Ab-Initio Description of p-Shell Hypernuclei

Hirschegg 2015

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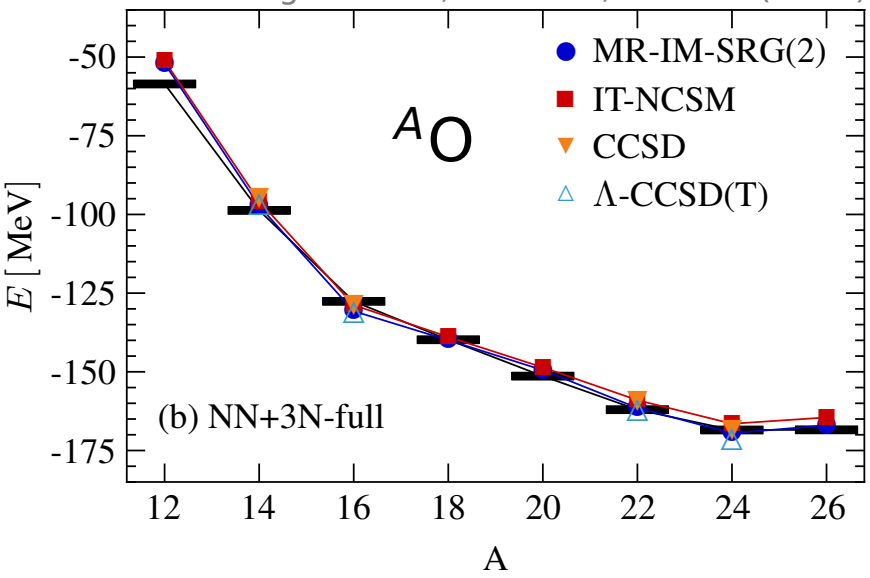
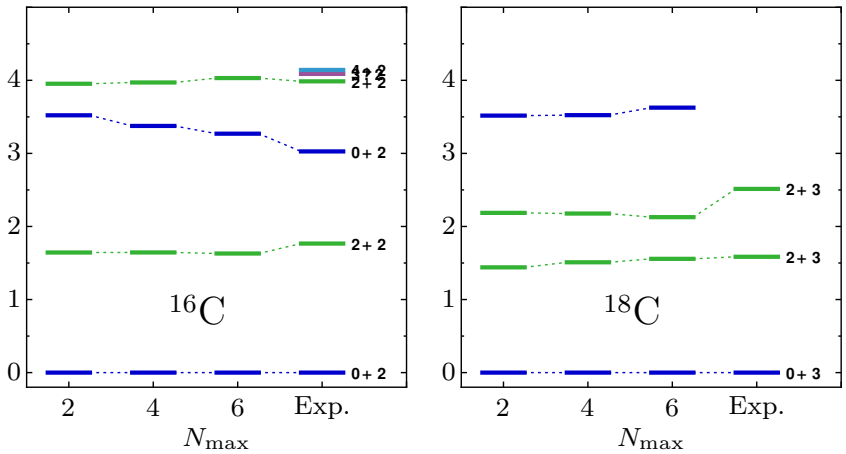
TECHNISCHE
UNIVERSITÄT
DARMSTADT

Motivation

Hergert *et al.*, PRL **110**, 242501 (2013)





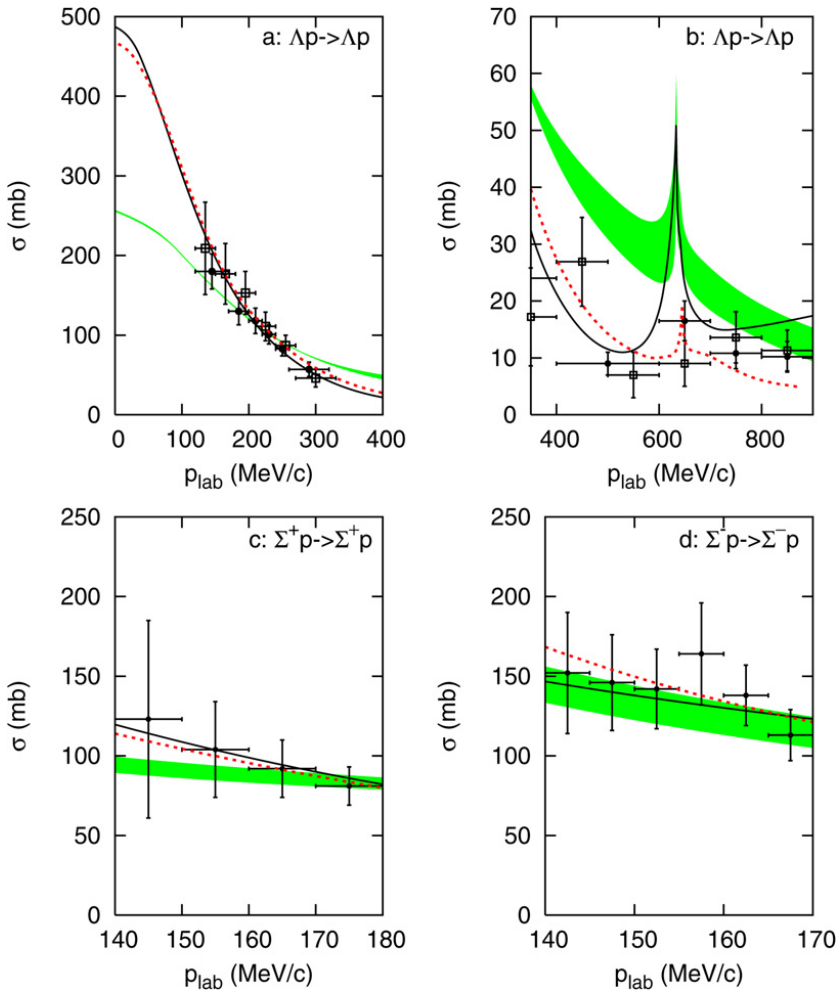
Roth *et al.*, Few-Body Syst. **55**, 659 (2014)

Status of *ab initio* nuclear structure

- Accurate NN+3N Hamiltonians from chiral EFT
- Unitary transformations for converged results
- Many-body methods with controlled uncertainties
- Access to binding energies, spectra, radii, transitions...

Extension to hypernuclei?

Hyperon-Nucleon Interaction



- Weaker than NN ($\sim 1\,\mathrm{MeV}/N$)
- No YN bound state
- $\Lambda N \leftrightarrow \Sigma N$ conversion
- Spin singlet—triplet transition

Meson-exchange models and chiral EFT interactions available

Polinder *et al.*, Nucl. Phys. A **779**, 244 (2006)

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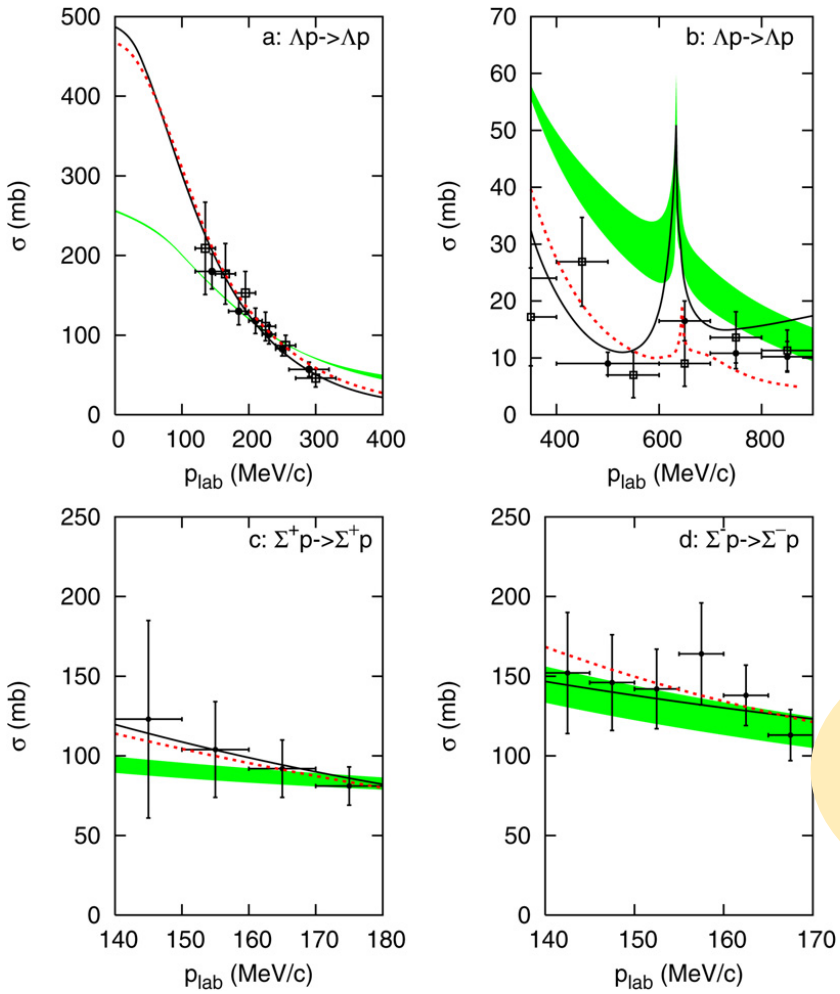

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Meson-exchange models and chiral EFT interactions available

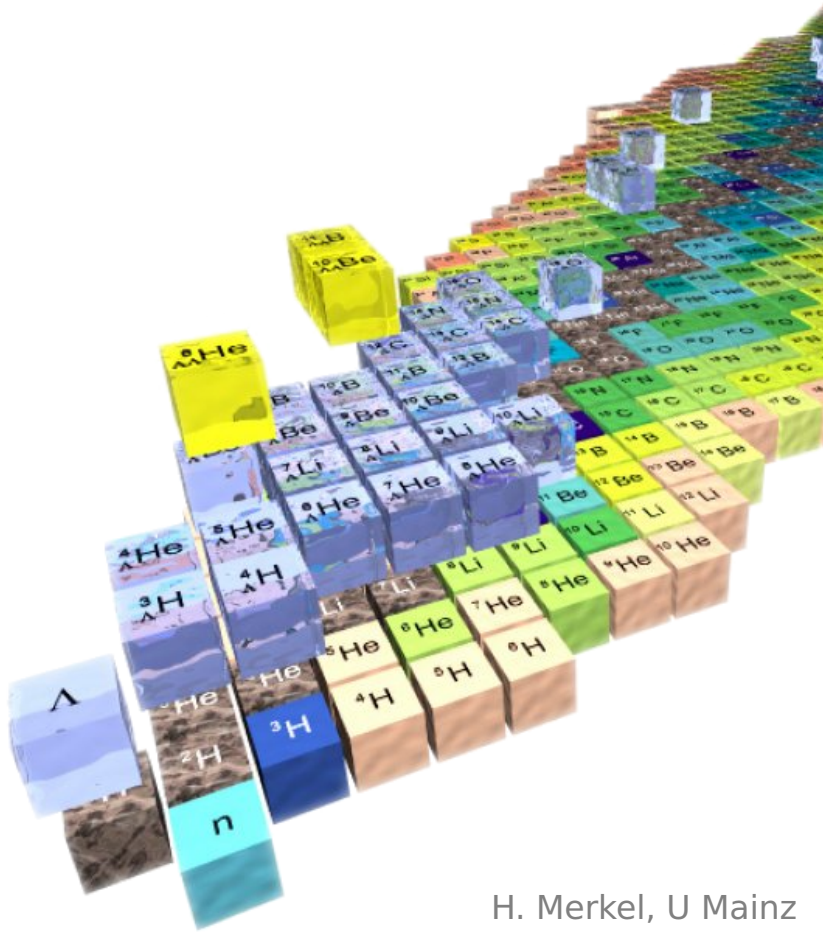
YN scattering experiments are challenging
⇒ Few data points
⇒ Large error bars

Polinder *et al.*, Nucl. Phys. A **779**, 244 (2006)

Necessary Developments

Extend No-Core Shell Model to hypernuclei

- Account for different particle masses
- Include fully active Λ and Σ
  ⇒ Coupled-channel problem
- Adapt unitary transformation framework



