Anisotropic hydrodynamics

Wojciech Florkowski

UJK Kielce and IFJ PAN Krakow, Poland

International Workshop XLIV on Gross Properties of Nuclei and Nuclear Excitations
Hirschegg, Kleinwalsertal, Austria, January 17 - 23, 2016

1. Introduction

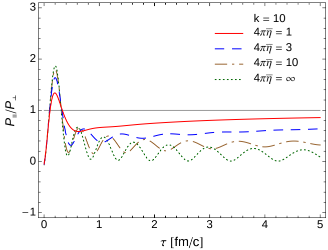
1.1 Plan

1.2 Motivation

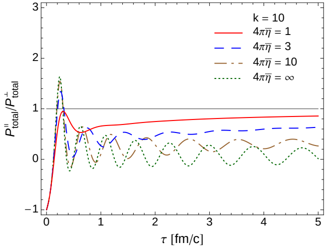
The results of microscopic models suggest that the initial state of matter produced in heavy-ion collisions is highly anisotropic in the momentum space (AdS/CFT, CGC, string models,...).

Color-flux-tube model results for different viscosity to entropy density ratios,
Ryblewski+WF, PRD 88, 034028 (2013)

ratio of pressures without the color field



ratio of pressures with the color field



1.2 Motivation

The viscous hydrodynamics is based on the gradient (Knudsen and inverse Reynolds numbers) expansion around the equilibrium state, this type of the expansion may be questioned in the situation where the space-time gradients are very large.

The goal of the anisotropic hydrodynamics program is to create a dissipative hydrodynamics framework that more accurately describes several features such as:

i) the early time dynamics of the QGP created in heavy-ion collisions
ii) dynamics near the transverse edges of the nuclear overlap region
iii) temperature-dependent (and potentially large) $\bar{\eta} = \eta/s$

2. Anisotropic hydrodynamics – an attempt of classification

2.1 Phenomenological vs. kinetic-theory formulations

Phenomenological formulation

R. Ryblewski, WF
PRC 83, 034907 (2011), JPG 38 (2011) 015104

1. energy-momentum conservation
   $\partial_\mu T^{\mu\nu} = 0$
2. ansatz for the entropy source, e.g.,
   $\partial(\sigma U^\mu) \propto (\lambda_\perp - \lambda_\parallel)^2/(\lambda_\perp \lambda_\parallel)$

Kinetic-theory formulation

M. Martinez, M. Strickland
NPA 848, 183 (2010), NPA 856, 68 (2011)

1. first moment of the Boltzmann equation = energy-momentum conservation
2. zeroth moment of the Boltzmann equation = specific form of the entropy source

3. Generalized form of the equation of state based on the Romatschke-Strickland (RS) form

generalization of equilibrium/isotropic distributions, frequently used in the studies of anisotropic quark-gluon plasma (here as a modified Boltzmann distribution in the local rest frame)

$$f_{RS} = \exp\left(-\sqrt{\frac{p_\perp^2}{\lambda_\perp^2} + \frac{p_\parallel^2}{\lambda_\parallel^2}}\right) = \exp\left(-\frac{1}{\lambda_\perp}\sqrt{p_\perp^2 + x\, p_\parallel^2}\right) = \exp\left(-\frac{1}{\Lambda}\sqrt{p_\perp^2 + (1+\xi)\, p_\parallel^2}\right)$$

anisotropy parameter $x = 1 + \xi = \left(\frac{\lambda_\perp}{\lambda_\parallel}\right)^2$ and transverse-momentum scale $\lambda_\perp = \Lambda$

2.1 Phenomenological vs. kinetic-theory formulations

4. Energy-momentum tensor (with single anisotropy parameter)

$$T^{\mu\nu} = (\varepsilon + P_\perp) U^\mu U^\nu - P_\perp g^{\mu\nu} - (P_\perp - P_\parallel) Z^\mu Z^\nu$$

$\varepsilon(\sigma, x)$ — energy density, $P_\perp(\sigma, x)$ — transv. pressure, $P_\parallel(\sigma, x)$ — long. pressure
alternatively one may use: $\varepsilon(\Lambda, \xi)$, $P_\perp(\Lambda, \xi)$, $P_\parallel(\Lambda, \xi)$

U — flow four-vector, Z — beam four-vector, $U^2 = 1$, $Z^2 = -1$, $U \cdot Z = 0$

this form of $T^{\mu\nu}$ follows from the covariant version of RS

$$f_{RS} = \exp\left(-\frac{1}{\Lambda}\sqrt{(p\cdot U)^2 + \xi\,(p\cdot Z)^2}\right), \quad U = (t/\tau, 0, 0, z/\tau), \quad Z = (z/\tau, 0, 0, t/\tau)$$

5. Early applications

5.1 (0+1)–D expansion with boost-invariance, transversely homogeneous matter
agreement with the Israel-Stewart second-order hydrodynamics has been demonstrated

5.2 (1+1)–D expansion without boost-invariance, transversely homogeneous matter

5.3 expansion in arbitrary number of dimensions, main physics result:
INSENSITIVITY OF HADRONIC OBSERVABLES TO INITIAL DYNAMICS
but: disagreement with Israel-Stewart for close-to-equilibrium situations for general expansion

2.2 Two expansion methods

Kinetic-theory formulation

Perturbative approach

Bazov, Heinz, Strickland
PRC 90, 044908 (2014)

$$f = f_{RS} + \delta f$$

- the leading order is still described by the Romatschke-Strickland form (accounting for the difference between the longitudinal and transverse pressures)
- advanced methods of traditional viscous hydrodynamics are used to restrict the form of the correction δf and to derive aHydro equations — non-trivial dynamics included in the transverse plane and, more generally, in (3+1)D case

Non-perturbative approach

Nopoush, Ryblewski, Strickland,Tinti, WF

$$f = f_{\mathrm{aniso}} + \dots$$

- all effects due to anisotropy included in the leading order, in the generalised RS form

1. (1+1)D conformal case, two anisotropy parameters
2. (1+1)D non-conformal case, two anisotropy parameters + one bulk parameter
3. full (3+1)D case, five anisotropy parameters + one bulk parameter (shear tensor and bulk pressure)

2.3 Non-perturbative approach

Boost-invariant and cylindrically symmetric expansion, (1+1)D non-perturbative approach

as much as possible, the momentum anisotropy is included in the leading order

$$f(x,p) = f_{\rm aniso} = f_{\rm iso}\left(\frac{\sqrt{p^\mu \Xi_{\mu\nu} p^\nu}}{\lambda}\right)$$

1. Conformal case, two anisotropy parameters
   L. Tinti, WF, Phys.Rev. C89 (2014) 034907

$$\Xi^{\mu\nu} = U^\mu U^\nu + \xi^{\mu\nu}$$
$$u_\mu \xi^{\mu\nu} = 0 \qquad \xi^\mu_\mu = 0$$
$$\xi^{\mu\nu} = \mathrm{diag}(0, \boldsymbol{\xi}) \qquad \boldsymbol{\xi} \equiv (\xi_x, \xi_y, \xi_z) \quad \text{(in the local rest frame)}$$

2. Non-conformal case, two anisotropy parameters + one bulk parameter
   M. Nopoush, R. Ryblewski, M. Strickland, Phys. Rev. C 90 (2014) 014908

$$\Xi^{\mu\nu} U^\mu U^\nu + \xi^{\mu\nu} - \Delta^{\mu\nu}\Phi$$
$$u_\mu \xi^{\mu\nu} = 0 \qquad u_\mu \Delta^{\mu\nu} = 0 \qquad \xi^\mu_\mu = 0 \qquad \Delta^\mu_\mu = 3$$
$$\xi^{\mu\nu} = \mathrm{diag}(0, \boldsymbol{\xi}) \qquad \boldsymbol{\xi} \equiv (\xi_x, \xi_y, \xi_z) \quad \text{(in the local rest frame)}$$

