Baryon-baryon interactions from lattice QCD

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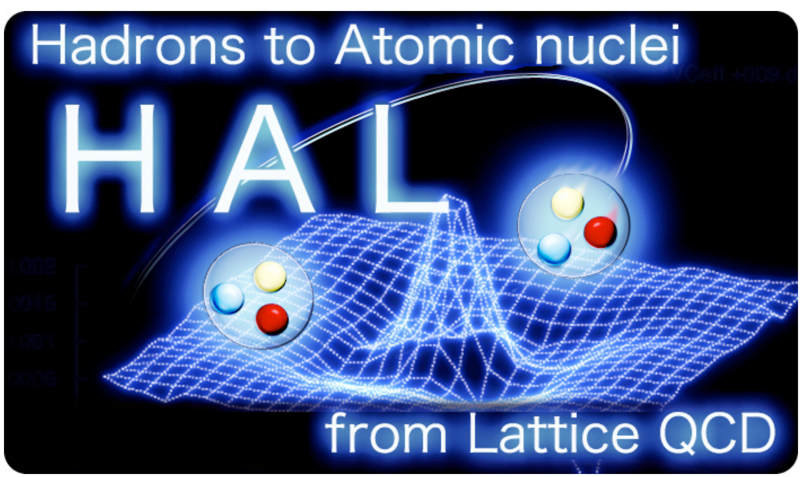
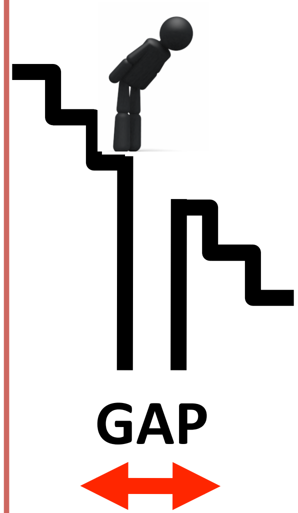


RCNP, Osaka Univ: N.Ishii, K.Murano
Univ. Tsukuba: H.Nemura, K.Sasaki, M.Yamada, F.Etminan, T.Miyamoto
RIKEN: T.Doi, T.Hatsuda, Y.Ikeda, V.Krejcirik
Nihon Univ: T.Inoue
Univ. Tokyo: B.Charron
YITP(Kyoto): S.Aoki, T.Iritani

There is a deep gap between QCD and Traditional/Hyper Nucl. Phys.

Quantum Chromodynamics (QCD)

- Input:
  - Fundamental DOF
    quarks and gluons
  - Interactions
    strong interaction from QCD
- Output:
  - Structures of nucleons/ hyperons
    [made of quarks and gluons]
  - their interactions



Traditional/Hyper Nucl. Phys.

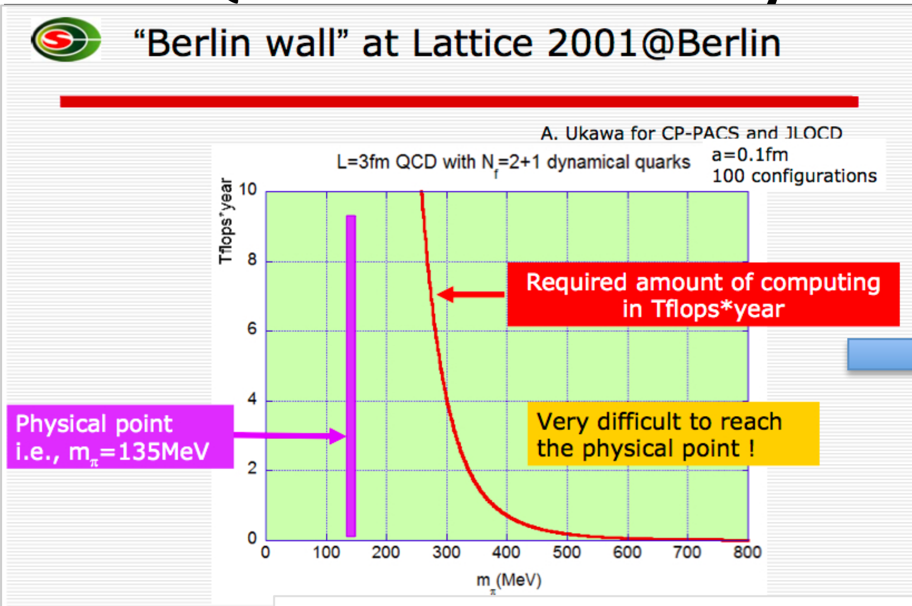
- Input:
  - Fundamental DOF
    nucleons(hyperon)
  - Interactions
    nuclear force from exp.
- Output:
  - Structures of (hyper)nuclei
    [made of nucleons/hyperons]
  - their reactions

The gap may begin to be filled by recent developments of lattice QCD.

- Lattice QCD at physical point
  PACSCS, BMW
- Lattice QCD calculations of atomic nuclei
  PACSCS, NPLQCD
- Lattice QCD calculations of nuclear/hyperon forces
  HALQCD

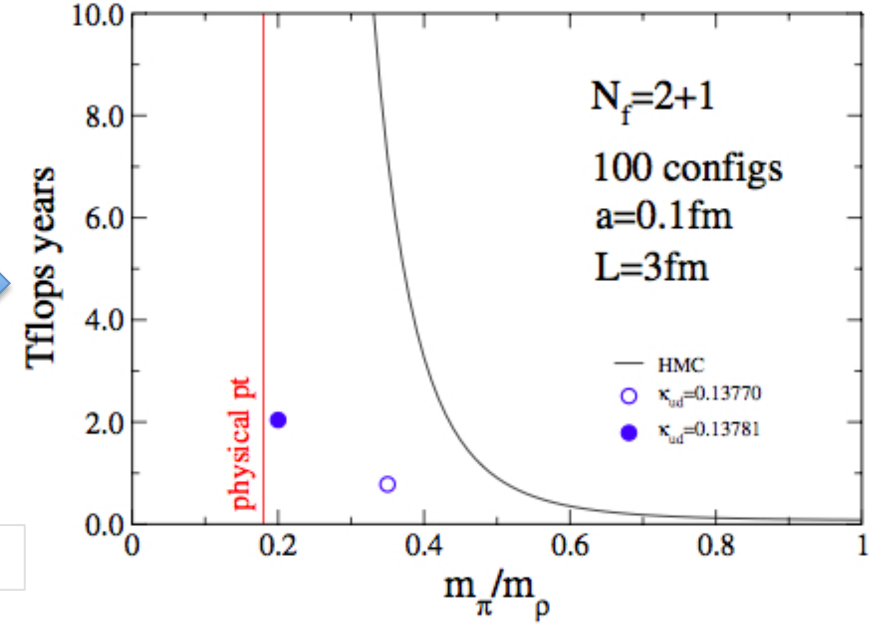
Lattice QCD at physical point

Lattice QCD simulation at Physical point



from Seminar at ICRR, 16 April 2010 by Ukawa.

Progress of LQCD algorithm



Lattice QCD simulation at Physical point is now possible.

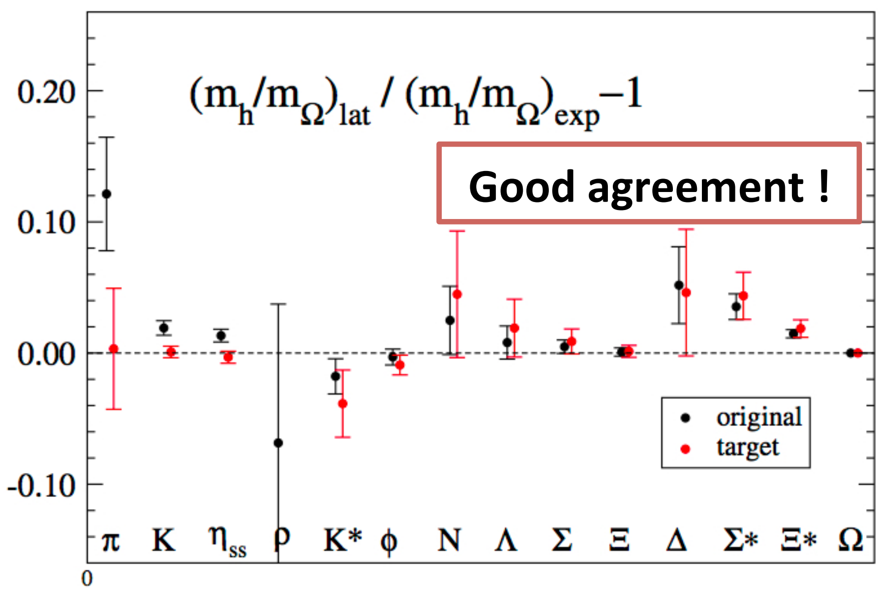


Fig. from PACSCS, Phys.Rev.D81,074503(2010). [L = 3 fm]
"Physical point simulation in 2+1 flavor lattice QCD"

See also:

- BMW, Science 322, 1224(2008)
- BMW, Phys.Lett.B701(2011)265. [L = 6fm]

Lattice QCD simulation at Physical point

To study multi-baryon systems at physical point,
spatial volume should be as huge as possible.

Such a physical point simulation is going on on K computer at AICS, RIKEN.

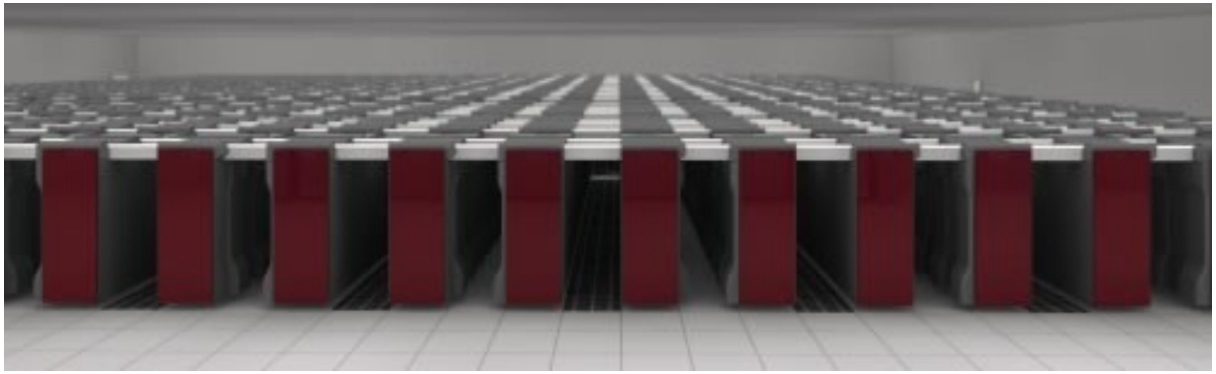
96^4 lattice, a = 0.1 fm, L = 9 fm, m_{pi}=135 MeV

The aims of the project:

- ◆ 2+1 flavor QCD → 1+1+1 flavor QCD+QED
- ◆ Various physical quantities
- ◆ Investigation of resonances
- ◆ Direct construction of light nuclei
- ◆ **Determination of baryon-baryon potentials**

← Nuclear Physics by LQCD

K computer (the 4th fastest in the world)



11.28 PFLOPS

Light Nuclei from Lattice QCD

Lattice calculation of light nuclei

There are several obstacles in LQCD calculation of atomic nuclei

- Signal to Noise
- Ground state saturation (at large volume)
- Careful volume extrapolation (Bound state ←→ scattering state)
- Computational cost for Wick contraction

Lattice calculation of light nuclei

Obstacle (1)

- Signal to Noise

$$G(t) \equiv \left\langle 0 \middle| O(t) \cdot \bar{O}(t=0) \middle| 0 \right\rangle$$
$$= \sum_n \left| \langle 0 | O | n \rangle \right|^2 e^{-E_n t}$$
$$\to \left| \langle 0 | O | \mathrm{G.S.} \rangle \right|^2 \times e^{-E_{\mathrm{G.S}} t} \ \text{for} \ \ t \to \text{large}$$

- pion $\quad \dfrac{\text{signal}}{\text{noise}} \sim \dfrac{\exp(-m_\pi t)}{\sqrt{\exp(-2 m_\pi t)}} \sim \text{const}$

- nucleon $\quad \dfrac{\text{signal}}{\text{noise}} \sim \dfrac{\exp(-m_N t)}{\sqrt{\exp(-3 m_\pi t)}} \sim \exp\left(-(m_N - 3/2 m_\pi) t\right)$

atomic nuclei with mass # A ➔ $\dfrac{\text{signal}}{\text{noise}} \sim \exp\left(-A(m_N - 3/2 m_\pi) t\right)$

For large nuclei, quality of the signal becomes bad quite rapidly.

Lattice calculation of light nuclei

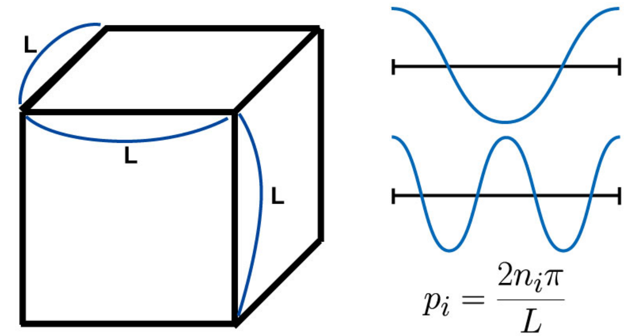
Obstacle (2)

◆ The ground state saturation is important in many LQCD calculations.

$$\langle 0|O(t)\cdot\bar{O}(t=0)|0\rangle$$
$$=\sum_n \left|\langle 0|O|n\rangle\right|^2 e^{-E_n t}$$
$$\to \left|\langle 0|O|\mathrm{G.S.}\rangle\right|^2 \times e^{-E_{\mathrm{G.S}}t} \text{ for } t\to\text{large}$$

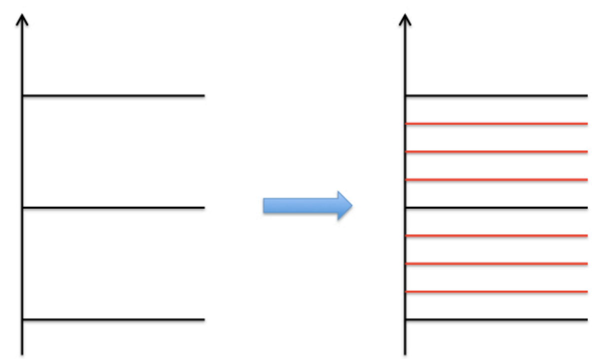
◆ **For multi-hadron systems, the ground state saturation becomes difficult as V → large.**

Spatial momentum is discretized due to the periodic BC.



◆ For L → large, energy gap shrinks as

$$\Delta E = E_{n+1} - E_n \sim \frac{1}{m_N}\left(\frac{2\pi}{L}\right)^2$$



	L=3 fm	L=6 fm	L=9 fm	L=12 fm
Δ E	181.5 MeV	45.3 MeV	20.2 MeV	11.3 MeV

If L becomes twice as large, ΔE becomes 4 times as small.

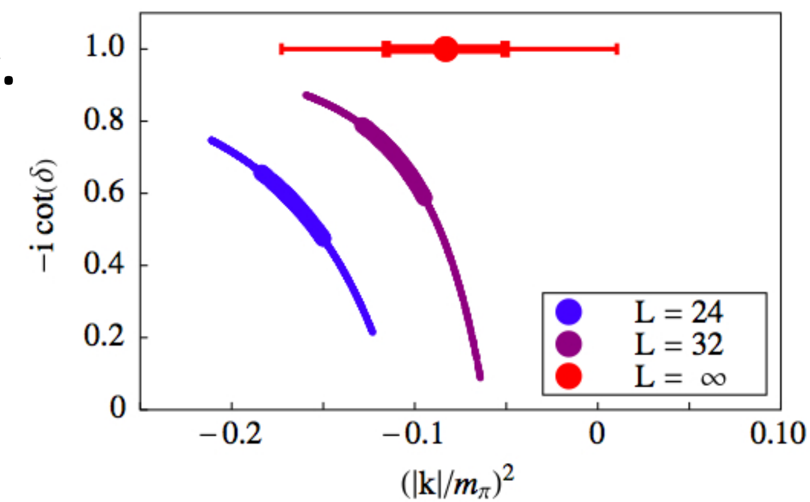
Lattice calculation of light nuclei

Obstacle (3)
Careful volume extrapolation.

ΔE < 0 in finite V means

i. bound state
ii. attractive interaction in finite volume.

Need to use several spatial volumes to extrapolate the results to V=∞ carefully.



volume extrapolation of deuteron:
from NPLQCD, PRD85,054511(2012)
(2+1 flavor QCD on anisotropic lattice.
m_{pi}=390 MeV, L=2.0,2.5,2.9,3.9 fm)

Lattice calculation of light nuclei

Obstacle (4)

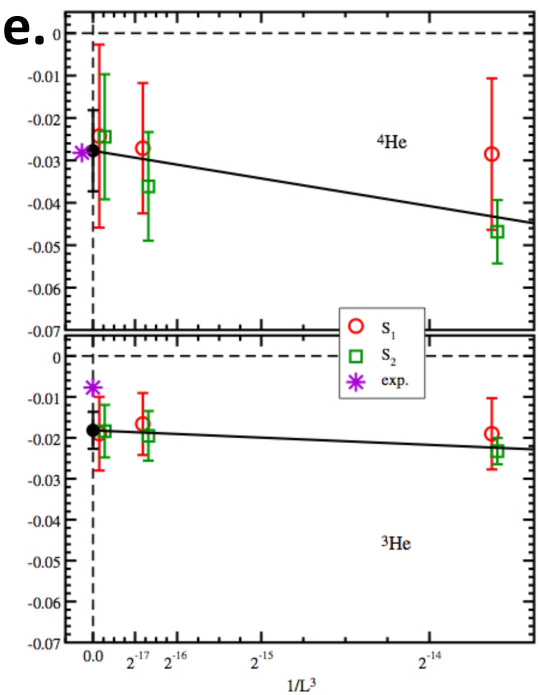
Computational cost for Wick contraction: grows with factorial.

Number of Wick contraction:

$$\propto (2N_n + N_p)! \times (N_n + 2N_p)!$$

	^1H	^2H	$^3H/^3He$	^4He	^6Li	...
#(Wick contraction)	2	36	2880	518400	131681894400	...

➔ **Naïve contraction algorithm will soon go into trouble.**



T.Yamazaki et al., PRD81,111504(2010) reduced these numbers as

- $^3H/^3He$: 360 ➔ 93
- ^4He: 518400 ➔ 1107

and gave the first LQCD results of bound light atomic nuclei.
(quenched QCD. m_{pi}=0.8 GeV, L=3.1-12.3 fm)

Light Nuclei from lattice QCD

Systematic reduction method is proposed by T.Doi, M.G.Endres, CPC184,117(2013).

$$N_\alpha(x;\xi_1,\xi_2,\xi_3) \equiv \langle N_\alpha(x)\cdot \bar{q}(\xi_1)\bar{q}(\xi_2)\bar{q}(\xi_3)\rangle$$

f :smearing function $\xi = (c,\alpha) \in \text{color} \times \text{Dirac}$

$$\langle N_\alpha(x)N_\beta(y)\cdot \bar{N}_{\alpha'}(f)\bar{N}_{\beta'}(f)\rangle$$

$$= \sum_\sigma \text{sign}(\sigma)\cdot \sum_{\xi_1,\cdots,\xi_6} N_\alpha(x;\xi_{\sigma(1)},\xi_{\sigma(2)},\xi_{\sigma(3)})N_\beta(y;\xi_{\sigma(4)},\xi_{\sigma(5)},\xi_{\sigma(6)})\cdot C(\xi_1,\cdots,\xi_6)$$

relabeling of summation var.: $\xi'_1 \equiv \xi_{\sigma(1)},\cdots,\xi'_6 \equiv \xi_{\sigma(6)}$

$$= \sum_{\xi'_1,\cdots,\xi'_6} N_\alpha(x;\xi'_1,\xi'_2,\xi'_3)N_\beta(y;\xi'_4,\xi'_5,\xi'_6)\left\{\sum_\sigma C(\xi'_{\sigma^{-1}(1)},\cdots,\xi'_{\sigma^{-1}(6)})\text{sign}(\sigma)\right\}$$

Permutation sum associated with Wick contraction can be carried out before the LQCD calculations.

Efficiency is significantly improved:

- ^{3}H/^{3}He: **x 192**
- ^{4}He: **x 20736**

See also:

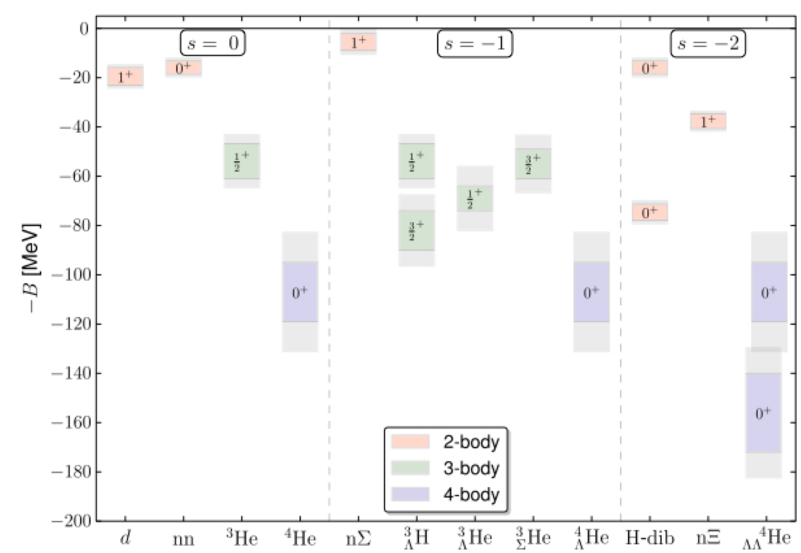
- W. Detmold, K.Orginos, PRD87,114512(2013)
- J.Gunter, B.C.Toth, L.Varnhorst, PRD87,094513(2013)

Lattice calculation of light nuclei

Several bound multi-baryon states are reported in heavy quark mass region (m_{pi} > 390 MeV). (Some agree, the others not.)

