Jet evolution in a dense QCD medium: wave turbulence and thermalization

Edmond Iancu
IPhT Saclay & CNRS

based on recent work by the Saclay collaboration

J.-P. Blaizot, F. Dominguez, M. Escobedo, Y. Mehtar-Tani, B. Wu

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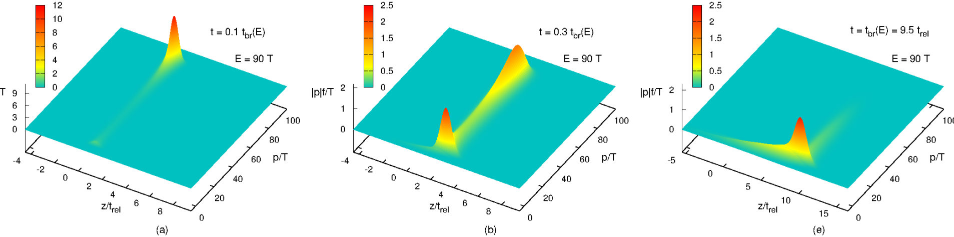
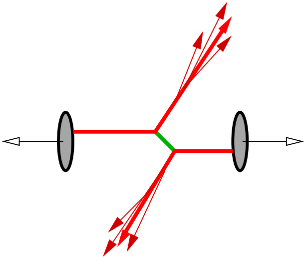
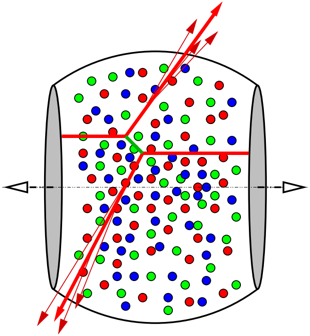


Jet quenching in heavy ion collisions

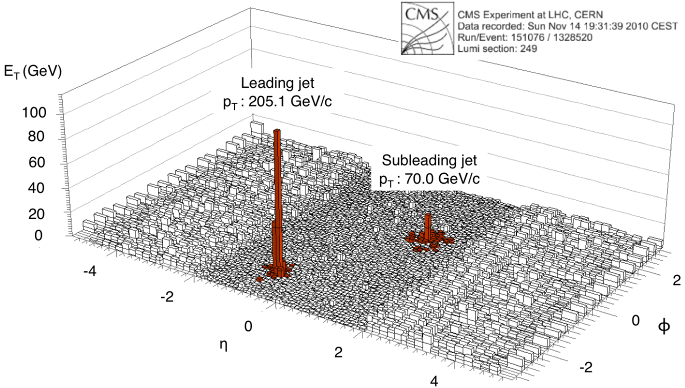
- Hard processes in QCD typically create pairs of partons which propagate back–to–back in the transverse plane
- In the vacuum, this leads to a pair of symmetric jets
- In a dense medium, the two jets can be differently affected by their interactions with the surrounding medium: 'di-jet asymmetry'





- The ensemble of medium-induced modifications: 'jet quenching'

Di–jet asymmetry at the LHC

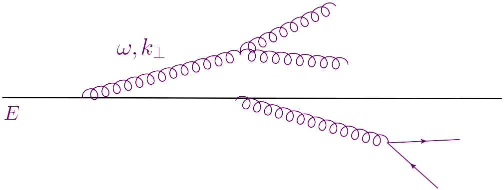


- Large energy imbalance between a pair of back-to-back jets
- Much larger than medium 'temperature' (average $p_\perp$): $T \sim 1$ GeV
- The 'missing energy' is found in the hemisphere of the subleading jet:

  ▷ many soft ($p_\perp < 2$ GeV) hadrons propagating at large angles

A challenge for the theorists

- Soft hadrons can be easily deviated towards large angles: elastic scatterings with the medium constituents
- Main question: how is that possible that a significant fraction of the jet energy be carried by its soft constituents ?
- Recall: bremsstrahlung in the vacuum ($\omega = xE$, $\theta \simeq k_\perp/\omega$)

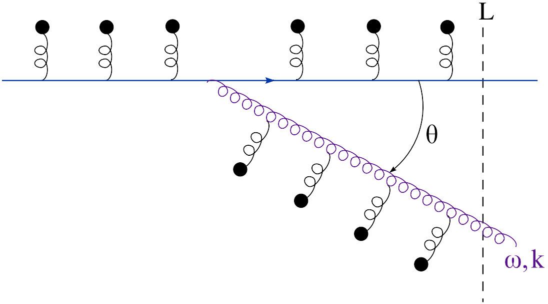


$$\mathrm{d}\mathcal{P} = \frac{\alpha_s C_R}{\pi} \frac{\mathrm{d}\omega}{\omega} \frac{\mathrm{d}k_\perp^2}{k_\perp^2} \simeq \frac{\alpha_s C_R}{\pi} \frac{\mathrm{d}x}{x} \frac{\mathrm{d}\theta^2}{\theta^2}$$

- many soft gluons ... which however carry very little energy
- asymmetric splittings $x \ll 1$: energy remains in the parent partons

Medium-induced radiation

- Additional radiation triggered by interactions in the medium: BDMPSZ



- Originally developed for a single gluon emission

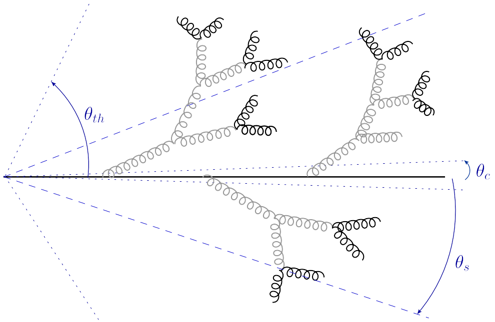
  Baier, Dokshitzer, Mueller, Peigné, and Schiff; Zakharov (96–97)
  Wiedemann (2000); Arnold, Moore, and Yaffe (2002–03); ...

- Sufficient for the relatively hard emissions which dominate the average energy loss by the leading particle (e.g. for computing R_{AA})

- However, hard emissions propagate at small angles ...