2.3 Non-perturbative approach

equations of motion for ξ_x, ξ_y, Φ, λ, T for (0+1)d case are obtained by taking moments of the Boltzmann equation in the relaxation time approximation

$$p^\mu \partial_\mu f = p^\mu \frac{u_\mu}{\tau_{\rm eq}}(f^{\rm eq} - f) \qquad \rightarrow \qquad \partial_{\mu_1} \int dP\, p^{\mu_1} \ldots p^{\mu_{n+1}} f = u_{\mu_1} \int dP\, p^{\mu_1} \ldots p^{\mu_n} \frac{1}{\tau_{\rm eq}}(f^{\rm eq} - f)$$

0th moment (1 eq.)

$$\partial_\mu N^\mu = \frac{u_\mu}{\tau_{\rm eq}}(N^\mu_{\rm eq} - N^\mu)$$

1st moment (2 eq.)

Landau matching condition for the energy

$$\begin{aligned} u_\nu \partial_\mu T^{\mu\nu} &= u_\nu \frac{u_\mu}{\tau_{\rm eq}}(T^{\mu\nu}_{\rm eq} - T^{\mu\nu}) \\ u_\mu T^{\mu\nu}_{\rm eq} &= u_\mu T^{\mu\nu} \end{aligned}$$

2nd moment (2 eq.)

$$X^i_\mu X^i_\nu \partial_\lambda \Theta^{\lambda\mu\nu} = X^i_\mu X^i_\nu \frac{u_\lambda}{\tau_{\rm eq}} \left(\Theta^{\lambda\mu\nu}_{\rm eq} - \Theta^{\lambda\mu\nu}\right)$$

two linear combinations of these equations with $i = 0, 1, 2, 3$

X, Y defined in addition to U and Z

...

2.3 Non-perturbative approach

3A. (3+1) dimensional framework for leading order anisotropic hydrodynamics
L. Tinti, Phys. Rev. C92 (2015) 014908

Testing different formulations of leading order anisotropic hydrodynamics
L. Tinti, R. Ryblewski, W. Florkowski, M. Strickland, Nucl. Phys. A946 (2016) 29

$$\Xi^{\mu\nu} = u^\mu u^\nu + \xi^{\mu\nu} - \Delta^{\mu\nu}\Phi$$
$$u_\mu \xi^{\mu\nu} = 0 \qquad \xi^\mu_\mu = 0 \quad \text{(5 parameters in } \xi^{\mu\nu}\text{)}$$

3B. Anisotropic matching principle for the hydrodynamic expansion, L. Tinti, arXiv:1506.07164

$$T^{\mu\nu} = \int dP\, p^\mu p^\nu f_{\rm aniso}(x,p) = \int dP\, p^\mu p^\nu f_{\rm iso}\left(\frac{\sqrt{p^\mu \Xi_{\mu\nu} p^\nu}}{\lambda}\right)$$

Instead of looking at the moments we can derive first the equations for the pressure corrections, following DNMR (Denicol, Niemi, Molnar, Rischke) strategy used for viscous hydrodynamics

This is the latest development for the leading order, that may be supplemented by NLO terms following the approach by Heinz et al.

3 Exact solutions of the Boltzmann equation

new technique: aHydro and viscous hydro predictions are checked agains exact solutions of the Boltzmann kinetic equation in the relaxation time approximation, important constraints on the structure of the hydro equations and the form of the kinetic coefficients

One dimensional expansion

Denicol, Maksymiuk, Ryblewski, Strickland, WF

1. conformal case
2. non-conformal case
3. non-conformal case with quantum statistics
4. mixtures

(1+1)D flow with Gubser symmetry

Denicol, Heinz, Martinez, Noronha, Strickland

3.1 Conformal case
Relaxation time approximation

- Boltzmann equation in the relaxation time approximation

$$p^\mu \partial_\mu f(x,p) = C[f(x,p)] \qquad C[f] = p^\mu U_\mu \, \frac{f^{\rm eq} - f}{\tau_{\rm eq}}$$

Bhatnagar, Gross, Krook, Phys. Rev. 94 (1954) 511

- background distribution (Boltzmann statistics)

$$f^{\rm eq} = \frac{g_s}{(2\pi)^3} \exp\left(-\frac{p^\mu U_\mu}{T}\right)$$

- boost-invariant variables (Bialas, Czyz)

$$w = t p_\parallel - zE \qquad\qquad v = tE - z p_\parallel$$

- for transversely homogeneous boost-invariant system

$$\frac{\partial f}{\partial \tau} = \frac{f^{\rm eq} - f}{\tau_{\rm eq}}$$

$$f^{\rm eq}(\tau, w, p_\perp) = \frac{g_s}{(2\pi)^3} \exp\left(-\frac{\sqrt{w^2 + (m^2 + p_\perp^2)\tau^2}}{T\tau}\right)$$

3.1 Conformal case
Formal solution

- formal solution (generalization of Baym's result)

$$f(\tau, w, p_\perp) = D(\tau, \tau_0) f_0(w, p_\perp) + \int\limits_{\tau_0}^{\tau} \frac{d\tau'}{\tau_{\rm eq}(\tau')} D(\tau, \tau') f^{\rm eq}(\tau', w, p_\perp)$$

$$D(\tau_2, \tau_1) = \exp\left[- \int\limits_{\tau_1}^{\tau_2} \frac{d\tau''}{\tau_{\rm eq}(\tau'')} \right]$$

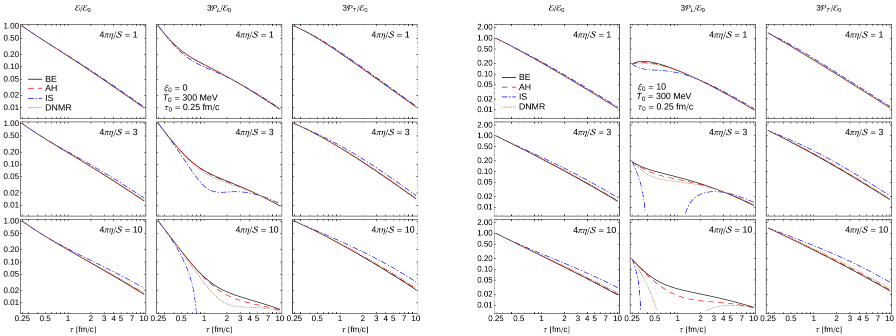
- initial condition (Romatschke-Strickland form)

$$f_0(w, p_\perp) = \frac{g_s}{(2\pi)^3} \exp\left[- \frac{\sqrt{(1+\xi_0) w^2 + (m^2 + p_\perp^2)\tau_0^2}}{\Lambda_0 \, \tau_0} \right]$$

$\xi_0 = \xi(\tau_0)$ - initial value of the anisotropy parameter
$\Lambda_0 = \Lambda(\tau_0)$ - initial transverse-momentum scale

3.1 Conformal case
Comparisons

WF, R. Ryblewski, M. Strickland, Phys.Rev. C88 (2013) 024903
$m = 0$, boost-invariant, transversally homogeneous system, (0+1) case

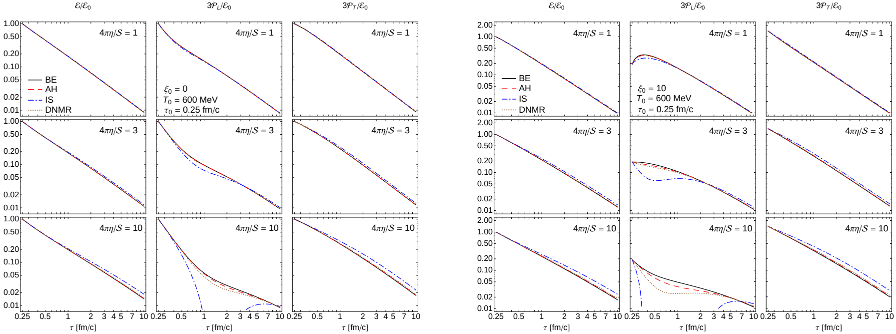


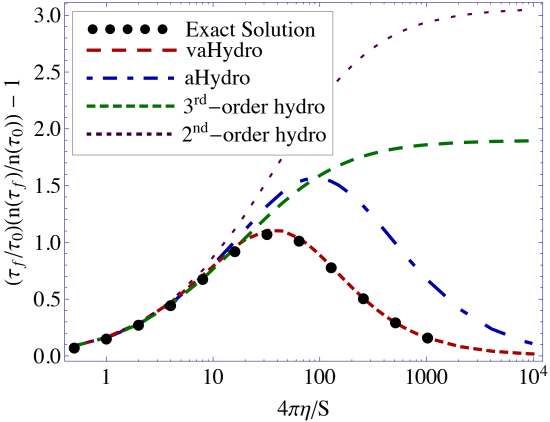
3.1 Conformal case

Further comparisons

WF, R. Ryblewski, M. Strickland, Phys. Rev. C88 (2013) 024903

$m = 0$, boost-invariant, transversally homogeneous system, (0+1) case





D. Bazow, U. W. Heinz, and M. Strickland, Phys.Rev. C90, 044908 (2014)

anisotropic hydro reproduces two limts: perfect fluid ($\bar{\eta} \to 0$) and free streaming ($\bar{\eta} \to \infty$)

3.2 Non-conformal case

Various, second order hydro equations for the bulk pressure

$$\tau_\Pi \dot{\Pi} + \Pi = -\frac{\zeta}{\tau} - \frac{1}{2}\tau_\Pi \Pi \left[\frac{1}{\tau} - \left(\frac{\dot{\zeta}}{\zeta} + \frac{\dot{T}}{T}\right)\right] \qquad (A)$$

Muronga, Phys. Rev. C69 (2004) 034903
Heinz, Song, Chaudhuri, Phys. Rev. C73 (2006) 034904

$$\tau_\Pi \dot{\Pi} + \Pi = -\frac{\zeta}{\tau} - \frac{4}{3}\tau_\Pi \Pi \frac{1}{\tau} \qquad (B)$$

Jaiswal, Bhalerao, Pal, Phys. Rev. C87 (2013) 021901

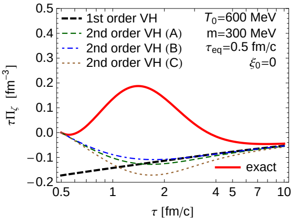
$$\tau_\Pi \dot{\Pi} + \Pi = -\frac{\zeta}{\tau} \qquad (C)$$

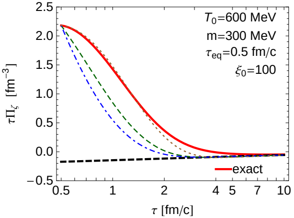
Heinz, Song, Chaudhuri, Phys. Rev. C73 (2006) 034904

3.2 Non-conformal case

Kinetic-theory vs. hydro

WF, E. Maksymiuk, R. Ryblewski, M.Strickland, Phys.Rev. C89 (2014) 054908





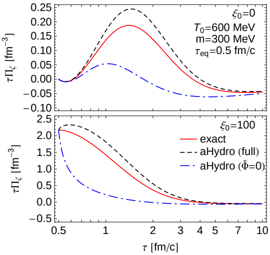
exact solution and all 2nd order viscous hydrodynamics variations tend toward the 1st order solution at late times

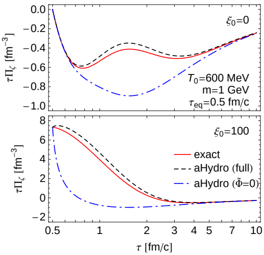
none of the 2nd order viscous hydrodynamics variations seems to qualitatively describe the early-time evolution of the bulk viscous pressure in all cases

there is something incomplete in the manner in which 2nd order viscous hydrodynamics treats the bulk pressure (neglected shear–bulk coupling)

3.2 Non-conformal case
aHydro predictions

Bulk viscous pressure evolution within LO anisotropic hydrodynamics
M. Nopoush, R. Ryblewski, M.Strickland, Phys. Rev. C 90, 014908 (2014)





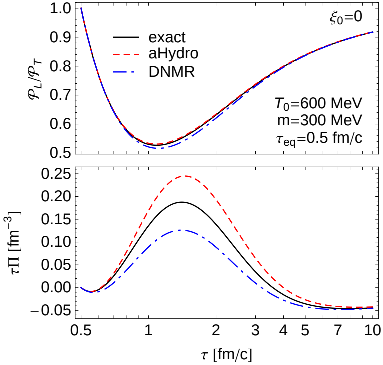
allowing for the bulk degree of freedom significantly improves agreement between anisotropic hydrodynamics and the exact solution

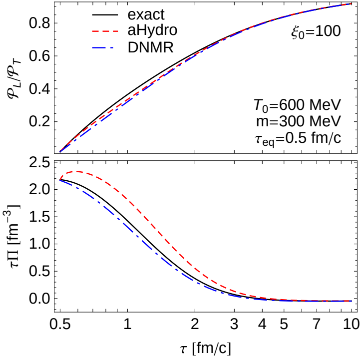
kinetic coefficients implicitly included

3.2 Non-conformal case

Viscous hydro predictions with shear-bulk coupling

Bulk viscous pressure evolution within full viscous anisotropic hydrodynamics with SHEAR-BULK COUPLING
G. Denicol, S. Jeon, C. Gale, Phys.Rev. C90 (2014) 024912
G. Denicol, R. Ryblewski, WF, M.Strickland, Phys.Rev. C90 (2014) 044905

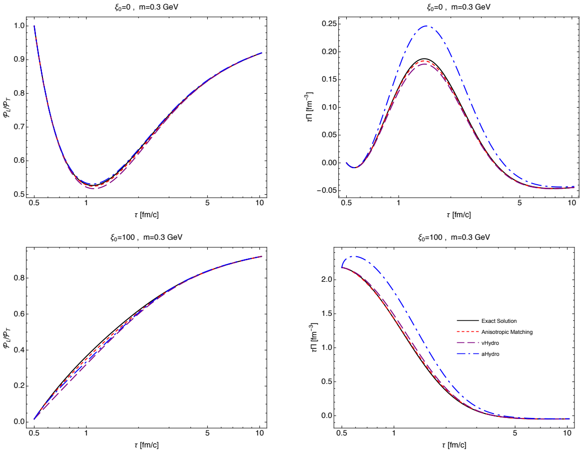




the shear-bulk couplings are extremly important for correct description of the bulk viscous correction

3.2 Non-conformal case
Anisotropic matching principle

Leonardo Tinti, arXiv:1506.07164



4. Summary and conclusions

4. Summary and conclusions

- A new framework of ANISOTROPIC HYDRODYNAMICS (aHydro) has been introduced. At the moment there exist two complementary formulations of aHydro based on the kinetic theory, and a phenomenological approach based on the entropy production form.
- Comparisons of aHydro and viscous hydrodynamics with the exact solutions of the kinetic theory confirmed usefulness of aHydro and gave information about the structure of hydrodynamic equations and the form of the kinetic coefficients (e.g., SHEAR-BULK COUPLING)
- aHydro may be matched with the microscopic models of early stages (as soon as the two pressures become positive), removes negative pressures, reproduces right limits for the entropy production, ...

