Fluctuations and correlations from lattice QCD: What have we learned ?

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Fluctuations and correlations: conserved charges

$$\chi_{mn}^{XY} = \left. \frac{\partial^{m+n} \ln \mathbb{Z}}{\partial^m \hat{\mu}_X \partial^n \hat{\mu}_Y} \right|_{\mu_X=\mu_Y=0}$$

$$\chi_n^Y \equiv \chi_{0n}^{XY}$$

B … net baryon
Q … net electric charge
S … net strangeness
C … net charm number

$$\hat{\mu}_X = \mu_X / T$$

number density: n_X

for example:

$$\chi_2^X = \langle n_X^2 \rangle$$

$$\chi_4^X = \langle n_X^4 \rangle - 3 \langle n_X^2 \rangle^2$$

$$\chi_{11}^{XY} = \langle n_X n_Y \rangle$$

Deconfinement: appearance of fractional charges

hadron gas: $P^S = P^S_M \cosh[\hat{\mu}_S] + \sum_{S=1,2,3} P^{S=k}_B \cosh[\hat{\mu}_B - S\hat{\mu}_S]$

$$\chi^{BS}_{11} = -1^1 \left(P^{S=1}_B + P^{S=2}_B + P^{S=3}_B\right)$$

from quantum # of the dof

$$\chi^{BS}_{31} = -1^3 \left(P^{S=1}_B + P^{S=2}_B + P^{S=3}_B\right)$$

depends on the hadron spectrum

$$\chi^{BS}_{31} - \chi^{BS}_{11} = \left(B^3 - B\right) \times f\left(m^{had}_S\right)$$

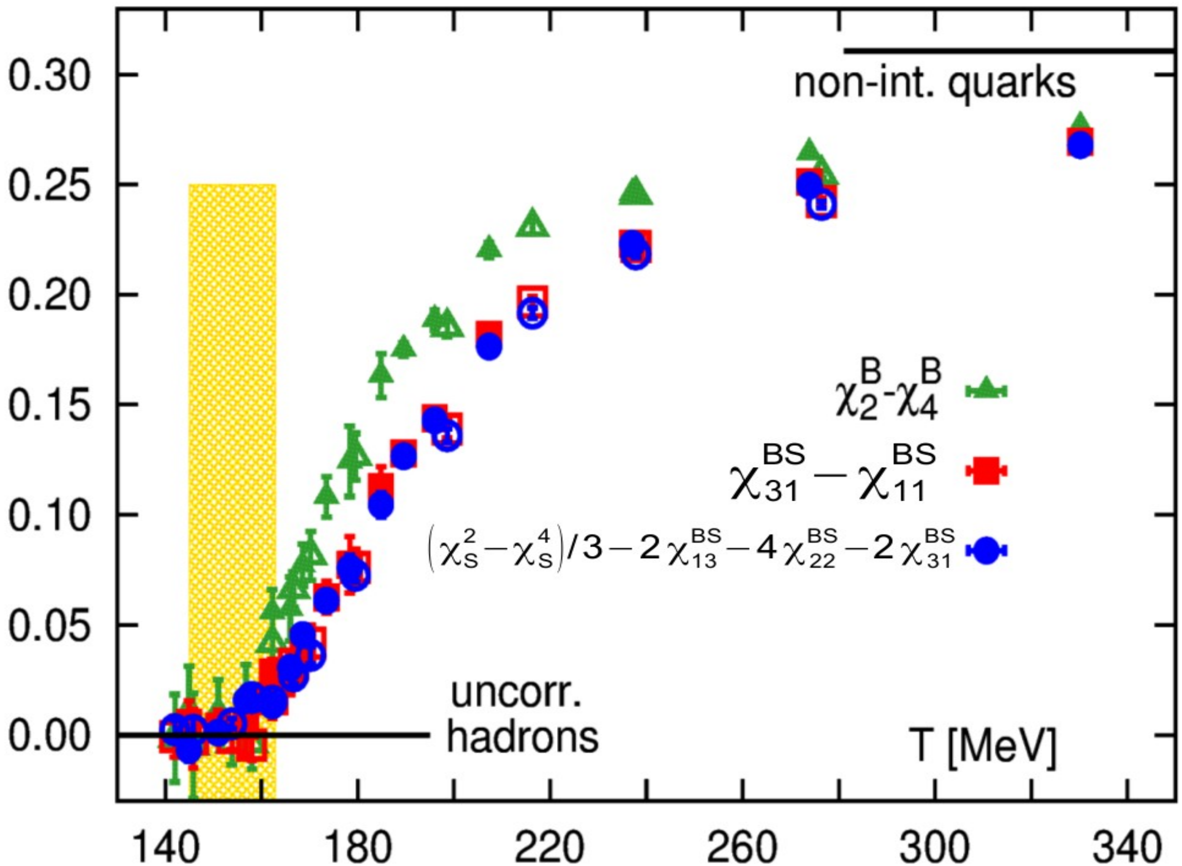
$$\chi^{XY}_{mn} = \left.\frac{\partial^{m+n} P}{\partial^m \hat{\mu}_X \partial^n \hat{\mu}_Y}\right|_{\mu_X=\mu_Y=0}$$

↓

= 0 for B=0,1

=/= 0 for quark dof with B=1/3

similarly: $\chi^B_4 - \chi^B_2 = \left(B^4 - B^2\right) \times f\left(m^{had}_{u,d,S}\right)$



appearance of fractional charges for T >

$T_c = 154 \pm 9$ MeV

deconfinement of light & strange quarks

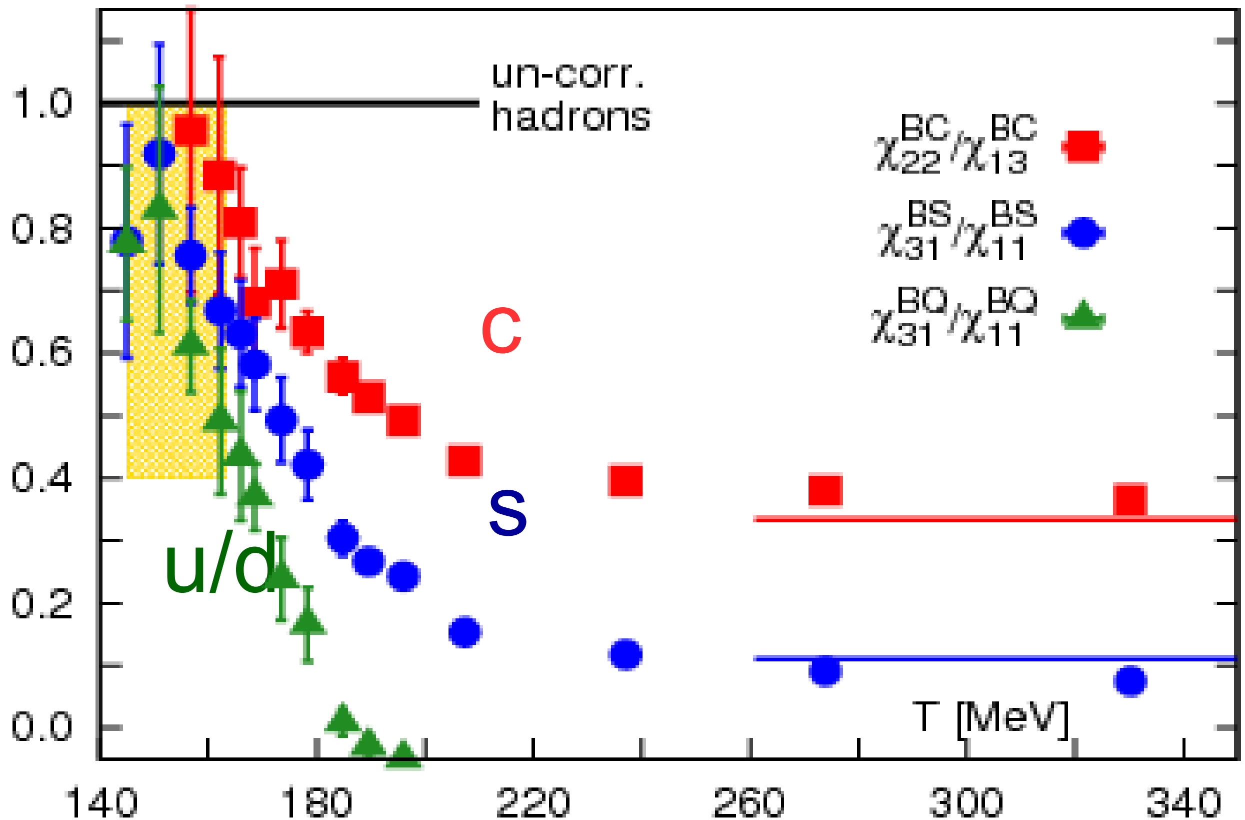
BNL-Bi: Phys. Rev. Lett. 111, 082301 (2013)

Flavor blind deconfinement ?

$\chi_{BX}^{nm} / \chi_{BX}^{km} = B^{n-k}$

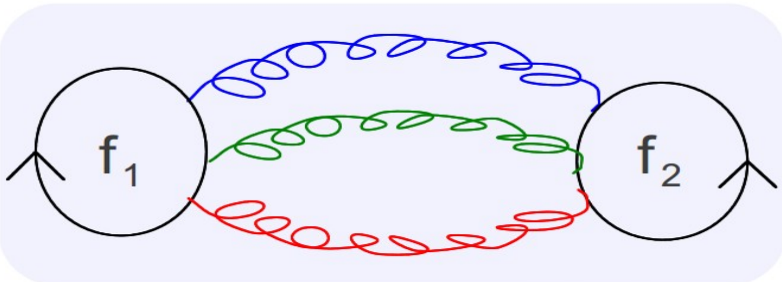
= 1 when DoF are hadronic

=/= 1 when DoF carries fractional B



BNL-Bi: Phys. Lett. B737 (2014) 210

BNL-Bi: Phys. Rev. Lett. 111, 082301 (2013)

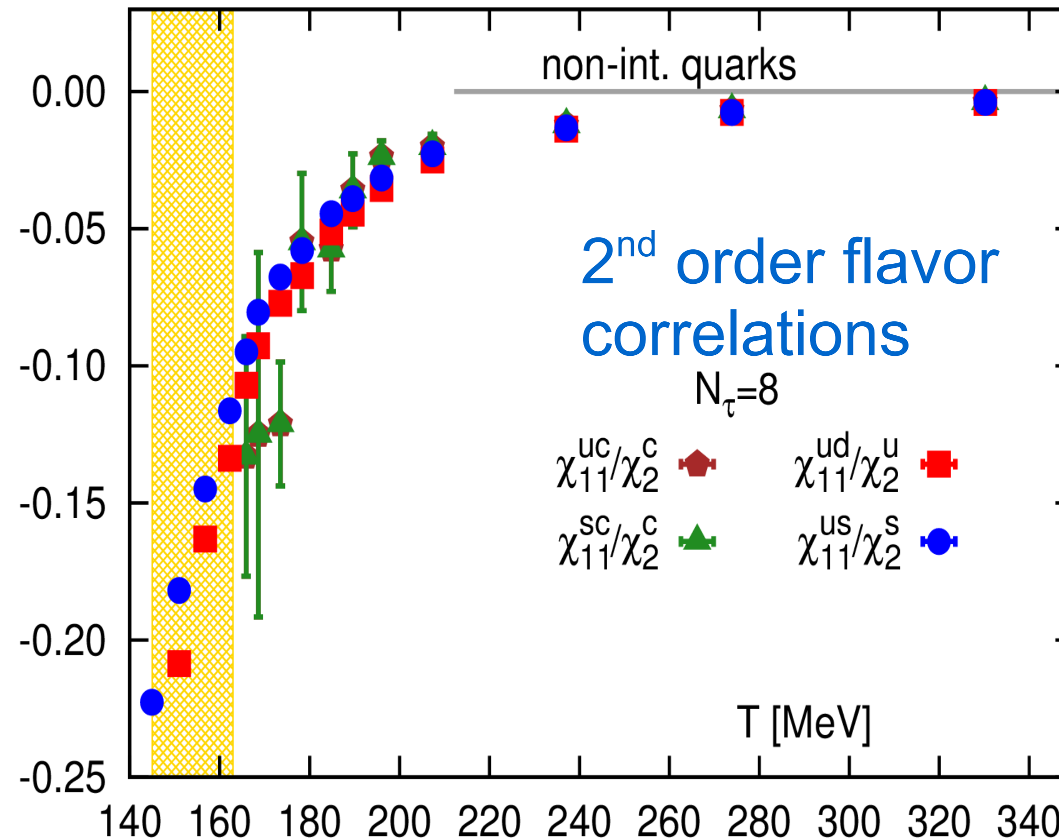


flavor correlations: $\chi_{mn}^{f_1 f_2} / \chi_{m+n}^{f_2}$

in deconfined phase gluon dominated interactions: flavor blind

$T_c \lesssim T \lesssim 2T_c$

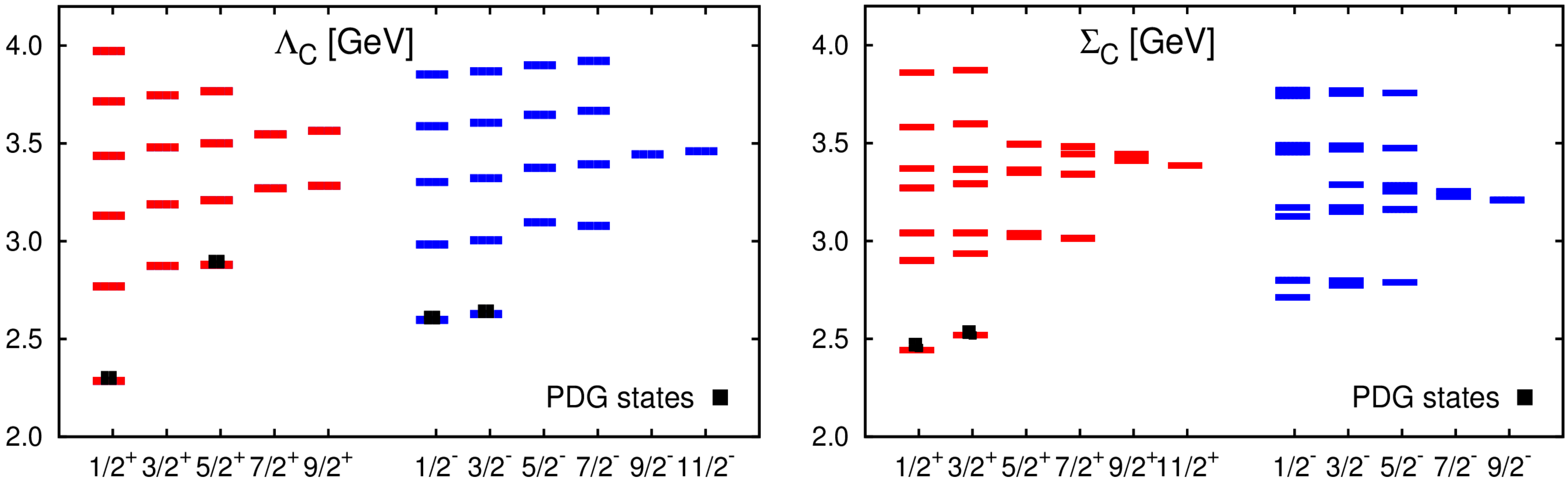
strong flavor correlations, but almost flavor blind