Medium-induced radiation

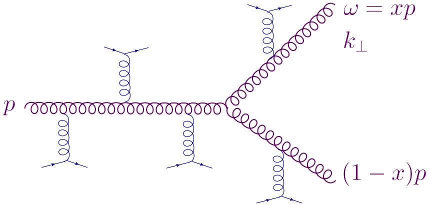
- Additional radiation triggered by interactions in the medium: BDMPSZ



- When gluons are sufficiently soft, multiple branching becomes important
  Blaizot, Dominguez, E.I., Mehtar-Tani (2012–13)
- Medium-induced gluons branchings are quasi-democratic ($x \sim 1/2$)
- Very efficient mechanism for energy transfer towards soft quanta: wave turbulence

The BDMPSZ mechanism *(cf. talk by Guy Moore)*

- Gluon emission is linked to transverse momentum broadening
- Uncertainty principle $\Longrightarrow$ gluon formation time $t_{\rm f}$



$$t_{\rm f} \simeq \frac{1}{\Delta E} \simeq \frac{\omega}{k_\perp^2}$$

- large when $k_\perp \to 0$

- $k_\perp$ cannot be arbitrarily small: it accumulates via collisions
  - independent multiple scattering $\Longrightarrow$ a random walk in $p_\perp$
  - during formation, the gluon acquires a momentum $k_\perp^2 \sim \hat{q} t_{\rm f}$

$$t_{\rm f} \simeq \frac{\omega}{k_\perp^2} \quad \& \quad k_\perp^2 \simeq \hat{q} t_{\rm f} \quad \Longrightarrow \quad t_{\rm f}(\omega) \simeq \sqrt{\frac{\omega}{\hat{q}}}$$

  - softer gluons are emitted faster: LPM effect

Democratic branchings

- Probability for emitting a gluon with energy $\geq \omega$ during a time L

$$\mathcal{P}(\omega, L) \simeq \alpha_s \frac{L}{t_{\rm f}(\omega)} \simeq \alpha_s L \sqrt{\frac{\hat{q}}{\omega}}$$

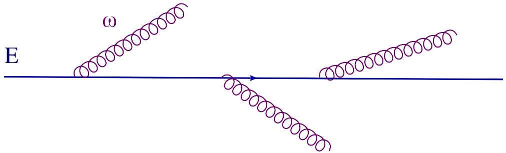
  - a function of the energy ω, not only of the splitting fraction x

- When $\mathcal{P}(\omega, L) \sim 1$, multiple branching becomes important

$$\omega \lesssim \omega_{\rm br}(L) \equiv \alpha_s^2 \hat{q} L^2 \quad \Longleftrightarrow \quad L \gtrsim t_{\rm br}(\omega) \equiv \frac{1}{\alpha_s}\sqrt{\frac{\omega}{\hat{q}}}$$

- LHC: the leading particle has $E \geq 100\,{\rm GeV} \gg \omega_{\rm br} \sim 10\,{\rm GeV}$

  - it abundantly emits soft $(x \ll 1)$ primary gluons with $\omega = xE \lesssim \omega_{\rm br}$

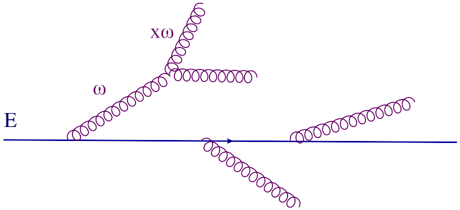


Democratic branchings

- Probability for emitting a gluon with energy $\geq \omega$ during a time L

$$\mathcal{P}(\omega, L) \simeq \alpha_s \frac{L}{t_{\rm f}(\omega)} \simeq \alpha_s L \sqrt{\frac{\hat{q}}{\omega}}$$

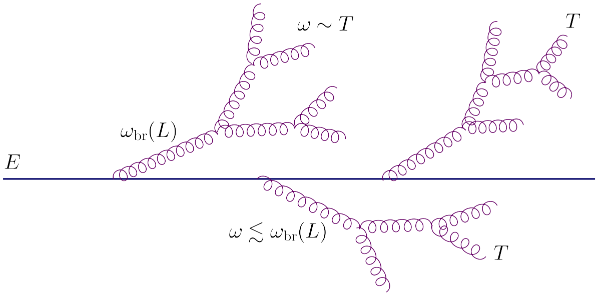
- Each such a primary gluon has a probability of $\mathcal{O}(1)$ to split again



- Their subsequent branchings are quasi-democratic: $x \sim 1/2$
  - when $\omega \lesssim \omega_{\rm br}$, $\mathcal{P}(x\omega) \sim 1$ independently of the value of x
  - daughter gluons are softer, so they disappear even faster
- $t_{\rm br}(\omega) \simeq$ the lifetime of the mini-jet initiated by ω

A typical jet event at the LHC

- Several primary gluons with energies $\omega \lesssim \omega_{\rm br}(L) \ll E$



- The primary gluons generate 'mini-jets' via democratic branchings
- Energy transmitted from primary gluons to soft gluons with $p \sim T$
- These soft gluons are expected to thermalize
- Energy loss at large angles is controlled by the hardest mini-jets : those initiated by primary gluons with $\omega \sim \omega_{\rm br}(L)$

Probabilistic picture

- Medium-induced jet evolution $\approx$ a Markovien stochastic process
  - $t_{\rm f} \sim \alpha_s t_{\rm br} \ll t_{\rm br}$: no overlap between successive branchings
  - interference phenomena could complicate the picture ...
    (in the vacuum, interference leads to angular ordering)
  - ... but they are suppressed by rescattering in the medium
    Mehtar-Tani, Salgado, Tywoniuk; Casalderrey-Solana, E. I. (10 –11)
    Blaizot, Dominguez, E.I., Mehtar-Tani (2012)
- Monte-Carlo studies: possible but generally complicated
  Schenke, Gale, Jeon (MARTINI, '09), based on AMY equations (03)
- Evolution equations for the gluon correlation functions

$$D(x,t) \equiv x \left\langle \frac{{\rm d}N}{{\rm d}x}(t) \right\rangle , \qquad D^{(2)}(x,x',t) \equiv xx' \left\langle \frac{{\rm d}N_{\rm pair}}{{\rm d}x\,{\rm d}x'}(t) \right\rangle$$

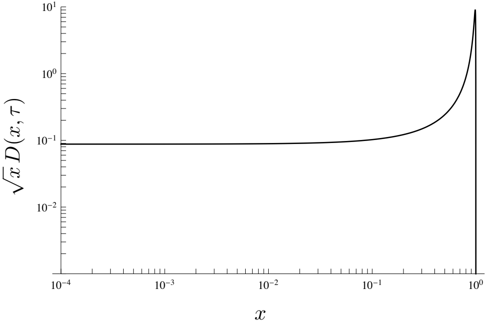
- Exact solutions for a pure branching dynamics (no elastic collisions)

Gluon spectrum

J.-P. Blaizot, E. I., Y. Mehtar-Tani, PRL 111, 052001 (2013)

$$D(x,\tau) = \frac{\tau}{\sqrt{x}(1-x)^{3/2}}\, e^{-\pi\frac{\tau^2}{1-x}}, \qquad x \equiv \frac{\omega}{E}, \quad \tau \equiv \frac{t}{t_{\rm br}(E)}$$

- Leading particle peak near $x=1$ and at early times $t \ll t_{\rm br}(E)$
- Power-law spectrum $D \propto 1/\sqrt{x}$ at $x \ll 1$
- The most interesting regime for the LHC: 'small-times' $\tau \sim 0.2 \div 0.4$

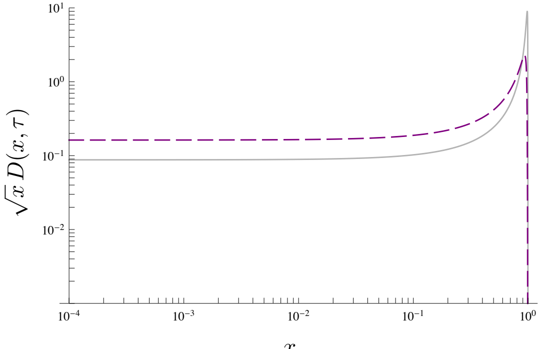


Early times: the LP peak

- Peak position: $1 - x_p \sim \pi\tau^2 \Longrightarrow$ Energy loss ΔE in a typical event
- Peak width: $\delta x \sim \pi\tau^2 \Longrightarrow$ LP broadening δE due to fluctuations

$$1 - x_p \sim \delta x \sim \pi\tau^2 \;\Longrightarrow\; \Delta E \sim \delta E \sim \alpha_s^2 \hat{q} t^2 = \omega_{\rm br}(t)$$

  - LP emits a number of $\mathcal{O}(1)$ primary gluons with energy $\omega_{\rm br}(t)$ each
  - these emissions are independent : fluctuations of $\mathcal{O}(1)$ as well



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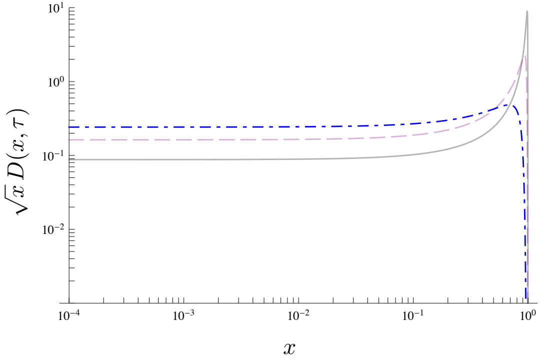
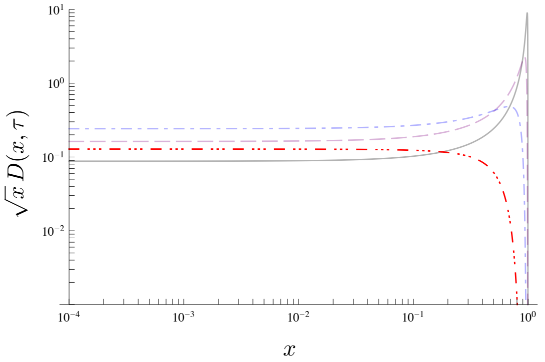
- Peak width: $\delta x \sim \pi\tau^2 \Longrightarrow$ LP broadening δE due to fluctuations
- Increasing t: the LP peaks decreases, broadens, and moves to the left
- When $t \sim t_{\rm br}(E)$, the LP disappears via democratic branching



Later times: the energy flow

- For $t > t_{\mathrm{br}}(E)$, the spectrum is suppressed at any x
- The energy flows out of the spectrum !
  - it actually happens at any τ, but easier to see when $\tau > 1$
  - formally, the energy accumulates into a condensate at $x = 0$
  - physically, it is expected to thermalize



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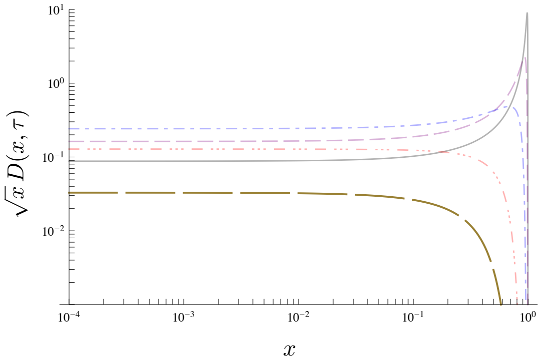

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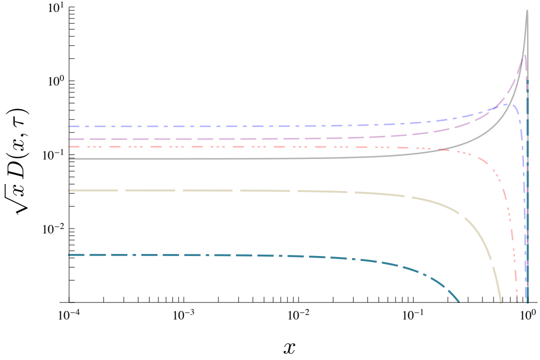
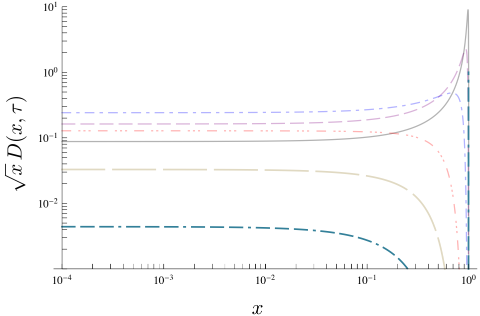


Wave turbulence

- The power-law behavior at $x \ll 1$ is never changing:

$$D(x,\tau) \simeq \frac{\tau}{\sqrt{x}}\, \mathrm{e}^{-\pi\tau^2}$$

- Kolmogorov-Zakharov fixed point: 'gain' = 'loss'
  - the energy flux associated with multiple branching is independent of x
  - via successive democratic branchings, the energy flows from one gluon generation to the next one, down to $x \sim T/E$



Energy loss at large angles

- The energy $\mathcal{E}$ which thermalizes ('accumulates at $x = 0$')

$$\int_0^1 \mathrm{d}x\, D(x,\tau) = \mathrm{e}^{-\pi\tau^2} \implies \langle\mathcal{E}\rangle = E\big(1 - \mathrm{e}^{-\pi\tau^2}\big)$$

- If $\tau \gtrsim 1$ (low energy jet/very large medium) : $\langle\mathcal{E}\rangle \sim E$
- For the LHC kinematics : $\tau \ll 1$, hence $\langle\mathcal{E}\rangle \ll E$
    - $\hat{q} \simeq 1\,\mathrm{GeV}^2/\mathrm{fm}\,, \quad L = 3 \div 6\,\mathrm{fm}\,, \quad E = 100\,\mathrm{GeV} \implies \tau \simeq 0.2 \div 0.4$

$$\langle\mathcal{E}\rangle \simeq \pi\tau^2 E = \pi\alpha_s^2 \hat{q} L^2 \simeq 14 \div 56\,\mathrm{GeV}$$