4. Summary and conclusions

more work has been done so far:

- mixtures of anisotropic fluids, interacting quark and gluon systems with different anisotropies
- comparisons with the Gubser flow, exact solutions of the Boltzmann equation with the transverse flow included
- NLO corrections
- inclusion of the medium dependent masses to reproduce QCD EOS
- photon, dilepton production from early anisotropic phase
- bottomonium suppresion
- ...

5. Back-up slides

Dissipative hydrodynamics

the system is never in local equilibrium (diffusion, friction, heat conduction,...)

- energy-momentum conservation

$$\partial_\mu T^{\mu\nu}_{vis} = 0 \qquad T^{\mu\nu}_{vis} = \mathcal{E} u^\mu u^\nu - \Delta^{\mu\nu}(\mathcal{P} + \Pi) + \pi^{\mu\nu}$$

- number of equations: 5 + 6 ($\mathcal{E}$, $\mathcal{P}$, u^μ(3), Π, $\pi^{\mu\nu}$(5))
- number of equations: 4 + 1 (equation of state $\mathcal{E}(\mathcal{P})$)
- we need 6 extra equations - different methods possible

$$\dot{\Pi} + \frac{\Pi}{\tau_\Pi} = -\beta_\Pi \theta - \delta_{\Pi\Pi} \Pi \theta + \lambda_{\Pi\pi} \pi^{\mu\nu} \sigma_{\mu\nu}$$

$$\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_\pi} = 2\beta_\pi \sigma^{\mu\nu} + 2\pi_\gamma^{\langle\mu} \omega^{\nu\rangle\gamma} - \delta_{\pi\pi} \pi^{\mu\nu} \theta - \tau_{\pi\pi} \pi_\gamma^{\langle\mu} \sigma^{\nu\rangle\gamma} + \lambda_{\pi\Pi} \Pi \sigma^{\mu\nu}$$

Israel-Stewart equations

$\tau_\Pi \beta_\Pi = \zeta \rightarrow$ bulk viscosity, $\tau_\pi \beta_\pi = \eta \rightarrow$ shear viscosity

Dissipative hydrodynamics

the system is never in local equilibrium (diffusion, friction, heat conduction,...)

- energy-momentum conservation

$$\partial_\mu T^{\mu\nu}_{vis} = 0 \qquad T^{\mu\nu}_{vis} = \mathcal{E} u^\mu u^\nu - \Delta^{\mu\nu}(\mathcal{P} + \Pi) + \pi^{\mu\nu}$$

- number of equations: 5 + 6 ($\mathcal{E}$, $\mathcal{P}$, u^μ(3), Π, $\pi^{\mu\nu}$(5))
- number of equations: 4 + 1 (equations of state $\mathcal{E}(\mathcal{P})$)
- we need 6 extra equations - different methods possible

$$\begin{aligned}
\dot{\Pi} + \frac{\Pi}{\tau_\Pi} &= -\beta_\Pi \theta - \delta_{\Pi\Pi}\Pi\theta \underline{+ \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu}} \\
\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_\pi} &= 2\beta_\pi \sigma^{\mu\nu} + 2\pi_\gamma^{\langle\mu}\omega^{\nu\rangle\gamma} - \delta_{\pi\pi}\pi^{\mu\nu}\theta \underline{- \tau_{\pi\pi}\pi_\gamma^{\langle\mu}\sigma^{\nu\rangle\gamma} + \lambda_{\pi\Pi}\Pi\sigma^{\mu\nu}}
\end{aligned}$$

New approaches (shear-bulk coupling $\eta - \zeta$)

$\tau_\Pi \beta_\Pi = \zeta \rightarrow$ bulk viscosity, $\tau_\pi \beta_\pi = \eta \rightarrow$ shear viscosity