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Bose-Einstein condensation of gluons within a transport approach

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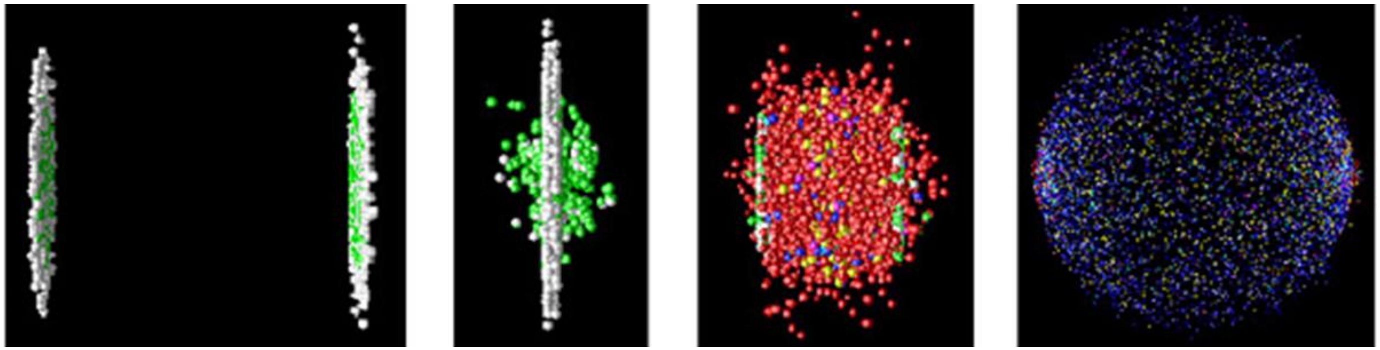
Outline

- Motivations
- Transport equation for Bose-Einstein condensation
- Bose statistics in BAMPS
- Recent results
- Summary and Outlook

Motivations



Ultrarelativistic Heavy Ion Collisions

CGC leads to overpopulated QGP

Blaizot, Gelis, Liao, McLerran, Venugopalan, NPA 873 (2012)

For CGC $\epsilon_0 \sim Q_s^4/\alpha_s, \quad n_0 \sim Q_s^3/\alpha_s; \qquad n_0 \epsilon_0^{-3/4} \sim \alpha_s^{-1/4}$

In equilibrium $f_{eq} = \frac{1}{exp(p/T)-1}, \qquad n_{eq}\epsilon_{eq}^{-3/4} \sim 1$

For small α_s CGC leads to overpopulated QGP with BEC assuming number conservation

$$f = f_{eq} + (2\pi)^3 n_c \delta^{(3)}(\vec{p})$$

Motivations

- Studies within the kinetic theory

  Onset of BEC, BUT no BEC

  Blaizot, Gelis, Liao, McLerran, Venugopalan, NPA 873 (2012)
  Blaizot, Liao, McLerran, NPA 920 (2013)
  Huang, Liao, arXiv:1303.7214
  Blaizot, Wu, Yan, arXiv:1402.5049
  Scardina, Perricone, Plumari, Ruggieri, Greco, arXiv:1408.1313

- Studies within the classical field theory (no number conservation)

  Epelbaum, Gelis, NPA 872 (2011)
  Berges, Sexty, PRL 108 (2012)
  Kurkela, Moore, PRD 86 (2012)
  Berges, Boguslavski, Schlichting, Venugopalan, arXiv:1408.1670

Motivations

We study the thermalization of gluons with BE condensation by using a transport model BAMPS.

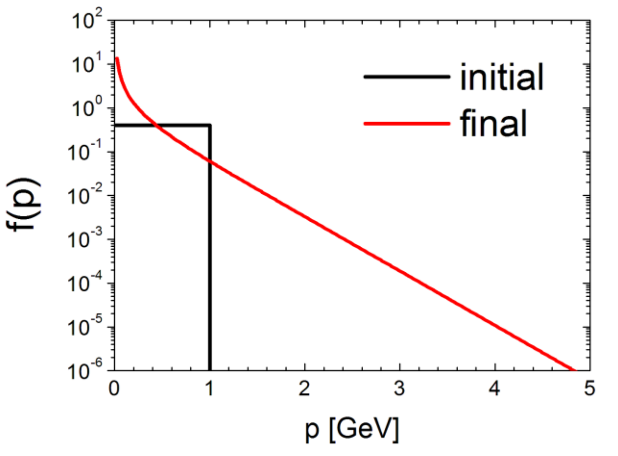
Assumptions:

- Static case: homogenous in space
- Gluon number conservation by considering only elastic scatterings
- Initial distribution: $f_{init}(\vec{p}) = f_0\theta(Q_s - p)$

  the condensation occurs when $f_0 > 0.154$

We know the final distribution

$$f_{eq}(\vec{p}) = \frac{1}{e^{p/T} - 1} + (2\pi)^3 n_c \delta^{(3)}(\vec{p})$$



We want to know how fast the thermalization occurs.

Transport Equation for BEC

Boltzmann Equation

$$\left(\partial_t + \frac{\vec{p}_1}{E_1}\cdot\vec{\nabla}\right) f_1(x,p_1) = \frac{1}{2E_1}\int \frac{d^3p_2}{(2\pi)^3 2E_2}\frac{d^3p_3}{(2\pi)^3 2E_3}\frac{d^3p_4}{(2\pi)^3 2E_4}\frac{1}{\nu}|\mathcal{M}_{12\to34}|^2$$

$$\times\ (2\pi)^4\delta^{(4)}(p_1+p_2-p_3-p_4)$$

$$\times\ [f_3 f_4 (\mathbf{1}+\boldsymbol{f_1})(\mathbf{1}+\boldsymbol{f_2}) - f_1 f_2 (\mathbf{1}+\boldsymbol{f_3})(\mathbf{1}+\boldsymbol{f_4})]$$

$$f = f^{gas} + f^{c}, \qquad f^{c} = (2\pi)^3 n_c \delta^{(3)}(\vec{p})$$

g: gas particle

c: condensate particle

Included	Not included
$g+g \leftrightarrow g+g$	$g+c \leftrightarrow g+c$
$g+g \leftrightarrow g+c$	$c+c \leftrightarrow c+c$

Transport Equation for BEC

For gas particles:

$$\left(\partial_t + \frac{\vec{p}_1}{E_1}\cdot\vec{\nabla}\right) f_1^{gas}(x,p_1) = \frac{1}{2E_1}\int \frac{d^3p_2}{(2\pi)^3 2E_2}\frac{d^3p_3}{(2\pi)^3 2E_3}\frac{d^3p_4}{(2\pi)^3 2E_4}\,|\mathcal{M}_{12\to 34}|^2$$

$$\times\ (2\pi)^4\delta^{(4)}(p_1+p_2-p_3-p_4)$$

$$\times\ [\tfrac{1}{2} f_3^{gas} f_4^{gas}(1+f_1^{gas})(1+f_2^{gas}) - \tfrac{1}{2} f_1^{gas} f_2^{gas}(1+f_3^{gas})(1+f_4^{gas})$$

$$+ f_3^{gas} f_4^{gas}(1+f_1^{gas}) f_2^c - \tfrac{1}{2} f_1^{gas} f_2^c (1+f_3^{gas})(1+f_4^{gas})$$

$$+ \tfrac{1}{2} f_3^c f_4^{gas}(1+f_1^{gas})(1+f_2^{gas}) - f_1^{gas} f_2^{gas} f_3^c (1+f_4^{gas})]$$

Transport Equation for BEC

For condensate particles:

$$\partial_t f_1^c(x,p_1) = \frac{1}{2E_1}\int \frac{d^3p_2}{(2\pi)^3 2E_2}\frac{d^3p_3}{(2\pi)^3 2E_3}\frac{d^3p_4}{(2\pi)^3 2E_4}|\mathcal{M}_{12\to 34}|^2$$

$$\times\ (2\pi)^4\delta^{(4)}(p_1+p_2-p_3-p_4)$$

$$\times\ \left[f_3^{gas}f_4^{gas}f_1^c(1+f_2^{gas}) - \tfrac{1}{2}f_1^c f_2^{gas}(1+f_3^{gas})(1+f_4^{gas})\right]$$

$$f^c = (2\pi)^3 n_c \delta^{(3)}(\vec{p})$$

$$\int \frac{d^3p_1}{(2\pi)^3}\partial_t f_1^c = \frac{\partial n_c}{\partial t} = R_c^{gain} - R_c^{loss}$$

A small phase space volume competes a δ function.

Transport Equation for BEC

Whether the condensation occurs?

$$R_c^{gain} = \frac{n_c}{(4\pi)^3} \int dp_3 dp_4 f_3 f_4 (1 + f_2) \frac{p_3 p_4}{E_3 E_4} E \left\{ \frac{|\mathcal{M}_{34\to12}|^2}{s} \right\}_{E-\sqrt{p^2+m^2}=m}$$

$$E = E_3 + E_4, \qquad p = |\vec{p}_3 + \vec{p}_4|, \qquad s = E^2 - p^2$$

m: particle mass

The kinematic constraint $E - \sqrt{p^2 + m^2} = m$ leads to $s = 2Em$.

For massless particles $s = 0$, i.e., $\vec{p}_3$ and $\vec{p}_4$ are parallel.

$\left\{ \frac{|\mathcal{M}_{34\to12}|^2}{s} \right\}_{s=0}$ can be zero, infinity, or has a finite value, which depends on the form of the scattering matrix.

Transport Equation for BEC

Whether the condensation occurs?

$$F = \left\{ \frac{|\mathcal{M}_{34\to12}|^2}{s} \right\}_{s=0} = ?$$

Case 1: interactions with the isotropic distribution of the collision angle.

$|\mathcal{M}_{34\to12}|^2 \sim s\sigma$, F can be a finite value, if the cross section is not diverge at $s = 0$.

Case 2: interactions with the pQCD cross section of gluons

$|\mathcal{M}_{34\to12}|^2 \sim \frac{s^2}{t^2} \approx \frac{s^2}{\left(t - m_D^2\right)^2}$, $F = 0$ at $s = 0$.

$|\mathcal{M}_{34\to12}|^2 \sim \frac{s^2}{t^2} \approx \frac{s^2}{t\left(t - m_D^2\right)}$, F has a finite value at $s = 0$.

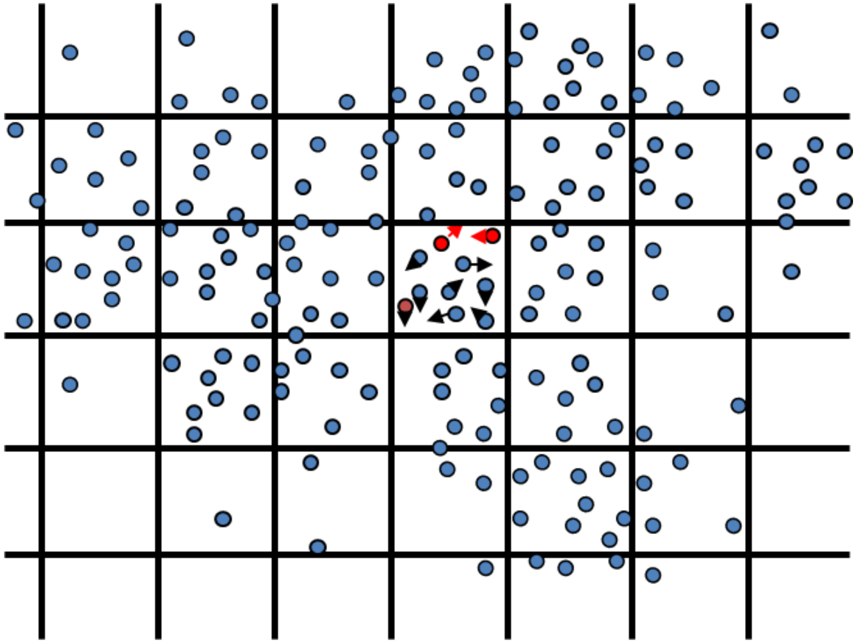
Aurenche, Gelis, Zaraket, JHEP 05 (2002)

Bose statistics in BAMPS

BAMPS: Boltzmann Approach of MultiParton Scatterings

solves the **semi-classical**, **relativistic** Boltzmann equation in the framework of **pQCD** by **Monte Carlo** simulations.

ZX and C. Greiner, PRC 71, 064901 (2005)



$$\left(\partial_t + \frac{\vec{p}_1}{E_1} \cdot \vec{\nabla}\right) f_1(x, p_1) = C$$

test particle representation of f

stochastic interpretation of the collision rates

Bose statistics in BAMPS

For $g + g \rightarrow g + c$ $\qquad P_{22} = v_{rel} \frac{\sigma_{22}^{c}}{N_{test}} \frac{\Delta t}{\Delta V}$

$$\sigma_{22}^{c} = \frac{1}{2s} \int \frac{d^3 p_1}{(2\pi)^3 2E_1} \frac{d^3 p_2}{(2\pi)^3 2E_2} |\mathcal{M}_{34\rightarrow 12}|^2 (2\pi)^7 \delta^{(4)}(\cdots) \boldsymbol{n_c} \boldsymbol{\delta^{(3)}(\vec{p}_1)} (1 + f_2)$$

$$= \frac{\pi}{2} n_c (1 + f_2) \frac{1}{p} \frac{|\mathcal{M}_{34\rightarrow 12}|^2}{s} \delta[(E - p)^2]$$

Approximation:

Particles with $\boldsymbol{p < \varepsilon}$ are regarded as condensate particles.
ε should be small to reduce the numerical uncertainty.
The rate for producing BEC does not depend on ε.

$$\delta^{(3)}(\vec{p}_1) \approx \frac{\theta(\varepsilon - p_1)}{4\pi p_1^2 \varepsilon} \quad \Rightarrow \quad \sigma_{22}^{c} \approx \frac{\pi}{2} n_c (1 + f_2) \frac{1}{p} \frac{|\mathcal{M}_{34\rightarrow 12}|^2}{s} \times \frac{1}{4\varepsilon} \left(\frac{2}{E - p} - \frac{1}{\varepsilon} \right) \theta \left(\varepsilon - \frac{E - p}{2} \right)$$

Recent results

Computation setups:

Box with volume: $3\ fm \times 3\ fm \times 3\ fm$

Cells with volume: $0.125\ fm \times 0.125\ fm \times 0.125\ fm$

Number of test particles per each real particle: $N_{test} = 6000$

Total particle number in computation: 2.23×10^6

MPI: 216 CPUs

Tests:

new scheme

collision rates

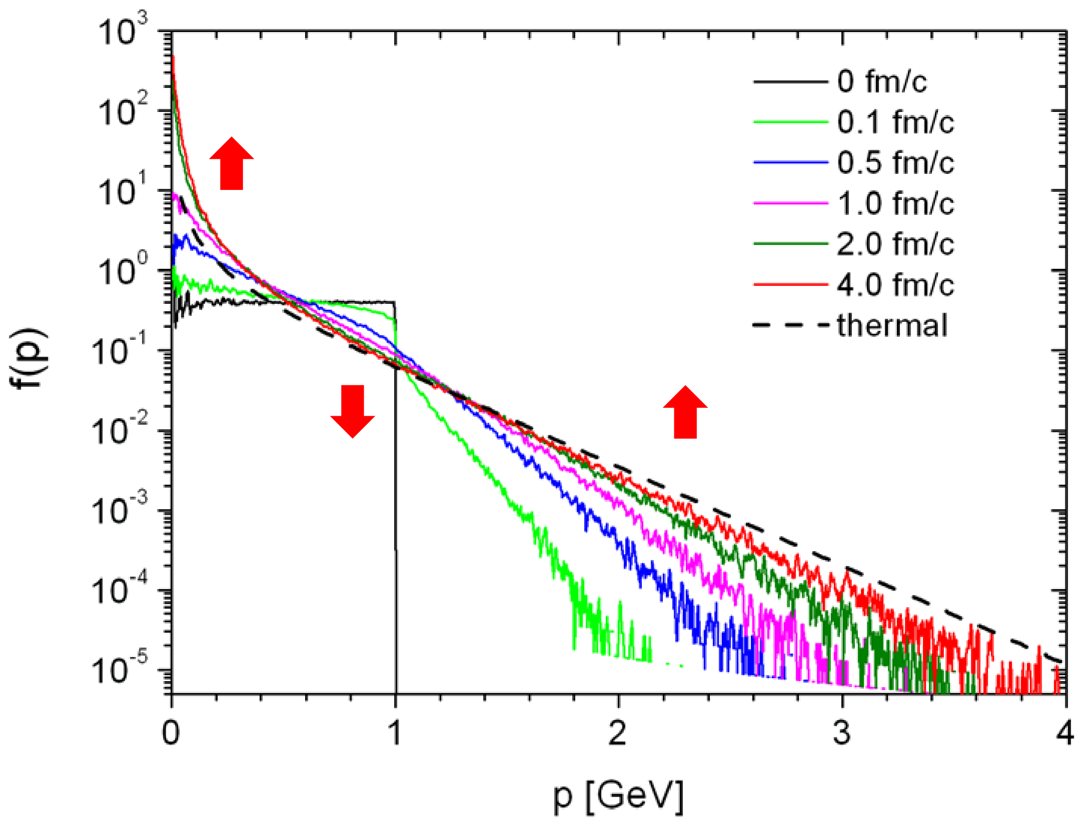
case of underpopulated gluons (negative chemical potential)

critical case (zero chemical potential and no condensation)

Recent results

$$|\mathcal{M}_{34\to 12}|^2 \approx (12\pi)^2 \alpha_s^2 \frac{s^2}{t(t-m_D^2)}, \qquad m_D^2 = 16\pi N_c \alpha_s \int \frac{d^3p}{(2\pi)^3}\frac{1}{p} f^{gas}$$

Onset of BEC: without $g + g \to g + c$ (BUT, there are still particles with energy smaller than ε.)



$f_0 = 0.4$

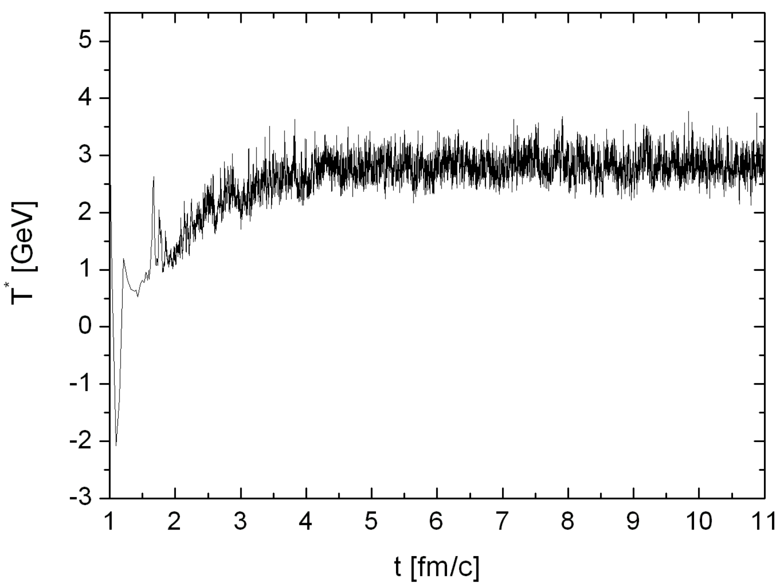
The distribution is frozen out at 4 fm/c (onset of BEC happens earlier) and is far from the equilibrium one.

Recent results

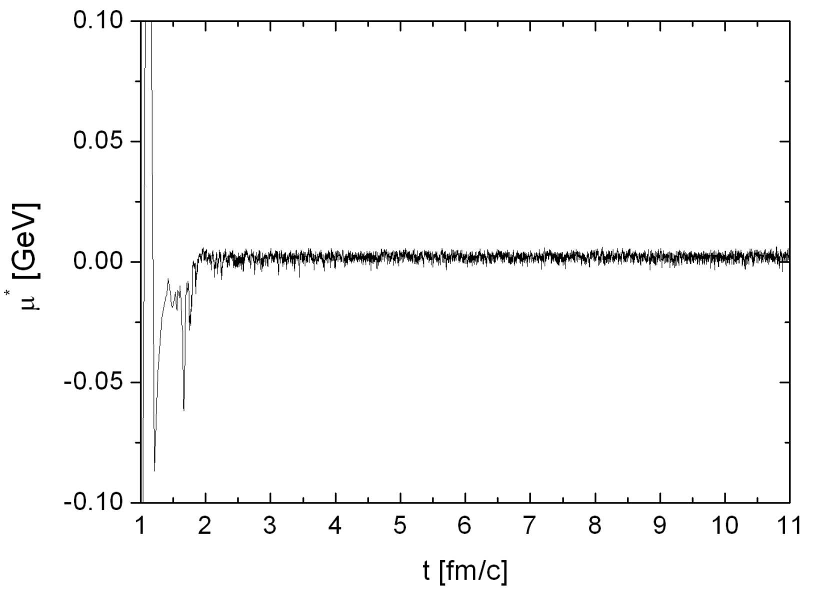
Onset of BEC: without $g + g \rightarrow g + c$

Assume: for small p, $$f \approx \frac{1}{exp\left(\frac{p-\mu^*}{T^*}\right)-1} \approx \frac{T^*}{p-\mu^*}$$