    - independent of the energy E of the leading particle
    - rapidly increasing with the medium size $\propto L^2$
    - carried by soft quanta ($x \simeq 0$) which propagate at large angles

Event-by-event fluctuations

M. Escobedo and E. I., arXiv:1601.03629 [hep-ph]

- Exact solution for the 2-point correlation (the gluon pair density)

$$D^{(2)}(x,x',\tau) = \frac{1}{2\pi}\frac{1}{\sqrt{xx'(1-x-x')}}\left[\mathrm{e}^{-\frac{\pi\tau^2}{1-x-x'}} - \mathrm{e}^{-\frac{4\pi\tau^2}{1-x-x'}}\right]$$

- The variance in the energy loss at large angles:

$$\sigma^2 \equiv \langle\mathcal{E}^2\rangle - \langle\mathcal{E}\rangle^2 \simeq \frac{\pi^2}{3}\,\omega_{\rm br}^2(L) = \frac{1}{3}\langle\mathcal{E}\rangle^2$$

- Large fluctuations: the dispersion σ comparable to the mean value $\langle\mathcal{E}\rangle$
  - fluctuations in the number of primary gluons with $p \sim \omega_{\rm br}(L)$

- Remarkable scaling properties:

$$\frac{\langle\mathcal{E}^2\rangle}{\langle\mathcal{E}\rangle^2} \simeq \frac{4}{3} \quad : \text{ independent of } E,\ L, \text{ and } \hat{q}$$

- Similar relations for the gluon number: Koba-Nielsen-Olesen scaling

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- Large fluctuations: the dispersion σ comparable to the mean value $\langle\mathcal{E}\rangle$
  - fluctuations in the number of primary gluons with $p \sim \omega_{\rm br}(L)$
- In experiments, L itself is a random quantity (geometry fluctuations)

$$\frac{\langle\mathcal{E}^2\rangle}{\langle\mathcal{E}\rangle^2} \simeq \frac{4}{3}\frac{\langle L^4\rangle}{\langle L^2\rangle^2} \quad : \text{constraint on the } L \text{ distribution}$$

- Additional constraints from the gluon multiplicities (via KNO scaling)

Thermalization of the soft constituents

- The jet constituents exchange energy and momentum with the medium via elastic collisions *(cf. talks by G. Moore and R. Rapp)*
- The soft jet constituents with $p \lesssim T$ are expected to thermalize
- Thermalization stops the branching process
  - detailed balance between splitting $(1 \to 2)$ and recombination $(2 \to 1)$
- Boltzmann equation with both elastic and inelastic collision terms
  Baier, Mueller, Schiff, Son '01 ('bottom-up'); Arnold, Moore, Yaffe, '03

$$\left(\frac{\partial}{\partial t} + \boldsymbol{v}\cdot\nabla_{\boldsymbol{x}}\right) f(t,\boldsymbol{x},\boldsymbol{p}) \;=\; \mathcal{C}_{\rm el}[f] + \mathcal{C}_{\rm br}[f]$$

  - $f(t,\boldsymbol{x},\boldsymbol{p})$: gluon occupation number (jet + medium)
- Used in studies of the QGP thermalization (homogeneity)
  Kurkela and Lu (2014); Kurkela and Zhu (2015)
- The jet problem is further complicated by its strong inhomogeneity

A longitudinal kinetic equation

E.I. and Bin Wu, arXiv:1506.07871

- A tractable, yet physically meaningful, equation can be obtained by projecting the dynamics along the longitudinal (jet) axis
  - the relevant time scales are controlled by the gluon energies

$$t_{\rm br}(\omega) = \frac{1}{\alpha_s}\sqrt{\frac{\omega}{\hat{q}}} \ll t_{\rm th}(\omega) \sim \frac{1}{\alpha_s^2 T \ln(1/\alpha_s)}\,\frac{\omega}{T} \quad \text{when } \omega \gg T$$

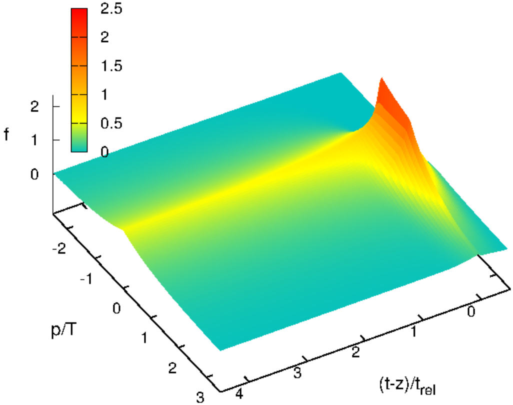
  - thermalization effects are irrelevant for the 'hard' ($\omega \gg T$) gluons
  - so long as $\omega \gg T$, transverse momenta are negligible: $\omega \simeq p_z \gg p_\perp$

$$\left(\frac{\partial}{\partial t} + v_z \frac{\partial}{\partial z}\right) f(t,z,p_z) = \frac{\hat{q}}{4}\,\frac{\partial}{\partial p_z}\left[\left(\frac{\partial}{\partial p_z} + \frac{v_z}{T}\right) f\right]$$

$$+\frac{1}{t_{\rm br}(p_z)} \int_{p_*} \frac{{\rm d}x}{[x(1-x)]^{\frac{3}{2}}}\left[\frac{1}{\sqrt{x}}\, f\left(\frac{p_z}{x}\right) - \frac{1}{2} f(p_z)\right]$$

- Parametrically correct down to $p_z \sim T$ (captures the right time scales)

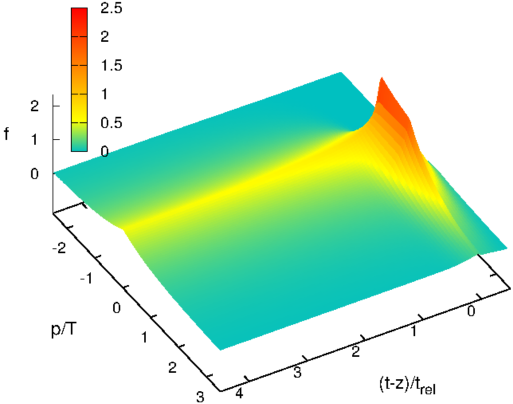
The front & the tail (exact solution)



- The branching process ≈ a source of gluons with $p = p_* \gtrsim T$

$$\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial z}\right) f = \frac{\hat{q}}{4}\frac{\partial}{\partial p}\left[\left(\frac{\partial}{\partial p} + \frac{v}{T}\right) f\right] + T\delta(t-z)\delta(p-p_*)$$