Probing hadron spectrum using thermodynamics

hadronic pressure: $P^C = \sum_{h \in \text{all hadrons}} P_h$ ⟸ expt. observed hadrons + unobserved ones

Quark Model charm baryons

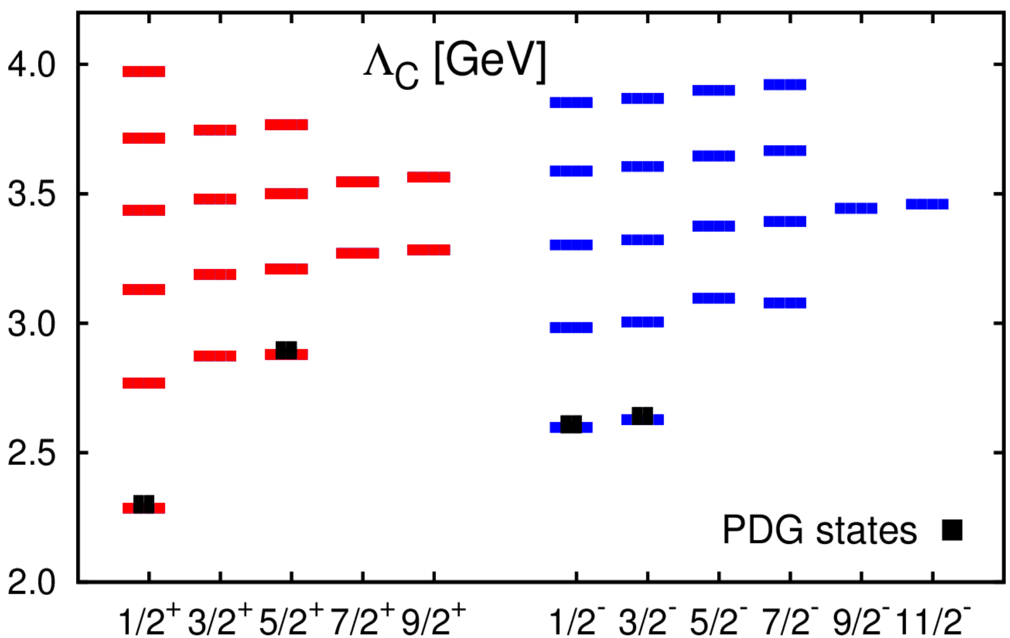


Ebert et. al.: Eur. Phys. J. C66, 197 (2010);
Phys. Rev. D84, 014025 (2011)

hadronic pressure: $P^C = \sum_{h \in \text{all hadrons}} P_h$ ⟸ expt. observed hadrons + unobserved ones

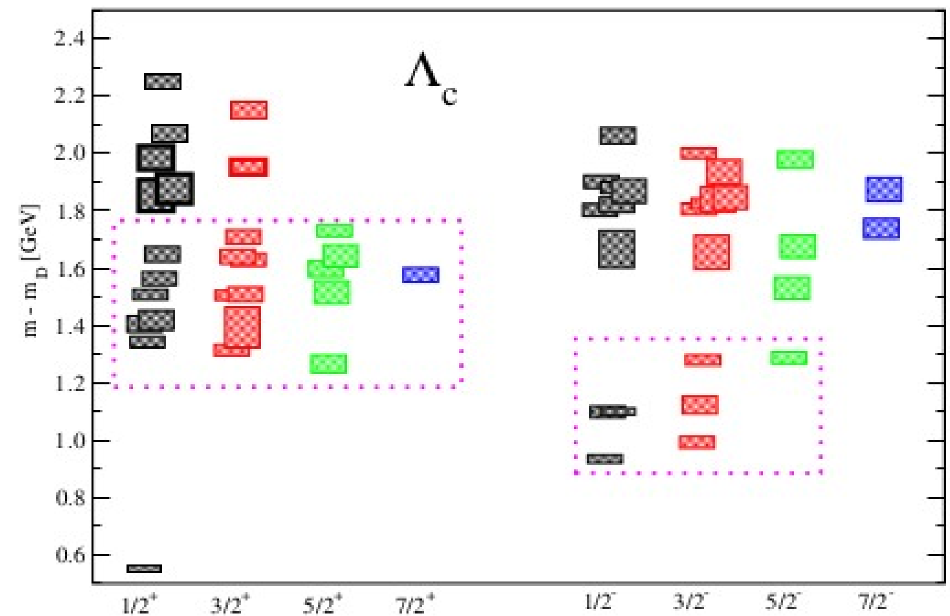
charm baryons

Quark Model



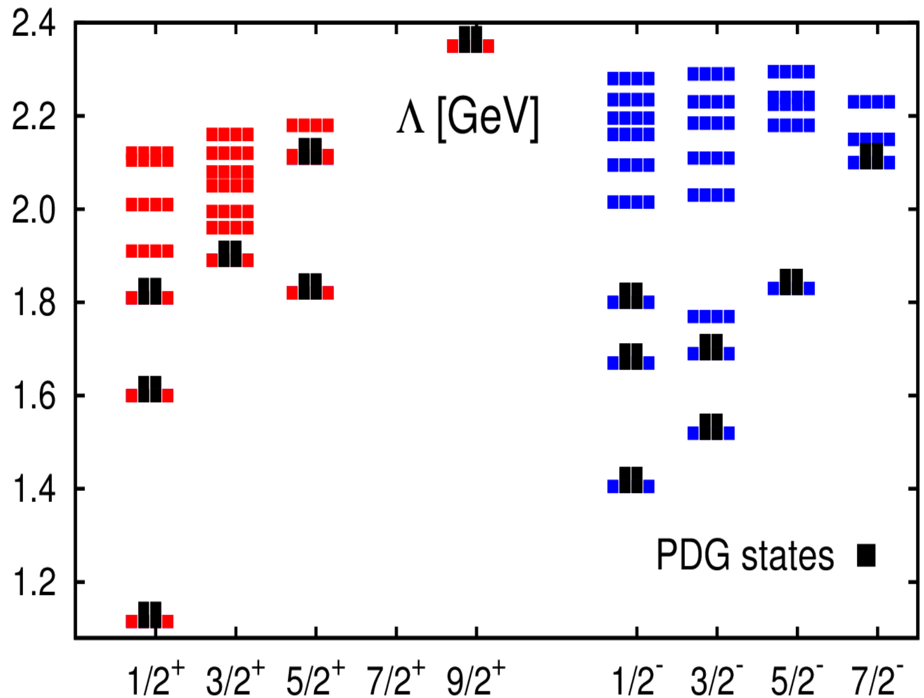
Ebert et. al.: Eur. Phys. J. C66, 197 (2010); Phys. Rev. D84, 014025 (2011)

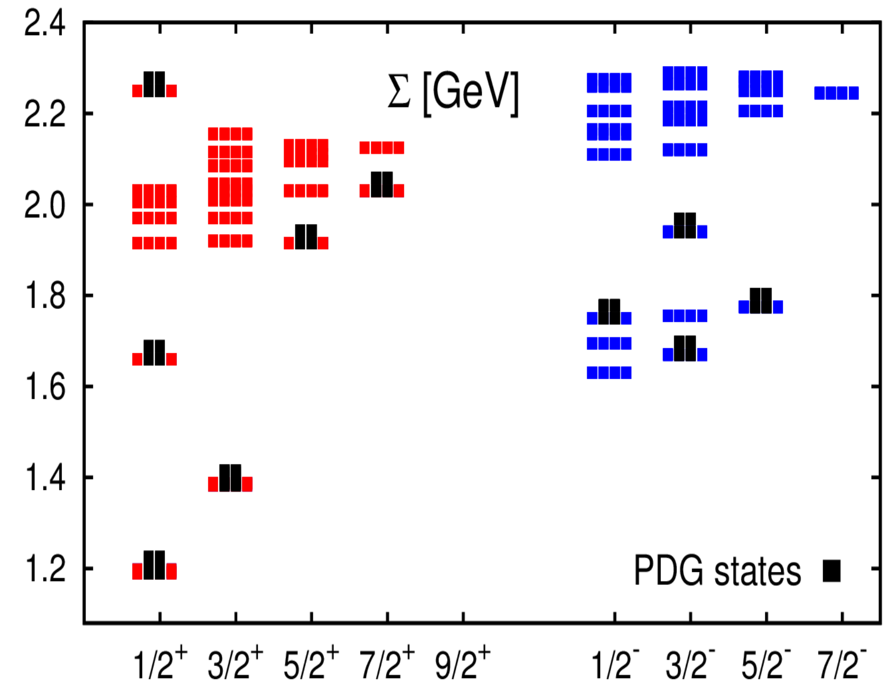
LQCD



Padmanath et.al.: arXiv:1311.4806 [hep-lat] hadronic pressure: $P^S = \sum_{h \in \text{all hadrons}} P_h$ ⟵ expt. observed hadrons + unobserved ones

Quark Model strange baryons



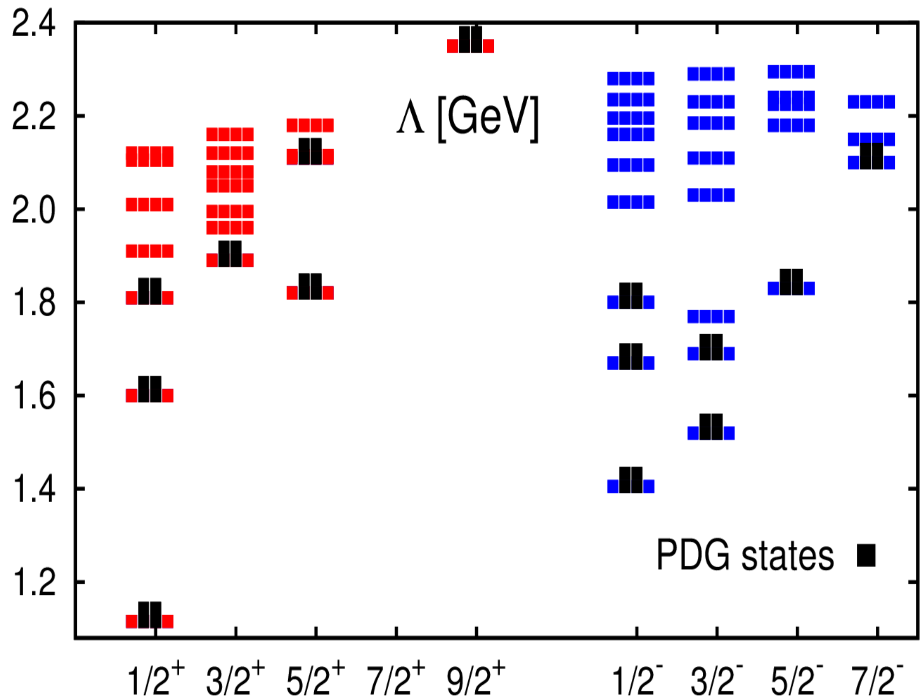


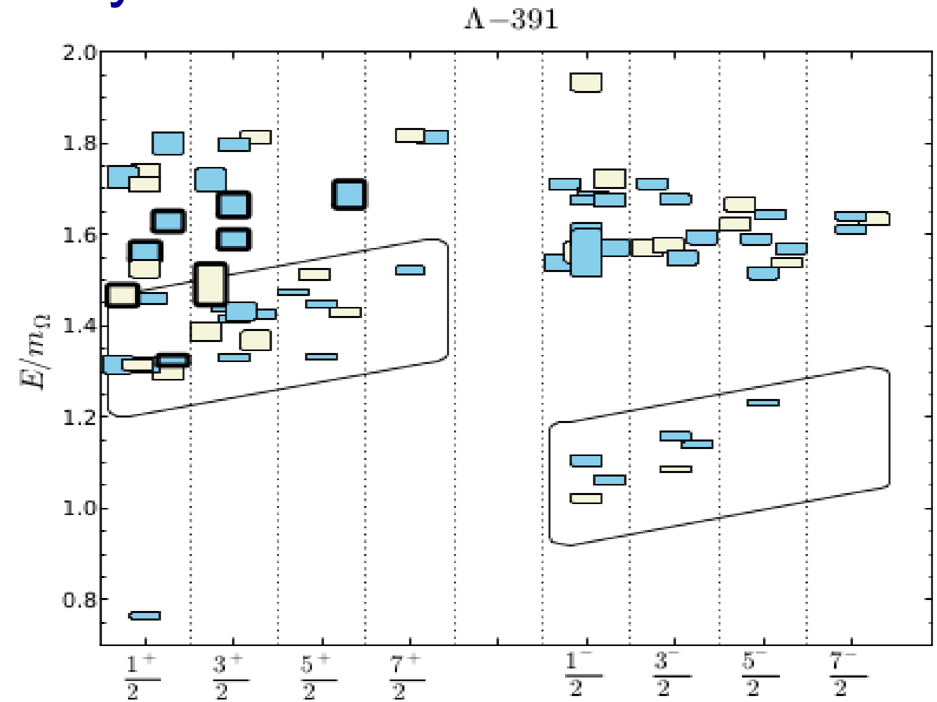
Capstick-Isgur: Phys. Rev. D34, 2809 (1986)

Probing hadron spectrum using thermodynamics

hadronic pressure: $P^S = \sum_{h \in \text{all hadrons}} P_h$ ⟵ expt. observed hadrons + unobserved ones