H. Merkel, U Mainz

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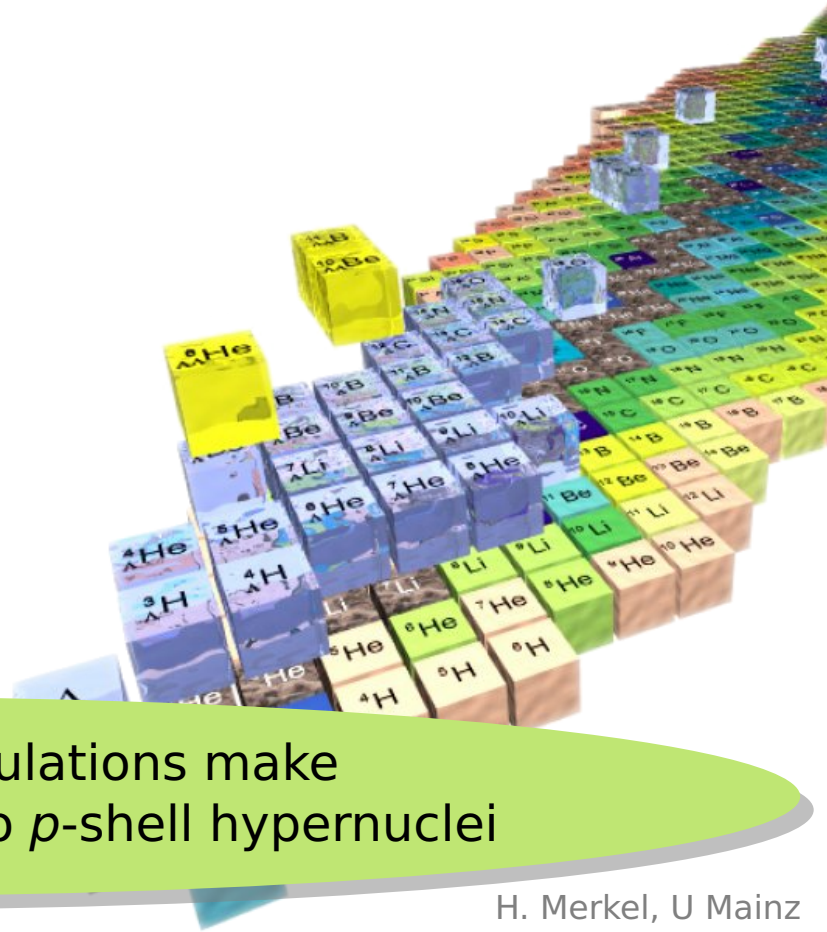

Ab-initio many-body calculations make connection from YN interactions to *p*-shell hypernuclei

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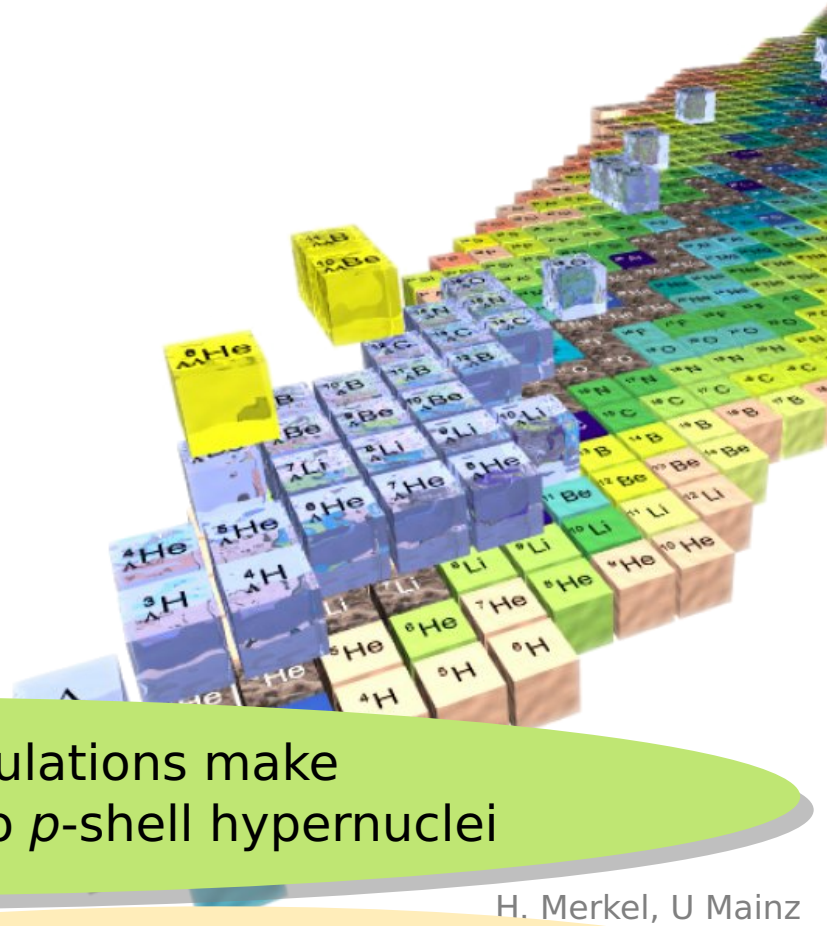


Ab-initio many-body calculations make connection from YN interactions to *p*-shell hypernuclei

H. Merkel, U Mainz

"→" prediction of observables
"←" constraints on YN interactions

Hypernuclear Hamiltonian

$$\boldsymbol{H} = \Delta\boldsymbol{M} + \boldsymbol{T}_{\text{int}} + \boldsymbol{V}_{\text{NN}} + \boldsymbol{V}_{\text{3N}} + \boldsymbol{V}_{\text{YN}}$$

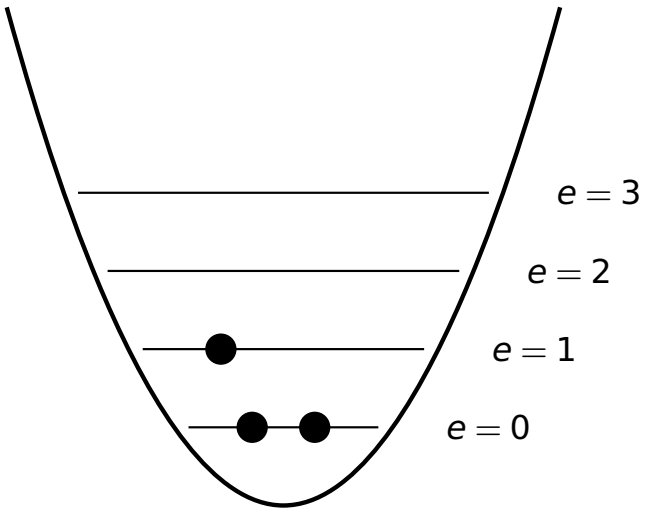
- NN: chiral N^{3}LO
  Entem & Machleidt
  Phys. Rev. C **68**, 041001(R) (2003)
  $\Lambda_{\text{NN}} = 500\,\text{MeV}$
- 3N: chiral N^{2}LO
  Navrátil
  Few-Body Syst. **41**, 117 (2007)
  $\Lambda_{\text{3N}} = 500\,\text{MeV}$

- YN: chiral LO
  Polinder, Haidenbauer & Meißner
  Nucl. Phys. A **779**, 244 (2006)
  $\Lambda_{\text{YN}} = 600\,\text{MeV}, 700\,\text{MeV}$

  OR Jülich'04
  Haidenbauer & Meißner
  Phys. Rev. C **72**, 044005 (2005)
  meson-exchange

NN+3N yields quantitative description of *p*-shell nuclei

Importance-Truncated No-Core Shell Model



^{6}Li proton orbits

- A-body Slater determinants from HO states

$$|s_1 s_2 \cdots s_A\rangle\,, \quad s_i \equiv |e(l\tfrac{1}{2})j\chi\rangle_i$$

- Λ-Σ conversion, e.g.

$$|pn\Lambda\rangle\,, |pp\Sigma^-\rangle\,, |nn\Sigma^+\rangle \in \mathcal{M}({}^3_\Lambda H)$$

- Impose $N_{\max}$ truncation
- Importance truncation: discard irrelevant states + *a posteriori* extrapolation
- Diagonalize Hamilton matrix ⇒ Energies & wave functions

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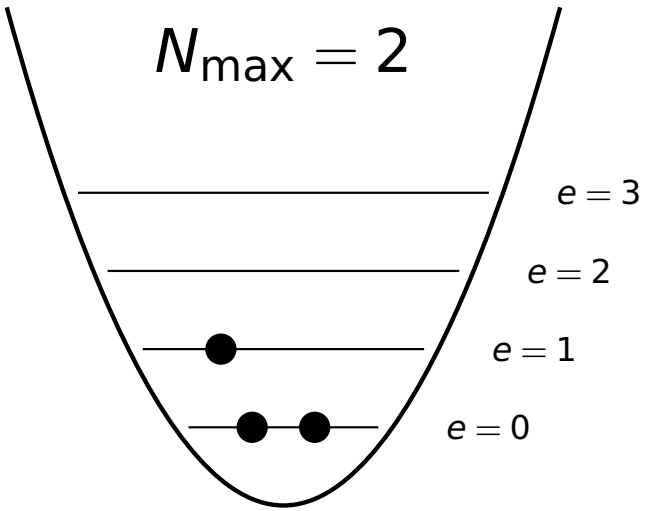

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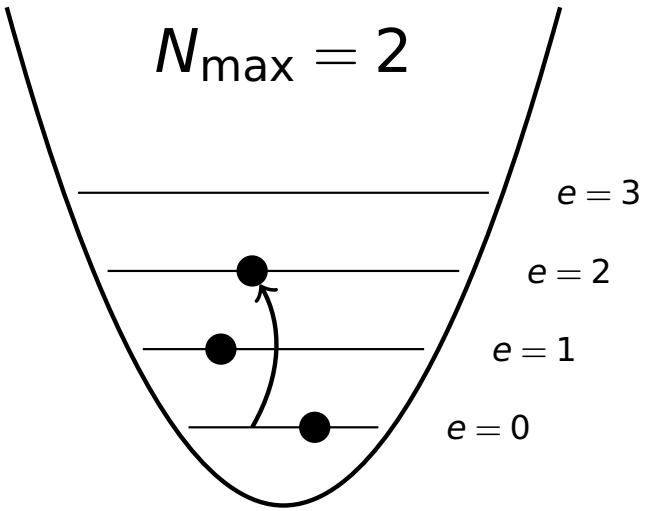

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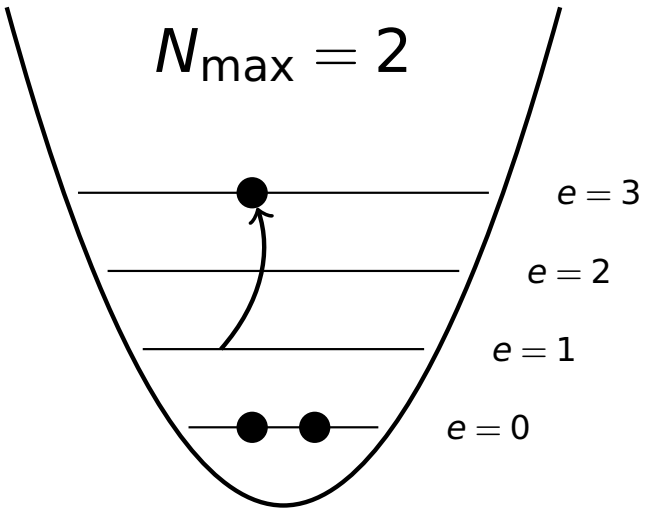

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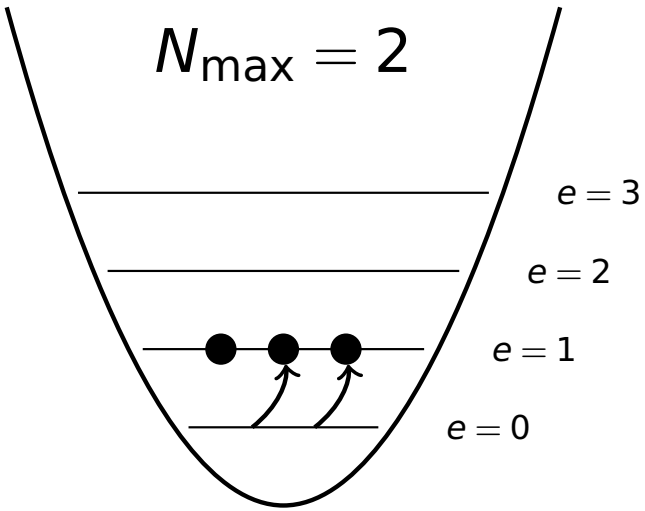

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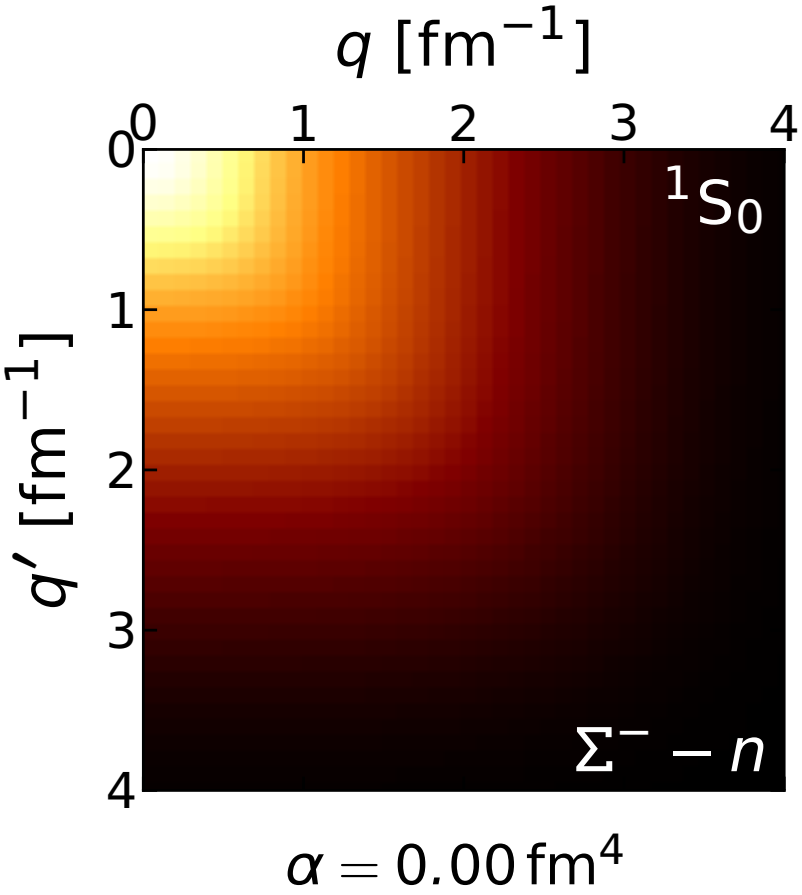
Similarity Renormalization Group



$$\partial_\alpha \boldsymbol{H}(\alpha) = [\boldsymbol{\eta}(\alpha), \boldsymbol{H}(\alpha)]$$

$$\boldsymbol{\eta}(\alpha) = m_N^2 [\boldsymbol{T}_{\mathrm{int}}, \boldsymbol{H}(\alpha)]$$

- Up to 6 coupled channels
- Decouples high and low momenta
  $\Rightarrow$ Improved $N_{\max}$ convergence
- BUT: Induced many-body terms
  $\Rightarrow$ Assess via α-dependence
- NN+3N: Induced terms negligible up to $A \approx 10$

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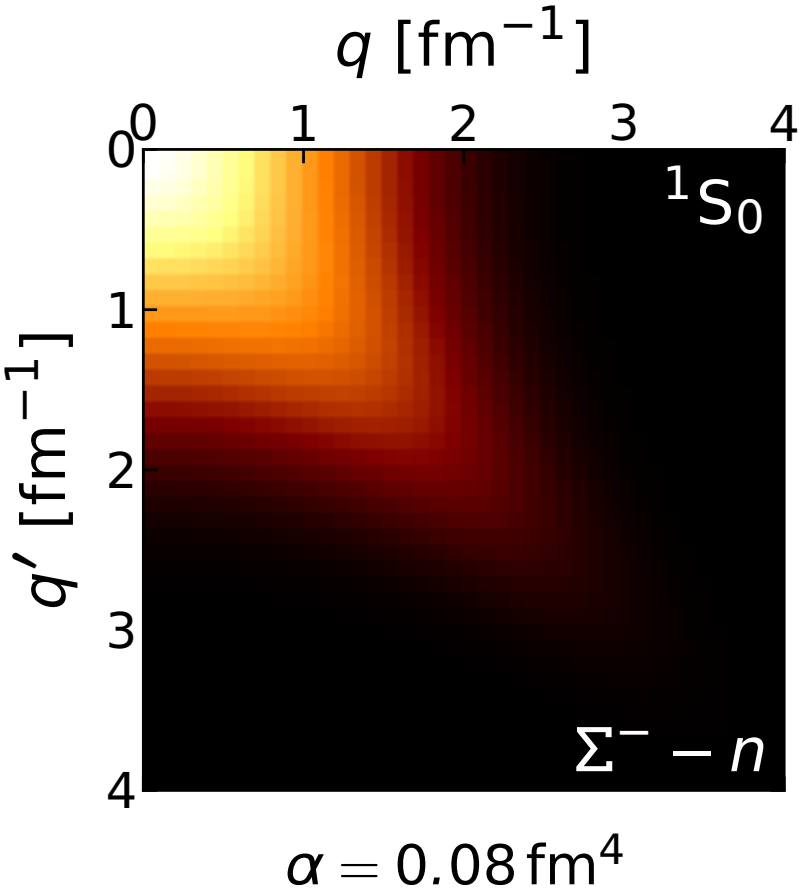

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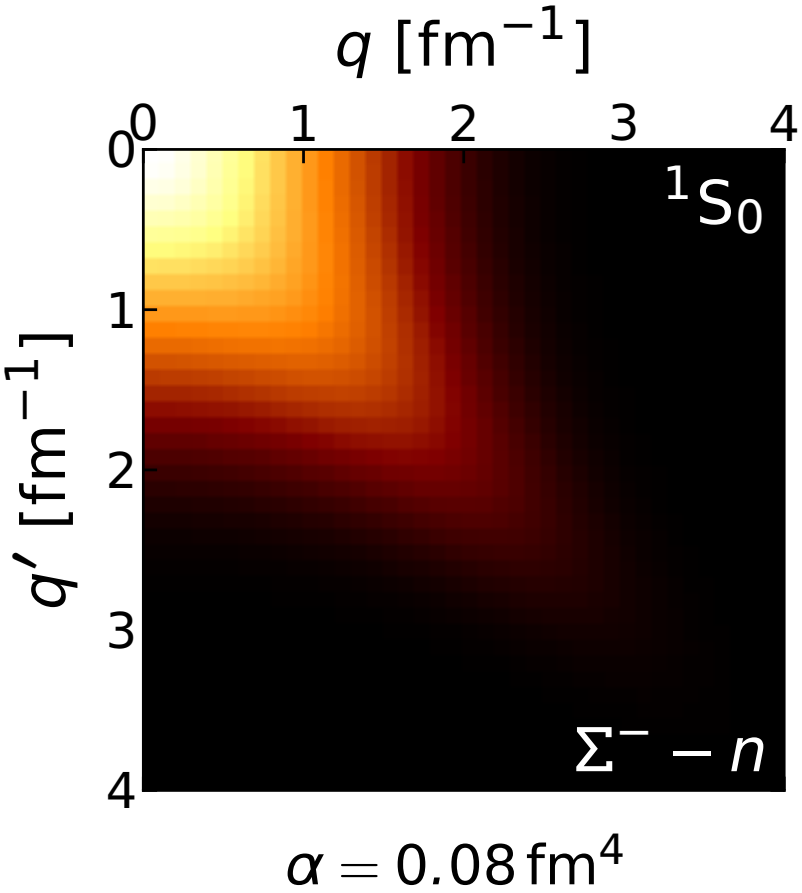

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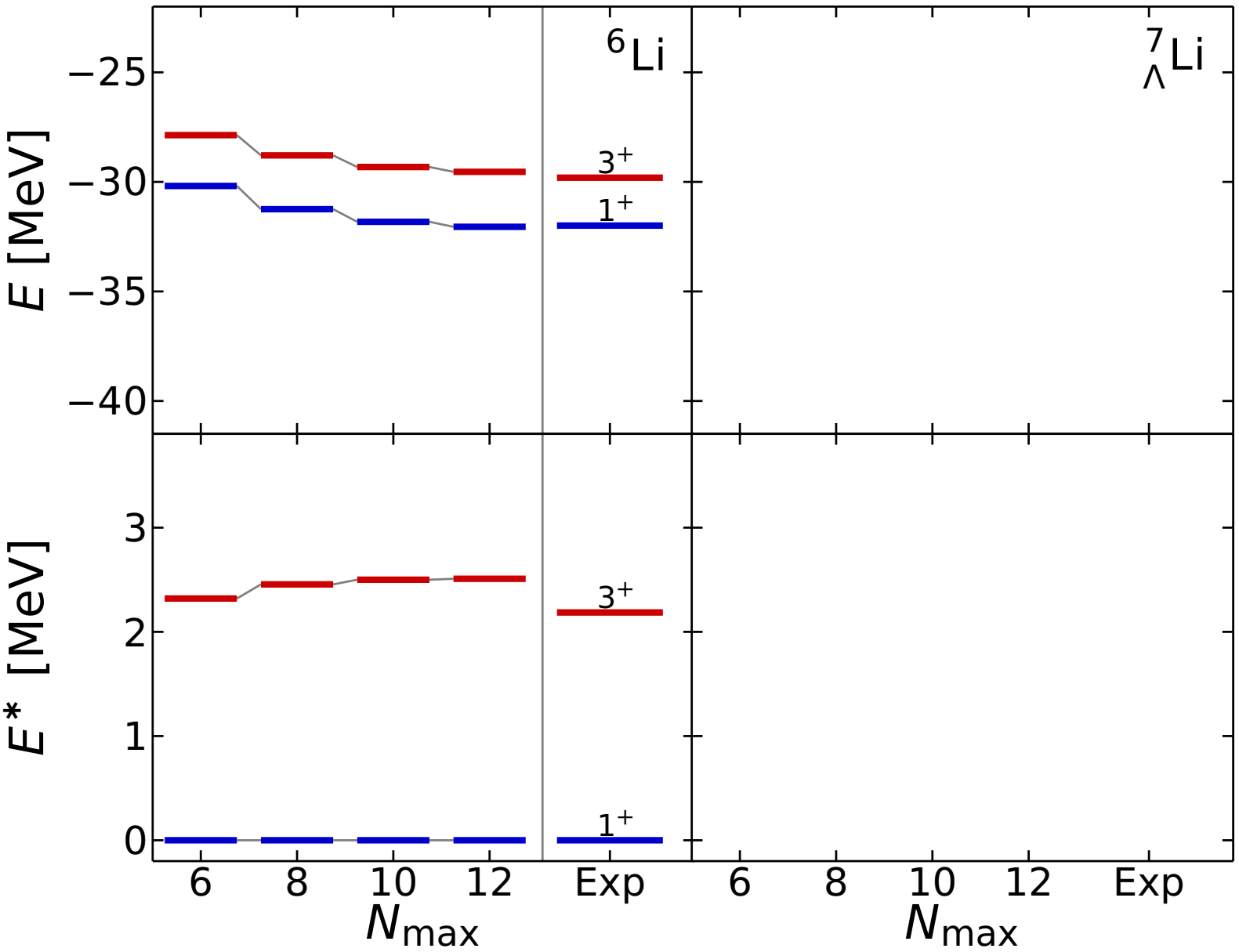
Use $\alpha_N = 0.08\,\mathrm{fm}^4$ for NN+3N
and $\alpha_Y = 0\,\mathrm{fm}^4$ for YN