- H-dibaryon
  - NPLQCD: NF=2+1: PRL106,162001(2011)
    NF=3:★
  - HALQCD: NF=3: PRL106,162002(2011)
- $^3H/^3He$, ^4He
  - PACSCS: both bound
    - NF=0: PRD81,111504(R)(2010)
    - NF=2+1: PRD86,074514(2012)
  - NPLQCD: NF=3: ★
  - HALQCD: NF=3: NPA881,28(2012).
    Only ^4He bound at m_{PS}=469MeV
- Deuteron, dineutron
  - NPLQCD: NF=2+1: PRD85,054511(2013)
    (also H-dibaryon and $\Xi^-\Xi^-$ bound)
    NF=3: ★
  - PACSCS: both bound
    - NF=0: PRD84,054506(2011)
    - NF=2+1: PRD86,074514(2013)
  - HALQCD: No bound states for both.
- Many others, light (hyper)nuclei
  - NF=3: ★

★ NPLQCD, PRD87,034506(2013)



Lattice QCD calculation of nuclear forces

HALQCD method (naïve introduction)

- Suppose it is possible to generate "NN wave functions" by LQCD.

$$\text{LQCD} \quad \Rightarrow \quad \psi(\vec{r})$$

- NN potential can be obtained by inversely solving Schrodinger eq. for V(r) as

$$\big(H_0 + V(r)\big)\psi(\vec{r}) = E\psi(\vec{r}) \quad \Longrightarrow \quad V(\vec{r}) \equiv E - \frac{H_0\psi(\vec{r})}{\psi(\vec{r})}$$

$$H_0 \equiv -\frac{\nabla^2}{m_N}$$

- If the potential has a more complicated structure

$$V = V_{\rm C}(r) + V_{\rm T}(r)S_{12} + V_{\rm LS}(r)\vec{L}\cdot\vec{S} + O(\nabla^2)$$

We have to do a more complicated inversion

$$\big(H_0 + V_{\rm C}(r) + V_{\rm T}(r)S_{12} + V_{\rm LS}(r)\vec{L}\cdot\vec{S}\big)\psi_n(\vec{r}) = E_n\psi_n(\vec{r}) \qquad (n = 0,1,2)$$

$$\Longrightarrow \begin{bmatrix} (E_0 - H_0)\psi_0(\vec{r}) \\ (E_1 - H_1)\psi_1(\vec{r}) \\ (E_2 - H_2)\psi_2(\vec{r}) \end{bmatrix} = \begin{bmatrix} \psi_0(\vec{r}) & S_{12}\psi_0(\vec{r}) & \vec{L}\cdot\vec{S}\psi_0(\vec{r}) \\ \psi_1(\vec{r}) & S_{12}\psi_1(\vec{r}) & \vec{L}\cdot\vec{S}\psi_1(\vec{r}) \\ \psi_2(\vec{r}) & S_{12}\psi_2(\vec{r}) & \vec{L}\cdot\vec{S}\psi_2(\vec{r}) \end{bmatrix} \cdot \begin{bmatrix} V_{\rm C}(\vec{r}) \\ V_{\rm T}(\vec{r}) \\ V_{\rm LS}(\vec{r}) \end{bmatrix}$$

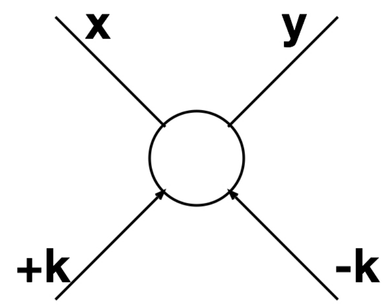
- Now, what is a suitable object in LQCD for "NN wave function" ?
  Our answer is **equal-time Nambu-Bethe-Salpeter(NBS) wave function**.

HALQCD method

[Aoki,Hatsuda,Ishii,PTP123(2010)89]

◆ Nambu-Bethe-Salpeter (NBS) wave function

$$\langle 0 | T[N(x)N(y)] | N(+k)N(-k), in \rangle$$



◆ Relation to S-matrix by reduction formula

Bosonic notation is to avoid lengthy notations.

$$\langle N(p_1)N(p_2), out | N(+k)N(-k), in \rangle$$
$$= \mathrm{disc} + \left(iZ_N^{-1/2}\right)^2 \int d^4x_1 d^4x_2 \, e^{ip_1x_1}\left(\Box_1 + m_N^2\right) e^{ip_2x_2}\left(\Box_2 + m_N^2\right) \langle 0 | T[N(x_1)N(x_2)] | N(+k)N(-k), in \rangle$$

◆ **Equal-time** restriction of **NBS wave function** behaves at long distance

[C.-J.D.Lin et al., NPB619,467(2001).]

$$\psi_k(\vec{x}-\vec{y}) \equiv \lim_{x_0 \to +0} Z_N^{-1} \langle 0 | T[N(\vec{x},x_0)N(\vec{y},0)] | N(+k)N(-k), in \rangle$$
$$= Z_N^{-1} \langle 0 | N(\vec{x},0)N(\vec{y},0) | N(+k)N(-k), in \rangle$$
$$\simeq e^{i\delta(k)} \frac{\sin\left(kr + \delta(k)\right)}{kr} + \cdots \quad \text{as } r \equiv |\vec{x}-\vec{y}| \to \text{large}$$

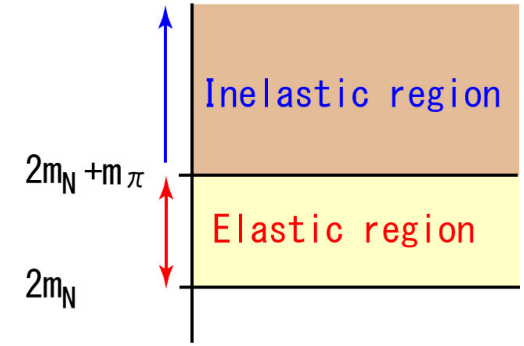
(for S-wave)

Exactly the same functional form
as that of scattering wave functions in quantum mechanics
(equal-time NBS wave function is a good candidate of “NN wave function”)

HALQCD method

◆ **Def.** of potential from **equal-time NBS wave functions**:

$$\left(k^2/m_N - H_0\right)\psi_k(\vec{r}) = \int d^3r' \, U(\vec{r},\vec{r}')\,\psi_k(\vec{r}')$$



$$\text{for } 2\sqrt{m_N^2 + k^2} < E_{\text{th}} \equiv 2m_N + m_\pi$$

$$H_0 \equiv -\frac{\nabla^2}{m_N}$$

◆ **U(r,r')** is demanded to be **E-indep**
so that the same U(r,r') can generate all the NBS wave functions
in the Elastic region.
(Proof of existence of such U(r,r') is given in the next slide)

◆ **U(r,r') reproduces the scattering phase δ(k),**
together with equal-time NBS wave functions

$$\psi_k(\vec{x}-\vec{y}) \simeq e^{i\delta(k)} \frac{\sin\left(kr + \delta(k)\right)}{kr} + \cdots \quad \text{as } r \equiv |\vec{x}-\vec{y}| \to \text{large}$$

HALQCD method

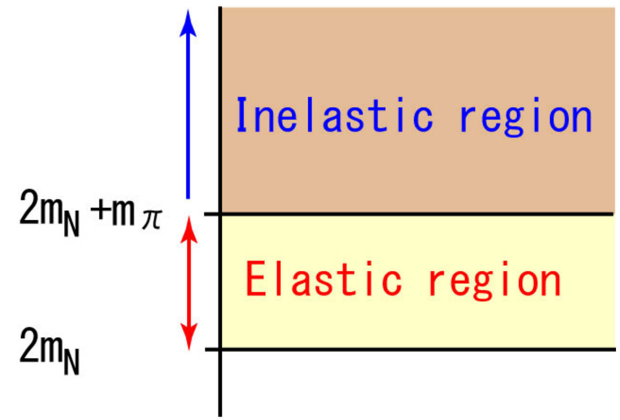
◆ Existence of E-indep. U(r,r')

◆ Assumption:

Linear independence of equal-time NBS wave func. for E < Eth.

➔ There exists dual basis:

$$\int d^3r\, \tilde{\psi}_{\vec{k}'}(\vec{r})\psi_{\vec{k}}(\vec{r}) = (2\pi)^3\delta^3(\vec{k}'-\vec{k})$$



$$H_0 \equiv -\frac{\nabla^2}{m_N}$$

◆ Proof:

$$K_{\vec{k}}(\vec{r}) \equiv \left(k^2/m_N - H_0\right)\psi_{\vec{k}}(\vec{r})$$

$$K_{\vec{k}}(\vec{r}) = \int \frac{d^3k'}{(2\pi)^3} K_{\vec{k}'}(\vec{r}) \int d^3r'\, \tilde{\psi}_{\vec{k}'}(\vec{r})\psi_{\vec{k}}(\vec{r})$$

$$= \int d^3r' \left\{ \int \frac{d^3k}{(2\pi)^3} K_{\vec{k}'}(\vec{r})\tilde{\psi}_{\vec{k}'}(\vec{r}') \right\} \psi_{\vec{k}}(\vec{r}')$$

$$\left(k^2/m_N - H_0\right)\psi_{\vec{k}}(\vec{r}) = \int d^3r'\, U(\vec{r},\vec{r}')\psi_{\vec{k}}(\vec{r}')$$

$$U(\vec{r},\vec{r}') \equiv \int \frac{d^3k'}{(2\pi)^3} K_{\vec{k}'}(\vec{r})\tilde{\psi}_{\vec{k}'}(\vec{r})$$

U(r,r') does not depend on E because of the intergration of k'.

HALQCD method

[Ishii et al.,PLB712(2012)437]

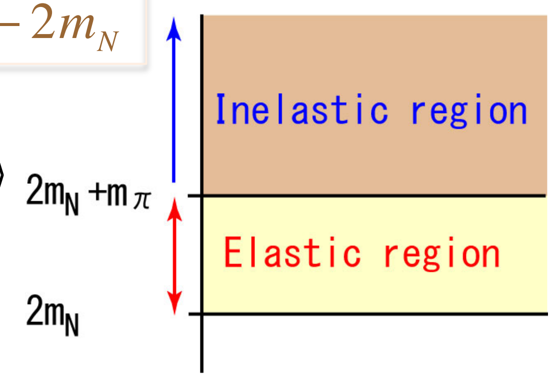
The ground state saturation is not needed to extract the potential.

$$\Delta W(k) \equiv 2\sqrt{m_N^2 + k^2} - 2m_N$$

- Normalized NN correlator

$$R(t,\vec{x}-\vec{y}) \equiv e^{2m_N \cdot t} \left\langle 0 \middle| T\left[N(\vec{x},t) N(\vec{y},t) \cdot \overline{NN}(t=0) \right] \middle| 0 \right\rangle$$

$$= \sum_k a_k \exp\left(-t \Delta W(k)\right) \cdot \psi_k(\vec{x}-\vec{y})$$



- All $\psi_k(\vec{r})$ satisfy the Schrodinger eq. with the same U(r,r') in the elastic region:

$$(H_0 + U)\psi_k(\vec{r}) = \frac{k^2}{m}\psi_k(\vec{r})$$

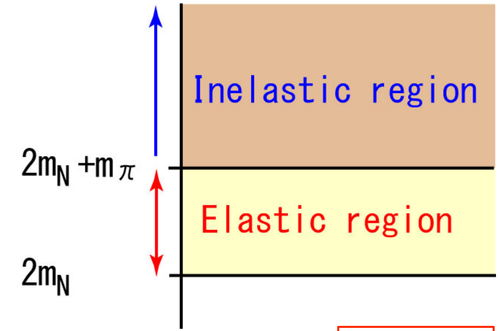
➔ **R(t,r) satisfies "Time-dependent" Schrodinger-like equation**
in the large "t" region where inelastic contributions are negligible.

$$\left(\frac{1}{4m_N}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} - H_0 \right) R(t,\vec{r}) = \int d^3r' U(\vec{r},\vec{r}') \cdot R(t,\vec{r}')$$

- Only **Elastic saturation** is required. (Gound state saturation is not needed.)
- **Elastic saturation** is much easier than **single state saturation**.

HALQCD method

- Rough idea of derivation



$$R(t,\vec{x}-\vec{y}) = \sum_{\vec{k}} a_{\vec{k}} \exp\left(-t\left\{2\sqrt{m_N^2+\vec{k}^2}-2m_N\right\}\right)\cdot\psi_{\vec{k}}(\vec{x}-\vec{y})$$

$$\simeq \sum_{\vec{k}} a_{\vec{k}} \exp\left(-t\frac{\vec{k}^2}{m_N}\right)\cdot\psi_{\vec{k}}(\vec{x}-\vec{y})$$

$$= \sum_{\vec{k}} a_{\vec{k}} \exp\left(-t\left\{H_0+U_{\mathrm{HALQCD}}\right\}\right)\cdot\psi_{\vec{k}}(\vec{x}-\vec{y})$$

$$= \exp\left(-t\left\{H_0+U_{\mathrm{HALQCD}}\right\}\right)\cdot R(t=0,\vec{x}-\vec{y})$$

Defining relation of HAL QCD potential (= Schroedinger eq)

$$\left(\vec{k}^2/m_N - H_0\right)\psi_{\vec{k}}(\vec{r}) = \int d^3r' U_{\mathrm{HALQCD}}(\vec{r},\vec{r}')\psi_{\vec{k}}(\vec{r}')$$

$$-\frac{\partial}{\partial t}R(t,\vec{x}-\vec{y}) \simeq \left(H_0+U_{\mathrm{HALQCD}}\right)R(t,\vec{x}-\vec{y})$$

$$\Rightarrow$$

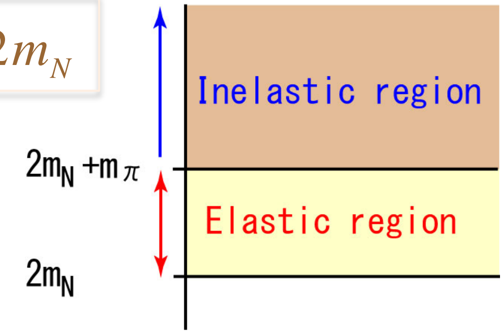
$$\left(-\frac{\partial}{\partial t}-H_0\right)R(t,\vec{r}) \simeq \int d^3r' U_{\mathrm{HALQCD}}(\vec{r},\vec{r}')R(t,\vec{r}')$$

HALQCD method

[Ishii et al.,PLB712(2012)437]

Full derivation.

$$\Delta W(k) \equiv 2\sqrt{m_N^2 + k^2} - 2m_N$$



◆ Normalized NN correlator

$$R(t, \vec{x} - \vec{y}) \equiv e^{2m_N \cdot t} \left\langle 0 \middle| T\left[N(\vec{x}, t) N(\vec{y}, t) \cdot \overline{NN}(t=0) \right] \middle| 0 \right\rangle$$

$$= \sum_k a_k \exp\left(-t\Delta W(k)\right) \cdot \psi_k(\vec{x} - \vec{y})$$

Assumption:

"t" is large enough so that **elastic contributions** can **dominate** intermediate states.

◆ **"Time-dependent" Schrodinger-like equation**

to extract our potential.

$$\left(\frac{1}{4m_N} \frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} \right) R(t, \vec{r}) = \sum_k a_k \frac{k^2}{m_N} \exp\left(-t\Delta W(k)\right) \cdot \psi_k(\vec{r})$$

An identity

$$\frac{\Delta W(k)^2}{4m_N} + \Delta W(k) = \frac{k^2}{m_N}$$

$$(H_0 + U)\psi_k(\vec{r}) = \frac{k^2}{m_N} \psi_k(\vec{r})$$

$$\left(\frac{1}{4m_N} \frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} - H_0 \right) R(t, \vec{r}) = \int d^3 r' U(\vec{r}, \vec{r}') \cdot R(t, \vec{r}')$$

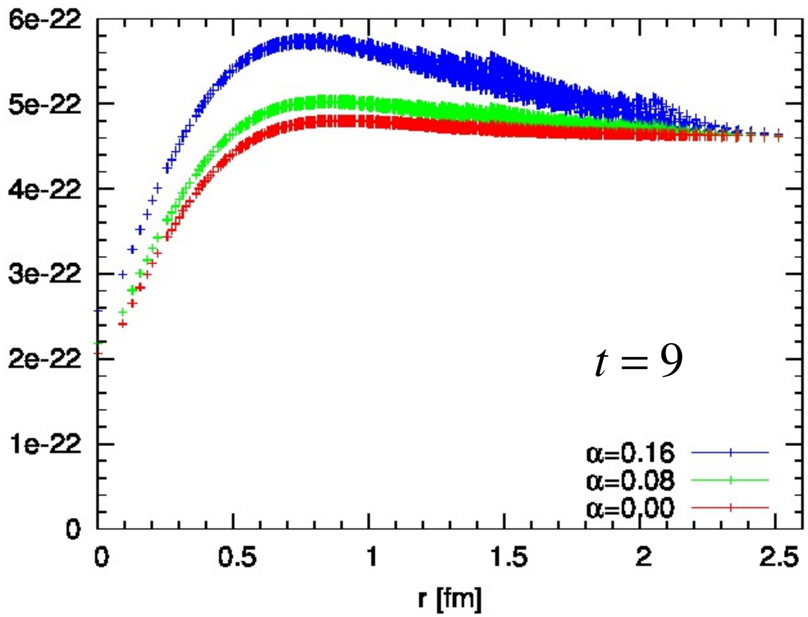
Only **Elastic saturation** is required to derive this equation.
(**Elastic saturation** is much easier than **single state saturation**.)

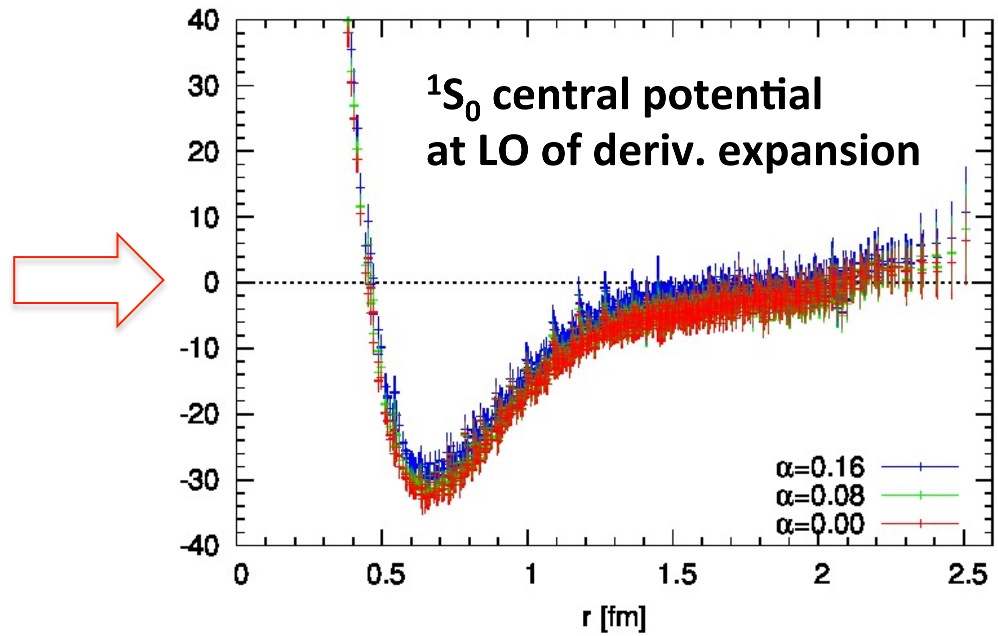
HALQCD method

Our potential is not affected by excited state contamination.

$$\langle 0|T[N(\vec{x},t)N(\vec{y},t)\cdot\overline{NN}(t=0;\alpha)]|0\rangle = \sum_n \psi_n(\vec{x}-\vec{y})\cdot a_n(\alpha)\cdot\exp(-E_n t)$$

$$V_C(\vec{x}) = -\frac{H_0 R(t,\vec{x})}{R(t,\vec{x})} - \frac{(\partial/\partial t)R(t,\vec{x})}{R(t,\vec{x})} + \frac{1}{4m_N}\frac{(\partial/\partial t)^2 R(t,\vec{x})}{R(t,\vec{x})}$$





Different mixture of NBS waves are generated by different α

$$f(x,y,z) = 1 + \alpha\left(\cos(2\pi x/L) + \cos(2\pi y/L) + \cos(2\pi z/L)\right)$$

HAL QCD method

[Murano et al., PTP125(2011)1225]

- General nonlocal potential is intractable ➔ We employ **derivative expansion**:

$$U(\vec{r},\vec{r}') \equiv V(\vec{r},\vec{\nabla})\delta(\vec{r}-\vec{r}')$$

$$V(\vec{r},\vec{\nabla}) \equiv V_C(r) + \underbrace{V_{ll}(r)\vec{L}^2 + \{V_{pp}(r),\nabla^2\}}_{O(\nabla^2)\ \text{term}} + O(\nabla^4)$$

Convergence of Derivative exp. can be checked by **E-dep. of potentials.**

(Example) $V(\vec{r},\vec{\nabla}) \equiv V_C(r) + O(\nabla^2)$ case:

We define

$$V_C(\vec{r};E) \equiv E - \frac{H_0\psi_E(\vec{r})}{\psi_E(\vec{r})}$$

⇔

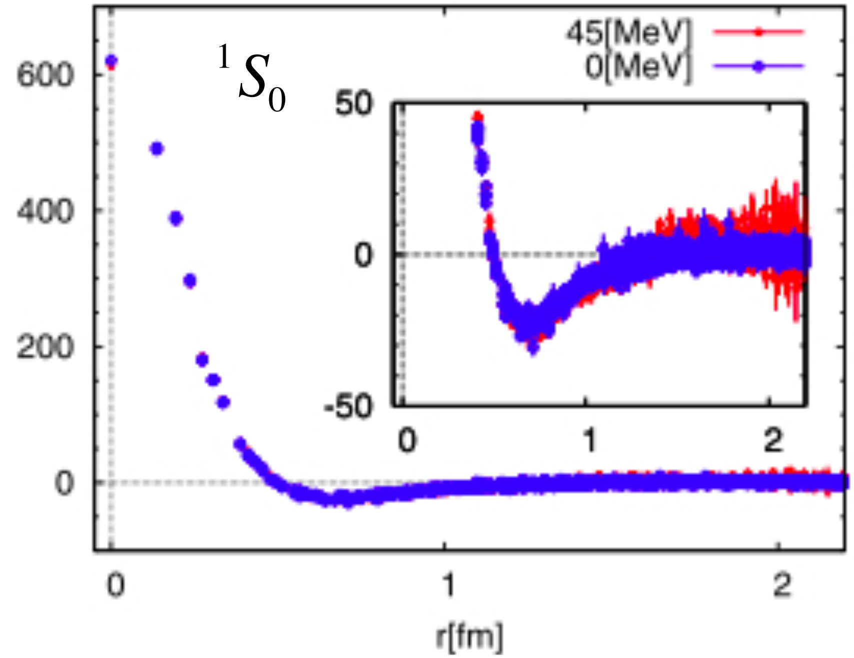
$$\left(k^2/m_N - H_0\right)\psi_E(\vec{r}) = V_C(r;E)\psi_E(\vec{r})$$

If $V_C(r;E)$ is E-indep. for $E_0 < E < E_1$,
then

$$V(\vec{r},\vec{\nabla}) \equiv V_C(r) + \cancel{O(\nabla^2)}$$

with $V_C(\vec{r}) \equiv V_C(\vec{r};E)$

➔ O(nabla^2) terms are negligible.



good agreement ! ➔ good convergence

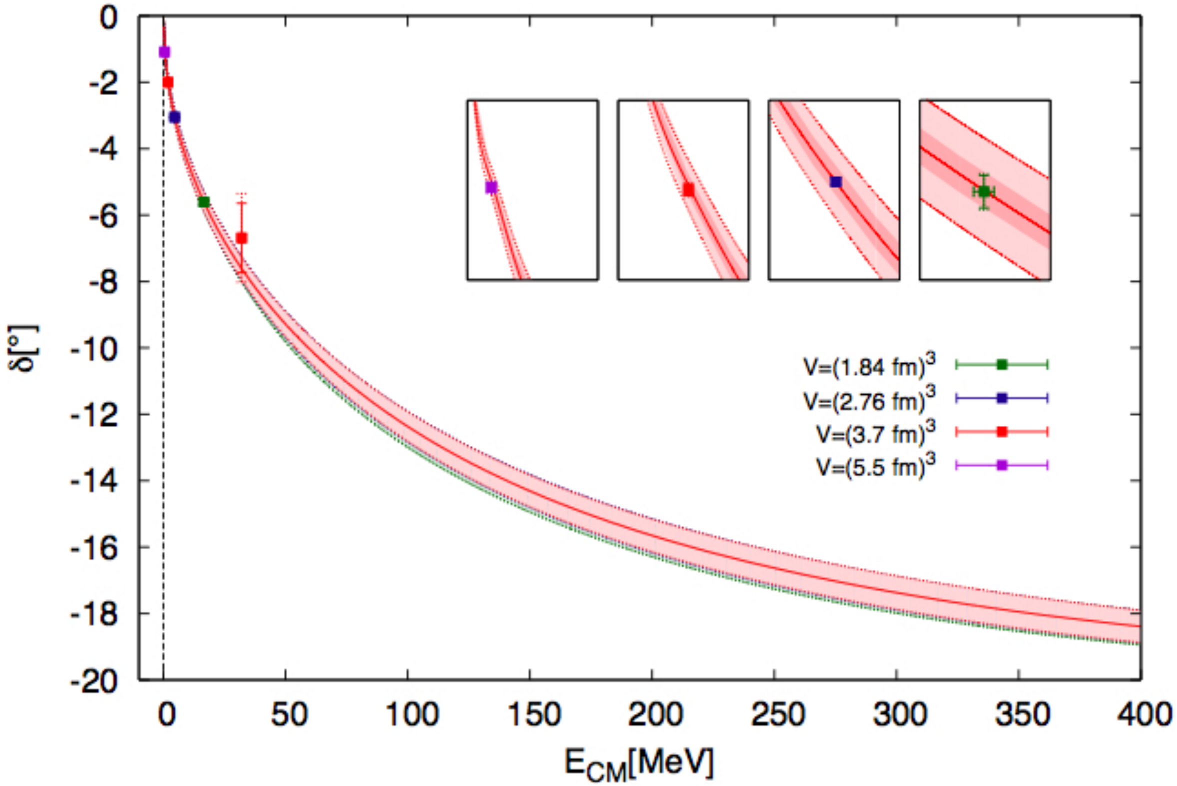
Comment: The current result is obtained based on an older method.
The result should be replaced by the new method. "time-dependent" Schrodinger-like eq.