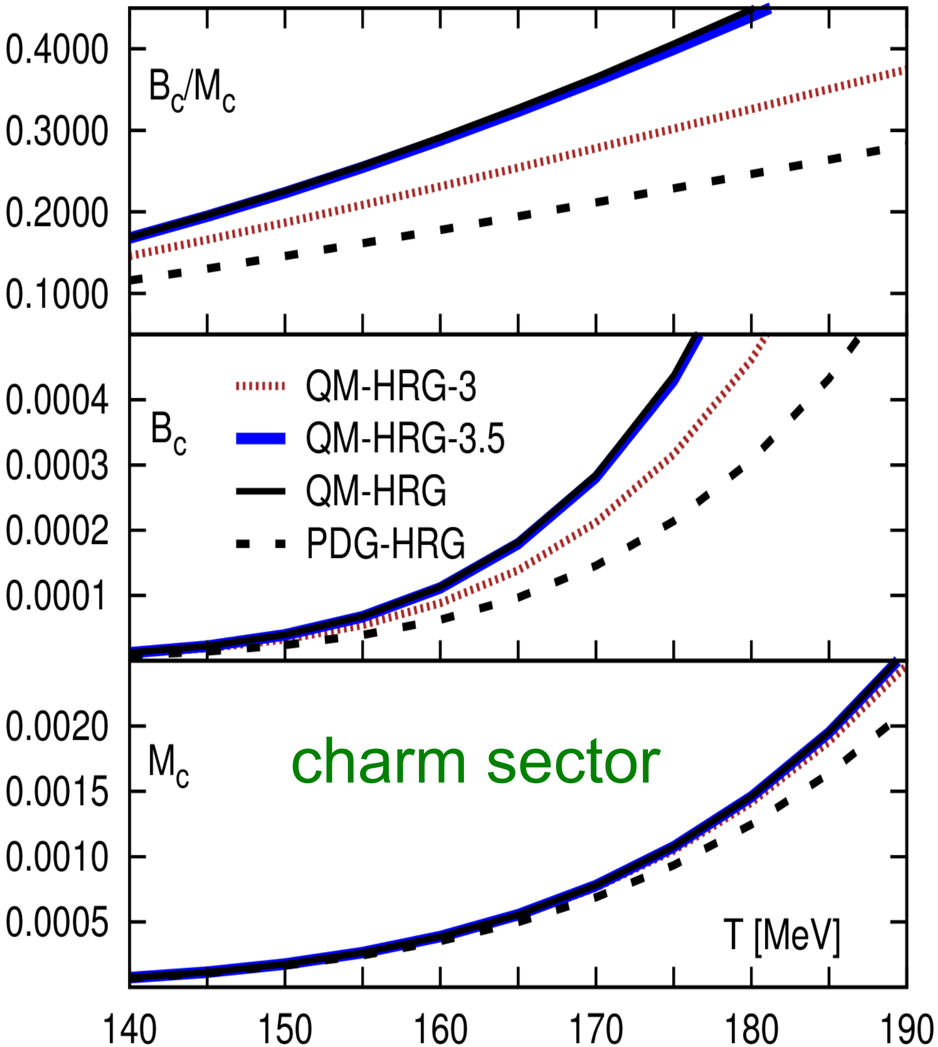
Quark Model strange baryons LQCD



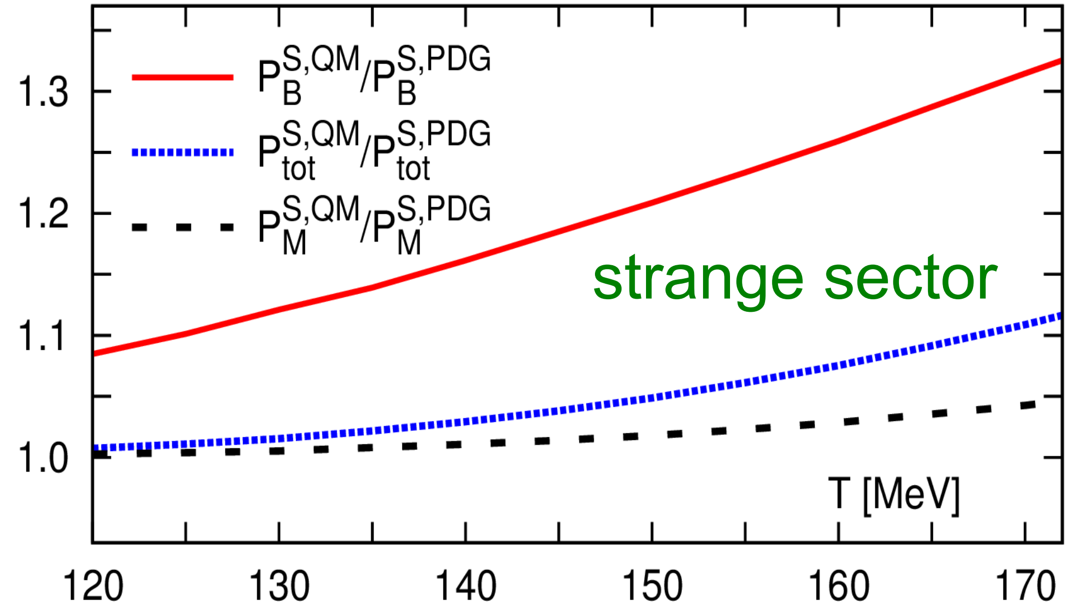


JLab: Phys. Rev. D87, 054506 (2013)

Capstick-Isgur: Phys. Rev. D34, 2809 (1986)



partial pressure of
baryon → B; meson → M



significant contributions of these unseen states to the ratios of partial pressures of baryon to meson near the QCD crossover

similar results with LQCD spectra

LQCD: operators to identify separate thermodynamic contributions of strange/charm baryons/mesons

suitable combinations of up to 4th order baryon – charm/strangeness correlations

a simplified example:

hadron gas $\rightarrow$ $\hat{P}^C \sim P_M^C \cosh[\hat{\mu}_C] + P_B^C \cosh[\hat{\mu}_B + \hat{\mu}_C]$

P_M^C: partial pressure of |C|=1 mesons

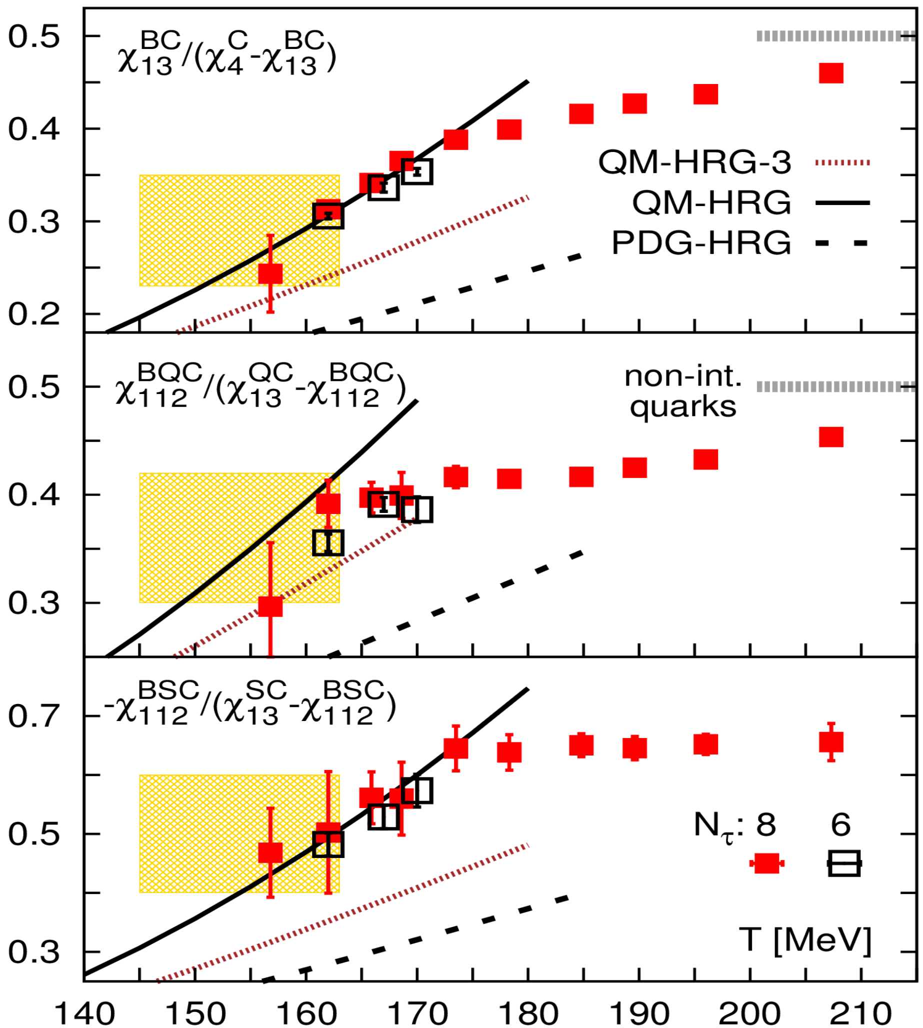
P_B^C: partial pressure of |C|=1 baryons

neglect contributions of heavier |C|=2,3 baryons, x1000 suppressed

$\chi_k^C \simeq P_M^C + P_B^C$

$\chi_{mn}^{BC} \simeq P_B^C$

Signatures of additional charm baryons



BNL-Bi: Phys. Lett. B737 (2014) 210

relative contributions:

charm baryons to charmed mesons

$$\chi_{13}^{BC}/(\chi_4^C-\chi_{13}^{BC}) = P_B^C/P_M^C$$

charged charm baryons to charged charmed mesons

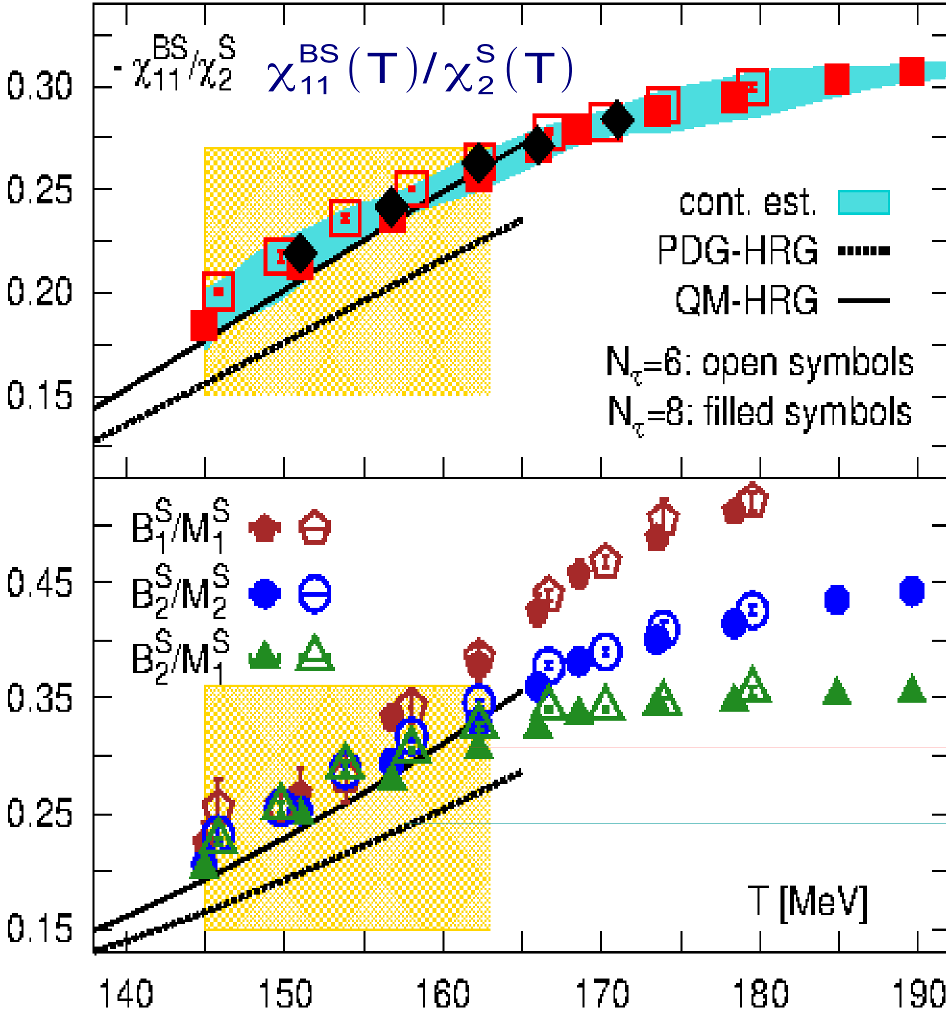
strange charm baryons to strange charmed mesons

signatures of additional, yet unobserved charm baryons from QCD thermodynamics

Signature of additional strange baryons

relative contributions of strange baryons to strange mesons

BNL-Bi: Phys. Rev. Lett. 113 (2014) 072001



partial pressure of strange mesons:

$$M_1^S = \chi_2^S - \chi_{22}^{BS}$$

$$M_2^S = \frac{1}{12}\left(\chi_4^S + 11\,\chi_2^S\right) + \frac{1}{2}\left(\chi_{22}^{BS} + \chi_{13}^{BS}\right)$$

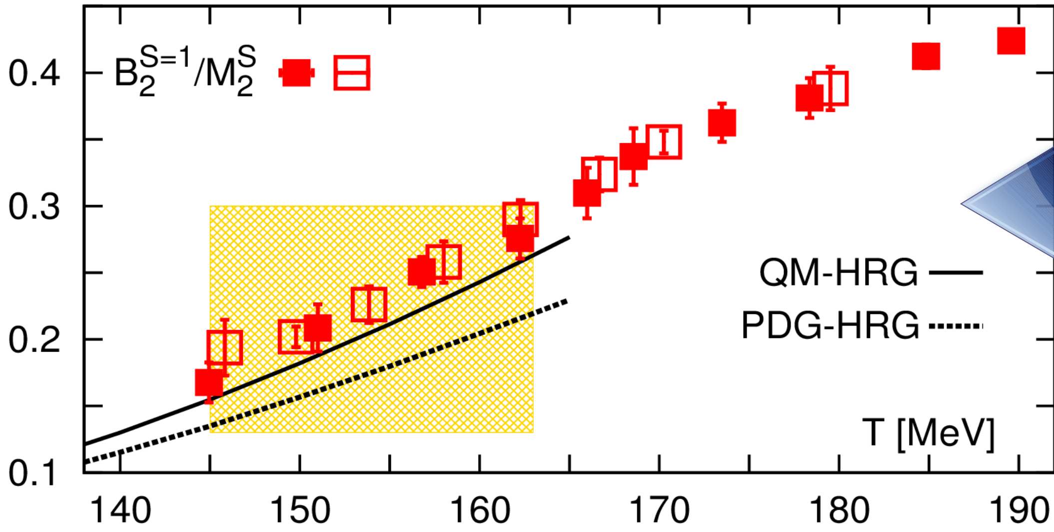
partial pressure of strange baryons:

$$B_1^S = -\frac{1}{6}\left(11\,\chi_{11}^{BS} + 6\,\chi_{22}^{BS} + \chi_{13}^{BS}\right)$$