Results for Selected Hypernuclei

RW, D. Gazda, P. Navrátil, A. Calci, J. Langhammer, R. Roth, PRL 113, 192502 (2014)

${}^{7}_{\Lambda}$Li — Energies and Spectra



$\alpha_N = 0.08\,\mathrm{fm}^4$
$\alpha_Y = 0\,\mathrm{fm}^4$
$\hbar\Omega = 20\,\mathrm{MeV}$

NN+3N
reproduces ^{6}Li

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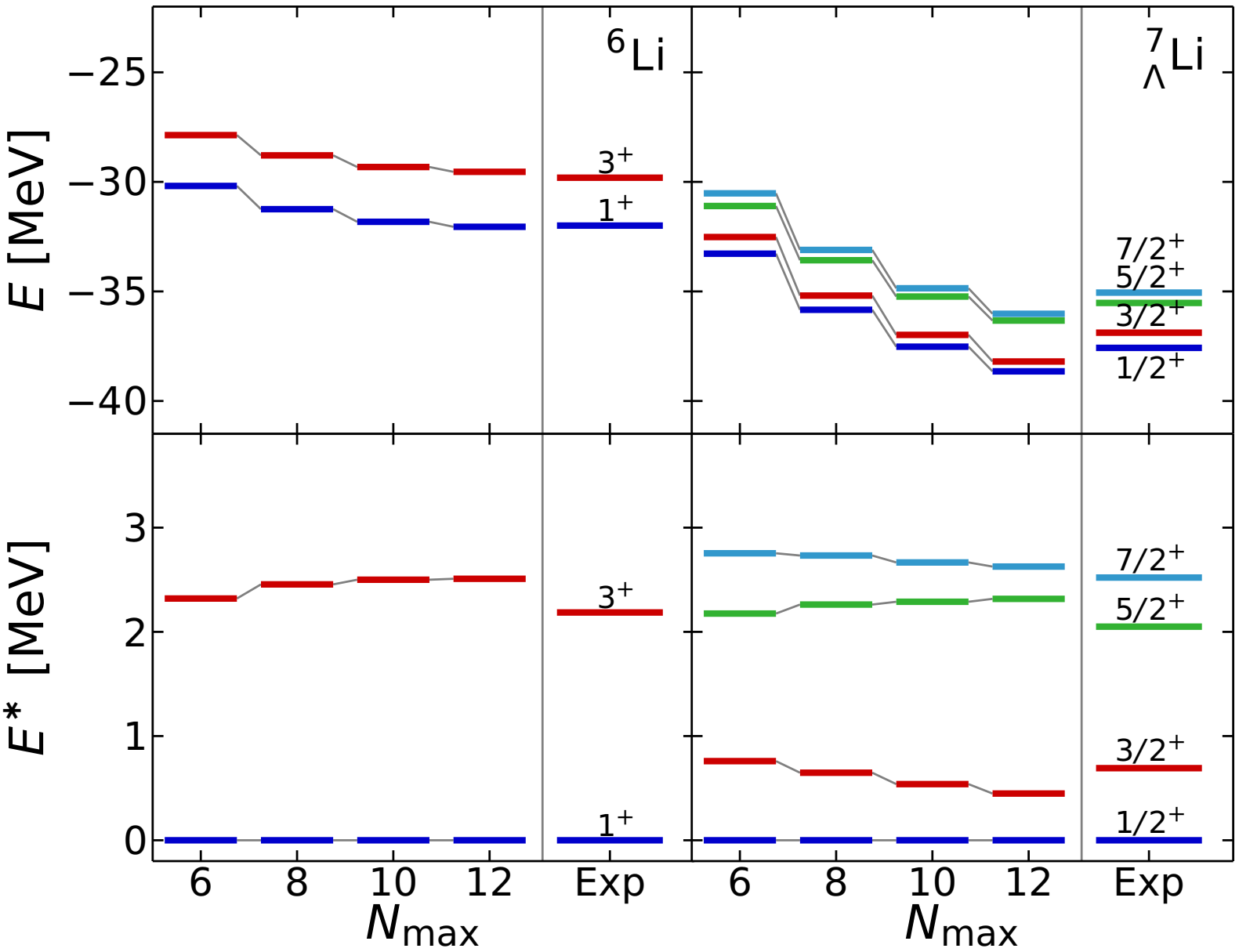

Chiral LO

— $\Lambda = 600\,\mathrm{MeV}$

$\alpha_N = 0.08\,\mathrm{fm}^4$
$\alpha_Y = 0\,\mathrm{fm}^4$
$\hbar\Omega = 20\,\mathrm{MeV}$

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reproduces ^{6}Li

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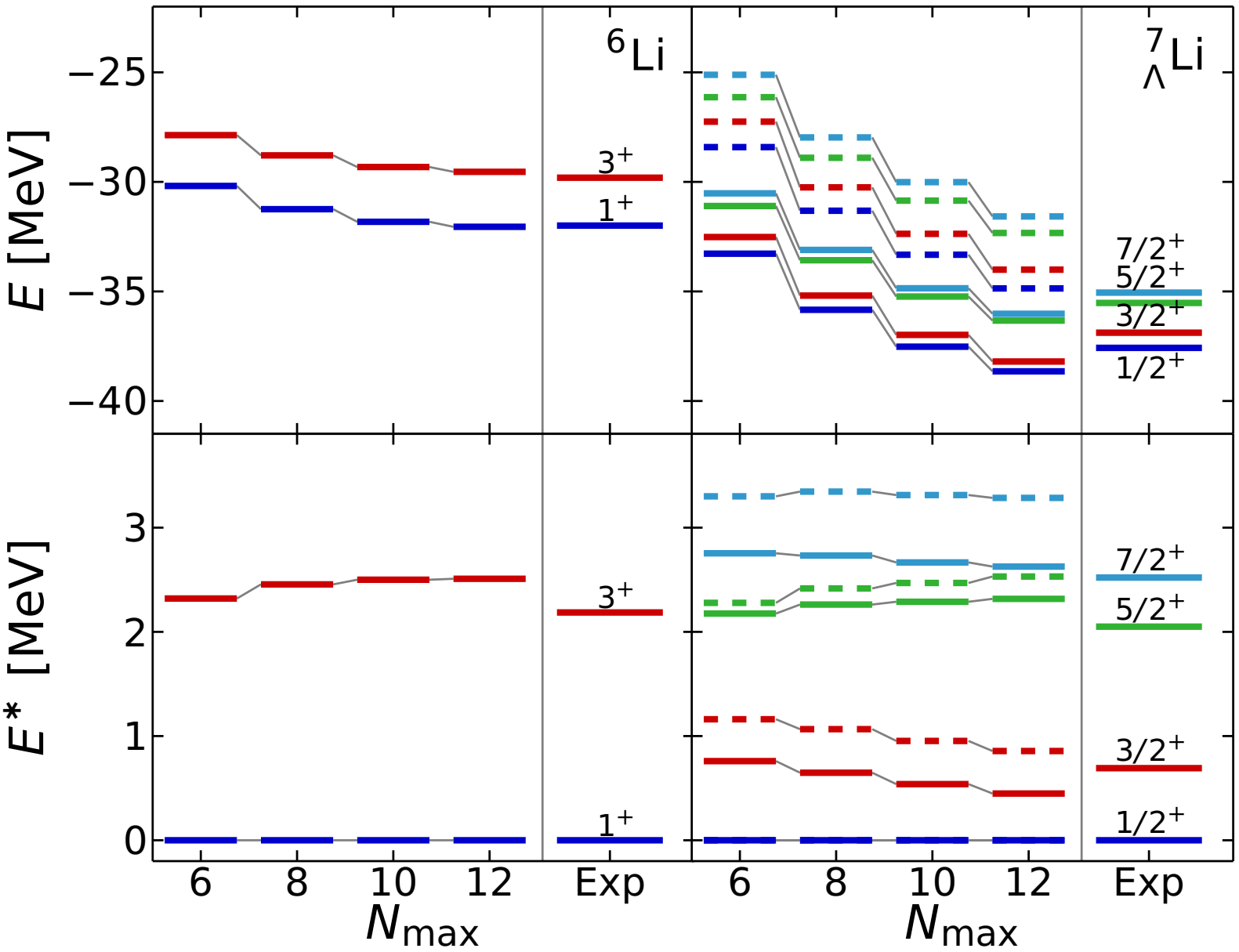

Chiral LO

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- - $\Lambda = 700\,\mathrm{MeV}$

$\alpha_N = 0.08\,\mathrm{fm}^4$

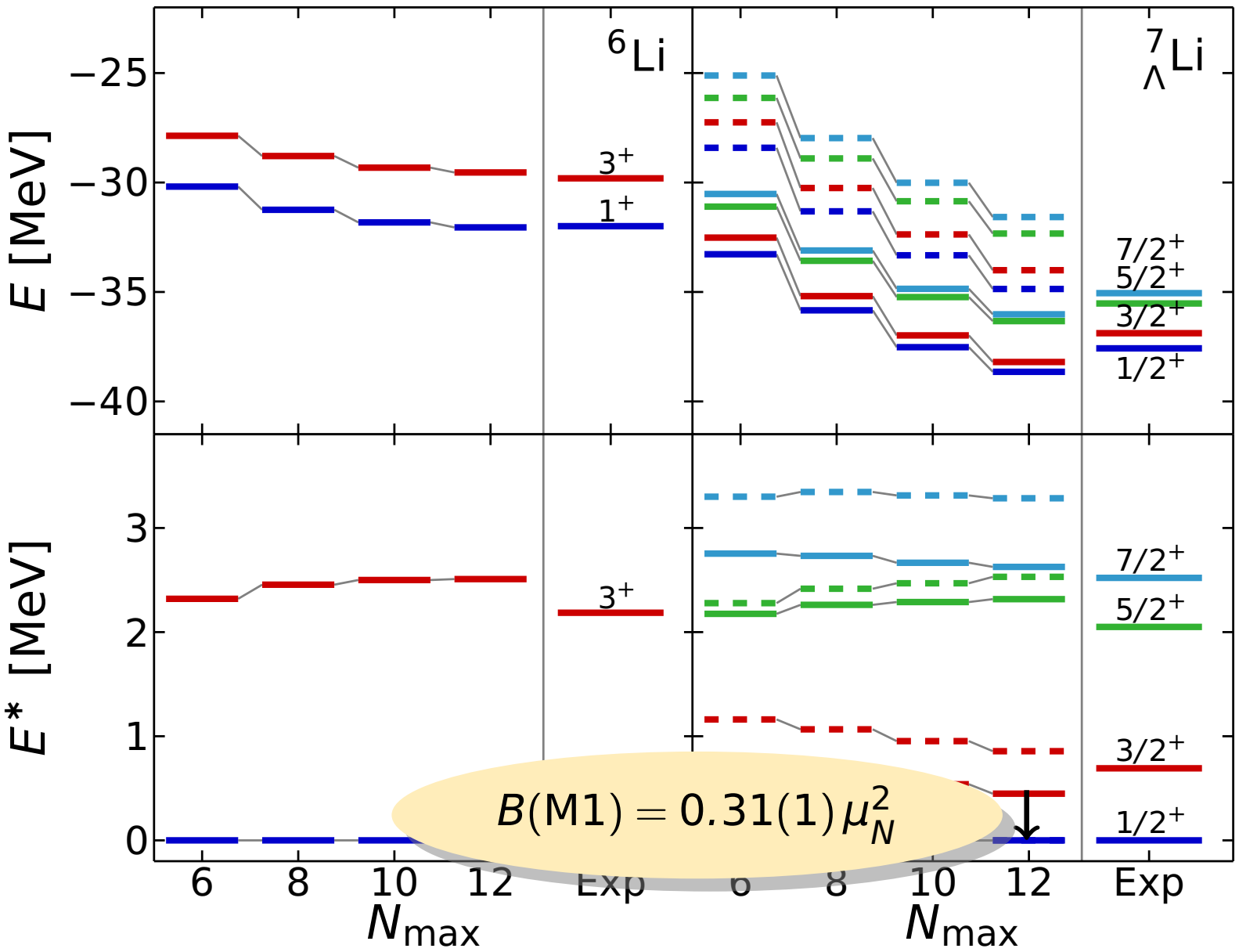
$\alpha_Y = 0\,\mathrm{fm}^4$

$\hbar\Omega = 20\,\mathrm{MeV}$

NN+3N reproduces ^{6}Li

$^{7}_{\Lambda}$Li: Sizable cutoff dependence

${}^{7}_{\Lambda}$Li — Energies and Spectra



Chiral LO

— $\Lambda = 600$ MeV

- - $\Lambda = 700$ MeV

$\alpha_N = 0.08\,\mathrm{fm}^4$

$\alpha_\gamma = 0\,\mathrm{fm}^4$

$\hbar\Omega = 20$ MeV

NN+3N reproduces ^{6}Li

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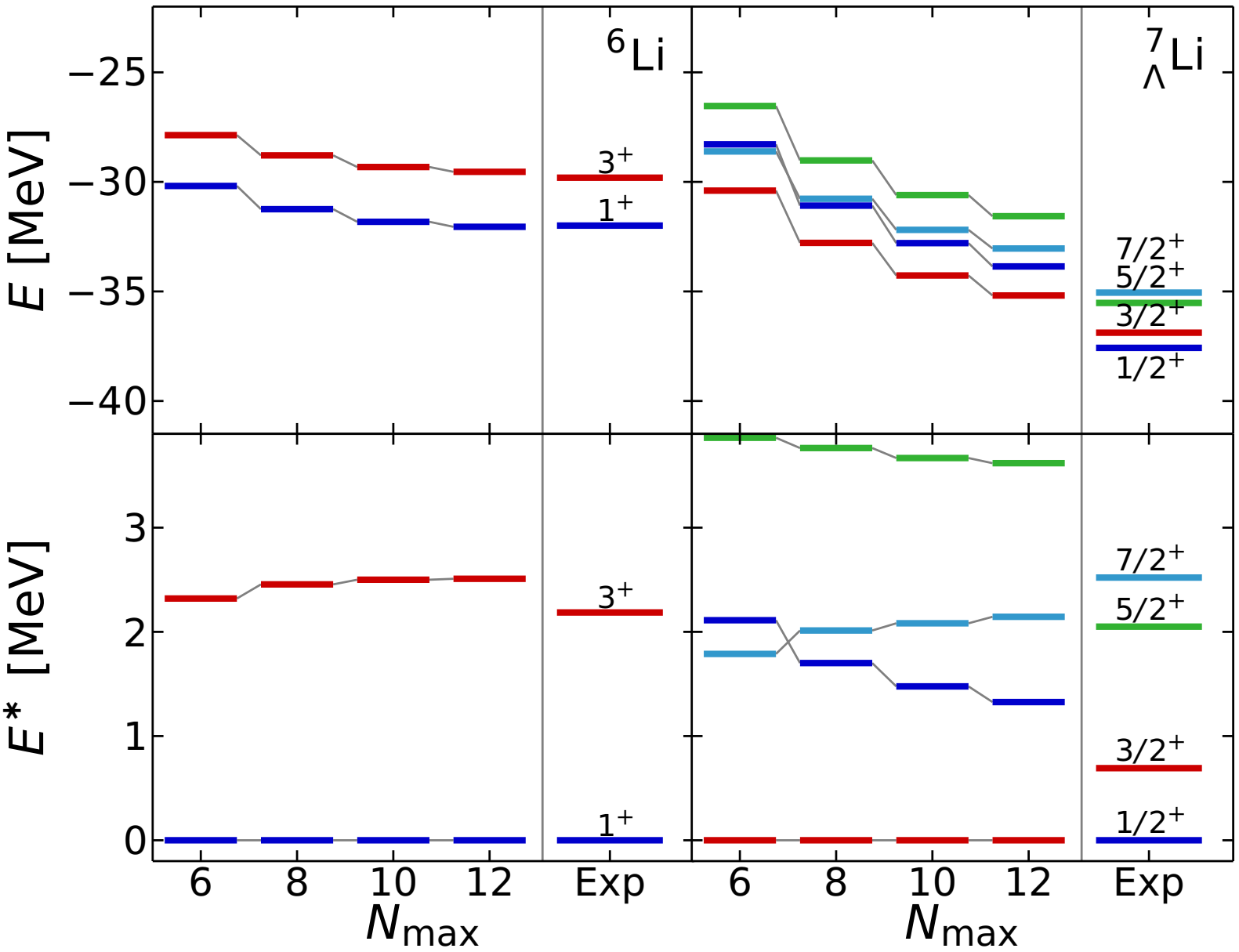
${}^{7}_{\Lambda}$Li — Energies and Spectra



Jülich'04

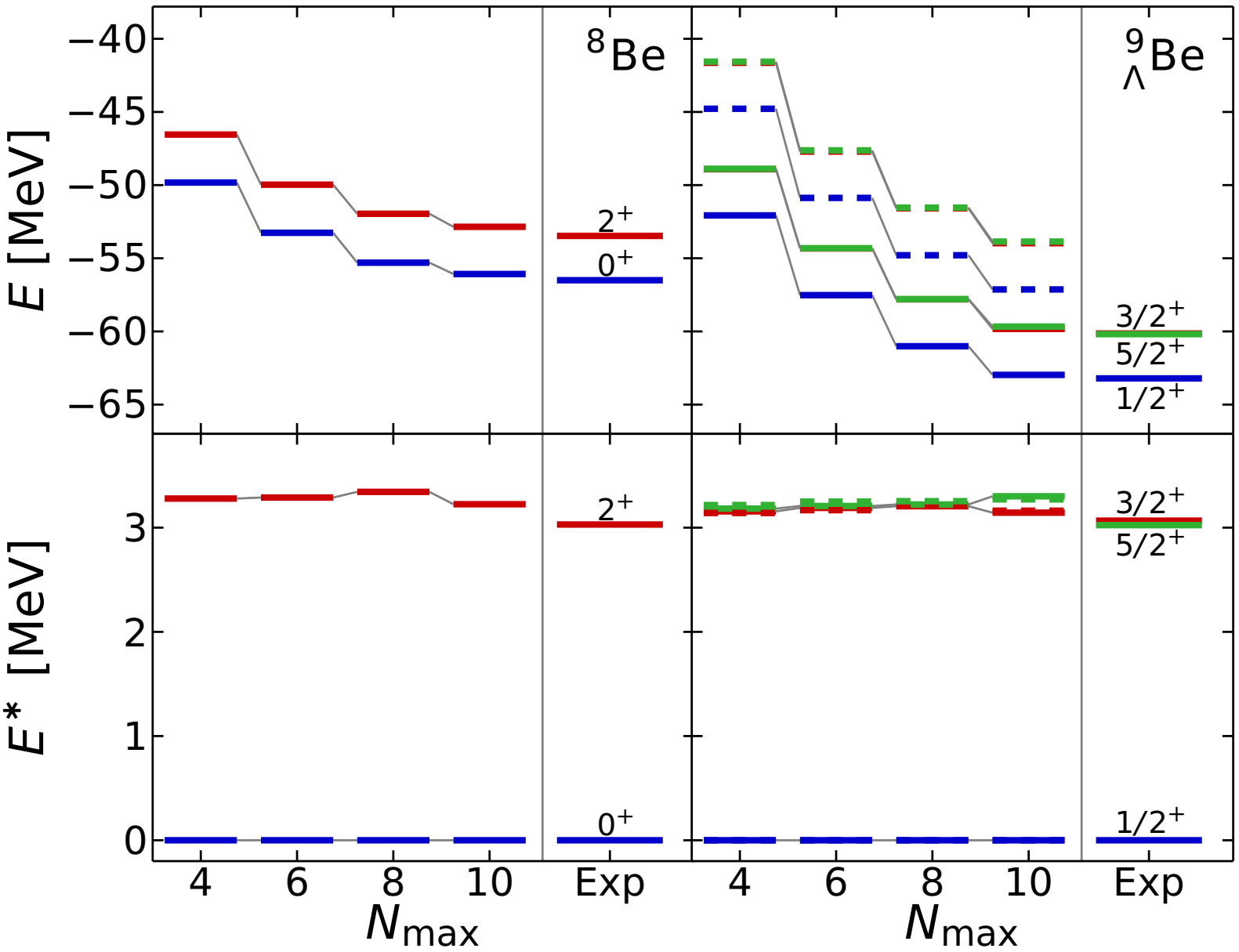
$\alpha_N = 0.08\,\mathrm{fm}^4$
$\alpha_Y = 0\,\mathrm{fm}^4$
$\hbar\Omega = 20\,\mathrm{MeV}$

