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Transverse Momentum Dependent Distribution (and Fragmentation) Functions of Hadrons

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ABSTRACT

Transverse Momentum Dependent Parton Distribution and Fragmentation Functions of Hadrons

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Transverse Momentum Dependent (TMD) parton distribution (PDF) and parton fragmentation (PFF) functions also take into account the intrinsic transverse momentum (p_T) of the partons. The p_T-integrated analogues can be linked directly to quark and gluon matrix elements using Operator Product Expansion in QCD, involving operators of definite twist. TMDs also involve operators of higher twist, which are not suppressed by powers of the hard scale, however. In this talk I will address the relevance of both quark and gluon TMDs in azimuthal and spin asymmetries and address theoretical issues related to gauge links, universality of the functions and the essential difference between T-odd PDFs and PFFs.

[collaborators: Maarten Buffing, Leonard Gamberg and Asmita Mukherjee]

Content

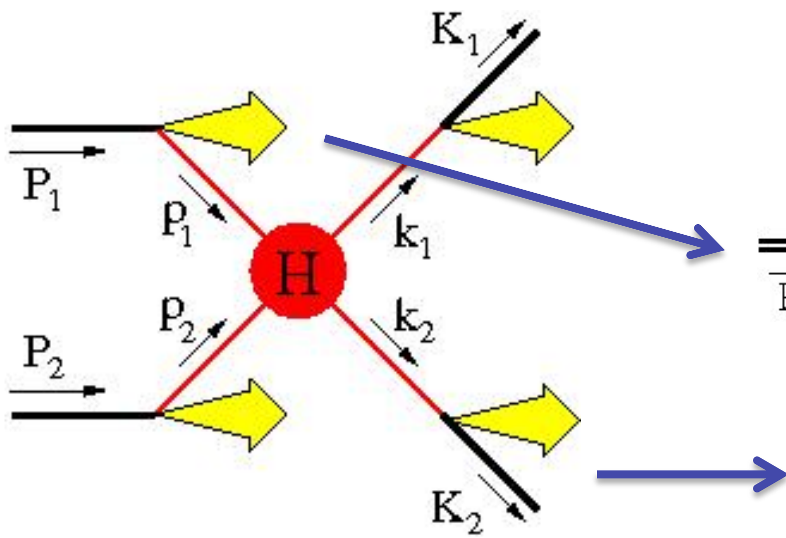
- Introduction: collinear (soft) x hard at high enegies
  - Collinear = hadron info (probabilities), hard = partonic cross section
  - Probabilities include spin-spin correlations
- Are TMD PDFs and PFFs relevant and can they be measured?
  - Yes, there are besides spin-spin also spin-orbit correlations
  - Yes, they can be measured (DY, SIDIS, …)
- Complications along the way!
  - Gauge links, universality, factorization
- QCD as the theoretical framework to help out
  - Extension of OPE
- The reward
  - Novel hadronic info on spin and orbital structure
  - Possible use of proton as tool (truly playing with partons)
  - TMD PFFs ‘simpler’ than TMD PDFs

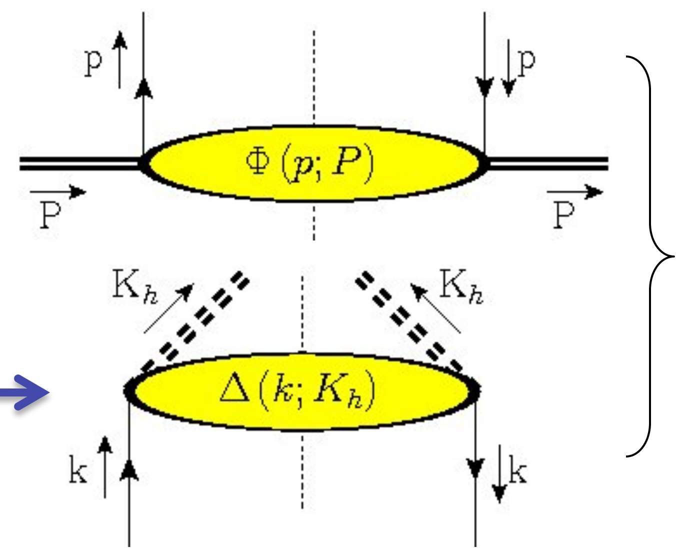
PDFs and PFFs

Basic idea of PDFs and PFFs (also for TMDs) is to obtain a full factorized description of high energy scattering processes

$$\hat{\sigma} = | H(p_1, p_2, \ldots) |^2$$

calculable





defined (!) & portable

$$\sigma(P_1, P_2, \ldots) = \iiint \ldots dp_1 \ldots \Phi_a(p_1, P_1; \mu) \otimes \Phi_b(p_2, P_2; \mu)$$

$$\otimes \hat{\sigma}_{ab,c\ldots}(p_1, p_2, \ldots; \mu) \otimes \Delta_c(k_1, K_1; \mu) \ldots.$$

Give a meaning to integration variables!

Hadron correlators

- At high energies no interference and squared amplitudes can be rewritten as correlators of forward matrix elements of parton fields

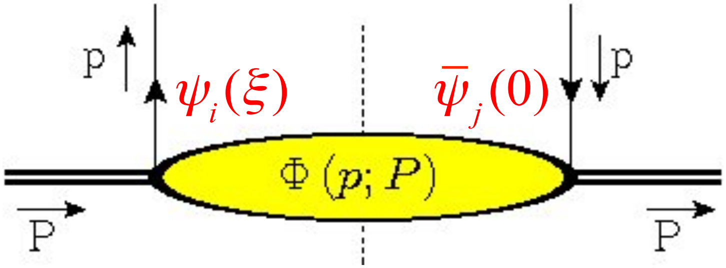
- Math: $u_i(p,s)\bar{u}_j(p,s) \;\Rightarrow\; \sum_X \langle P|\bar{\psi}_j(0)\,|X><X|\,\psi_i(0)|P\rangle\,\delta(p-P+P_X)$

$$= \sum_X \int \frac{d\xi}{2\pi} \langle P|\bar{\psi}_j(0)\,|X><X|\,\psi_i(0)|P\rangle\, e^{i(p-P+P_X).\xi}$$

$$= \sum_X \int \frac{d\xi}{2\pi} \langle P|\bar{\psi}_j(0)\,|X><X|\,\psi_i(\xi)|P\rangle\, e^{i\,p.\xi}$$

$$= \int \frac{d\xi}{2\pi} e^{i\,p.\xi} \langle P|\bar{\psi}_j(0)\,\psi_i(\xi)|P\rangle$$

- Picture:



Hadron correlators

- At high energies no interference and squared amplitudes can be rewritten as correlators of matrix elements of parton fields

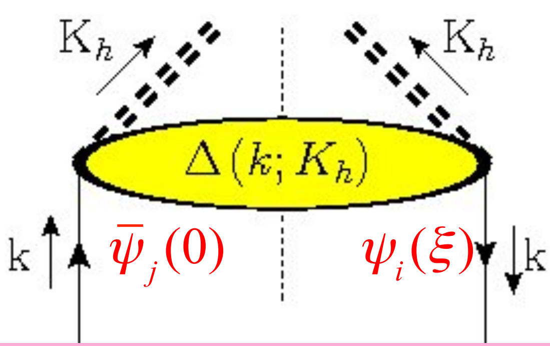
- Math: $u_i(k,s)\bar{u}_j(k,s) \;\Rightarrow\; \sum_X \langle 0 | \psi_i(0) | K_h X \rangle\langle K_h X | \bar{\psi}_j(0) | 0 \rangle \delta(k - K_h - K_X)$

$$= \sum_X \int \frac{d\xi}{2\pi} \langle 0 | \psi_i(0) | K_h X \rangle\langle K_h X | \bar{\psi}_j(0) | 0 \rangle e^{i(k-K_h-K_X).\xi}$$

$$= \sum_X \int \frac{d\xi}{2\pi} \langle 0 | \psi_i(\xi) | K_h X \rangle\langle K_h X | \bar{\psi}_j(0) | 0 \rangle e^{ik.\xi}$$

$$= \int \frac{d\xi}{2\pi} e^{ik.\xi} \langle 0 | \psi_i(\xi) a_h^+ a_h \bar{\psi}_j(0) | 0 \rangle$$