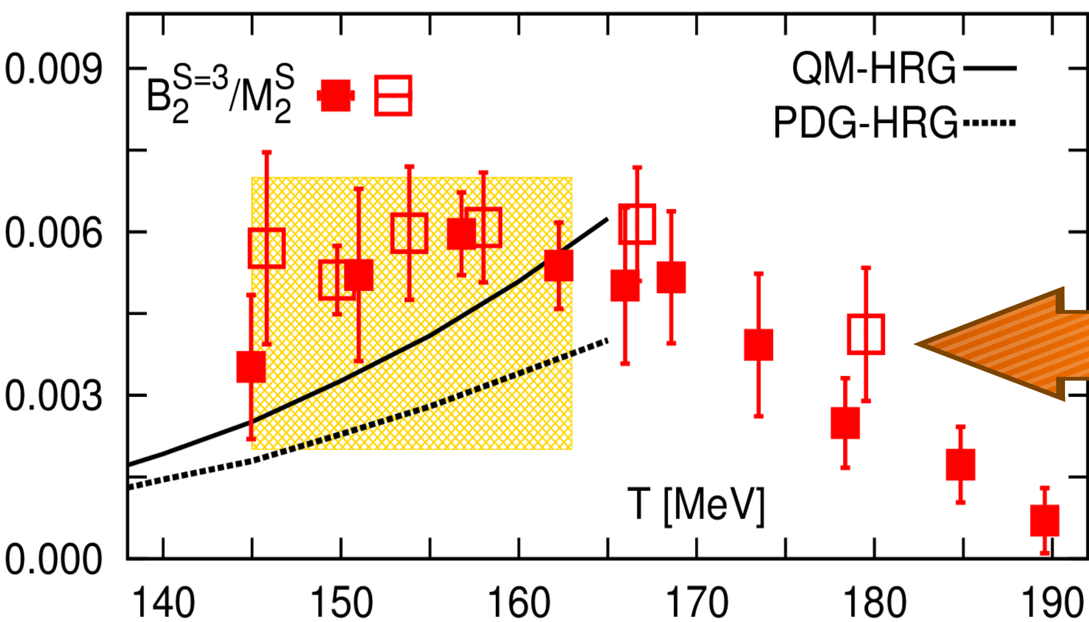
$$B_2^S = \frac{1}{12}\left(\chi_4^S - \chi_2^S\right) + \frac{1}{3}\left(4\,\chi_{11}^{BS} - \chi_{13}^{BS}\right)$$

+ undiscovered strange baryons

contributions of all expt. observed strange hadrons



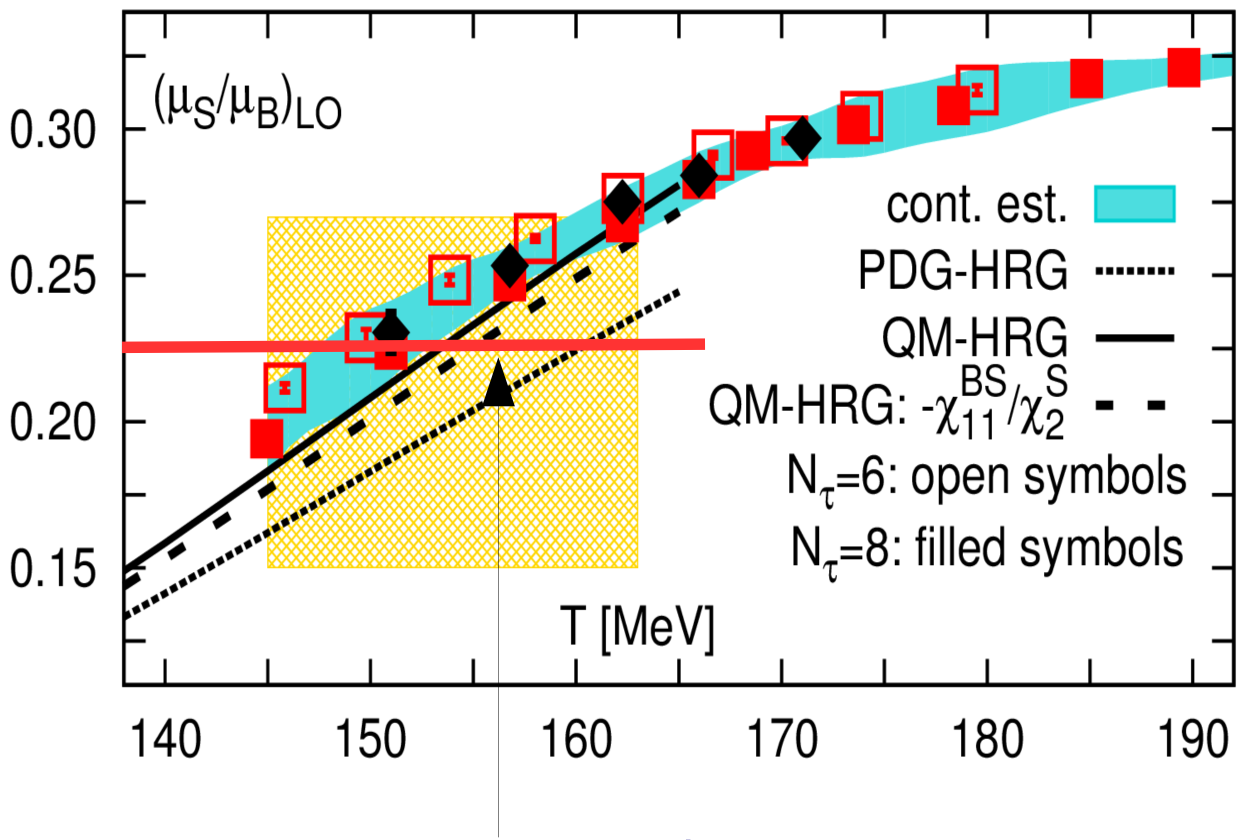
relative contributions of S=1 baryons to strange mesons



relative contributions of S=3 baryons to strange mesons

Strangeness chemical potential in HIC

medium formed in HIC is strangeness neutral: $\langle n_S \rangle = 0$



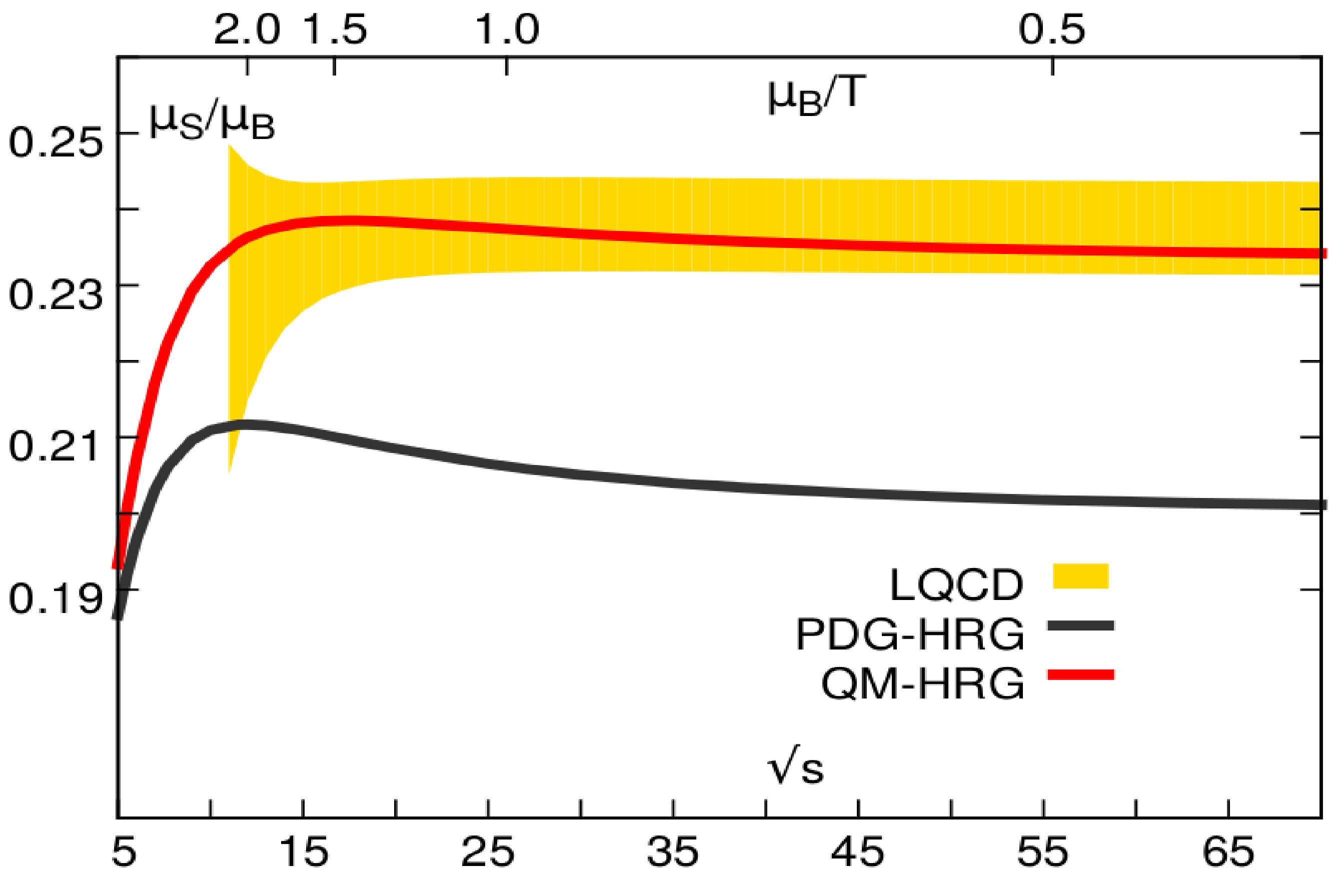
$$\frac{\mu_S}{\mu_B}(T, \mu_B/T) \simeq \frac{\chi_{11}^{BS}(T)}{\chi_2^S(T)} + \ldots$$

relative contribution of strange baryons to mesons

LQCD results are reproduced by including additional Quark Model states

a given value of μ_S/μ_B is realized at a lower temperature

BNL-Bi: Phys. Rev. Lett. 113 (2014) 072001



signature for unobserved strange baryons persists for RHIC BES-II

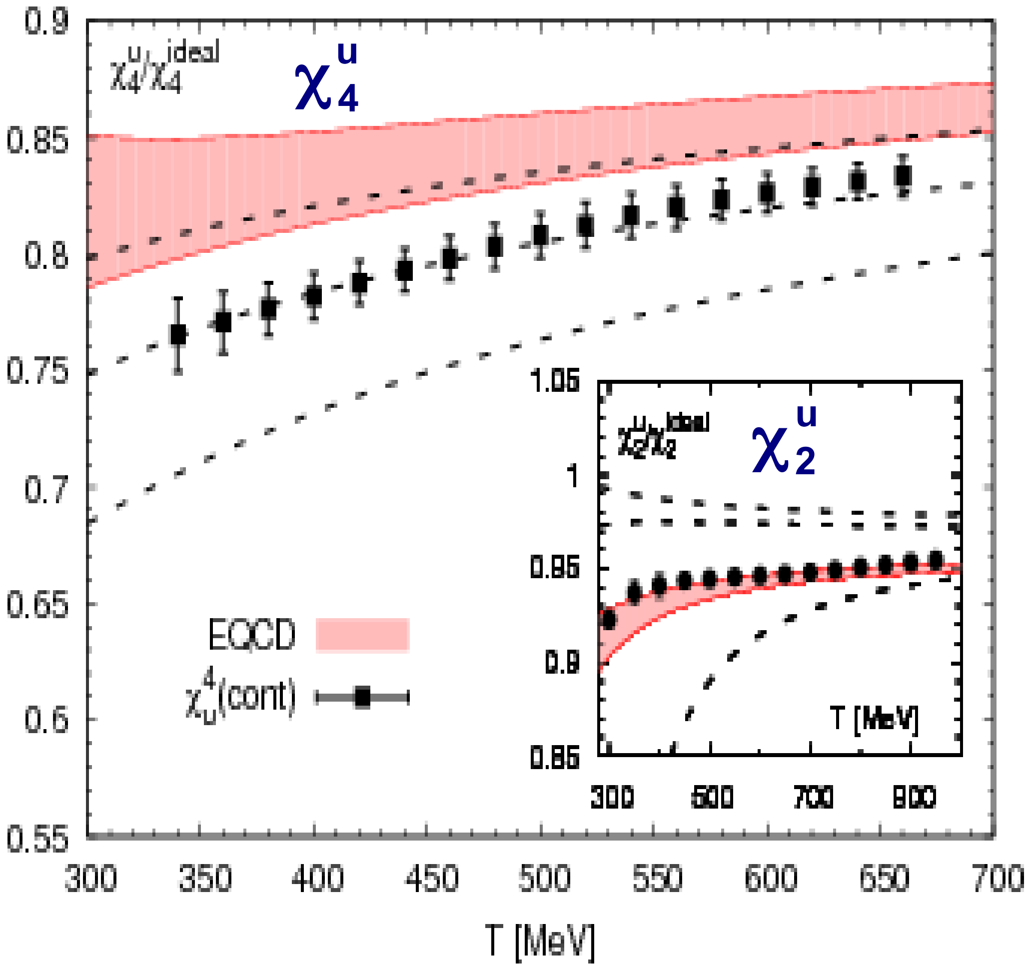
can also be extracted from expt. measured