HAL QCD method

◆ Comparison of **the HAL QCD method** and **Lueschcer's finite volume method.**

The standard method to calculate scattering phases in Lattice QCD

ππ scattering in I = 2 channel



Ns=16,24,32,48, Nt=128, a=0.115 fm.

m_{pi} = 940 MeV by Quenched QCD

Good agreement !

[Kurth et al., JHEP **1312**(2013)015.]

Nuclear Forces

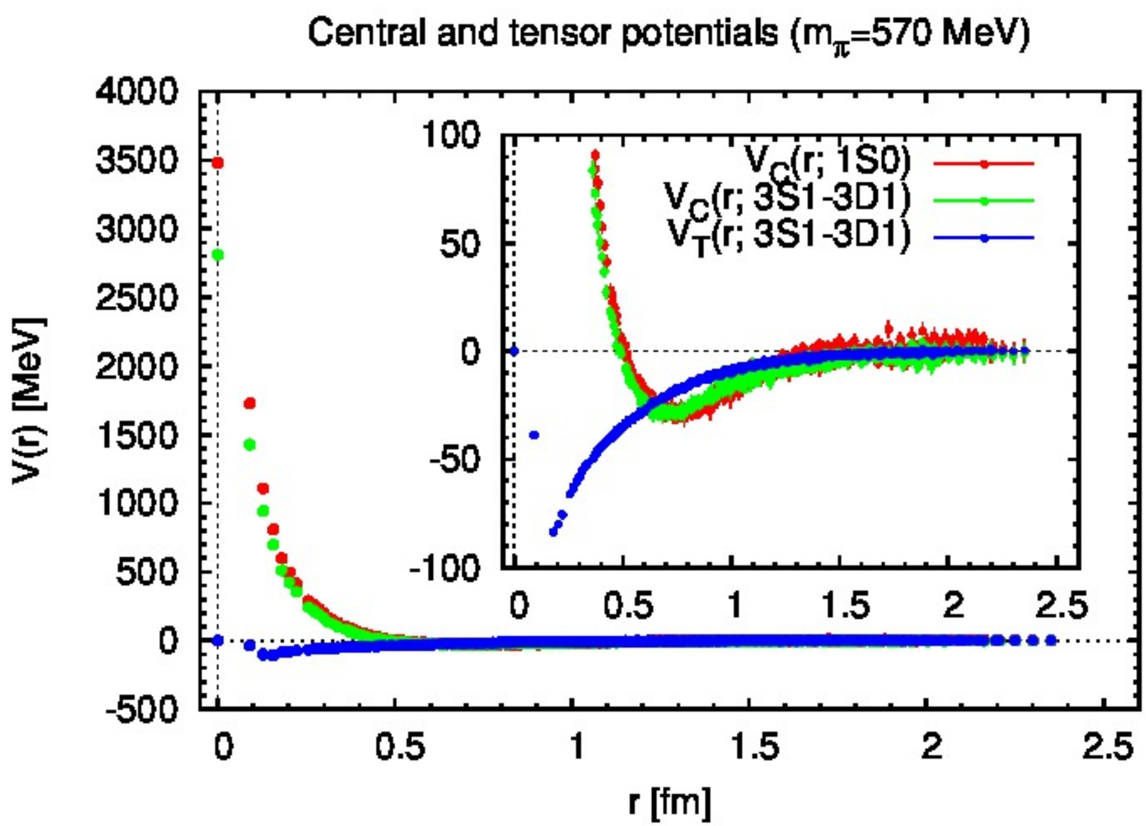
Nuclear Forces

Nuclear Force at LO (parity-even sector):

$$V_{NN} = V_{C;S=0}(r)\mathbb{P}^{(S=0)} + V_{C;S=1}(r)\mathbb{P}^{(S=1)} + V_T(r)S_{12} + O(\nabla)$$

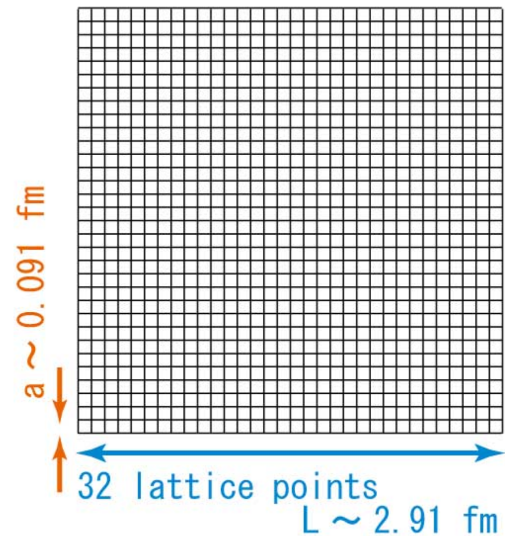
$$S_{12} \equiv 3(\hat{r}\cdot\vec{\sigma}_1)(\hat{r}\cdot\vec{\sigma}_2) - \vec{\sigma}_1\cdot\vec{\sigma}_2$$

2+1 flavor QCD result of nuclear forces at LO for m(pion)=570 MeV.



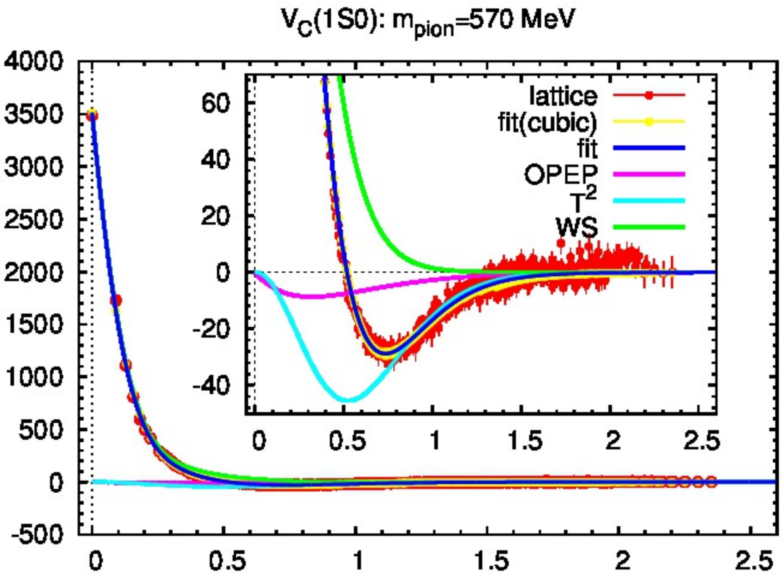
Japan Lattice Data Grid

2+1 flavor config by PACS-CS Coll.
m(pion) = 570 MeV, m(N)=1412MeV



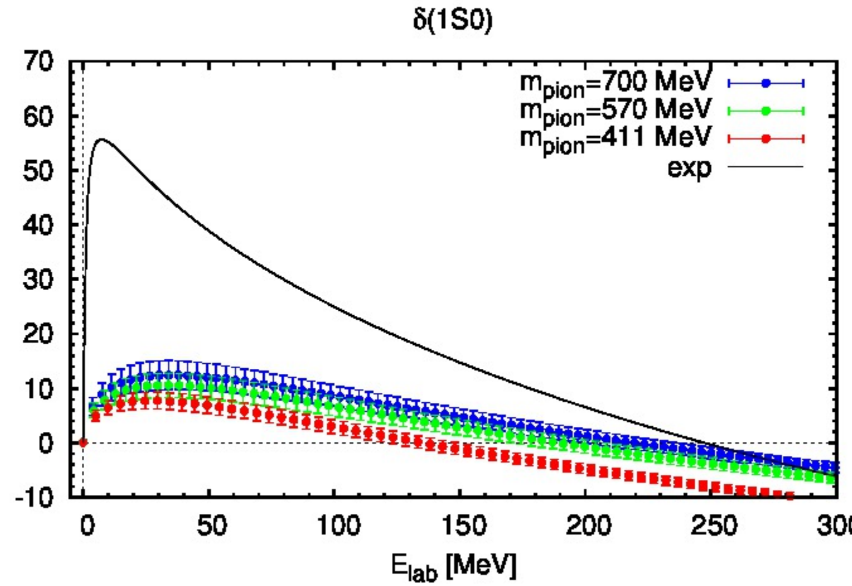
Nuclear Forces

Fit

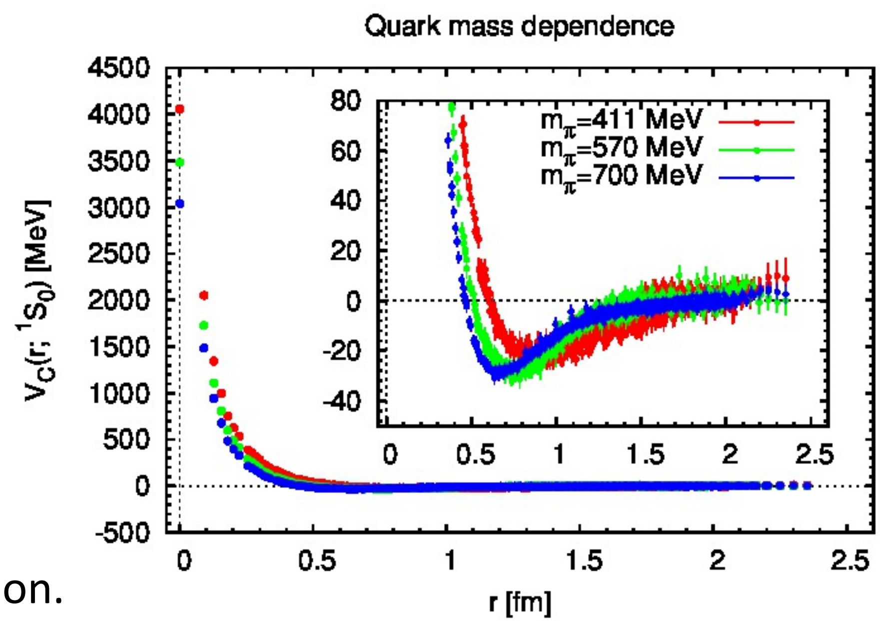


Schroedinger eq.

$^1S_0$ phase shift from Schrodinger eq.



- Qualitatively reasonable behavior. (Attractive. No bound state.) But the strength is significantly weak.
- Attraction shrinks as m_{pion} decreases.
  Reason:
  The **repulsive core** grows more rapidly than the **attractive pocket** in the region m_{pion} = 411-700 MeV.



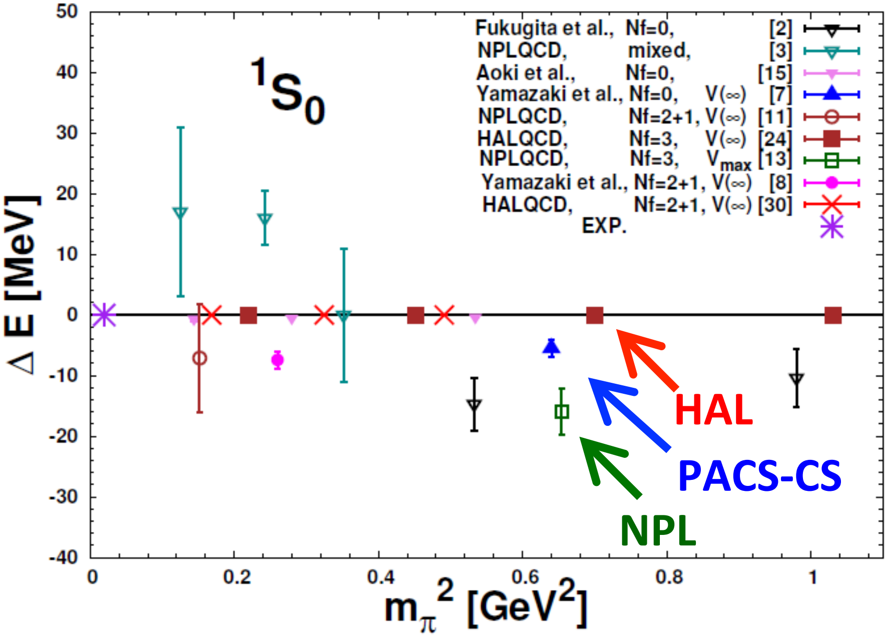
- It is important to go to smaller quark mass region.

Nuclear Forces

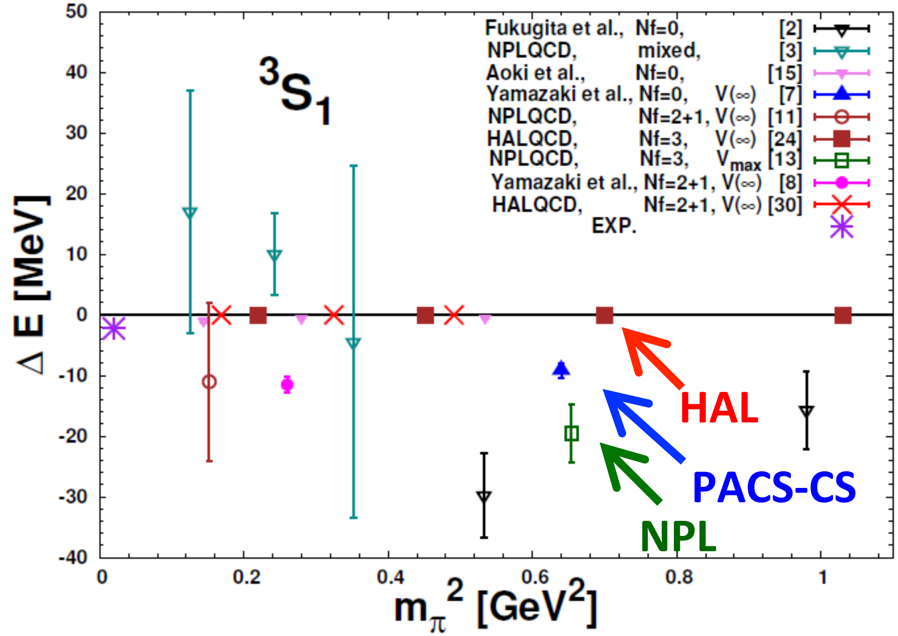
[T.Doi, PoS LAT2012,009]

Comparison with other collaborations (two-nucleon ΔE)

"di-neutron"



"deuteron"



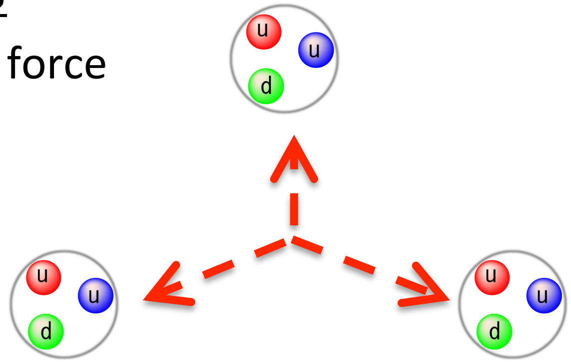
Comments:

YN/YY are also inconsistent between HAL and NPL

HAL: B.E.(H) = 37.8(3.1)(4.2) MeV
NPL: B.E.(H) = 74.6(3.3)(3.3)(0.8) MeV

Nuclear Forces

◆ Three-nucleon force



◆ Few body calculations shows its relevance
--- To understand qualitative trend,
two-nucleon force is enough.
For quantitative argument,
three-nucleon force is needed.

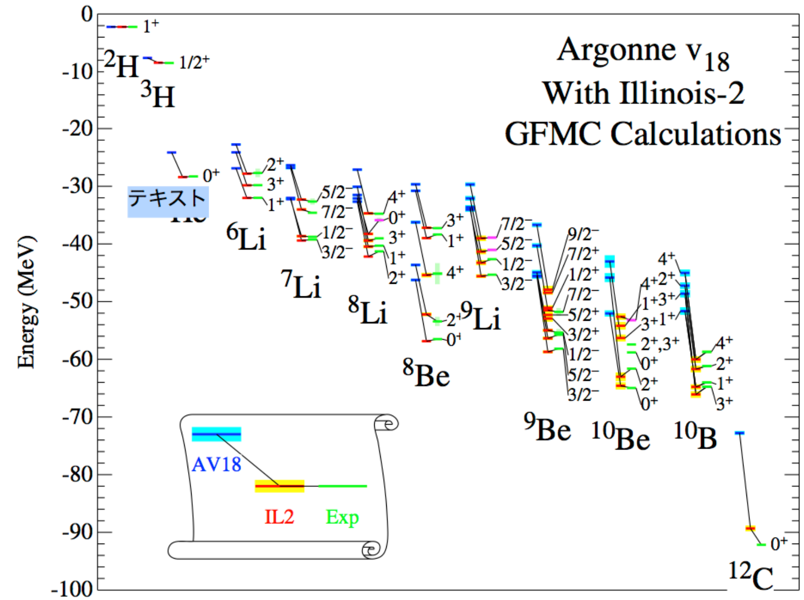
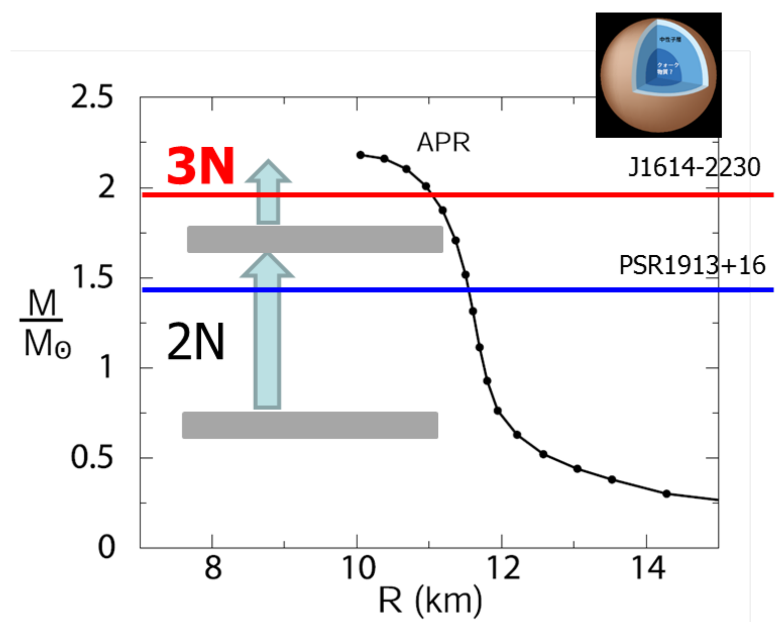


Fig. 3. – GFMC computations of energies for the AV18 and AV18+IL2 Hamiltonians compared with experiment.

◆ Important influence on neutron-rich nuclei.
---the magic number and the drip line.

◆ Important at higher density.
➔ supernova explosion and neutron star.

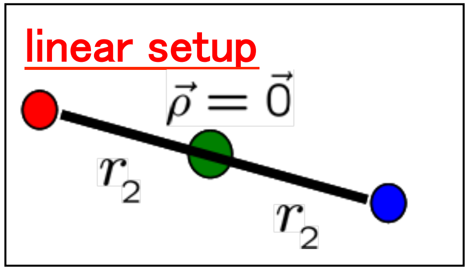


◆ Experimental information is limited

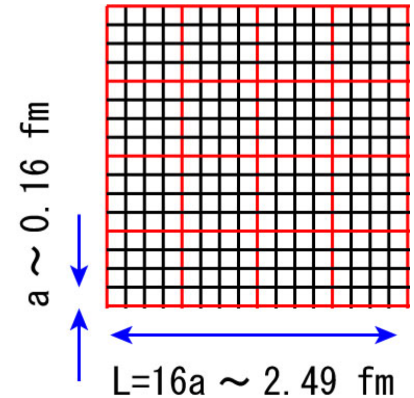
Nuclear Forces

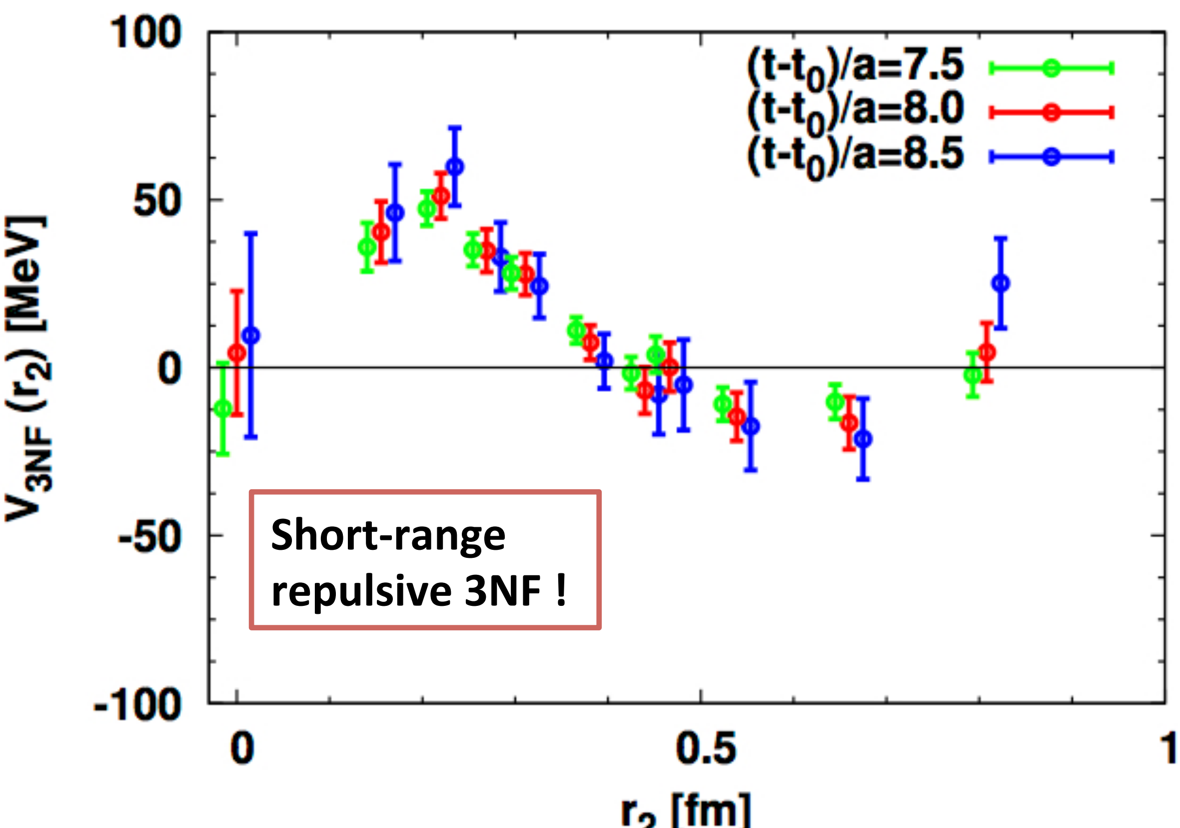
[T.Doi et al, PTP127,723(2012)]