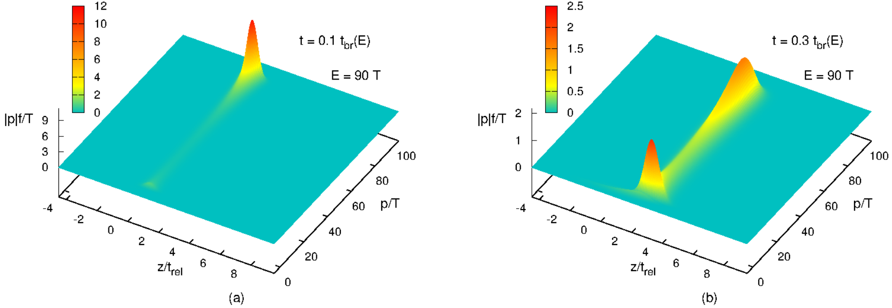
The front & the tail (exact solution)



- The 'front' $\propto \delta(t-z)$: gluons with $T \lesssim p < p_*$
  - gluons recently injected that had no time to thermalize
- The 'tail' at $z \lesssim t - t_{\rm th}$: $f_p \propto {\rm e}^{-|p|/T}$
  - gluons in thermal equilibrium with the medium

Numerical studies of the full dynamics (1)

- Parameters: $T = 0.5\,\text{GeV}$, $E = 90\,T$, $t_{\rm br}(E) \simeq 10\,\text{fm}$



- $t = 0.1\,t_{\rm br}(E) \simeq 1$ fm is representative for the 'leading jet' at the LHC
- $t = 0.3\,t_{\rm br}(E) \simeq 3$ fm : the 'subleading jet' (partially quenched)
- With increasing time, the jet substructure is softening and broadening

Numerical studies of the full dynamics (1)

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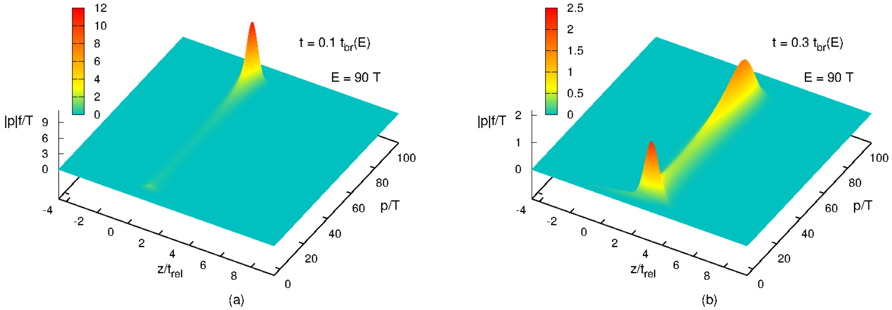
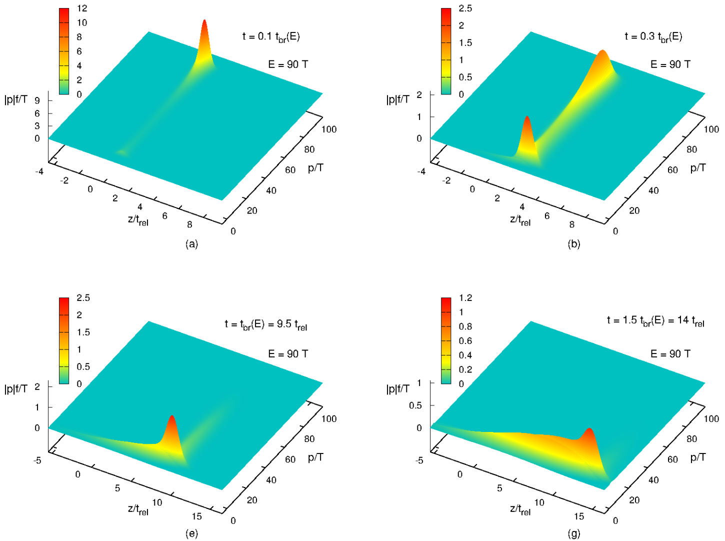


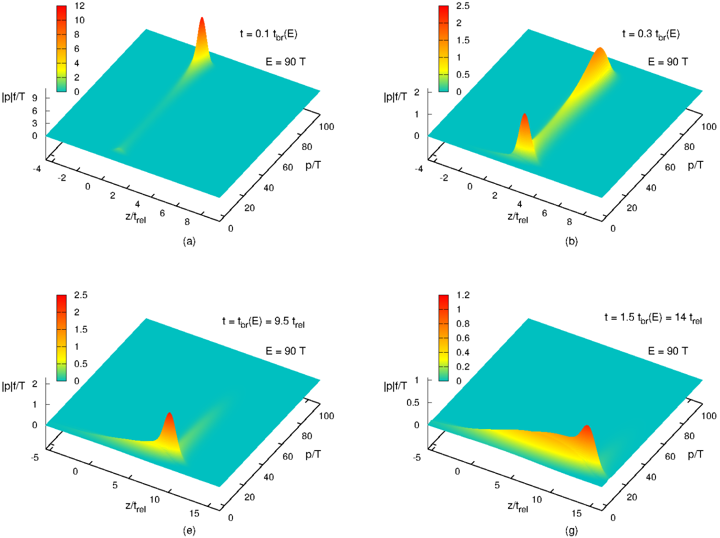
- $t = 0.3\,t_{\rm br}(E) \simeq 3\,\text{fm}$
  - the LP peak still visible around $p = E$
  - a second peak emerges near $p = T$ (branchings)
  - this second peak develops a thermalized tail at $z < t$ (collisions)

Numerical studies of the full dynamics (2)



- $t = t_{\rm br}(E)$: the leading particle disappears (democratic branching)

Numerical studies of the full dynamics (2)

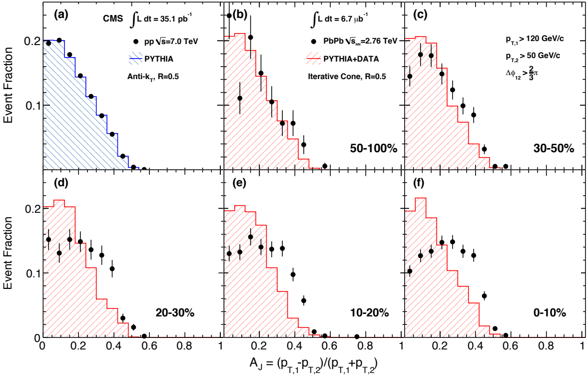


- $t = 1.5\, t_{\rm br}(E) \simeq 15$ fm : the jet is fully quenched

Conclusions

- Effective theory and physical picture for jet quenching from pQCD
  - event-by-event production of 'mini-jets' via democratic branchings
  - thermalization of the soft branching products with $p \sim T$
- Characteristic branching pattern, different from that in the vacuum
  - democratic branchings leading to wave turbulence
  - efficient transmission of energy to large angles
  - large event-by-event fluctuations, strong correlations
- Characteristic, 'front + tail', structure of the (partially) quenched jet
  - 'front' : the leading particle, but also soft gluons radiated at late times
  - 'tail' : soft partons in local thermal equilibrium with the medium
- Qualitative and semi-quantitative agreement with the phenomenology of di-jet asymmetry at the LHC