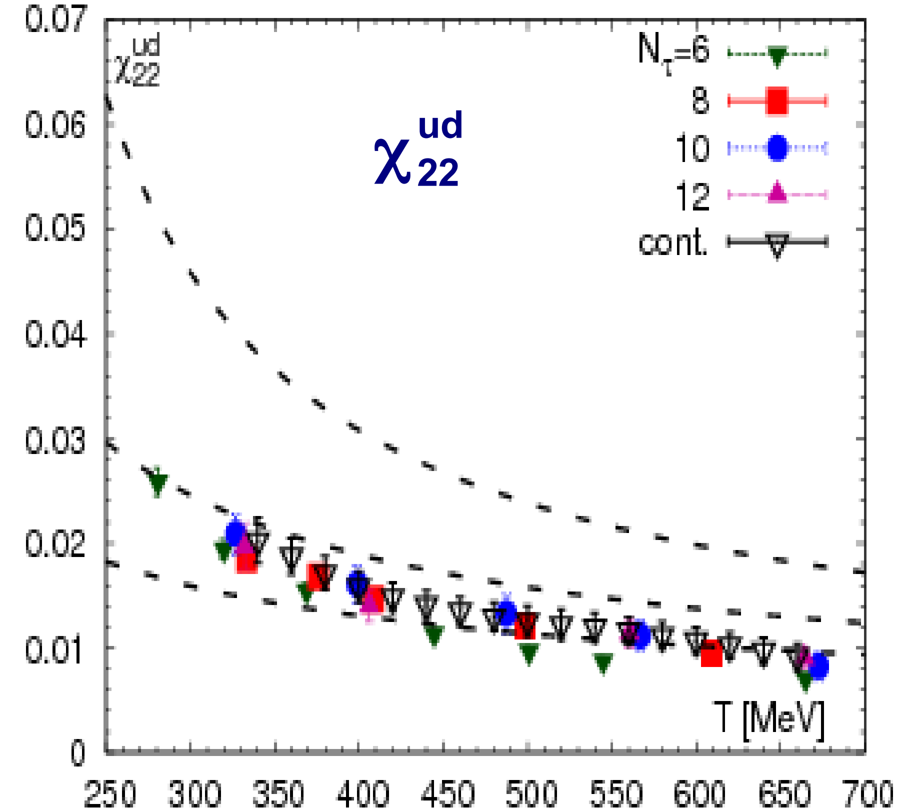
$$\frac{\ln\left[N_{K^-}/N_{K^+}\right]}{\ln\left[N_{\bar{p}}/N_p\right]} = \frac{\mu_S^f}{\mu_B^f}$$

need accurate expt. measurements & feed-down corrections

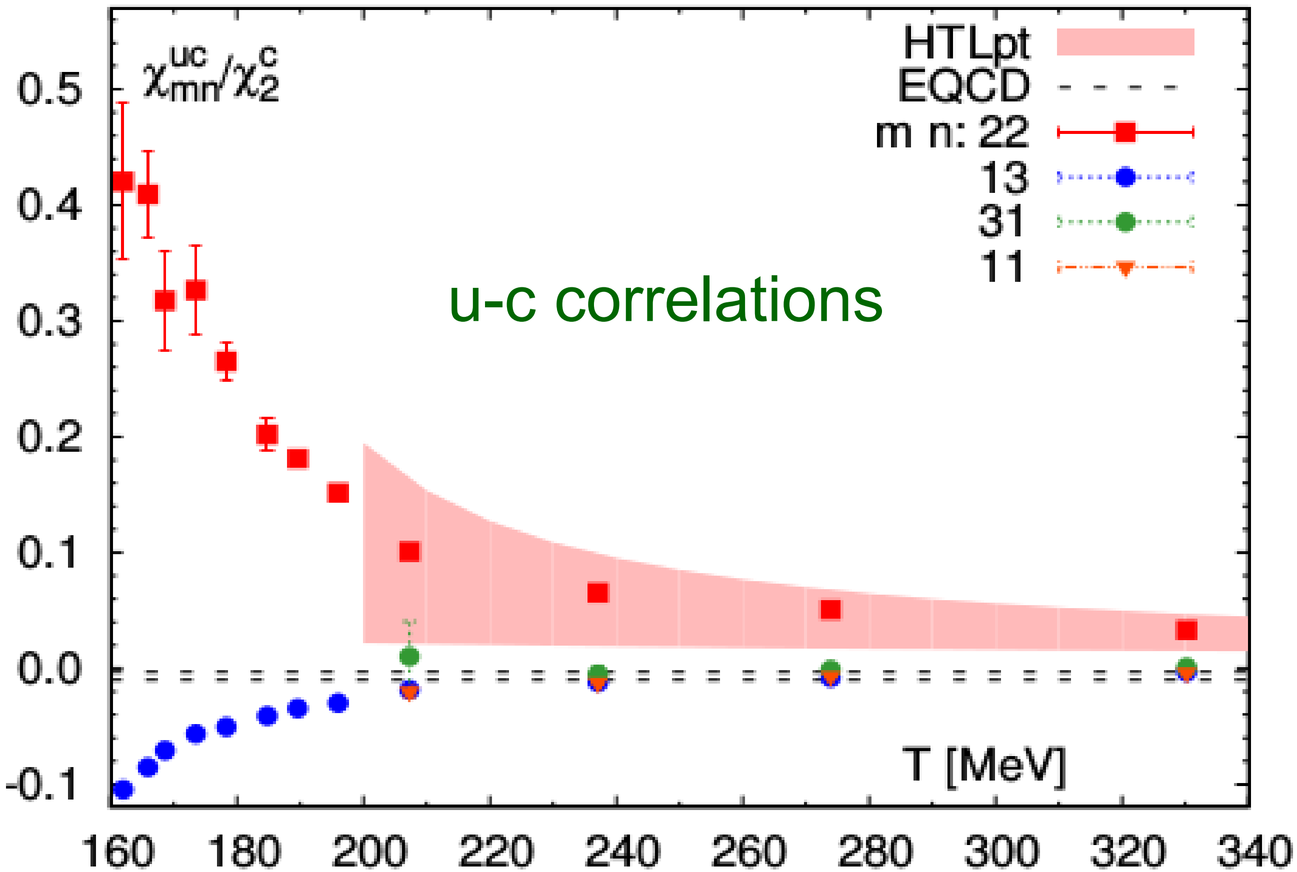
DoF at high temperatures

BNL-Bi-CCNU: Phys. Rev. D88 (2013) 9, 094021
Phys. Rev. D92 (2015) 7, 074043





agreements with weak coupling 3-loop HTL results: $T \gtrsim 2T_c$



agreements with weak coupling calculations:

$T \geqslant 200$ MeV

BNL: Phys. Rev. D93 (2016) 1, 014502

Test possible charm dof in QGP

naive postulate: non-interacting gas of charm quark, meson & baryon-like excitations in QGP

charm quark & its possible bound states much heavy compared to T
→ can be treated as quasi-particles within classical/Boltzmann approximation

$$P^C = P^C_q \cosh\left[\frac{\hat{\mu}_B}{3} + \hat{\mu}_C\right] + P^C_M \cosh\left[\hat{\mu}_C\right] + P^C_B \cosh\left[\hat{\mu}_B + \hat{\mu}_C\right]$$

$$p^C_q = 9\left(\chi^{BC}_{13} - \chi^{BC}_{22}\right)/2$$

$$p^C_B = \left(3\chi^{BC}_{22} - \chi^{BC}_{13}\right)/2$$

$$p^C_M = \chi^C_2 + 3\chi^{BC}_{22} - 4\chi^{BC}_{13}$$

naive postulate: non-interacting gas of charm quark, meson & baryon-like excitations in QGP

charm quark & its possible bound states
much heavy compared to T
→ can be treated as quasi-particles within classical/Boltzmann approximation

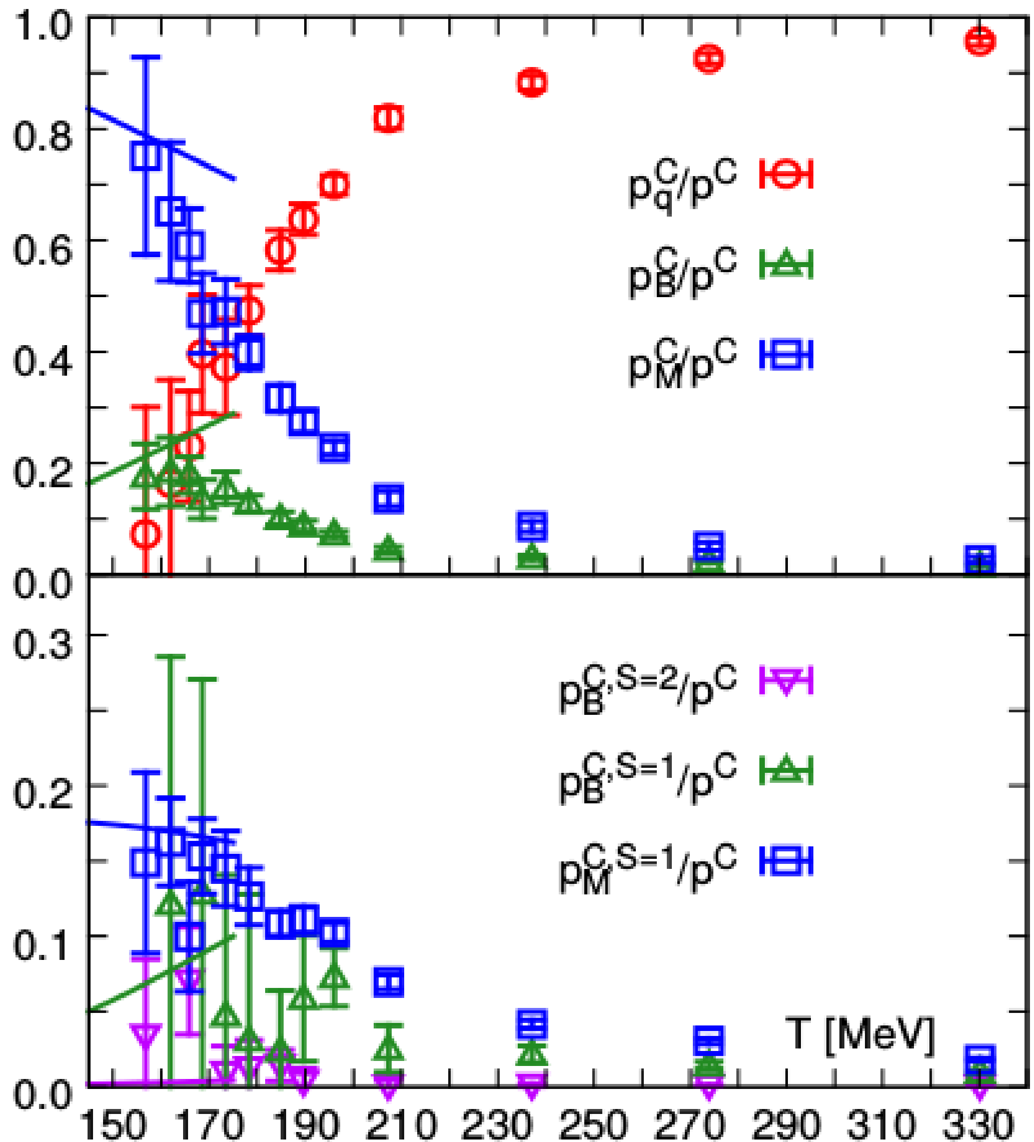
strangeness sub-sector: charm quarks do not carry S, S-C correlations from possible bound states

$$P^{C,S} = P_M^{C,S=1} \cosh[\hat{\mu}_S + \hat{\mu}_C] + \sum_{k=1,2} P_B^{C,S=k} \cosh[\hat{\mu}_B - k\hat{\mu}_S + \hat{\mu}_C]$$

$$p_M^{C,S=1} = \chi_{13}^{SC} - \chi_{112}^{BSC}$$

$$p_B^{C,S=1} = \chi_{13}^{SC} - \chi_{22}^{SC} - 3\chi_{112}^{BSC}$$

$$p_B^{C,S=2} = (2\chi_{112}^{BSC} + \chi_{22}^{SC} - \chi_{13}^{SC})/2$$



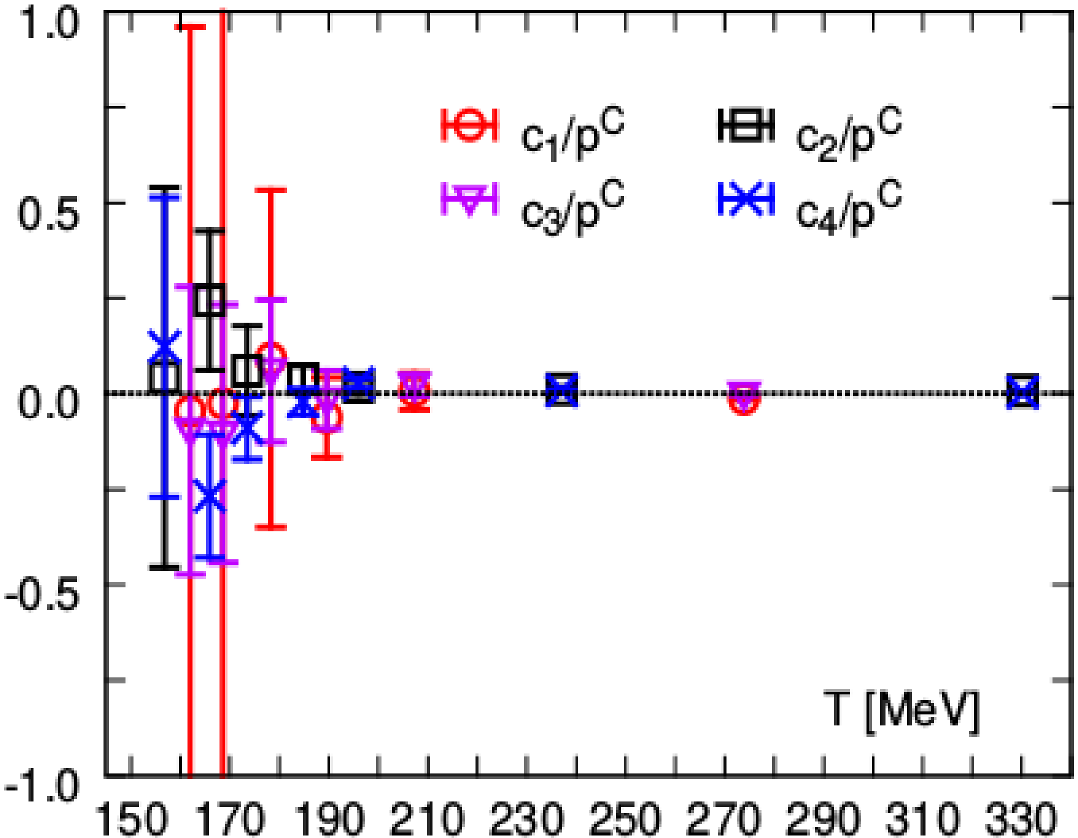
contributions of quark-like excitations dominant for $T \gtrsim 200\,\mathrm{MeV}$

contributions of meson- & baryon-like excitations dominant for $T \lesssim 200\,\mathrm{MeV}$

meson- & baryon-like excitations are not vacuum hadrons

BNL: Phys. Rev. D93 (2016) 1, 014502

Test possible charm dof in QGP: consistency



BNL: Phys. Rev. D93 (2016) 1, 014502

$$c_1 \equiv \chi_{13}^{BC} - 4\chi_{22}^{BC} + 3\chi_{31}^{BC} = 0$$

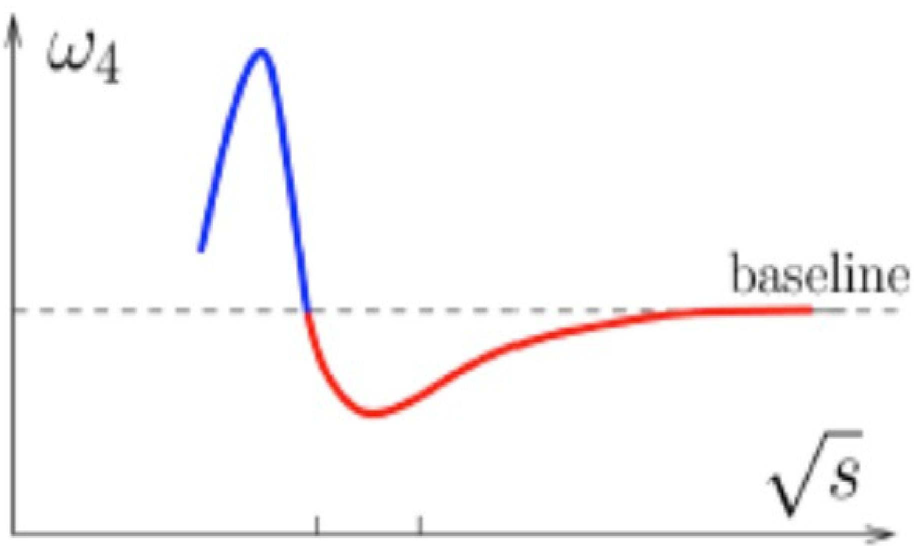
$$c_2 \equiv 2\chi_{121}^{BSC} + 4\chi_{112}^{BSC} + \chi_{22}^{SC} - 2\chi_{13}^{SC} + \chi_{31}^{SC} = 0$$

$$c_3 \equiv 3\chi_{112}^{BSC} + 3\chi_{121}^{BSC} - \chi_{13}^{SC} + \chi_{31}^{SC} = 0$$