Three-nucleon potential (on the linear setup)



2 flavor gauge config by CP-PACS Coll.
m(pion) = 1136 MeV, m(N) = 2165 MeV





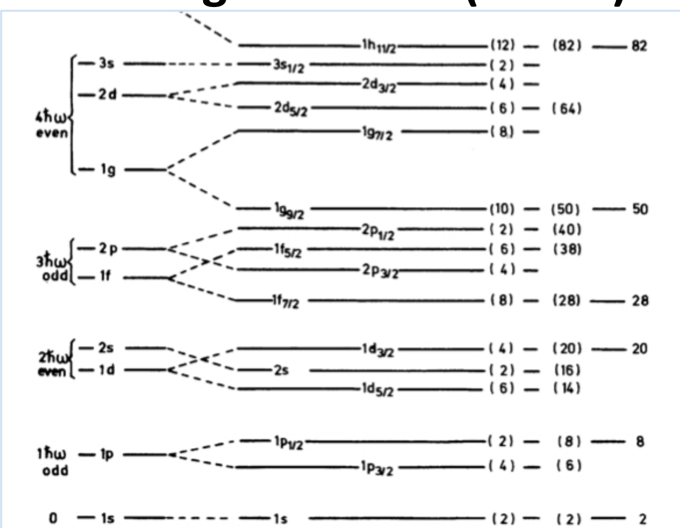
Nuclear Forces

◆ Nuclear Force up to NLO

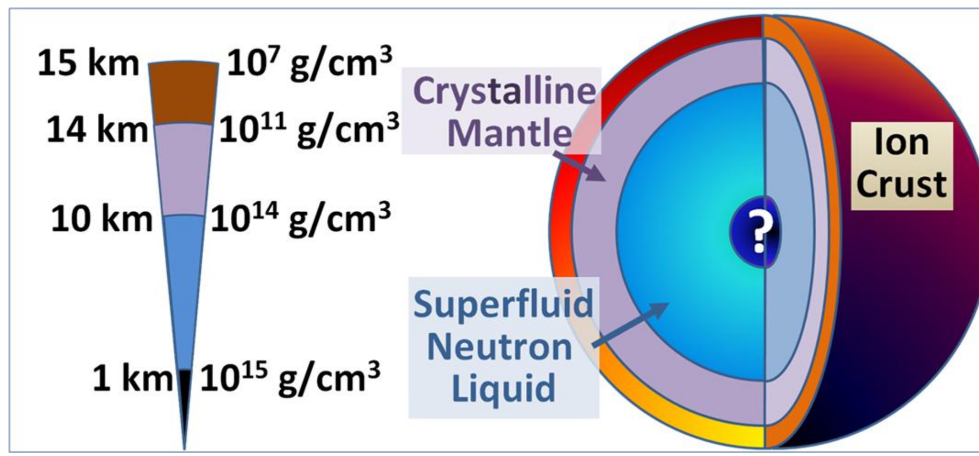
$$V^{(\pm)}(\vec{r},\vec{\nabla}) = \underbrace{V^{(\pm)}_{\mathrm{C;S=0}}(r)\mathbb{P}^{(S=0)} + V^{(\pm)}_{\mathrm{C;S=1}}(r)\mathbb{P}^{(S=1)} + V^{(\pm)}_{\mathrm{T}}(r)S_{12}(\hat{r})}_{\mathrm{LO}:O(\nabla^0)} + \underbrace{V^{(\pm)}_{\mathrm{LS}}(r)\vec{L}\cdot\vec{S}}_{\mathrm{NLO}:O(\nabla^1)} + O(\nabla^2)$$

◆ **Spin orbit (LS) force** is important in phenomenology.

Magic number (nuclei)



$^3P_2$ neutron superfuluid (neutron star cooling)



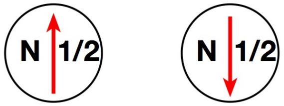
Momentum wall source

- Wall source:

$$\overline{\mathcal{J}}_{\alpha\beta} \equiv \sum_{\vec{x}_1,\cdots,\vec{x}_6} \bar{N}_\alpha(\vec{x}_1,\vec{x}_2,\vec{x}_3)\bar{N}_\beta(\vec{x}_4,\vec{x}_5,\vec{x}_6)$$

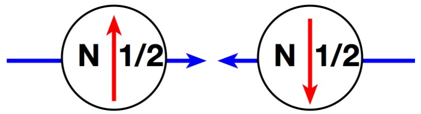
$$N_\alpha(x_1,x_2,x_3) \equiv \begin{cases} q_{abc}\left(u_a(x_1)C\gamma_5 d_b(x_2)\right)u_{c;\alpha}(x_3) & \text{(proton)} \\ q_{abc}\left(u_a(x_1)C\gamma_5 d_b(x_2)\right)d_{c;\alpha}(x_3) & \text{(neutron)} \end{cases}$$

accessible only to $J^P = A_1^+(\sim 0^+)$ and $T_1^+(\sim 1^+)$.



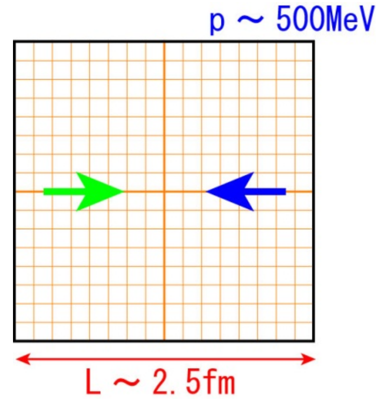
➔ Only LO potentials in even parity sector are calculable.

- Momentum wall source:
(Non-vanishing momentum **p** is carried by ``spectator quarks''.)



$$\overline{\mathcal{J}}_{\alpha\beta}(\vec{p}) \equiv \sum_{\vec{x}_1,\cdots,\vec{x}_6} \bar{N}_\alpha(\vec{x}_1,\vec{x}_2,\vec{x}_3)\bar{N}_\beta(\vec{x}_4,\vec{x}_5,\vec{x}_6)\cdot\exp\left(i\,\vec{p}\cdot(\vec{x}_3-\vec{x}_6)\right)$$

$$\overline{\mathcal{J}}^{\Gamma}_{\alpha\beta}(|\,\vec{p}\,|) \equiv \frac{1}{48}\sum_{g\in O_h}\chi^{(\Gamma)}(g^{-1})\cdot\overline{\mathcal{J}}_{\alpha'\beta'}(g\cdot\vec{p})S_{\alpha'\alpha}(g^{-1})S_{\beta'\beta}(g^{-1})$$



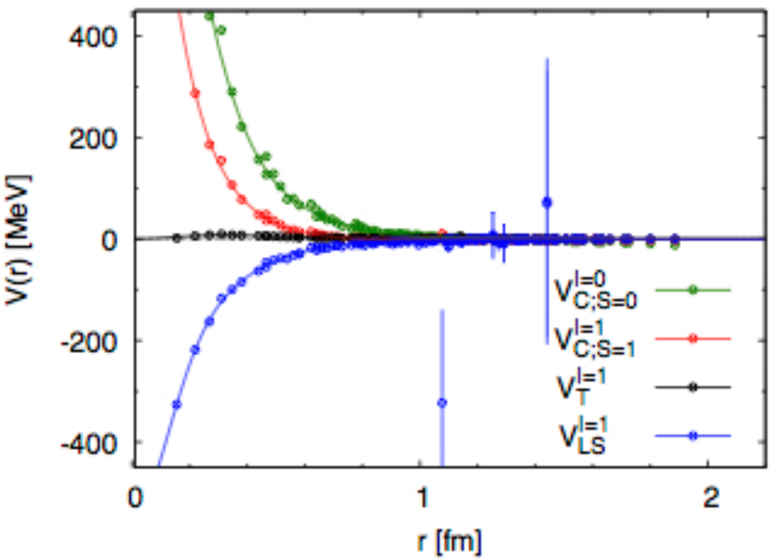
allows us to access varieties of **cubic group irreps. $J^P=\Gamma$.**

➔ Potentials beyond NLO can be calculable.

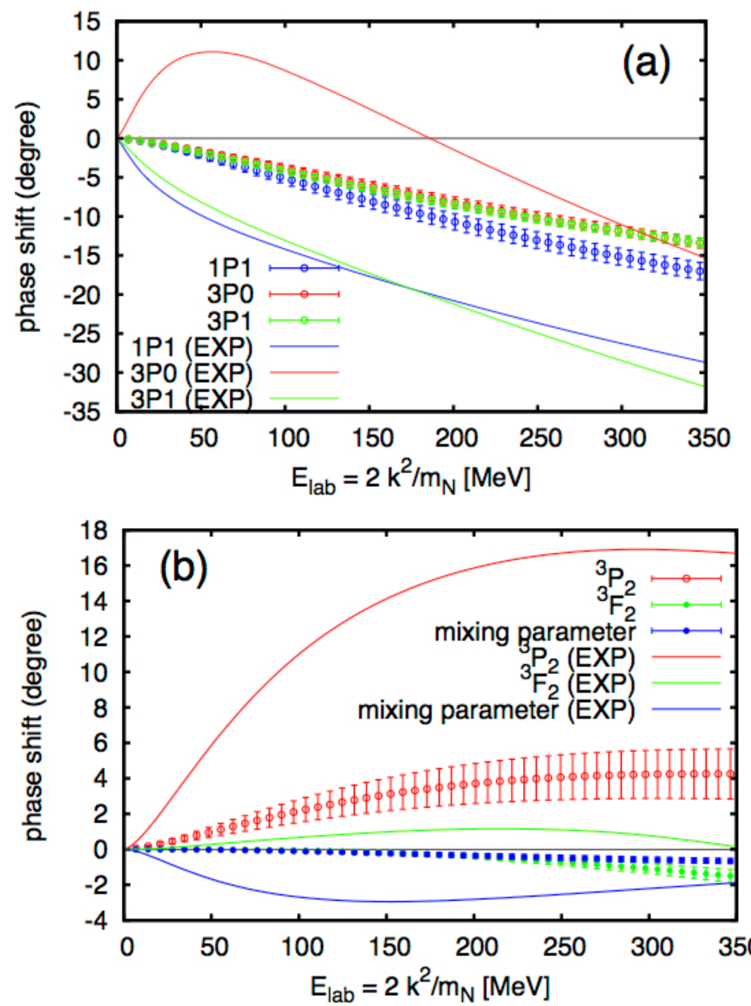
Nuclear Forces

[K.Murano et al., PLB735(2014)19.]

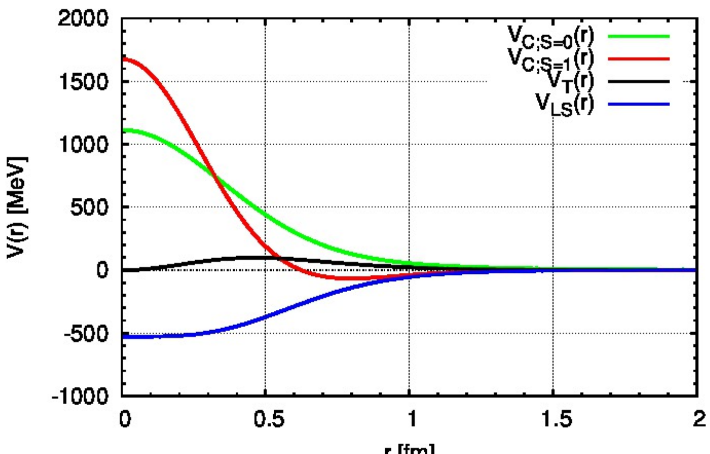
◆ Nuclear forces and **LS force** in **parity-odd** sector



Scattering phase shifts

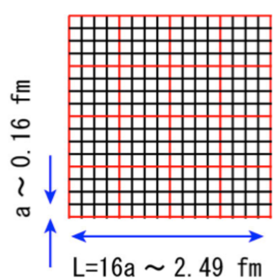


Phemenological one (AV18)
for comparison



ILDG

2 flavor gauge config
by CP-PACS Coll.
m(pion) = 1136 MeV,
m(N) = 2165 MeV



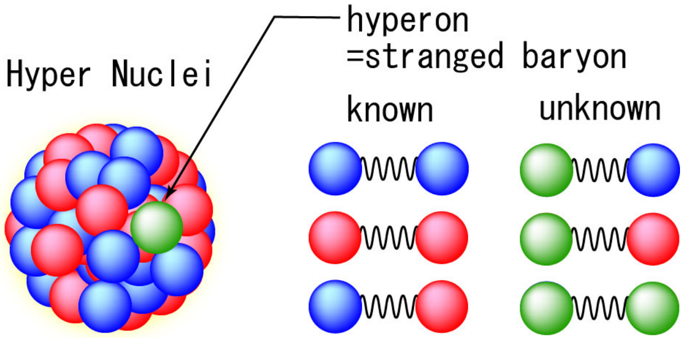
Hyperon Forces

Hyperon Forces

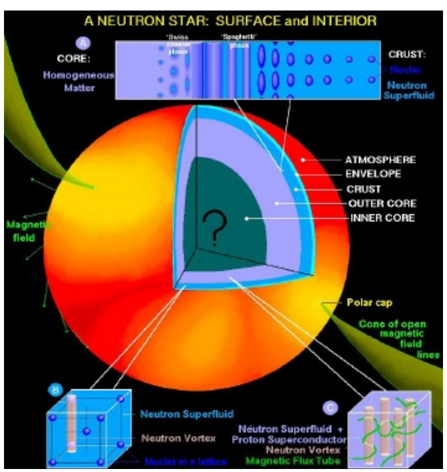
Our best target is hyperon force.

- ◆ Experimental information is limited due to the short life time of hyperons.

- ◆ Structure of hypernuclei

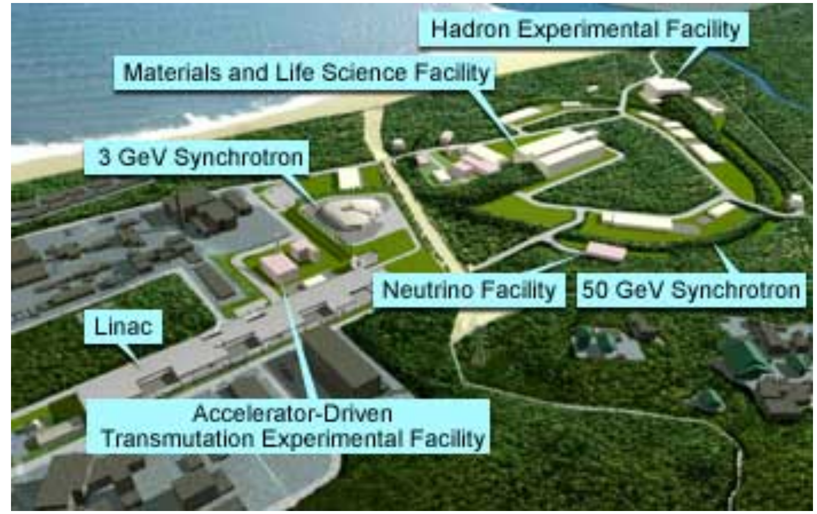


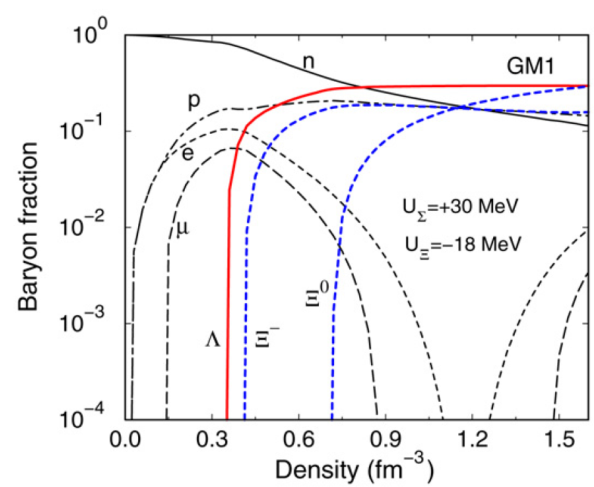
- ◆ Eq. of state of hyperon matter



J-PARC

Exploration of multi-strangeness world



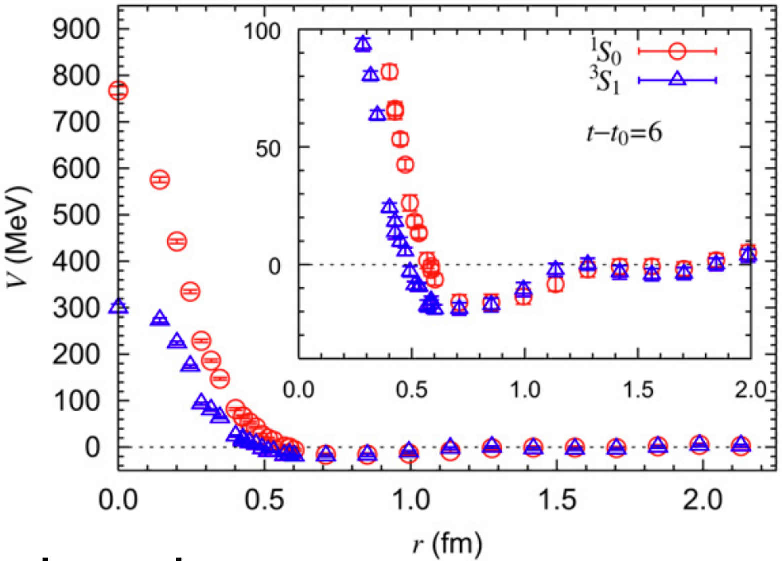


J.Schaffner-Bielich, NPA804('08)309.

Hyperon Forces

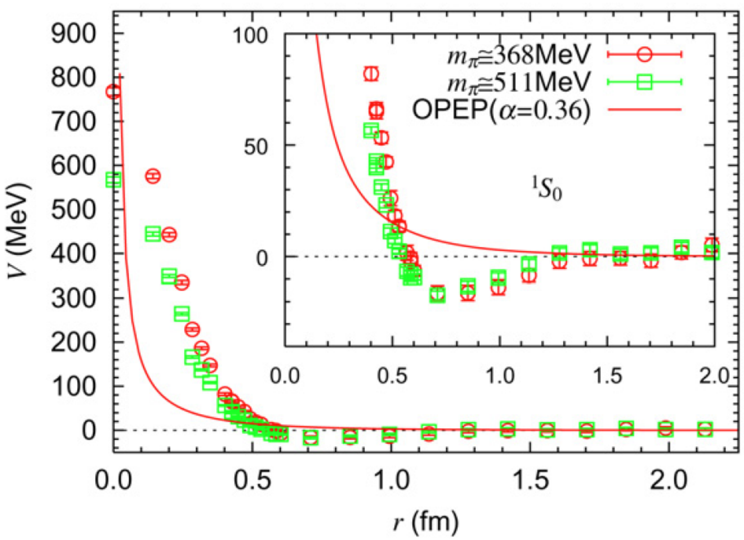
Nemura, Ishii, Aoki, Hatsuda, PLB673(2009)136.

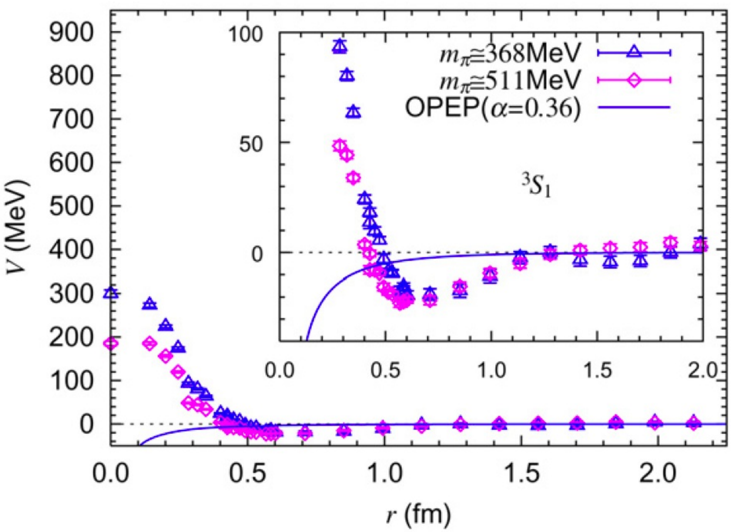
$\Xi N(I=1)$



- Repulsive core is surrounded by attraction like NN case.
- Strong spin dependence of repulsive core.

quark mass dependence





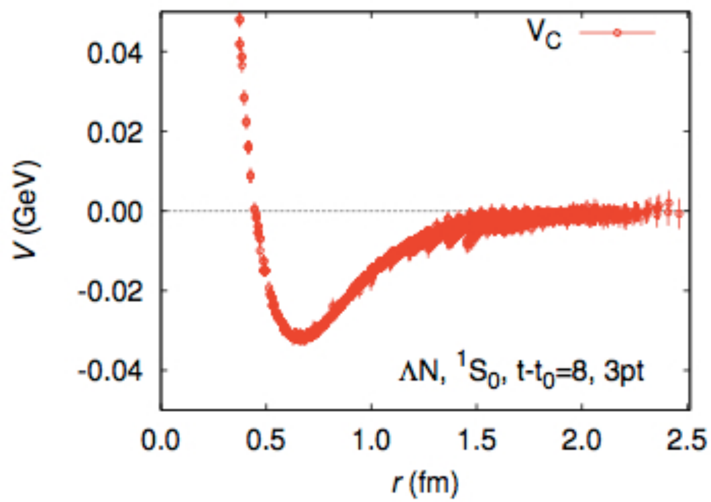
Repulsive core grows with decreasing quark mass.
No significant change in the attraction.

Hyperon Forces

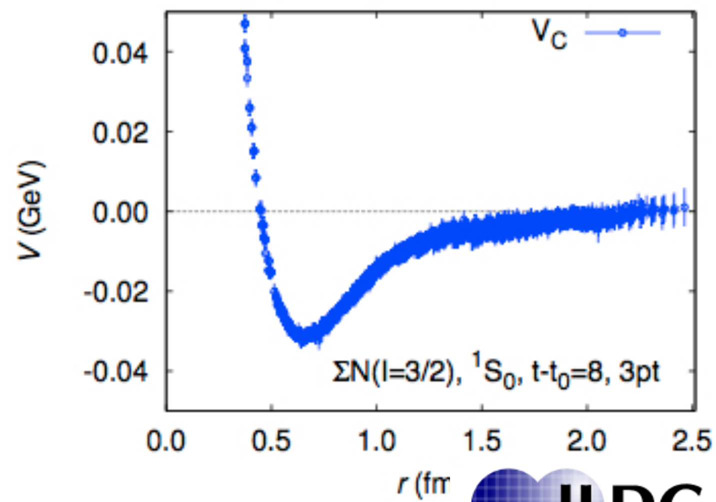
Spin-singlet sector

$^1S_0$

ΛN



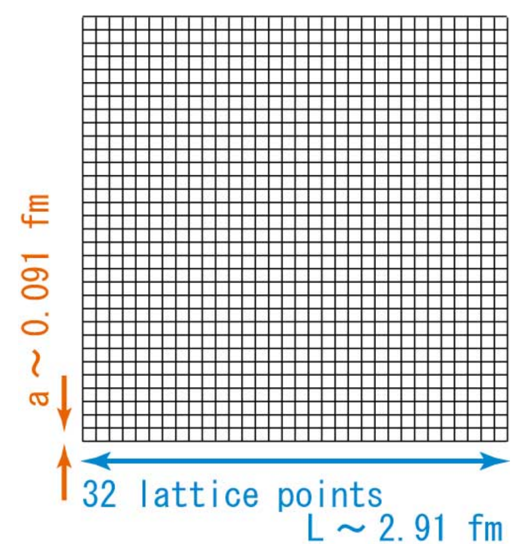
$\Sigma N(I = 3/2)$



ILDG Japan Lattice Data Grid

2+1 flavor config by PACS-CS Coll.
m(pion) = 570 MeV, m(N)=1412MeV

- Repulsive core is surrounded by attraction like NN case.
- These two potentials looks similar.
  (This may be due to small flavor SU(3) breaking.)



