

Recent developments in hadron structure: from electron scattering to atomic physics

Marc Vanderhaeghen (JGU Mainz)



[XLII International Workshop: Hirschegg 2014](#)
Hirschegg, Austria, January 12 - 18, 2014

Unraveling the structure of mesons and baryons

- **Basic question: unraveling strong QCD**

Origin of mass, spin, imaging of hadrons

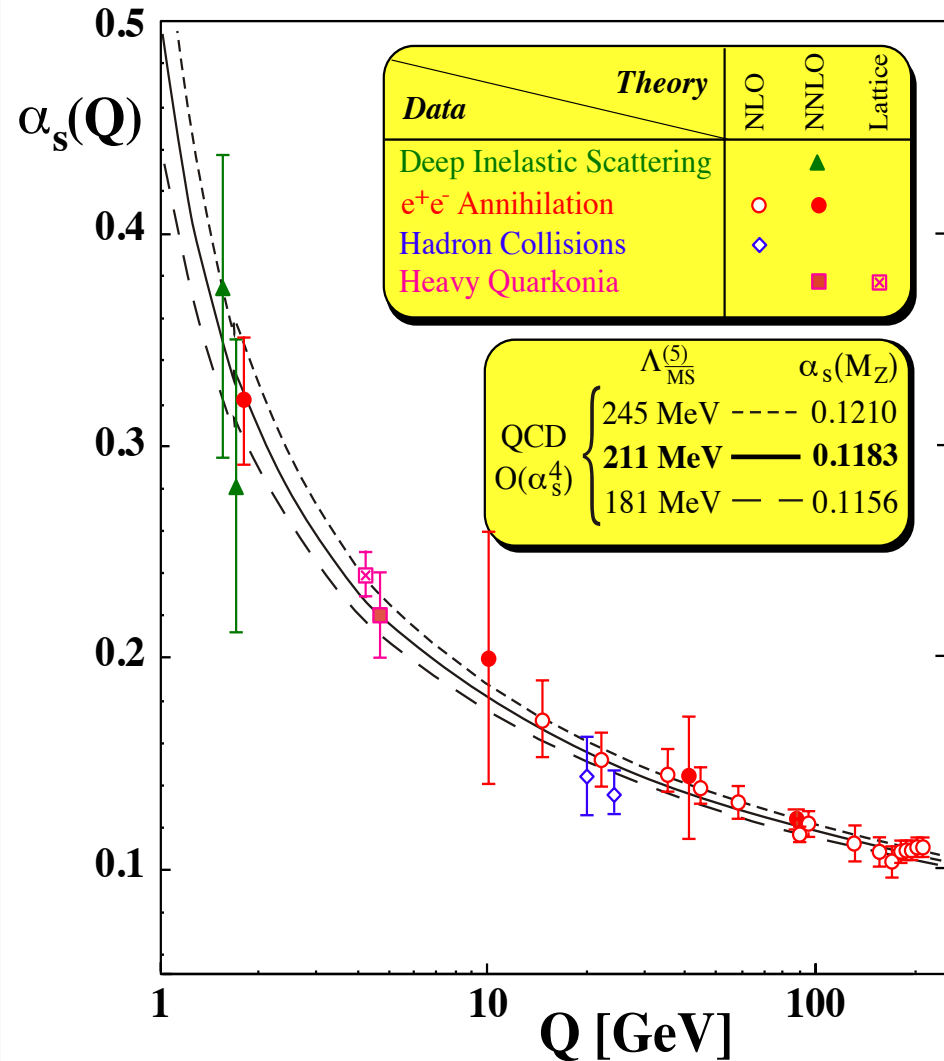
- **Precision hadron physics**

Impact on new physics searches: $(g-2)_\mu$, dark photon search, proton radius puzzle, weak mixing angle

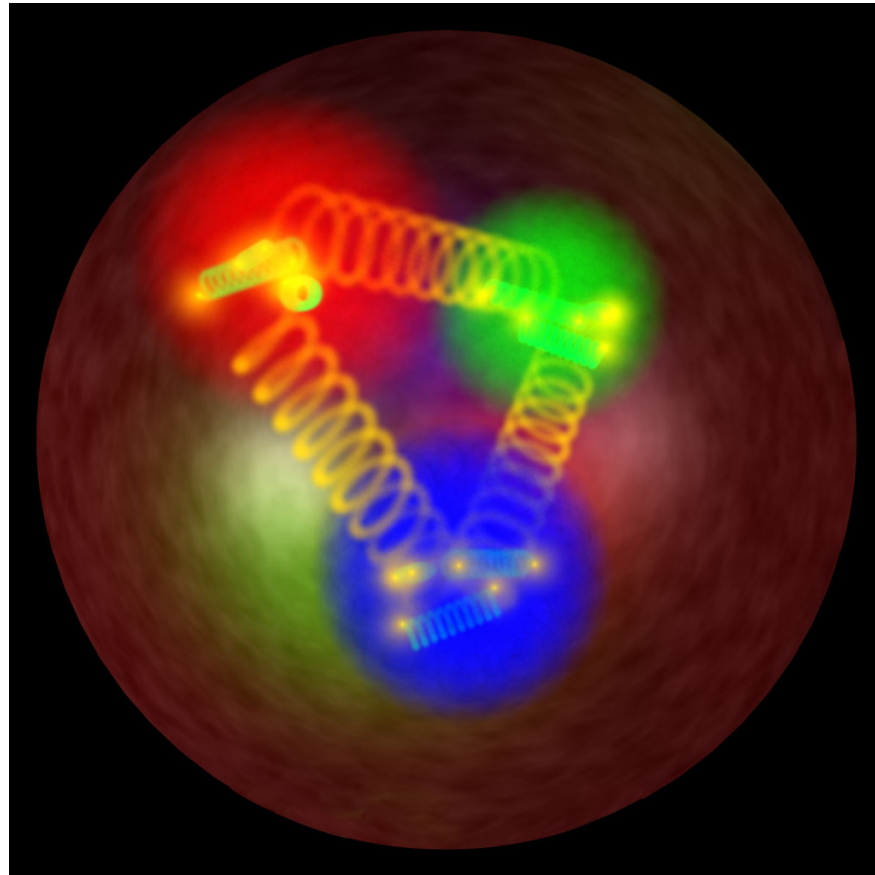
- **Theory tools**

lattice QCD: ab initio

EFT/phenomenology: interplay with precision hadron data

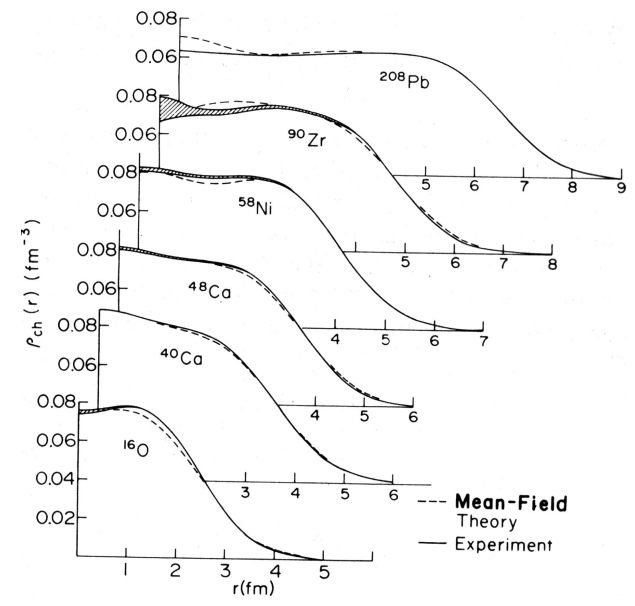
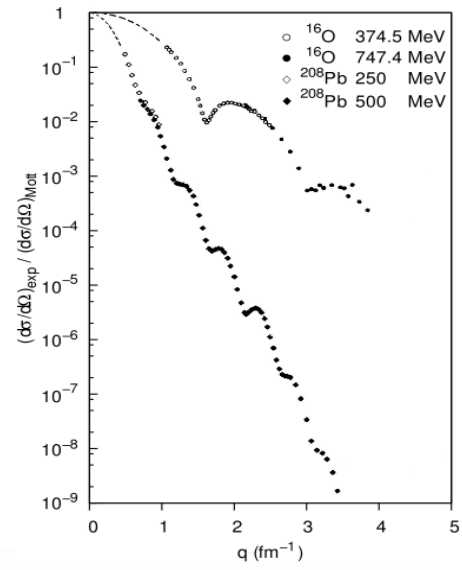


What is the size of the proton?

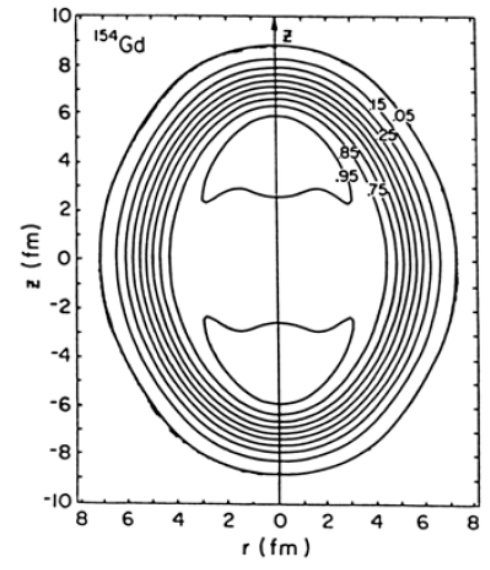
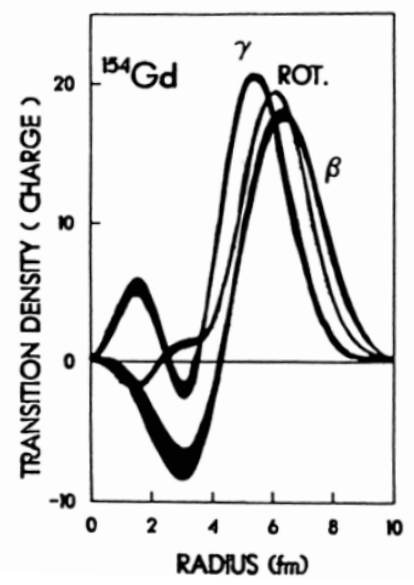


Sizes and shapes of non-relativistic many-body systems

Sizes of nuclei
 as revealed through
elastic electron scattering



Shapes of deformed nuclei
 as revealed through
inelastic electron scattering



Electron scattering facilities MAMI, JLab:

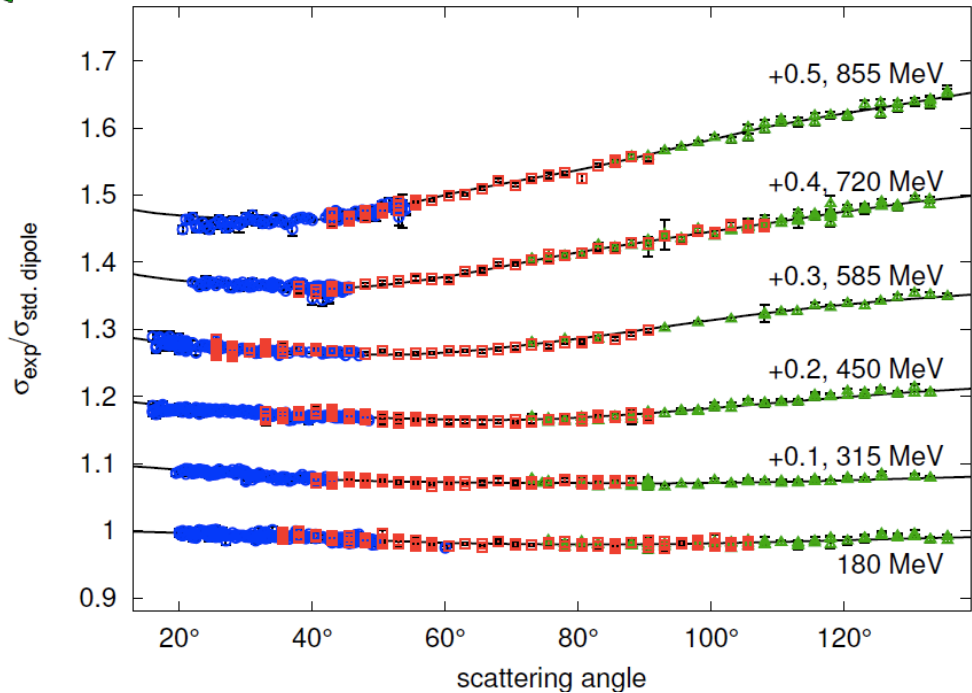
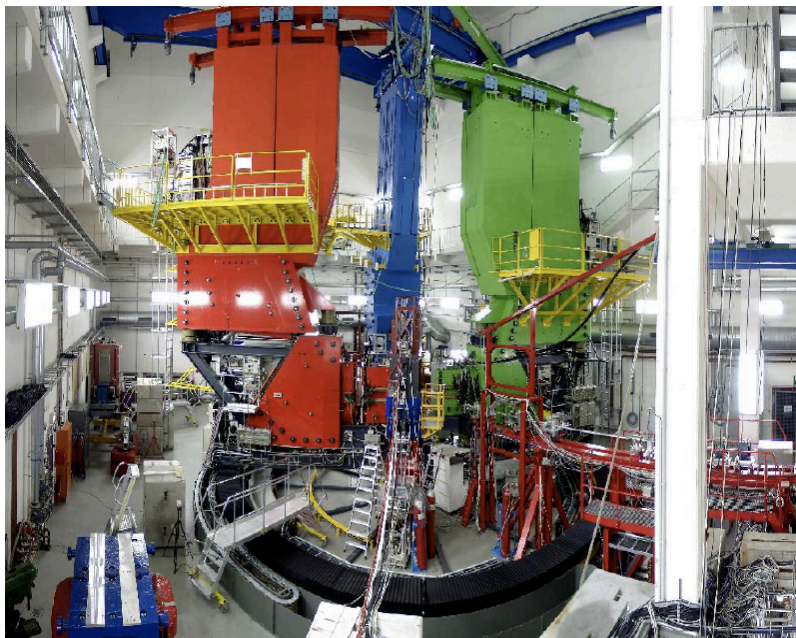
uniquely positioned to deliver high-precision hadron data

recent cross section data **A1@MAMI**

High momentum resolution $\sim 10^{-4}$

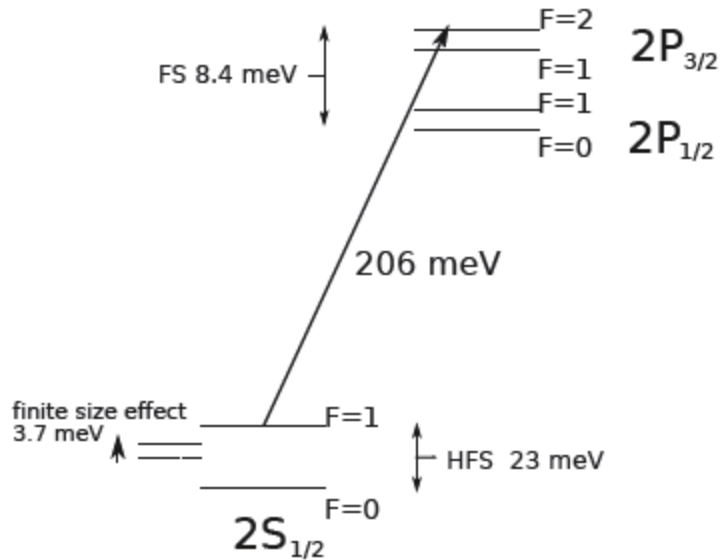


MAMI achieved 1% measurement of R_E



Bernauer et al. (2010)

extraction of R_E from μH Lamb shift



- Lamb shift is dominated by vacuum polarization : drops 2S state by a lot
 - Experiment measures 2S $F=1$ to 2P $_{3/2}$ $F=2$ state (F is total angular momentum)
 - Finite size effect on s-wave states ($l=0$)
- Non-relativistic 1γ -exchange calculation

$$\Delta E = \frac{2\pi\alpha}{3} R_E^2 \phi_n^2(0)$$

Karplus, Klein, Schwinger (1952)

- Leading term of order $O(\alpha^4)$: $\phi_n^2(0) = m_r^3 \alpha^3 / (\pi n^3)$

Lamb Shift

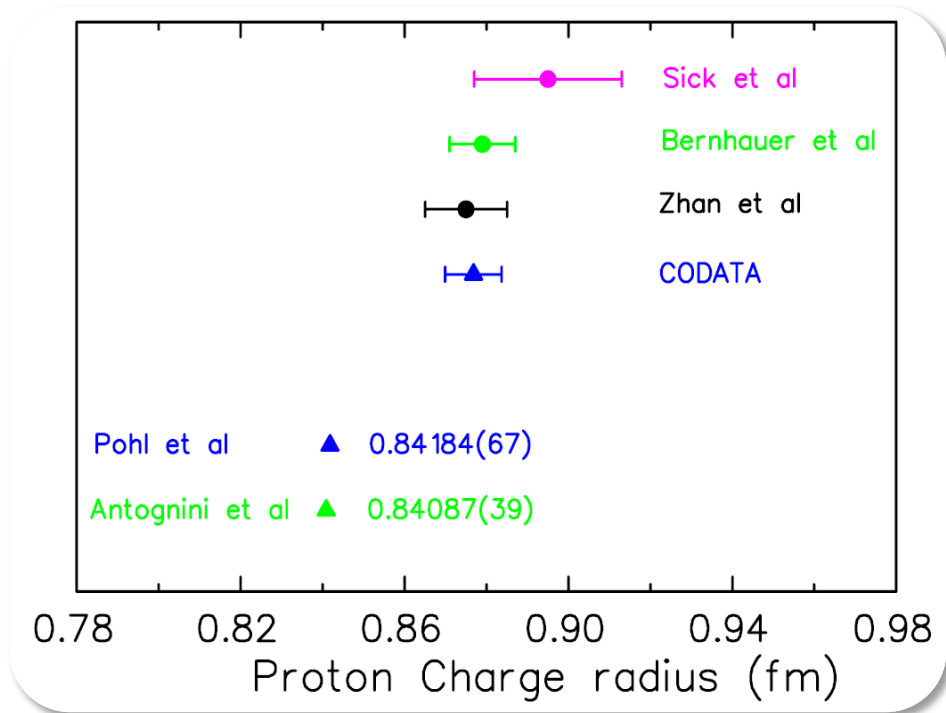
$$\Delta E_{LS} = 209.9779 (49) - 5.2262 R_E^2 + 0.00913 R_{(2)}^3 \text{ meV}$$

3.70 meV

0.026 meV

$R_{(2)}^3$: $O(\alpha^5)$ correction term

Proton radius puzzle ?



μH data:

$$R_E = 0.8409 \pm 0.0004 \text{ fm}$$

Pohl et al. (2010)

Antognini et al. (2013)



**7.7σ
difference !?**

ep-data :

CODATA

$$R_E = 0.8772 \pm 0.0046 \text{ fm}$$

Bernauer et al. (2010)

Zhan et al. (2011)

Experimental precision $\approx 2 \mu\text{eV}$

**Energy shift ascribed to finite
proton size is $310 \mu\text{eV}$ less
than expected !!!**

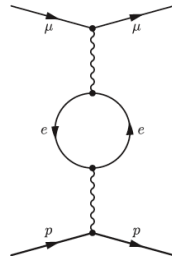
Lamb shift: QED corrections

- Calculated by several groups

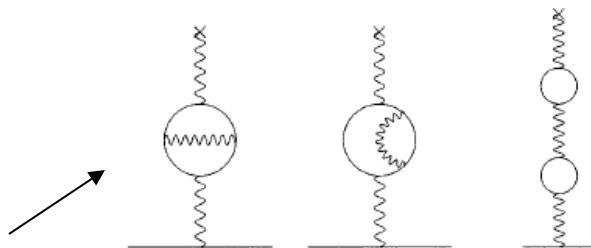
Pachucki (1996, 1999)

Borie (1976, 2005)

- 1 loop electron



$$\Delta E = 205.0282 \text{ meV}$$



$$\Delta E = 1.5081 \text{ meV}$$

- 2 loop electron



$$\Delta E = 0.1509 \text{ meV}$$

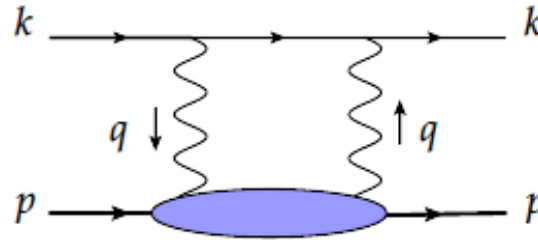
- Muon self-energy, vacuum polarization $\Delta E = -0.6677 \text{ meV}$

- other QED corrections calculated : all of size 0.005 meV or smaller $\ll 0.3 \text{ meV}$

Lamb shift: hadronic corrections (I)

- **Finite-size** correction:

$\gamma\gamma$ box diagram



- “3rd Zemach moment”

non-rel. calculation

Friar (1979)

$$R_{(2)}^3 = \int d^3\vec{r}_1 d^3\vec{r}_2 |\vec{r}_1 - \vec{r}_2|^3 \rho_E(r_1) \rho_E(r_2)$$

$$= \frac{48}{\pi} \int_0^\infty \frac{dQ}{Q^4} \left[G_E^2(Q) - 1 - 2Q^2 G_E(0) \frac{dG_E}{dQ^2}(0) \right]$$

recent evaluation

**Distler, Bernauer,
Walcher (2011)**

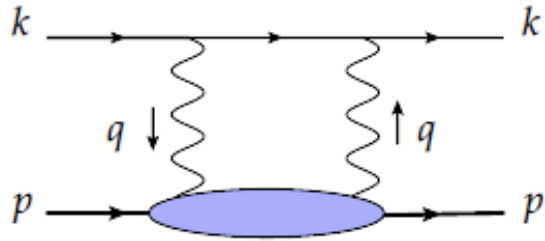
$$R_{(2)}^3 = 2.85 (8) \text{ fm}^3 \longrightarrow \Delta E \approx -0.026 \text{ meV}$$

- What do we know model independently ?