Di–jet asymmetry : $A_{\rm J}$ (CMS)

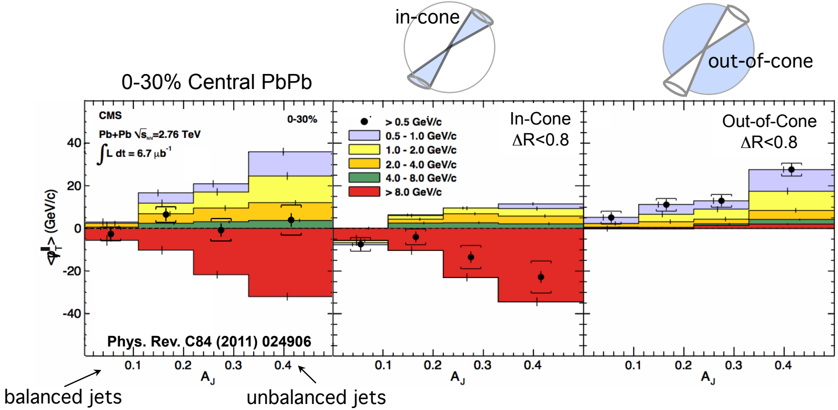


- Event fraction as a function of the di–jet energy imbalance in p+p (a) and Pb+Pb (b–f) collisions for different bins of centrality

$$A_{\rm J} = \frac{E_1 - E_2}{E_1 + E_2} \qquad (E_i \equiv p_{T,i} = \text{ jet energies})$$

- N.B. A pronounced asymmetry already in the p+p collisions !

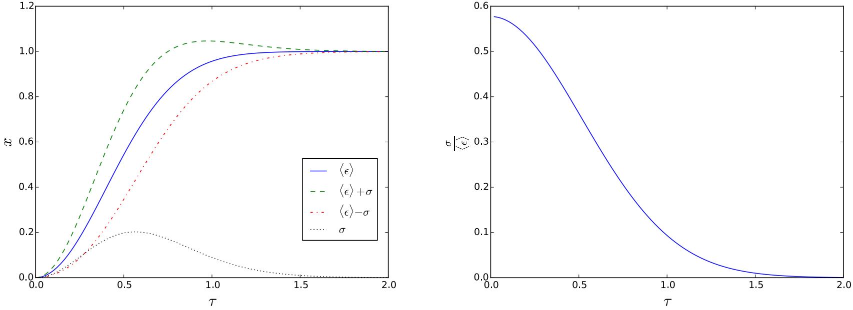
Energy imbalance @ large angles: $R = 0.8$



- No missing energy : $E_{\rm Lead}^{\rm in+out} = E_{\rm SubLead}^{\rm in+out}$
- In-Cone : $E_{\rm Lead}^{\rm in} > E_{\rm SubLead}^{\rm in}$: di-jet asymmetry, hard particles
- Out-of-Cone : $E_{\rm Lead}^{\rm out} < E_{\rm SubLead}^{\rm out}$: soft hadrons @ large angles

Koba-Nielsen-Olesen scaling

- Fractional energy loss at large angles $\langle\epsilon\rangle \equiv \langle\mathcal{E}\rangle/E$ and its dispersion



- Some typical values for $L = 4\,\text{fm}$: $10 < \mathcal{E} < 40\,\text{GeV},\ 3 < N(\omega_0) < 15$

- $N(\omega_0)$: # of gluons with $\omega \geq \omega_0$ (above: $\omega_0 = 0.5\,\text{GeV}$)

$$\langle N(\omega_0)\rangle \simeq 2\left[\frac{\omega_{\rm br}(L)}{\omega_0}\right]^{1/2}, \quad \frac{\langle N^2\rangle}{\langle N\rangle^2} \simeq \frac{3}{2}, \quad \frac{\langle N^p\rangle}{\langle N\rangle^p} \simeq C_p \quad : \text{KNO scaling}$$

Koba-Nielsen-Olesen scaling

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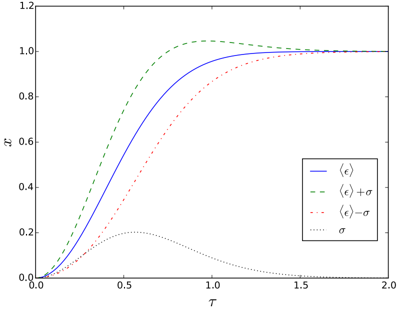

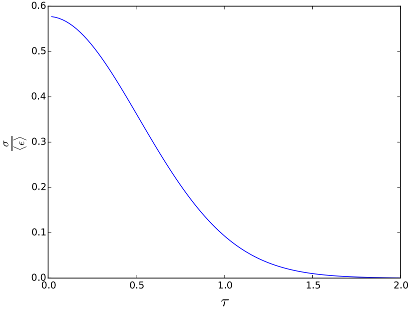

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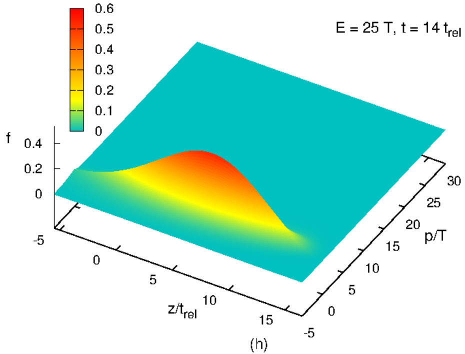
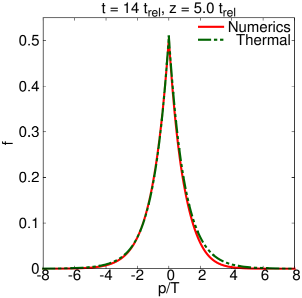
$$\frac{\langle N^2 \rangle}{\langle N \rangle^2} \simeq \frac{3}{2} \frac{\langle L^2 \rangle}{\langle L \rangle^2}$$: further constraints on the L distribution

The late stages: thermalizing a mini-jet

- At late times $t \gg t_{\rm br}(E)$, the jet is 'fully quenched'
  - no trace of the leading particle, just a thermalized tail
  - the typical situation for a mini-jet : $E \lesssim \omega_{\rm br}(L)$

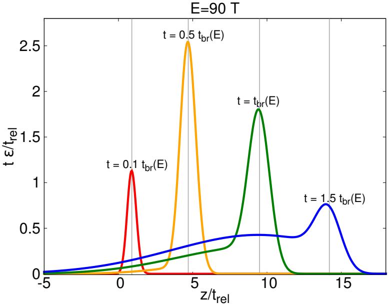


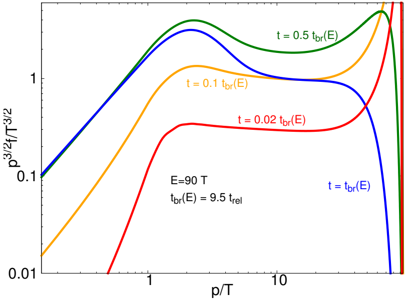


$$f(t,z,p) \simeq \mathrm{e}^{-\frac{|p|}{T}}\,\mathrm{e}^{-\frac{z^2}{4tt_{\rm th}}}$$

(spatial diffusion $\Longrightarrow$ hydro)

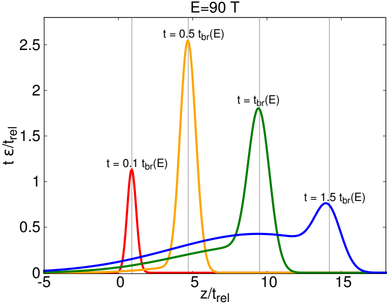
Energy distribution

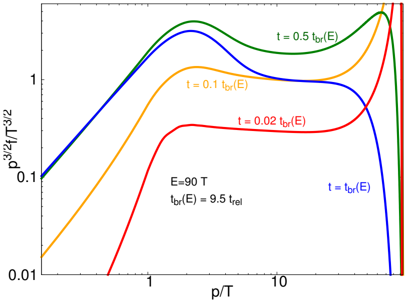




- Left: z-distribution of the energy density $\varepsilon(t,z) = \int \mathrm{d}p\, |p|\, f(t,z,p)$
  - even for $t = t_{\rm br}(E)$, most of the energy is still carried by the front ...
  - but the respective 'front' is mostly made with soft gluons $(p \sim T)$
  - branching products which did not yet have the time to thermalize
  - a thermalized tail is clearly visible at $z < t$

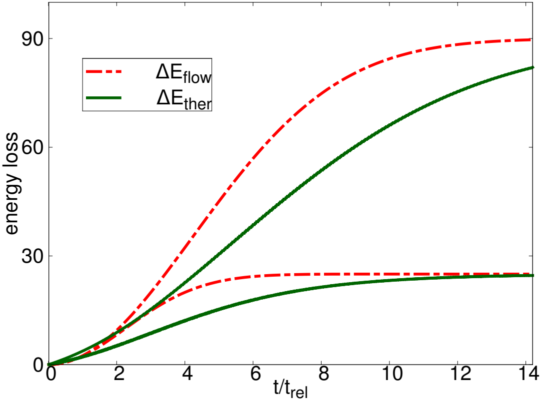
Energy distribution





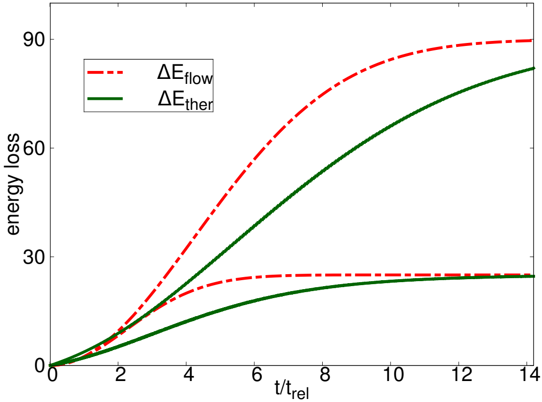
- Left: z-distribution of the energy density $\varepsilon(t,z) = \int \mathrm{d}p\,|p|\,f(t,z,p)$
- Right: p-distribution near the front: $f(t,z,p)$ for $z = t$
  - scaling window at $t \ll t_{\rm br}(E)$: $p^{3/2} f \approx$ const. $\Longrightarrow$ wave turbulence
  - for $t \gtrsim 0.5\,t_{\rm br}(E)$, some pile-up is visible around $p = T$
  - thermalization is efficient but certainly not instantaneous

Energy loss towards the medium



- Upper curves: $E = 90\,T$; lower curves: $E = 25\,T$
- $\Delta E_{\rm ther}$: the energy carried by the thermalized tail $(t - z \geq t_{\rm th})$
- $\Delta E_{\rm flow} = \pi\omega_{\rm br}(t) \propto t^2$: only branchings

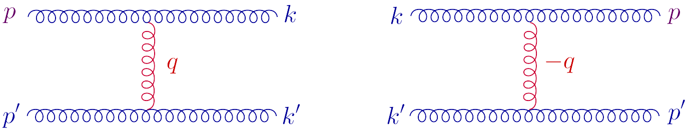
Energy loss towards the medium



- The energy carried by the thermalized tail is ...
  - slightly smaller than the 'flow' energy (medium is not a perfect sink)
  - ... but still substantial: $5 \div 15$ GeV for $L = 3 \div 6$ fm
  - ... and in the right ballpark to explain the LHC data ☺

Elastic collisions

- The usual Boltzmann collision term adapted to QCD



- 'Gain' and 'loss' terms: the gluon p is the one which is measured

$$\mathcal{C}_{\rm el}[f] \,=\, \int_{\boldsymbol{p}',\boldsymbol{k},\boldsymbol{k}'} \frac{|\mathcal{M}|^2}{(2p)(2p')(2k)(2k')}\,\Phi[f]$$

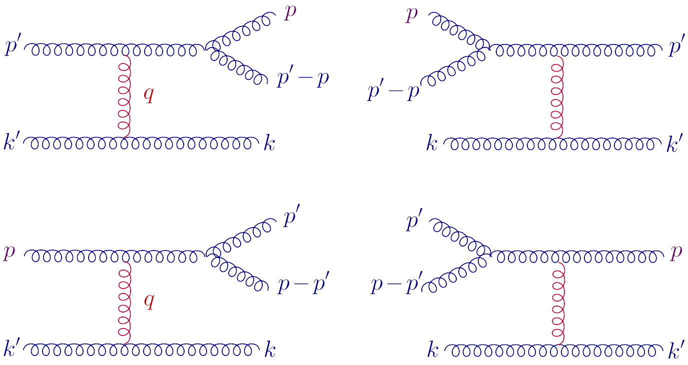
$$-\Phi[f] = f_{\boldsymbol{p}} f_{\boldsymbol{p}'}[1+f_{\boldsymbol{k}}][1+f_{\boldsymbol{k}'}] - f_{\boldsymbol{k}} f_{\boldsymbol{k}'}[1+f_{\boldsymbol{p}}][1+f_{\boldsymbol{p}'}]$$

- 5 conserved quantities: particle number and the 4–momentum
- Detailed balance $\Longrightarrow$ local thermal equilibrium: $\mathcal{C}_{\rm el}[f_{\rm loc}] = 0$

$$f_{\rm eq}(p) = \frac{1}{{\rm e}^{\beta(p-\mu)}-1} \;\rightarrow\; f_{\rm loc}(x,\boldsymbol{p}) = \frac{1}{{\rm e}^{\beta(x)[p-\boldsymbol{p}\cdot\boldsymbol{u}(x)-\mu(x)]}-1}$$

Inelastic collisions: Medium-induced branchings

- The prototype: $2 \to 3$ (single scattering)



- 'Gain' - recombination; 'loss' - recombination
- Particle number is obviously not conserved
- Fixed point: zero chemical potential: $f_{\rm loc}(x, \boldsymbol{p}) = \frac{1}{e^{\beta(x)[p - \boldsymbol{p}\cdot\boldsymbol{u}(x)]} - 1}$

An exact solution to the 1-D FP equation

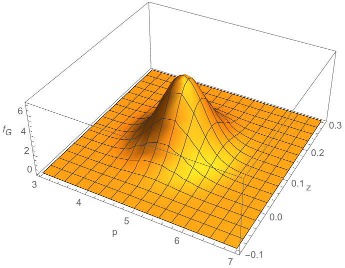
- The Green's function for ultrarelativistic FP in D=1+1+1

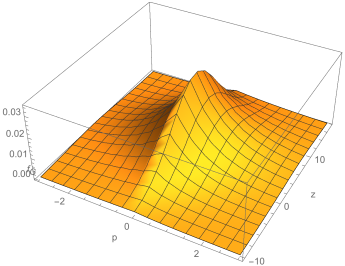
$$\left(\frac{\partial}{\partial t}+v\frac{\partial}{\partial z}\right)f=\frac{\hat{q}}{4}\frac{\partial}{\partial p}\left(\frac{\partial}{\partial p}+\frac{v}{T}\right)f\,,\qquad f(t=0,z,p)=\delta(z)\delta(p-p_0)$$

$$\begin{aligned}f(t,z,p>0)\;&=\frac{\mathrm{e}^{-\frac{p_0-p}{2}-\frac{t}{4}}}{2\sqrt{\pi t}}\left[\mathrm{e}^{-\frac{(p-p_0)^2}{4t}}-\mathrm{e}^{-\frac{(p+p_0)^2}{4t}}\right]\delta(t-z)\\&+\frac{\mathrm{e}^{-\frac{(p+p_0-z)^2}{4t}-p}}{8\sqrt{\pi}t^{5/2}}\left[t(t+2)-(p+p_0-z)^2\right]\times\\&\times\ \mathrm{erfc}\left(\frac{1}{2}\sqrt{t-\frac{z^2}{t}}\left(\frac{p+p_0}{t+z}-1\right)\right)\\&+\frac{(t+z)(p+p_0+t-z)}{4\pi t^2\sqrt{t^2-z^2}}\,\mathrm{e}^{-\frac{(p+p_0)^2}{2(t+z)}+\frac{p_0-p}{2}-\frac{t}{4}}\end{aligned}$$

Limiting behaviors

- Physics transparent at both small and large times: $t_{\rm drag}(p_0) = \frac{p_0}{T}\, t_{\rm th}$





- small times: $t_{\rm th} \lesssim t \ll t_{\rm drag}(p_0)$

$$f(t,z,p) \propto \mathrm{e}^{-\frac{(p-\langle p(t)\rangle)^2}{\hat{q}t}}\, \delta(t-z)$$

$$\langle p(t)\rangle = p_0 - \frac{t}{t_{\rm rel}}\, T$$

- energy loss & diffusion

- large times: $t > t_{\rm drag}(p_0) \gg t_{\rm th}$

$$f \simeq \mathrm{e}^{-\frac{|p|}{T}}\, \exp\left\{-\frac{\big(z-(p_0/T)t_{\rm th}\big)^2}{4tt_{\rm th}}\right\}$$

- equilibrium & spatial diffusion

- plots: $p_0 = 5T$, $t_1 = t_{\rm th}$, $t_2 = 20t_{\rm th}$

Formation time & emission angle

$$t_{\mathrm{f}}(\omega) \simeq \sqrt{\frac{\omega}{\hat{q}}} \qquad \& \qquad \theta_{\mathrm{f}}(\omega) \simeq \frac{\sqrt{\hat{q} t_{\mathrm{f}}}}{\omega} \sim \left(\frac{\hat{q}}{\omega^3}\right)^{1/4}$$

- This mechanism applies so long as

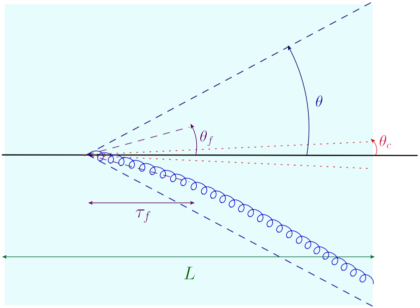
$$\lambda \ll t_{\mathrm{f}}(\omega) \leq L \implies T \ll \omega \leq \omega_c \equiv \hat{q} L^2$$

- Soft gluons : short formation times & large emission angles

$$\omega \ll \omega_c \implies t_{\mathrm{f}}(\omega) \ll L$$

$$\theta_{\mathrm{f}}(\omega) \gg \theta_c$$

$$\theta(\omega) \simeq \frac{\sqrt{\hat{q} L}}{\omega} \gg \theta_{\mathrm{f}}(\omega)$$



- The emission angle keeps increasing with time, via rescattering

Formation time & emission angle

$$t_{\rm f}(\omega) \simeq \sqrt{\frac{\omega}{\hat q}} \qquad \& \qquad \theta_{\rm f}(\omega) \simeq \frac{\sqrt{\hat q t_{\rm f}}}{\omega} \sim \left(\frac{\hat q}{\omega^3}\right)^{1/4}$$

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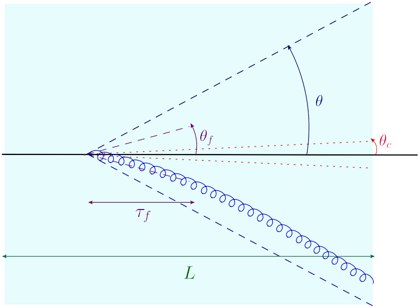
$$\theta_{\rm f}(\omega) \gg \theta_c$$

$$\theta(\omega) \simeq \frac{\sqrt{\hat q L}}{\omega} \gg \theta_{\rm f}(\omega)$$



- Emissions can effectively be treated as collinear