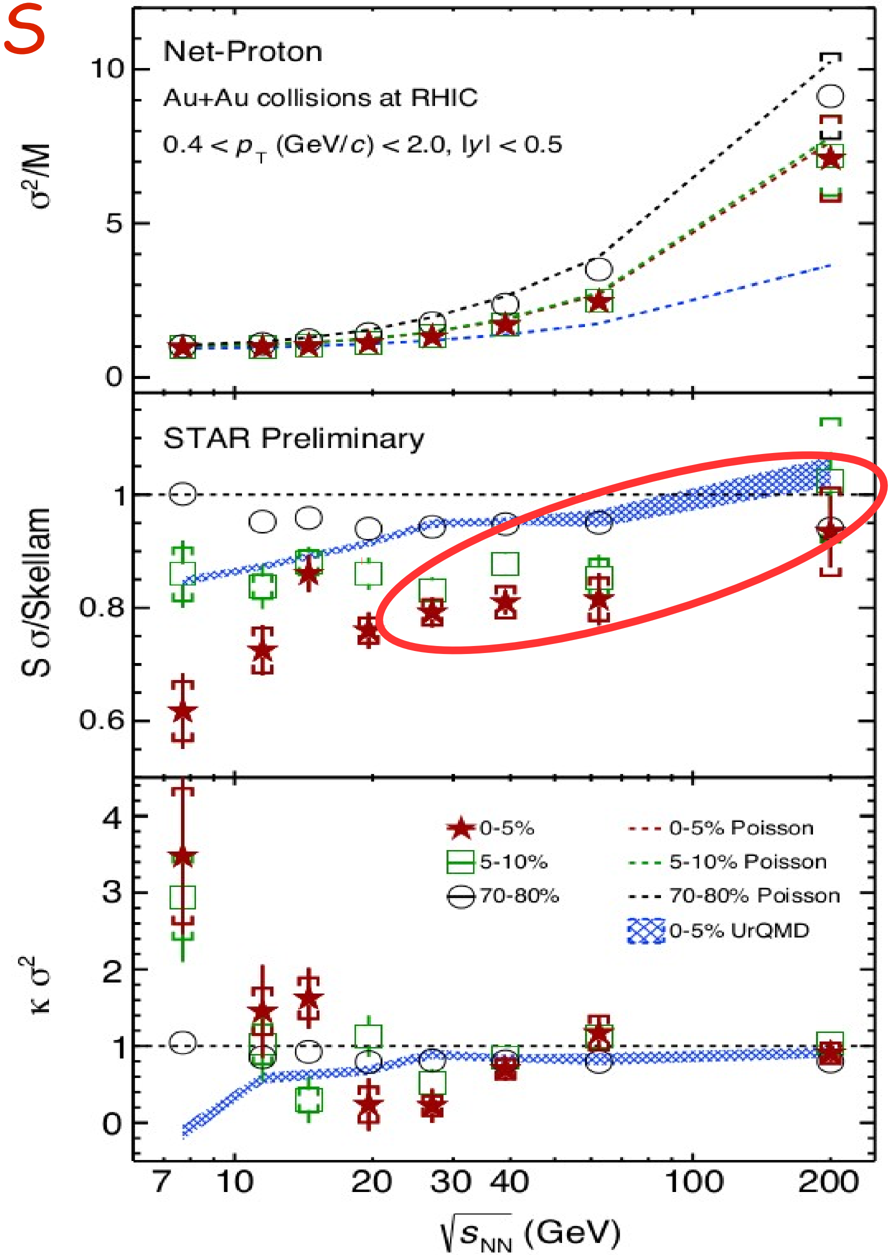
$$c_4 \equiv \chi_{211}^{BSC} - \chi_{112}^{BSC} = 0$$

negligible contributions of S-C di-quarks:
S=C=1 but B=2/3

Equilibrium QCD baseline for BES



sketch: net-baryon kurtosis across the QCD critical point



(L)QCD cumulants of conserved charge fluctuations along:

$T_f(\mu_B^f)$

↑

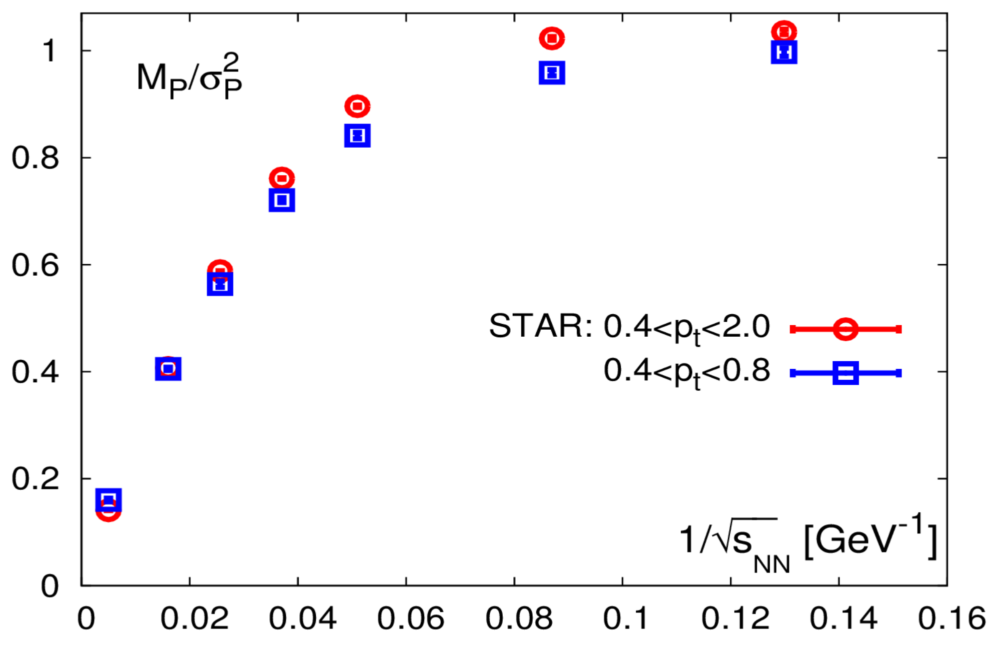
not a fundamental QCD parameter: expt. input for a given colliding system, phase space cuts, $\sqrt{s}$ ...

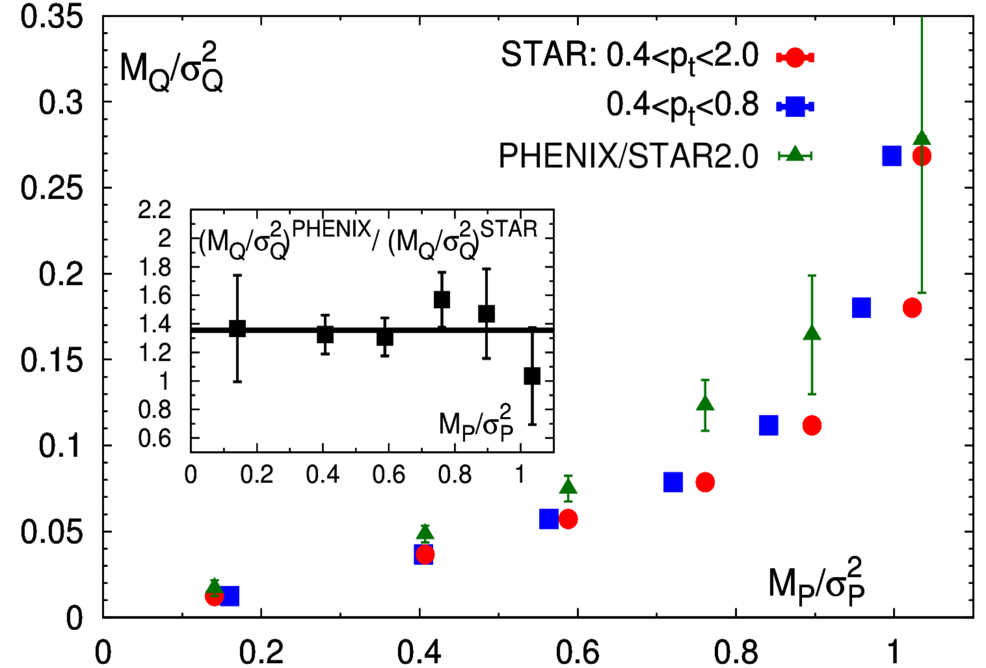
underlying assumption: expt. observables can be mapped into thermodynamic parameters T_f, μ_B^f

for consistency:

estimate $T_f(\mu_B^f)$ by matching expt. lower cumulants $M_Q/\sigma_Q^2 [M_p/\sigma_p^2]$ with equilibrium QCD $M_Q/\sigma_Q^2 [M_B/\sigma_B^2]$ despite all known/unknown caveats

equilibrium QCD baseline for higher cumulants along this $T_f(\mu_B^f)$





$$R_{12}^{P} \equiv \frac{M_P}{\sigma_P^2}$$

monotonic functions of $\sqrt{s}$

$$R_{12}^{Q} \equiv \frac{M_Q}{\sigma_Q^2}$$

μ_B/T

$$\frac{\mu_B}{T} = m_1^B R_{12}^B + m_3^B \left(R_{12}^B\right)^3 + \mathcal{O}\left(\left(R_{12}^B\right)^5\right)$$

M_X/σ_X along the freeze-out line: $T_f(\mu_B) = T_{f,0}\left(1 - \kappa_2^f\left(\frac{\mu_B}{T}\right)^2\right)$

in practice: $M_s=0$, $M_Q/M_B=0.4$ → $\mu_Q(T,\mu_B)$, $\mu_S(T,\mu_B)$

for simplicity of discussion:

$\mu_Q=\mu_S=0$

$$\frac{M_B}{\sigma_B^2} = \frac{\mu_B}{T}\frac{1+\frac{1}{6}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2}{1+\frac{1}{2}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2}$$

$$\frac{M_Q}{\sigma_Q^2} = \frac{\mu_B}{T}\frac{\chi_{11}^{BQ}}{\chi_2^Q}\frac{1+\frac{1}{6}\frac{\chi_{31}^{BQ}}{\chi_{11}^{BQ}}\left(\frac{\mu_B}{T}\right)^2}{1+\frac{1}{2}\frac{\chi_{22}^{BQ}}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2}$$

$$\chi(T_f) = \chi(T_{f,0}) - \kappa_2^f\left(\frac{d\chi}{dT}\right)_{T_{f,0}}\left(\frac{\mu_B}{T}\right)^2$$

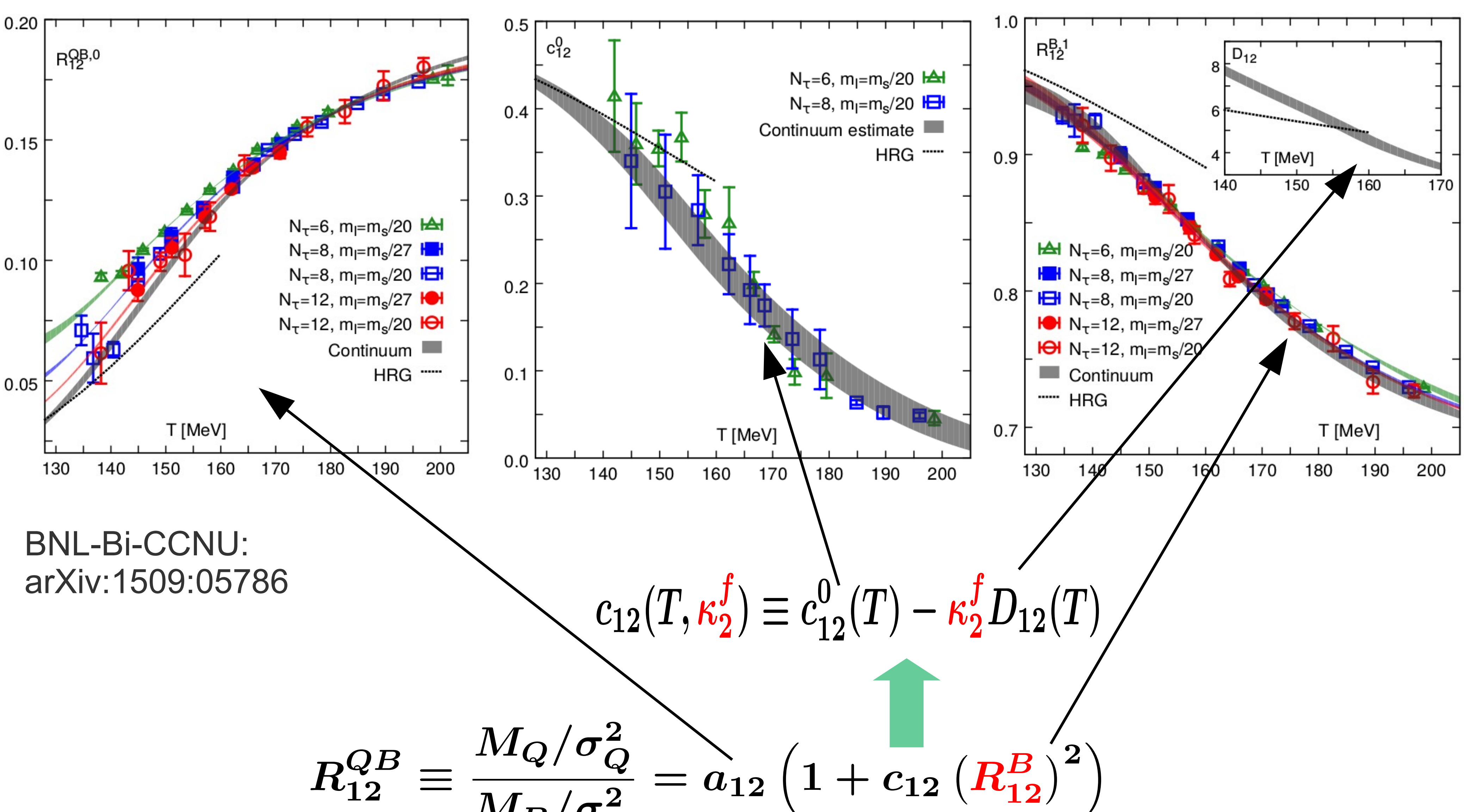
$$R_{12}^{QB,0}(T) = r\,\frac{\chi_2^B(T)}{\chi_2^Q(T)}$$

↑

$$R_{12}^{QB} \equiv \frac{M_Q/\sigma_Q^2}{M_B/\sigma_B^2} = a_{12}\left(1 + c_{12}\left(R_{12}^B\right)^2\right)$$

↓

$$c_{12}(T,\kappa_2^f) \equiv c_{12}^0(T) - \kappa_2^f D_{12}(T)$$



BNL-Bi-CCNU:
arXiv:1509:05786

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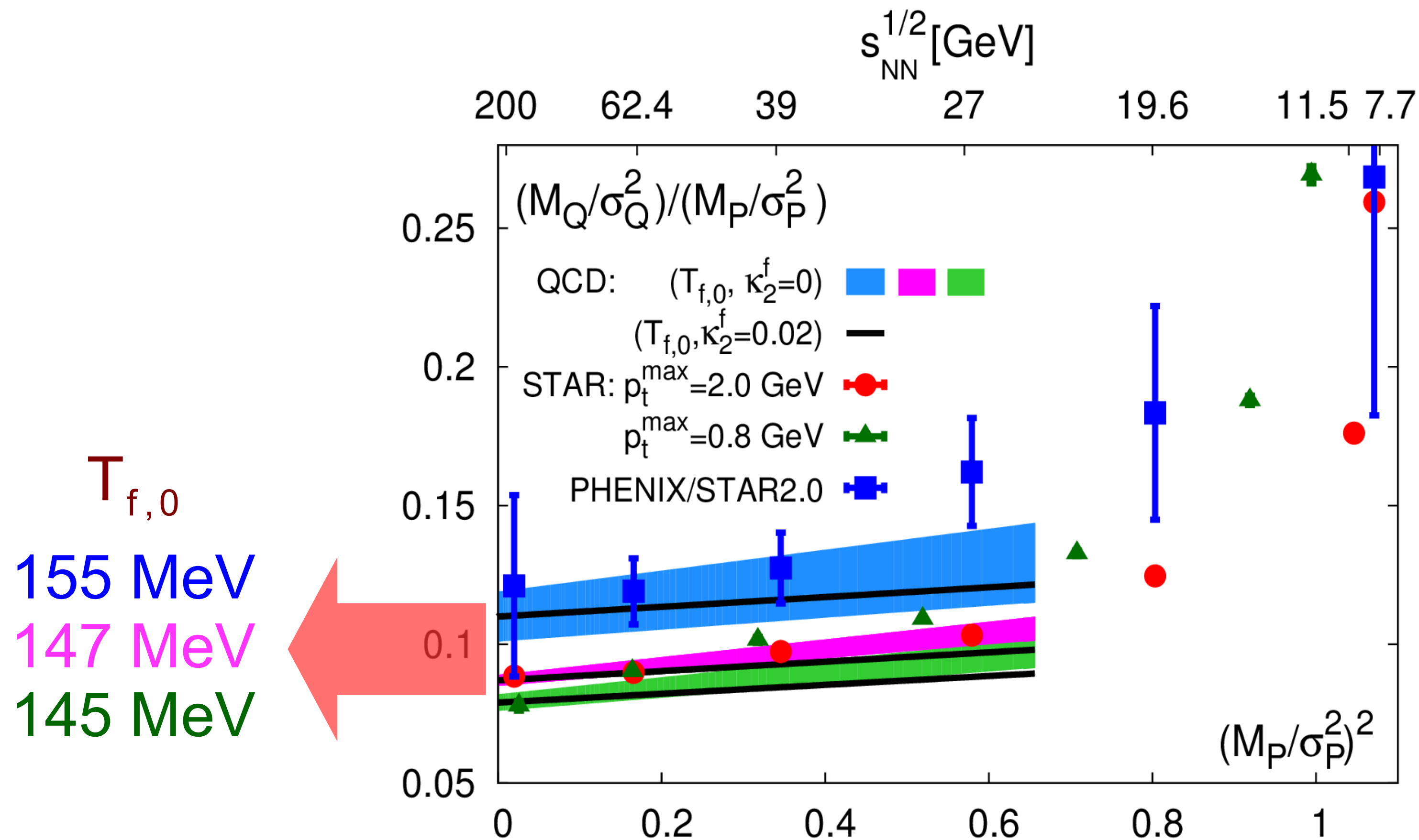
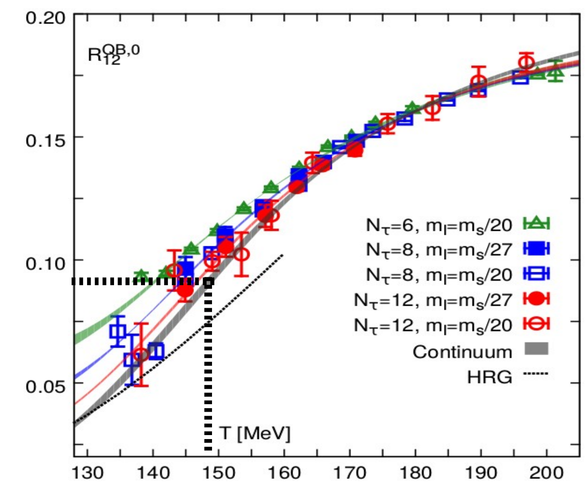




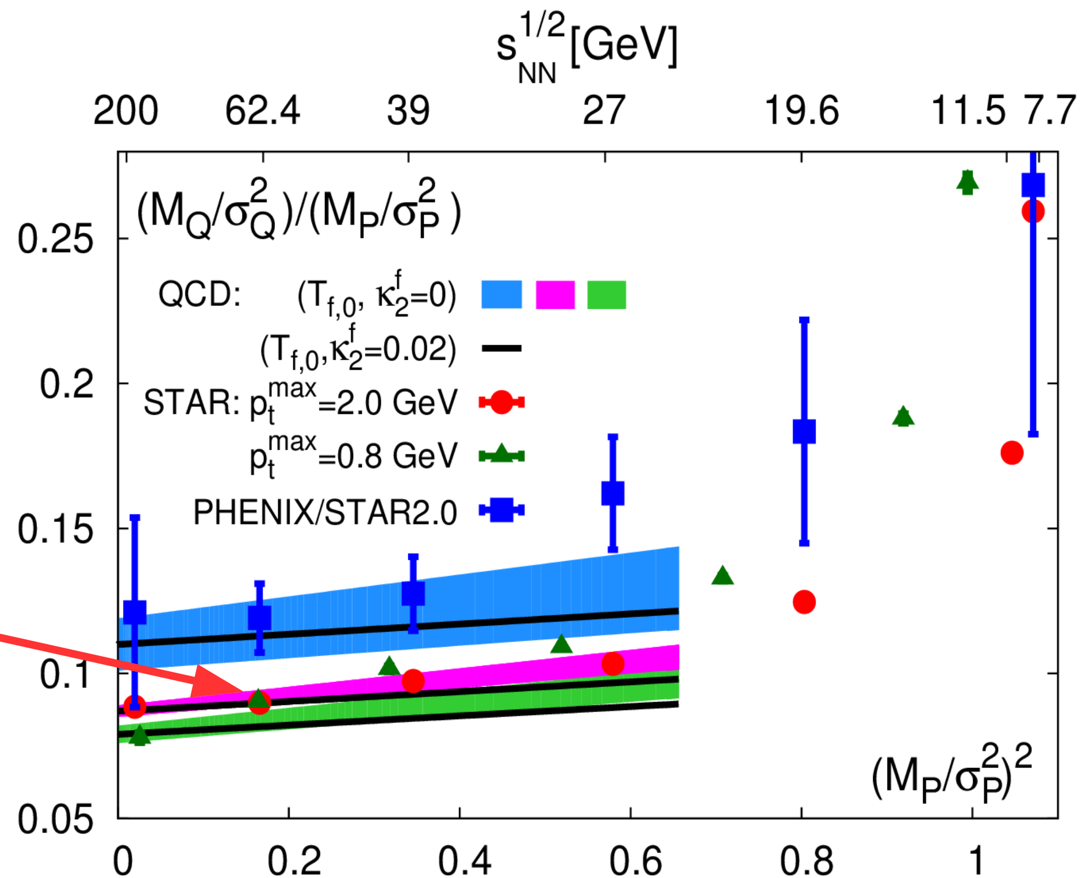
$$R_{12}^{QB} \equiv \frac{M_Q/\sigma_Q^2}{M_B/\sigma_B^2} = a_{12}\left(1 + c_{12}\left(R_{12}^{B}\right)^2\right)$$

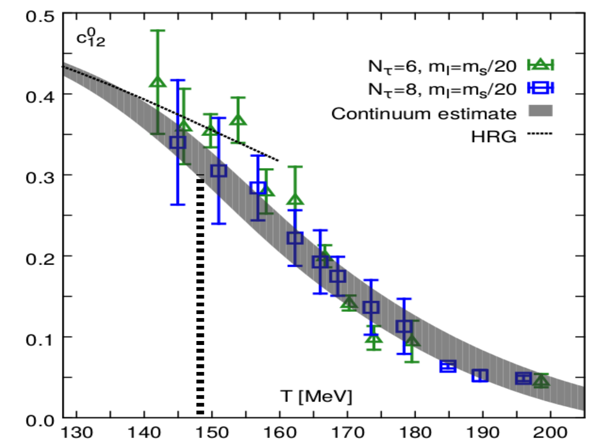
BNL-Bi-CCNU: arXiv:1509:05786

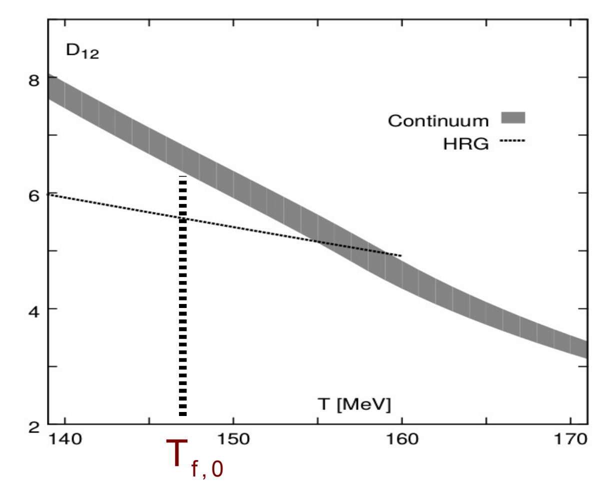
$$c_{12}(T,\kappa_2^f) \equiv c_{12}^0(T) - \kappa_2^f D_{12}(T)$$

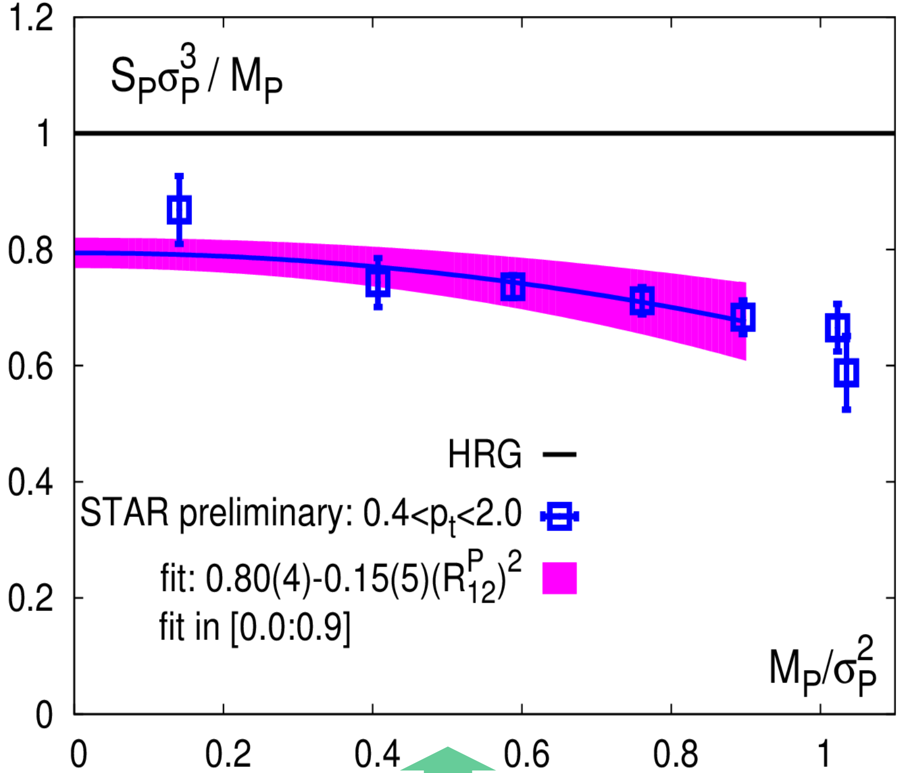
slope:

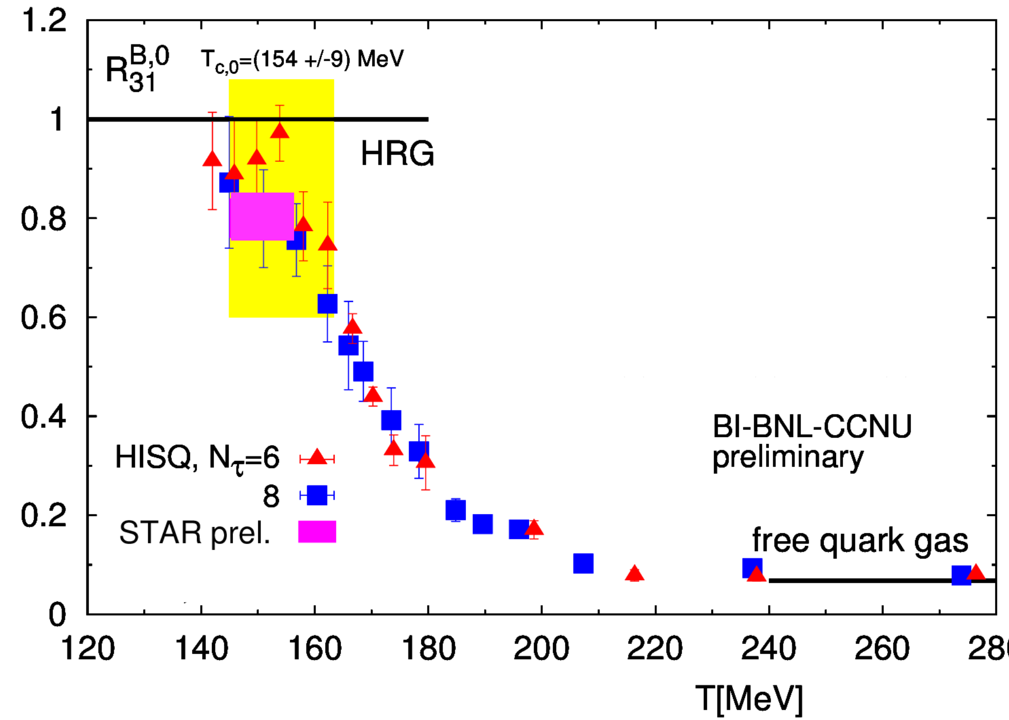
$\kappa_2^f < 0.011$









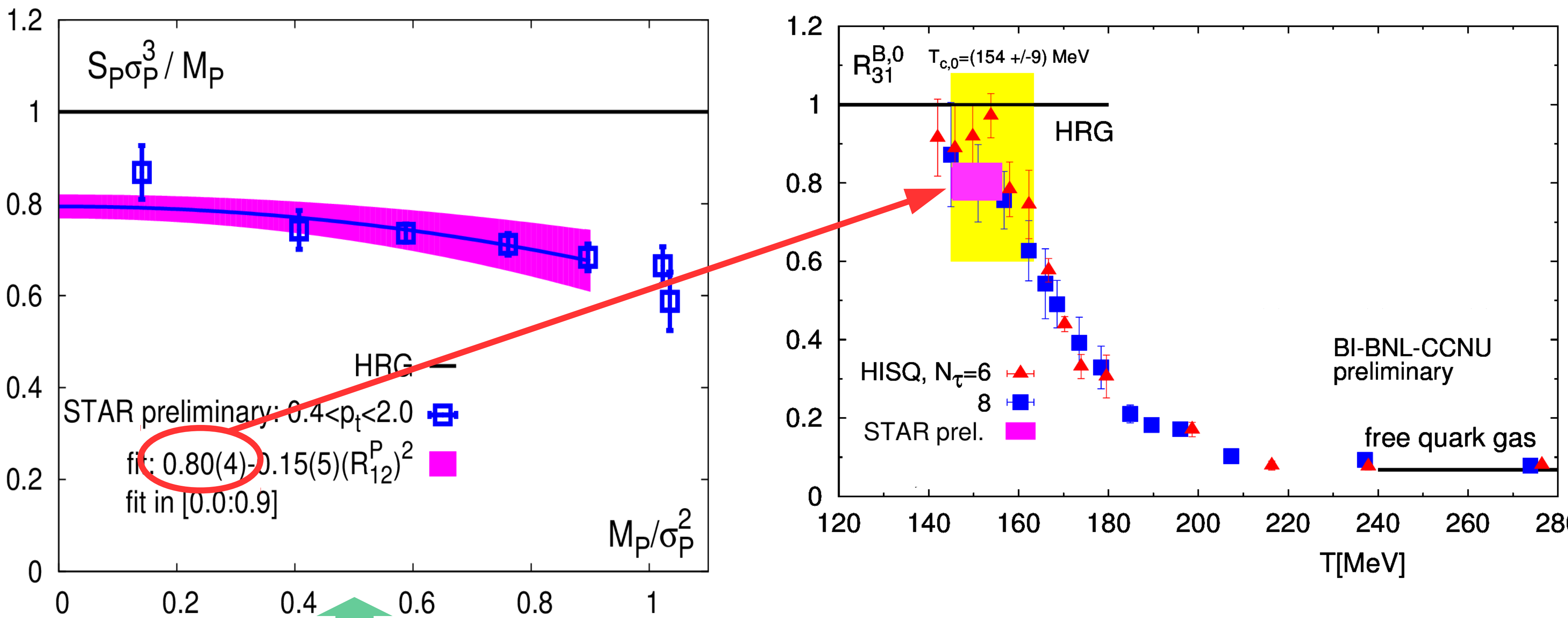


$$\frac{S_B\sigma_B^3}{M_B} = R_{31}^{B,0} + R_{31}^{B,2}\left(R_{12}^B\right)^2$$

choosing for simplicity:

$\mu_Q=\mu_S=\kappa_2^f=0$

$$S_B\sigma_B = \frac{\chi_4^B}{\chi_2^B}\frac{M_B}{\sigma_B^2} + \frac{1}{6}\left(\frac{\chi_6^B}{\chi_2^B} - \left(\frac{\chi_4^B}{\chi_2^B}\right)^2\right)\left(\frac{M_B}{\sigma_B^2}\right)^3 + \ldots$$

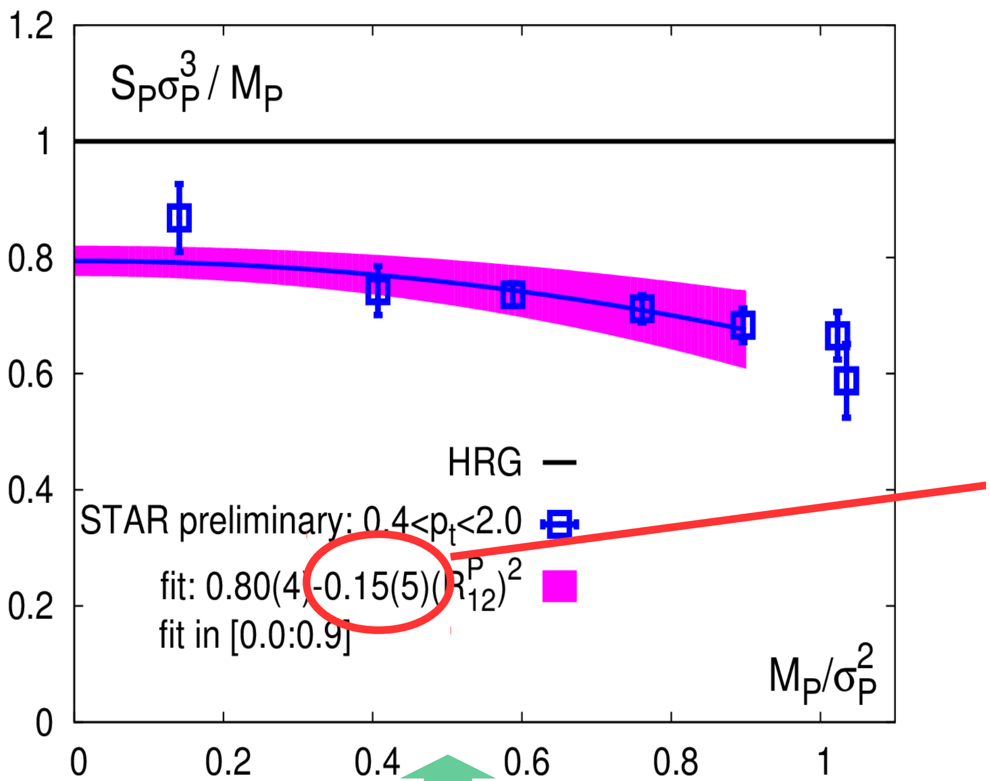


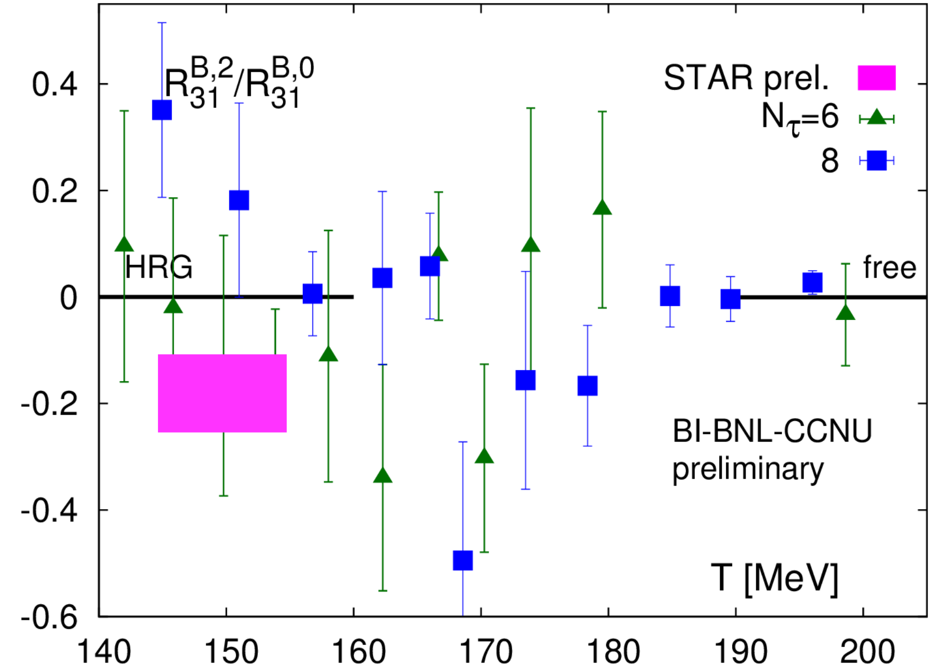
$$\frac{S_B \sigma_B^3}{M_B} = R_{31}^{B,0} + R_{31}^{B,2} \left(R_{12}^B\right)^2$$

choosing for simplicity:

$\mu_Q = \mu_S = \kappa_2^f = 0$

$$S_B \sigma_B = \frac{\chi_4^B}{\chi_2^B} \frac{M_B}{\sigma_B^2} + \frac{1}{6} \left(\frac{\chi_6^B}{\chi_2^B} - \left(\frac{\chi_4^B}{\chi_2^B} \right)^2 \right) \left(\frac{M_B}{\sigma_B^2} \right)^3 + \dots$$





$$\frac{S_B\sigma_B^3}{M_B} = R_{31}^{B,0} + R_{31}^{B,2}\left(R_{12}^B\right)^2$$

choosing for simplicity:

$\mu_Q=\mu_S=\kappa_2^f=0$

$$S_B\sigma_B = \frac{\chi_4^B}{\chi_2^B}\frac{M_B}{\sigma_B^2} + \frac{1}{6}\left(\frac{\chi_6^B}{\chi_2^B} - \left(\frac{\chi_4^B}{\chi_2^B}\right)^2\right)\left(\frac{M_B}{\sigma_B^2}\right)^3 + \ldots$$

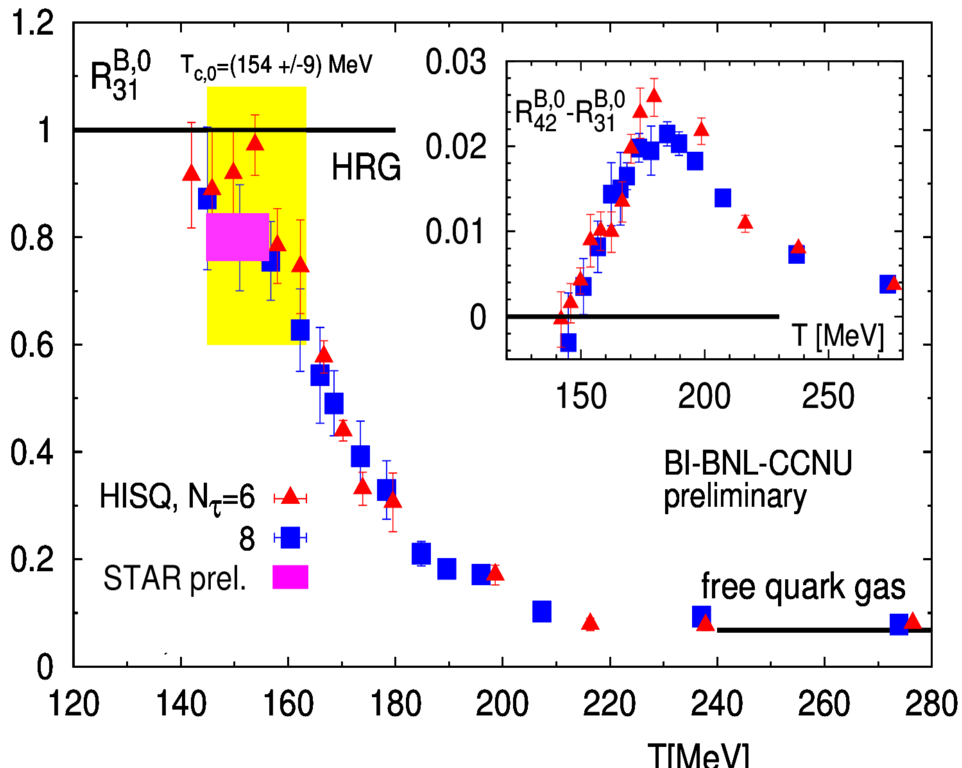
$R^B_{31} \equiv S_B \sigma^3_B / M_B$

$R^B_{31} = R^{B,0}_{31} + R^{B,2}_{31} \left(R^B_{12}\right)^2$

$R^B_{42} \equiv \kappa_B \sigma^2_B$

$R^B_{42} = R^{B,0}_{42} + R^{B,2}_{42} \left(R^B_{12}\right)^2$

$R^{B,0}_{42} \simeq R^{B,0}_{31}$



$$R^{B,2}_{42} = 3R^{B,2}_{31} = \frac{1}{2}\left(\frac{\chi^B_6}{\chi^B_2} - \left(\frac{\chi^B_4}{\chi^B_2}\right)^2\right)$$

choosing for simplicity:

$\mu_Q = \mu_S = \kappa^f_2 = 0$

$$R_{31}^{B} \equiv S_B \sigma_B^3 / M_B$$

$$R_{31}^{B} = R_{31}^{B,0} + R_{31}^{B,2} \left(R_{12}^{B}\right)^2$$

$$R_{42}^{B} \equiv \kappa_B \sigma_B^2$$

$$R_{42}^{B} = R_{42}^{B,0} + R_{42}^{B,2} \left(R_{12}^{B}\right)^2$$

$$R_{42}^{B,0} \simeq R_{31}^{B,0}$$

$$R_{42}^{B,2} = 3R_{31}^{B,2} = \frac{1}{2}\left(\frac{\chi_6^B}{\chi_2^B} - \left(\frac{\chi_4^B}{\chi_2^B}\right)^2\right)$$