- Picture:



no T-constraint

$T|K_h,X\rangle_{out} = |K_h,X\rangle_{in}$

Collins & Soper, NP B 194 (1982) 445

Role of the hard scale

- In high-energy processes hard momenta are available, such that P.P′ ~ s with a hard scale s >> M^2
- Employ light-like vectors P and n, such that P.n = 1 (e.g. n = P′/P.P′) to make a Sudakov expansion of parton momentum (write s = Q^2)

$$p = \underbrace{x P^\mu}_{\sim Q} + \underbrace{p_T^\mu}_{\sim M} + \underbrace{\sigma n^\mu}_{\sim M^2/Q}$$

$$x = p^+ = p.n \qquad (0 \le x \le 1)$$

$$\sigma = p^- = p.P - xM^2 \sim O(M^2)$$

- Enables expansion in inverse hard scale (twist analysis) for integrated correlators,

$$\Phi(p) = \Phi(x, p_T, p.P) \;\Rightarrow\; \Phi(x, p_T) \;\Rightarrow\; \Phi(x) \;\Rightarrow\; \Phi$$

(Un)integrated correlators

$$\Phi(x, p_T, p.P) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{i\,p.\xi} \langle P | \bar{\psi}(0)\, \psi(\xi) | P \rangle$$

- unintegrated

$$\Phi(x, p_T; n) = \int \frac{d(\xi.P)\, d^2\xi_T}{(2\pi)^3}\, e^{i\,p.\xi} \langle P | \bar{\psi}(0)\, \psi(\xi) | P \rangle_{\boxed{\xi.n=\xi^+=0}}$$

- TMD (light-front)

- σ = p⁻ integration makes time-ordering automatic. The soft part is simply sliced at the light-front

$$\Phi(x) = \int \frac{d(\xi.P)}{(2\pi)}\, e^{i\,p.\xi} \langle P | \bar{\psi}(0)\, \psi(\xi) | P \rangle_{\boxed{\xi.n=\xi_T=0} \text{ or } \boxed{\xi^2=0}}$$

- Is already equivalent to a point-like interaction

- collinear (light-cone)

$$\Phi = \langle P | \bar{\psi}(0)\, \psi(\xi) | P \rangle_{\boxed{\xi=0}}$$

- local

- Local operators with calculable anomalous dimension relevance and measurability of TMDs

Transverse momentum dependence

- Mismatch of hadronic and partonic momenta

$$p - xP = p_T + \ldots = -xP_\perp + \ldots$$

$$k - \tfrac{1}{z}K_h = k_T + \ldots = -\tfrac{1}{z}K_{h\perp} + \ldots$$

- Momentum fractions are linked to scaling variables, e.g. SIDIS (up to 1/Q^2 corrections):

$$x = p.n / P.n = Q^2 / 2P.q = x_B$$

$$z = K.n / k.n = P.K / P.q = z_h$$

- Transverse momenta are convoluted into a measurable off-collinearity,

$$\boxed{q_T = q + x_B P - z_h^{-1} K = k_T - p_T}$$

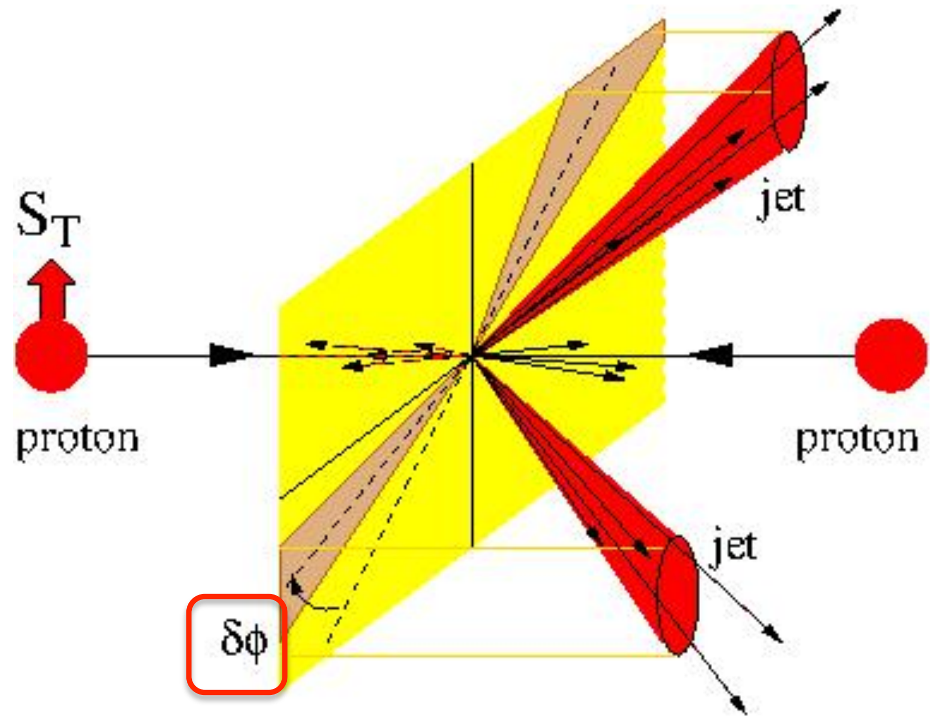
- … or non-alignment of jets in hadron + hadron → jet + jet.

Access to transverse momenta

- Also in more complex situations like hadron-hadron collisions

$$p_1 \approx x_1 P_1 + p_{1T}$$
$$p_2 \approx x_2 P_2 + p_{2T}$$

$$x_1 = p_1.n = \frac{p_1.P_2}{P_1.P_2} = \frac{(k_1 + k_2).P_2}{P_1.P_2}$$



$$q_T = k_{jet,1} + k_{jet,2} - x_1 P_1 - x_2 P_2$$
$$= p_{1T} + p_{2T}$$

Second scale!

relevance and measurability of TMDs

New information in TMD's: $f(x,p_T)$ or $D(1/z,k_T)$

- Quarks in polarized nucleon: $S = S_L\left(\frac{P}{M} + Mn\right) + S_T \qquad S_L^2 + S_T^2 = -1$

$$\Phi^q(p;P,S) \propto x f_1^q(x,p_T^2)\,\not{P} + S_L\, x g_{1L}^q(x,p_T^2)\,\not{P}\gamma_5$$
$$+ x h_{1T}^q(x,p_T^2)\,\not{S}_T\not{P}\gamma_5 \quad + \quad \ldots$$

unpolarized quarks

T-polarized quarks in T-polarized N

chiral quarks in L-polarized N

compare

$$u(p,s)\bar{u}(p,s) = \tfrac{1}{2}(\not{p} + m)(1 + \gamma_5\not{s})$$

- … but also

$$\Phi^q(p;P,S) \propto \ldots + \frac{(p_T \cdot S_T)}{M} x g_{1T}^q(x,p_T^2)\,\not{P}\gamma_5 \quad + \quad \ldots$$

spin ←→ spin

chiral quarks in T-polarized N

New information in TMD's: f(x,p_T) or D(1/z,k_T)