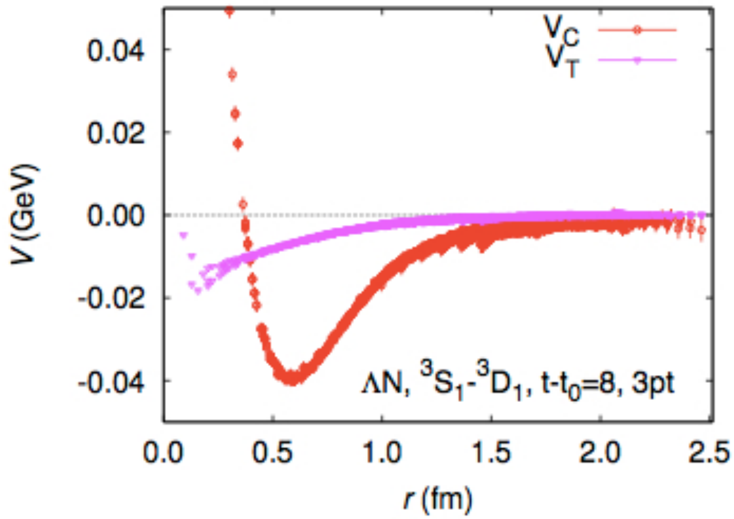
Hyperon Forces

[Nemura,PoS(LAT2011)]

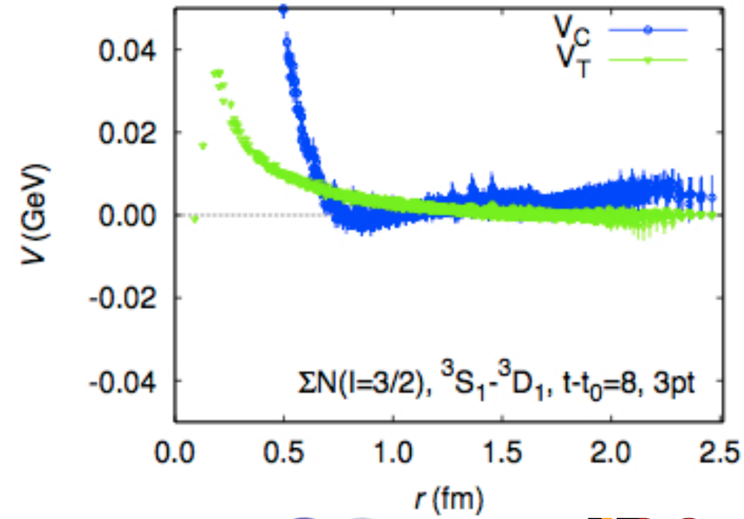
Spin-triplet sector

$^3S_1 - ^3D_1$

ΛN

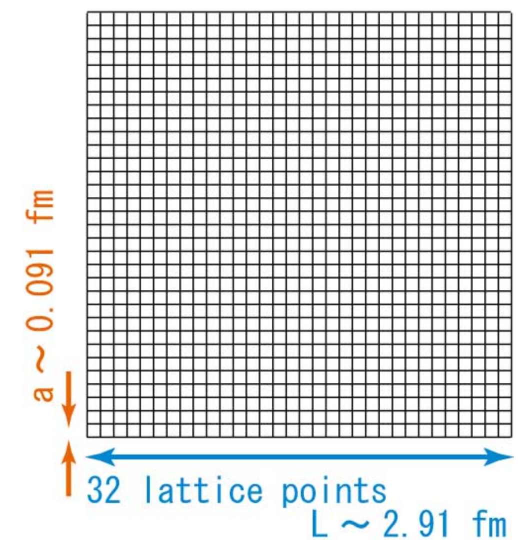


$\Sigma N(I = 3/2)$



ILDG JLDG Japan Lattice Data Grid

2+1 flavor config by PACS-CS Coll.
m(pion) = 570 MeV, m(N)=1412MeV



- N-Lambda
  - Repulsive core is surrounded by attraction
  - The attraction is deeper than 1S0
  - Weak tensor force (no one-pion exchange is allowed)

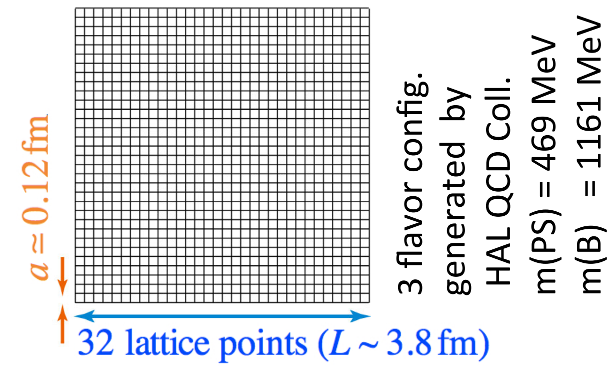
- N-Sigma
  - Repulsive core at short distance
  - No clear attractive well
    (Repulsive nature is consistent with the quark model)

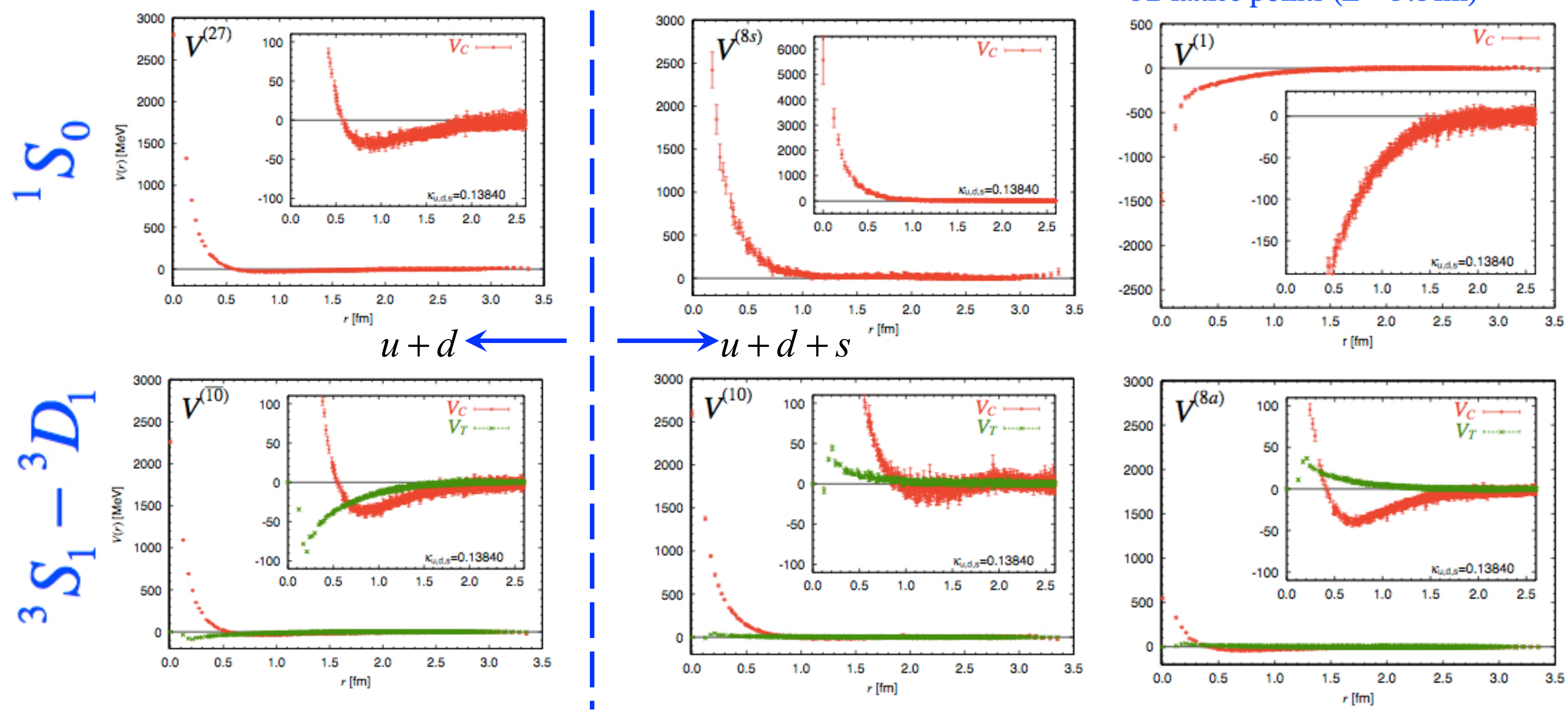
Hyperon Forces

[T.Inoue et al, PTP124,591(2010)]

Flavor SU(3) limit to understand a general trend.

$$\underbrace{8\otimes 8 = 27\oplus 8_S\oplus 1}_{\text{symmetric}}\oplus \underbrace{\overline{10}\oplus 10\oplus 8_A}_{\text{anti-symmetric}}$$



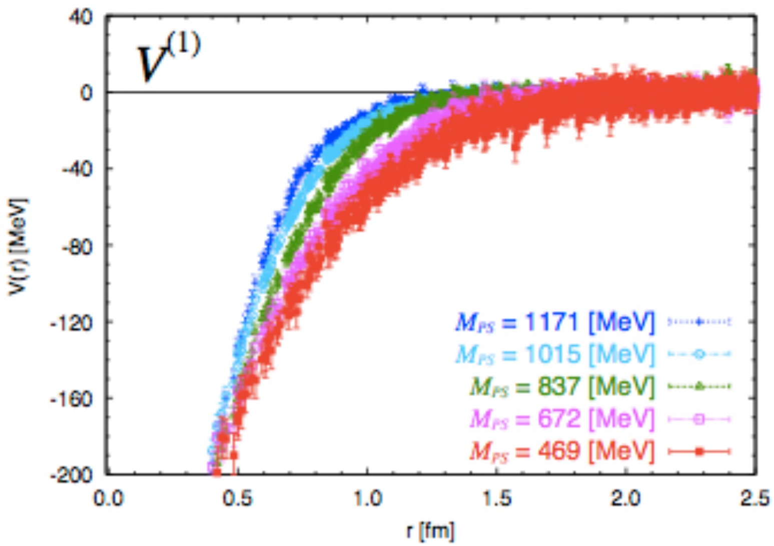


- Strong flavor dependence
- These short distance behaviors are consistent with quark Pauli blocking picture.

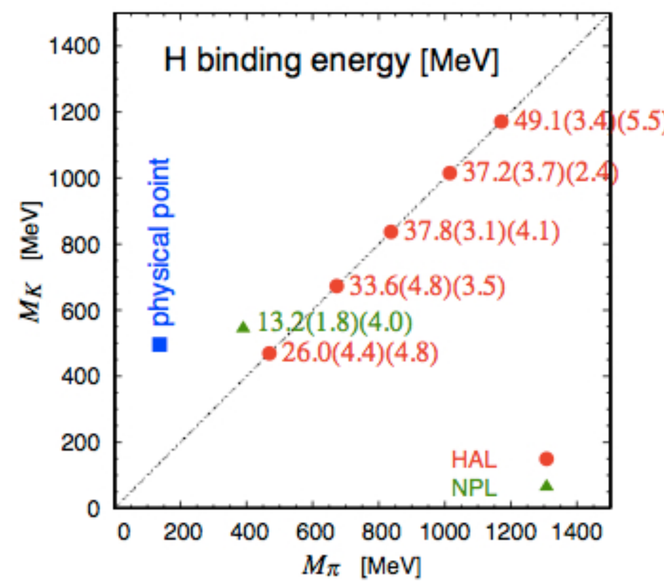
Hyperon Forces

[T.Inoue et al., PRL106(2011)162002.]

◆ Bound H-dibaryon in flavor SU(3) limit



Entirely attractive potential in flavor 1 channel leads to a bound H-dibaryon

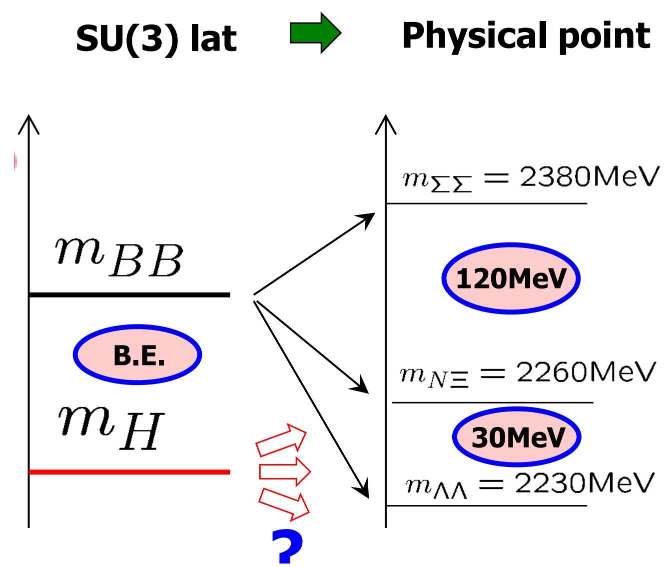


◆ Flavor SU(3) breaking for real world.

➔ BB threshold in flavor SU(3) limit splits into three, i.e., ΛΛ, NΞ, ΣΣ thresholds

➔ Coupled channel system of

$$\Lambda\Lambda - N\Xi - \Sigma\Sigma$$



Hyperon Forces

[S.Aoki et al., Proc.Japan Acad.B87(2011)509.]

- In finite volume,
  it is not possible to impose incoming B.C.'s separately.

$$|n,in\rangle = |\Lambda\Lambda,in\rangle, |N\Xi,in\rangle, |\Sigma\Sigma,in\rangle$$

➔ **Coupled ch. extension** of **the finite volume method** is **NOT straightforward**.

- **Coupled ch. extension** of **HAL QCD method** is **straightforward.**

$$\Psi_n(\vec{x}-\vec{y}) \equiv \begin{bmatrix} \langle 0|\Lambda(\vec{x})\Lambda(\vec{y})|n,in\rangle \\ \langle 0|N(\vec{x})\Xi(\vec{y})|n,in\rangle \\ \langle 0|\Sigma(\vec{x})\Sigma(\vec{y})|n,in\rangle \end{bmatrix}$$

$$\begin{cases} E \equiv 2\sqrt{m_\Lambda^2 + \vec{p}_{\Lambda\Lambda}^2} \\ \quad = \sqrt{m_N^2 + \vec{p}_{N\Xi}^2} + \sqrt{m_\Xi^2 + \vec{p}_{N\Xi}^2} \\ \quad = 2\sqrt{m_\Sigma^2 + \vec{p}_{\Sigma\Sigma}^2} \end{cases}$$

"Coupled channel Schrodinger eq."

$$\begin{bmatrix} \left(\frac{\vec{p}_{\Lambda\Lambda}^2}{2\mu_{\Lambda\Lambda}} + \frac{\Delta}{2\mu_{\Lambda\Lambda}}\right)\psi_{\Lambda\Lambda}(\vec{r};n) \\ \left(\frac{\vec{p}_{N\Xi}^2}{2\mu_{N\Xi}} + \frac{\Delta}{2\mu_{N\Xi}}\right)\psi_{N\Xi}(\vec{r};n) \\ \left(\frac{\vec{p}_{\Sigma\Sigma}^2}{2\mu_{\Sigma\Sigma}} + \frac{\Delta}{2\mu_{\Sigma\Sigma}}\right)\psi_{\Sigma\Sigma}(\vec{r};n) \end{bmatrix} = \int d^3r' \begin{bmatrix} U_{\Lambda\Lambda;\Lambda\Lambda}(\vec{r},\vec{r}') & U_{\Lambda\Lambda;N\Xi}(\vec{r},\vec{r}') & U_{\Lambda\Lambda;\Sigma\Sigma}(\vec{r},\vec{r}') \\ U_{N\Xi;\Lambda\Lambda}(\vec{r},\vec{r}') & U_{N\Xi;N\Xi}(\vec{r},\vec{r}') & U_{N\Xi;\Sigma\Sigma}(\vec{r},\vec{r}') \\ U_{\Sigma\Sigma;\Lambda\Lambda}(\vec{r},\vec{r}') & U_{\Sigma\Sigma;N\Xi}(\vec{r},\vec{r}') & U_{\Sigma\Sigma;\Sigma\Sigma}(\vec{r},\vec{r}') \end{bmatrix} \cdot \begin{bmatrix} \psi_{\Lambda\Lambda}(\vec{r}';n) \\ \psi_{N\Xi}(\vec{r}';n) \\ \psi_{\Sigma\Sigma}(\vec{r}';n) \end{bmatrix}$$

- ☐ U(r,r') is state-independent, i.e.,
  It works for any linear combinations |n,in> = |ΛΛ,in>α+ |NΞ,in>β+ |ΣΣ,in>γ.
  ➔ We can extract U(r,r') in the **finite** volume.

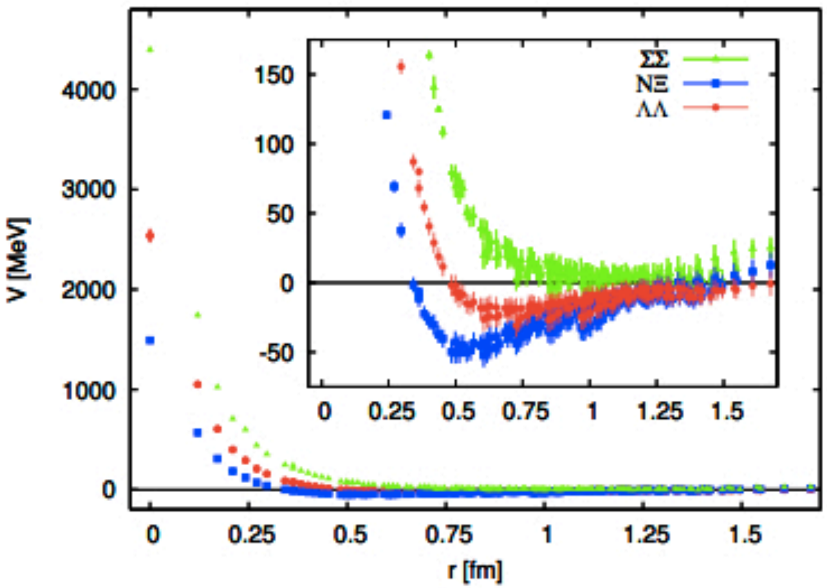
- ☐ We use U(r,r') in the **inifinite** volume
  to obtain the NBS wave funcs. of these states separately. ➔ **S-matrix**.

Hyperon Forces

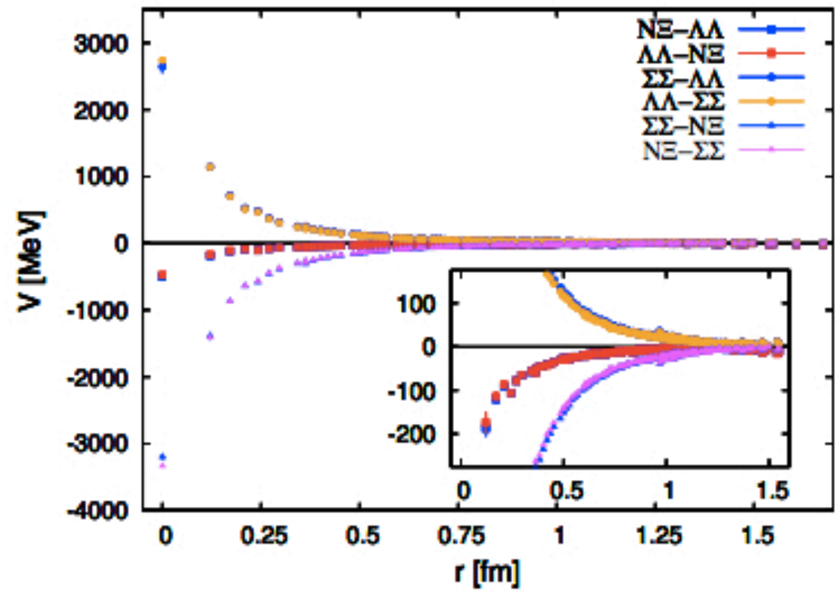
[K.Sasaki@Lattice2012,
K.Sasaki et al., coming soon]

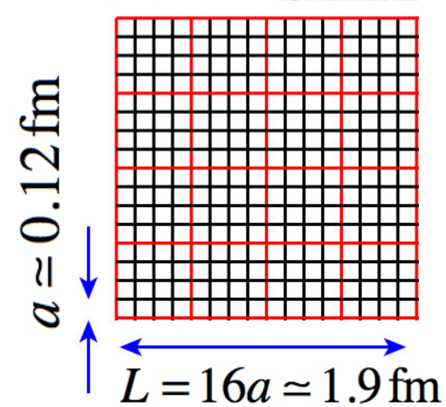
The numerical calculation is tough.
But it is doable.

diagonal part



off-diagonal part





2+1 flavor gauge config
by CP-PACS/JLQCD Coll.
m(pion) = 875 MeV
m(K) = 916 MeV
m(N) = 1806 MeV
m(Lambda) = 1835 MeV
m(Sigma) = 1841 MeV
m(Xi) = 1867 MeV

Hyperon Forces

- ΛN forces up to NLO

$$V_{\Lambda N} = V_{\mathrm{C;S=0}}(r)\mathbb{P}^{(S=0)} + V_{\mathrm{C;S=1}}(r)\mathbb{P}^{(S=1)} + V_{\mathrm{T}}(r)\left(3(\hat{r}\cdot\vec{\sigma}_\Lambda)(\hat{r}\cdot\vec{\sigma}_N) - \vec{\sigma}_\Lambda\cdot\vec{\sigma}_N\right)$$

$$+V_{\mathrm{LS}}(r)\vec{L}\cdot(\vec{s}_\Lambda + \vec{s}_N) + \underbrace{V_{\mathrm{ALS}}(r)\vec{L}\cdot(\vec{s}_\Lambda - \vec{s}_N)}_{\text{NEW TERM: Anti-symmetric LS}} + O(\nabla^2)$$

- Spin-orbit puzzle in ΛN sector

Convensional Nuclei

Large Spin-Orbit Splitting

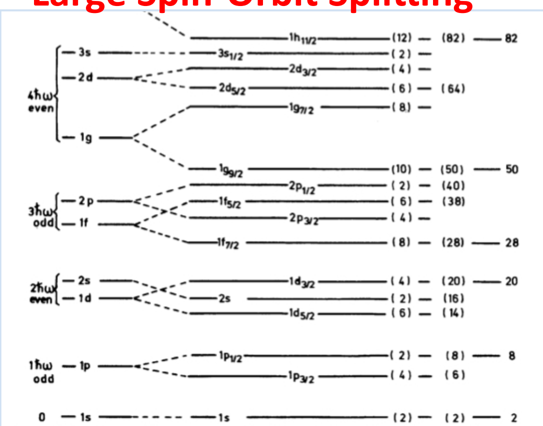
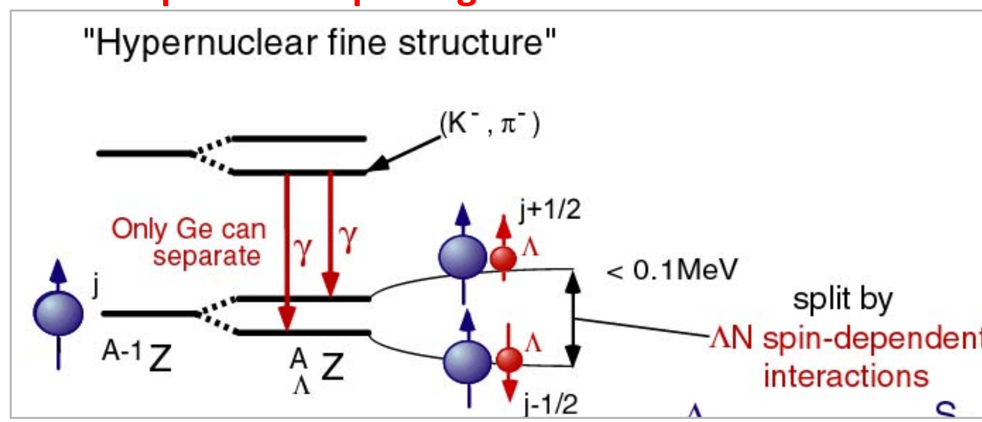


Figure 2.5. Schematic nuclear levels of the shell model with spin orbit term.