Lower blob contains both elastic (nucleon) and in-elastic states

Information is contained in **forward, double virtual Compton scattering**

Lamb shift: hadronic corrections (II)



$$\begin{aligned}
 T^{\mu\nu}(p, q) &= \frac{i}{8\pi M} \int d^4x e^{iqx} \langle p | T j^\mu(x) j^\nu(0) | p \rangle \\
 &= \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) T_1(\nu, Q^2) \\
 &+ \frac{1}{M^2} \left(p^\mu - \frac{p \cdot q}{q^2} q^\mu \right) \left(p^\nu - \frac{p \cdot q}{q^2} q^\nu \right) T_2(\nu, Q^2)
 \end{aligned}$$

- Lower blob contains both elastic (nucleon) and in-elastic states



**Hadron physics
input required**

Information contained in **forward, double virtual Compton scattering**

- Described by two amplitudes **T1** and **T2**: function of energy ν and virtuality Q^2

$$\text{Im } T_1(\nu, Q^2) = \frac{1}{4M} F_1(\nu, Q^2)$$

- Imaginary parts of **T1**, **T2**: unpolarized structure functions of proton

$$\text{Im } T_2(\nu, Q^2) = \frac{1}{4\nu} F_2(\nu, Q^2)$$

- ΔE evaluated through an integral over Q^2 and ν

$$\begin{aligned}
 \Delta E &= \Delta E^{el} \quad \rightarrow \text{Elastic state: involves } \mathbf{nucleon \ form \ factors} \\
 &+ \Delta E^{subtr} \quad \rightarrow \text{Subtraction: involves } \mathbf{nucleon \ polarizabilities} \\
 &+ \Delta E^{inel} \quad \rightarrow \text{Inelastic, dispersion integrals: involves } \mathbf{structure \ functions \ F1, \ F2}
 \end{aligned}$$

Lamb shift: hadronic corrections (III)

- Low-energy expansion of forward, doubly virtual Compton scattering constrains subtraction term $T_1(0, Q^2)$

effective Hamiltonian : $\mathcal{H} = -\frac{1}{2}4\pi\alpha_E\vec{E}^2 - \frac{1}{2}4\pi\beta_M\vec{B}^2$

↓ electric ↓ magnetic polarizabilities

$$\lim_{\nu^2, Q^2 \rightarrow 0} T_1^{\text{non-Born}}(\nu, Q^2) = \frac{\nu^2}{e^2}(\alpha_E + \beta_M) + \frac{Q^2}{e^2}\beta_M$$

$$\lim_{\nu^2, Q^2 \rightarrow 0} T_2^{\text{non-Born}}(\nu, Q^2) = \frac{Q^2}{e^2}(\alpha_E + \beta_M)$$

subtraction term for T_1

- Numerical evaluations :

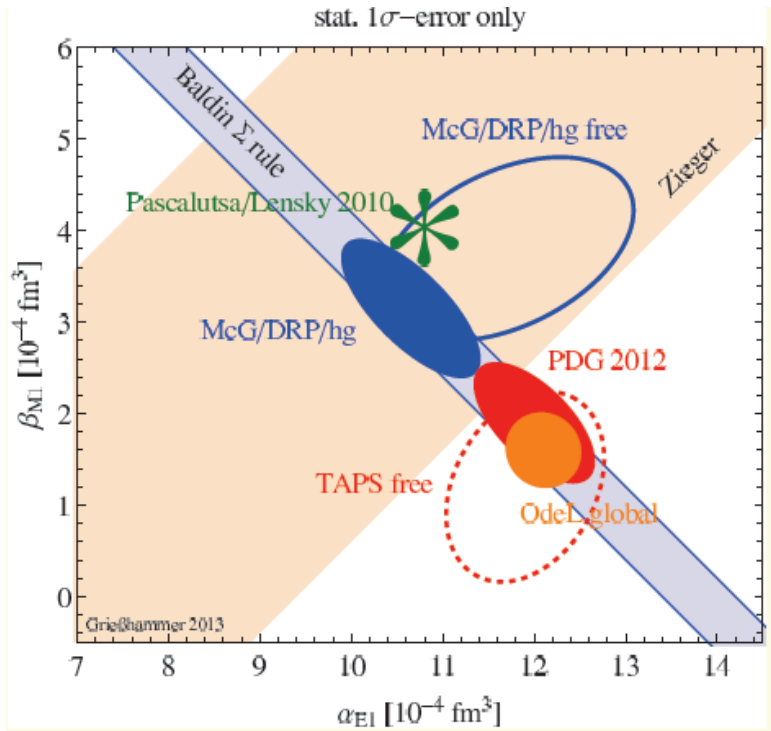
(μeV)	Carlson, Vdh (2011)	Pachucki (1999)	Martyntenko (2006)
ΔE^{subt}	5.3 ± 1.9	1.8	2.3
ΔE^{inel}	-12.7 ± 0.5	-13.9	-13.8
ΔE^{cl}	-29.5 ± 1.3	-23.0	-23.0
ΔE	-36.9 ± 2.4	-35.1	-34.5

Carlson, Vdh (2011)
+ Birse, McGovern (2012)

$$\Delta E = (-33 \pm 2) \mu\text{eV}$$

or about 12% of the needed correction ...
present experimental precision: $2 \mu\text{eV}$

Static Polarizability Status



Theory analyses:

BChPT: Lensky, Pascalutsa (2010)

HBChPT: Griesshammer, McGovern, Phillips (2013)

Disp. Rel.: Olmos de Leon (2001)

New (2013) PDG values:

$$\alpha = (11.2 \pm 0.4) \times 10^{-4} \text{ fm}^3$$

$$\beta = (2.5 \pm 0.4) \times 10^{-4} \text{ fm}^3$$

Dispersion based and chiral extractions of α_E - β_M disagree at about 2σ

In the low-energy range: linear photon beam asymmetry in Compton scattering is purely dependent on β_M

Krupina, Pascalutsa (PRL 110, 2013)

More precise measurement of β_M underway at A2@MAMI using linearly polarized photons

Proton radius puzzle: what could it mean ?

- unknown correction ? ...after known constraints have been built in !

- Change in Rydberg constant ?

In absence of further (sizeable) corrections, use of muonic extraction of R_E plugged into electron H Lamb shift yields R_∞ which is 4.9σ away from CODATA value (and factor 4.6 more precise)

Pohl et al. (2010)

- New physics ?

- explain 3σ $(g-2)_\mu$ discrepancy AND 7σ R_E discrepancy from μ H Lamb shift simultaneously invoking a correction by a hypothetical light boson ?

- $(g-2)_e$ puts strong limit on coupling to e -> much smaller,

Non-universality e – μ ?

- New parity violating muonic forces ?

- Can rare Kaon decay data help ?

Tucker, Smith (2010)

Barger, Chiang, Keung, Marfatia (2011)

Batell, McKeen, Pospelov (2011)

Brax, Burrage (2011)

Rislow, Carlson (2012)

Proton radius puzzle: what's next ?

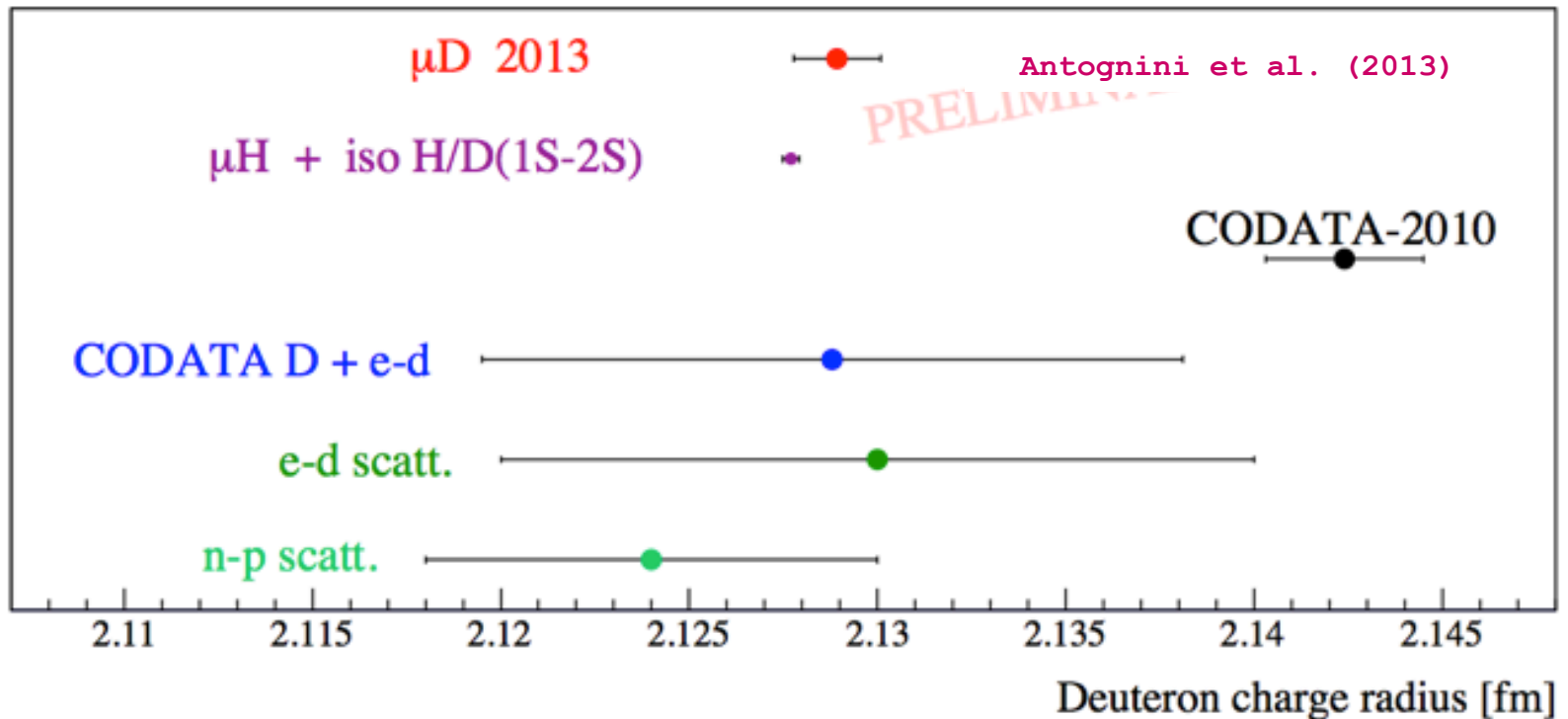
- **Muonic** Lamb shift : muonic D, muonic ^3He measurements planned