- … and T-odd functions

$$\Phi^q(p;P,S) \propto \ldots + i h_1^{\perp q}(x,p_T^2)\frac{\not p_T}{M}\not P + i\frac{(p_T \times S_T)}{M} x f_{1T}^{\perp q}(x,p_T^2)\not P + \ldots$$

T-polarized quarks in unpolarized N (Boer-Mulders)

unpolarized quarks in T-polarized N (Sivers)

compare

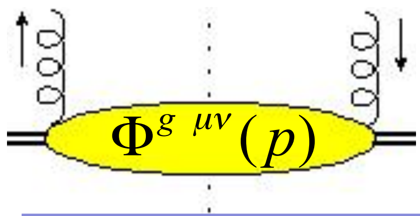
$$u(p,s)\bar{u}(p,s) = \tfrac{1}{2}(\not p + m)(1+\gamma_5 \not s)$$

spin ←→ orbit

- Yes, definitely there is new information and even very interesting spin-orbit correlations (single spin!). These are T-odd and because of T-conservation show up in T-odd observables, such as single spin asymmetries, e.g. left-right asymmetry in $p(P_1)p_\uparrow(P_2) \rightarrow \pi(K)X$

New information in gluon TMD's: f(x,p$_T$) or D(1/z,k$_T$)

- Also for gluons there are new features in TMD's



circularly polarized gluons in L-pol. N

spin ←→ spin

$$\Phi^{g\;\mu\nu}(p;P,S) \propto -g_T^{\mu\nu}\, x f_1^{\,g}(x,p_T^2) + iS_L \varepsilon_T^{\mu\nu}\, x g_{1L}^{\,g}(x,p_T^2) + \left(\frac{p_T^\mu p_T^\nu}{M^2} - g_T^{\mu\nu}\frac{p_T^2}{2M^2}\right) x h_1^{\perp g}(x,p_T^2) + \ldots$$

unpolarized gluons in unpol. N quarks

linearly polarized gluons in unpol. N (Gluon Boer-Mulders)

spin ←→ orbit

compare

$$\varepsilon^\mu(p,\lambda)\varepsilon^{\nu *}(p,\lambda) = -g_T^{\mu\nu} + \ldots$$

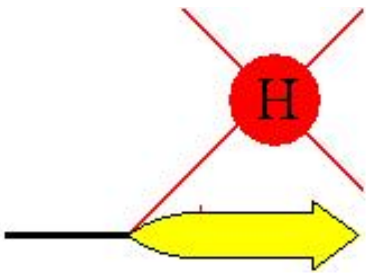
Complications for TMDs

Hadron correlators

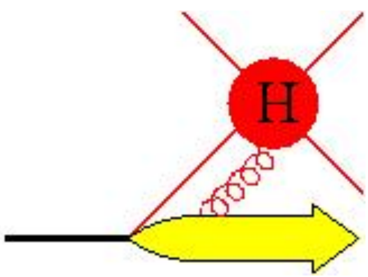
- Hadronic correlators establish the diagrammatic link between hadrons and partonic hard scattering amplitude
- Quark, quark + gluon, gluon, …

- Disentangling a hard process into collinear parts involving hadrons, hard scattering amplitude and soft factors is non-trivial

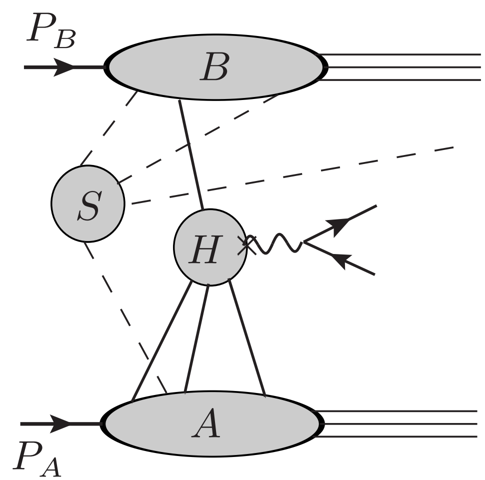
$$\left\langle 0 \middle| \psi_i\,(\xi) \middle| p,s \right\rangle = u_i(p,s)\, e^{-ip.\xi}$$



$$\left\langle X \middle| \psi_i\,(\xi) \middle| P \right\rangle e^{+ip.\xi}$$



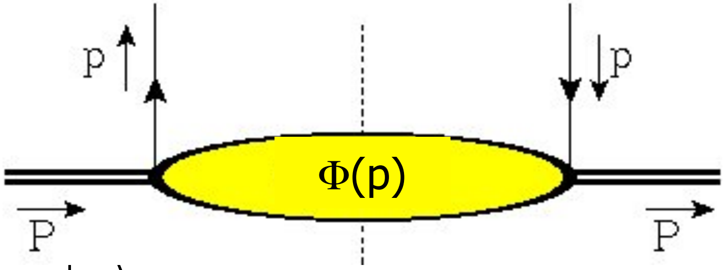
$$\left\langle X \middle| \psi_i\,(\xi) A^\mu(\eta) \middle| P \right\rangle e^{+i(p-p_1).\xi + ip_1.\eta}$$



J.C. Collins, Foundations of Perturbative QCD, Cambridge Univ. Press 2011

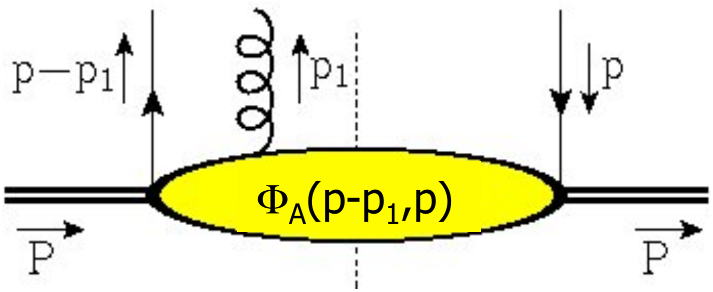
Soft part: hadron correlators

- Forward matrix elements of parton fields describe distribution (and fragmentation) parts



$$\Phi_{ij}(p;P) = \Phi_{ij}(p\,|\,p) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{i\,p.\xi} \left\langle P \right| \bar{\psi}_j(0)\, \psi_i(\xi) \left| P \right\rangle$$

- Also needed are multi-parton correlators



$$\Phi^{\alpha}_{A;ij}(p-p_1,p_1\,|\,p) = \int \frac{d^4\xi\, d^4\eta}{(2\pi)^8}\, e^{i(p-p_1).\xi + ip_1.\eta} \left\langle P \right| \bar{\psi}_j(0)\, A^{\alpha}(\eta)\, \psi_i(\xi) \left| P \right\rangle$$

Color gauge invariance

- Gauge invariance in a non-local situation requires a gauge link U(0,ξ)

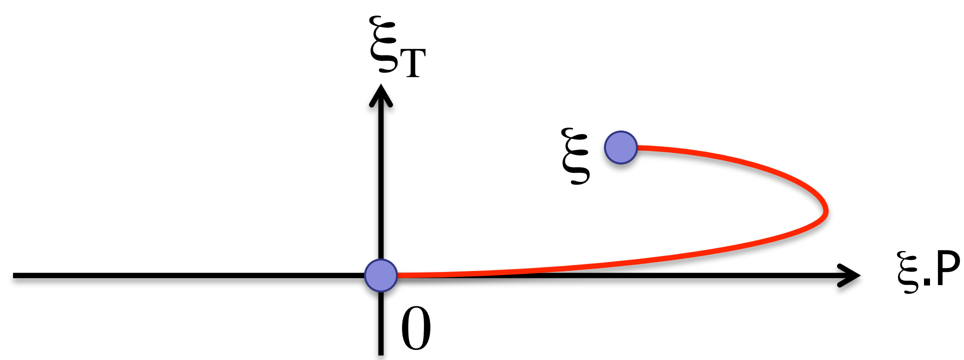
$$\bar{\psi}(0)\psi(\xi) = \sum_n \frac{1}{n!}\xi^{\mu_1}\ldots\xi^{\mu_N}\bar{\psi}(0)\,\partial_{\mu_1}\ldots\partial_{\mu_N}\psi(0)$$

$$U(0,\xi) = \mathcal{P}\,\exp\left(-ig\int_0^\xi ds^\mu A_\mu\right)$$

$$\bar{\psi}(0)U(0,\xi)\psi(\xi) = \sum_n \frac{1}{n!}\xi^{\mu_1}\ldots\xi^{\mu_N}\bar{\psi}(0)\,D_{\mu_1}\ldots D_{\mu_N}\psi(0)$$

- Introduces path dependence for $\Phi(x,p_T)$

$$\Phi^{[U]}(x,p_T) \;\Rightarrow\; \Phi(x)$$



Which gauge links?

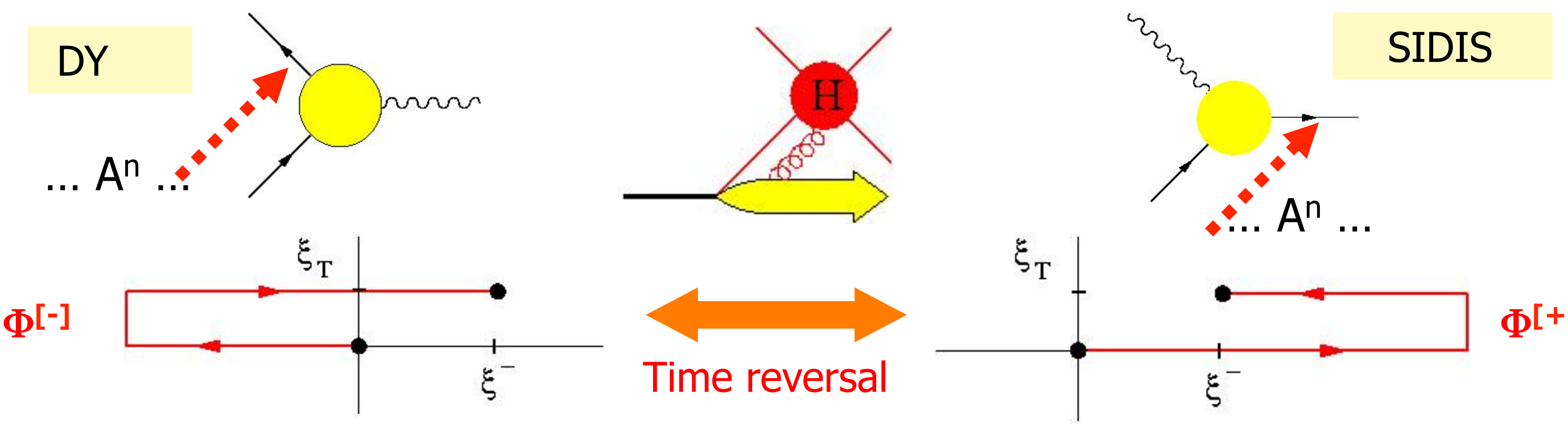
$$\Phi_{ij}^{q[C]}(x,p_T;n) = \int \frac{d(\xi.P)\,d^2\xi_T}{(2\pi)^3}\, e^{i\,p.\xi} \left\langle P\middle|\bar{\psi}_j(0)\,U_{[0,\xi]}^{[C]}\,\psi_i(\xi)\middle|P\right\rangle_{\xi.n=0}$$

TMD

$$\Phi_{ij}^{q}(x;n) = \int \frac{d(\xi.P)}{(2\pi)}\, e^{i\,p.\xi} \left\langle P\middle|\bar{\psi}_j(0)\,U_{[0,\xi]}^{[n]}\,\psi_i(\xi)\middle|P\right\rangle_{\xi.n=\xi_T=0}$$

collinear

- Gauge links come from dimension zero (not suppressed!) collinear A.n gluons, but leads for TMD correlators to process-dependence:



AV Belitsky, X Ji and F Yuan, NP B 656 (2003) 165
D Boer, PJM and F Pijlman, NP B 667 (2003) 201

Which gauge links?

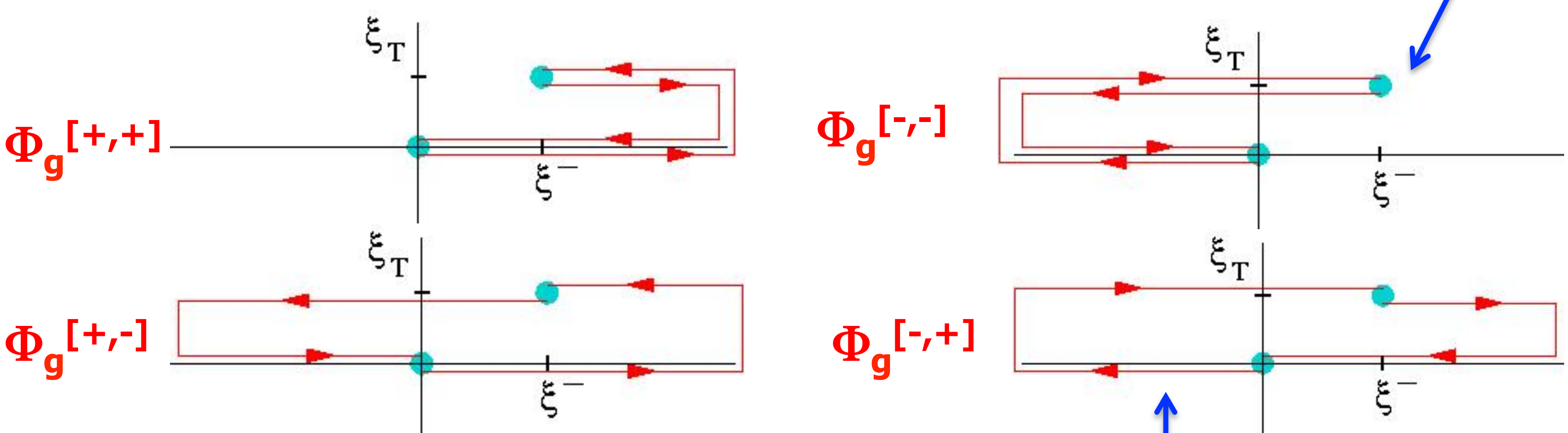
$$\Phi_g^{\alpha\beta[C,C']}(x,p_T;n) = \int \frac{d(\xi.P)\,d^2\xi_T}{(2\pi)^3}\, e^{i\,p.\xi} \left\langle P \right| U^{[C]}_{[\xi,0]} F^{n\alpha}(0)\, U^{[C']}_{[0,\xi]} F^{n\beta}(\xi) \left| P \right\rangle_{\xi.n=0}$$

- The TMD gluon correlators contain two links, which can have different paths. Note that standard field displacement involves C = C'

$$F^{\alpha\beta}(\xi) \rightarrow U^{[C]}_{[\eta,\xi]} F^{\alpha\beta}(\xi)\, U^{[C]}_{[\xi,\eta]}$$