Λ Hyper Nuclei

Small Spin-Orbit Splitting for Λ



- Λ-spin dependent Spin-orbit force

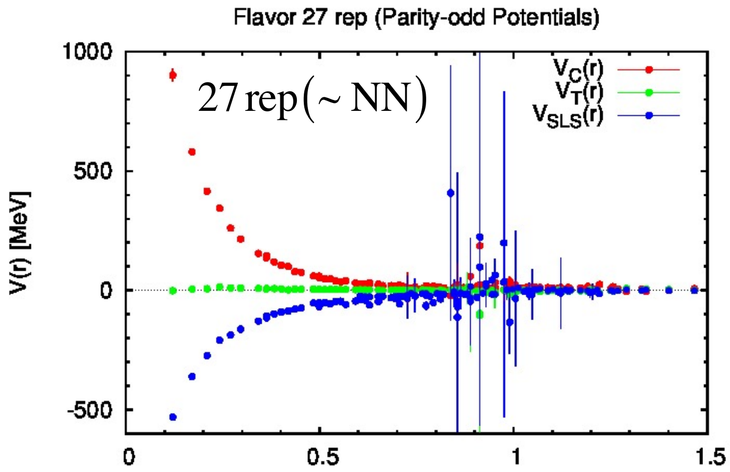
$$V_{\mathrm{LS}}^{(\Lambda)}(r) \equiv V_{\mathrm{LS}}(r) + V_{\mathrm{ALS}}(r) \sim 0 \quad \rightarrow \quad \text{LS-ALS cancellation}$$

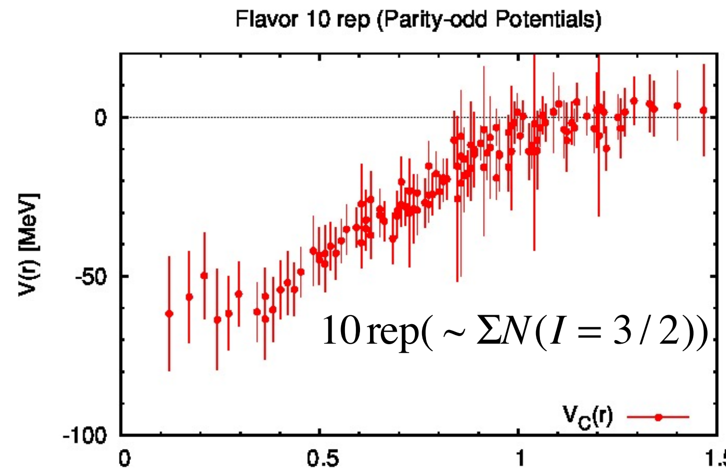
✧ Quark model → Strong cancellation

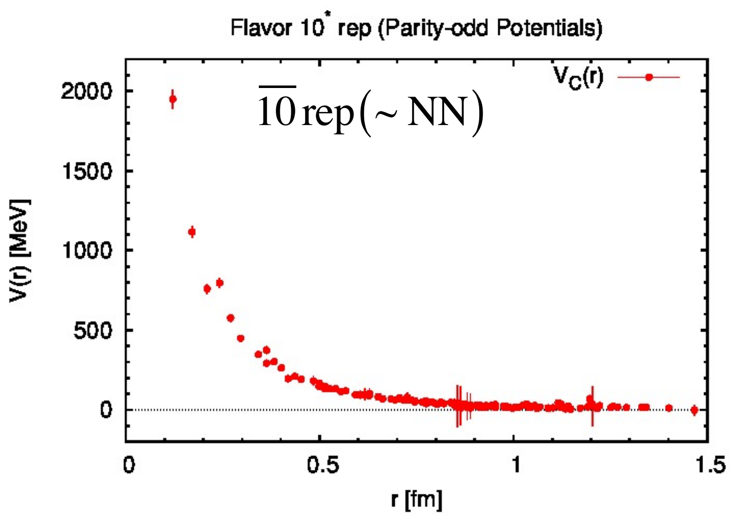
✧ Meson exch. Model → Weak cancellation

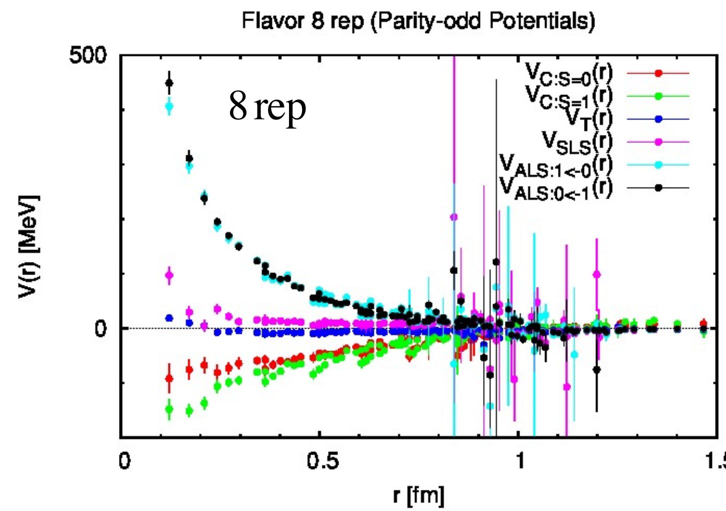
Hyperon Forces

Parity-odd hyperon potentials in the **flavor SU(3) limit**. [N.Ishii@Lattice 2013]







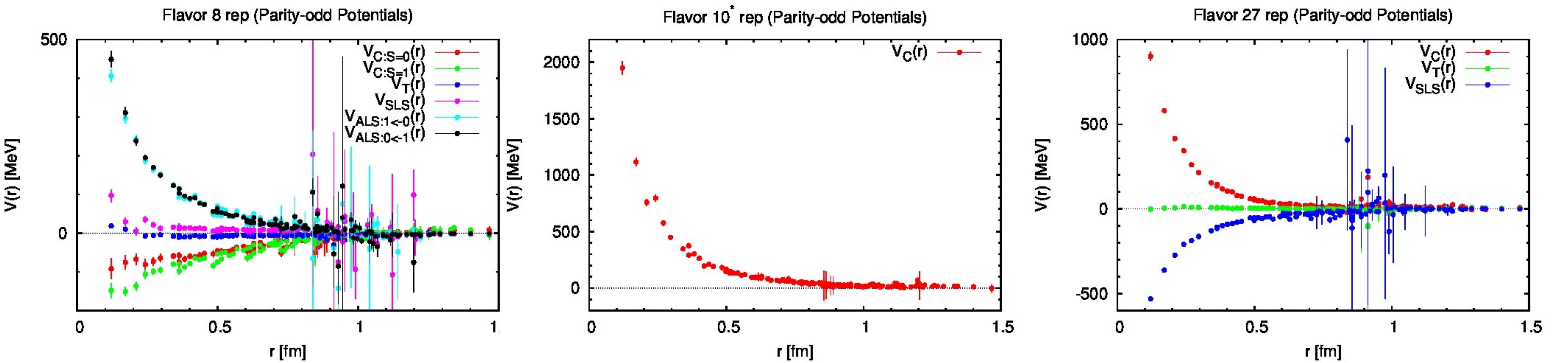


- Repulsive core for irreps. 27 and 10^* (∼NN). (consistent with quark model)
- Strong LS for irrep. 27 (∼NN). Weak LS for irrep. 8.
- Strong anti-symmetric LS (irrep. 8).

No repulsive core for irreps. 10 and 8.

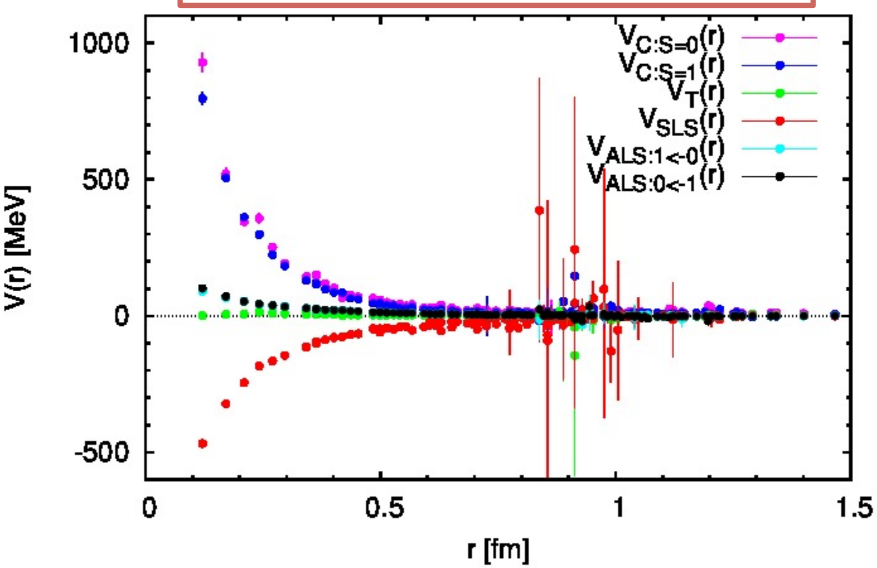
Parity-odd ΛN potential

- Flavor SU(3) irreps. : 8, 10^*, 27 ➔ potential



$$V_{\Lambda N} = \left(\frac{1}{2}V_{\mathrm{C}}^{(\overline{10})} + \frac{1}{2}V_{\mathrm{C;S=0}}^{(8)}\right)\mathbb{P}^{(S=0)} + \left(\frac{1}{10}V_{\mathrm{C:S=1}}^{(8)} + \frac{9}{10}V_{\mathrm{C}}^{(27)}\right)\mathbb{P}^{(S=1)} + \left(\frac{1}{10}V_{\mathrm{T}}^{(8)} + \frac{9}{10}V_{\mathrm{T}}^{(27)}\right)S_{12}(\hat{r}) + \left(\frac{1}{10}V_{\mathrm{LS}}^{(8)} + \frac{9}{10}V_{\mathrm{LS}}^{(27)}\right)\vec{L}\cdot\vec{S}_{+} + \frac{1}{2\sqrt{5}}V_{\mathrm{ALS}}^{(8)}\cdot\vec{L}\cdot\vec{S}_{-}$$

ΛN potential (odd parity)



- ΛN component in ΛN-ΣN coupled channel potential
- Strong symmetric LS potential
  It comes from 27 irrep. (90%), i.e., NN LS

$$V_{\mathrm{LS}}^{(\Lambda N)} = \frac{1}{10}V_{\mathrm{LS}}^{(8)} + \frac{9}{10}V_{\mathrm{LS}}^{(27)}$$

- Weak anti-symmetric LS potential
  SU(3) Clebsch-Gordan factor: 1/(2*sqrt(5))

$$V_{\mathrm{ALS}}^{(\Lambda N)} = \frac{1}{2\sqrt{5}}V_{\mathrm{ALS}}^{(8)}$$

ΣN component in anti-symmetric LS potential large

- ΛN-ΣN coupled channel potential

$$\begin{pmatrix} V_{\Lambda N,\Lambda N} & V_{\Lambda N,\Sigma N} \\ V_{\Sigma N,\Lambda N} & V_{\Sigma N,\Sigma N} \end{pmatrix} = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix} V_{\mathrm{C}}^{(10^*)}(r)\mathbb{P}^{(S=0)} + \begin{pmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{pmatrix} V_{\mathrm{C}}^{(8A)}(r)\mathbb{P}^{(S=0)}$$

$$+ \begin{pmatrix} 1/10 & 3/10 \\ 3/10 & 9/10 \end{pmatrix} \left(V_{\mathrm{C}}^{(8S)}(r)\mathbb{P}^{(S=1)} + V_{\mathrm{T}}^{(8S)}(r)S_{12}(\hat{r}) + V_{\mathrm{LS}}^{(8S)}(r)\vec{L}\cdot\vec{S}_+ \right)$$

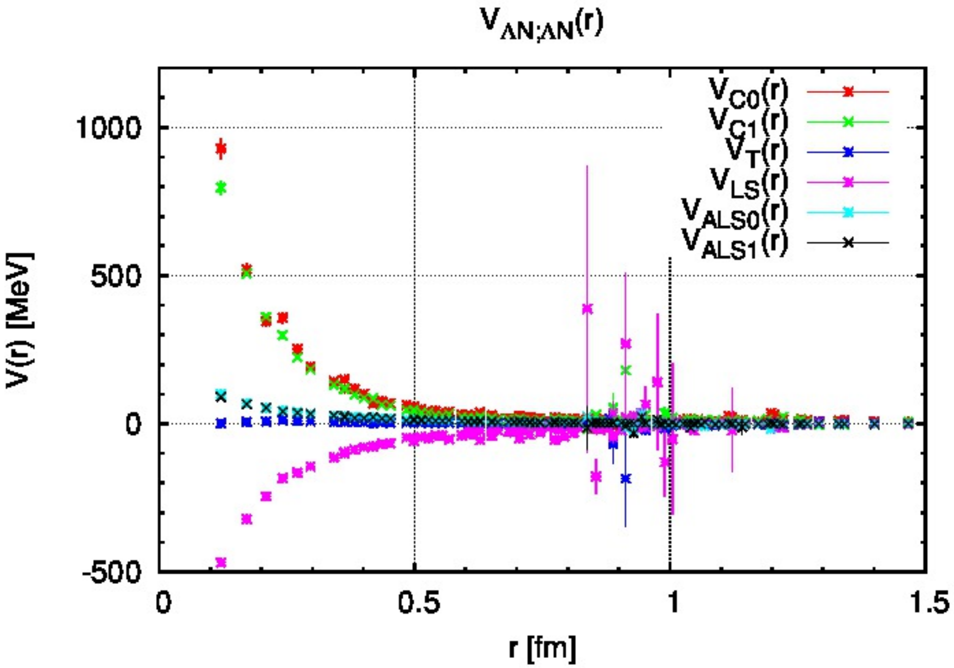
$$+ \begin{pmatrix} 9/10 & -3/10 \\ -3/10 & 1/10 \end{pmatrix} \left(V_{\mathrm{C}}^{(27)}(r)\mathbb{P}^{(S=1)} + V_{\mathrm{T}}^{(27)}(r)S_{12}(\hat{r}) + V_{\mathrm{LS}}^{(27)}(r)\vec{L}\cdot\vec{S}_+ \right)$$

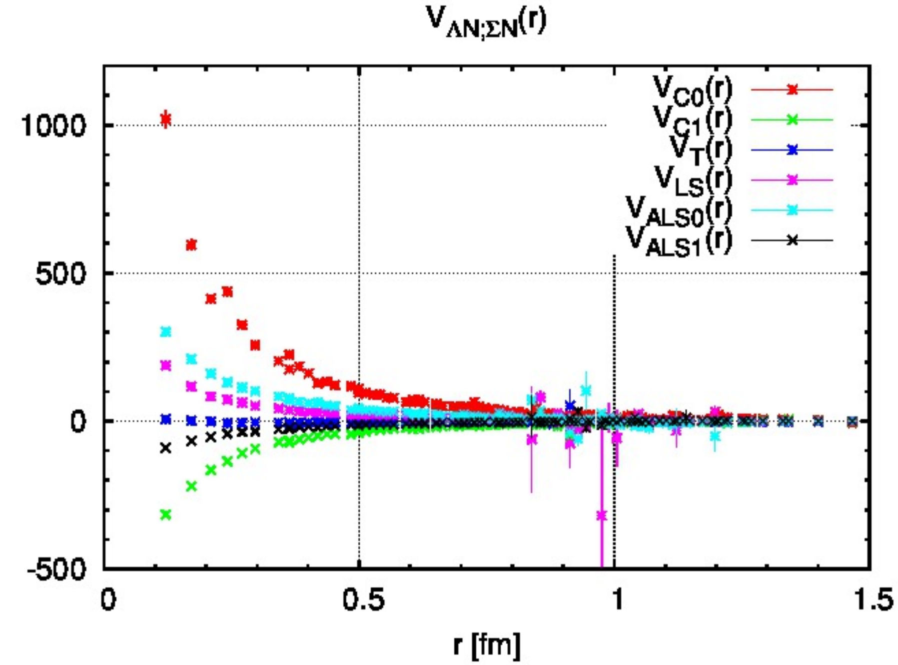
$$+ \frac{1}{2\sqrt{5}} \begin{pmatrix} 1 & 3 \\ -1 & -3 \end{pmatrix} V_{\mathrm{ALS}}(r)\mathbb{P}^{(S=0)}\vec{L}\cdot\vec{S}_-\mathbb{P}^{(S=1)}$$

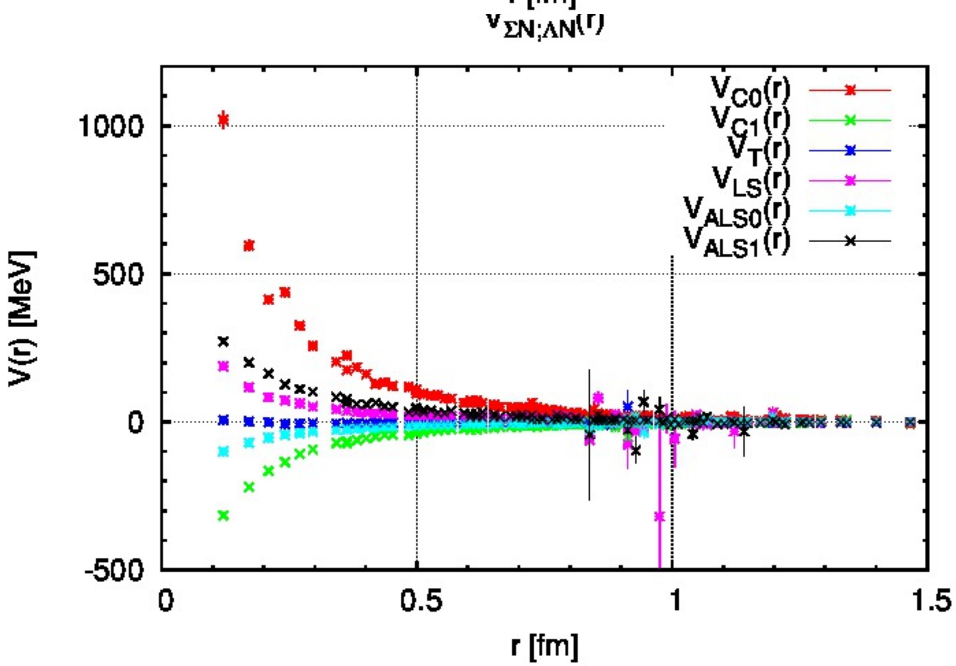
$$+ \frac{1}{2\sqrt{5}} \begin{pmatrix} 1 & -1 \\ 3 & -3 \end{pmatrix} V_{\mathrm{ALS}}(r)\mathbb{P}^{(S=1)}\vec{L}\cdot\vec{S}_-\mathbb{P}^{(S=0)}$$

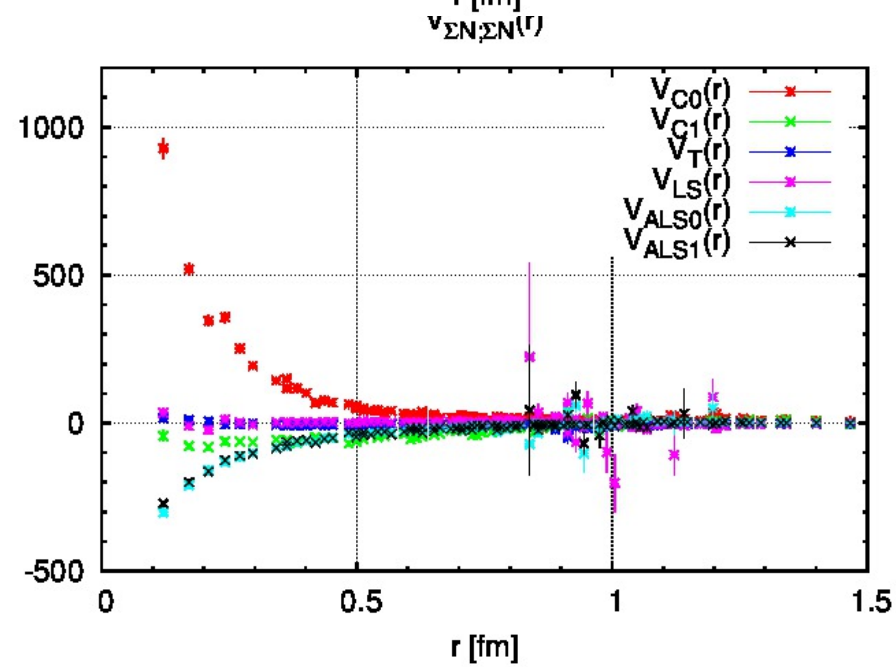
ALS of ΣN is 3 times stronger than **ALS of ΛN.**

ΛN-ΣN coupled channel potentials(odd parity sector)



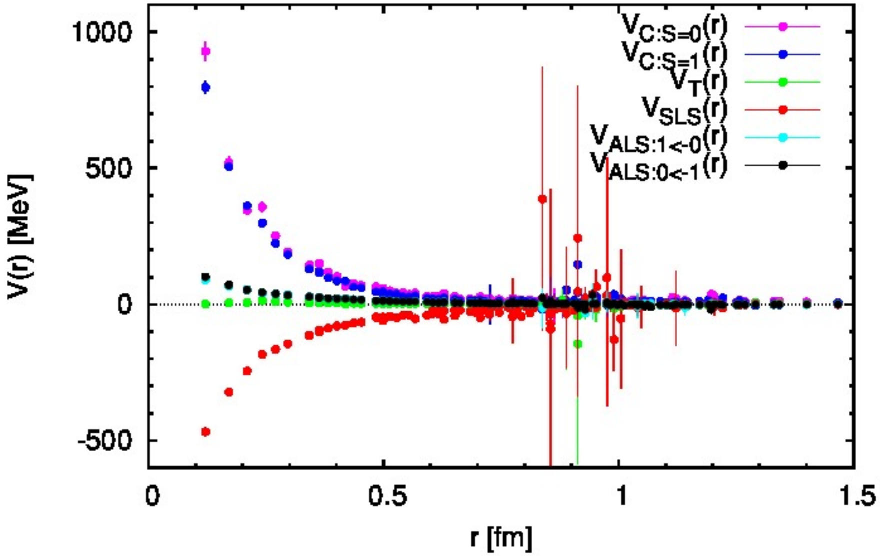




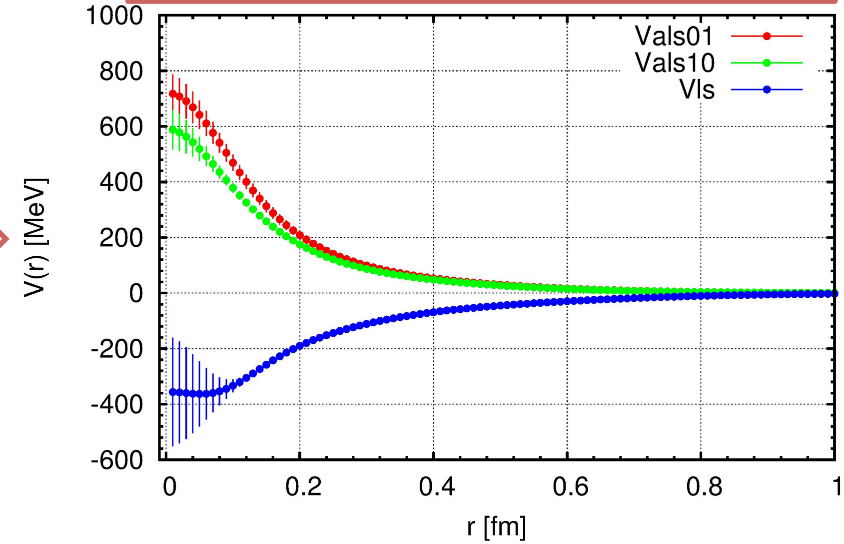


LS v.s. ALS (Integrating out ΣN channel)

Before integrating out ΣN channel



After integrating out ΣN channel



- The ALS in the effective ΛN potential becomes comparably large as that of SLS.