Slight underbinding

Reversed level ordering

Large doublet splittings

${}^{9}_{\Lambda}$Be — Energies and Spectra



Chiral LO

— $\Lambda = 600\,\mathrm{MeV}$

- - $\Lambda = 700\,\mathrm{MeV}$

$\alpha_N = 0.08\,\mathrm{fm}^4$

$\alpha_Y = 0\,\mathrm{fm}^4$

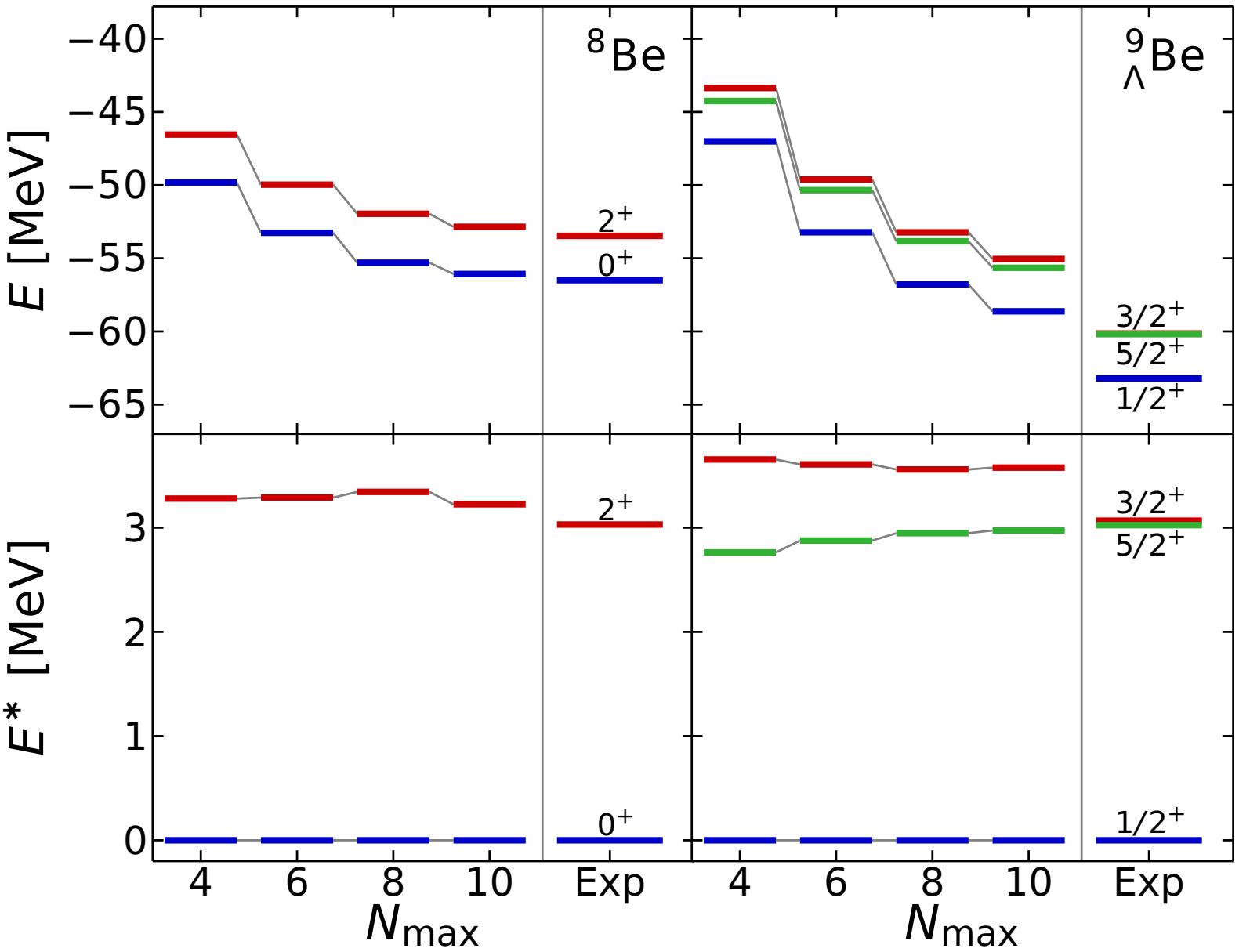
$\hbar\Omega = 20\,\mathrm{MeV}$

Small splitting reproduced

Tiny cutoff dependence

Reversed level ordering

${}^{9}_{\Lambda}$Be — Energies and Spectra



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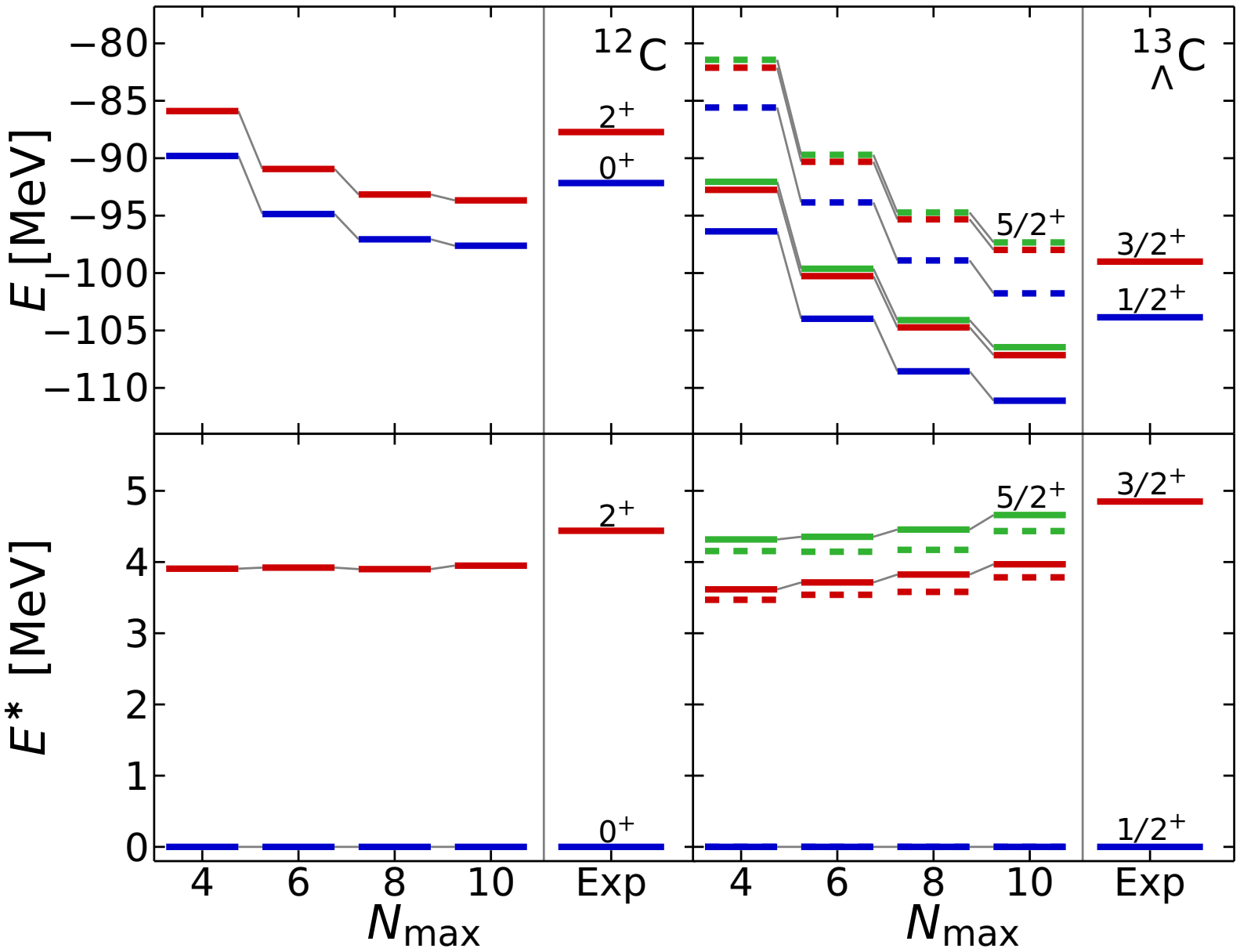
Slight underbinding

Large doublet splittings

Correct level ordering

$^{13}_{\Lambda}$C — Energies and Spectra



Chiral LO

— $\Lambda = 600\,\mathrm{MeV}$

- - $\Lambda = 700\,\mathrm{MeV}$

$\alpha_N = 0.08\,\mathrm{fm}^4$

$\alpha_Y = 0\,\mathrm{fm}^4$

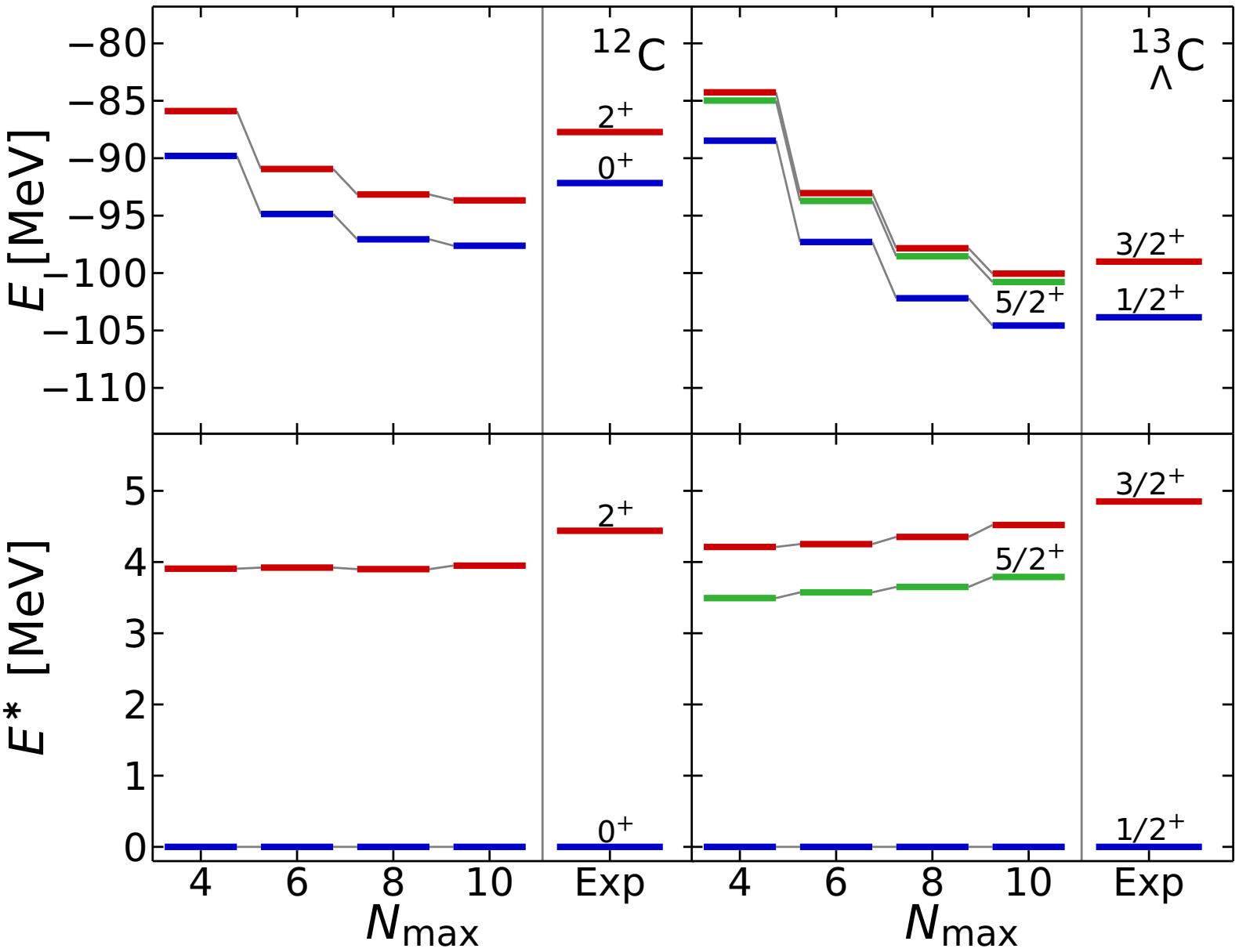
$\hbar\Omega = 20\,\mathrm{MeV}$

Overbinding due to induced 4N

E^* lower for $\Lambda = 700\,\mathrm{MeV}$

0.7 MeV splitting

$^{13}_{\Lambda}$C — Energies and Spectra



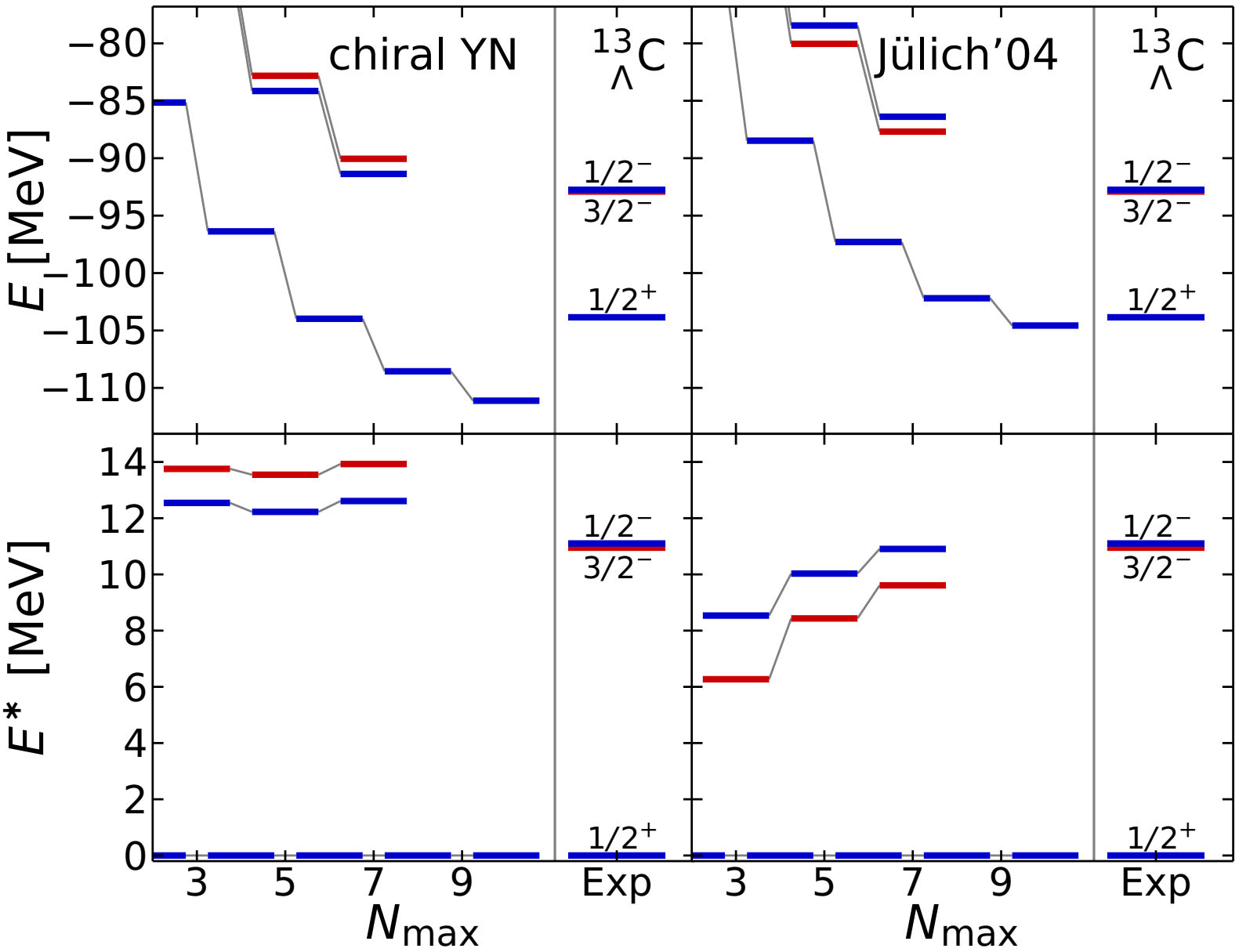
Jülich'04

$\alpha_N = 0.08\,\mathrm{fm}^4$
$\alpha_Y = 0\,\mathrm{fm}^4$
$\hbar\Omega = 20\,\mathrm{MeV}$

Very similar to chiral LO

Opposite level ordering

$^{13}_{\Lambda}$C — Negative-Parity States



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Chiral LO

Excitation energies and splitting too large

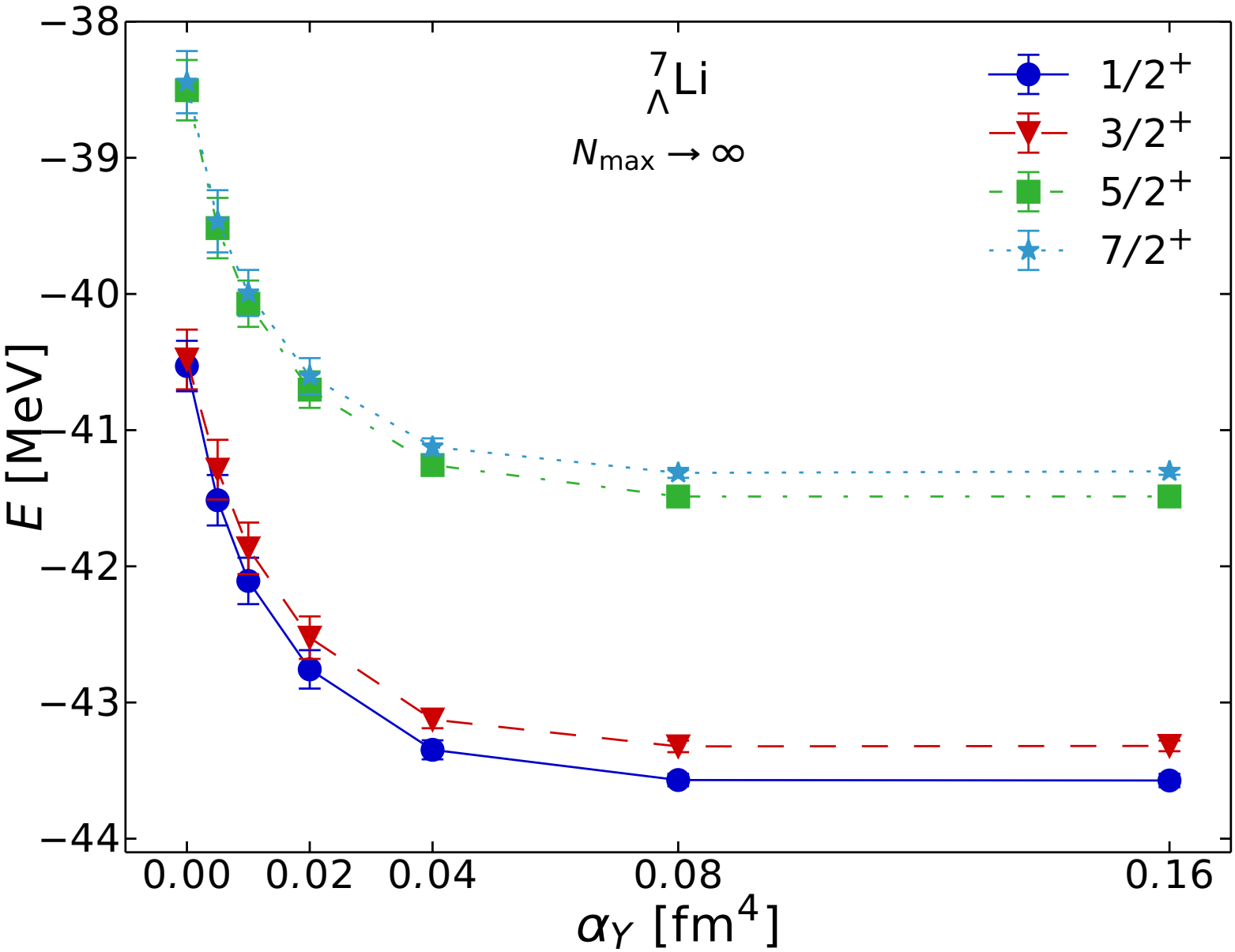
Jülich'04

Slow convergence

Correct level ordering

SRG-Induced YNN Interactions

Variation of α_Y

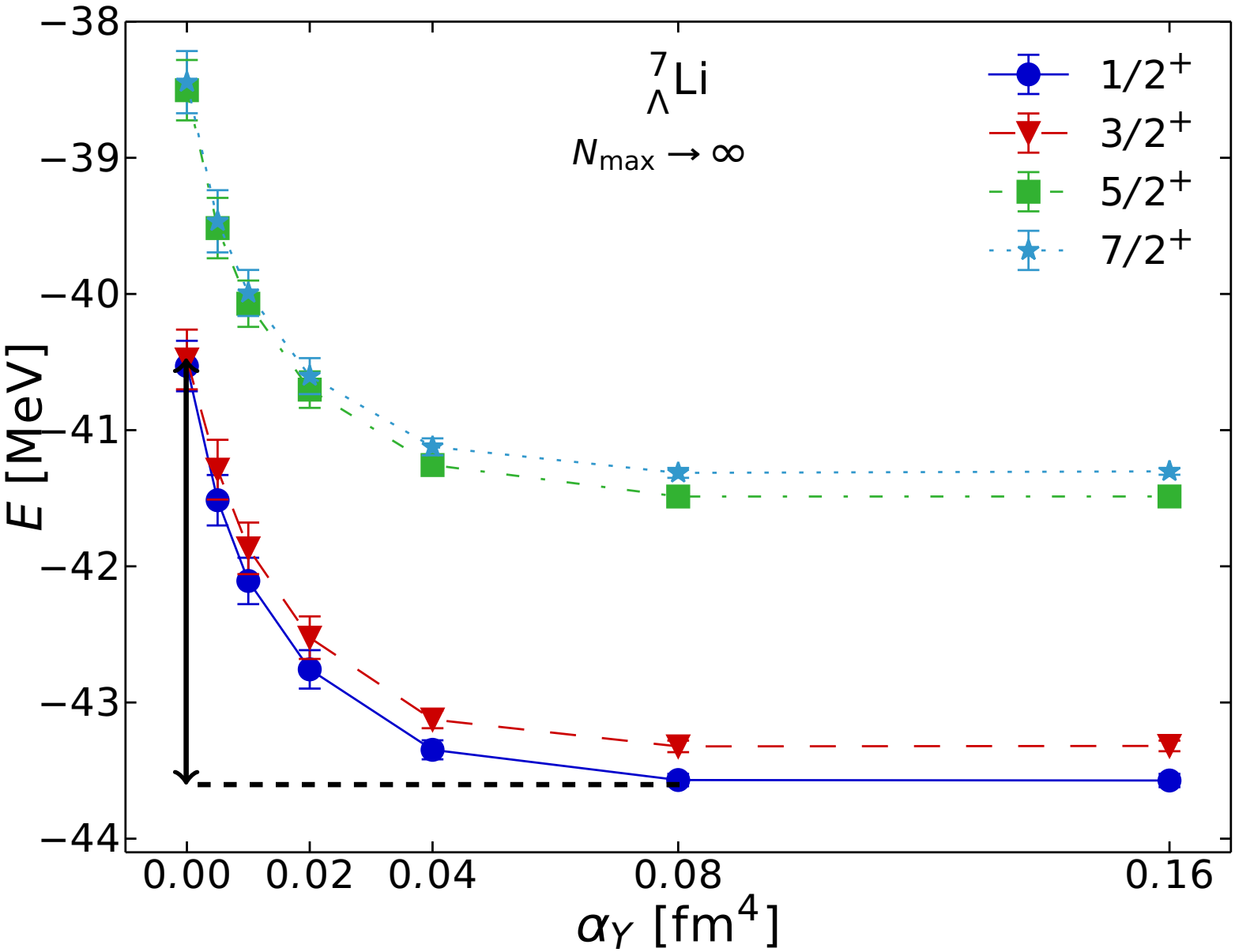


Strong α_Y-dependence of extrapolated energies

Induced YNN strongly repulsive

Spectra stable

Variation of α_Y



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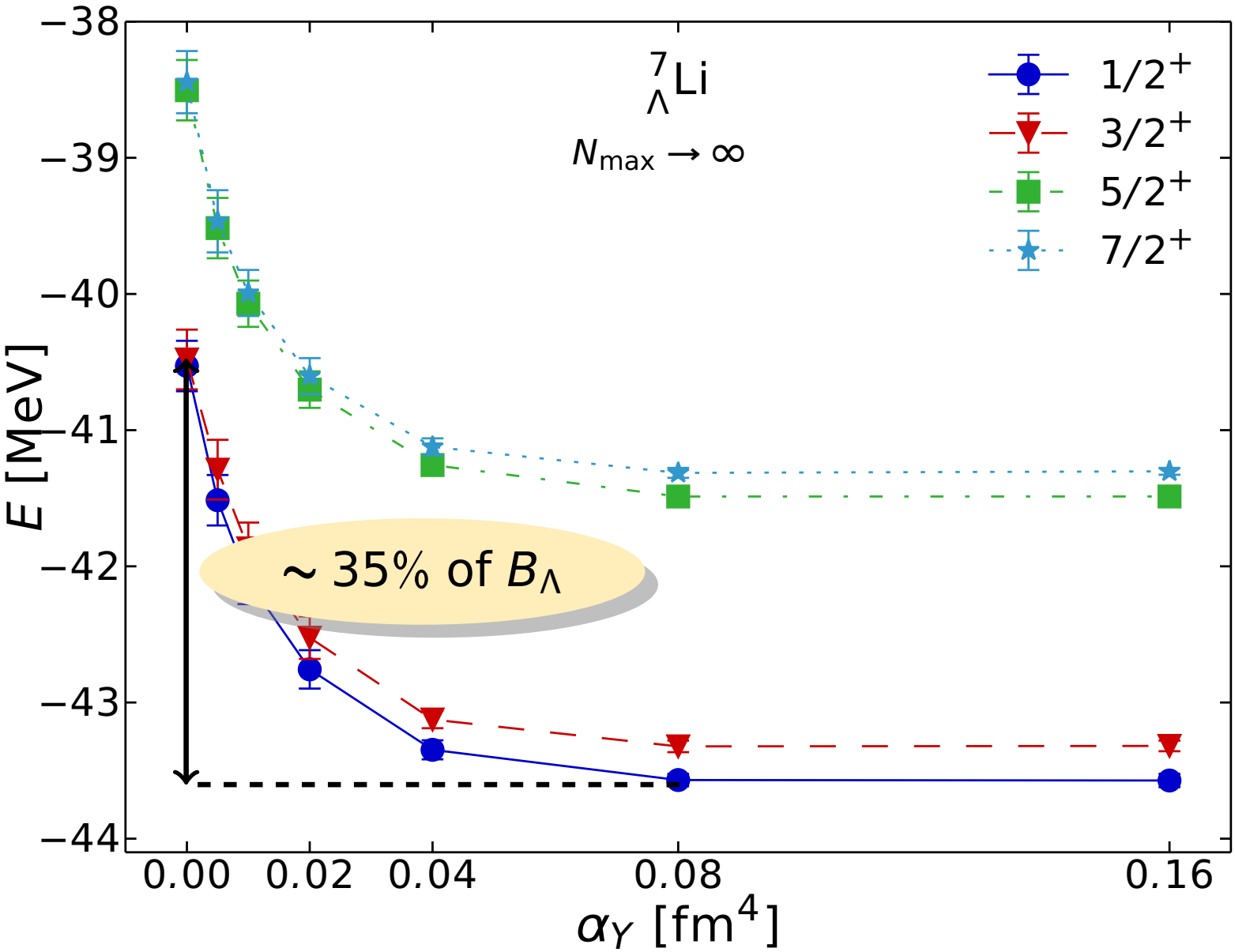