- Basic (simplest) gauge links for gluon TMD correlators:

gg ➔ H

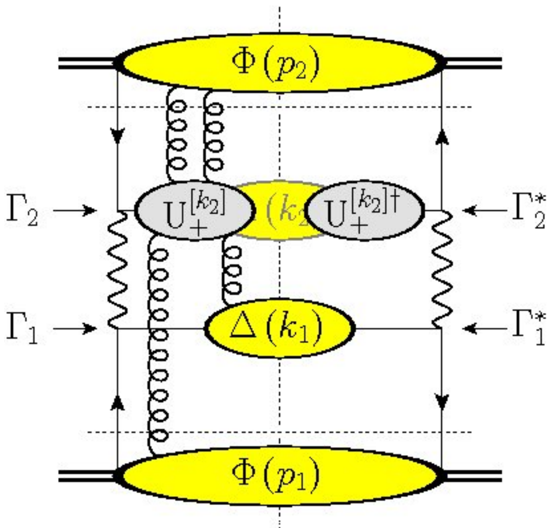


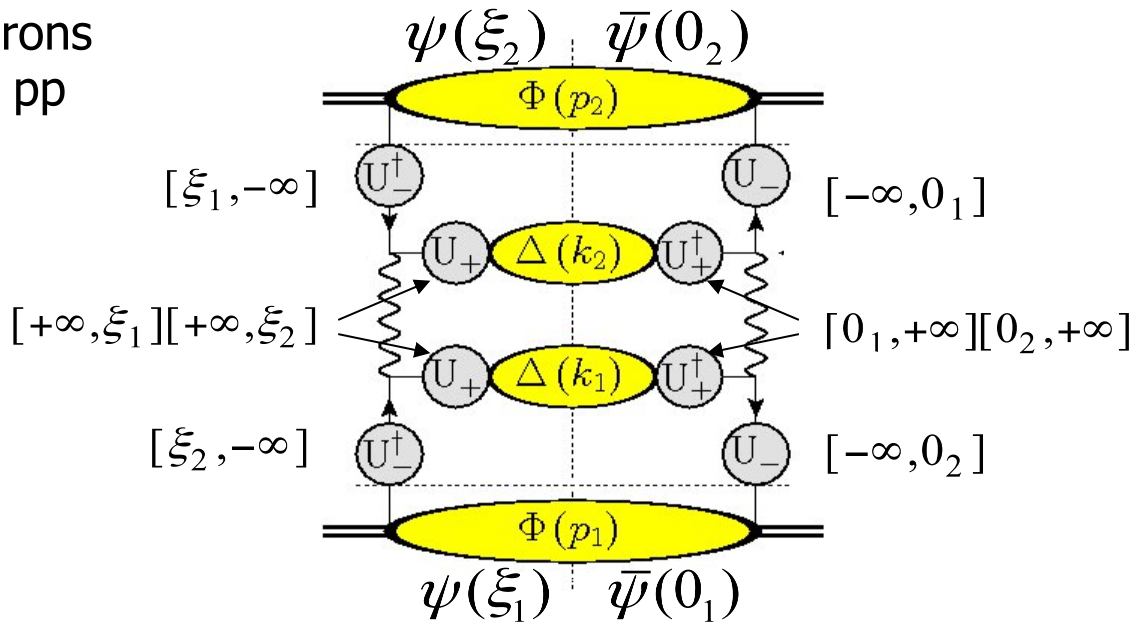
in gg ➔ $Q\bar{Q}$

C Bomhof, PJM, F Pijlman; EPJ C 47 (2006) 147
F Dominguez, B-W Xiao, F Yuan, PRL 106 (2011) 022301

Which gauge links?

- With more (initial state) hadrons color gets entangled, e.g. in pp





- Outgoing color contributes future pointing gauge link to $\Phi(p_2)$ and future pointing part of a loop in the gauge link for $\Phi(p_1)$

- Can be color-detangled if only p_T of one correlator is relevant (using polarization, …) but must include Wilson loops in final U

T.C. Rogers, PJM, PR D81 (2010) 094006

MGA Buffing, PJM, JHEP 07 (2011) 065

Summarizing: color gauge invariant correlators

- So it looks that at best we have well-defined matrix elements for TMDs but including multiple possiblities for gauge links and each process or even each diagram its own gauge link (depending on flow of color)
- Leading quark TMDs

$$\Phi^{[U]}(x,p_T;n) = \Big\{ f_1^{[U]}(x,p_T^2) - f_{1T}^{\perp[U]}(x,p_T^2)\,\frac{\epsilon_T^{p_T S_T}}{M} + g_{1s}^{[U]}(x,p_T)\gamma_5 + h_{1T}^{[U]}(x,p_T^2)\,\gamma_5\,\not{S}_T + h_{1s}^{\perp[U]}(x,p_T)\,\frac{\gamma_5\,\not{p}_T}{M} + i h_1^{\perp[U]}(x,p_T^2)\,\frac{\not{p}_T}{M}\Big\}\frac{\not{P}}{2},$$

- Leading gluon TMDs:

$$2x\,\Gamma^{\mu\nu[U]}(x,p_T) = -g_T^{\mu\nu}\,f_1^{g[U]}(x,p_T^2) + g_T^{\mu\nu}\frac{\epsilon_T^{p_T S_T}}{M}\,f_{1T}^{\perp g[U]}(x,p_T^2) + i\epsilon_T^{\mu\nu}\,g_{1s}^{g[U]}(x,p_T) + \left(\frac{p_T^\mu p_T^\nu}{M^2} - g_T^{\mu\nu}\frac{p_T^2}{2M^2}\right) h_1^{\perp g[U]}(x,p_T^2) - \frac{\epsilon_T^{p_T\{\mu}\,p_T^{\nu\}}}{2M^2}\,h_{1s}^{\perp g[U]}(x,p_T) - \frac{\epsilon_T^{p_T\{\mu}\,S_T^{\nu\}} + \epsilon_T^{S_T\{\mu}\,p_T^{\nu\}}}{4M}\,h_{1T}^{g[U]}(x,p_T^2).$$

Next step

Basic strategy: operator product expansion

- Taylor expansion for functions around zero

$$f(z) = \sum_n \frac{f^n}{n!} z^n \qquad f^n = \left. \frac{\partial^n f}{\partial z^n} \right|_{z=0}$$

- Mellin transform for functions on [-1,1] interval

$$f(x) = -\frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} dn\, x^{-n} M_n \qquad M_n = \int_0^1 dx\, x^{n-1} f(x)$$

- functions in (transverse) plane

$$f(p_T) = \sum_n \sum_{\alpha_1 \ldots \alpha_n} p_T^{\alpha_1} \ldots p_T^{\alpha_n} f_{\alpha_1 \ldots \alpha_n} \qquad f_{\alpha_1 \ldots \alpha_n} = \partial_{\alpha_1} \ldots \partial_{\alpha_n} f(p_T) \Big|_{p_T = 0}$$

Operator structure in collinear case (reminder)

- Collinear functions and x-moments

$$\Phi^q(x) = \int \frac{d(\xi.P)}{(2\pi)} e^{i\,p.\xi} \left\langle P \middle| \overline{\psi}(0)\, U_{[0,\xi]}^{[n]}\, \psi(\xi) \middle| P \right\rangle_{\xi.n=\xi_T=0}$$

$$x^{N-1}\,\Phi^q(x) = \int \frac{d(\xi.P)}{(2\pi)} e^{i\,p.\xi} \left\langle P \middle| \overline{\psi}(0)\, (\partial_\xi^n)^{N-1}\, U_{[0,\xi]}^{[n]}\, \psi(\xi) \middle| P \right\rangle_{\xi.n=\xi_T=0}$$

x = p.n

$$= \int \frac{d(\xi.P)}{(2\pi)} e^{i\,p.\xi} \left\langle P \middle| \overline{\psi}(0)\, U_{[0,\xi]}^{[n]}\, (D_\xi^n)^{N-1}\, \psi(\xi) \middle| P \right\rangle_{\xi.n=\xi_T=0}$$

- Moments correspond to local matrix elements of operators that all have the same twist since dim(D^n) = 0

$$\Phi^{(N)} = \left\langle P \middle| \overline{\psi}(0) (D^n)^{N-1}\, \psi(0) \middle| P \right\rangle$$

- Moments are particularly useful because their anomalous dimensions can be rigorously calculated and these can be Mellin transformed into the splitting functions that govern the QCD evolution.

Operator structure in TMD case

- For TMD functions one can consider transverse moments

$$\Phi(x,p_T;n) = \int \frac{d(\xi.P)d^2\xi_T}{(2\pi)^3} e^{i\,p.\xi} \left\langle P \middle| \bar{\psi}(0)\, U_{[0,\xi]}^{[\pm]}\, \psi(\xi) \middle| P \right\rangle_{\xi.n=0}$$

$$p_T^{\alpha}\Phi^{[\pm]}(x,p_T;n) = \int \frac{d(\xi.P)d^2\xi_T}{(2\pi)^3} e^{i\,p.\xi} \left\langle P \middle| \bar{\psi}(0) U_{[0,\pm\infty]} D_T^{\alpha} U_{[\pm\infty,\xi]} \psi(\xi) \middle| P \right\rangle_{\xi.n=0}$$

$$p_T^{\alpha_1} p_T^{\alpha_2}\Phi^{[\pm]}(x,p_T;n) = \int \frac{d(\xi.P)d^2\xi_T}{(2\pi)^3} e^{i\,p.\xi} \left\langle P \middle| \bar{\psi}(0) U_{[0,\pm\infty]} D_T^{\alpha_1} D_T^{\alpha_2} U_{[\pm\infty,\xi]} \psi(\xi) \middle| P \right\rangle_{\xi.n=0}$$

- Upon integration, these do involve collinear twist-3 multi-parton correlators

MGA Buffing, A Mukherjee, PJM, PRD 86 (2012) 074030 , Arxiv: 1207.3221 [hep-ph]

Operator structure in TMD case

- For first transverse moment one needs quark-gluon correlators

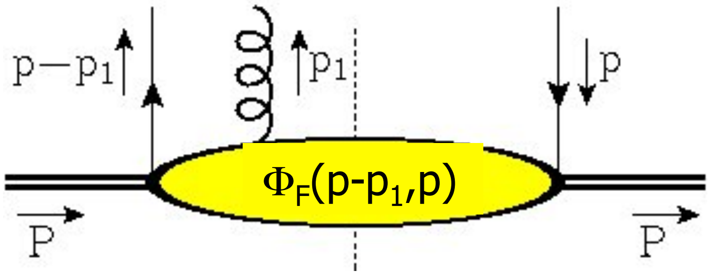
$$\Phi_D^\alpha(x-x_1,x_1\,|\,x) = \int \frac{d\xi.P\, d\eta.P}{(2\pi)^2} e^{i(p-p_1).\xi+ip_1.\eta} \langle P|\overline{\psi}(0)D_T^\alpha(\eta)\,\psi(\xi)|P\rangle_{\xi.n=\xi_T=0}$$

$$\Phi_F^\alpha(x-x_1,x_1\,|\,x) = \int \frac{d\xi.P\, d\eta.P}{(2\pi)^2} e^{i(p-p_1).\xi+ip_1.\eta} \langle P|\overline{\psi}(0)F^{n\alpha}(\eta)\,\psi(\xi)|P\rangle_{\xi.n=\xi_T=0}$$

- In principle multi-parton, but we need

$$\Phi_D^\alpha(x) = \int dx_1\, \Phi_D^\alpha(x-x_1,x_1\,|\,x)$$

$$\Phi_A^\alpha(x) = PV\int dx_1\, \frac{1}{x_1}\Phi_F^{n\alpha}(x-x_1,x_1\,|\,x)$$



$$\boxed{\tilde{\Phi}_\partial^\alpha(x) = \Phi_D^\alpha(x) - \Phi_A^\alpha(x)}$$ T-even (gauge-invariant derivative)

$$\boxed{\Phi_G^\alpha(x) = \pi\,\Phi_F^{n\alpha}(x,0\,|\,x)}$$ T-odd (soft-gluon or gluonic pole)

Efremov, Teryaev; Qiu, Sterman; Brodsky, Hwang, Schmidt; Boer, Teryaev, M; Bomhof, Pijlman, M

Operator structure in TMD case

- Transverse moments can be expressed in these particular collinear multi-parton twist-3 correlators (which are not suppressed!)

$$\Phi_{\partial}^{\alpha[U]}(x) = \int d^2p_T\; p_T^\alpha \Phi^{[U]}(x,p_T;n) = \tilde{\Phi}_{\partial}^{\alpha}(x) + C_G^{[U]}\Phi_G^\alpha(x)$$

T-even

T-odd

T-even

T-even

T-odd

$$\Phi_{\partial\partial}^{\alpha\beta[U]}(x) = \tilde{\Phi}_{\partial\partial}^{\alpha\beta}(x) + C_{GG,c}^{[U]}\,\Phi_{GG,c}^{\alpha\beta}(x) + C_G^{[U]}\left(\tilde{\Phi}_{\partial G}^{\alpha\beta}(x) + \tilde{\Phi}_{G\partial}^{\alpha\beta}(x)\right)$$

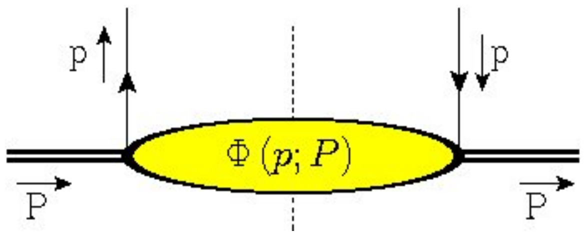
$\mathrm{Tr}_c(GG\,\psi\bar{\psi})$

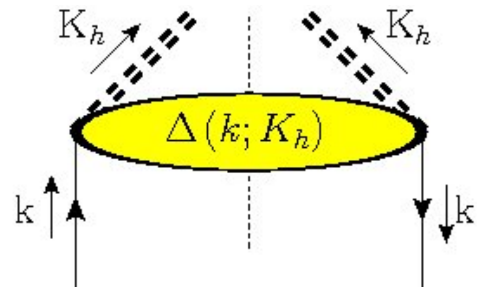
$\mathrm{Tr}_c(GG)\,\mathrm{Tr}_c(\psi\bar{\psi})$

- $C_G^{[U]}$ calculable gluonic pole factors

U	$U^{[\pm]}$	$U^{[+]}\,U^{[\square]}$	$\frac{1}{N_c}\,\mathrm{Tr}_c(U^{[\square]})\,U^{[+]}$
$\Phi^{[U]}$	$\Phi^{[\pm]}$	$\Phi^{[+\square]}$	$\Phi^{[(\square)+]}$
$C_G^{[U]}$	± 1	3	1
$C_{GG,1}^{[U]}$	1	9	1
$C_{GG,2}^{[U]}$	0	0	4