- Technical comment:
  Violation of Hermiticity of ALS（ALS[S=0→S=1] ≠ ALS[S=1→S=0]）

  Two wave functions of ΛN-ΣN coupled channel are orthogonal

$$\delta_{nm} = \langle n \mid m \rangle = \int d^3x \left(\psi^*_{\Lambda N}(\vec{x};n)\psi_{\Lambda N}(\vec{x};m) + \psi^*_{\Sigma N}(\vec{x};n)\psi_{\Sigma N}(\vec{x};m) \right)$$

→ orthogonality in ΛN component fails at short distance.

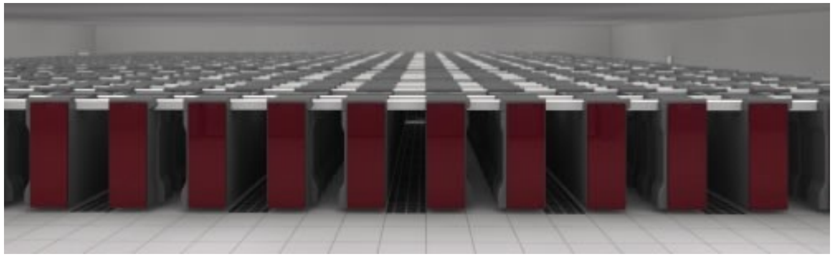
$$\int d^3x\, \psi^*_{\Lambda N}(\vec{x};n)\psi_{\Lambda N}(\vec{x};m) = \delta_{nm} - \int d^3x \psi^*_{\Sigma N}(\vec{x};n)\psi_{\Sigma N}(\vec{x};m)$$

Summary

Summary

- The gap between QCD and traditional/hyper nuclear physics is being filled by recent progress of LQCD
  - LQCD simulation at physical point
    - Spatial volume is becoming huge (for multi-baryon system)
  - LQCD calculation of light nuclei
    - LQCD calculations are accumulating at heavier quark mass region.
    - New techniques are being developed.
  - LQCD calculation of nuclear forces
    - The method does not need ground state saturation
    - Wide range of application to baryon interaction
      NN, NY, YY, NNN, negative parity, LS and anti-symmetric LS potential, etc.
    - LQCD simulation at the physical point in large spatial volume is going on.
      (m_{pion}=135 MeV, L∼9 fm.)

K computer (4th fastest in the world)

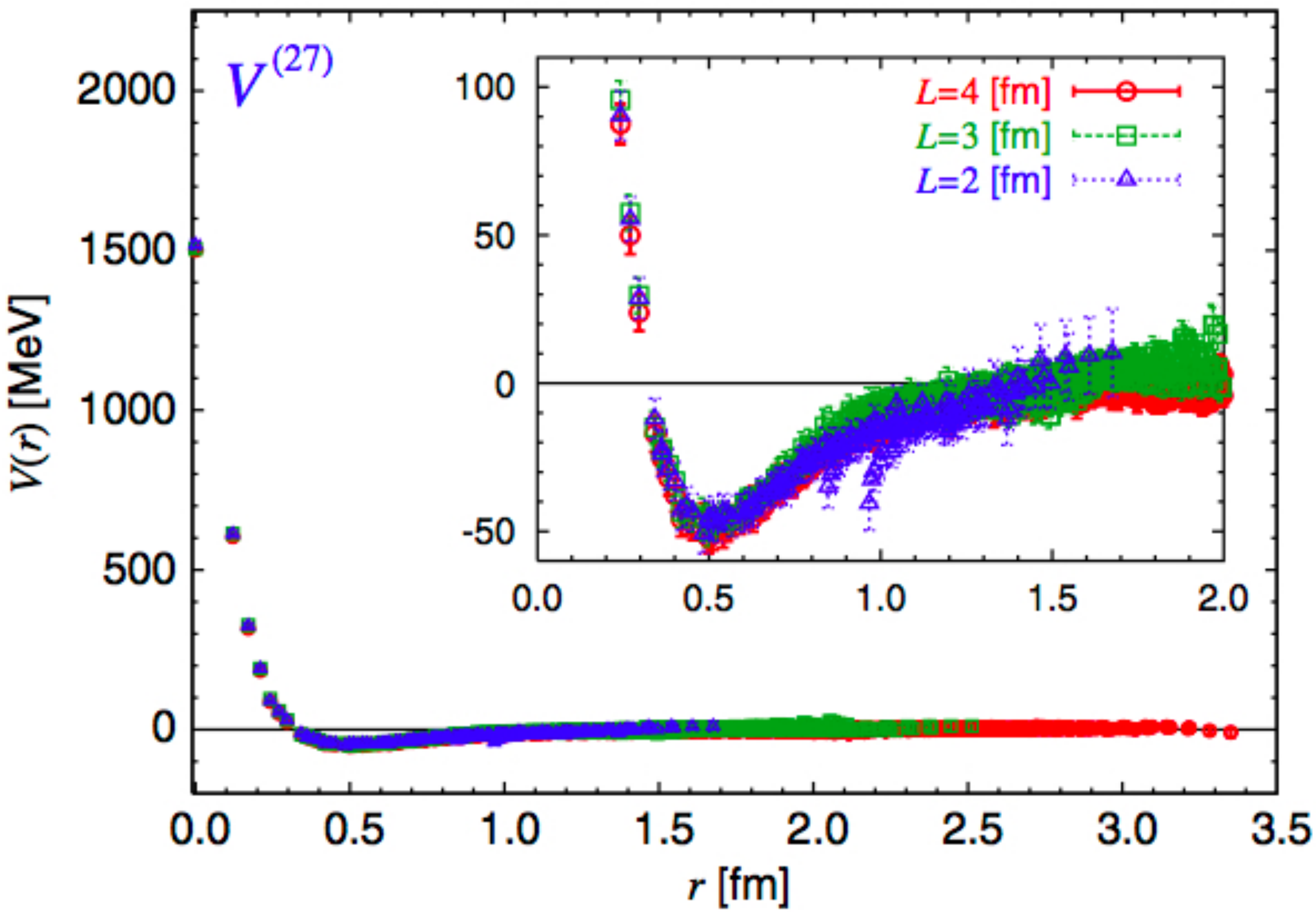


11.28 PFLOPS

Backup Slides

Nuclear Force from Lattice QCD

Volume dependence of the potential.

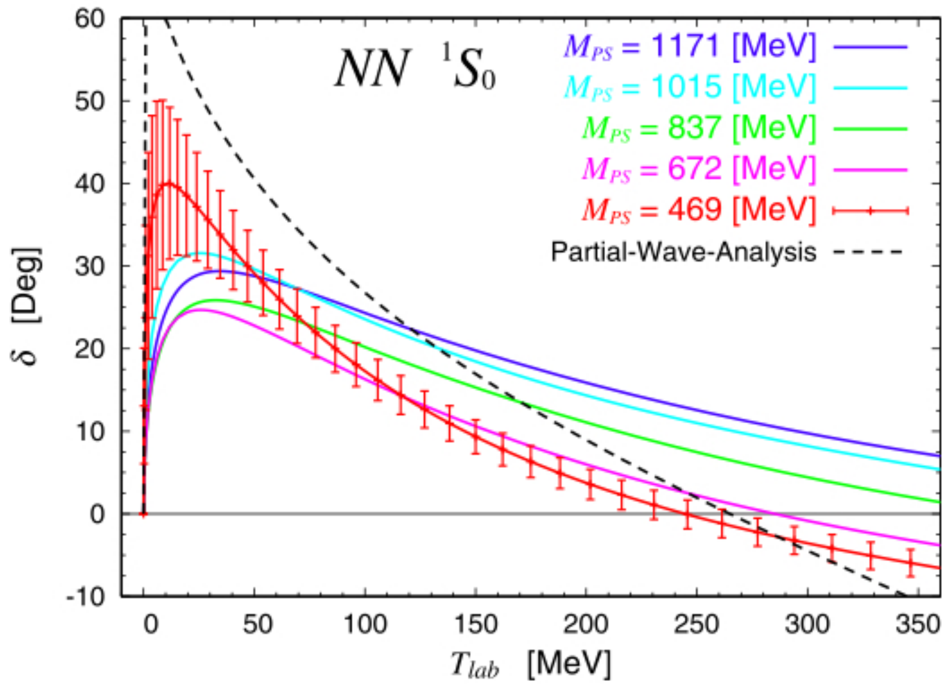


Nuclear Forces

[Inoue et al.,NPA881(2012)28]

Similar behavior is seen in NF=3 calculation (flavor SU(3) limit)

- m_{PS}=672-1171 MeV: attraction shrinks as decreasing quark mass.
- m_{PS}=**469**-672 MeV: **turning point**: attraction starts to increase.
- m_{PS}=0 -**469** MeV: attraction increase (← Our expectation !)



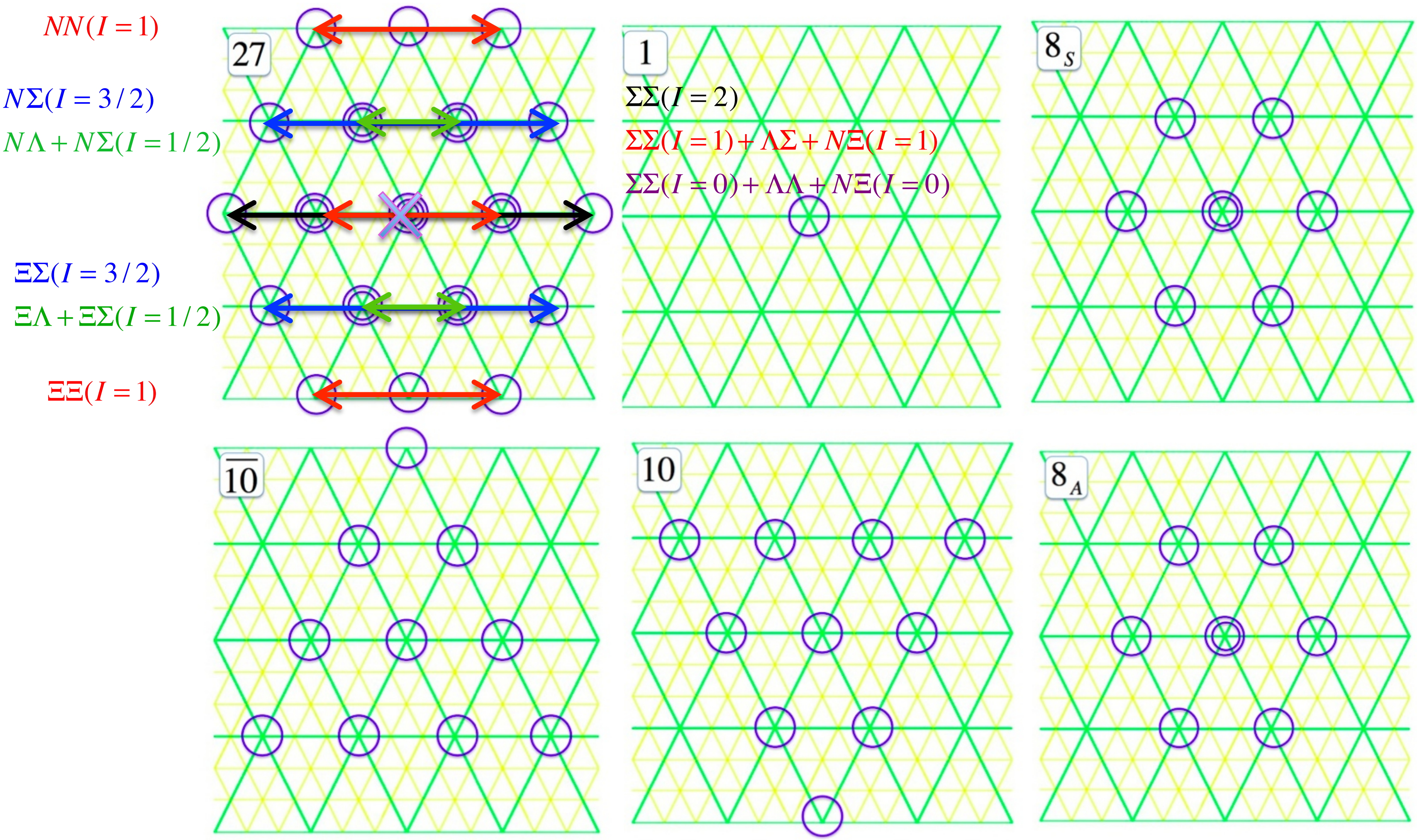
- For the similar thing to happen for NF=2+1,pion mass has to be smaller. Nuclear force for NF=3 is generally more attractive than NF=2+1.

$$\#(\text{Goldstone mode}) = \begin{cases} 3 & (N_F = 2+1) \\ 8 & (N_F = 3) \end{cases}$$

Hyperon Forces

◆ Flavor SU(3) limit to understand a general trend.

$$8 \otimes 8 = \underbrace{27 \oplus 8_S \oplus 1}_{\text{symmetric}} \oplus \underbrace{\overline{10} \oplus 10 \oplus 8_A}_{\text{anti-symmetric}}$$

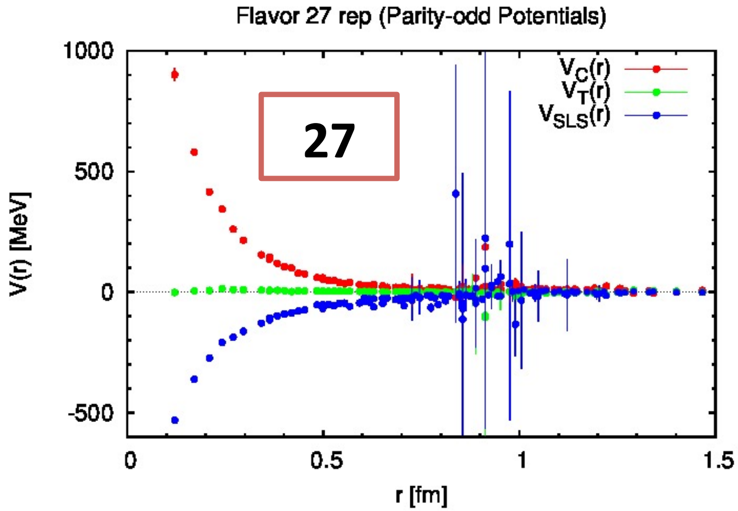


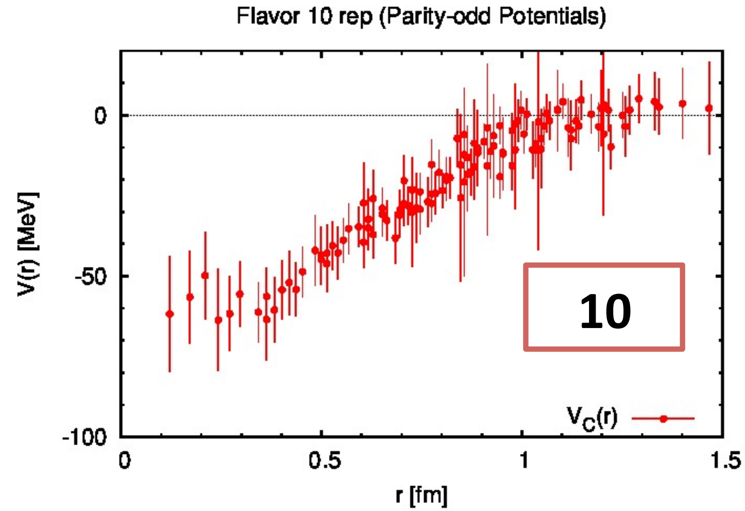
Defining relation of HAL QCD potential (= Schroedinger eq)

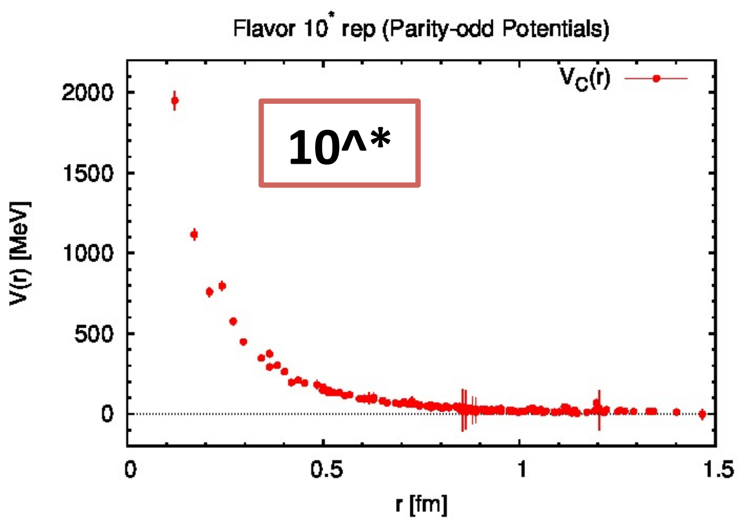
$$\left(\vec{k}^{2}/m_{N}-H_{0}\right)\psi_{\vec{k}}(\vec{r})=\int d^{3}r'\,U_{\mathrm{HALQCD}}(\vec{r},\vec{r}')\psi_{\vec{k}}(\vec{r}')$$

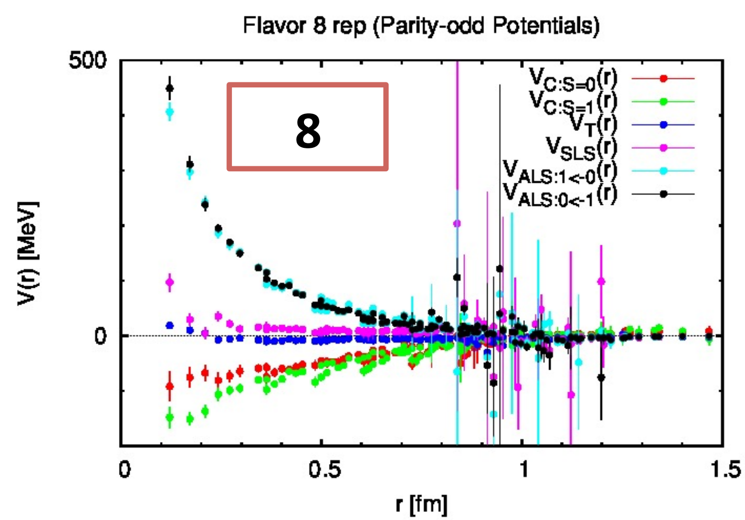
Numerical Results

Parity-odd potentials for flavor SU(3) irreps.







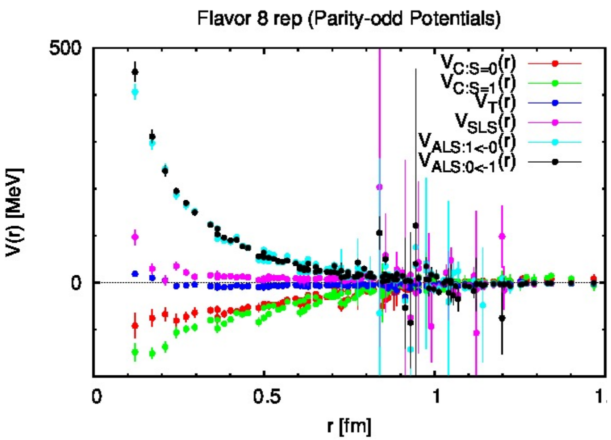


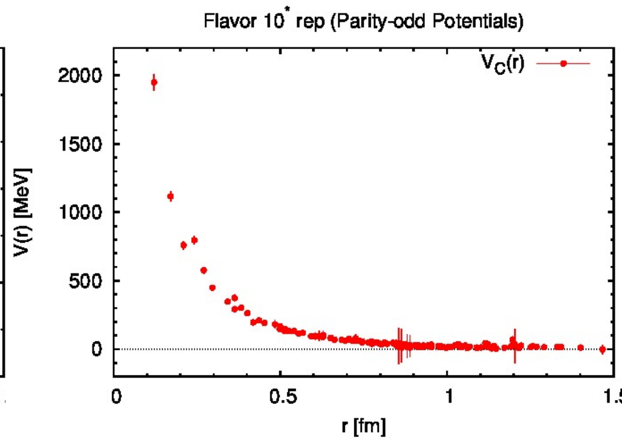
- These two correspond to NN sector.
- Qualitatively good
  - Repulsive core at short distance
  - Strong LS potential

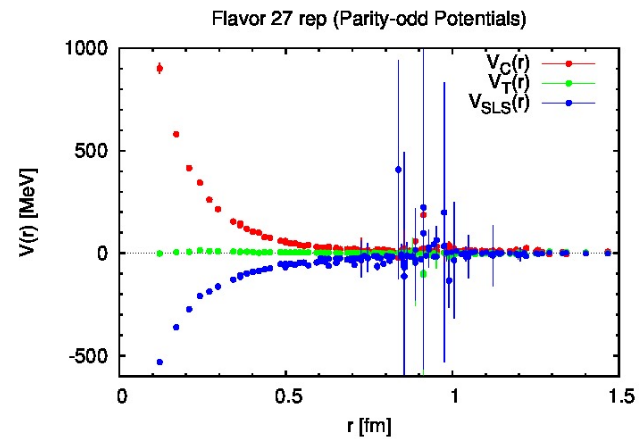
- No repulsive core at short distance (consistent with quark model)
- Strong anti-symmetric LS potential

