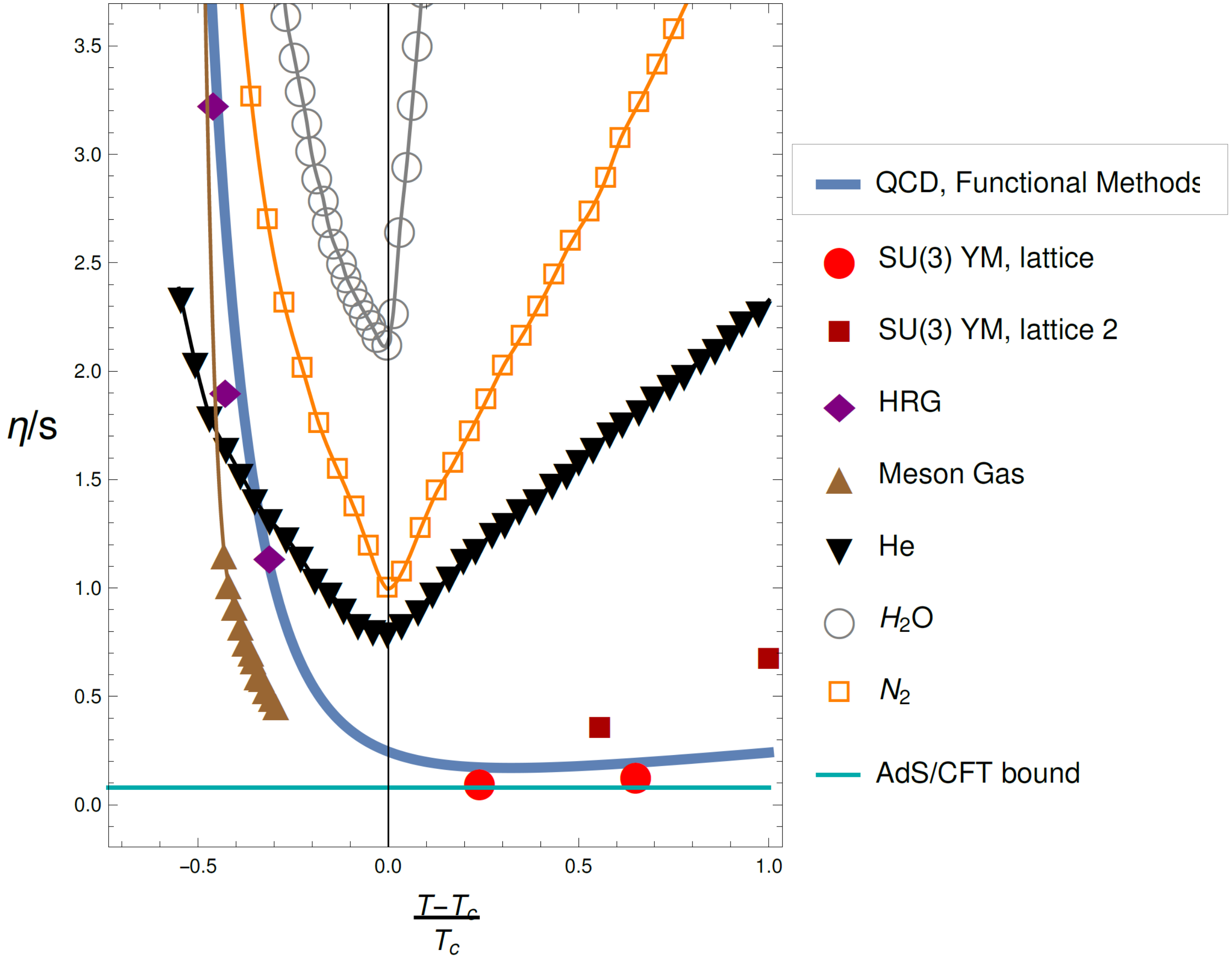
Viscosity over Entropy in QCD

Estimate for full QCD:

based on the global fit with:

$\alpha_{\mathrm{YM}} \longrightarrow \alpha_{\mathrm{QCD}}$

genuine quark contributions from pert. theory



gluon resonance gas → hadron resonance gas

$a_{\mathrm{QCD}} \approx 4/3a$

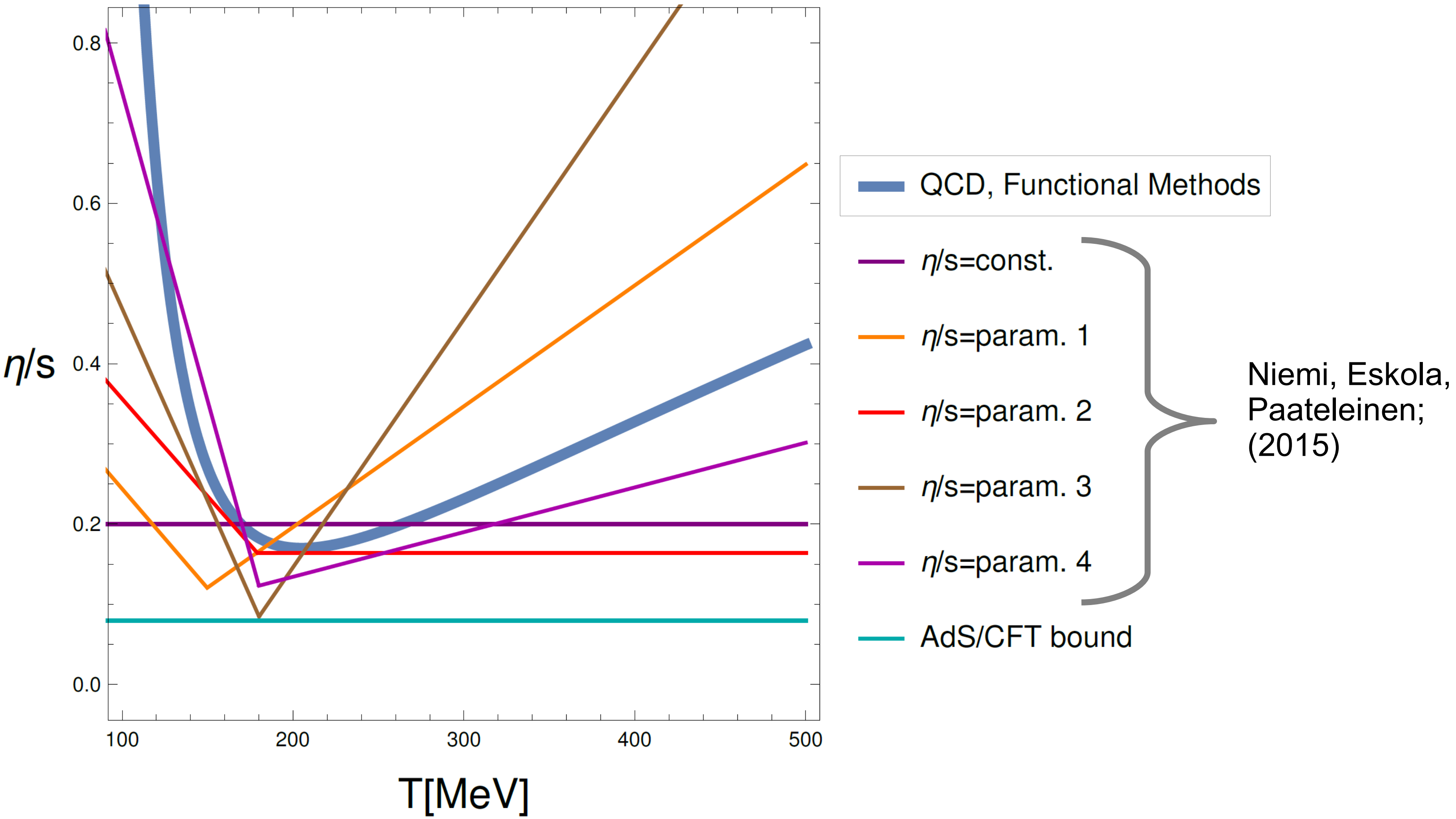
$b_{\mathrm{QCD}} \approx 0.16$

$c_{\mathrm{QCD}} \approx 0.79$

$\delta_{QCD} \approx 5$

Facing Reality

fitting ν_2 with $\frac{\eta}{s}(T)$



Summary and Outlook

- η/s in SU(3) YM theory in the non-perturbative regime and over a wide range of temperatures
- estimate of η/s in full QCD

Outlook

- Genuine quark dynamics
- Gluon spectral functions without Euclidean detour
- The formalism presented is very general !!!

→ other transport coefficients

→ glueball masses, ...

Thank You!!!