Distribution versus fragmentation functions





- Operators:

$$\Phi^{[U]}(p\,|\,p) \sim \left\langle P\,|\,\bar{\psi}(0)U_{[0,\xi]}\psi(\xi)\,|\,P\right\rangle$$

$$\Phi_{\partial}^{\alpha[U]}(x) = \tilde{\Phi}_{\partial}^{\alpha}(x) + C_G^{[U]}\,\Phi_G^{\alpha}(x)$$

T-even T-odd (gluonic pole)

$$\boxed{\Phi_G^{\alpha}(x) = \pi\Phi_F^{n\alpha}(x,0\,|\,x) \neq 0}$$

- Operators:

out state

$$\Delta(k\,|\,k) \sim \sum_X \left\langle 0\,|\,\psi(\xi)\,|\,K_h X\right\rangle\left\langle K_h X\,|\,\bar{\psi}(0)\,|\,0\right\rangle$$

$$\boxed{\Delta_G^{\alpha}(x) = \pi\Delta_F^{n\alpha}\left(\tfrac{1}{Z},0\,|\,\tfrac{1}{Z}\right) = 0}$$

$$\Delta_{\partial}^{\alpha[U]}(x) = \tilde{\Delta}_{\partial}^{\alpha}(x)$$

T-even operator combination, but still T-odd functions!

Collins, Metz; Meissner, Metz; Gamberg, M, Mukherjee, PR D 83 (2011) 071503

Classifying Quark TMDs

- Collecting right moments gives expansion into full TMD PDFs of definite rank

$$\Phi^{[U]}(x,p_T) = \Phi(x,p_T^2) + p_{Ti}\tilde{\Phi}_\partial^i(x,p_T^2) + p_{Tij}\tilde{\Phi}_{\partial\partial}^{ij}(x,p_T^2) + \ldots$$
$$+\sum_c C_{G,c}^{[U]}\left[p_{Ti}\Phi_{G,c}^i(x,p_T^2) + C_{G,c}^{[U]}p_{Tij}\tilde{\Phi}_{\{\partial G\}}^{ij}(x,p_T^2) + \ldots\right]$$
$$+\sum_c C_{GG,c}^{[U]}\left[p_{Tij}\Phi_{GG,c}^{ij}(x,p_T^2) + \ldots\right]$$

- but for TMD PFFs

$$\Delta^{[U]}(z^{-1},k_T) = \Delta(z^{-1},k_T^2) + k_{Ti}\tilde{\Delta}_\partial^i(z^{-1},k_T^2) + k_{Tij}\tilde{\Delta}_{\partial\partial}^{ij}(z^{-1},k_T^2) + \ldots$$

Classifying Quark TMDs

factor	TMD PDF RANK			
	0	1	2	3
1	$\Phi(x, p_T^2)$	$\tilde{\Phi}_{\partial}(x, p_T^2)$	$\tilde{\Phi}_{\partial\partial}(x, p_T^2)$	$\tilde{\Phi}_{\partial\partial\partial}(x, p_T^2)$
$C_{G,c}^{[U]}$		$\Phi_{G,c}(x, p_T^2)$	$\tilde{\Phi}_{\{G\partial\},c}(x, p_T^2)$	$\tilde{\Phi}_{\{G\partial\partial\},c}(x, p_T^2)$
$C_{GG,c}^{[U]}$			$\Phi_{GG,c}(x, p_T^2)$	$\tilde{\Phi}_{\{GG\partial\},c}(x, p_T^2)$
$C_{GGG,c}^{[U]}$				$\Phi_{GGG,c}(x, p_T^2)$

- Only a finite number needed: rank up to $2(S_{\text{hadron}}+s_{\text{parton}})$
- Rank m shows up as $\cos(m\phi)$ and $\sin(m\phi)$ azimuthal asymmetries
- No gluonic poles for PFFs

factor	TMD PFF RANK			
	0	1	2	3
1	$\Delta(z^{-1}, k_T^2)$	$\tilde{\Delta}_{\partial}(z^{-1}, k_T^2)$	$\tilde{\Delta}_{\partial\partial}(z^{-1}, k_T^2)$	$\tilde{\Delta}_{\partial\partial\partial}(z^{-1}, k_T^2)$

MGA Buffing, A Mukherjee, PJM, PRD2012 , Arxiv: 1207.3221 [hep-ph]

Classifying Quark TMDs

factor	QUARK TMD PDF RANK UNPOLARIZED HADRON			
	0	1	2	3
1	f_1			
$C_G^{[U]}$		$h_1^{\perp}$		
$C_{GG,c}^{[U]}$				

- Only a finite number needed: rank up to $2(S_{hadron}+s_{parton})$
- Rank m shows up as cos(mϕ) and sin(mϕ) azimuthal asymmetries
- Example: quarks in an unpolarized target are described by just 2 functions

$$\Phi(x,p_T^2) = \left(f_1(x,p_T^2)\right)\frac{\not{P}}{2}$$

T-even

$$\Phi_G^{\alpha}(x,p_T^2) = \left(i h_1^{\perp}(x,p_T^2)\frac{\gamma_T^{\alpha}}{M}\right)\frac{\not{P}}{2}$$

T-odd

[B-M function]

Classifying Quark TMDs

factor	QUARK TMD PDFs RANK SPIN ½ HADRON			
	0	1	2	3
1	$f_1,\ g_1,\ h_1$	$g_{1T},\ h_{1L}^{\perp}$	$h_{1T}^{\perp(A)}$	
$C_G^{[U]}$		$h_1^{\perp},\ f_{1T}^{\perp}$		
$C_{GG,c}^{[U]}$			$h_{1T}^{\perp(B1)},\ h_{1T}^{\perp(B2)}$	

Three pretzelocities:

$$A:\ \ \bar{\psi}\partial\partial\psi = Tr_c\left[\partial\partial\psi\bar{\psi}\right]$$

$$B1:\ Tr_c\left[GG\psi\bar{\psi}\right]$$

$$B2:\ Tr_c\left[GG\right]Tr_c\left[\psi\bar{\psi}\right]$$

Classifying Quark TMDs

factor	QUARK TMD PDFs RANK SPIN ½ HADRON			
	0	1	2	3
1	$f_1,\ g_1,\ h_1$	$g_{1T},\ h_{1L}^{\perp}$	$h_{1T}^{\perp(A)}$	
$C_G^{[U]}$		$h_1^{\perp},\ f_{1T}^{\perp}$		
$C_{GG,c}^{[U]}$			$h_{1T}^{\perp(B1)},\ h_{1T}^{\perp(B2)}$	

factor	QUARK TMD PFFs RANK SPIN ½ HADRON			
	0	1	2	3
1	$D_1,\ G_1,\ H_1$	$D_{1T}^{\perp}, G_{1T}, H_1^{\perp}, H_{1L}^{\perp}$	$H_{1T}^{\perp}$	

