Cold quarks stars from hot lattice QCD

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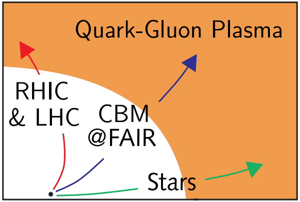
1. hot lattice QCD and quasiparticles
2. quasiparticle model: going to $\mu > 0$
3. cold quark stars



Effective QPM

- quasiparticle model:

$$s = \sum_i s_i \qquad i = g, u, d, s$$

$$s_i \sim \int_{\mathrm{d}^4 k} \frac{\partial\, n_{\mathrm{B/F}}}{\partial T}\, \Theta\big(-\omega^2 + k^2 + m_\infty^2(\mu, T, G^2(\mu, T)\big)$$

  (derived from 1-loop QCD)

Blaizot, Iancu, Rebhan: PRD'01
RS, Bluhm, Kämpfer: JPPNP'09

- running/effective coupling

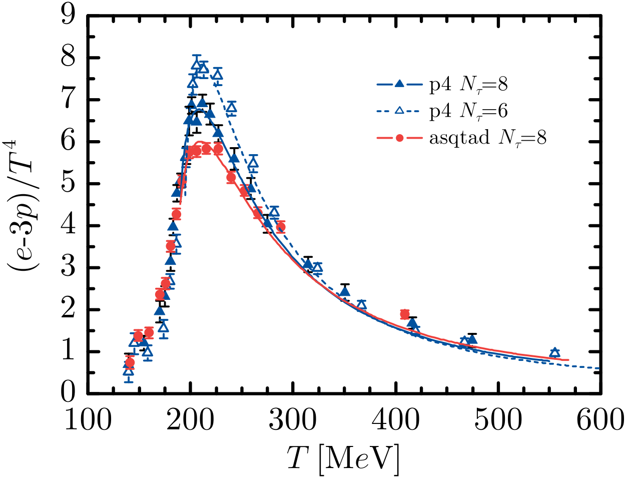
$$G^2(x^2) = \frac{16\pi^2}{\beta_0 \ln(x^2)} \quad \xrightarrow{\mu=0} \quad x = \frac{\bar{\mu}}{\Lambda_{\mathrm{QCD}}} \to \frac{T\text{-}T_s}{\lambda}$$

Bluhm, Kämpfer, RS, Seipt: EPJC'07

- fit to e-$3p$ with fixed $p(T_c)$: T_s, λ, p_0

At $\mu=0$

- quasiparticle model (QPM) fit to lattice results



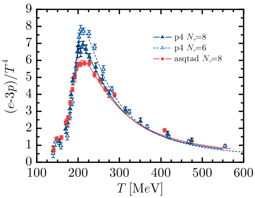
Bazavov et al.: PRD '09

state variables s, n, p, $(e\text{-}3p)$, ...

effective coupling G^2

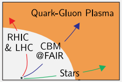
@ $\mu = 0$:

@ $\mu \neq 0$:



??

Into the T-μ-plane



- $\mu > 0$: stationary potential, self-consistent model
  $\rightarrow$ impose Maxwell's relation

$$\frac{\partial s}{\partial \mu} = \frac{\partial n}{\partial T} \qquad \longrightarrow \qquad a_T \frac{\partial G^2}{\partial T} + a_\mu \frac{\partial G^2}{\partial \mu} = b$$

Peshier, