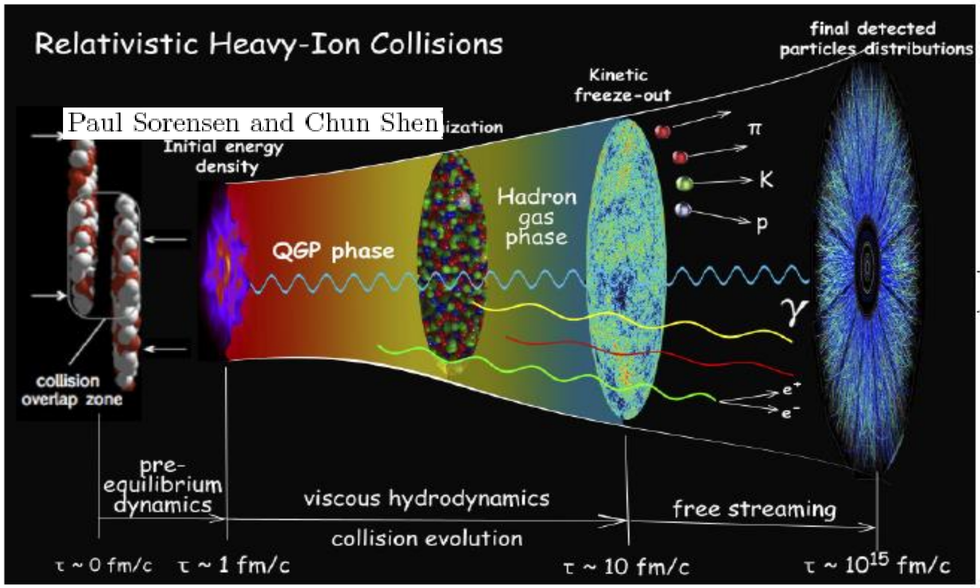
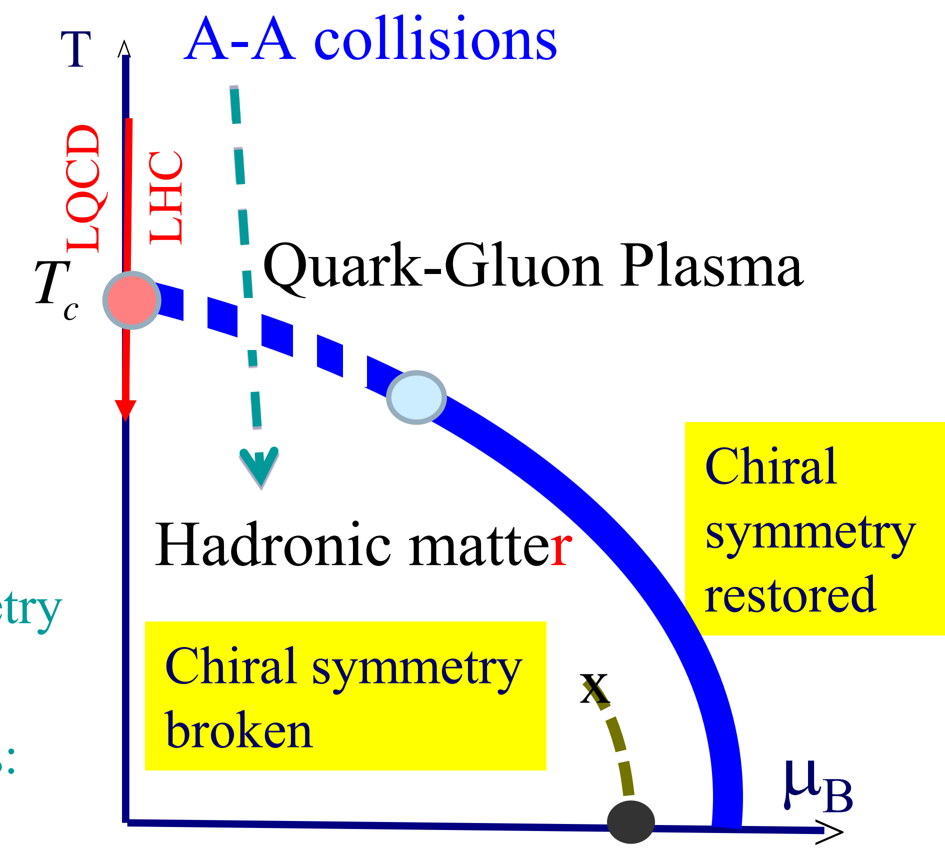
Probing phase diagram of QCD with fluctuations of conserved charges

Krzysztof Redlich, Wroclaw Uni., EMMI





Interplay between deconfinement & chiral symmetry breaking in QCD: LQCD & chiral models

Confronting 2nd order fluctuations and correlations: ALICE LHC ⇔ LQCD

Work done with: Pok M. Lo, B. Friman, O. Kaczmarek, C. Sasaki & K.R. Phys .Rev. D90 (2014) 7, 074035

P. Braun-Munzinger, A. Kalweit , J. Stachel, & K.R. Phys.Lett. B747 (2015) 292

K. Morita, B. Friman & K.R. Phys.Lett. B741 (2015) 178

Polyakov loop on the lattice needs renormalization

- Introduce Polyakov loop:

$$\boxed{L \Rightarrow c_N L} \quad \boxed{c_N = e^{2\pi i k/N} \in Z(N)}$$

$$L_{\vec{x}}^{\rm bare} = \frac{1}{N_c} Tr \prod_{\tau=1}^{N_\tau} U_{(\vec{x},\tau),4}$$

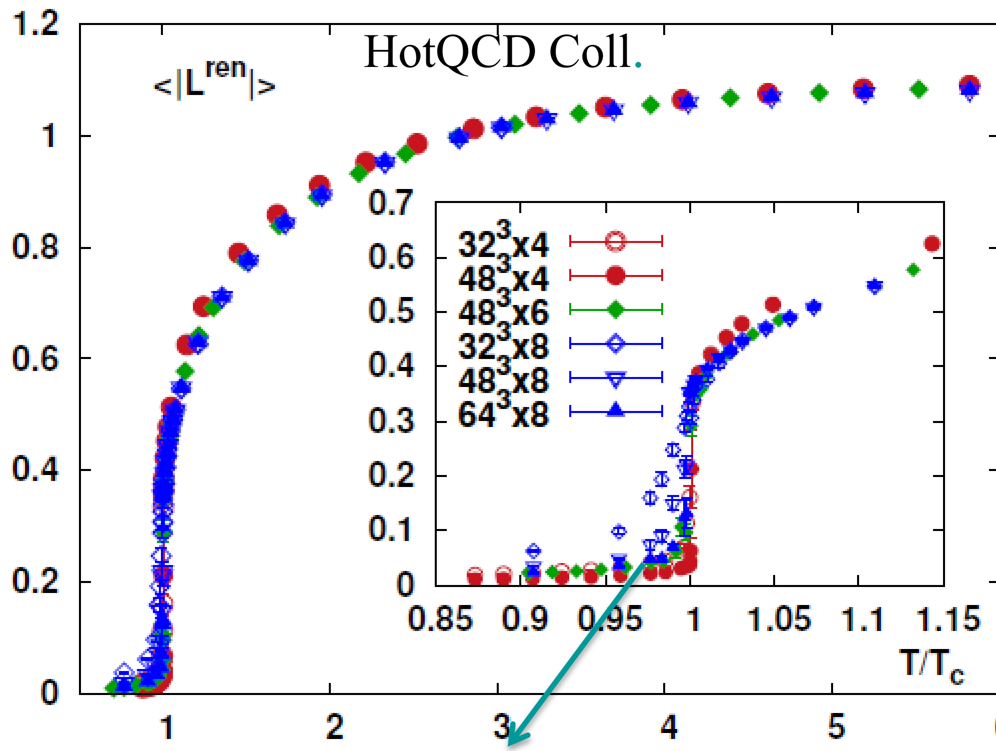
$$\langle |L^{\rm ren}| \rangle = e^{-\beta F_q^{\rm ren}} \rightarrow \begin{cases} \neq 0 \Leftrightarrow \text{deconfined } T > T_c \\ 0 \Leftrightarrow \text{confined } T < T_c \end{cases}$$

$$L^{\rm bare} = \frac{1}{N_\sigma^3} \sum_{\vec{x}} L_{\vec{x}}^{\rm bare}$$

- Renormalized ultraviolet divergence

$$L^{\rm ren} = \left(Z(g^2)\right)^{N_\tau} L^{\rm bare}$$

- Usually one takes $\langle |L^{\rm ren}| \rangle$ as an order parameter



$1/\sqrt{V}$

To probe deconfinement : consider fluctuations

- Fluctuations of modulus of the Polyakov loop

$$T^3 \chi_A = \frac{N_\sigma^3}{N_\tau^3} \left(\langle |L^{\rm ren}|^2 \rangle - \langle |L^{\rm ren}| \rangle^2 \right)$$

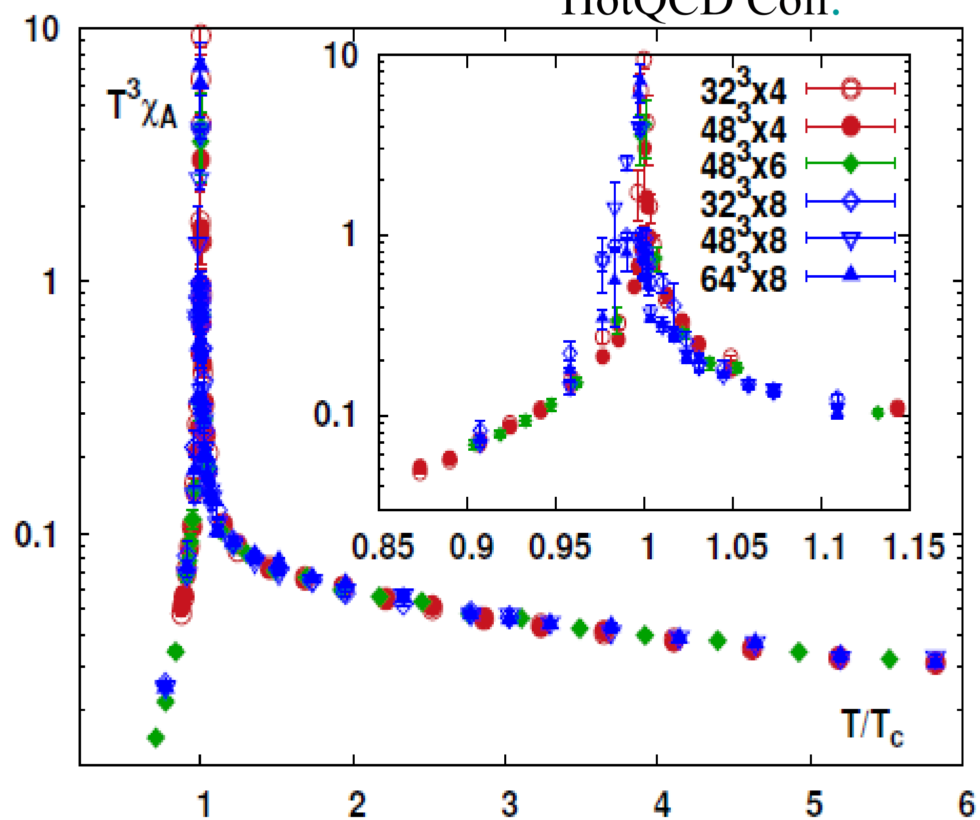
However, the Polyakov loop

$$L = L_R + i L_I$$

Thus, one can consider fluctuations of the real χ_R and the imaginary part χ_I of the Polyakov loop.

SU(3) pure gauge: LGT data

HotQCD Coll.



Fluctuations of the real and imaginary part of the renormalized Polyakov loop

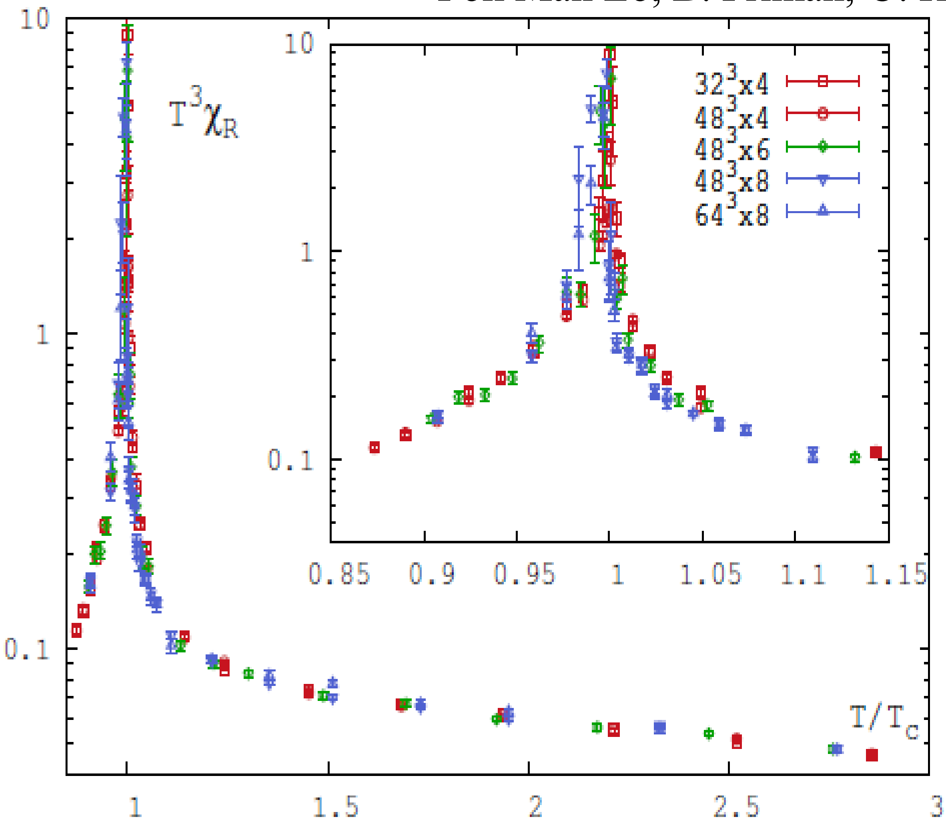
- Real part fluctuations

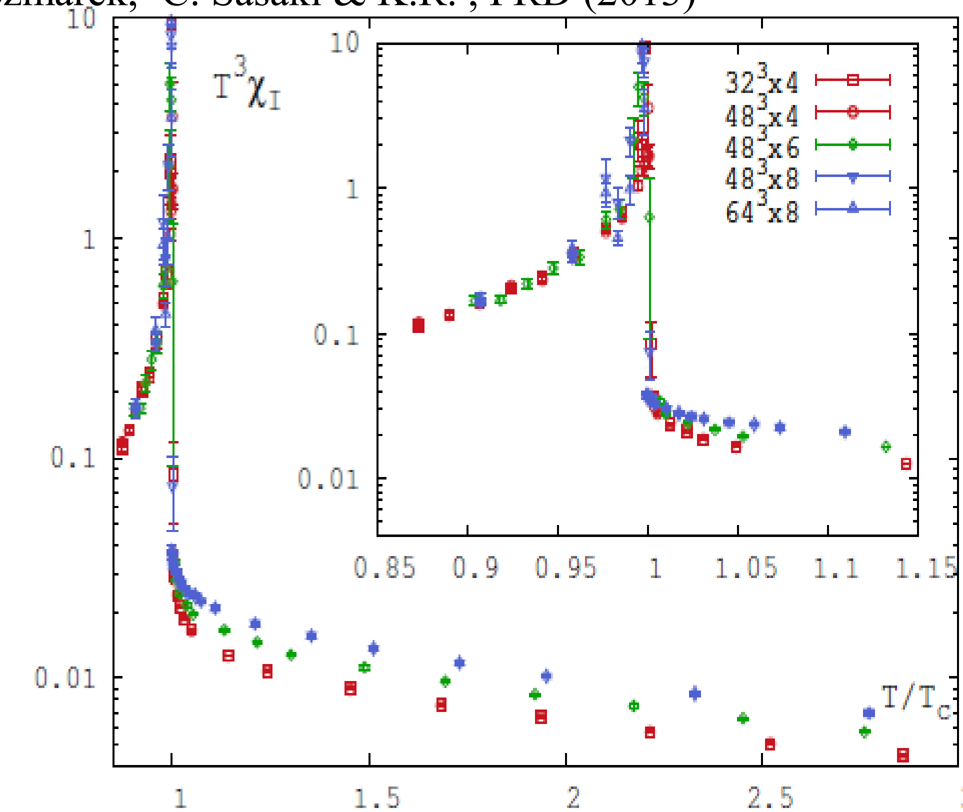
$$T^3\chi_R = \frac{N_\sigma^3}{N_\tau^3}\left[\langle (L_R^{\rm ren})^2\rangle - \langle L_R^{\rm ren}\rangle^2\right]$$

- Imaginary part fluctuations

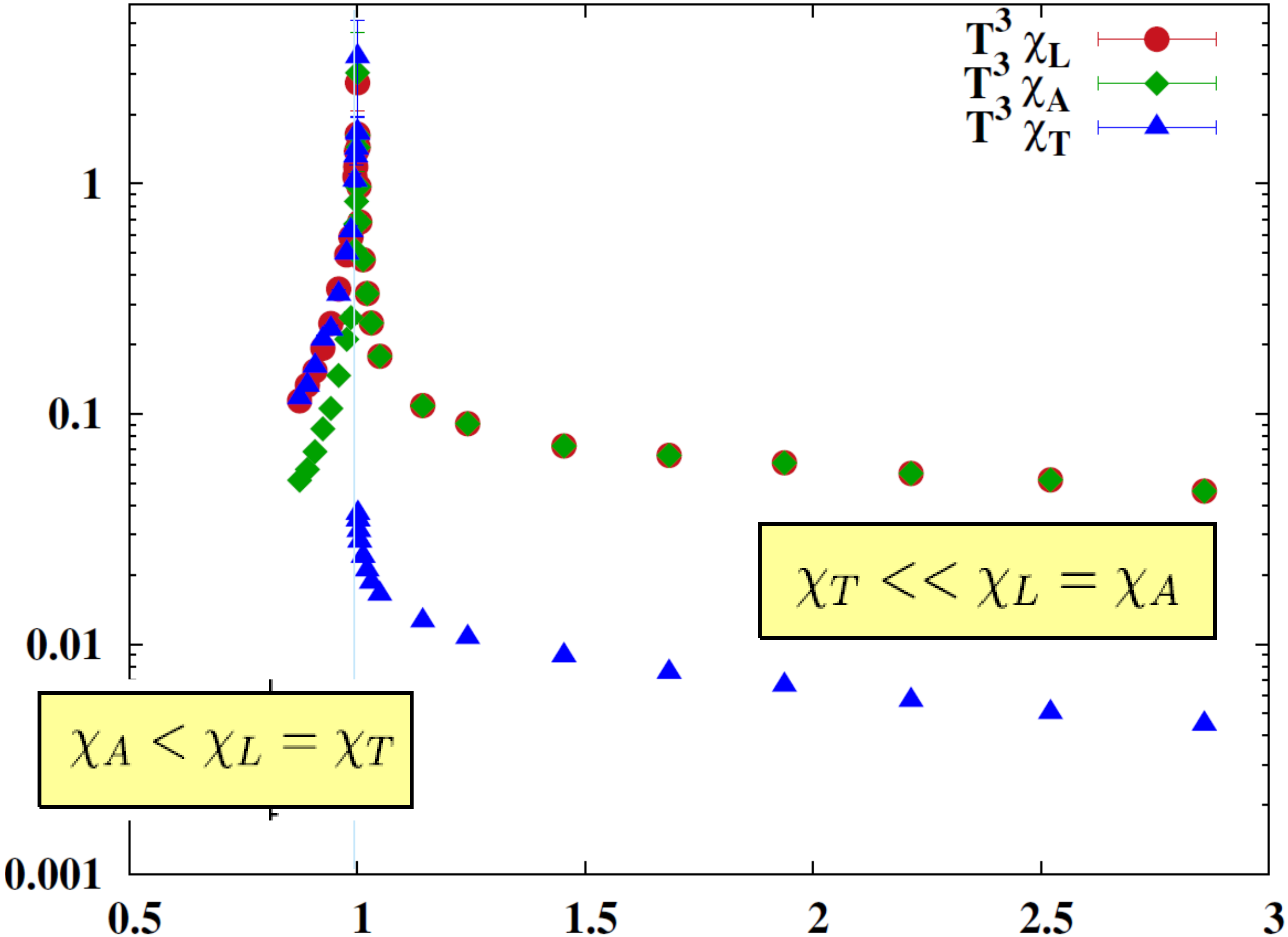
$$T^3\chi_I = \frac{N_\sigma^3}{N_\tau^3}\left[\langle (L_I^{\rm ren})^2\rangle - \langle L_I^{\rm ren}\rangle^2\right]$$

Pok Man Lo, B. Friman, O. Kaczmarek, C. Sasaki & K.R. , PRD (2013)





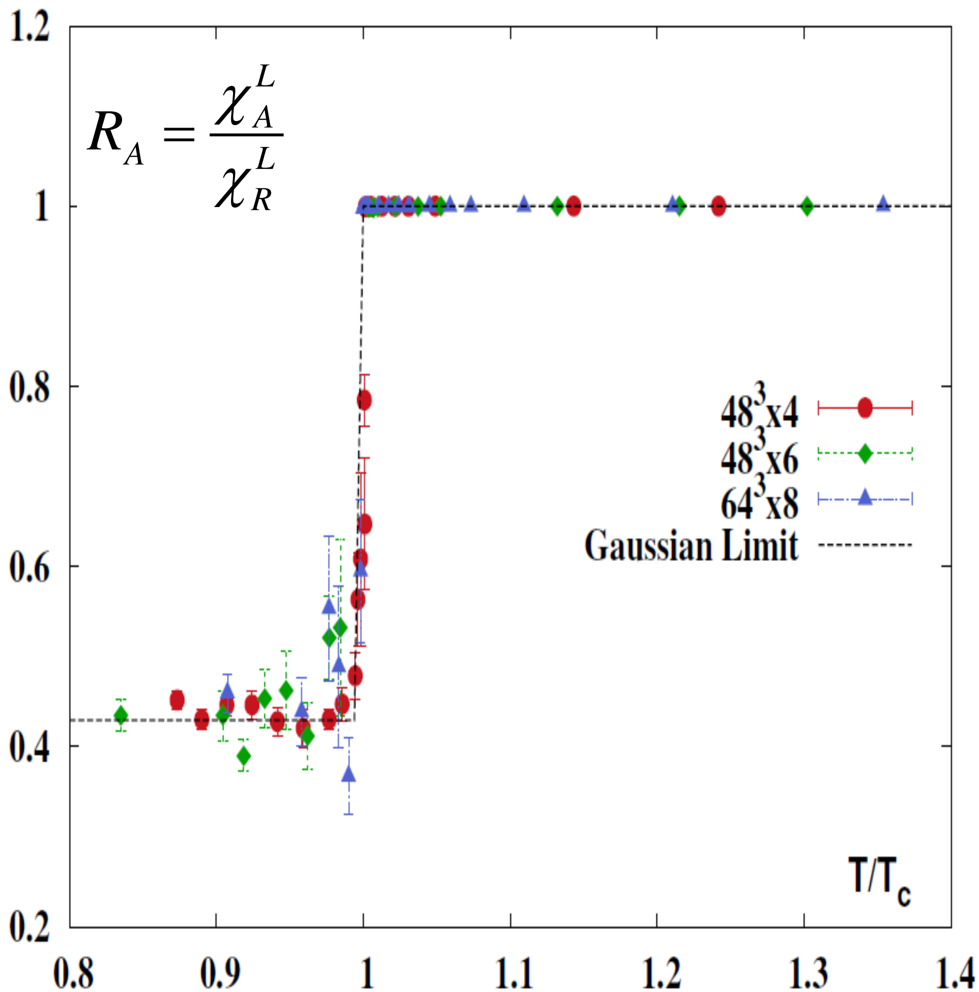
Compare different Susceptibilities:



- Systematic differences/similarities of the Polyakov lopp susceptibilities
- Consider their ratios!

Ratios of the Polyakov loop fluctuations as an excellent probe for deconfinement

Pok Man Lo, B. Friman, O. Kaczmarek, C. Sasaki & K.R. , PRD (2013)



- In the deconfined phase $R_A \approx 1$

Indeed, in the real sector of Z(3)

$$L_R \approx L_0 + \delta L_R \quad \text{with} \quad L_0 = < L_R >$$

$$L_I \approx L_0^I + \delta L_I \quad \text{with} \quad L_0^I = 0, \quad \text{thus}$$

$$\chi_R^L = V < (\delta L_R)^2 >, \quad \chi_I^L = V < (\delta L_I)^2 >$$

Expand the modulus,

$$| L | = \sqrt{L_R^{\ 2} + L_I^{\ 2}} \approx L_0 (1 + \frac{\delta L_R}{L_0} + \frac{(\delta L_I)^2}{2 L_0^{\ 2}})$$

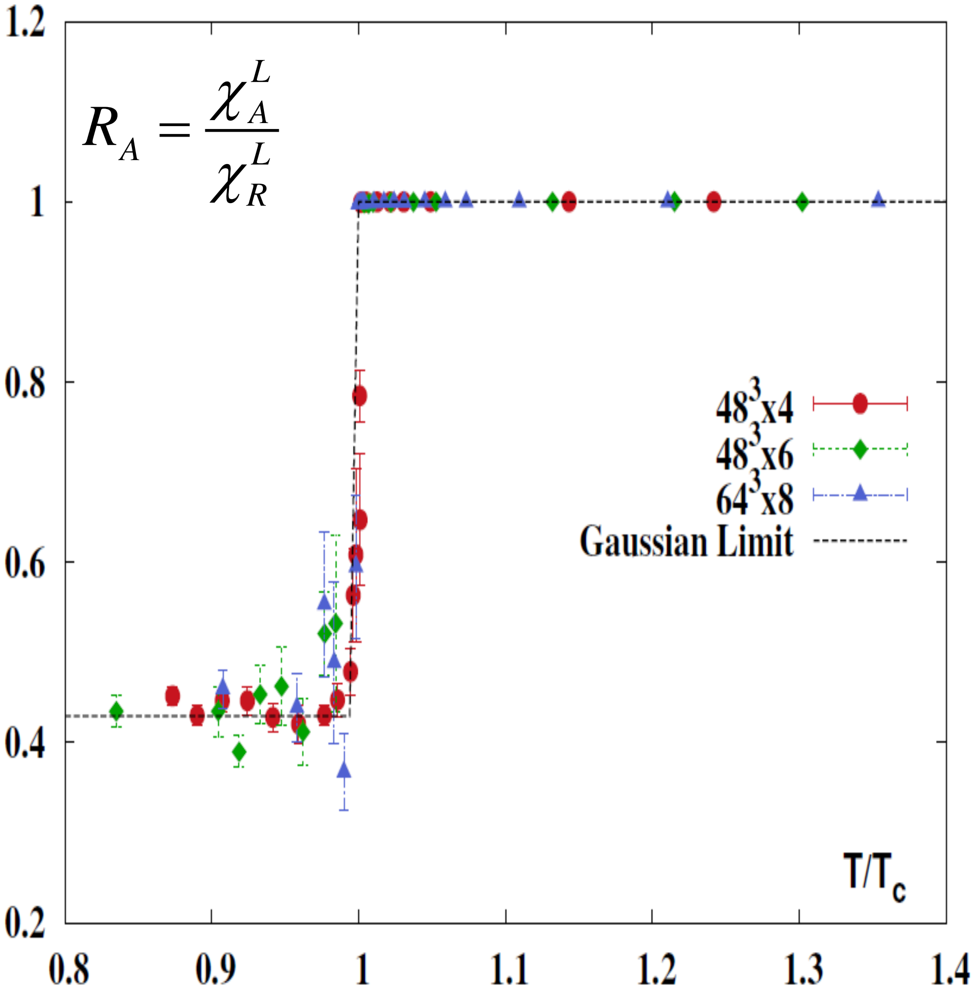
get in the leading order

$$\boxed{< | L |^2 > - < | L | >^2 \approx < (\delta L_R)^2 >}$$

thus $\chi_A \approx \chi_R$

Ratios of the Polyakov loop fluctuations as an excellent probe for deconfinement

Pok Man Lo, B. Friman, O. Kaczmarek, C. Sasaki & K.R. , PRD (2013)



- In the confined phase $R_A \approx 0.43$

Indeed, in the Z(3) symmetric phase, the probability distribution is Gaussian to the first approximation, with the partition function

$$Z = \int dL_R dL_I e^{VT^3[\alpha(T)(L_R^2 + L_I^2)]}$$

Thus $\chi_R = \frac{1}{2\alpha T^3}$, $\chi_I = \frac{1}{2\alpha T^3}$ and

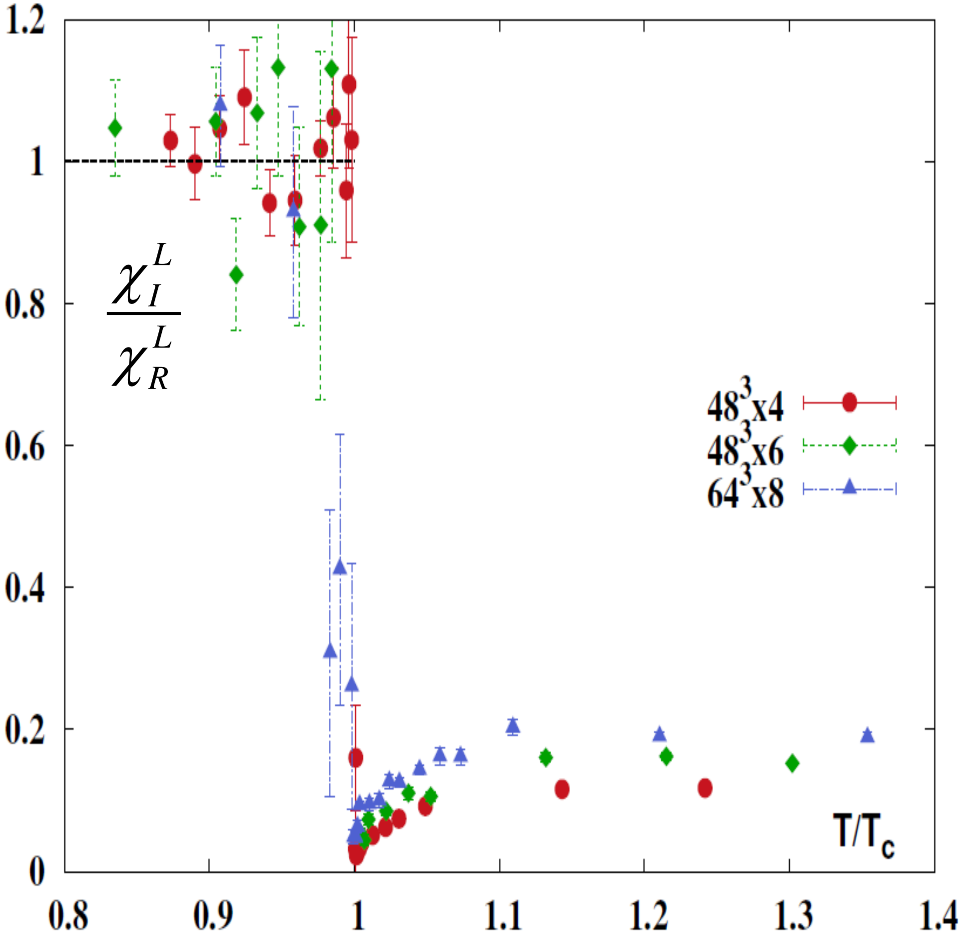
$\chi_A = \frac{1}{2\alpha T^3}(2 - \frac{\pi}{2})$, consequently

$$R_A^{SU(3)} = (2 - \frac{\pi}{2}) = 0.429$$

In the SU(2) case $R_A^{SU(2)} = (2 - \frac{2}{\pi}) = 0.363$ is in agreement with MC results

Ratio Imaginary/Real of Polyakov loop fluctuations

Pok Man Lo, B. Friman, O. Kaczmarek,
C. Sasaki & K.R. , PRD (2013)



- In the confined phase for any symmetry breaking operator its average vanishes, thus

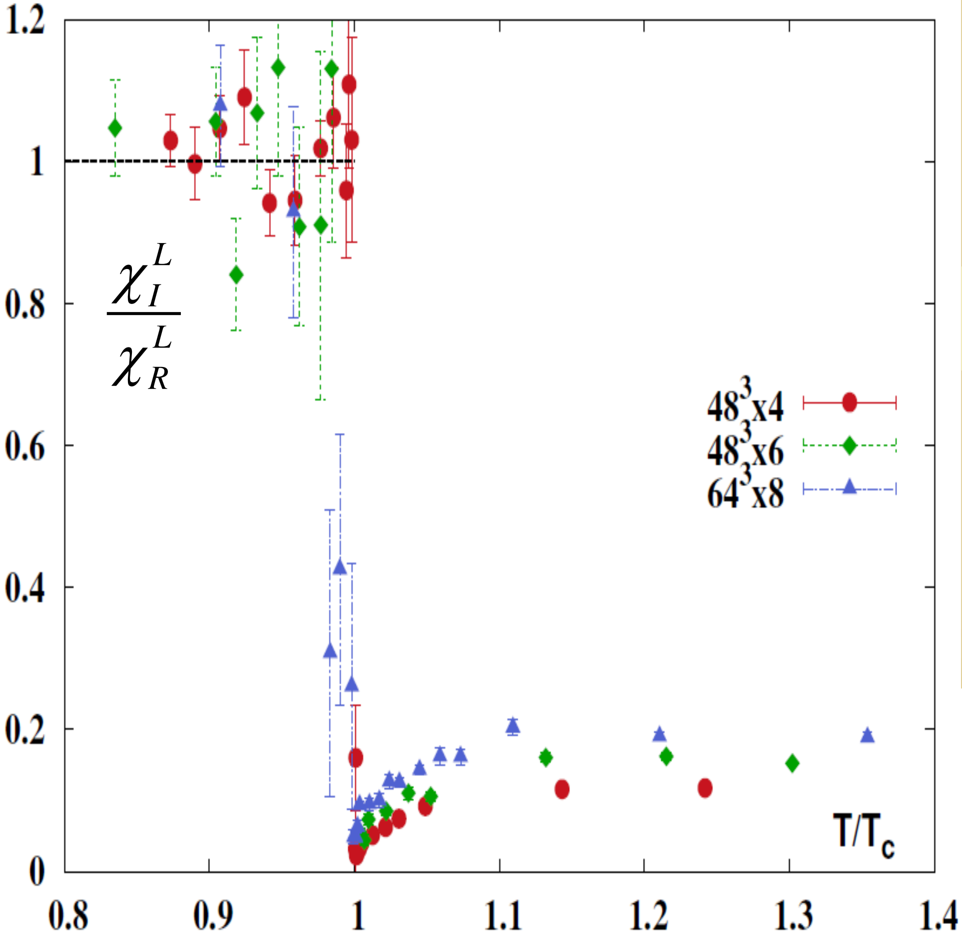
  $\chi_{LL} = < L^2 > - < L >^2 = 0$ and

  $\chi_{LL} = \chi_R - \chi_I$ thus $\boxed{\chi_R = \chi_I}$

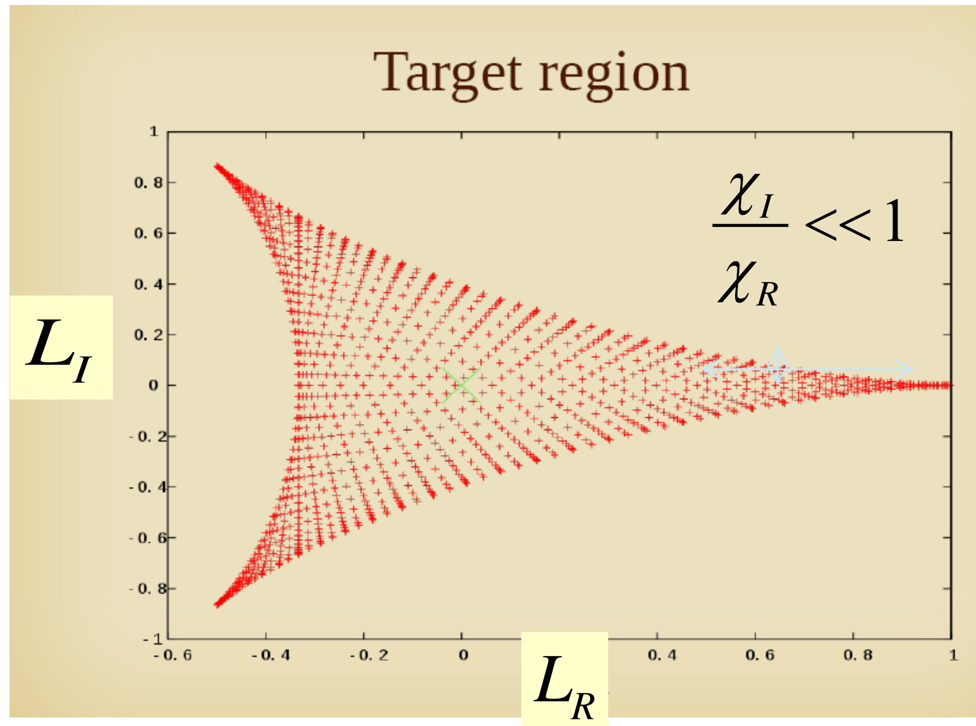
- In deconfined phase the ratio of $\chi_I / \chi_R \neq 0$ and its value is model dependent

Ratio Imaginary/Real of Polyakov loop fluctuations

Pok Man Lo, B. Friman, O. Kaczmarek, C. Sasaki & K.R. , PRD (2013)



- In deconfined phase



Due to Z(3) symmetry breaking in deconfined phase, the fluctuations of transverse Polyakov loop strongly suppressed,

Deconfinement phase transition in a large quark mass limit: effective approach

Start with QCD thermodynamic potential

Employ the background field approach

$$A_\mu \approx A_\mu^{cl} + gA_\mu^{quantum}$$

Consider a uniform background

$$A_\mu = A_\mu \delta_{4\mu}$$

Consider effective gluon potential $U(L, L^+)$ with parameters fixed to reproduce a pure LQCD thermodynamic as:

C. Sasaki &K.R. Phys.Rev. D86 (2012) 014007

- Effective partition function

$$Z = \int dL dL^+ e^{-\beta V U(L,L^+) + \ln \det[Q_f]}$$

where

$$\hat{Q}_F = (-\partial_\tau + \mu + igA_4)\gamma^0 + i\vec{\gamma}\cdot\nabla - M_Q$$

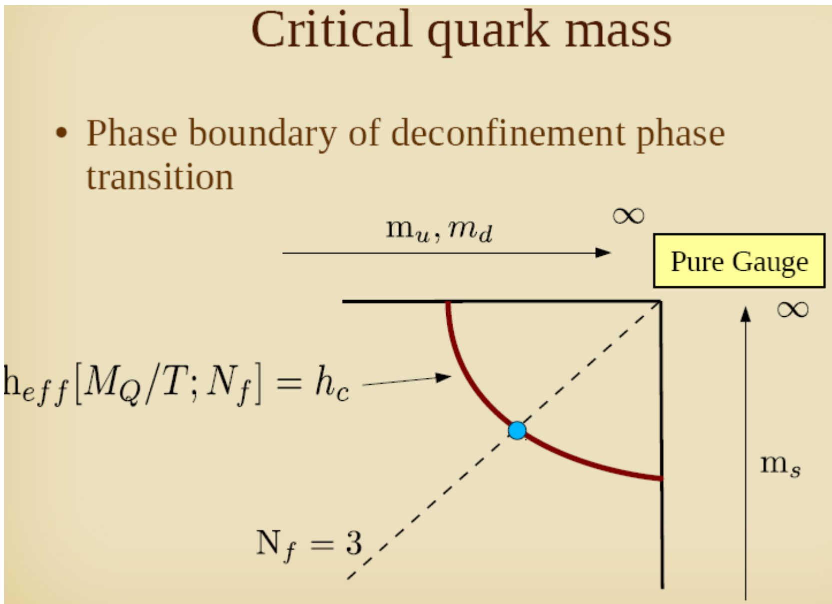
$$T \ln \det[\hat{Q}_F] = V 2 N_f T \int \frac{d^3k}{(2\pi)^3} [3\beta E[k] + \ln g^+ + \ln g^-]$$

$$g^\pm = 1 + 3\{L, \bar{L}\} e^{-\beta E^\pm} + 3\{\bar{L}, L\} e^{-2\beta E^\pm} + e^{-3\beta E^\pm}$$

$$E^\pm = E[k] \mp \mu$$

$$E[k] = (k^2 + M_Q^2)^{1/2}$$

PL and heavy quark coupling



- Effective potential

$$\ln \det[Q_f] = -VT^3 U_q[L, L^+; M_q]$$

- Tree level result $M_q >> T$

$$U_q = -h_{eff}[M_q / T] L_R$$

G. Green & F. Karsch (83) $\boxed{U_G \to U_G - h_{eff} L_R}$

where $h_{eff} \approx N_f (M_q / T)^2 K_2(M_q / T)$

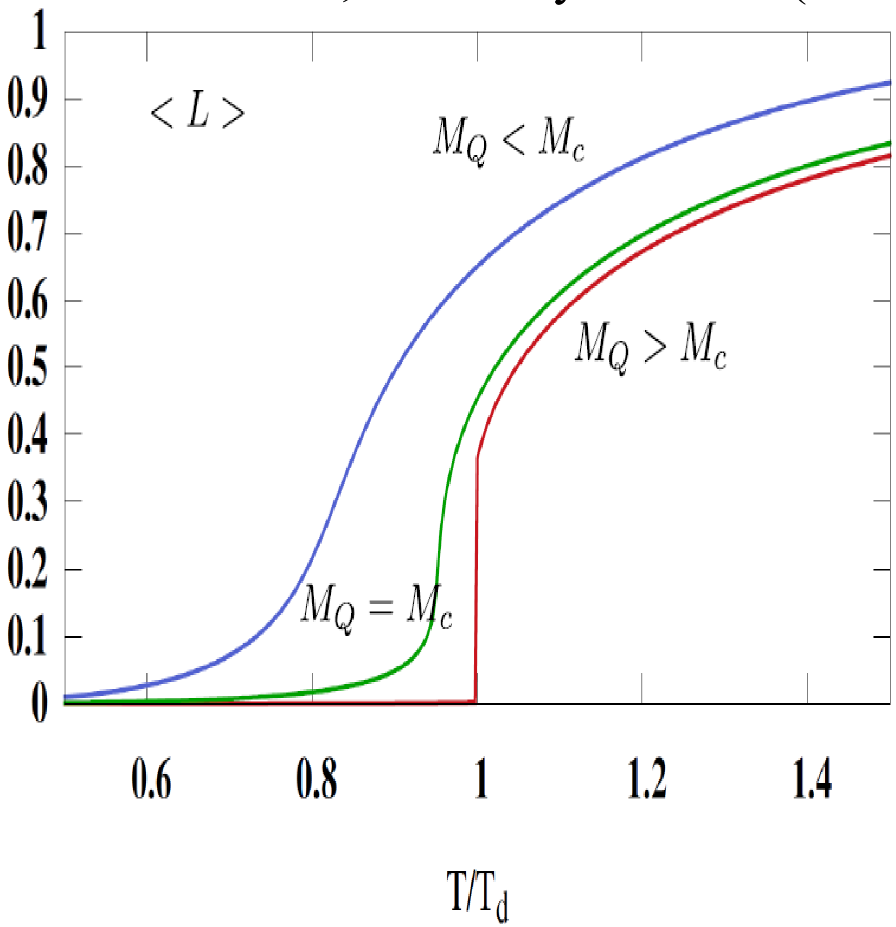
Compare with LGT:

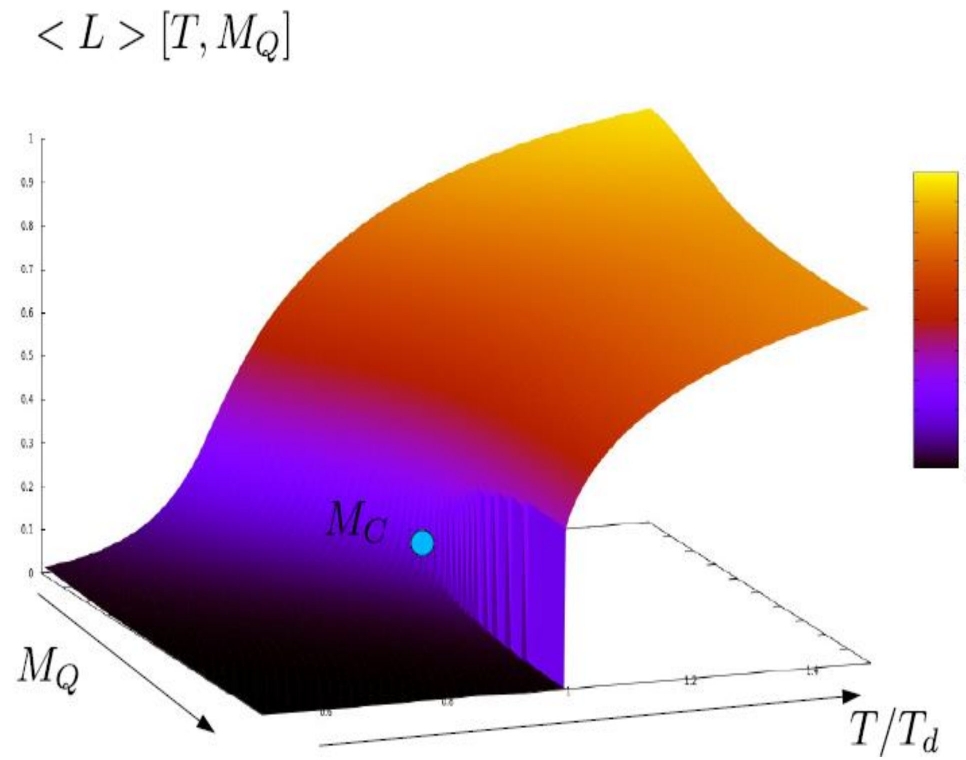
$$h_{eff}^{LGT} = (2N_f)(2N_c)(2\kappa(N_\tau))^{N_\tau} N_\tau^{\ 3}$$

H. Saito, S. Ejiri, S. Aoki, T. Hatsuda, K. Kanaya, Y. Maezawa, H. Ohno, and T. Umeda, Phys. Rev. D **84** (2011) 054502

The critical point of the 2nd order transition

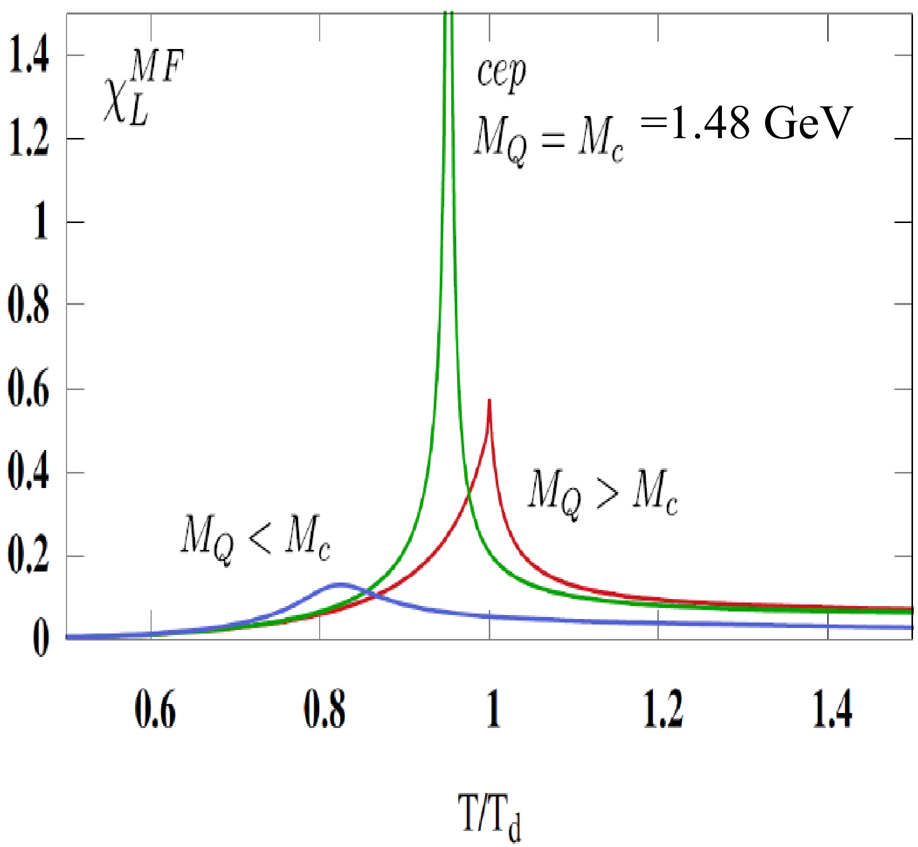
Pok Man Lo, et al. Phys. Rev. D (2014)

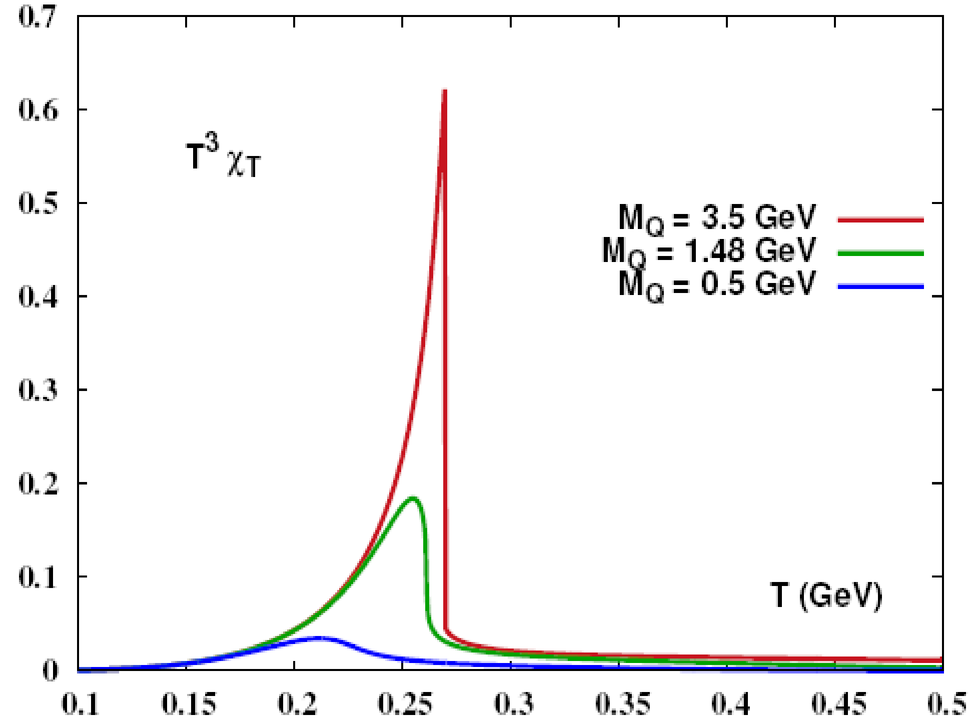




Susceptibility at the critical point

Pok Man Lo, et al. Phys. Rev. D (2014)

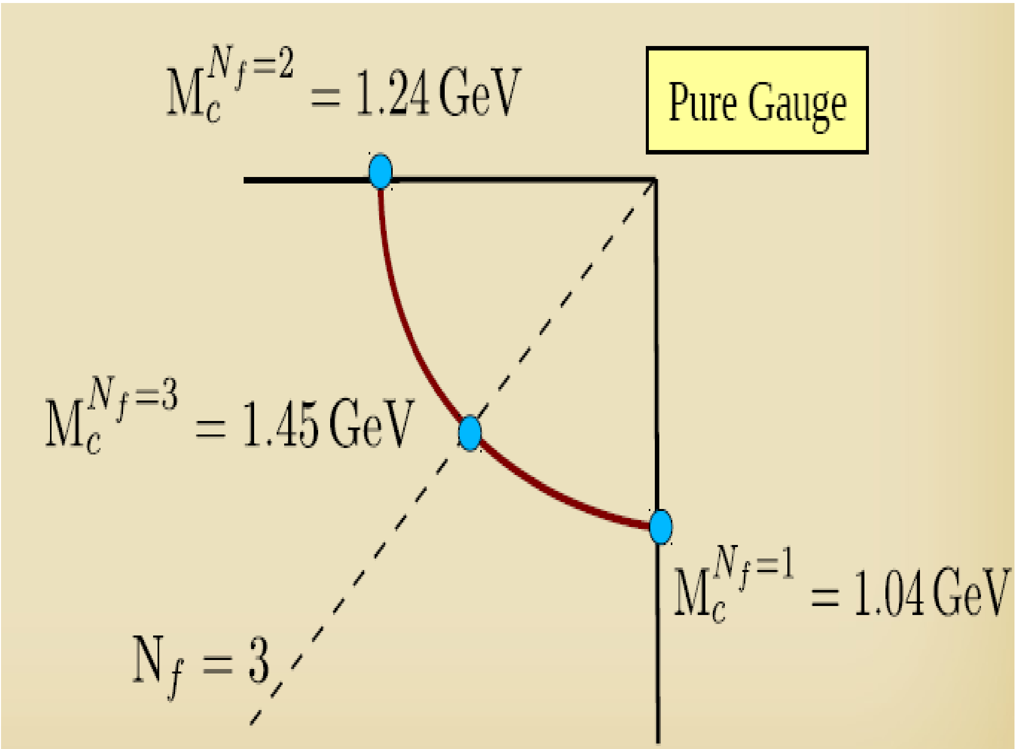


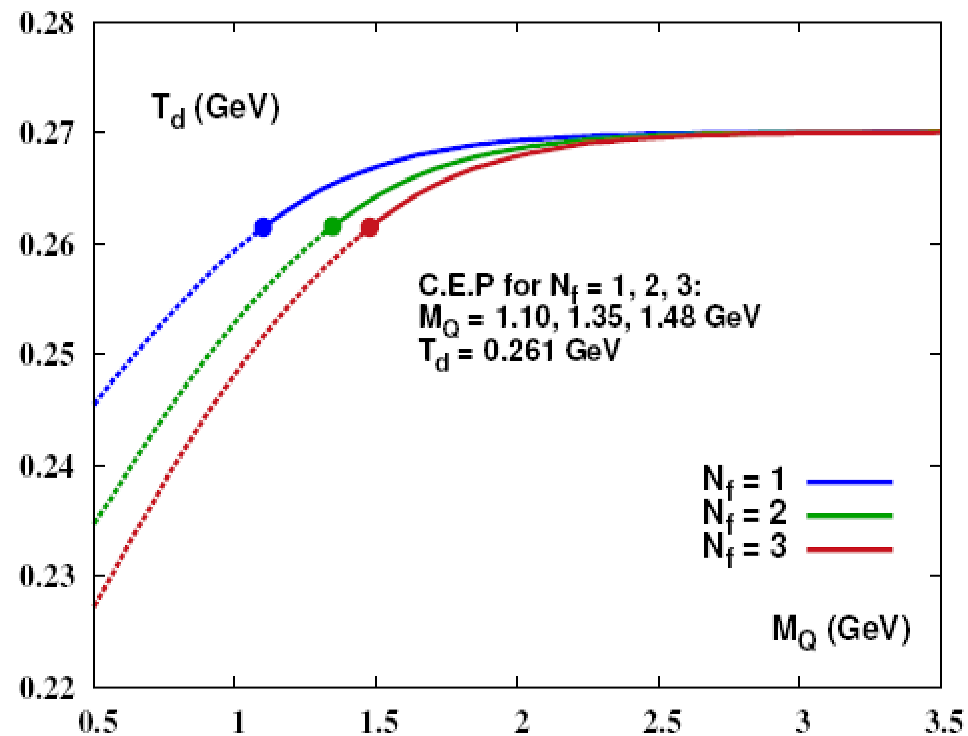


- Divergent longitudinal susceptibility at the critical point for decondinement

Critical masses and temperature values

Pok Man Lo, et al. Phys. Rev. D (2014)





- Different values then in the matrix model by

$$M_c^{N_f=3} \approx 2.5\,\mathrm{GeV}$$

$$T_c^{\mathrm{de}} \approx 0.27\,\mathrm{GeV}$$

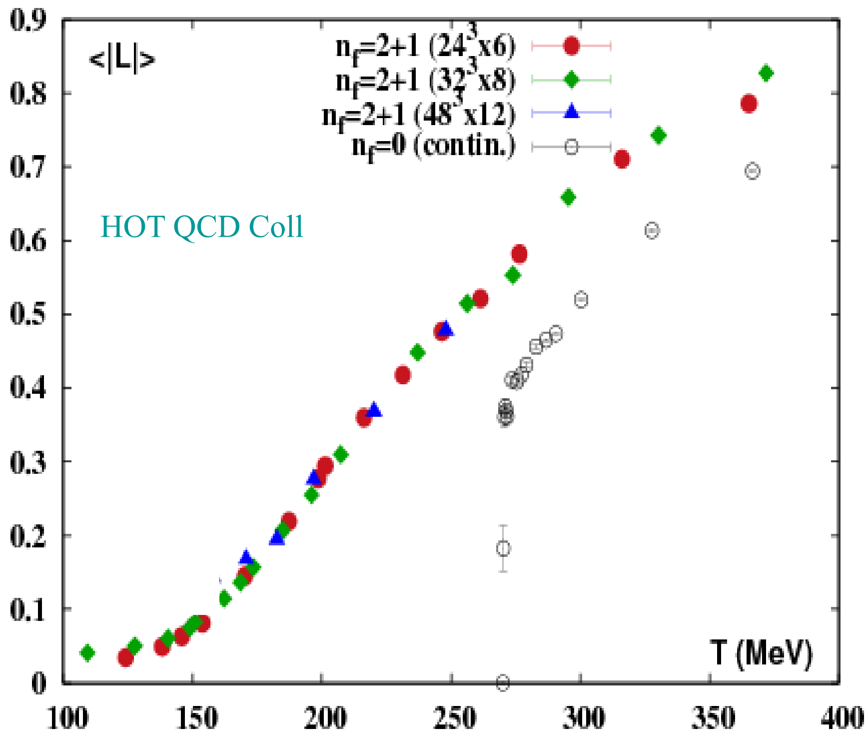
K. Kashiwa, R. Pisarski and V. Skokov,
Phys. Rev. D85 (2012)

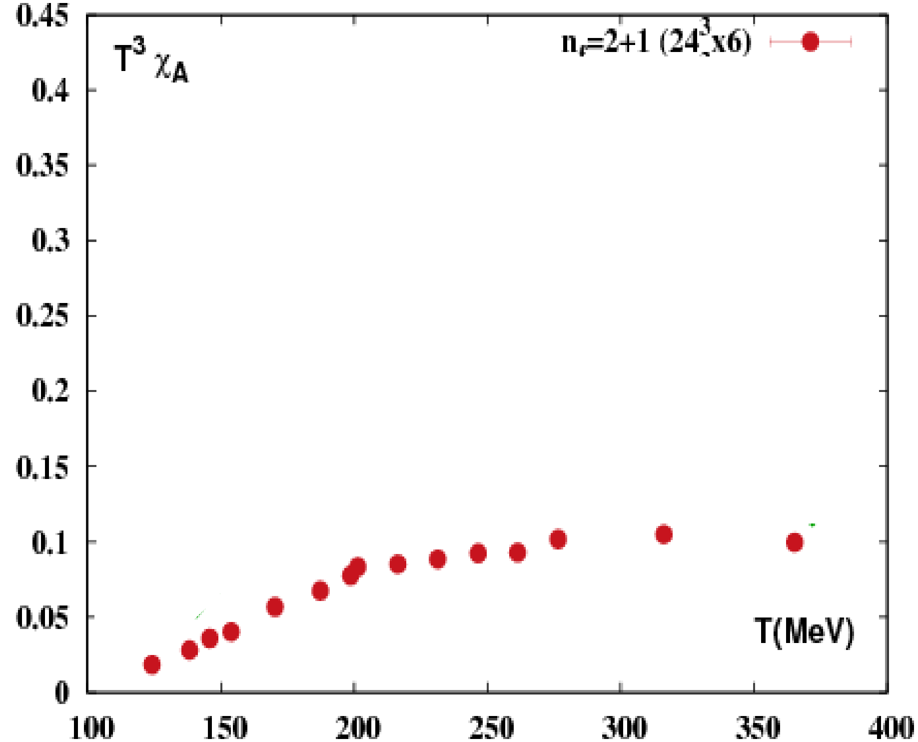
LGT C. Alexandrou et al. (99) $M_c^{N_f=1} \approx 1.4 GeV$

Polyakov loop and fluctuations in QCD

- Smooth behavior for the Polyakov loop and fluctuations

  ⇨ difficult to determine where is “deconfinement”



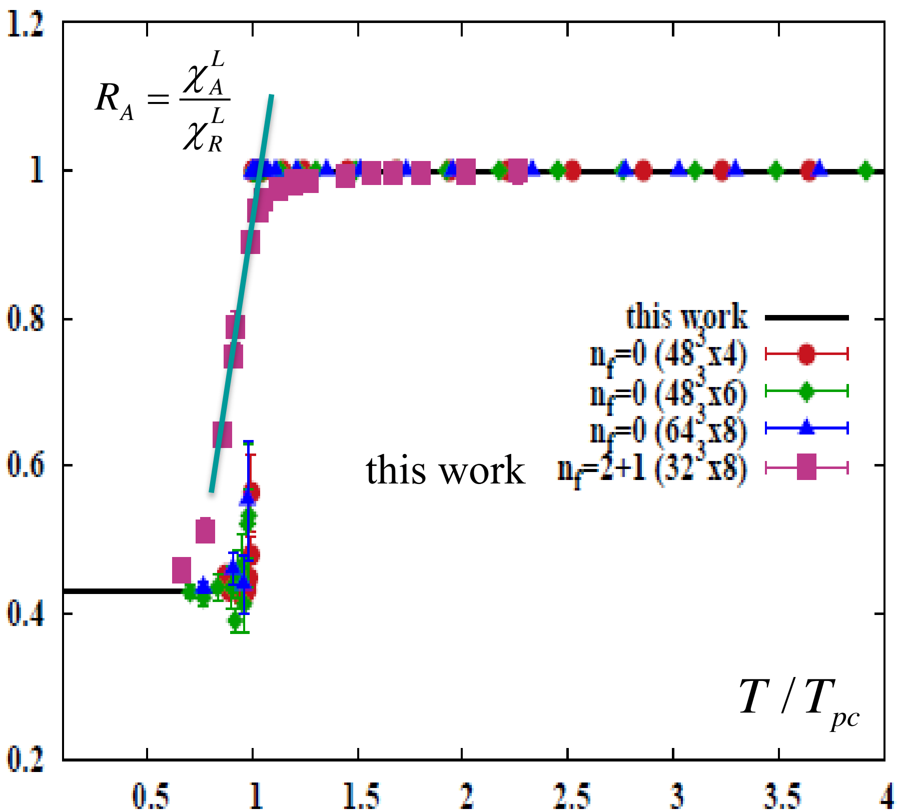


The inflection point at $T_{dec} \approx 0.22 GeV$

The influence of fermions on the Polyakov loop susceptibility ratio

- Z(3) symmetry broken, however ratios still showing deconfinement

Pok Man Lo, B. Friman, O. Kaczmarek, C. Sasaki & K.R.



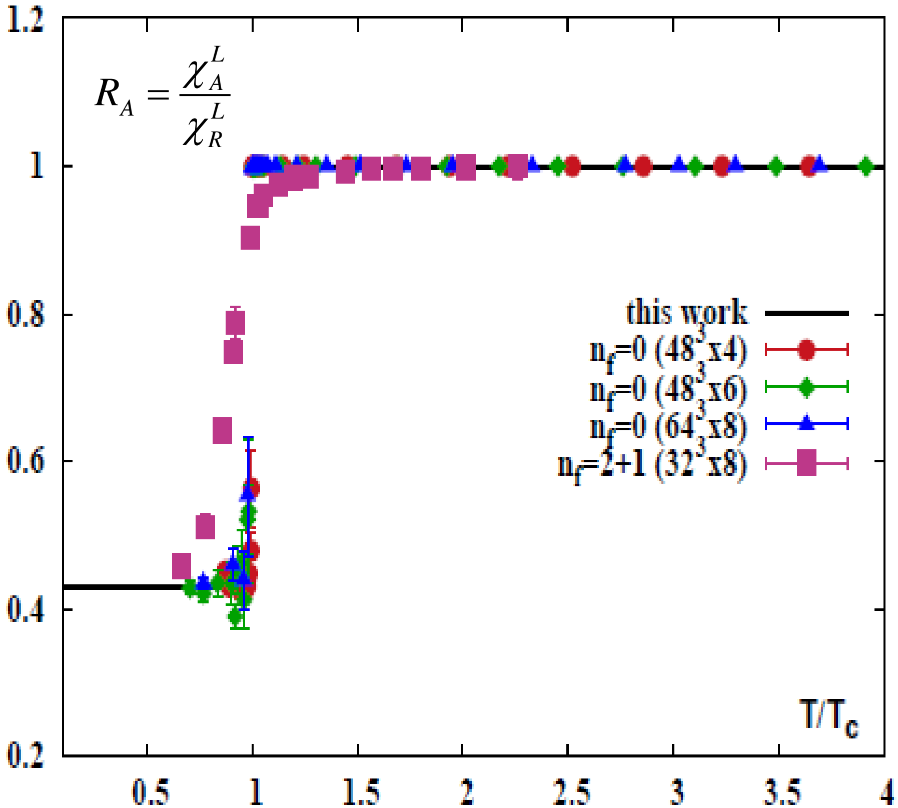
- Change of the slope in the narrow temperature range signals color deconfinement

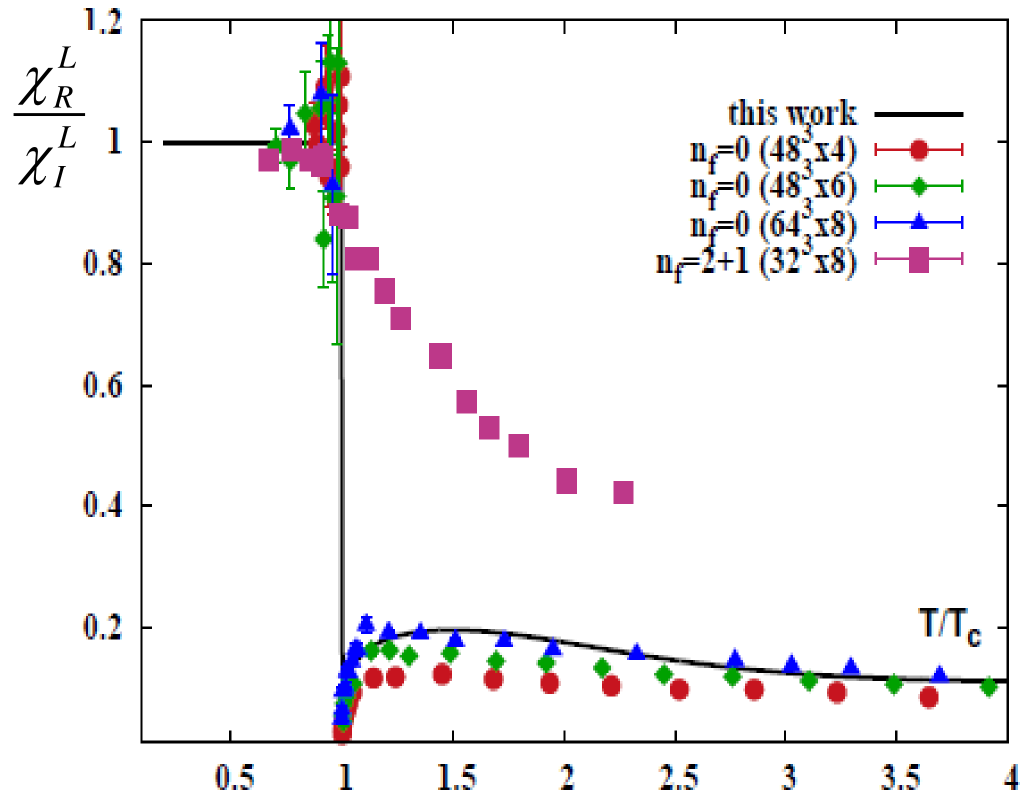
- Dynamical quarks imply smoothening of the susceptibilities ratio, between the limiting values as in the SU(3) pure gauge theory

The influence of fermions on ratios of the Polyakov loop susceptibilities

- Z(3) symmetry broken, however ratios still showing the transition
- Change of the slopes at fixed T

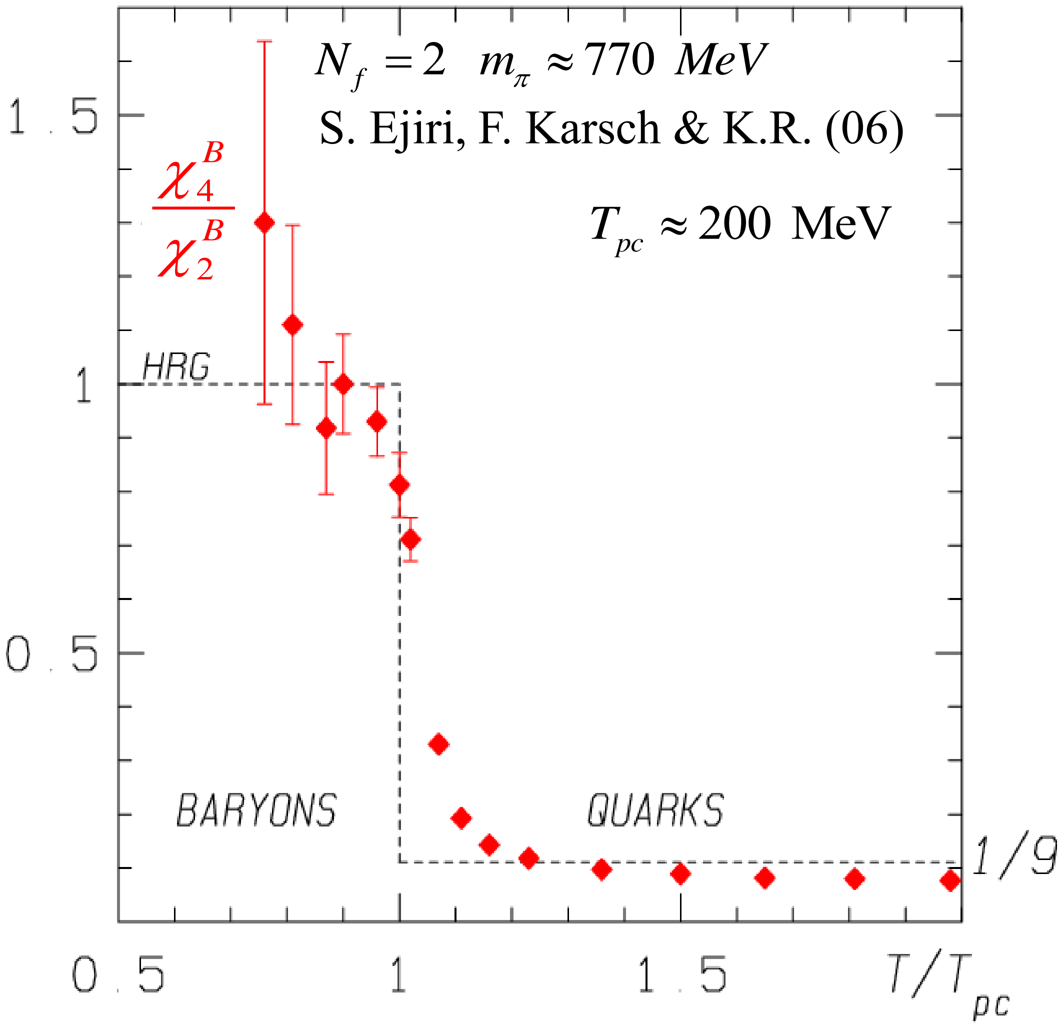
Pok Man Lo, B. Friman, O. Kaczmarek, C. Sasaki & K.R.





Fluctuations of net baryon number sensitive to deconfinement in QCD

$16^3 \times 4$ lattice with p4 fermion action



$$\chi_n^B = \frac{\partial^n (P/T^4)}{\partial(\mu_B/T)^n}$$

- HRG factorization of pressure:

$$\boxed{P^B(T,\mu_q) = F(T)\cosh(B\mu_B/T)}$$

- Kurtosis measures the squared of the baryon number carried by leading particles in a medium

S. Ejiri, F. Karsch & K.R. (06)

$$\kappa\,\sigma^2 = \frac{\chi_4^B}{\chi_2^B} \approx B^2 = \begin{pmatrix} 1 & T < T_{PC} \\ \frac{1}{9} & T > T_{PC} \end{pmatrix}$$

Kurtosis of net quark number density in PQM model

V. Skokov, B. Friman &K.R.

- For $T < T_c$

  the assymptotic value

  due to „confinement" properties

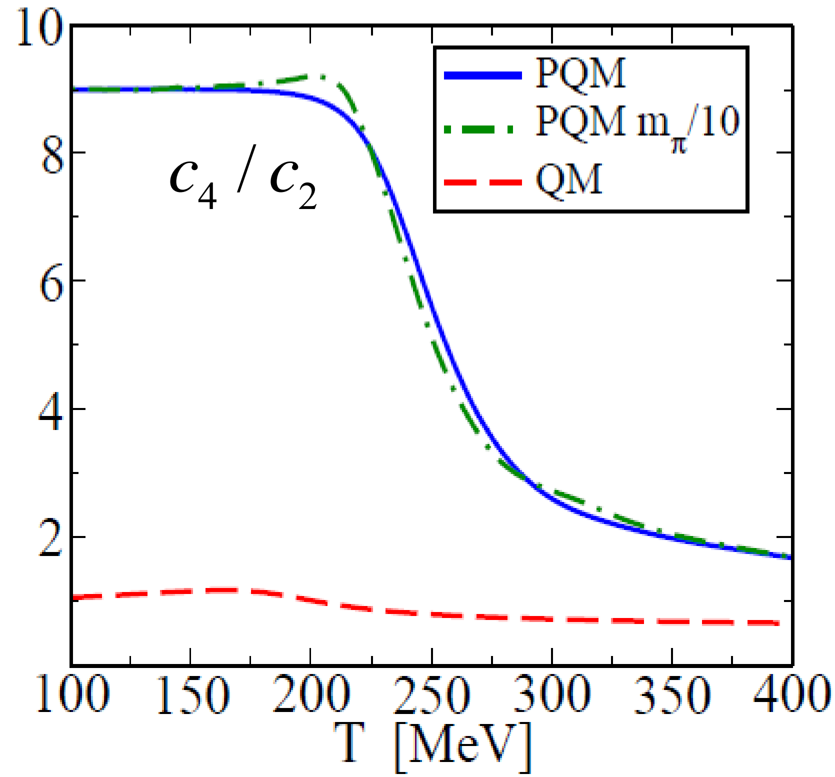
$$\frac{P_{q\bar{q}}(T)}{T^4} \approx \frac{2N_f}{N_c^2}\left(\frac{3m_q}{T}\right)^2 K_2\left(\frac{3m_q}{T}\right)\cosh\frac{3\mu_q}{T}$$

$$\Rightarrow \quad c_4 / c_2 = 9$$

- For $T >> T_c$

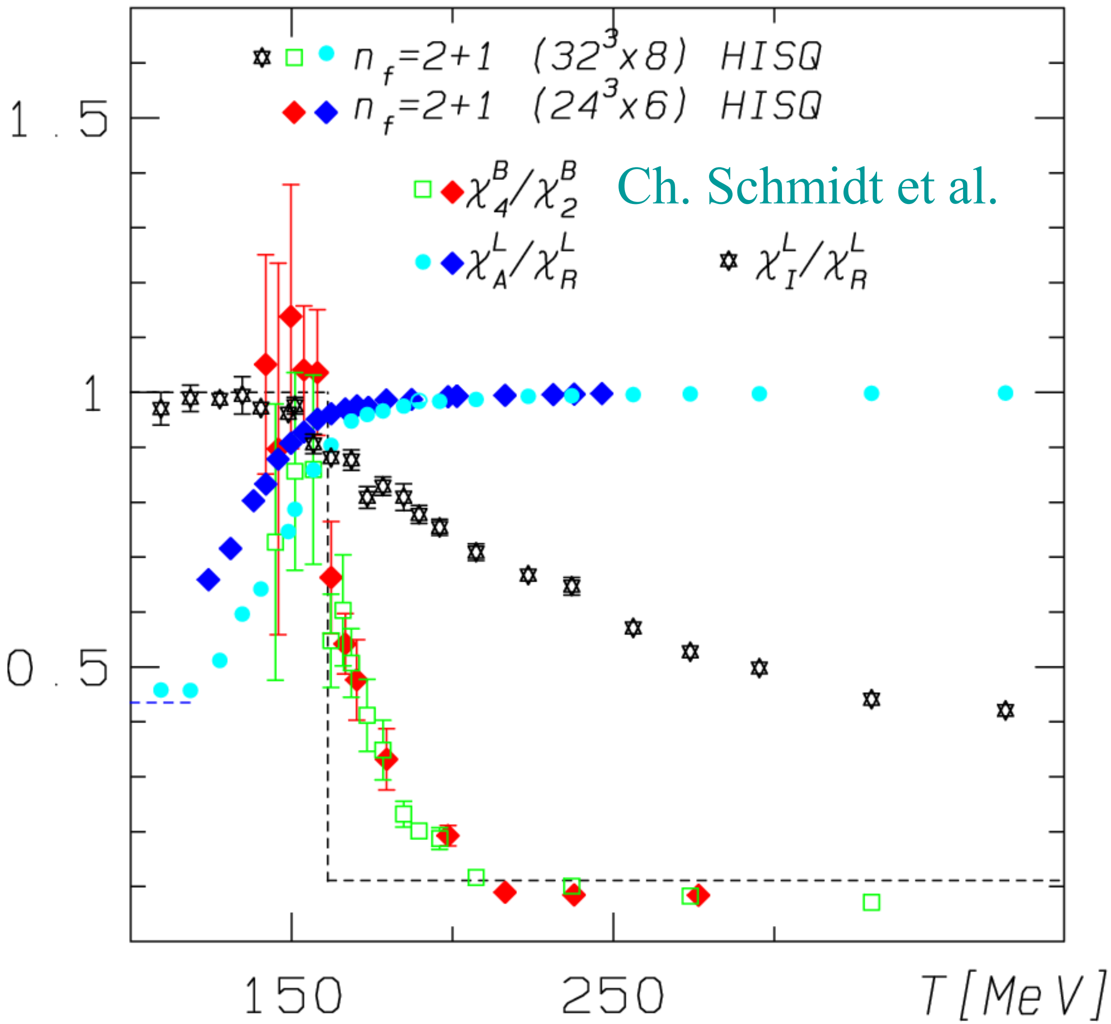
$$\frac{P_{q\bar{q}}(T)}{T^4} = N_f N_c [\frac{1}{2\pi^2}\left(\frac{\mu}{T}\right)^4 + \frac{1}{6}\left(\frac{\mu}{T}\right)^2 + \frac{7\pi^2}{180}]$$

$$\Rightarrow c_4 / c_2 = 6 / \pi^2$$



- Smooth change with a very weak dependence on the pion mass

Polyakov loop susceptibility ratios still away from the continuum limit:

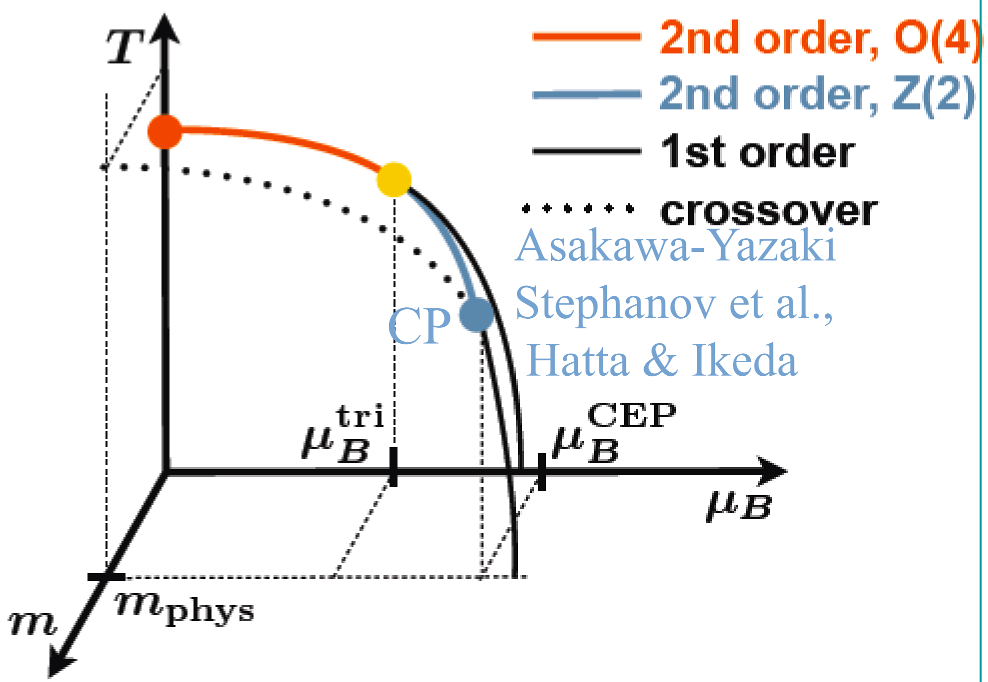


- The renormalization of the Polyakov loop susceptibilities is still not well described:

Still strong dependence on N_τ in the presence of quarks.

The phase diagram at finite quark masses

At the CP: Divergence of Fluctuations, Correlation Length and Specific Heat



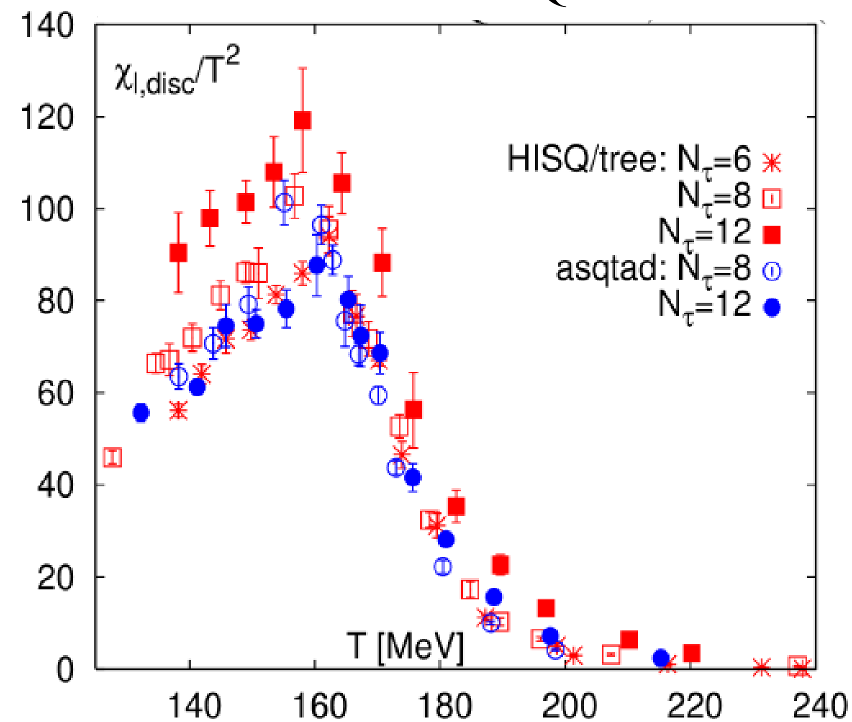
The existence of the CP in QCD has not been established yet

- To identify chiral crossover consider fluctuations of

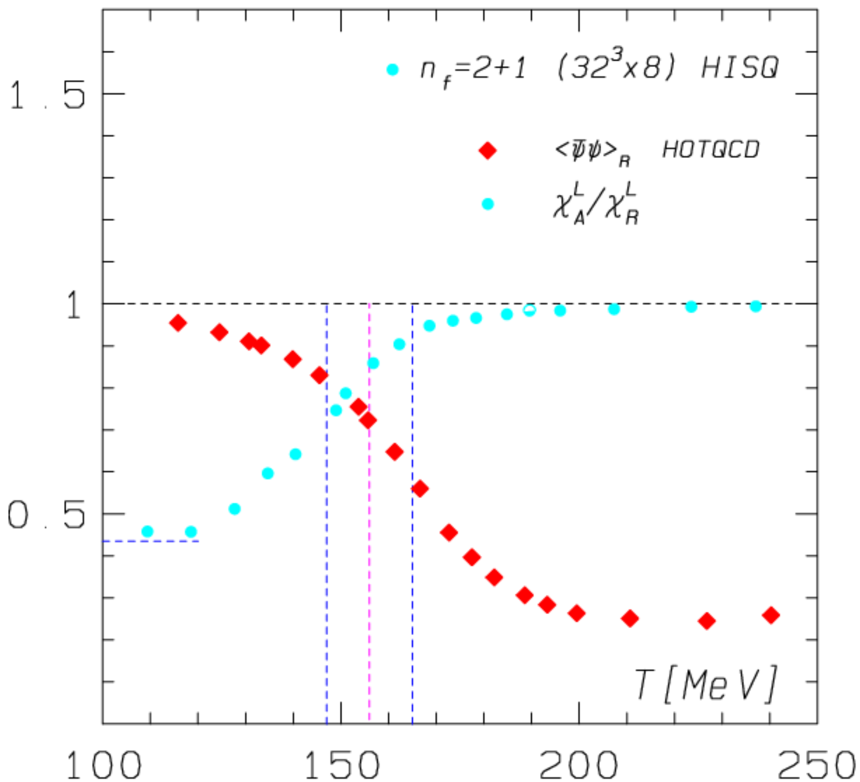
$$<\bar{\psi}\psi>=\frac{T}{V}\frac{\partial \ln Z}{\partial m} \qquad \chi_{\bar{\psi}\psi}=\frac{T}{V}\frac{\partial^2 \ln Z}{\partial m^2}$$

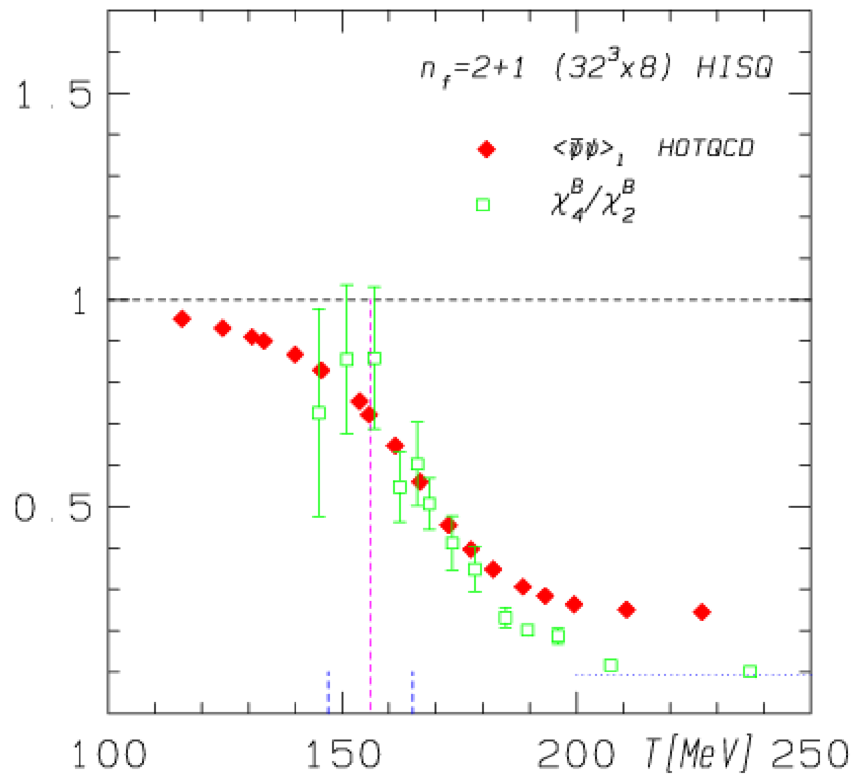
S. Borsayi et al. Wuppertal-Budapest Coll.

A. Bazavov et al. HotQCD Coll.



The interplay between deconfinement and the chiral crossover





- The change of properties of observables which are sensitive to deconfinement and chiral transition appear in the same narrow temperature range

.

Quark fluctuations and O(4) universality class

- Due to the expected O(4) scaling in QCD the free energy:

$$\boxed{F = F_R(T,\mu_q,\mu_I) + b^{-1} F_S(b^{(2-\alpha)^{-1}} t, b^{\beta\delta/\nu} h)}$$

- Consider generalized susceptibilities of the net-quark number

$$c_B^{(n)} = \frac{\partial^n (P/T^4)}{\partial(\mu_B/T)^n} = c_R^{(n)} + c_S^{(n)} \quad \text{with} \quad c_s^{(n)} = d\ h^{(2-\alpha-n)/\beta\delta}\ f_\pm^{(n)}(z)$$

- Since for $T < T_{pc}$, $c_R^{(n)}$ are well described by the HRG

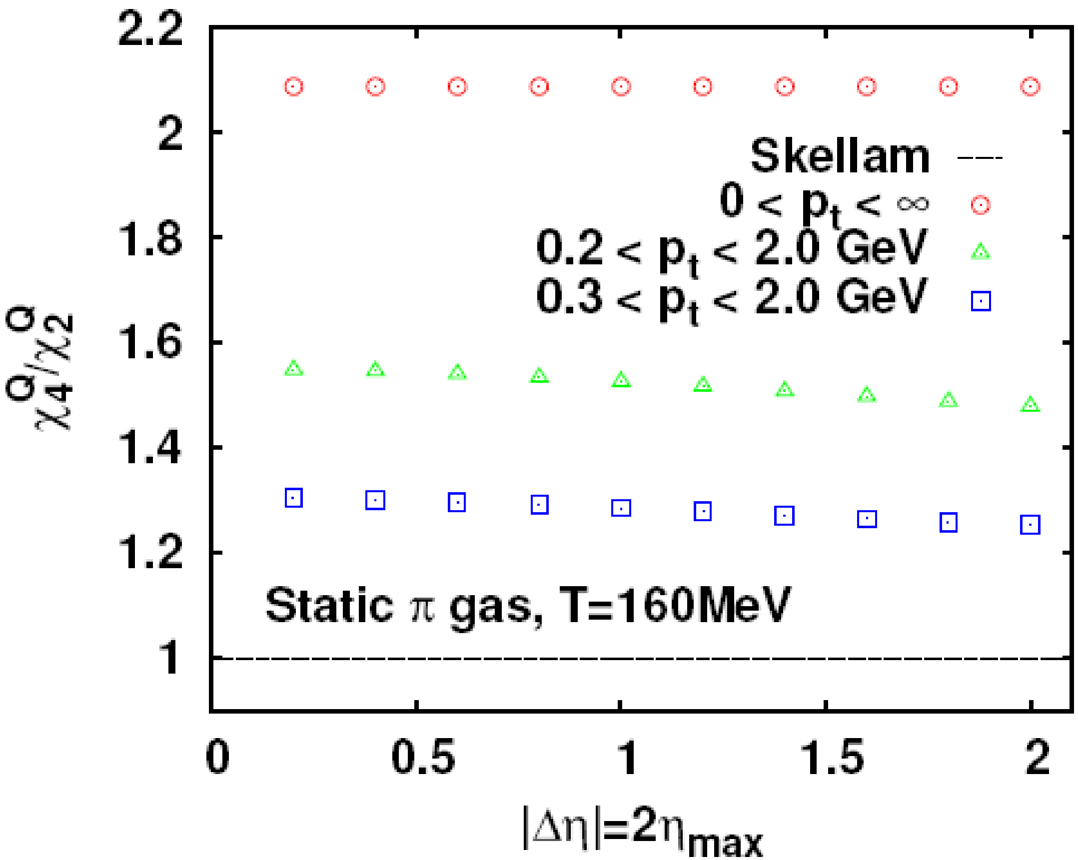
search for deviations (in particular for larger n) from HRG

⇨ to quantify the contributions of $c_S^{(n)}$, i.e. the O(4) criticality

S. Ejiri, F. Karsch & K.R. Phys. Lett. B633, (2006) 275

M. Asakawa, S. Ejiri and M. Kitazawa, Phys. Rev. Lett. 103 (2009) 262301
V. Skokov, B. Stokic, B. Friman &K.R. Phys. Rev. C82 (2010) 015206
F. Karsch & K. R. Phys.Lett. B695 (2011) 136
B. Friman, et al. . Phys.Lett. B708 (2012) 179, Nucl.Phys. A880 (2012) 48

Momentum cut dependence of the cumulant ratio of the electric charge fluctuations



- Strong dependence of the electric charge kurtosis on the low momentum cut, thus
- direct comparisons of LQCD and STAR data with low momentum cut contain unknown systematic errors

Consider fluctuations and correlations of conserved charges to be compared with LQCD

Excellent probe of:

- QCD criticality

A. Asakawa at. al.

S. Ejiri et al.,…

M. Stephanov et al.,

K. Rajagopal et al.

B. Frimann et al.

- freezeout conditions in HIC

F. Karsch &

S. Mukherjee et al.,

P. Braun-Munzinger et al.,,

- They are quantified by susceptibilities:

If $P(T,\mu_B,\mu_Q,\mu_S)$ denotes pressure, then

$$\frac{\chi_N}{T^2}=\frac{\partial^2(P)}{\partial(\mu_N)^2} \qquad \frac{\chi_{NM}}{T^2}=\frac{\partial^2(P)}{\partial\mu_N\partial\mu_M}$$

$$N=N_q-N_{-q},\ \ N,M=(B,S,Q),\quad \mu=\mu/T,\quad P=P/T^4$$

- Susceptibility is connected with variance

$$\frac{\chi_N}{T^2}=\frac{1}{VT^3}(<N^2>-<N>^2)$$

- If $\mathrm{P}(N)$ probability distribution of N then

$$<N^n>=\sum_N N^n\mathrm{P}(N)$$

Consider special case:

P. Braun-Munzinger,
B. Friman, F. Karsch,
V Skokov &K.R.
Phys .Rev. C84 (2011) 064911
Nucl. Phys. A880 (2012) 48)

$$< N_q > \equiv \overline{N}_q$$

=>

- Charge and anti-charge uncorrelated and Poisson distributed, then
- $\mathrm{P}(N)$ the Skellam distribution

$$\mathrm{P}(N) = \left(\frac{\overline{N}_q}{\overline{N}_{-q}} \right)^{N/2} I_N(2\sqrt{\overline{N}_{-q}\overline{N}_q}) \exp[-(\overline{N}_{-q} + \overline{N}_q)]$$

- Then the susceptibility

$$\frac{\chi_N}{T^2} = \frac{1}{VT^3}(< N_q > + < N_{-q} >)$$

Consider special case: particles carrying $q = \pm 1, \pm 2, \pm 3$

P. Braun-Munzinger,
B. Friman, F. Karsch,
V Skokov &K.R.
Phys .Rev. C84 (2011) 064911
Nucl. Phys. A880 (2012) 48)

- The probability distribution

$$< S_{-q} > \equiv \bar{S}_{-q}$$

$$q = \pm 1, \pm 2, \pm 3$$

$$\begin{aligned} P(S) \;=\; & (\frac{\bar{S}_1}{\bar{S}_{\bar{1}}})^{\frac{S}{2}} \exp[\sum_{n=1}^{3}(\bar{S}_n + \bar{S}_{\bar{n}})] \\ & \sum_{i=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} (\frac{\bar{S}_3}{\bar{S}_{\bar{3}}})^{k/2} I_k(2\sqrt{\bar{S}_3 \bar{S}_{\bar{3}}}) \\ & (\frac{\bar{S}_2}{\bar{S}_{\bar{2}}})^{i/2} I_i(2\sqrt{\bar{S}_2 \bar{S}_{\bar{2}}}) \\ & (\frac{\bar{S}_1}{\bar{S}_{\bar{1}}})^{-i-3k/2} I_{2i+3k-S}(2\sqrt{\bar{S}_1 \bar{S}_{\bar{1}}}) \end{aligned}$$

Fluctuations

$$\frac{\chi_S}{T^2} = \frac{1}{VT^3} \sum_{n=1}^{|q|} n^2 (\langle S_n \rangle + \langle S_{-n} \rangle)$$

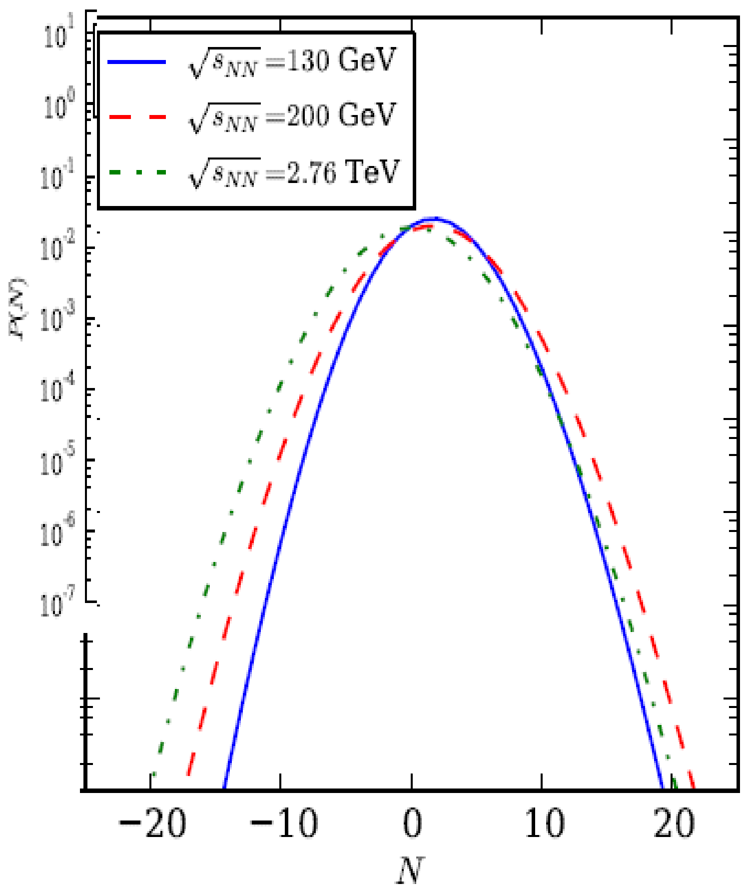
Correlations

$$\frac{\chi_{NM}}{T^2} = \frac{1}{VT^3} \sum_{n=-q_N}^{q_N} \sum_{m=-q_M}^{q_M} nm \langle N_{n,m} \rangle$$

$\langle N_{n,m} \rangle$, is the mean number of particles
; carrying charge $N = n$ and $M = m$.

Variance at 200 GeV AA central coll. at RHIC

P. Braun-Munzinger, et al.
Nucl. Phys. A880 (2012) 48)



STAR Collaboration data in central coll. 200 GeV

- Consistent with Skellam distribution

$$\boxed{\frac{<p> + <\bar{p}>}{\sigma^2} = 1.022 \pm 0.016} \qquad \frac{\chi_1}{\chi_3} = 1.076 \pm 0.035$$

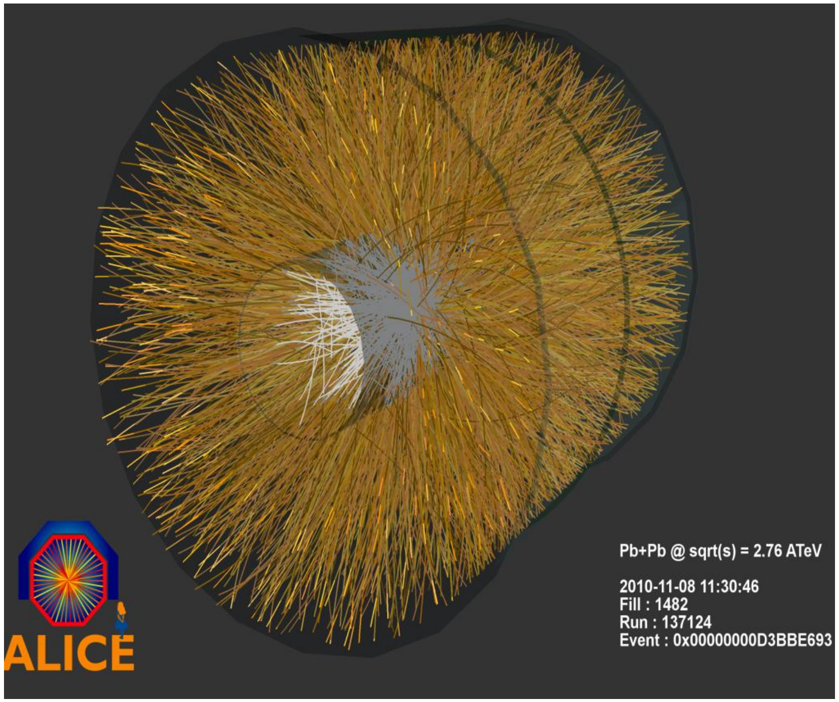
- Consider ratio of cumulants in in the whole momentum range:

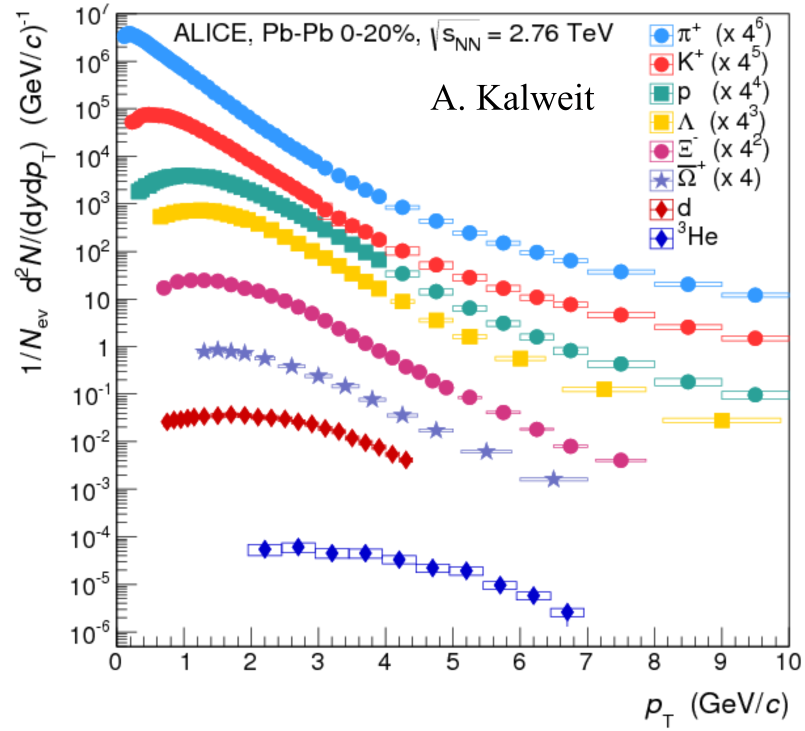
$$\frac{\sigma^2}{p - \bar{p}} = 6.18 \pm 0.14 \ \ in \ \ 0.4 < p_t < 0.8 GeV$$

$$\frac{p + \bar{p}}{p - \bar{p}} = 7.67 \pm 1.86 \ \ in \ \ 0.0 < p_t < \infty \ GeV$$

Particle production yields in Heavy Ion Collisions at LHC

Paolo Giubellino & Jürgen Schukraft for ALICE Collaboration





Use ALICE data to quantify the 2nd order correlations and fluctuations of conserved charges

Constructing net charge fluctuations and correlation from ALICE data

- Net baryon number susceptibility

$$\frac{\chi_B}{T^2} \approx \frac{1}{VT^3}(\langle p\rangle + \langle N\rangle + \langle \Lambda + \Sigma_0\rangle + \langle \Sigma^+\rangle + \langle \Sigma^-\rangle + \langle \Xi^-\rangle + \langle \Xi^0\rangle + \langle \Omega^-\rangle + \overline{par}\ )$$

- Net strangeness

$$\frac{\chi_S}{T^2} \approx \frac{1}{VT^3}(\langle K^+\rangle + \langle K_S^0\rangle + \langle \Lambda + \Sigma_0\rangle + \langle \Sigma^+\rangle + \langle \Sigma^-\rangle + 4\langle \Xi^-\rangle + 4\langle \Xi^0\rangle + 9\langle \Omega^-\rangle + \overline{par}$$
$$-(\Gamma_{\varphi\to K^+} + \Gamma_{\varphi\to K^-} + \Gamma_{\varphi\to K_S^0} + \Gamma_{\varphi\to K_L^0})\langle \varphi\rangle\ )$$

- Charge-strangeness correlation

$$\frac{\chi_{QS}}{T^2} \approx \frac{1}{VT^3}(\langle K^+\rangle + 2\langle \Xi^-\rangle + 3\langle \Omega^-\rangle + \overline{par}$$
$$-(\Gamma_{\varphi\to K^+} + \Gamma_{\varphi\to K^-})\langle \varphi\rangle - (\Gamma_{K_0^*\to K^+} + \Gamma_{K_0^*\to K^-})\langle K_0^*\rangle\ )$$

χ_B, χ_S, χ_{QS} constructed from ALICE particle yields

- use also $\Sigma^0/\Lambda = 0.278$ from pBe at $\sqrt{s} = 25\ GeV$

- Net baryon fluctuations $$\frac{\chi_B}{T^2} \approx \frac{1}{VT^3}(203.7 \pm 11.4)$$

- Net strangeness fluctuations $$\frac{\chi_S}{T^2} \approx \frac{1}{VT^3}(504.2 \pm 24)$$

- Charge-Strangeness corr. $$\frac{\chi_{QS}}{T^2} \approx \frac{1}{VT^3}(178 \pm 17)$$

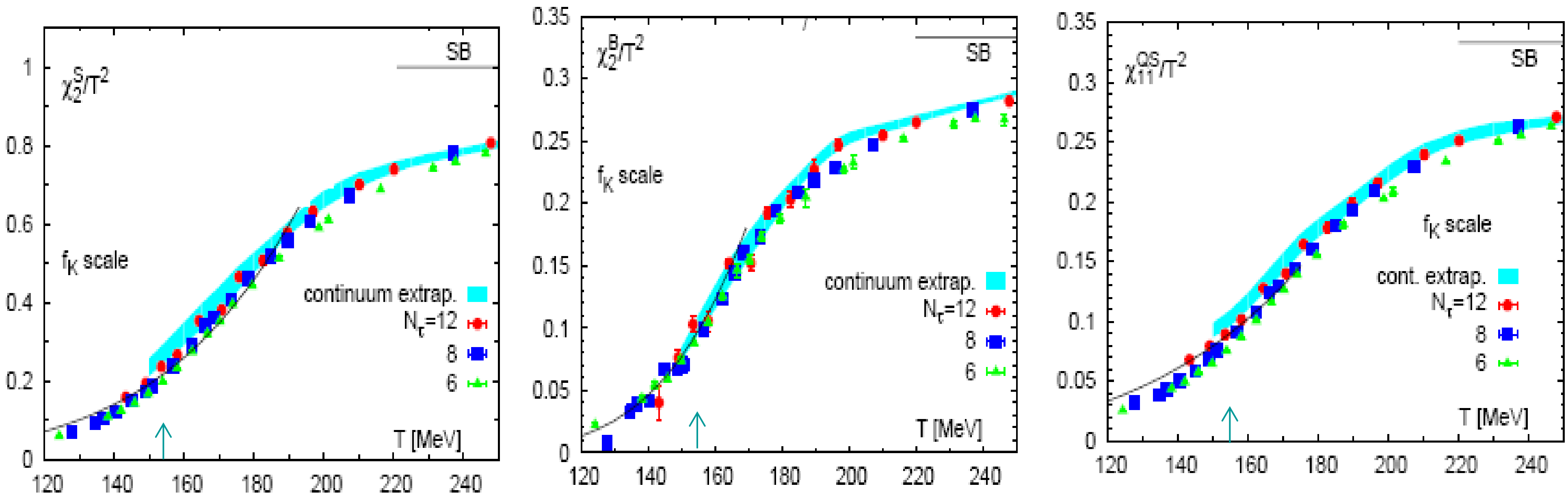
- Ratios is volume independent

$$\boxed{\frac{\chi_B}{\chi_S} = 0.404 \pm 0.028} \quad \text{and} \quad \boxed{\frac{\chi_B}{\chi_{QS}} = 1.14 \pm 0.13}$$

Compare the ratio with LQCD data:

A. Bazavov, H.-T. Ding, P. Hegde, O. Kaczmarek, F. Karsch, E. Laermann, Y. Maezawa and S. Mukherjee

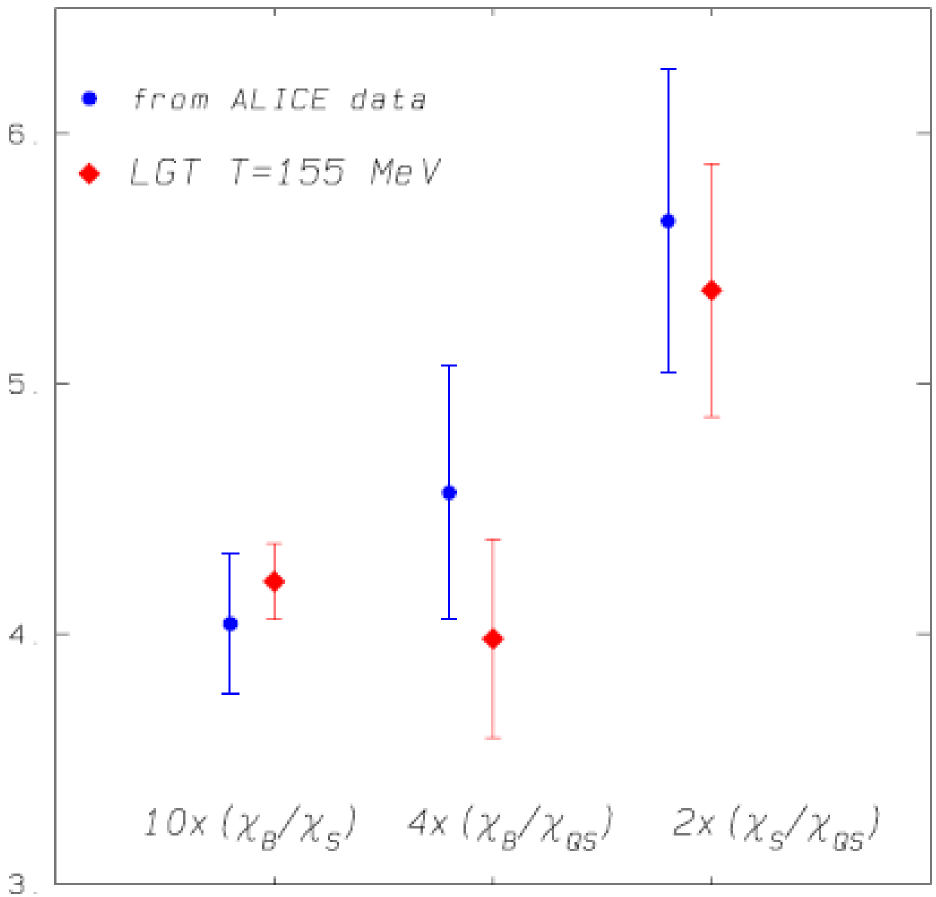
Phys.Rev.Lett. 113 (2014) and HotQCD Coll. A. Bazavov et al. Phys.Rev. D86 (2012) 034509



- Is there a temperature where calculated ratios from ALICE data agree with LQCD?

Baryon number, strangeness and Q-S correlations

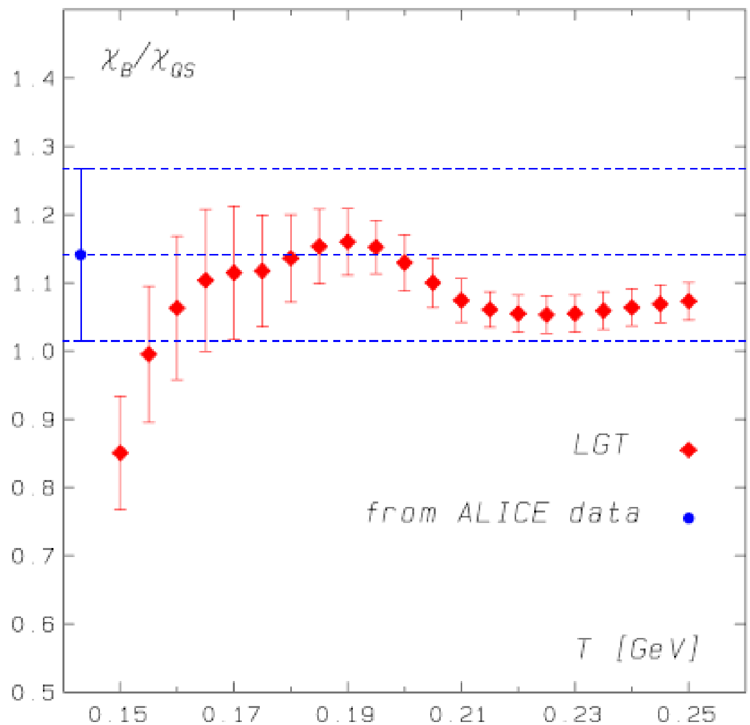
Compare ratios at the chiral crossover

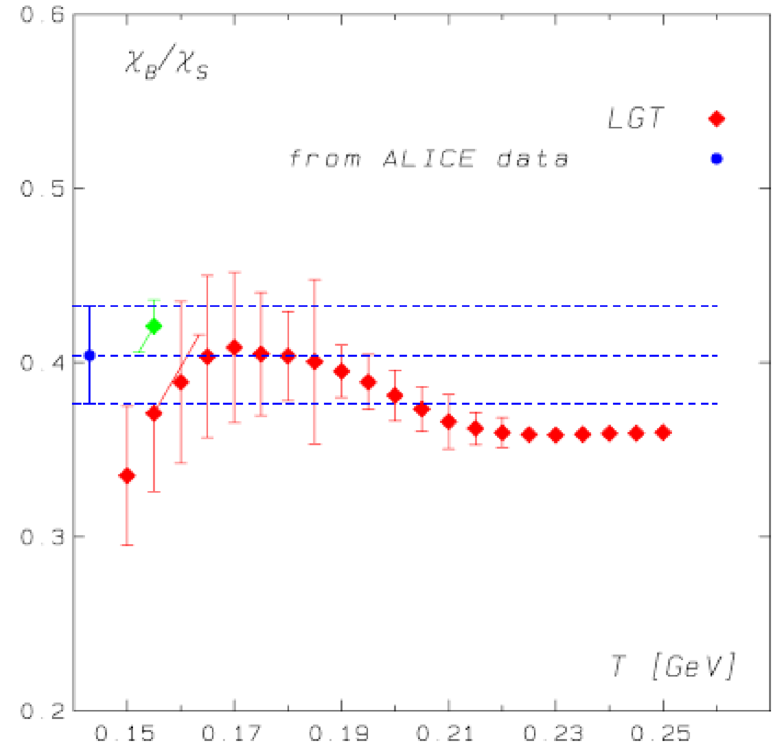


- There is a very good agreement, within systematic uncertainties, between extracted susceptibilities from ALICE data and LQCD at the chiral crossover

- How unique is the determination of the temperature at which such agreement holds?

Consider T-dependent LQCD ratios and compare with ALICE data





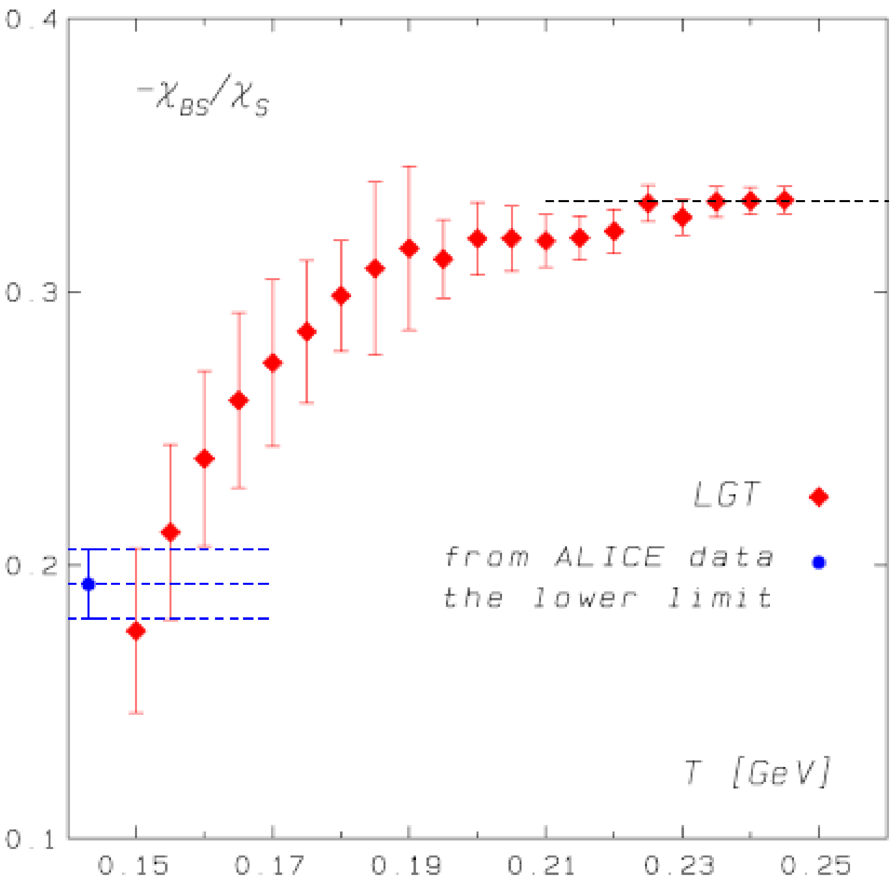
- The LQCD susceptibilities ratios are weakly T-dependent for $T \geq T_c$
- We can reject $T \leq 0.15\ GeV$ for saturation of χ_B, χ_S *and* χ_{QS} at the LHC, and can fix T to be in the range $0.15 < T \leq 0.21\ GeV$, however
- LQCD => for $T > 0.163\ GeV$ thermodynamics cannot be anymore described by the hadronic degrees of freedom

Baryon-Strangeness Correlations

Consider

$$C_{BS} = -\frac{<(\delta B)(\delta S)>}{<(\delta S)^2>} = -\frac{\chi_{BS}}{\chi_S}$$