H/D isotope shift (1S-2S): $r_d^2 - r_p^2 = 3.82007 \pm 0.00065 \text{ fm}^2$ Parthey et al. (2010)

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New radius measurement from e-d scattering planned at MAMI (2014)

Caveat: error bar for μD does not include polarization correction

Polarization correction for μD Lamb shift

Data based dispersion relation analysis:

Carlson, Gorchtein, Vdh (2013)

Elastic	$\Delta\bar{E}^{el}$	-0.417(2) meV	
Nuclear (QE)	ΔE^{PWBA}	-1.616(739) meV	→ largest uncertainty
	ΔE^{FSI}	-0.391(44) meV	
	ΔE^{\perp}	-0.322(3) meV	
Hadronic	ΔE^{hadr}	-0.028(2) meV	
Subtraction	ΔE^{subt}	0.740(40) meV	
	$\Delta E^{Thomson}$	0.023(1) meV	
	ΔE_{total}	-2.011(740) meV	Polarization correction to $2S_{1/2}$ level

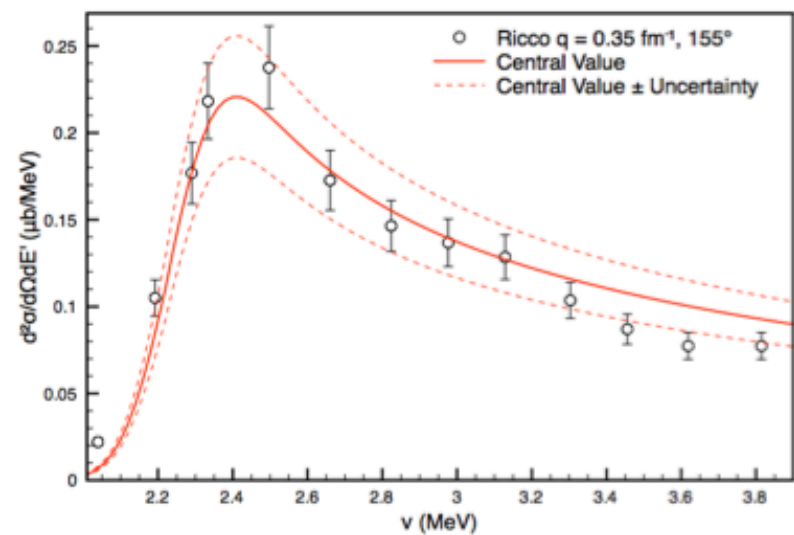
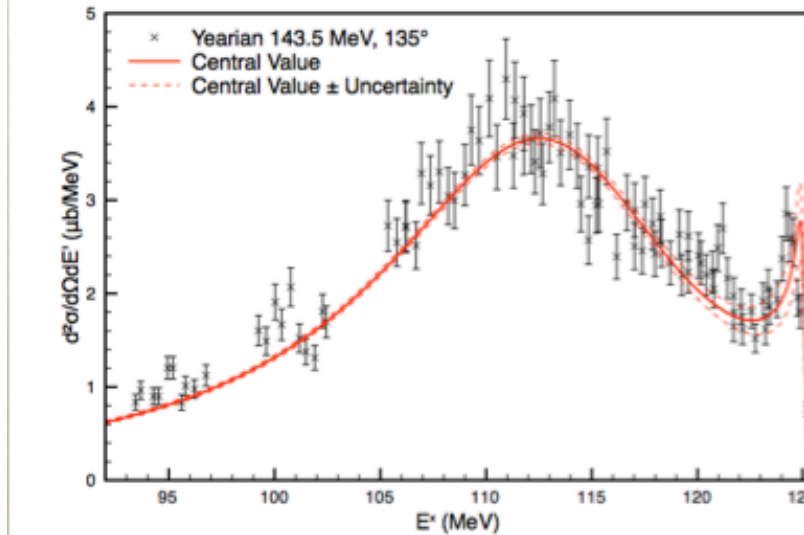
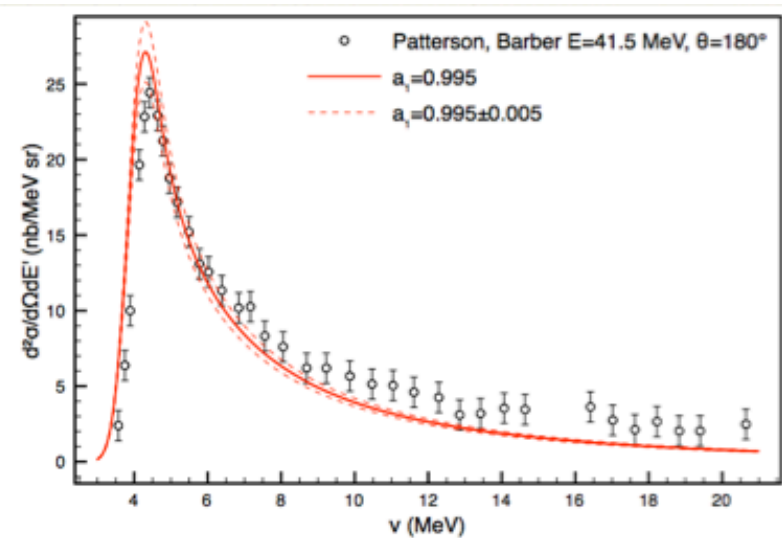
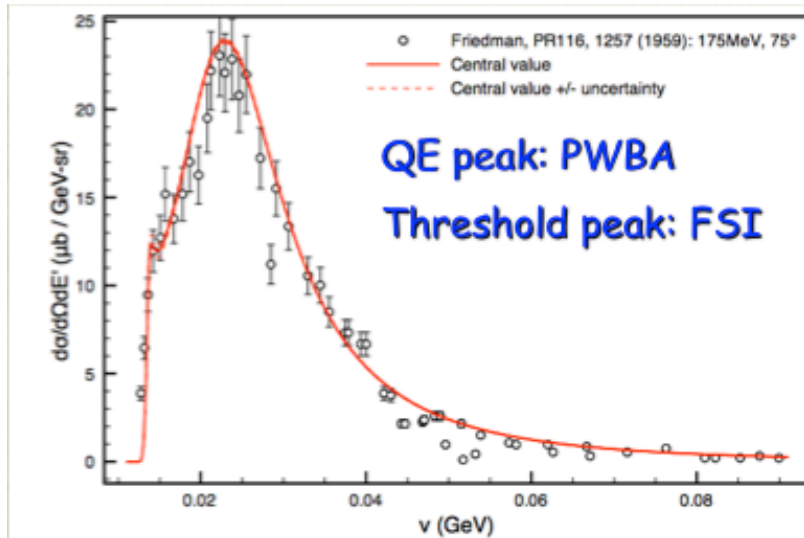
Compared to NR calculation: $\Delta E_{total} = -1.680 (16) \text{ meV}$

Pachucki (2011)

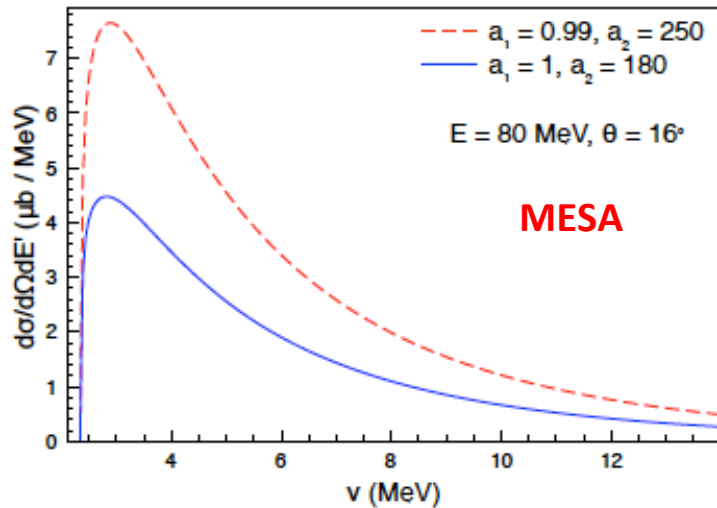
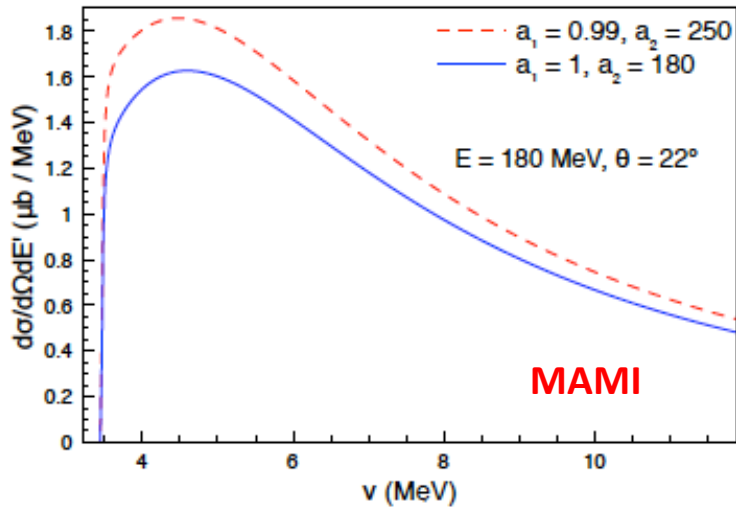
The bulk of the uncertainty in DR approach comes from a very limited kinematic region

$$\begin{aligned} \langle Q^2 \rangle &= 0.003 - 0.006 \text{ (GeV/c)}^2 \\ \langle \nu \rangle &= 6 - 10 \text{ MeV} \end{aligned}$$

Fit of Quasi-elastic e-D scattering data: $0.005 \text{ GeV}^2 < Q^2 < 3 \text{ GeV}^2$