Parity-odd ΛN potential

- Flavor SU(3) irreps. : 8, 10^*, 27 ➔ potential

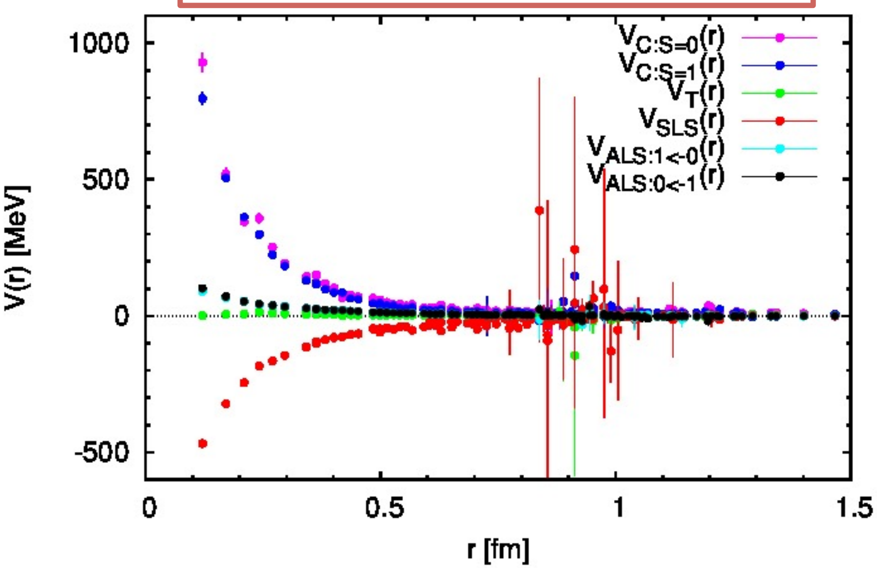






$$V_{\Lambda N} = \left(\frac{1}{2}V_{C}^{(\overline{10})} + \frac{1}{2}V_{C;S=0}^{(8)}\right)\mathbb{P}^{(S=0)} + \left(\frac{1}{10}V_{C:S=1}^{(8)} + \frac{9}{10}V_{C}^{(27)}\right)\mathbb{P}^{(S=1)} + \left(\frac{1}{10}V_{T}^{(8)} + \frac{9}{10}V_{T}^{(27)}\right)S_{12}(\hat{r}) + \left(\frac{1}{10}V_{LS}^{(8)} + \frac{9}{10}V_{LS}^{(27)}\right)\vec{L}\cdot\vec{S}_{+} + \frac{1}{2\sqrt{5}}V_{ALS}^{(8)}\cdot\vec{L}\cdot\vec{S}_{-}$$

ΛN potential (odd parity)



- ΛN component in ΛN-ΣN coupled channel potential
- Strong symmetric LS potential
  It comes from 27 irrep. (90%), i.e., NN LS

$$V_{LS}^{(\Lambda N)} = \frac{1}{10}V_{LS}^{(8)} + \frac{9}{10}V_{LS}^{(27)}$$

- Weak anti-symmetric LS potential
  SU(3) Clebsch-Gordan factor: 1/(2*sqrt(5))

$$V_{ALS}^{(\Lambda N)} = \frac{1}{2\sqrt{5}}V_{ALS}^{(8)}$$

ΣN component in anti-symmetric LS potential large

- ΛN-ΣN coupled channel potential

$$\begin{pmatrix} V_{\Lambda N,\Lambda N} & V_{\Lambda N,\Sigma N} \\ V_{\Sigma N,\Lambda N} & V_{\Sigma N,\Sigma N} \end{pmatrix} = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix} V_{\mathrm{C}}^{(10^*)}(r)\mathbb{P}^{(S=0)} + \begin{pmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{pmatrix} V_{\mathrm{C}}^{(8A)}(r)\mathbb{P}^{(S=0)}$$

$$+ \begin{pmatrix} 1/10 & 3/10 \\ 3/10 & 9/10 \end{pmatrix} \left(V_{\mathrm{C}}^{(8S)}(r)\mathbb{P}^{(S=1)} + V_{\mathrm{T}}^{(8S)}(r)S_{12}(\hat{r}) + V_{\mathrm{LS}}^{(8S)}(r)\vec{L}\cdot\vec{S}_+ \right)$$

$$+ \begin{pmatrix} 9/10 & -3/10 \\ -3/10 & 1/10 \end{pmatrix} \left(V_{\mathrm{C}}^{(27)}(r)\mathbb{P}^{(S=1)} + V_{\mathrm{T}}^{(27)}(r)S_{12}(\hat{r}) + V_{\mathrm{LS}}^{(27)}(r)\vec{L}\cdot\vec{S}_+ \right)$$

$$+ \frac{1}{2\sqrt{5}} \begin{pmatrix} 1 & 3 \\ -1 & -3 \end{pmatrix} V_{\mathrm{ALS}}(r)\mathbb{P}^{(S=0)}\vec{L}\cdot\vec{S}_-\mathbb{P}^{(S=1)}$$

$$+ \frac{1}{2\sqrt{5}} \begin{pmatrix} 1 & -1 \\ 3 & -3 \end{pmatrix} V_{\mathrm{ALS}}(r)\mathbb{P}^{(S=1)}\vec{L}\cdot\vec{S}_-\mathbb{P}^{(S=0)}$$

ALS of ΣN is 3 times stronger than **ALS of ΛN.**

Effective ΛN potential (Integrating out ΣN)

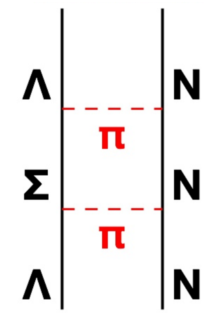
◆ **Model : ΛN-ΣN coupled channel Schrodinger eq.** $m_N < m_\Lambda < m_\Sigma$

$$\begin{pmatrix} \left(E + \dfrac{\nabla^2}{2\mu_{\Lambda N}}\right)\psi_{\Lambda N}(\vec{r}) \\ \left(E - m_\Sigma + m_\Lambda + \dfrac{\nabla^2}{2\mu_{\Sigma N}}\right)\psi_{\Sigma N}(\vec{r}) \end{pmatrix} = \begin{pmatrix} V_{\Lambda N;\Lambda N}(\vec{r},\vec{\nabla}) & V_{\Lambda N;\Sigma N}(\vec{r},\vec{\nabla}) \\ V_{\Sigma N;\Lambda N}(\vec{r},\vec{\nabla}) & V_{\Sigma N;\Sigma N}(\vec{r},\vec{\nabla}) \end{pmatrix} \cdot \begin{pmatrix} \psi_{\Lambda N}(\vec{r}) \\ \psi_{\Sigma N}(\vec{r}) \end{pmatrix}$$

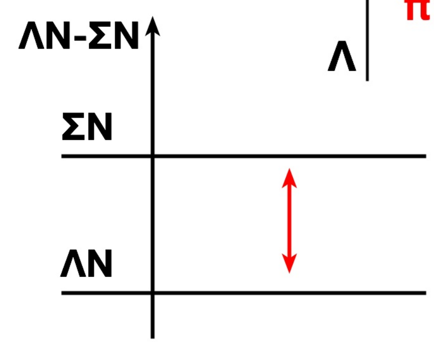
- ☐ We use the potentials in the flavor SU(3) limit.
  Assuming that Baryon-Baryon potential is not so sensitive to the change of quark mass.

◆ **Method: (HAL QCD method)**

- ☐ Solve it for $J^P=0^-,1^-,2^-$ in the ΛN elastic region with ΛN incoming BC.
- ☐ Focus on the ΛN component.



$$\begin{pmatrix} \psi_{\Lambda N}(\vec{r}) \\ \psi_{\Sigma N}(\vec{r}) \end{pmatrix} \Rightarrow \psi_{\Lambda N}(\vec{r})$$



- ☐ Effective ΛN potential by requiring that $\psi_{\Lambda N}(r)$ is reproduced.

$$\left(E + \frac{\nabla^2}{2\mu_{\Lambda N}}\right)\psi_{\Lambda N}(\vec{r}) = V(\vec{r},\vec{\nabla})\psi_{\Lambda N}(\vec{r})$$

$$V(\vec{r},\vec{\nabla}) = V_{\mathrm{C;S=0}}(\vec{r})\mathbb{P}^{(S=0)} + V_{\mathrm{C;S=1}}(\vec{r})\mathbb{P}^{(S=1)} + V_{\mathrm{T}}(\vec{r})S_{12}(\hat{r}) + V_{\mathrm{LS}}(\vec{r})\vec{L}\cdot\vec{S}_+ + V_{\mathrm{ALS}}(\vec{r})\vec{L}\cdot\vec{S}_- + \cdots$$

Effective ΛN potential (Integrating out ΣN)

- Baryon mass: (choice seems to be almost arbitrary)

$$m_B = 2051\,\mathrm{MeV}(\text{original. SU(3) limit}) \Rightarrow \begin{cases} m_\Sigma = 2150\,\mathrm{MeV} \\ m_\Lambda = 2100\,\mathrm{MeV} \\ m_N = 2000\,\mathrm{MeV} \end{cases}$$

- Energy : E=1MeV.

- 5 asymptotic states are used.
  - 3P0: |ΛN(3P0),in>
  - 3P1-1P1: |ΛN(3P1),in> & |ΛN(1P1), in>
  - 3P2-3F2: |ΛN(3P2),in> & |ΛN(3F2), in>

	$S=1$	$S=0$
$J^P=0^-$	${}^3P_0$	
$J^P=1^-$	${}^3P_1$ —	${}^1P_1$
$J^P=2^-$	${}^3P_2-{}^3F_2$	

- Schrodinger eq.

$$\left(E+\frac{\nabla^2}{2\mu_{\Lambda N}}\right)\psi_{\Lambda N}(\vec r)=\left(V_{C;S=0}(\vec r)\mathbb{P}^{(S=0)}+V_{C;S=1}(\vec r)\mathbb{P}^{(S=1)}+V_T(\vec r)S_{12}(\hat r)+V_{LS}(\vec r)\vec L\cdot\vec S_+ +V_{ALS}(\vec r)\vec L\cdot\vec S_-\right)\psi_{\Lambda N}(\vec r)$$

➔ spin “singlet” component

$$\left(E+\frac{\nabla^2}{2\mu_{\Lambda N}}\right)\mathbb{P}^{(S=0)}\psi_{\Lambda N}(\vec r)=V_{C;S=0}(\vec r)\mathbb{P}^{(S=0)}\psi_{\Lambda N}(\vec r)+V_{ALS}(\vec r)\mathbb{P}^{(S=0)}\vec L\cdot\vec S_-\mathbb{P}^{(S=1)}\psi_{\Lambda N}(\vec r)$$

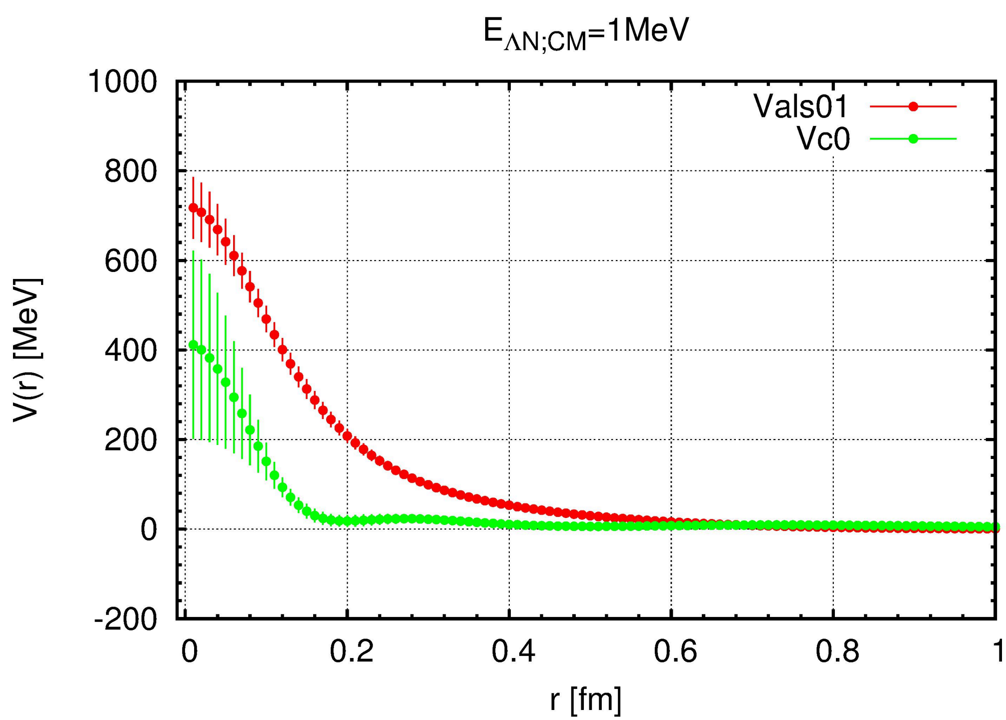
➔ spin “triplet” component

$$\left(E+\frac{\nabla^2}{2\mu_{\Lambda N}}\right)\mathbb{P}^{(S=1)}\psi_{\Lambda N}(\vec r)=\left(V_{C;S=1}(\vec r)+V_T(\vec r)S_{12}(\hat r)+V_{LS}(\vec r)\vec L\cdot\vec S_+ +\right)\mathbb{P}^{(S=1)}\psi_{\Lambda N}(\vec r)+V_{ALS}(\vec r)\mathbb{P}^{(S=1)}\vec L\cdot\vec S_-\mathbb{P}^{(S=0)}\psi_{\Lambda N}(\vec r)$$

"Spin singlet" part

$$\left(E+\frac{\nabla^2}{2\mu_{\Lambda N}}\right)\mathbb{P}^{(S=0)}\psi_{\Lambda N}(\vec{r}) = V_{\mathrm{C;S=0}}(\vec{r})\mathbb{P}^{(S=0)}\psi_{\Lambda N}(\vec{r}) + V_{\mathrm{ALS}}(\vec{r})\mathbb{P}^{(S=0)}\vec{L}\cdot\vec{S}\_\mathbb{P}^{(S=1)}\psi_{\Lambda N}(\vec{r})$$

$$\Rightarrow \begin{pmatrix} \left(E+\frac{1}{2\mu_{\Lambda N}}\left(\frac{d^2}{dr^2}-\frac{2}{r^2}\right)\right)\phi_{^1P_1}(r;{}^1P_1) \\ \left(E+\frac{1}{2\mu_{\Lambda N}}\left(\frac{d^2}{dr^2}-\frac{2}{r^2}\right)\right)\phi_{^1P_1}(r;{}^3P_1) \end{pmatrix} = \begin{pmatrix} \phi_{^1P_1}(r;{}^1P_1) & \sqrt{2}\phi_{^3P_1}(r;{}^1P_1) \\ \phi_{^1P_1}(r;{}^3P_1) & \sqrt{2}\phi_{^3P_1}(r;{}^3P_1) \end{pmatrix} \cdot \begin{pmatrix} V_{\mathrm{C;S=0}}(r) \\ V_{\mathrm{ALS}}(r) \end{pmatrix}$$

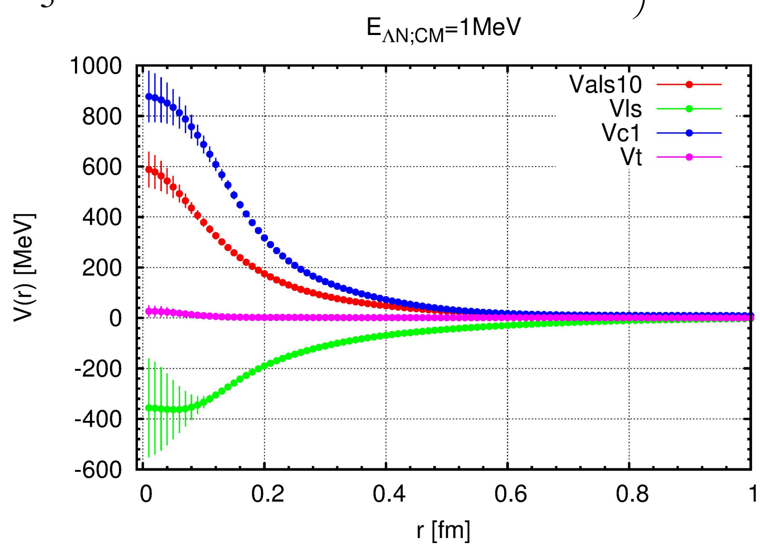


- 2 equations
- 2 unknowns (potentials)
  - ☐ Central pot. (spin singlet)
  - ☐ Anti-sym. LS pot. (S=1 ➔ S=0)

"Spin triplet" part

$$\left(E+\frac{\nabla^2}{2\mu_{\Lambda N}}\right)\mathbb{P}^{(S=1)}\psi_{\Lambda N}(\vec{r}) = \left(V_{C;S=1}(\vec{r}) + V_T(\vec{r})S_{12}(\hat{r}) + V_{LS}(\vec{r})\vec{L}\cdot\vec{S}_+ +\right)\mathbb{P}^{(S=1)}\psi_{\Lambda N}(\vec{r}) + V_{ALS}(\vec{r})\mathbb{P}^{(S=1)}\vec{L}\cdot\vec{S}_-\mathbb{P}^{(S=0)}\psi_{\Lambda N}(\vec{r})$$

$$\begin{pmatrix}
\left(E+\frac{1}{2\mu_{\Lambda N}}\left(\frac{d^2}{dr^2}-\frac{2}{r^2}\right)\right)\phi_{^3P_0}(r;{}^3P_0) \\
\left(E+\frac{1}{2\mu_{\Lambda N}}\left(\frac{d^2}{dr^2}-\frac{2}{r^2}\right)\right)\phi_{^3P_1}(r;{}^3P_1) \\
\left(E+\frac{1}{2\mu_{\Lambda N}}\left(\frac{d^2}{dr^2}-\frac{2}{r^2}\right)\right)\phi_{^3P_1}(r;{}^1P_1) \\
\left(E+\frac{1}{2\mu_{\Lambda N}}\left(\frac{d^2}{dr^2}-\frac{2}{r^2}\right)\right)\phi_{^3P_2}(r;{}^3P_2) \\
\left(E+\frac{1}{2\mu_{\Lambda N}}\left(\frac{d^2}{dr^2}-\frac{2}{r^2}\right)\right)\phi_{^3P_2}(r;{}^3F_2) \\
\left(E+\frac{1}{2\mu_{\Lambda N}}\left(\frac{d^2}{dr^2}-\frac{12}{r^2}\right)\right)\phi_{^3F_2}(r;{}^3P_2) \\
\left(E+\frac{1}{2\mu_{\Lambda N}}\left(\frac{d^2}{dr^2}-\frac{12}{r}\right)\right)\phi_{^3F_2}(r;{}^3F_2)
\end{pmatrix}
=
\begin{pmatrix}
\phi_{^3P_0}(r;{}^3P_0) & -4\phi_{^3P_0}(r;{}^3P_0) & -2\phi_{^3P_0}(r;{}^3P_0) & 0 \\
\phi_{^3P_1}(r;{}^3P_1) & 2\phi_{^3P_1}(r;{}^3P_1) & -\phi_{^3P_1}(r;{}^3P_1) & \sqrt{2}\phi_{^1P_1}(r;{}^3P_1) \\
\phi_{^3P_1}(r;{}^1P_1) & 2\phi_{^3P_1}(r;{}^1P_1) & -\phi_{^3P_1}(r;{}^1P_1) & \sqrt{2}\phi_{^1P_1}(r;{}^1P_1) \\
\phi_{^3P_2}(r;{}^3P_2) & -\frac{2}{5}\phi_{^3P_2}(r;{}^3P_2)+\frac{6\sqrt{6}}{5}\phi_{^3F_2}(r;{}^3P_2) & \phi_{^3P_2}(r;{}^3P_2) & 0 \\
\phi_{^3P_2}(r;{}^3F_2) & -\frac{2}{5}\phi_{^3P_2}(r;{}^3F_2)+\frac{6\sqrt{6}}{5}\phi_{^3F_2}(r;{}^3F_2) & \phi_{^3P_2}(r;{}^3F_2) & 0 \\
\phi_{^3F_2}(r;{}^3P_2) & -\frac{8}{5}\phi_{^3F_2}(r;{}^3P_2)+\frac{6\sqrt{6}}{5}\phi_{^3P_2}(r;{}^3P_2) & -4\phi_{^3F_2}(r;{}^3P_2) & 0 \\
\phi_{^3F_2}(r;{}^3F_2) & -\frac{8}{5}\phi_{^3F_2}(r;{}^3F_2)+\frac{6\sqrt{6}}{5}\phi_{^3P_2}(r;{}^3F_2) & -4\phi_{^3F_2}(r;{}^3F_2) & 0
\end{pmatrix}
\cdot
\begin{pmatrix}
V_{C;S=0}(r) \\ V_T(r) \\ V_{LS}(r) \\ V_{ALS}(r)
\end{pmatrix}$$

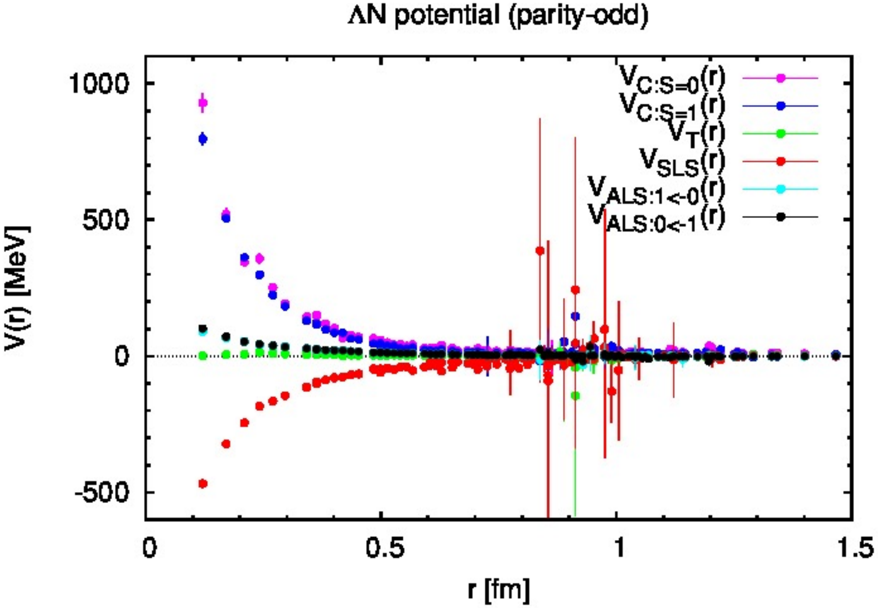


- 7 equations
- 4 unkowns (potentials)
  - ☐ Central potential (spin triplet)
  - ☐ Tensor potential
  - ☐ LS potential
  - ☐ Anti-symmetric LS potential (S=0 ➔ S=1)

Hyperon Forces

[N.Ishii et al. coming soon]

① ΛN part of ΛN-ΣN coupled channel potential (parity-odd sector) is obtained as linear combination of 8, 10^* and 27.



$$V_{\Lambda N}=\left(\frac{1}{2}V_{\mathrm{C}}^{(\overline{10})}+\frac{1}{2}V_{\mathrm{C;S=0}}^{(8)}\right)\mathbb{P}^{(S=0)}+\left(\frac{1}{10}V_{\mathrm{C:S=1}}^{(8)}+\frac{9}{10}V_{\mathrm{C}}^{(27)}\right)\mathbb{P}^{(S=1)}$$
$$+\left(\frac{1}{10}V_{\mathrm{T}}^{(8)}+\frac{9}{10}V_{\mathrm{T}}^{(27)}\right)\left(3(\hat{r}\cdot\vec{\sigma}_\Lambda)(\hat{r}\cdot\vec{\sigma}_N)-\vec{\sigma}_\Lambda\cdot\vec{\sigma}_N\right)$$
$$+\left(\frac{1}{10}V_{\mathrm{LS}}^{(8)}+\frac{9}{10}V_{\mathrm{LS}}^{(27)}\right)\vec{L}\cdot(\vec{s}_\Lambda+\vec{s}_N)$$
$$+\frac{1}{2\sqrt{5}}V_{\mathrm{ALS}}^{(8)}\cdot\vec{L}\cdot(\vec{s}_\Lambda-\vec{s}_N)$$

② ΣN channel is integrated out from ΛN-ΣN coupled channel potential.