Volker Koch 05

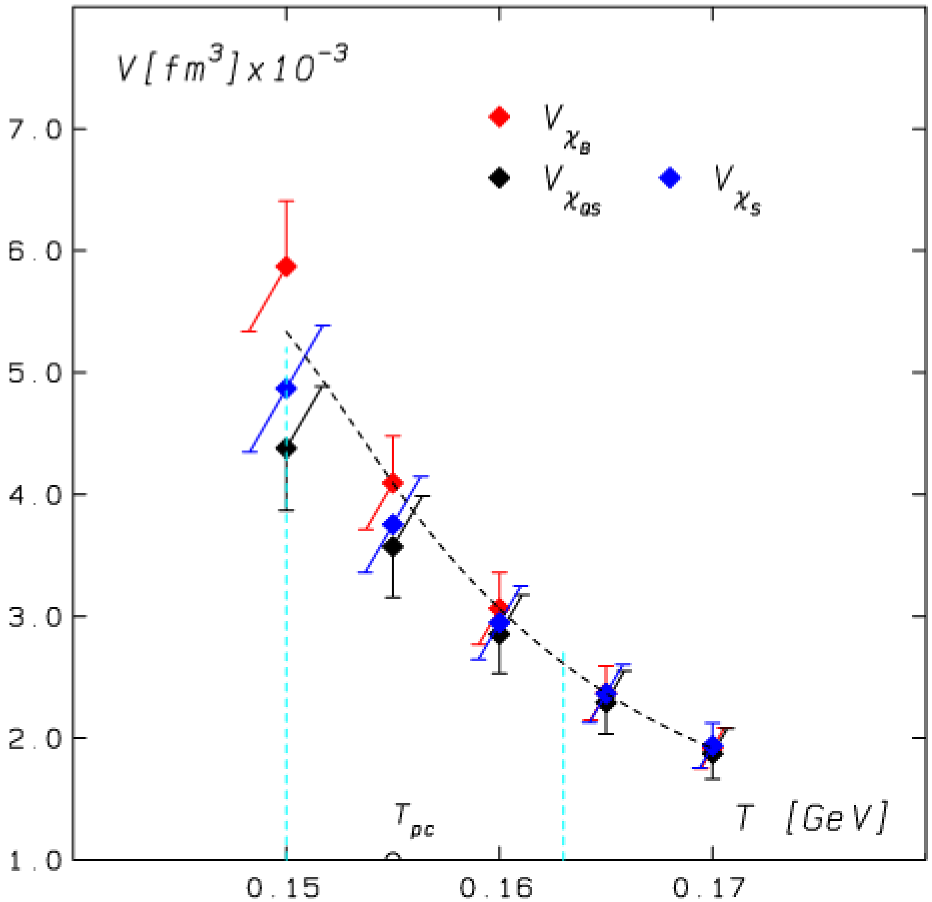


- Excellent observable to fix temperature

$$-\frac{\chi_{BS}}{T^2} > \frac{1}{VT^3}[2\langle\Lambda+\Sigma^0\rangle + 4\langle\Sigma^+\rangle + 8\langle\Xi\rangle + 6\langle\Omega^-\rangle] = 97.4 \pm 5.8.$$

- Data fix only the lower limit since e.g. $\Sigma^* \xrightarrow{-} N\bar{K}$ not included

Extract the volume by comparing data with LQCD



- Since

$$(\chi_N / T^2)_{LQCD} = \frac{(<N^2> - <N>^2)_{LHC}}{V_N T^3}$$

thus

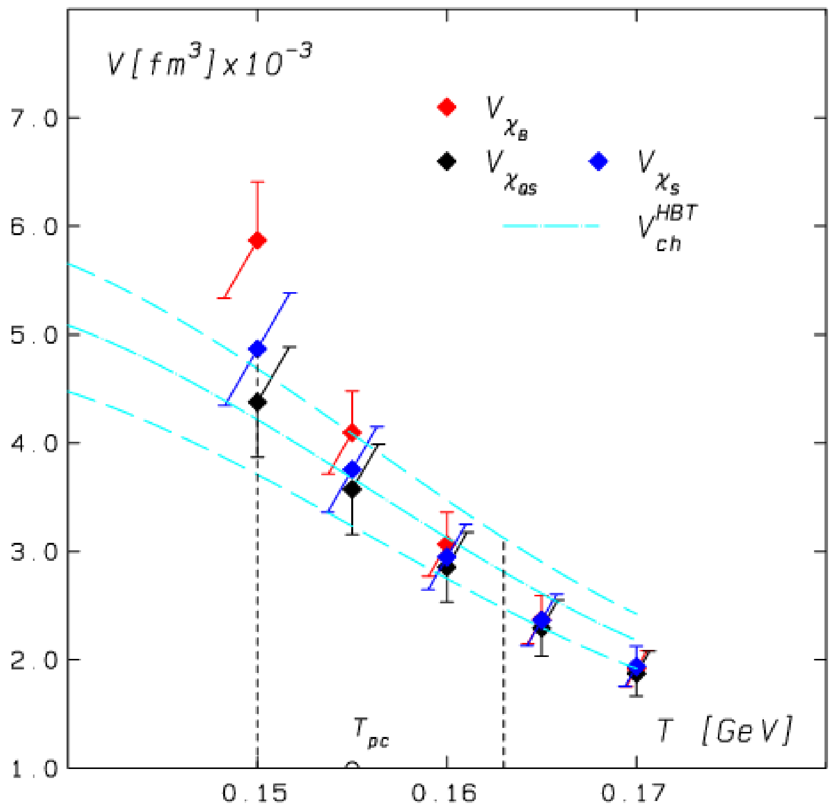
$$V_{\chi_B}(T) = \frac{203.7 \pm 11.4}{T^3 (\chi_B / T^2)_{LQCD}} \qquad V_{\chi_S}(T) = \frac{504.2 \pm 24.2}{T^3 (\chi_B / T^2)_{LQCD}}$$

$$V_{\chi_{QS}}(T) = \frac{178 \pm 17}{T^3 (\chi_B / T^2)_{LQCD}}$$

- All volumes, should be equal at a given temperature if originating from the same source, thus

$$T > 150\ MeV$$

Constraining the volume from HBT and percolation theory



D. Adamova, et al (CERES) Phys Rev. Lett 90, 022301 (2003).

S.V. Akkelin, P. Braun-Munzinger & Y. Sinyukov Nucl. Phys. A710 (2002).

- Some limitation on volume from Hanbury-Brown–Twiss: HBT

$$V_{HBT} = (2\pi)^{3/2} R_l R_o R_s.$$

Take ALICE data from pion interferometry $V_{HBT} = 4800 \pm 640\, fm^{-3}$

Use 3D hydro to transfer

V_{HBT}: the volume of the homogeneity at the last interaction

$V_{th}(T_{th})$:the volume at the thermal freezeout $T_{th} \approx 100\ MeV$

$V_{ch}(T)$: the volume at temperature $T_{ch} > T_{th}$

Thermal origin of particle yields with respect to HRG

Rolf Hagedorn => the Hadron Resonace Gas (HRG):

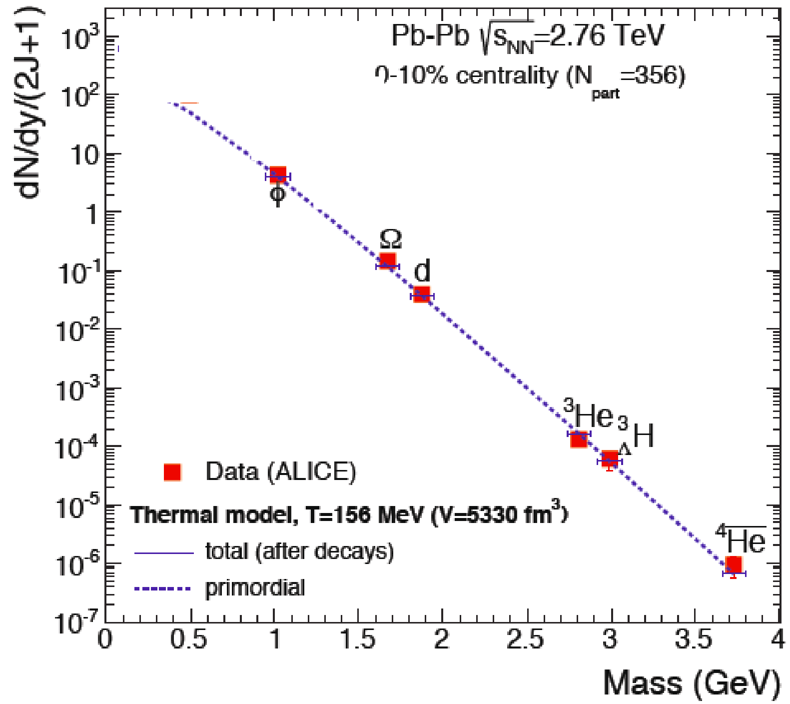
"uncorrelated" gas of hadrons and resonances

$$< N_i > = V\,[n_i^{th}(T,\vec{\mu}) + \sum_K \Gamma_{K\to i}\, n_i^{th-\mathrm{Res.}}(T,\vec{\mu})]$$

A. Andronic, Peter Braun-Munzinger, & Johanna Stachel, et al.

Particle yields with no resonance decay contributions at the LHC:

$$\frac{1}{2j+1}\frac{dN}{dy} = V(m/T)^2 K_2(m/T)$$



- Measured yields are reproduced with HRG at $T \approx 156$ MeV

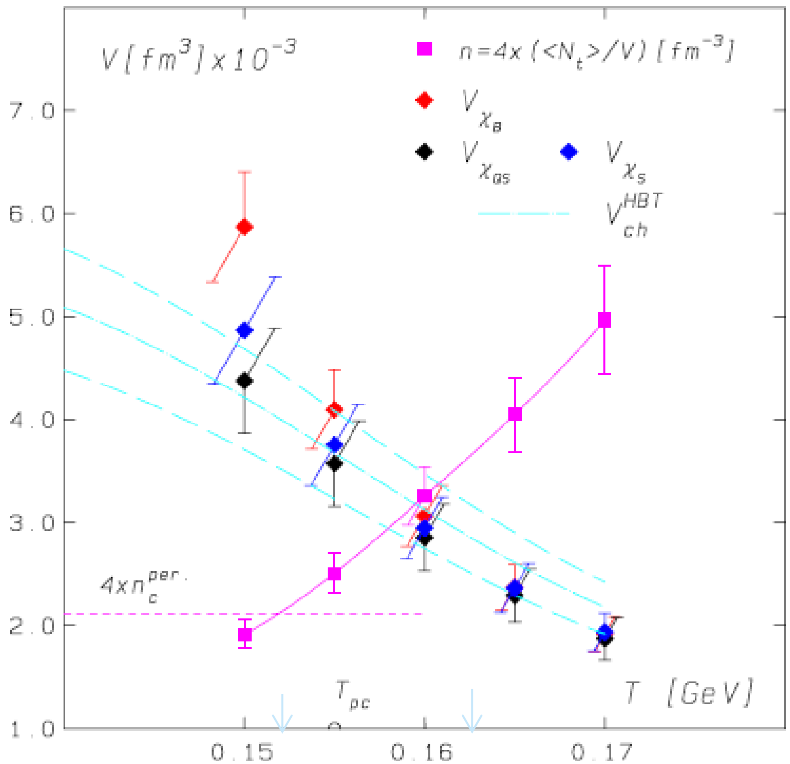
Conclusions:

- Ratios of the Polyakov loop and the Net-charge susceptibilities are excellent probes for deconfinement and/or the O(4) chiral crossover in QCD
- From direct comparisons of 2^{nd} order fluctuations and correlations constructed from ALICE data and LQCD results one concludes that:

  - there is thermalization in heavy ion collisions at the LHC and the 2nd order charge fluctuations and correlations are saturated at the chiral crossover temperature

Skellam distribution, and its generalization, is a good approximation of the net charge probability distribution P(N) for small N<10. The chiral criticality sets in at larger N>10 and implies shrinking of the Skellam distribution.

Particle density and percolation theory



- Density of particles at a given volume $n(T) = \dfrac{N^{\exp}_{total}}{V(T)}$

- Total number of particles in HIC at LHC, ALICE

$$\langle N_t \rangle = 3\langle \pi \rangle + 4\langle p \rangle + 4\langle K \rangle + (2 + 4 \times 0.2175)\langle \Lambda_\Sigma \rangle + 4\langle \bar{\Xi} \rangle + 2\langle \bar{\Omega} \rangle,$$

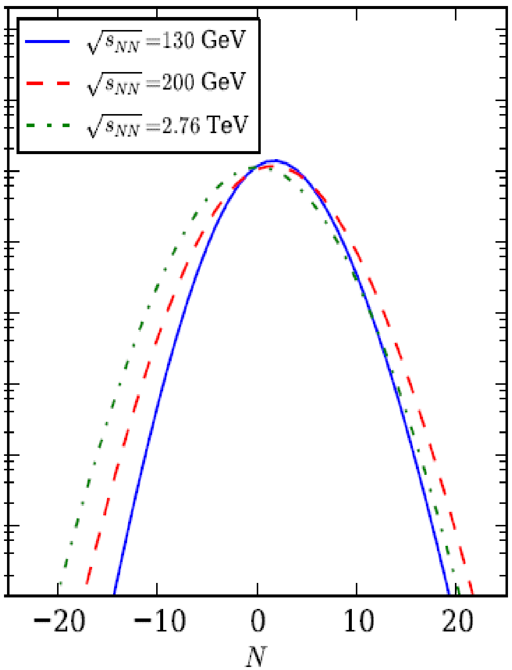
$$\boxed{\langle N_t \rangle = 2486 \pm 146}$$

- Percolation theory: 3-dim system of objects of volume $V_0 = 4/3\pi R_0^3$

$$n_c = \frac{1.22}{V_0} \quad \text{take} \quad R_0 \approx 0.82\, fm \Rightarrow n_c \approx 0.52\ [fm^{-3}] \Rightarrow T_c^p \approx 152\ [MeV]$$

P. Castorina, H. Satz &K.R. Eur.Phys.J. C59 (2009)

What is the influence of O(4) criticality on P(N)?



- For the net baryon number use the Skellam distribution (HRG baseline)

$$P(N) = \left(\frac{B}{\bar{B}}\right)^{N/2} I_N(2\sqrt{B\bar{B}})\exp[-(B+\bar{B})]$$

as the reference for the non-critical behavior

- Calculate P(N) in an effective chiral model which exhibits O(4) scaling and compare to the Skellam distribtuion

$$P(N) = \frac{Z_C(N)}{Z_{GC}} e^{\frac{\mu N}{T}}$$

Moments obtained from probability distributions

- Moments obtained from probability distribution

$$< N^k >= \sum_N N^k P(N)$$

- Probability quantified by all cumulants

$$P(N) = \frac{1}{2\pi}\int_0^{2\pi} dy \exp[iyN - \chi(iy)]$$

Cumulants generating function: $\chi(y) = \beta V[p(T, y+\mu) - p(T,\mu)] = \sum_k \chi_k y^k$

- In statistical physics

$$P(N) = \frac{Z_C(N)}{Z_{GC}} e^{\frac{\mu N}{T}}$$

What is the influence of O(4) criticality on P(N)?

P. Braun-Munzinger,
B. Friman, F. Karsch,
V Skokov &K.R.
Phys .Rev. C84 (2011) 064911
Nucl. Phys. A880 (2012) 48)

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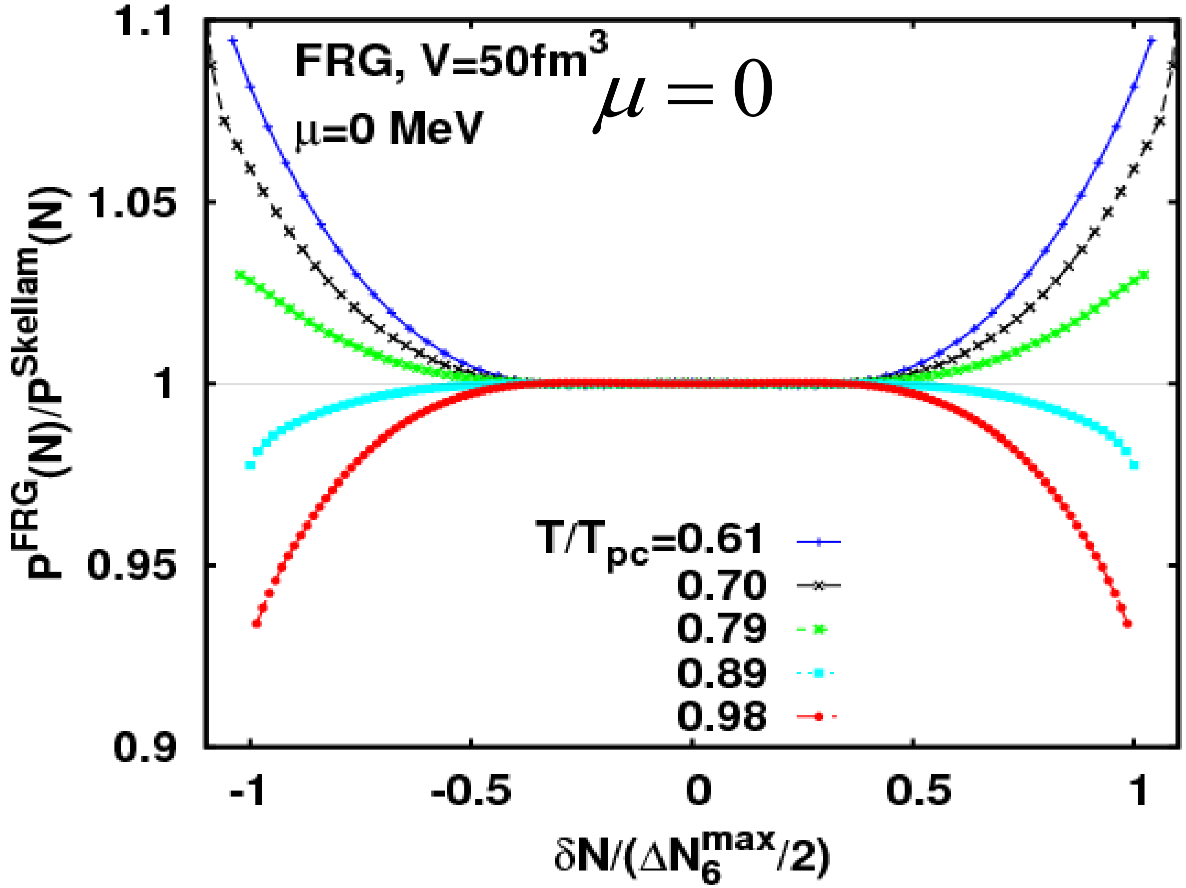
- Calculate P(N) in an effective chiral model which exhibits O(4) scaling and compare to the Skellam distribtuion

The influence of O(4) criticality on P(N) for $\mu = 0$

- Take the ratio of $P^{FRG}(N)$ which contains O(4) dynamics to Skellam distribution with the same Mean and Variance at different T/T_{pc}

K. Morita, B. Friman &K.R. (PQM model within renormalization group FRG)

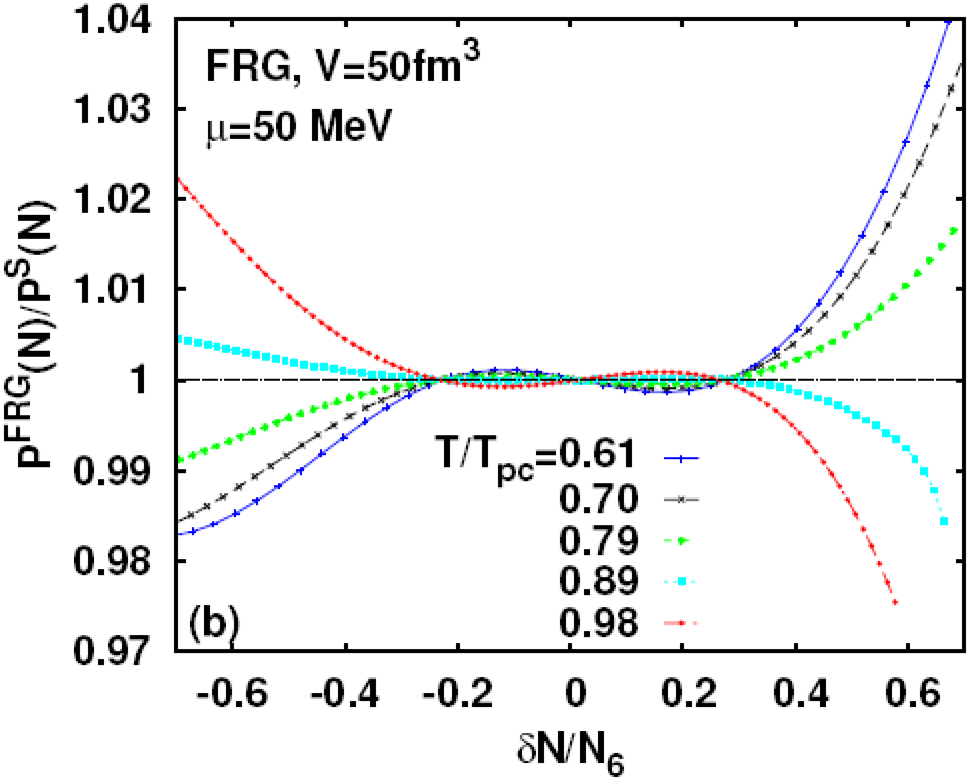


- Ratios less than unity near the chiral crossover, indicating the contribution of the O(4) criticality to the thermodynamic pressure

The influence of O(4) criticality on P(N) for $\mu \neq 0$

- Take the ratio of $P^{FRG}(N)$ which contains O(4) dynamics to Skellam distribution with the same Mean and Variance near $T_{pc}(\mu)$

K. Morita, B. Friman et al.

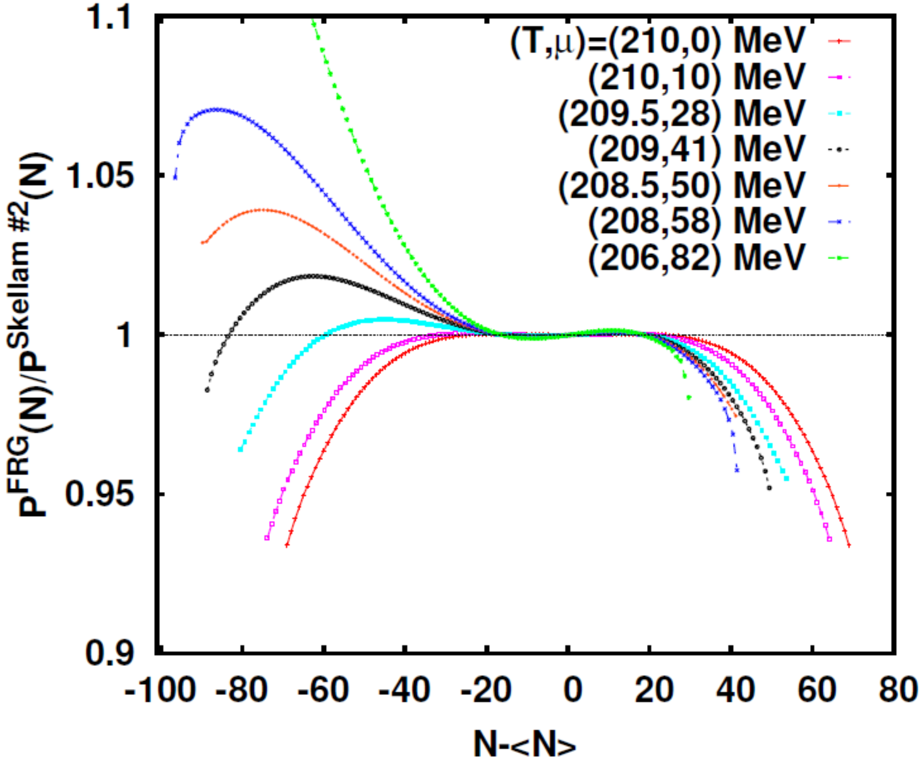


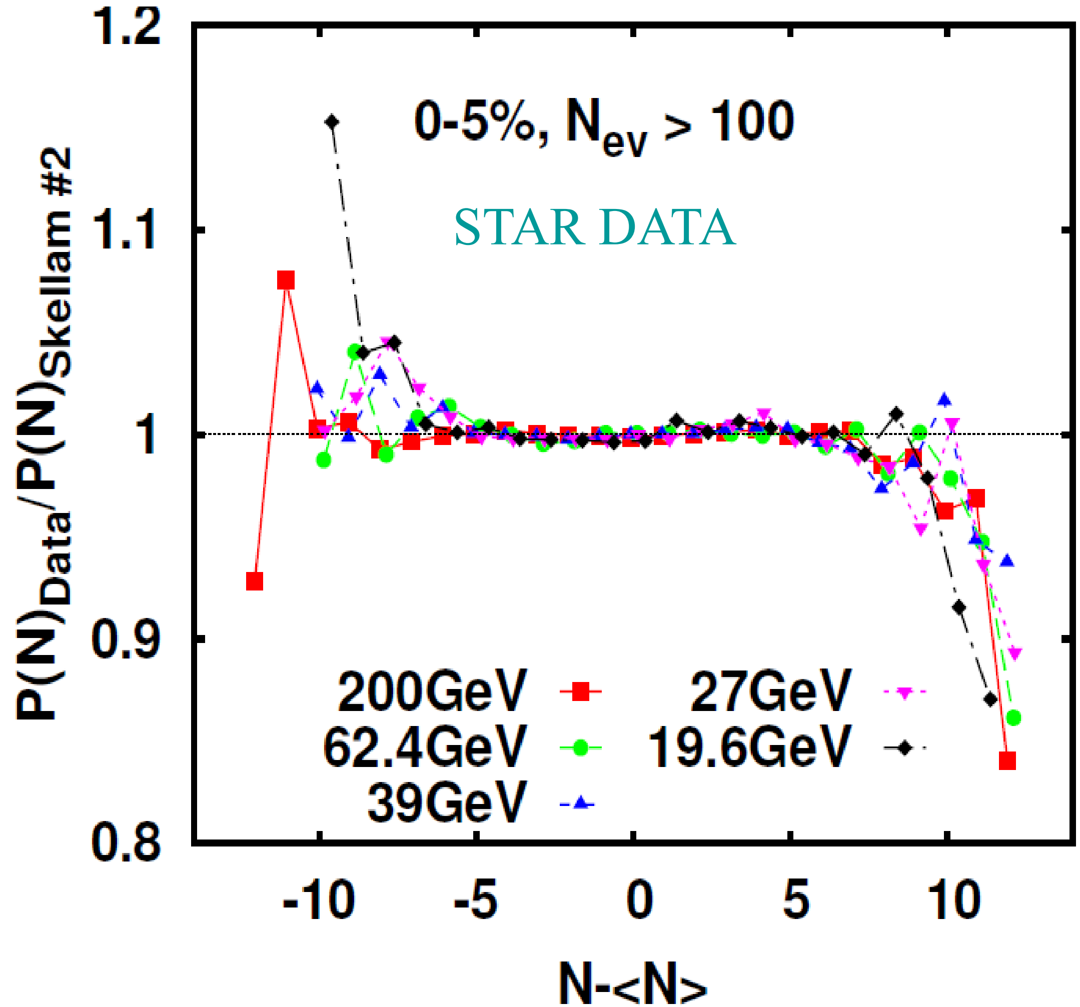
- Asymmetric P(N)
- Near $T_{pc}(\mu)$ the ratios less than unity for $N > \langle N \rangle$

The influence of O(4) criticality on P(N) for $\mu \neq 0$

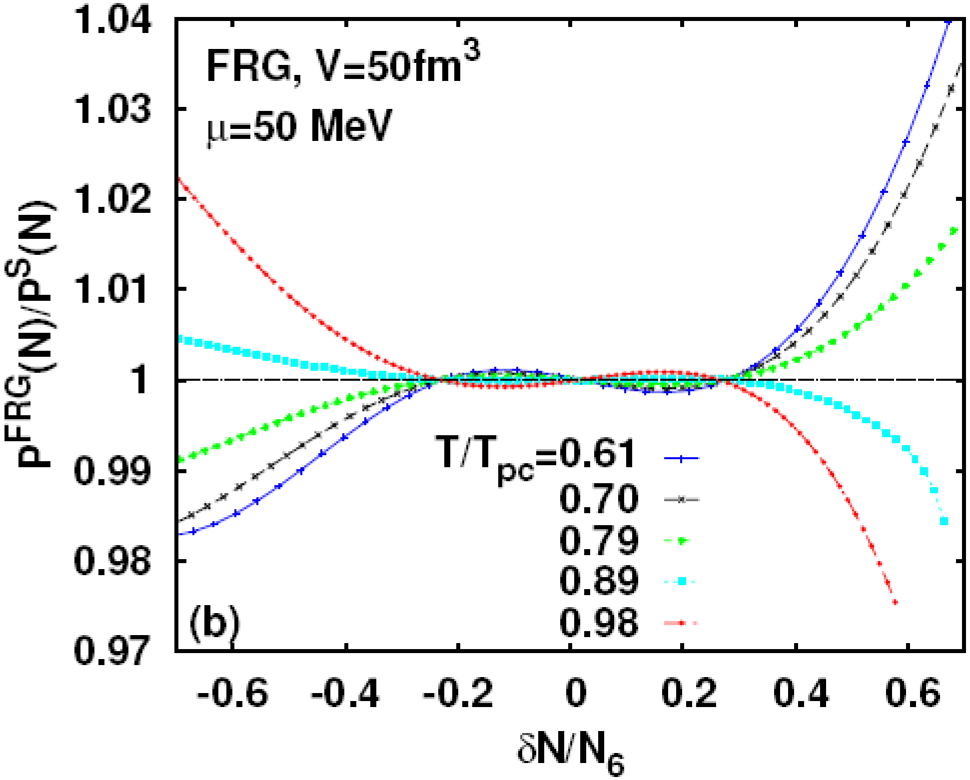
- In central collisions the probability behaves as being influenced by the chiral transition

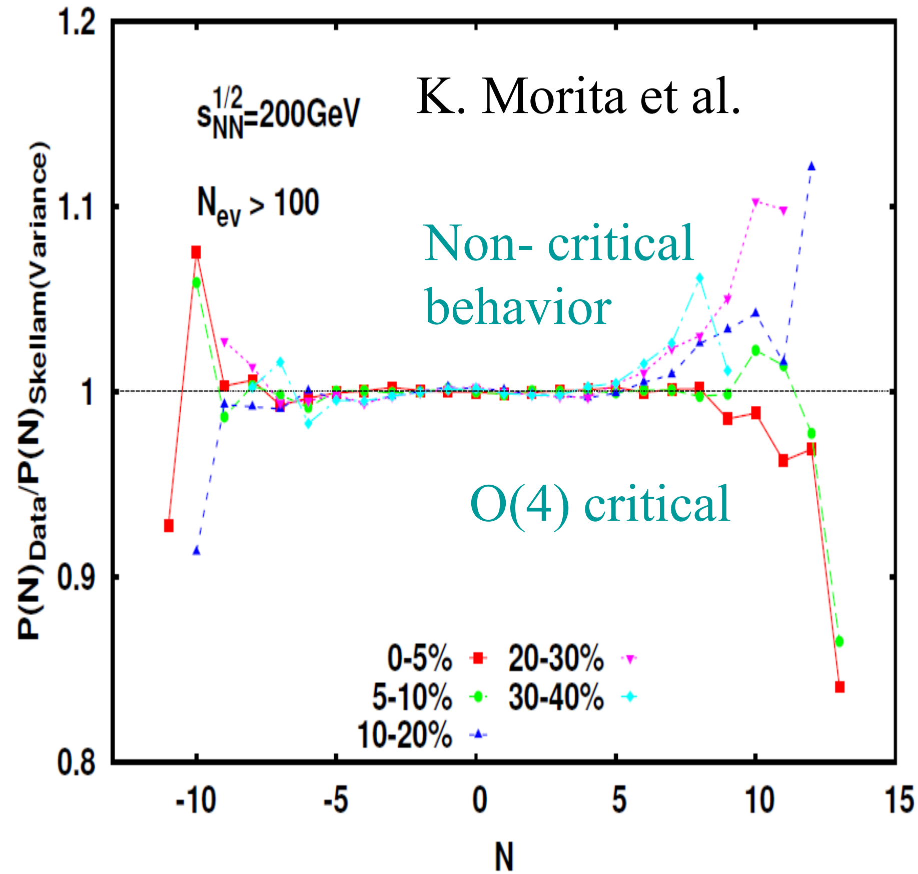
K. Morita, B. Friman & K.R.





Centrality dependence of probability ratio

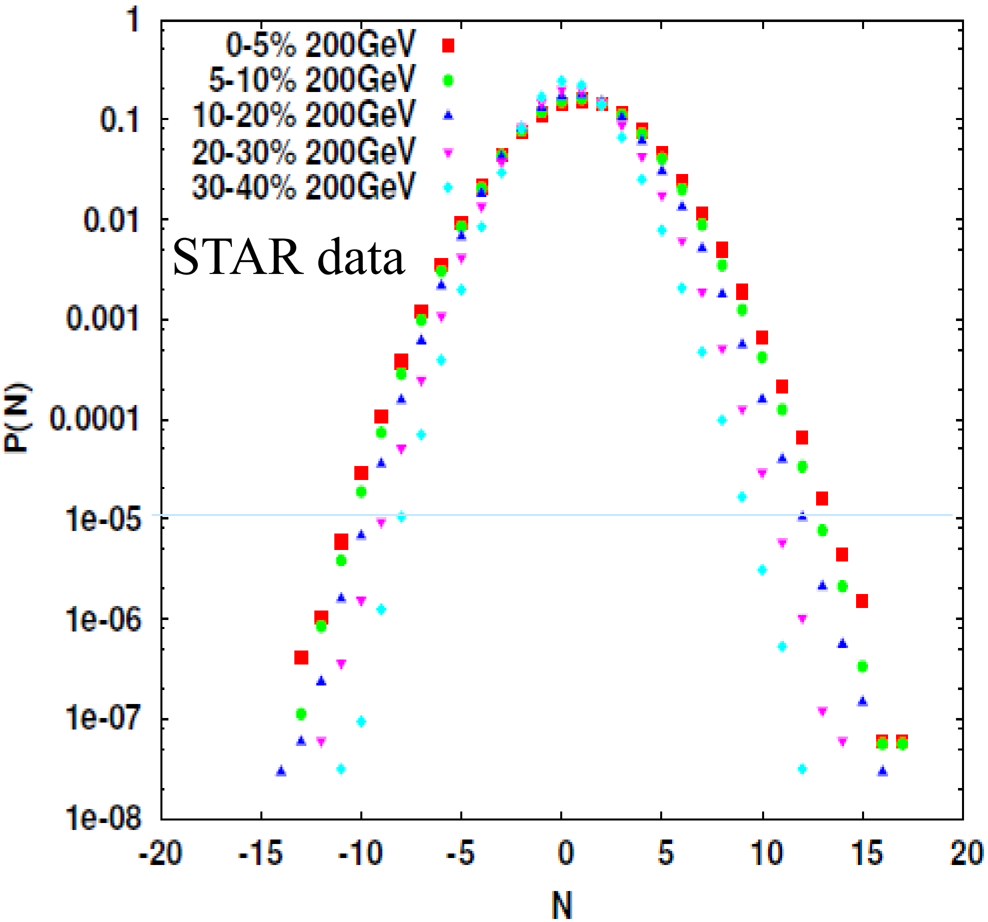


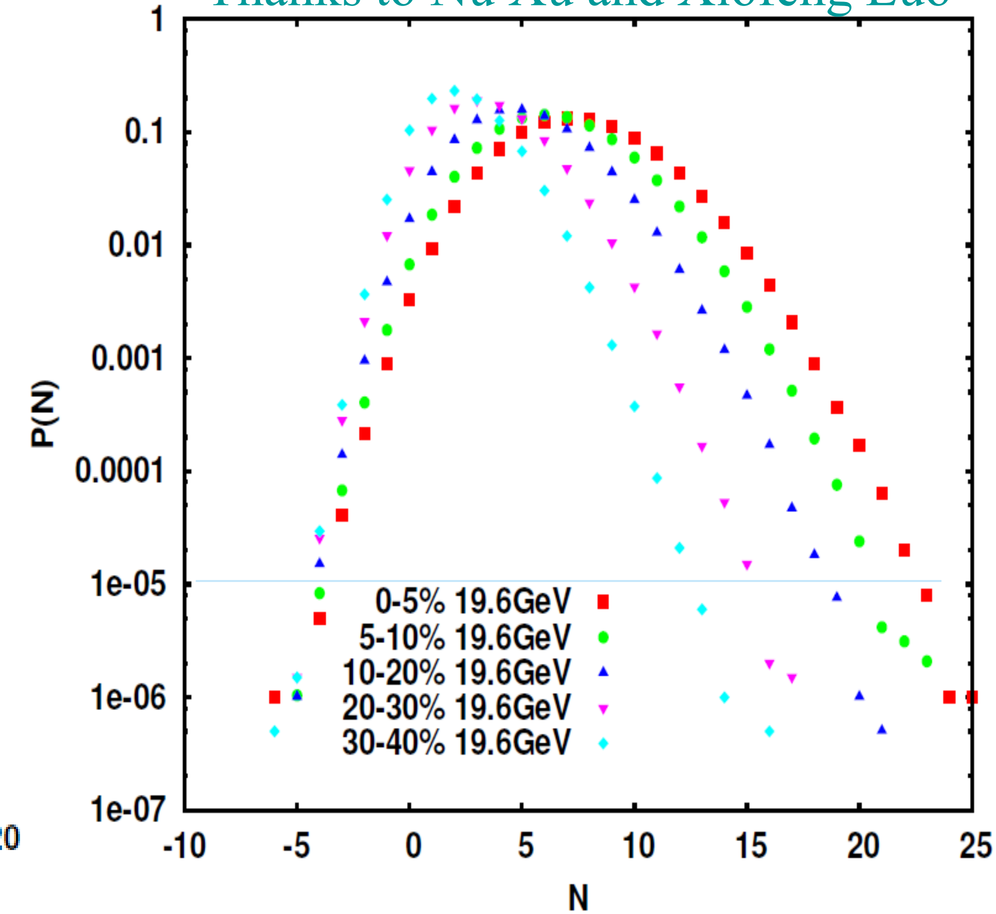


- For less central collisions, the freezeout appears away of the pseudocritical line, resulting in an absence of the O(4) critical structure in the probability ratio.

Probability distribution of net proton number STAR Coll. data at RHIC

Thanks to Nu Xu and Xiofeng Luo





Do we also see the O(4) critical structure in these probability distributions ?
Efficiency uncorrected data!!

Effective Polyakov loop Potential from Y-M Lagrangian

Chihiro Sasaki & K.R.

Deriving partition function from YM Lagrangian

$$Z = \int \mathcal{D}A_\mu \mathcal{D}C\mathcal{D}\bar{C} \exp\left[i\int d^4x\mathcal{L}\right]\,, \quad \mathcal{L} = \mathcal{L}_{\rm kin} + \mathcal{L}_{\rm GF} + \mathcal{L}_{\rm FP}$$

1. employ background field method. (Gross, Pisarski & Yaffe)

$$A_\mu = \bar{A}_\mu + g\check{A}_\mu$$

2. collect terms quadratic in quantum fields.

$$\mathcal{L}^{(2)} = -\frac{1}{2}\check{A}^a_\alpha\left[\delta_{ab}g^{\alpha\beta}\partial^2 - f_{abc}\left(\partial^\beta\bar{A}^{\alpha,c} + 2g^{\alpha\beta}\bar{A}^c_\mu\partial^\mu\right)\right.$$
$$\left. + f_{ac\bar{c}}f_{cb\bar{d}}g^{\alpha\beta}\bar{A}^{\bar{c}}_\mu\bar{A}^{\mu,\bar{d}} + 2f_{abc}\bar{A}^{\alpha\beta,c}\right]\check{A}_\beta{}^b$$

3. consider a constant uniform background $\bar{A}_0$.

$$\bar{A}^a_\mu = \bar{A}^a_0\delta_{\mu 0}\,, \quad \bar{A}_0 = \bar{A}^3_0T^3 + \bar{A}^8_0T^8$$

4. calculate propagator inverse and diagonalize it.

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