New low Q^2 (forward angle) data needed to constrain F_2 for D

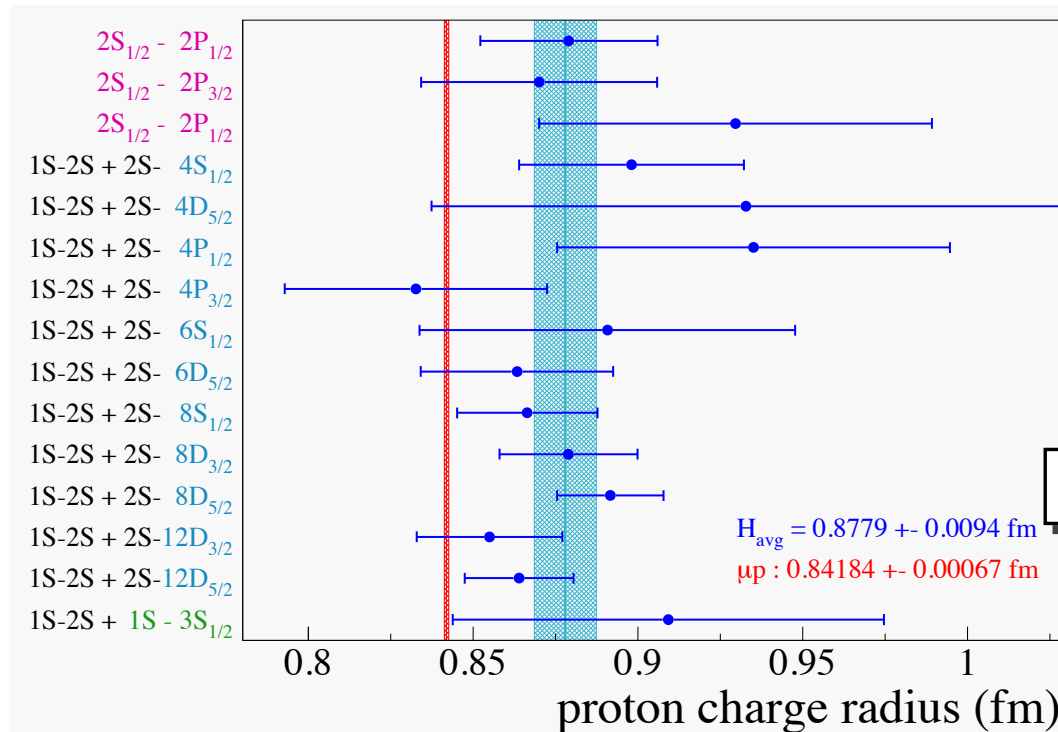


E_{lab}, θ_{lab}	Exp. precision	$\delta(\Delta E_{2S-2P}^{\mu D})$	$\delta(\Delta E_{1S-2S}^{eD})$
180 MeV, 30°	2%	740 μeV	12 kHz
	1%	370 μeV	6 kHz
180 MeV, 22°	2%	390 μeV	6.32 kHz
	1%	195 μeV	3.16 kHz
180 MeV, 16°	2%	211 μeV	3.36 kHz
	1%	110 μeV	1.68 kHz
80 MeV, 16°	2%	67 μeV	1.078 kHz
	1%	48 μeV	0.780 kHz



Proton radius puzzle: what's next ?

- **Muonic** Lamb shift : muonic D, muonic ^3He measurements planned
 - **Electronic** H Lamb shift : higher accuracy measurement very timely
- New proposal (York Univ, Canada) : R_E to 0.7%



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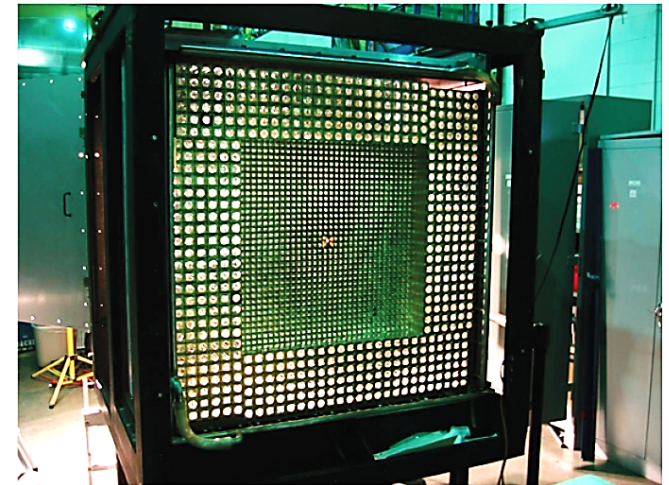
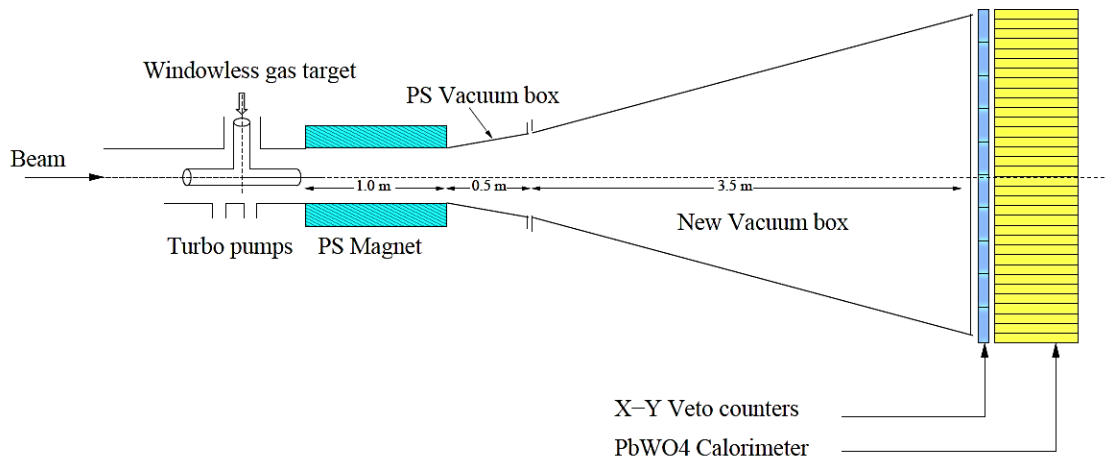
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- new G_{Ep} measurements at very low Q^2 down to $Q^2 \approx 2 \times 10^{-4} \text{ GeV}^2$

JLAB/Hall B approved expt : magnetic-spectrometer-free experiment (HyCal)

$$Q^2 = 2 \times 10^{-4} - 2 \times 10^{-2} \text{ GeV}^2$$

$ep \rightarrow ep$ cross sections normalized to Moller scattering



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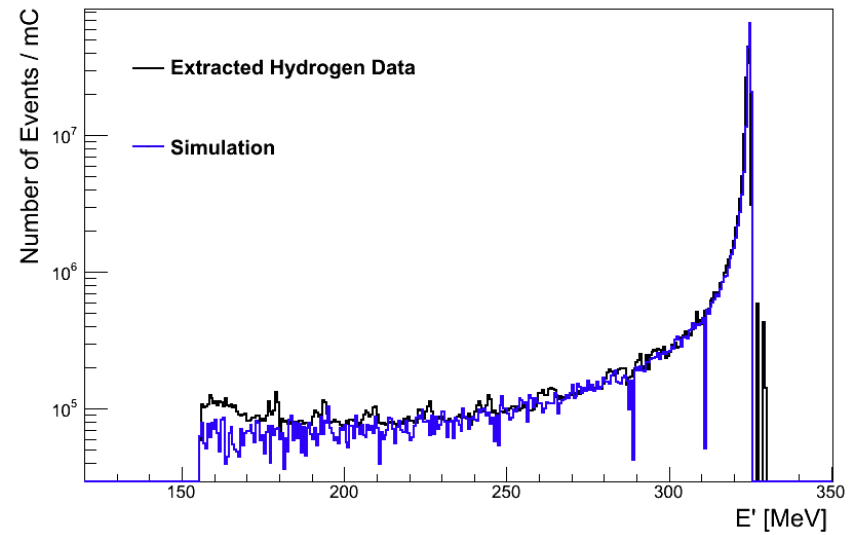
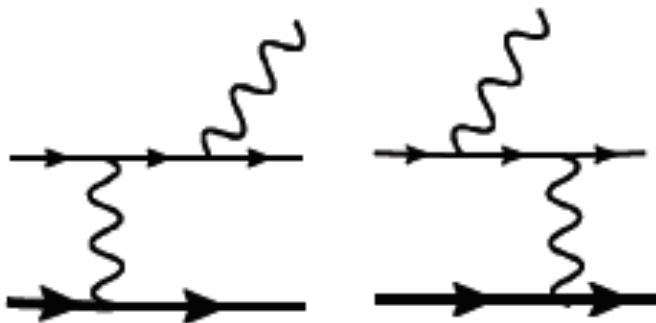
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MAMI/A1 : using initial state radiation (2013)



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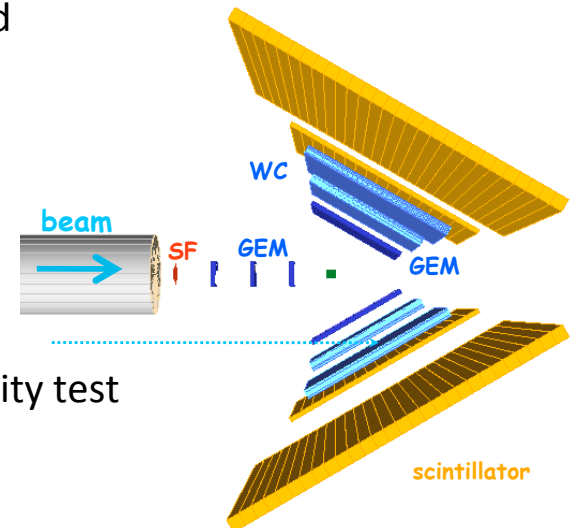
MESA : low-energy, high resolution spectrometers

- $\mu - p$ scattering (MUSE) at low Q^2 at PSI: (2015 – 2017)

simultaneous measurement of $\mu^\pm p$ and $e^\pm p$: lepton universality test

$$0.002 \text{ GeV}^2 < Q^2 < 0.07 \text{ GeV}^2$$

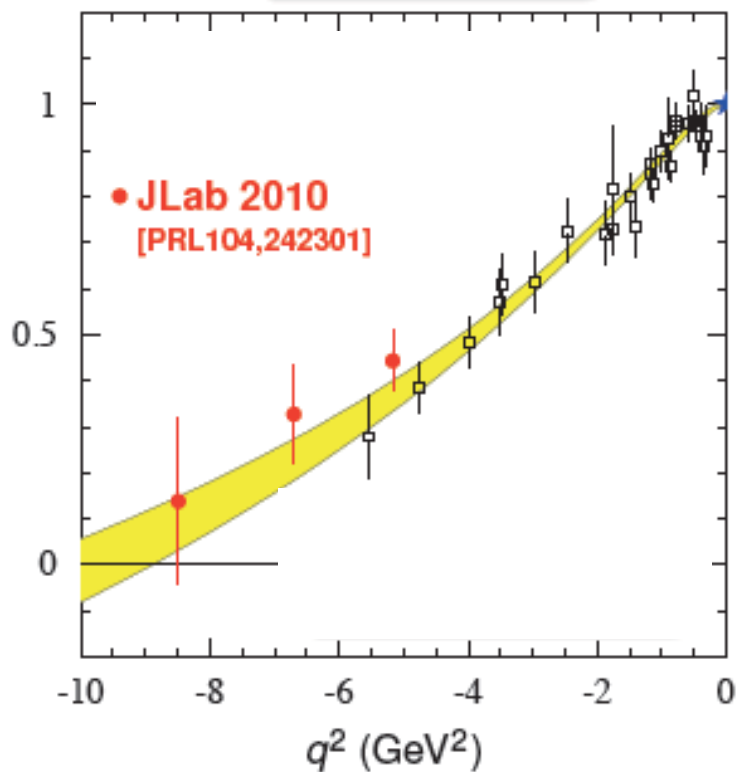
2 beam polarities give 2γ exchange test



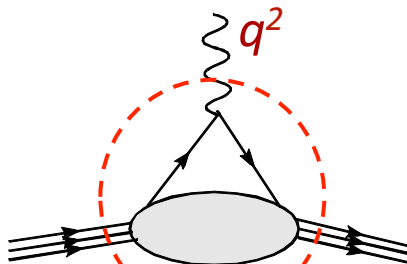
Complementing hadron structure in space- and timelike regions

Proton spacelike form factors

$$e^- p \rightarrow e^- p$$



JLab, MAMI

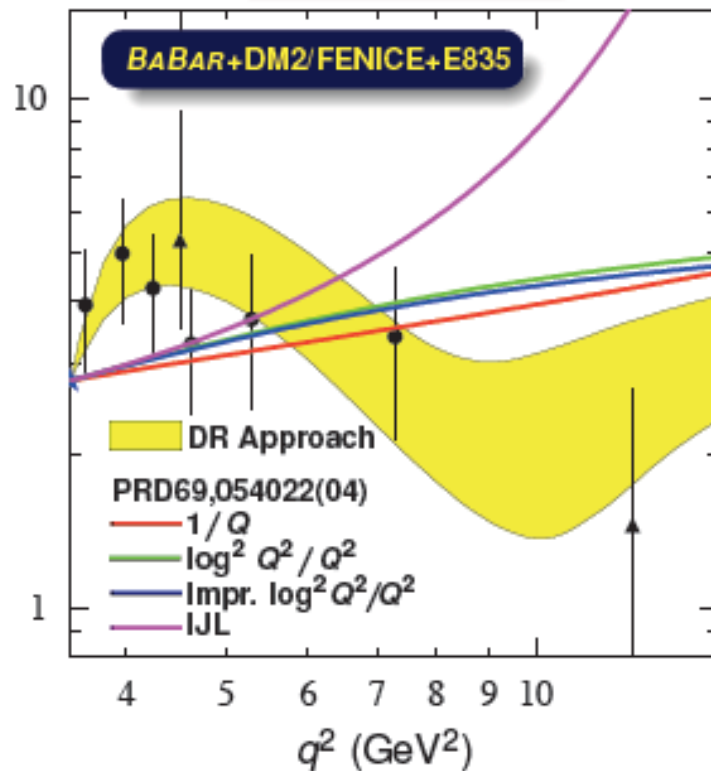


y-axis:

$$\mu_p G_E / G_M$$

Proton timelike form factors

$$e^+ e^- \rightarrow p \bar{p}$$

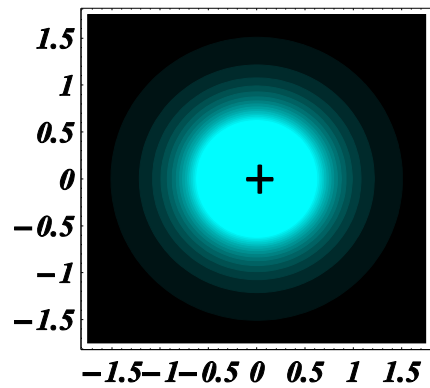


BES-III, PANDA

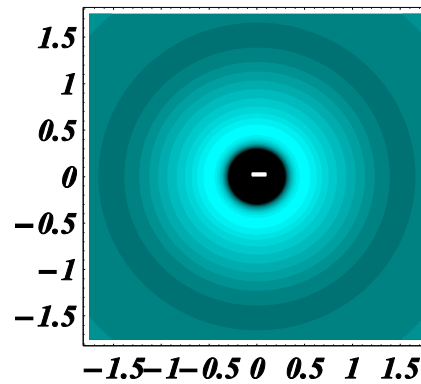
Spatial imaging of hadrons

Charge, mass, spin densities of quarks in a hadron

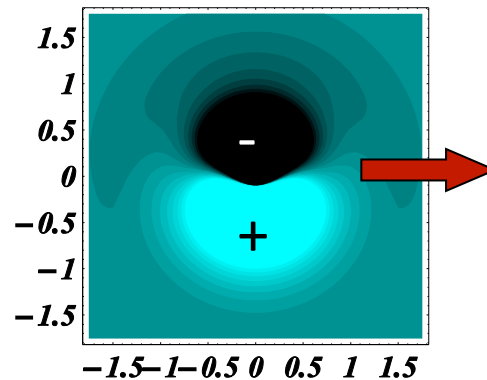
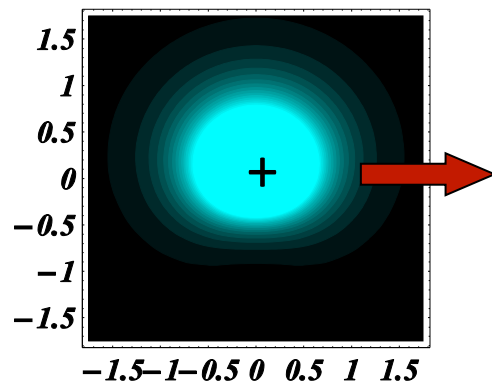
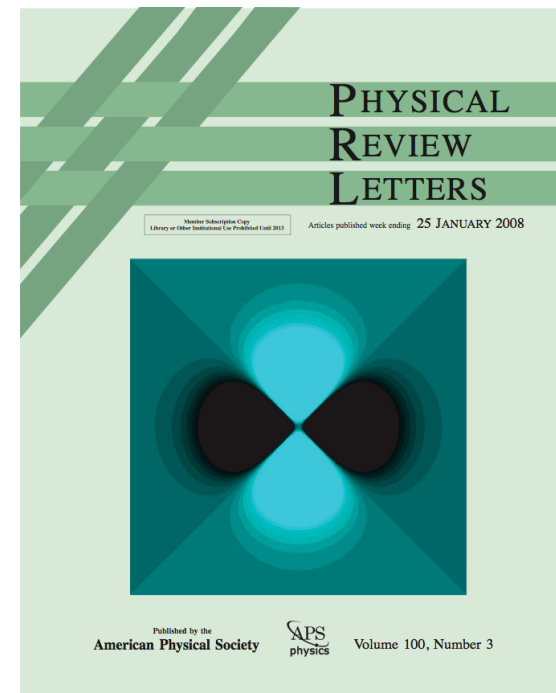
proton



neutron



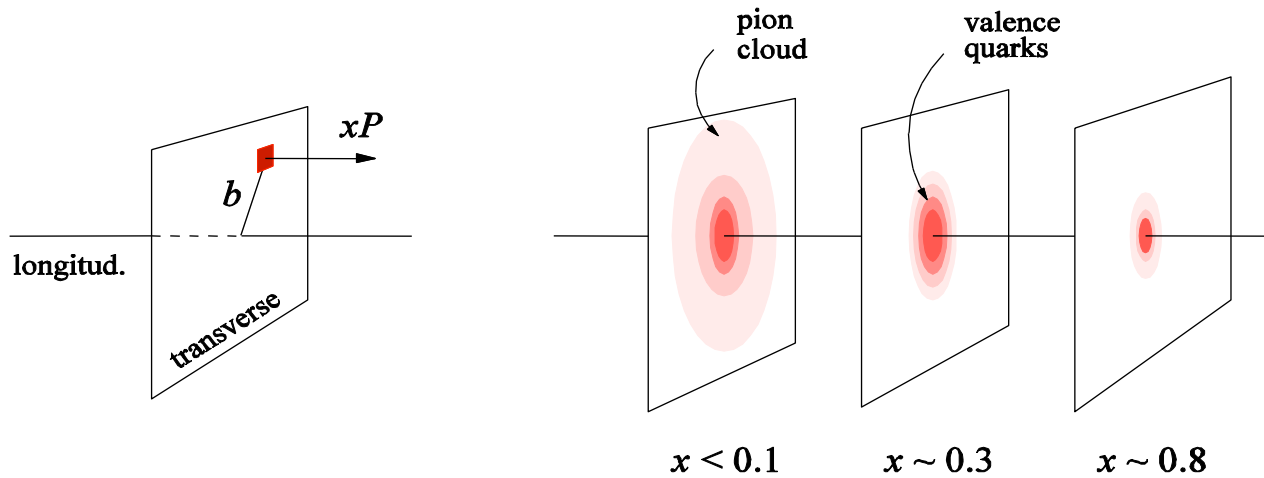
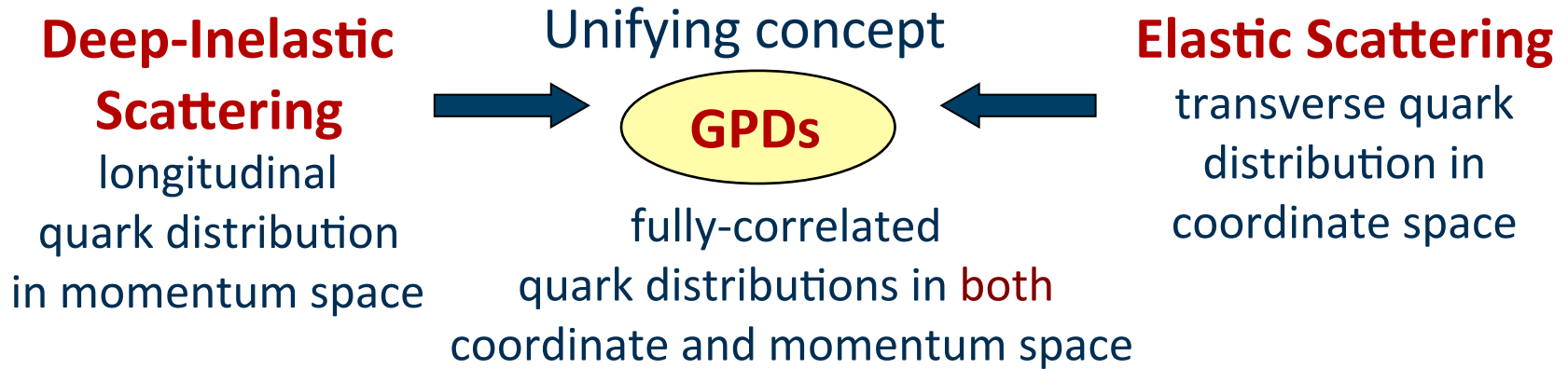
$p \rightarrow \Delta^+ (1232)$



Miller (2007)

Carlson, Vdh (2008)

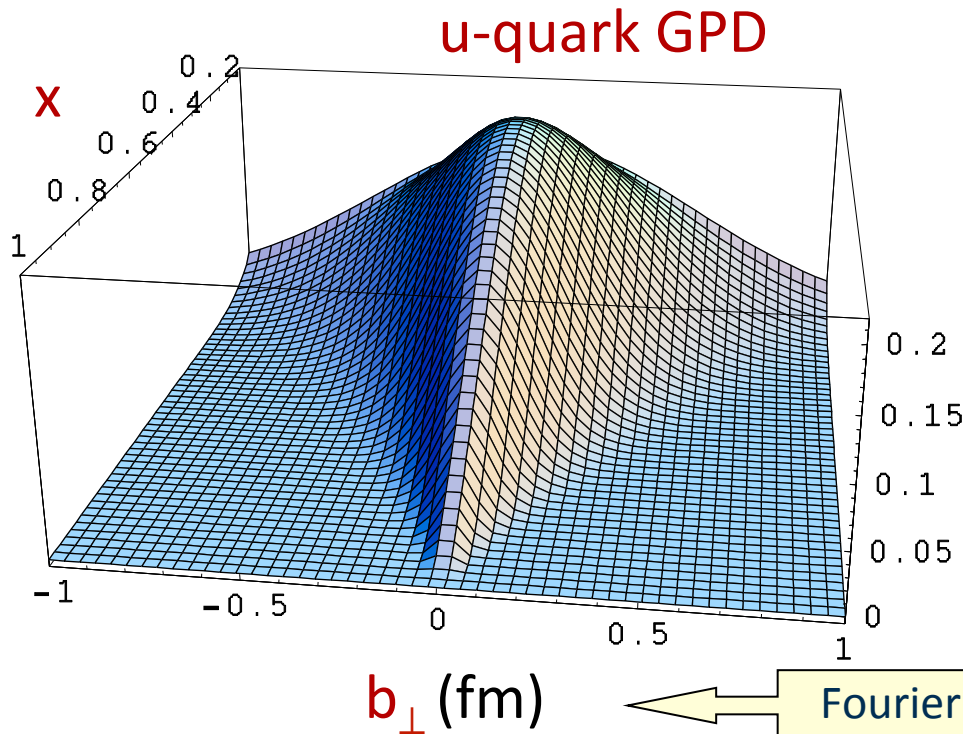
Generalized Parton Distributions (GPDs): 3D image of hadrons



Burkardt (2000, 2003),
Belitsky, Ji, Yuan (2004)

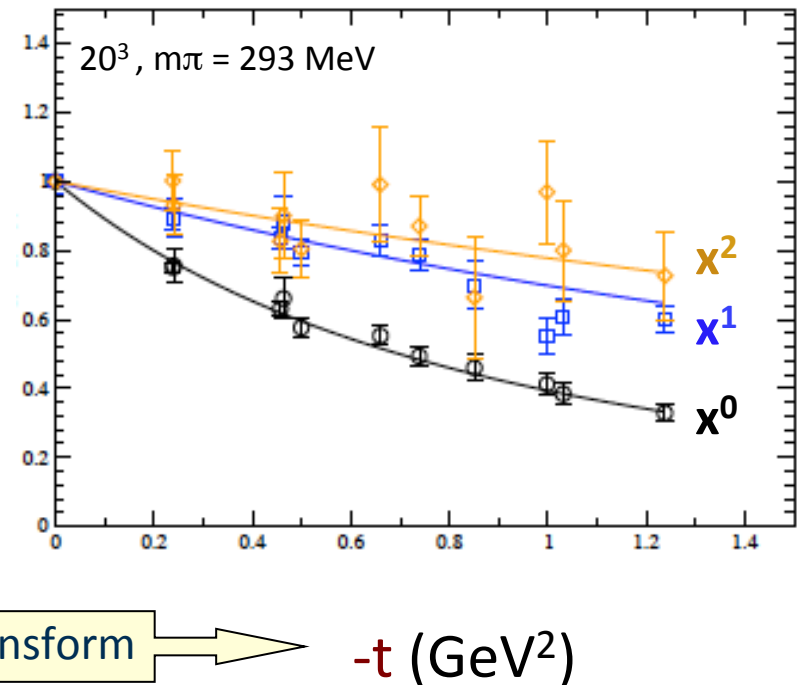
GPDs: transverse image of hadrons

GPDs: quark distributions w.r.t. longitudinal momentum x and transverse position b_{\perp}



lattice QCD:
moments of GPDs

x^n moment of u-d GPD



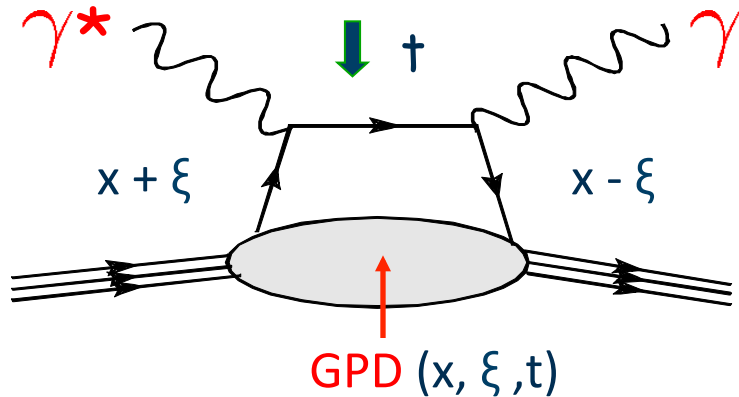
Guidal, Polyakov, Radyushkin, Vdh (2005)

Diehl, Feldmann, Jakob, Kroll (2005)

LHPC Coll.

QCD factorization: tool to access GPDs

$Q^2 \gg 1 \text{ GeV}^2$

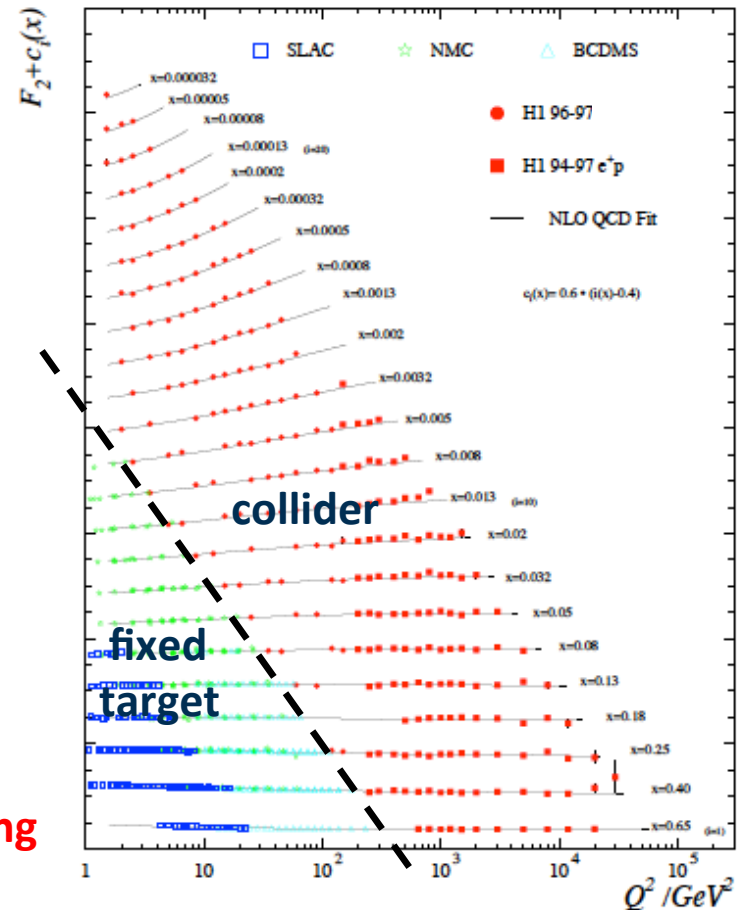


➔ at large Q^2 : **QCD factorization theorem** :
hard exclusive process described by **GPDs**
model independent !

Müller et al.(1994), Ji(1995), Radyushkin(1995),
Collins, Frankfurt, Strikman (1996)

➔ **KEY** Q^2 leverage required to test **QCD scaling**

world data on proton **F2**



„complete“ picture of nucleon

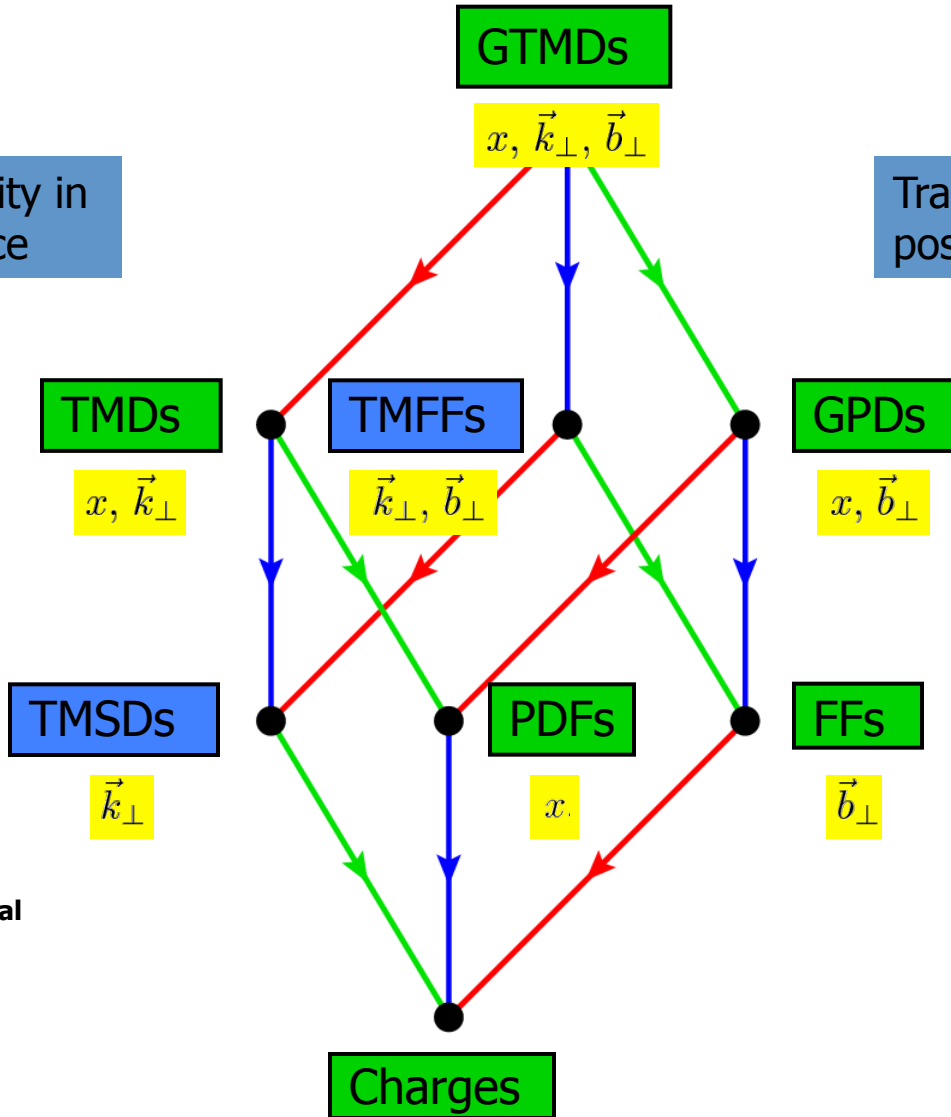
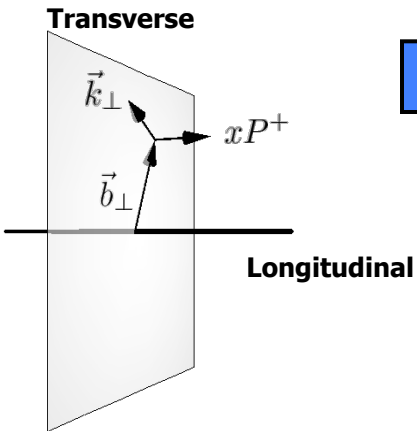
$$\xi = 0$$

Momentum space	$\vec{k}_\perp \leftrightarrow \vec{z}_\perp$	Position space
	$\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$	

Transverse density in momentum space

Transverse density in position space

Lorcé (2011)

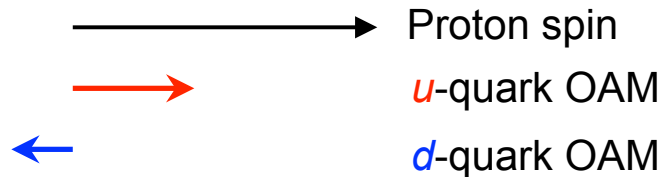
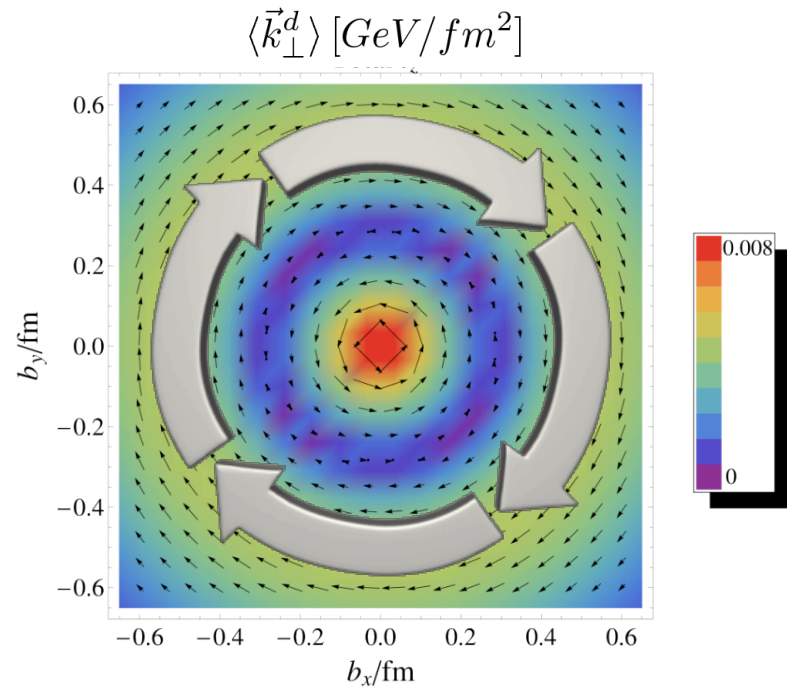
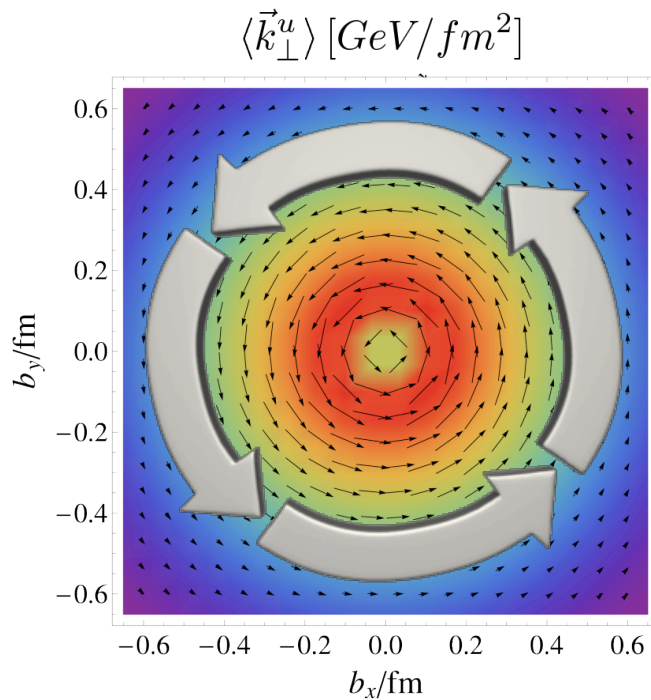


- $\int d^2 b_\perp$
- $\int dx$
- $\int d^2 k_\perp$

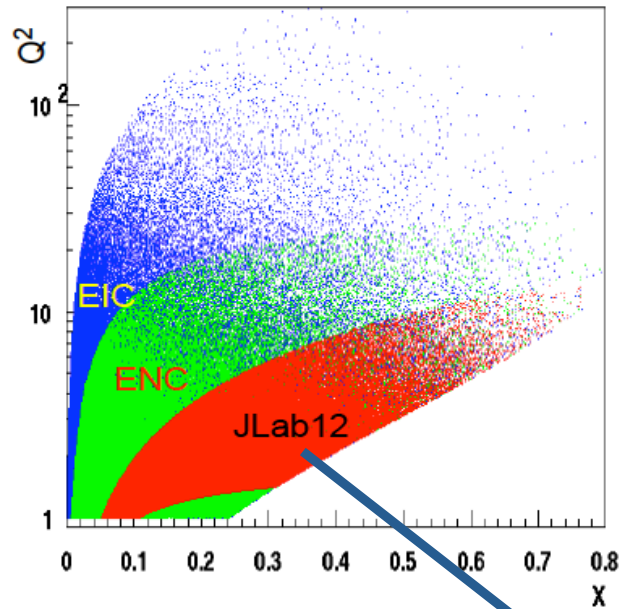
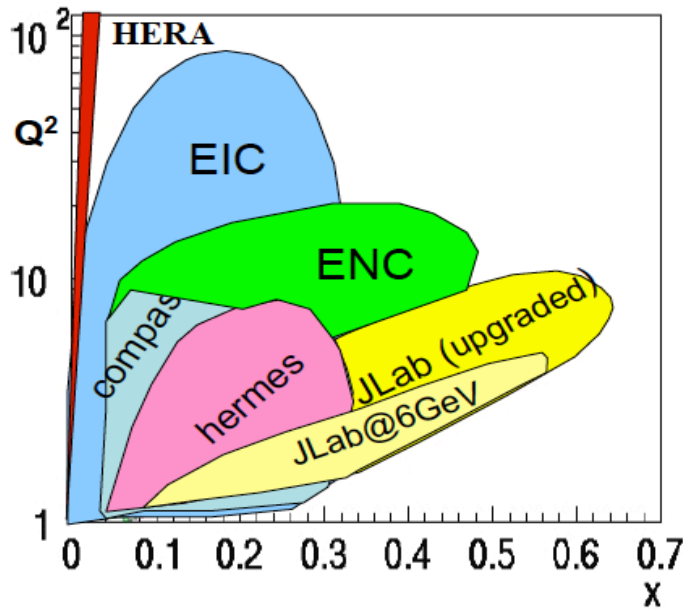
Quark orbital angular momentum in proton

$$\ell_z^q = \int d^2\vec{b}_\perp (\vec{b}_\perp \times \langle \vec{k}_\perp^q \rangle)_z$$

$$\langle \vec{k}_\perp^q \rangle = \int dx d^2\vec{k}_\perp \vec{k}_\perp \rho_{LU}^q(x, \vec{b}_\perp, \vec{k}_\perp)$$



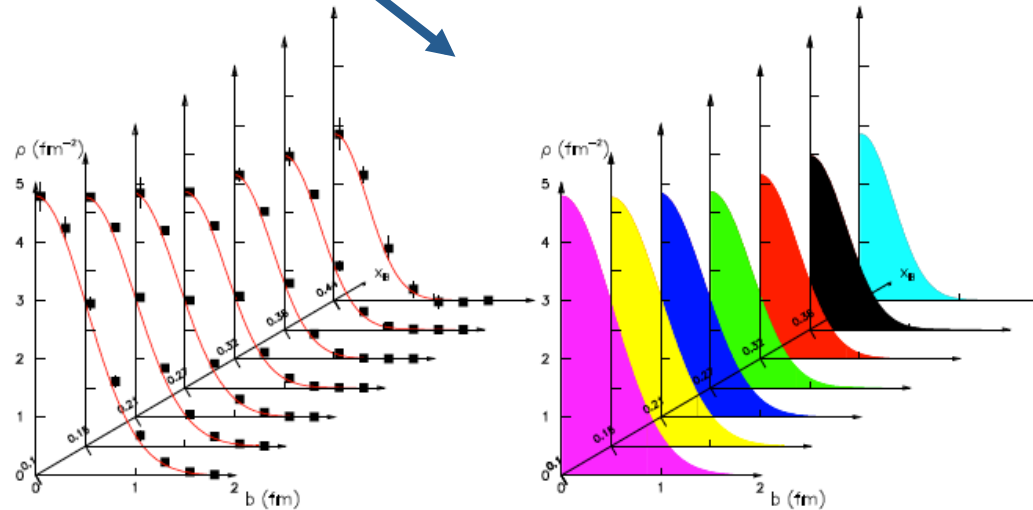
Energy-luminosity frontier in lepton-nucleon physics



JLab 11 GeV
projections:

Guidal, Moutarde, Vdh
(2013)

- **High-energy, high-luminosity facilities:** Compass, JLab@11 GeV, collider projects (EIC, ENC, ...)
- **Global nucleon structure analysis effort required**



CONCLUSIONS

- **Strong interplay between high-energy ↔ precision ↔ low-energy frontiers**
- **Impact of hadron physics on new physics searches:**
 $(g-2)_\mu$, Q_{weak} , new dark photon searches
- **Unraveling hadron structure in strong QCD:**
 - proton radius puzzle has shaken textbook beliefs
 - combination of new experiments + theory opens perspectives for an imaging of hadrons to an unprecedented level of detail