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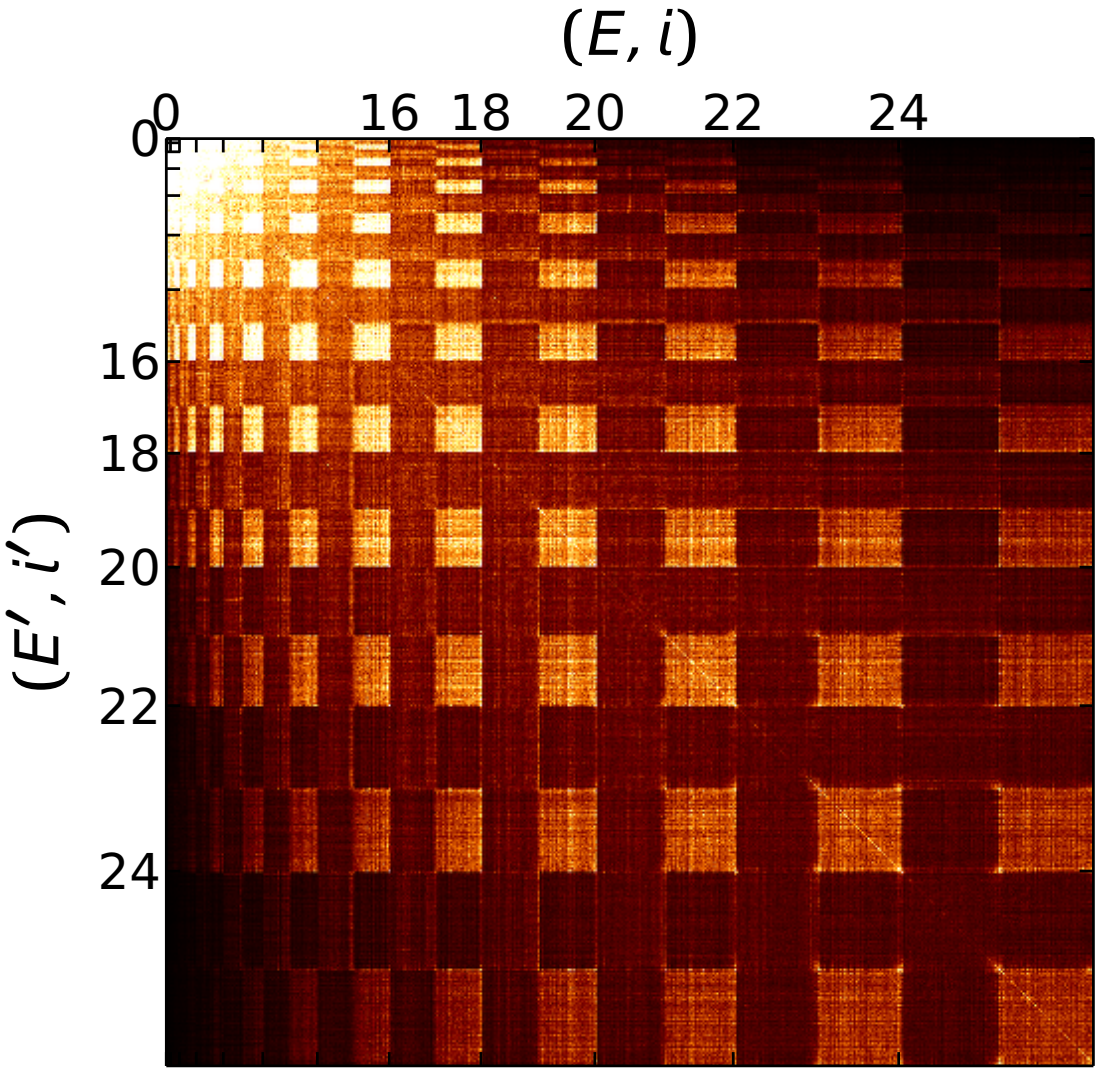
WIP: SRG at the Three-Body Level

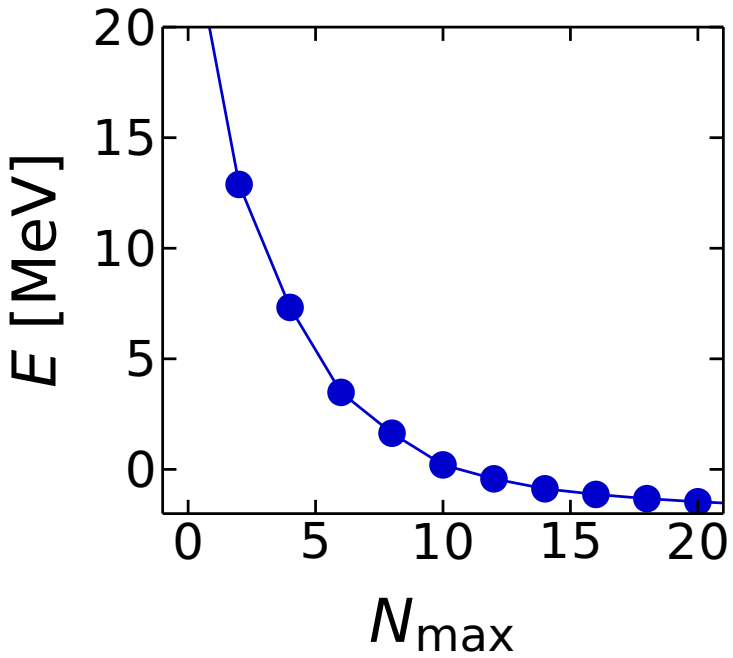


$$ {}_a\langle Ei|\boldsymbol{H} - \boldsymbol{T}_{\text{int}} - \boldsymbol{M}|E'i'\rangle_a $$

$$ \hbar\Omega = 32\,\text{MeV} $$

$$ J^{\Pi}T = \tfrac{1}{2}^{+}0 $$

$$ \alpha = 0.00\,\text{fm}^4 $$



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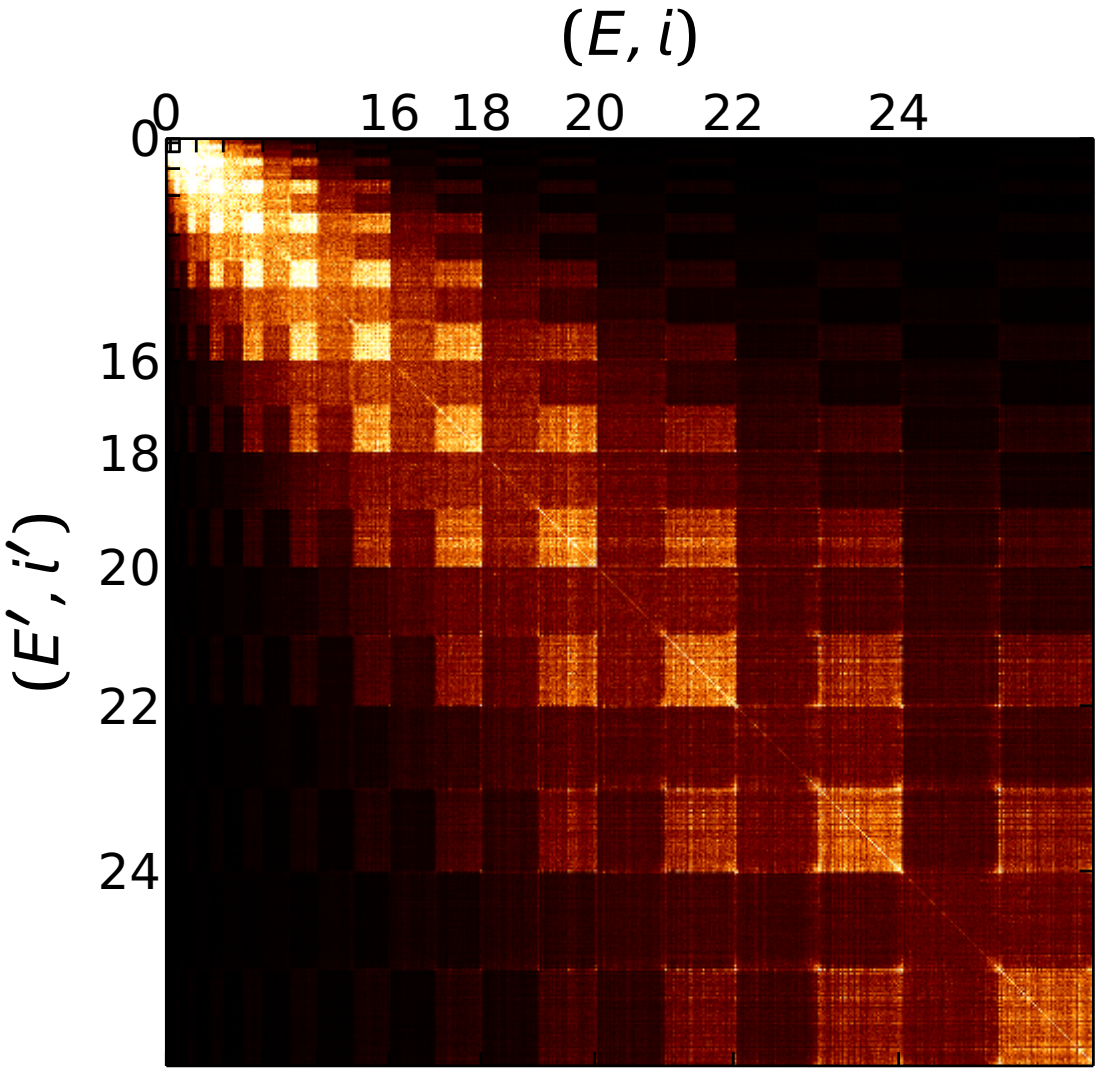

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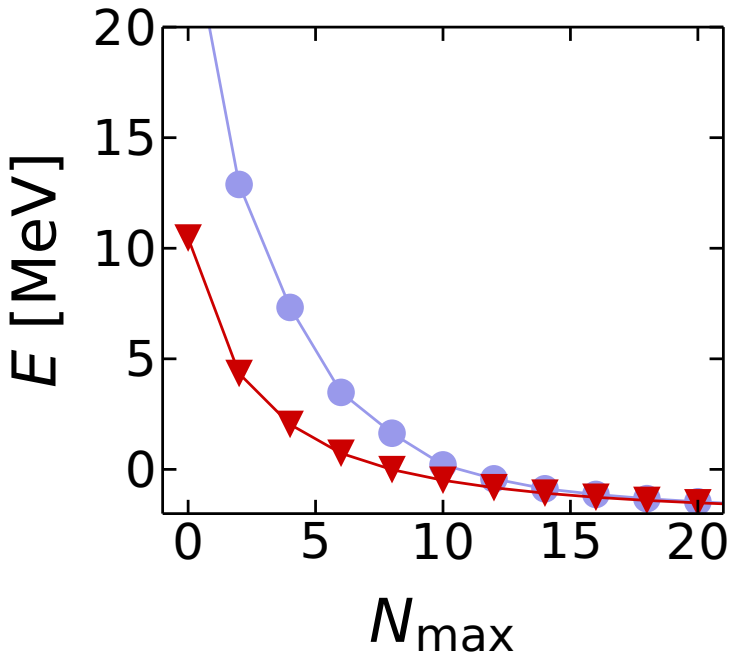

Summary & Outlook

- First ab-initio calculations in the *p* shell
- Prediction of observables
- Dependence on YN interaction ⇒ constraints
- SRG of YN induces strong YNN terms
  Possible implications for soft ΛN interactions and the hyperon puzzle
- Next: NLO chiral YN interaction & treat induced YNN terms explicitly

Epilog

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