Just a single ‘pretzelocity’ PFF

Classifying Gluon TMDs

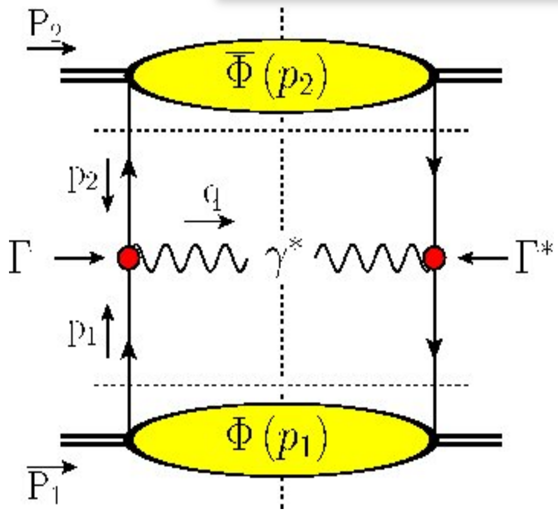
factor	GLUON TMD PDF RANK UNPOLARIZED HADRON			
	0	1	2	3
1	f_1		$h_1^{\perp(A)}$	
$C_{GG,c}^{[U]}$			$h_1^{\perp(Bc)}$	

factor	GLUON TMD PDF RANK SPIN ½ HADRON			
	0	1	2	3
1	$f_1,\ g_1$	g_{1T}	$h_1^{\perp(A)}$	
$C_{G,c}^{[U]}$		$f_{1T}^{\perp(Ac)},\ h_{1T}^{(Ac)}$	$h_{1L}^{\perp(A,c)}$	$h_{1T}^{\perp(Ac)}$
$C_{GG,c}^{[U]}$			$h_1^{\perp(Bc)}$	
$C_{GGG,c}^{[U]}$				$h_{1T}^{\perp(Bc)}$

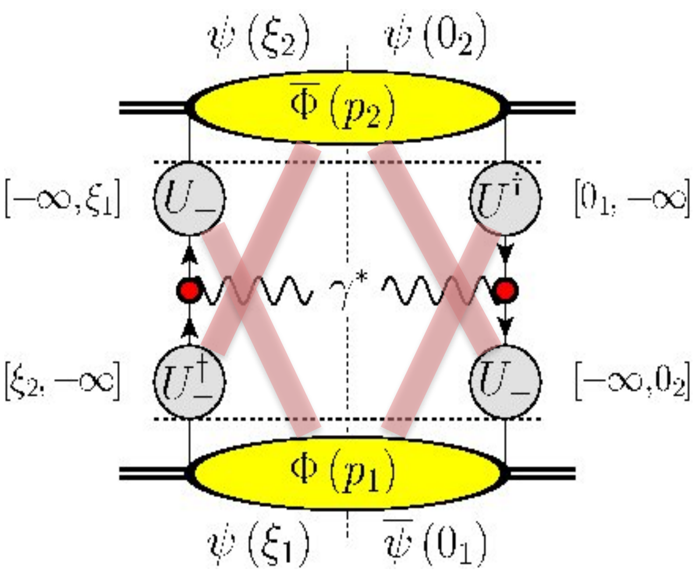
MGA Buffing, A Mukherjee, PJM, PRD (2013) Arxiv: 1306.6513 [hep-ph]

Multiple TMDs in cross sections

Correlators in description of hard process (e.g. DY)



$$\xcancel{\begin{aligned} d\sigma_{\rm DY} &\sim \mathrm{Tr}_c\left[\Phi(x_1,p_{1T})\Gamma^*\overline{\Phi}(x_2,p_{2T})\Gamma\right] \\ &= \frac{1}{N_c}\Phi(x_1,p_{1T})\Gamma^*\overline{\Phi}(x_2,p_{2T})\Gamma, \end{aligned}}$$



- Complications if the transverse momentum of two initial state hadrons is involved, resulting for DY at measured Q_T in

$$\begin{aligned} d\sigma_{\rm DY} &= \mathrm{Tr}_c\Big[U_-^\dagger[p_2]\Phi(x_1,p_{1T})U_-[p_2]\Gamma^* \\ &\qquad \times U_-^\dagger[p_1]\overline{\Phi}(x_2,p_{2T})U_-[p_1]\Gamma\Big] \\ &\neq \frac{1}{N_c}\Phi^{[-]}(x_1,p_{1T})\Gamma^*\overline{\Phi}^{[-^\dagger]}(x_2,p_{2T})\Gamma, \end{aligned}$$

Just as for twist-3 squared in collinear DY

Classifying Quark TMDs

factor	TMD RANK			
	0	1	2	3
1	$\Phi(x,p_T^2)$	$\tilde{\Phi}_{\partial}(x,p_T^2)$	$\tilde{\Phi}_{\partial\partial}(x,p_T^2)$	$\tilde{\Phi}_{\partial\partial\partial}(x,p_T^2)$
$C_{G,c}^{[U]}$		$\Phi_{G,c}(x,p_T^2)$	$\tilde{\Phi}_{\{G\partial\},c}(x,p_T^2)$	$\tilde{\Phi}_{\{G\partial\partial\},c}(x,p_T^2)$
$C_{GG,c}^{[U]}$			$\Phi_{GG,c}(x,p_T^2)$	$\tilde{\Phi}_{\{GG\partial\},c}(x,p_T^2)$
$C_{GGG,c}^{[U]}$				$\Phi_{GGG,c}(x,p_T^2)$

$$\sigma(x_1,x_2,q_T) \sim \frac{1}{N_c} f^{[U_1,U_2]}_{R_{G1}R_{G2}}\, \Phi^{[U_1]}(x_1,p_{1T}) \otimes \overline{\Phi}^{[U_2]}(x_2,p_{2T})\, \hat{\sigma}(x_1,x_2),$$

R_G for $\overline{\Phi}^{[-^\dagger]}$	R_G for $\Phi^{[-]}$: 0	1	2
0	1	1	1
1	1	$-\frac{1}{N_c^2-1}$	$\frac{N_c^2+2}{(N_c^2-2)(N_c^2-1)}$
2	1	$\frac{N_c^2+2}{(N_c^2-2)(N_c^2-1)}$	$\frac{3N_c^4-8N_c^2-4}{(N_c^2-2)^2(N_c^2-1)}$

$$\frac{\mathrm{Tr}_c[T^aT^bT^aT^b]}{\mathrm{Tr}_c[T^aT^a]\,\mathrm{Tr}_c[T^bT^b]} = -\frac{1}{N_c^2-1}\frac{1}{N_c}.$$

Remember classification of Quark TMDs

factor	QUARK TMD RANK UNPOLARIZED HADRON			
	0	1	2	3
1	f_1			
$C_G^{[U]}$		$h_1^\perp$		
$C_{GG,c}^{[U]}$				

- Example: quarks in an unpolarized target needs only 2 functions
- Resulting in cross section for unpolarized DY at measured Q_T

$$\sigma_{DY}(x_1,x_2,q_T) = \frac{1}{N_c}\Phi(x_1,p_{1T})\otimes\bar{\Phi}(x_2,p_{2T})$$

contains f_1

$$-\frac{1}{N_c}\frac{1}{N_c^2-1}\,q_T^{\alpha\beta}\Phi_G^{\alpha}(x_1,p_{1T})\otimes\bar{\Phi}_G^{\beta}(x_2,p_{2T})$$

contains $h_1^{perp\cdot}$

D. Boer, PRD 60 (1999) 014012; MGA Buffing, PRL (2014) PJM, Arxiv: 1309.4681 [hep-ph]

Conclusion with (potential) rewards

- (Generalized) universality studied via operator product expansion, extending the well-known collinear distributions (including polarization 3 for quarks and 2 for gluons) to novel TMD PDF and PFF functions, ordered into functions of definite rank.
- Knowledge of operator structure is important for lattice calculations.
- The rank m is linked to specific cos(mϕ) and sin(mϕ) azimuthal asymmetries.
- TMDs encode aspects of hadronic structure, e.g. spin-orbit correlations, such as T-odd transversely polarized quarks or T-even longitudinally polarized gluons in an unpolarized hadron, thus possible applications for precision probing at the LHC, but for sure at a polarized EIC.
- The TMD PDFs appear in cross sections with specific calculable factors that deviate from (or extend on) the naïve parton universality for hadron-hadron scattering.

Thank you