effective temperature

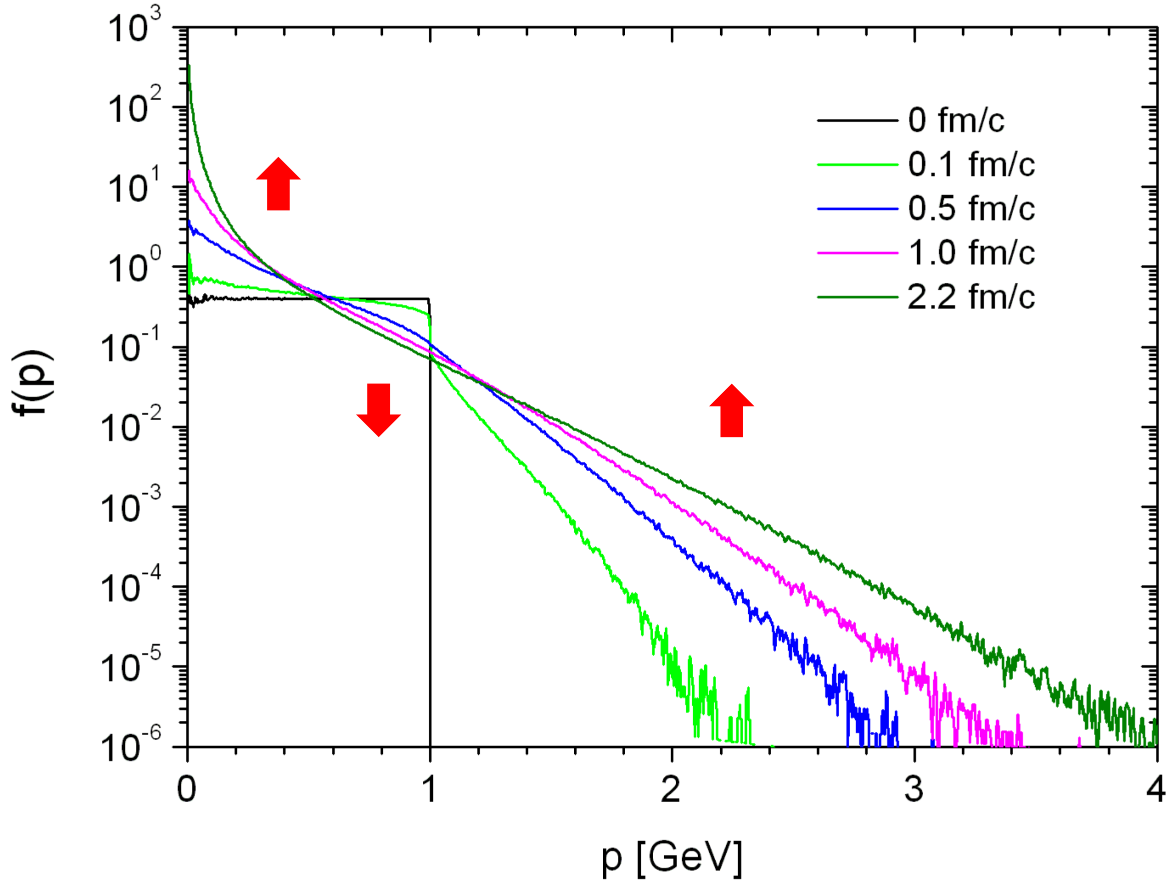


effective chemical potential



Recent results

BE Condensation: with $g + g \to g + c$, when μ^* is sufficiently close to zero and is positive due to numerical fluctuation.

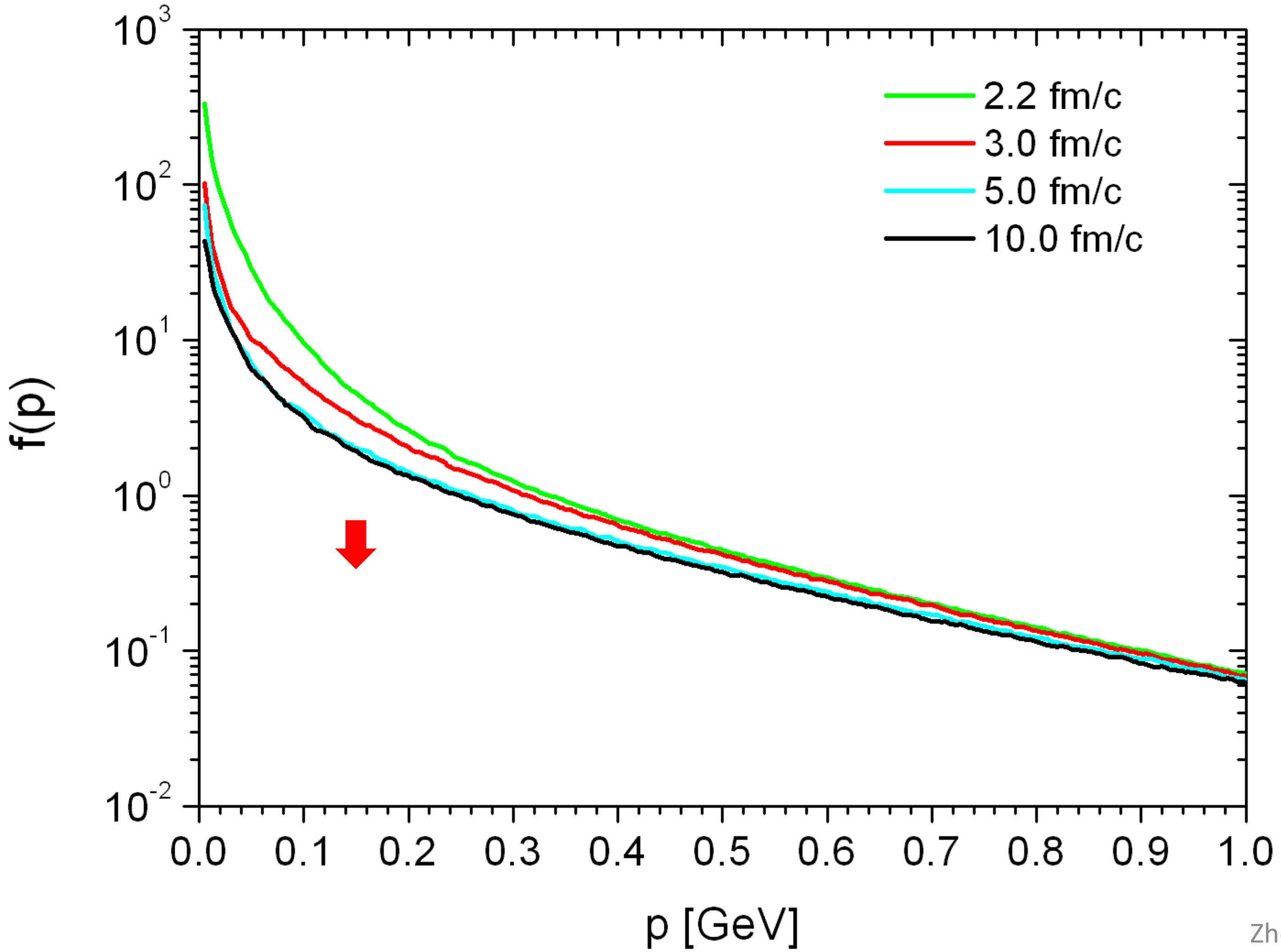


$f_0 = 0.4$

The condensation begins at 1.6 fm/c. The distribution at small p is increasing until 2.2 fm/c.

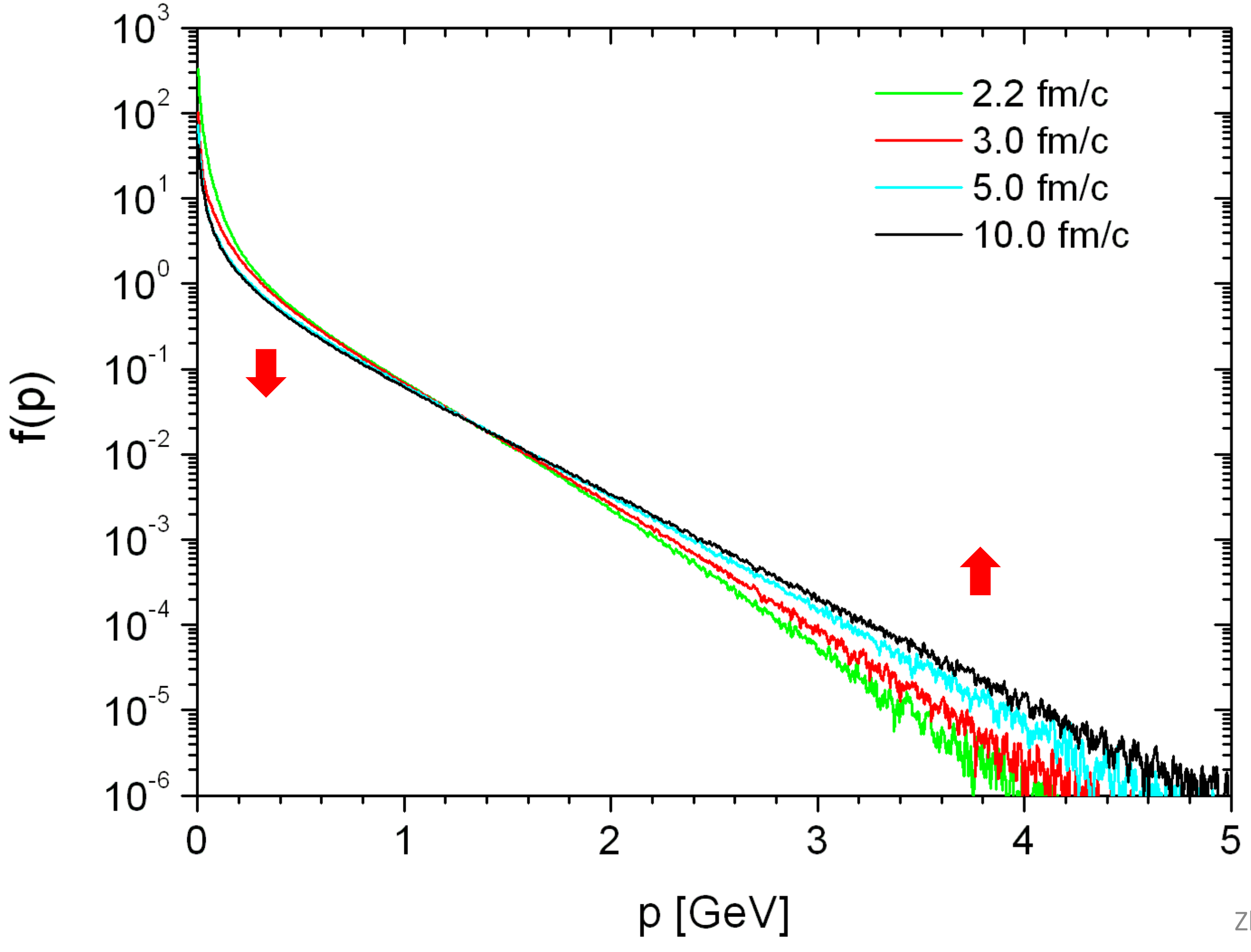
Recent results

Thermlization with BE Condensation



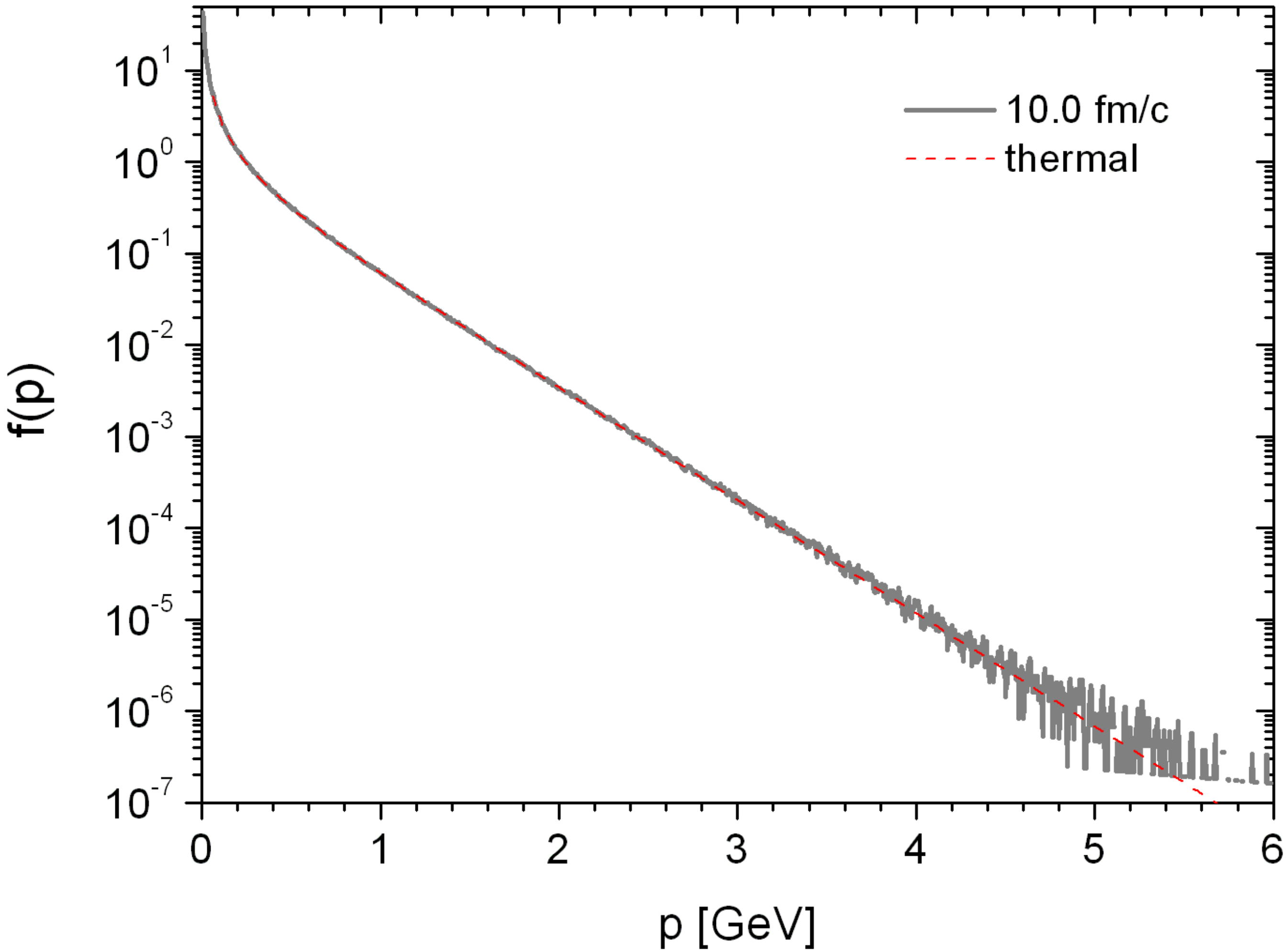
Recent results

Thermlization with BE Condensation



Recent results

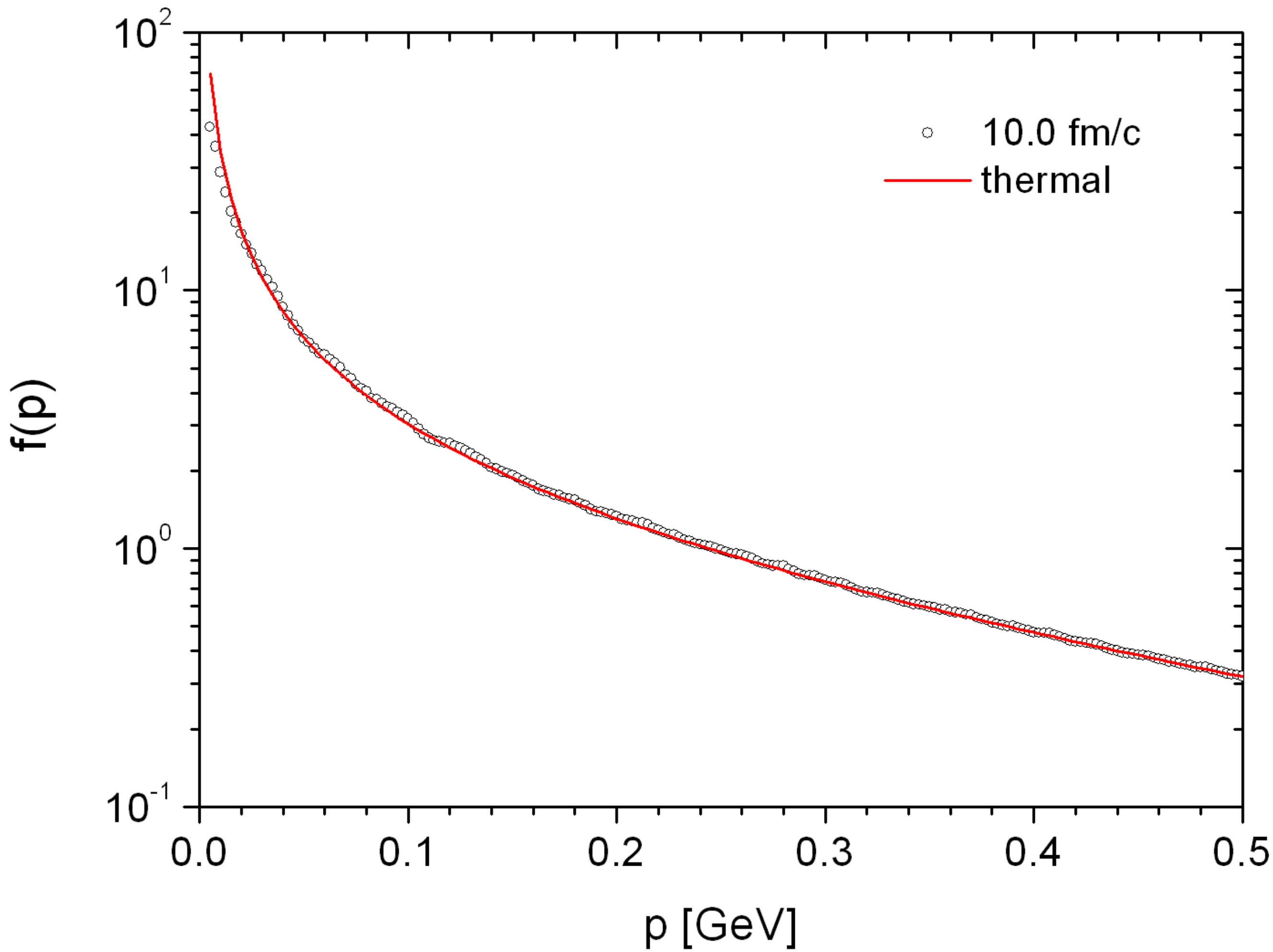
Thermlization with BE Condensation



perfect agreement !

Recent results

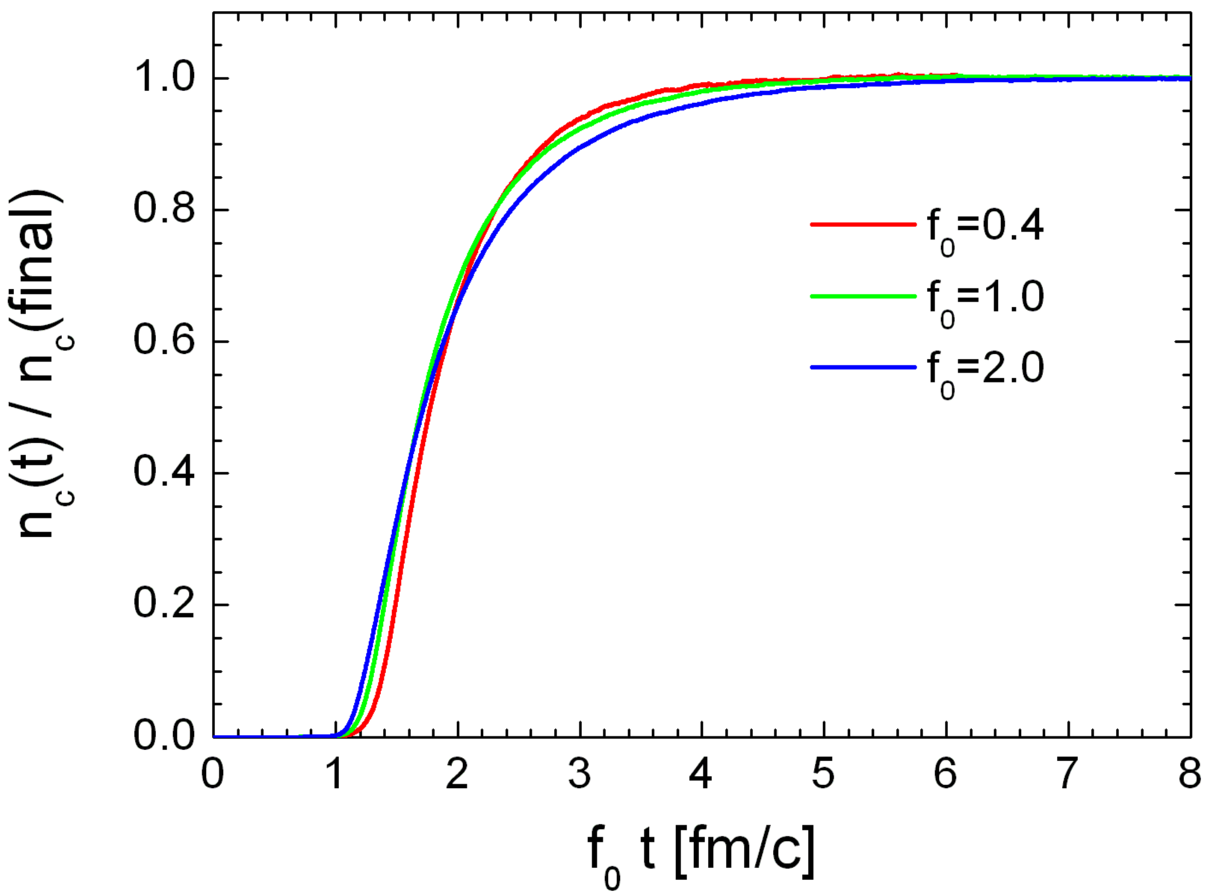
Thermlization with BE Condensation



perfect agreement !

Recent results

Scaling behaviour of BE condensation



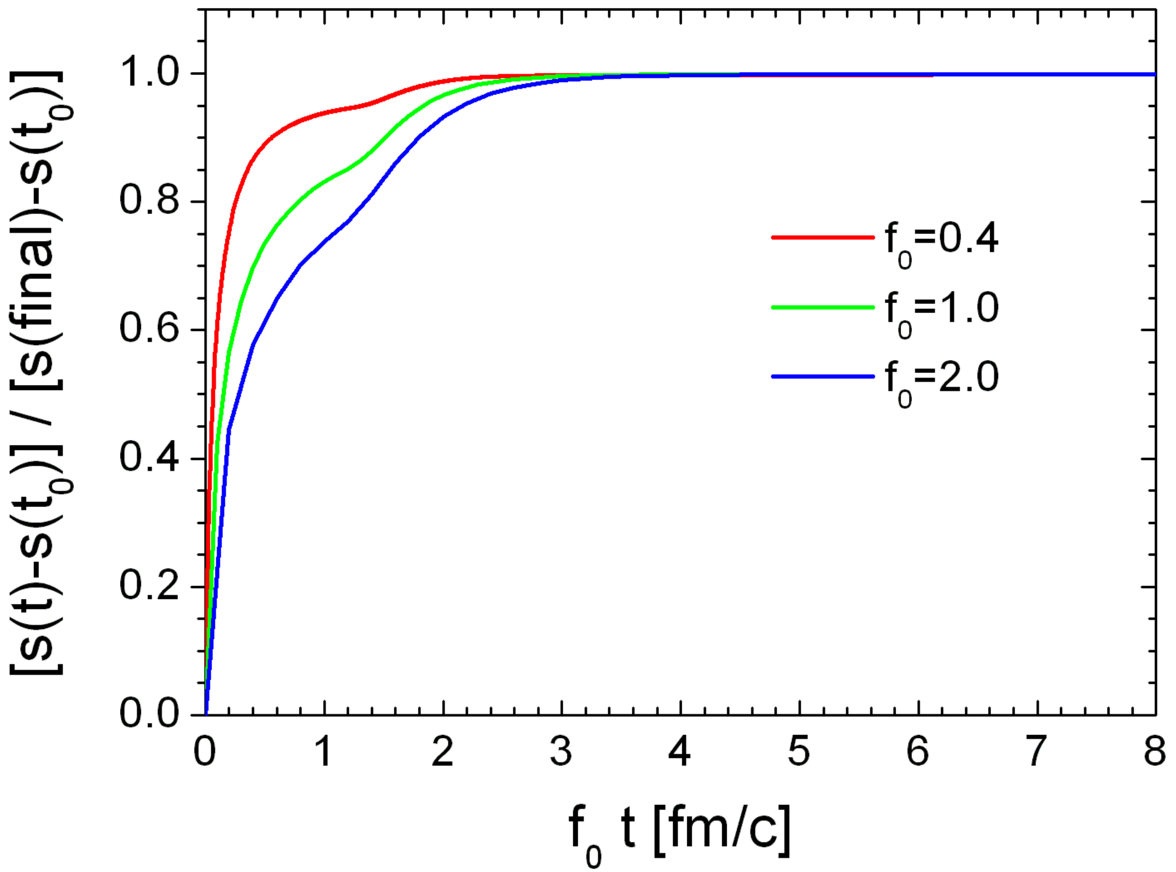
BEC completion

at $f_0 t \approx 6\ fm/c$

ZX, Zhou, Zhuang, Greiner, PRL 114, 2015

Recent results

Scaling behaviour of entropy production during BE condensation



Two step entropy production

Full thermalization at $f_0 t \approx 3\ fm/c$

Summary and Outlook

- show the onset and full process of BE condensation in BAMPS
- BE condensation is complete at $f_0 t \approx 6\ fm/c$
- almost full thermalization at $f_0 t \approx 3\ fm/c$

Further studies:

Influence of momentum anisotropy, 2<->3 processes, quarks, and expansion on BE condensation

Backup

Bose statistics in BAMPS

Collision scheme: stochastic method

For $g+g\to g+g$ $\quad P_{22}=v_{rel}\frac{\sigma'_{22}}{N_{test}}\frac{\Delta t}{\Delta V}$

$g+c\to g+g$

$$\sigma'_{22}=\frac{1}{4s}\int\frac{d^3p_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2E_2}|\mathcal{M}_{34\to 12}|^2(2\pi)^4\delta^{(4)}(\cdots)(\mathbf{1}+\boldsymbol{f}_1)(\mathbf{1}+\boldsymbol{f}_2)$$

$$=\int d\Omega\frac{d\sigma_{22}}{d\Omega}(\mathbf{1}+\boldsymbol{f}_1)(\mathbf{1}+\boldsymbol{f}_2)=\int d\Omega\frac{d\sigma'_{22}}{d\Omega}$$

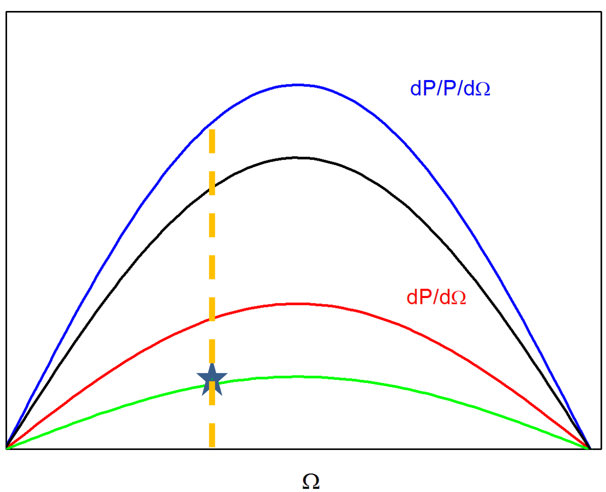
used by Scardina, Perricone, Plumari, Ruggieri, Greco, arXiv:1408.1313

Disadvantages:

1. Uncertainties from the extraction of f, and the integration in σ'_{22}
2. Time consuming due to the integrations in σ'_{22} for each particle pairs, whether or not they collide.

Bose statistics in BAMPS

$$\frac{dP_{22}}{d\Omega} = v_{rel}\frac{1}{N_{test}}\frac{d\sigma'_{22}}{d\Omega}\frac{\Delta t}{\Delta V}$$



New Scheme:

1. sample first the collision angle Ω
2. sample a random number between 0 and $dP/P\ /d\Omega$. If this number is smaller than $dP/d\Omega$, a collision occurs.

Transport Equation for BEC

$$R_c^{gain} = \frac{n_c}{(4\pi)^3} \int dp_3 dp_4 f_3 f_4 (1 + f_2) \frac{p_3 p_4}{E_3 E_4} E \left\{ \frac{|\mathcal{M}_{34\to\mathbf{1}2}|^2}{s} \right\}_{s=0}$$

$$R_c^{loss} = \frac{n_c}{(4\pi)^3} \int dp_3 dp_4 (1 + f_3)(1 + f_4) f_2 \frac{p_3 p_4}{E_3 E_4} E \left\{ \frac{|\mathcal{M}_{\mathbf{1}2\to 34}|^2}{s} \right\}_{s=0}$$

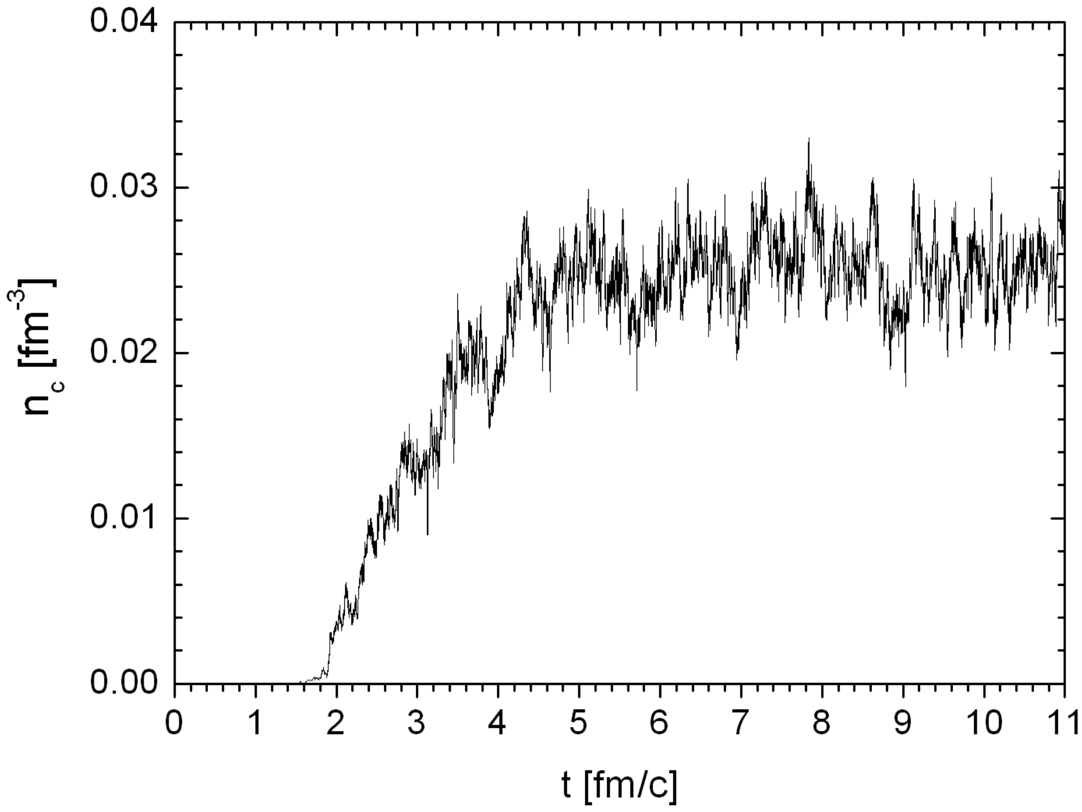
- $\frac{\partial n_c}{\partial t} \sim n_c$ need an initial seed for the condensation
- At thermal equilibrium, $f_{2,3,4} = \frac{1}{exp\left(\frac{p_{2,3,4}}{T}\right) - 1}$

In this case $R_c^{gain} - R_c^{loss} = 0$, but, both R_c^{gain} and R_c^{gain} are divergent.

The divergence is logarithmically with a lower cutoff ε for p.

Recent results

Onset of BEC: without $g + g \rightarrow g + c$ (BUT, there are still particles with energy smaller than ε.)



$\varepsilon = 0.0025\ GeV$

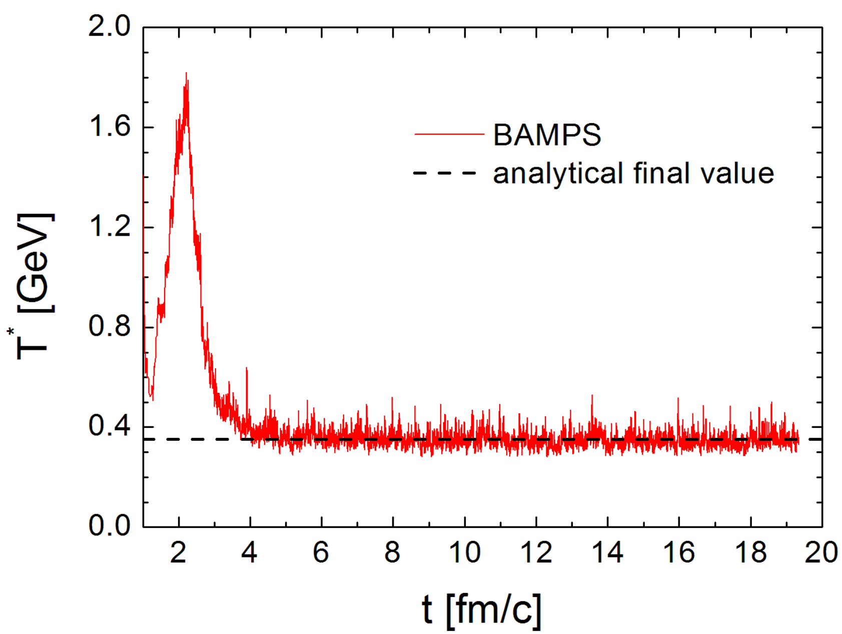
Density of particles with energy smaller than ε, about 1% of the density of the real condensate Particles ($n_c = 2.98\ fm^{-3}$).

Recent results

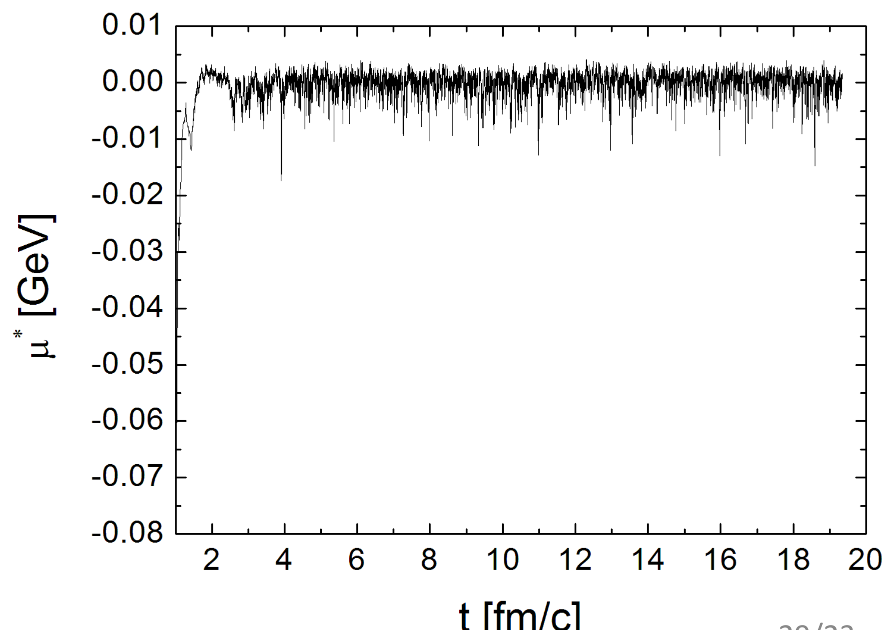
BEC: with $g + g \rightarrow g + c$

Assume: for small p, $$f \approx \frac{1}{exp\left(\frac{p-\mu^*}{T^*}\right)-1} \approx \frac{T^*}{p-\mu^*}$$

effective temperature

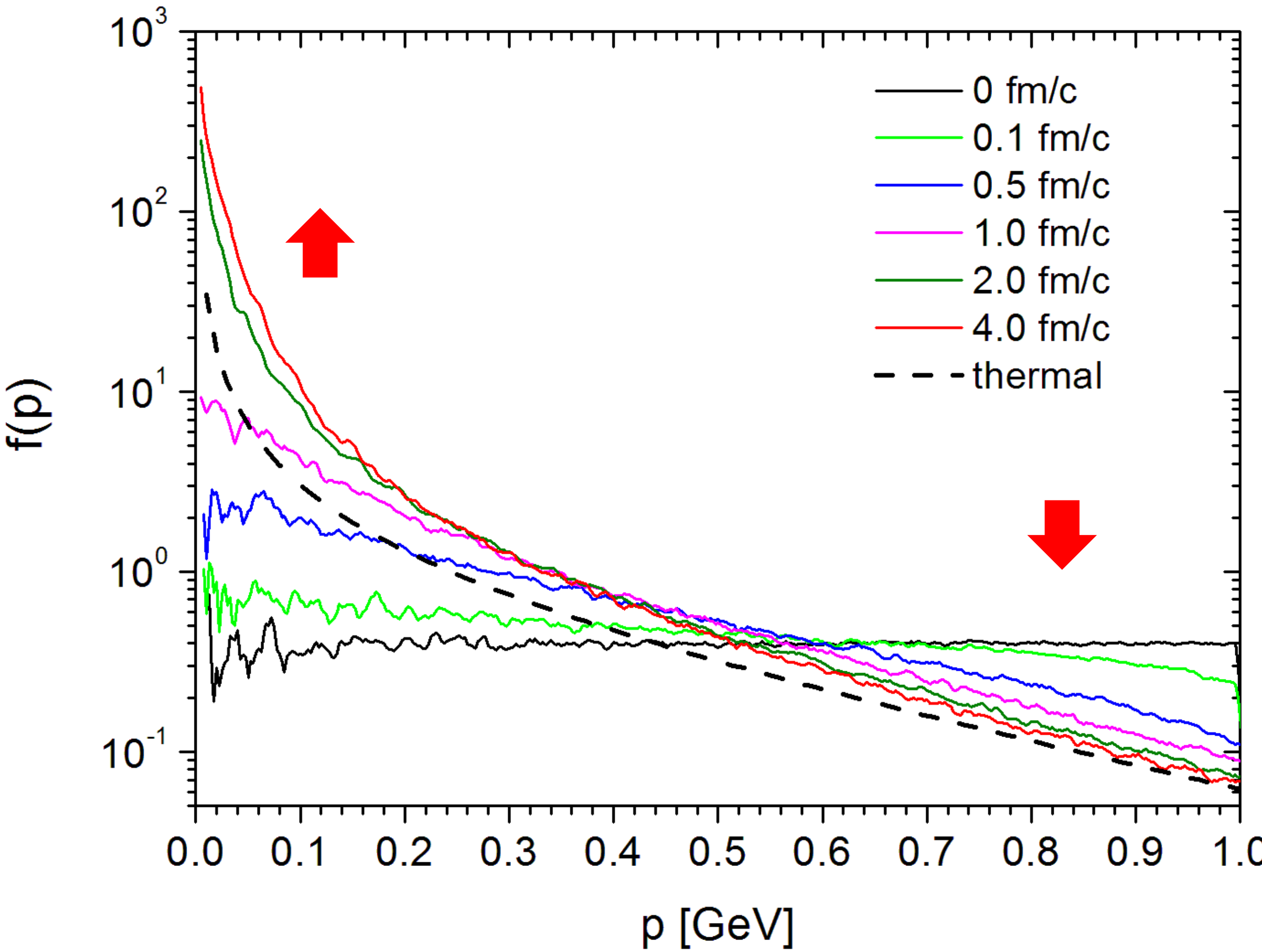


effective chemical potential



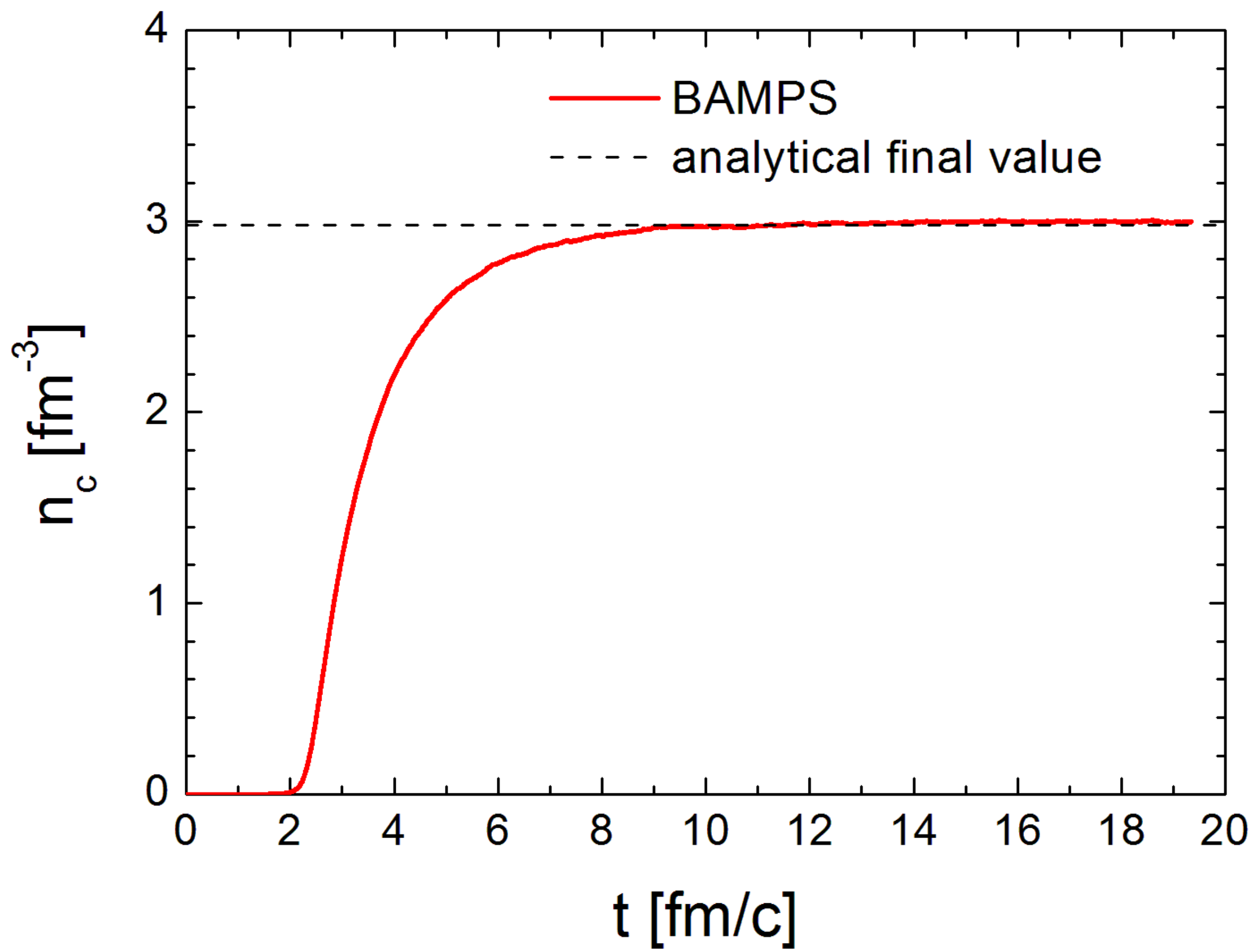
Recent results

Onset of BEC: without $g + g \to g + c$



Recent results

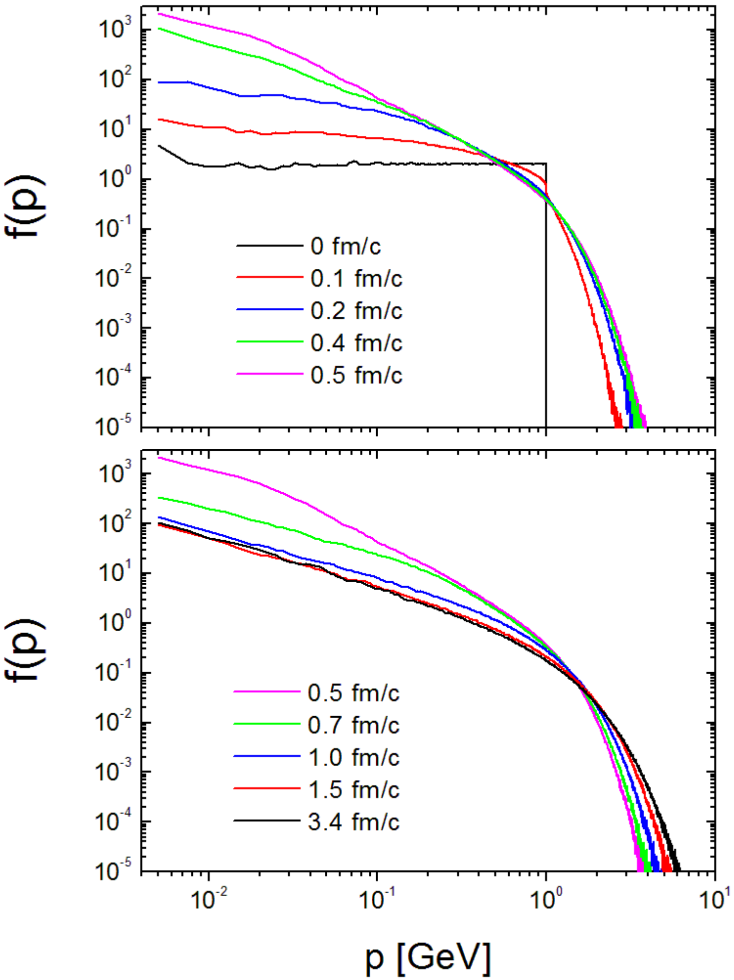
BE Condensation is complete at 10 fm/c for $f_0 = 0.4$.



perfect agreement !

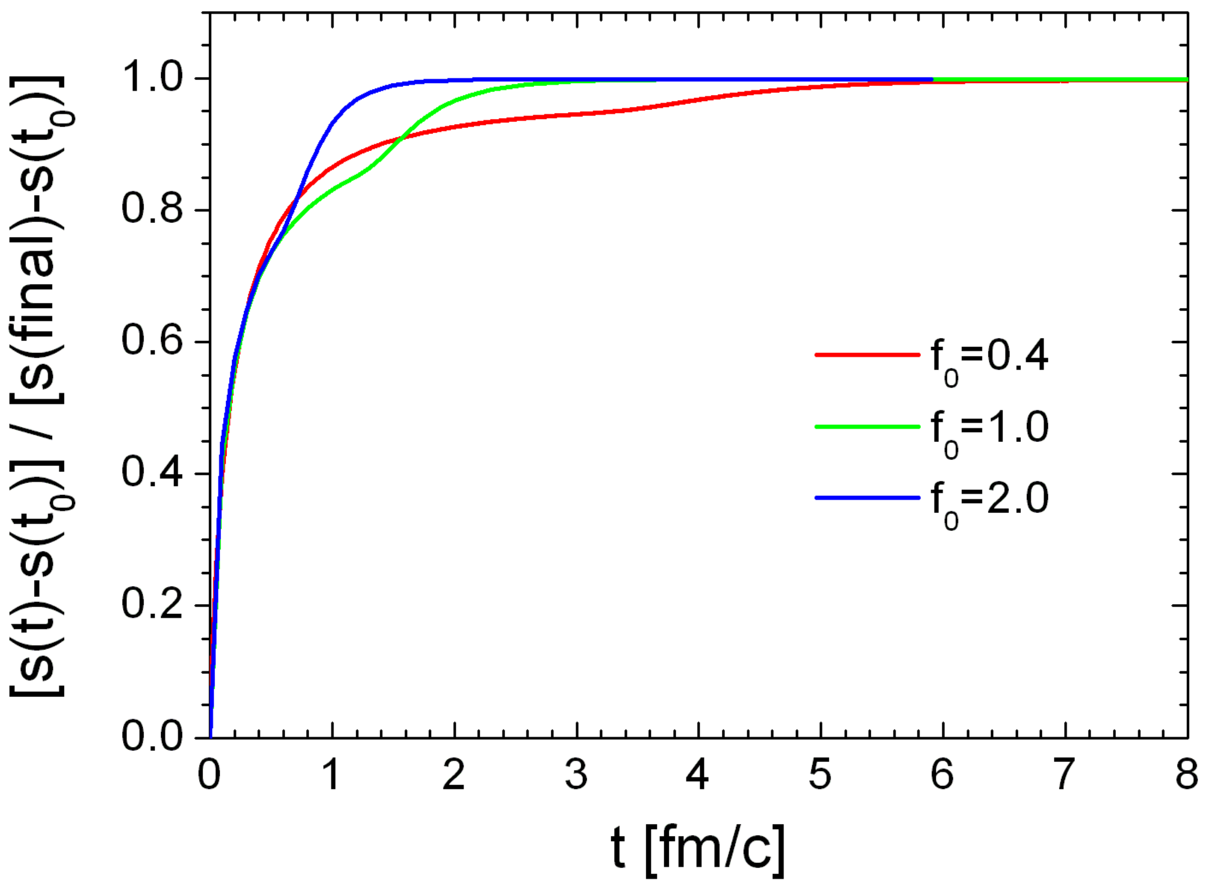
Recent results

Thermlization with BE Condensation ($f_0 = 2$)



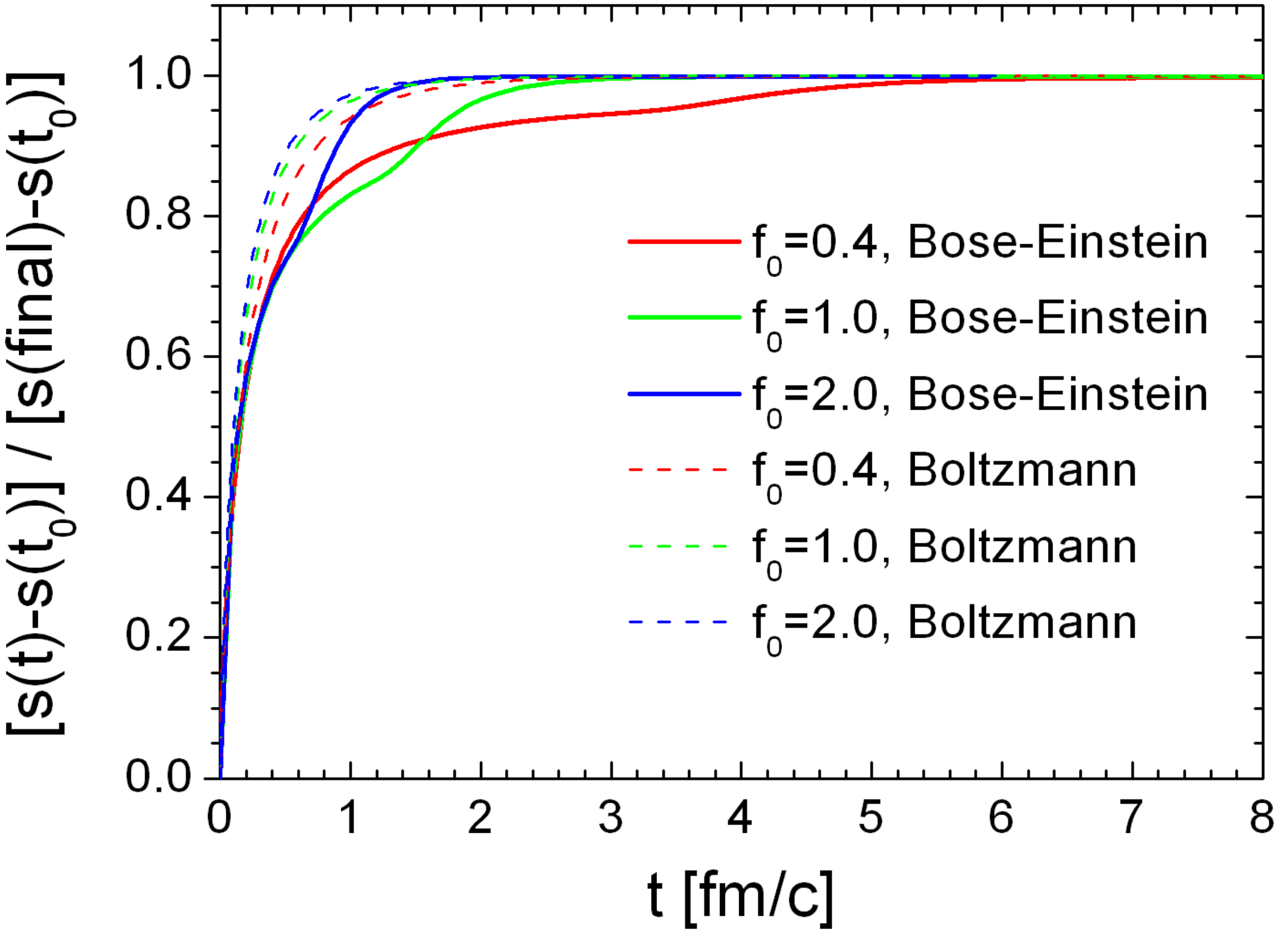
Recent results

Entropy production before BE condensation



Recent results

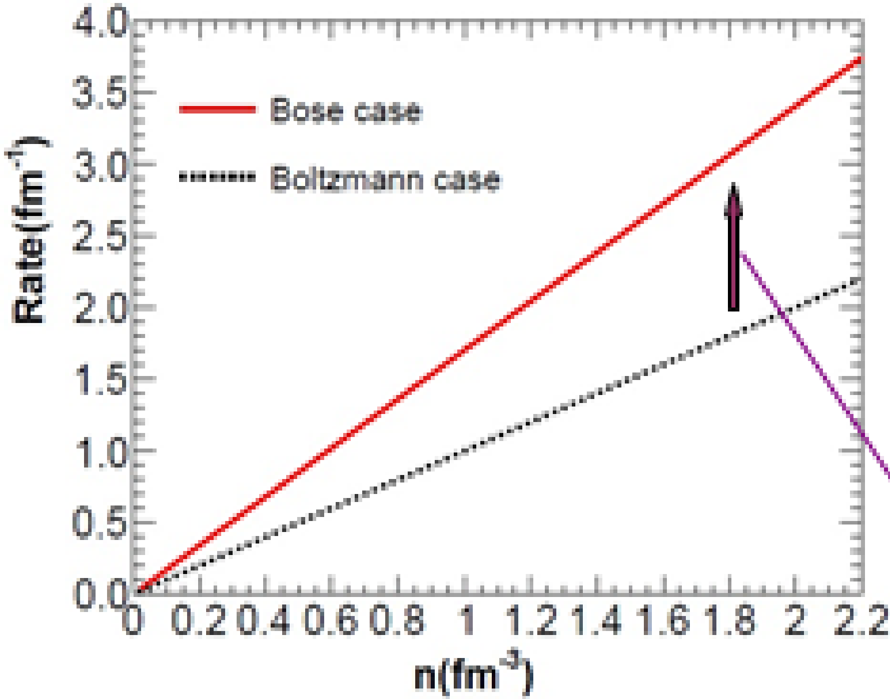
BE condensation slows down the entropy production.

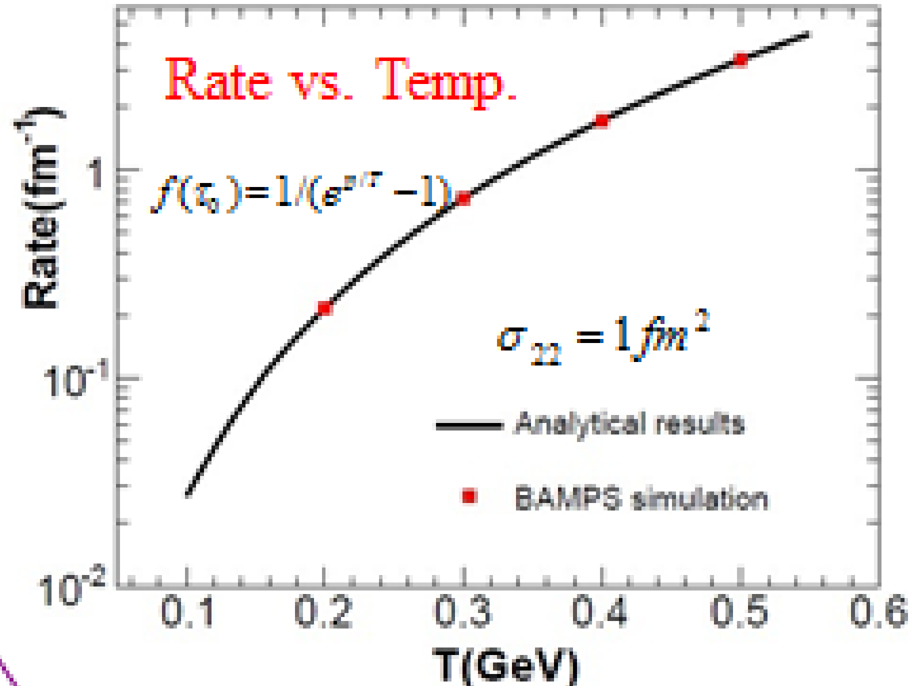


Transport model: Reaction Rate test

$$Rate = n \cdot \langle v_{rel}\sigma_{22} \rangle^{medium} = \frac{1}{n}\int \frac{d^3\vec{p}_1}{(2\pi)^3}\frac{d^3\vec{p}_2}{(2\pi)^3} f_1 f_2 \int d\Omega^* \frac{d\sigma_{22}}{d\Omega^*} v_{rel}(1+f_3)(1+f_4)$$

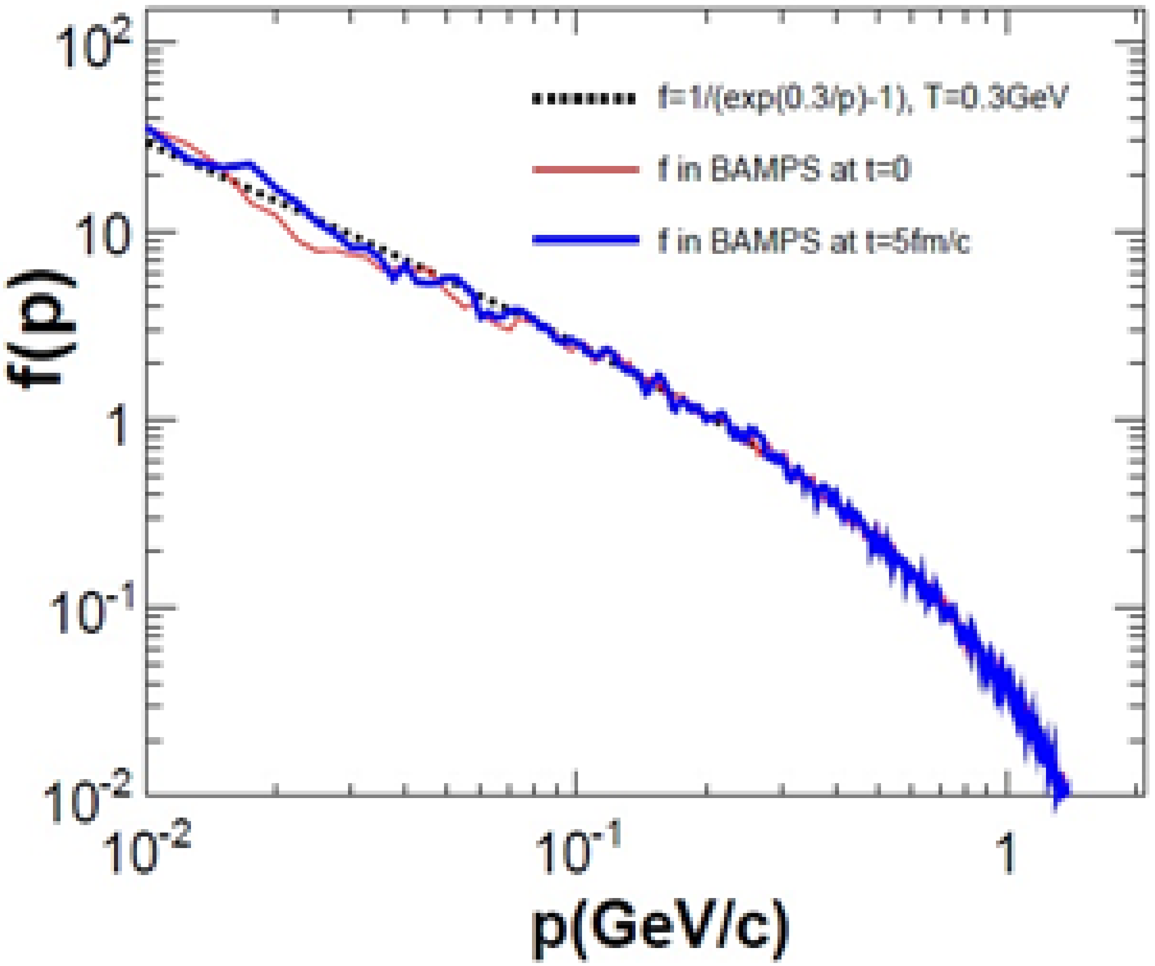
$$= N_{coll}/(t \cdot N_g/2)$$





Bose Enhancement effect

Transport model: Equlibrium Test

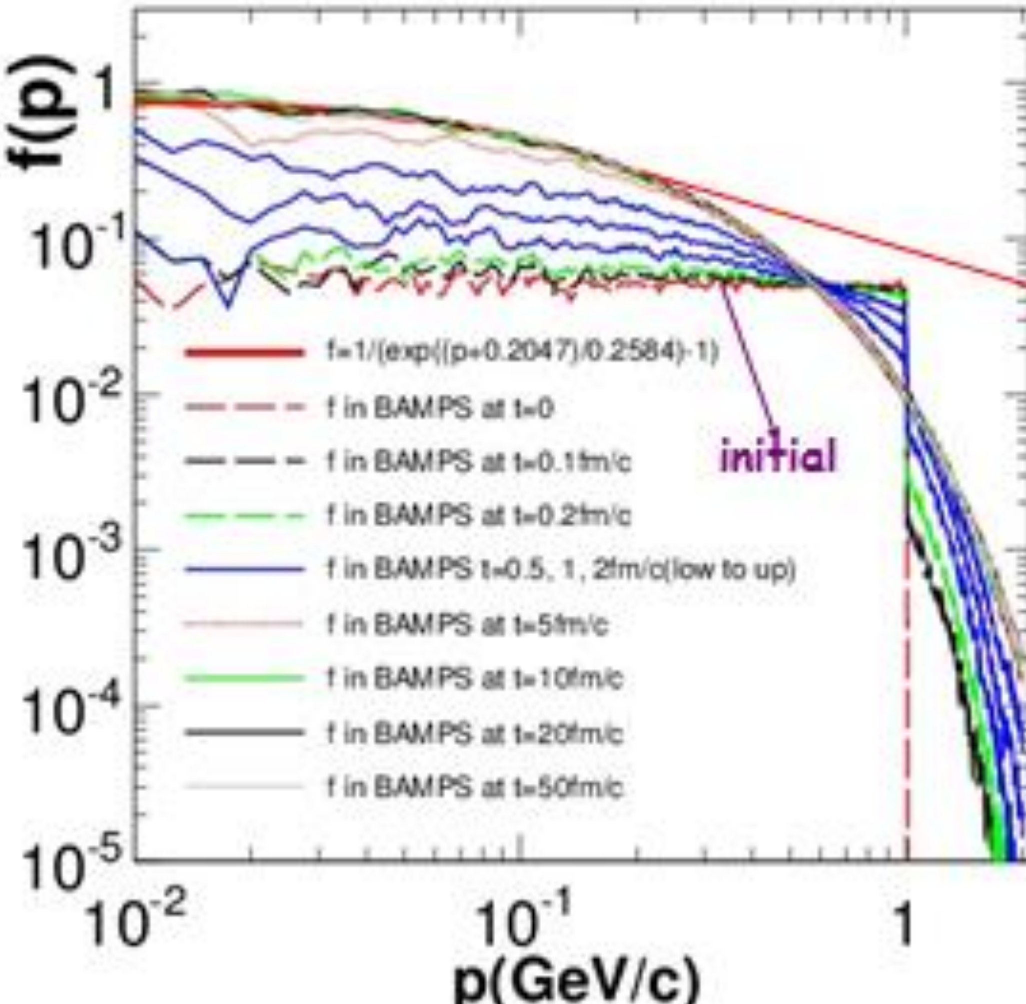


$$f(\tau_0) = 1/(e^{p/T} - 1)$$

$$T = 300MeV$$

within acceptable fluctuation , the equilibrium state can be maintained <->detailed balance ok and BE distribution is fix-point of the syst-em from simulation

f_0 = 0.05 simulation results



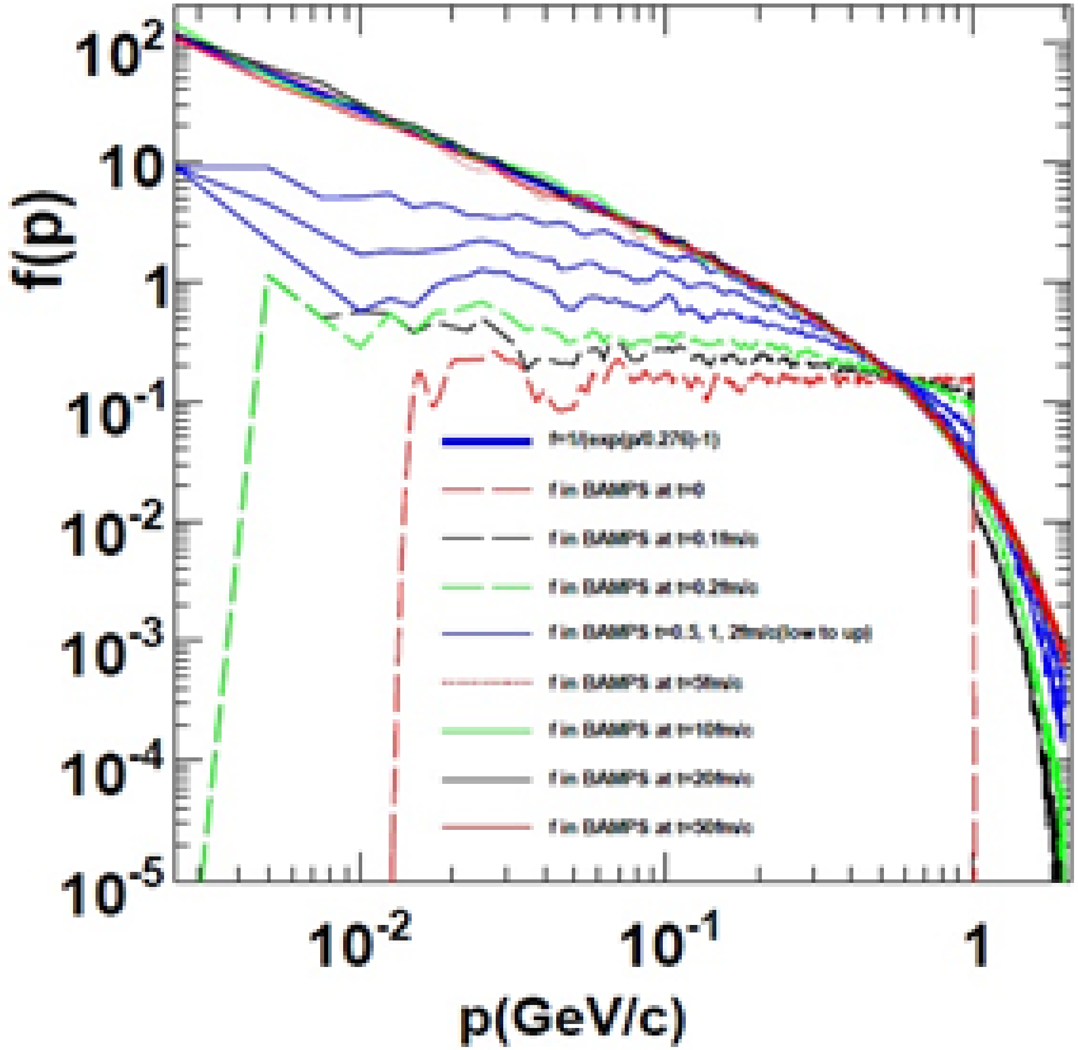
$3\times3\times3\,fm^3$ static box

$f_0 = 0.05 < f_{critica}$

$$\begin{cases} T_{eq} = 0.258 MeV \\ \mu_{eq} = -0.205 MeV \end{cases}$$

the system thermalizes to the fix-point: thermal BE distribution

f_0 = 0.154 simulation results



$3\times3\times3\,fm^3$ static box

$f_0 = 0.154 = f_{critical}$

$T_{eq} = 0.276\,MeV$

$\mu_{eq} = 0\,MeV$

the system thermalizes to the fix-point: thermal BE distribution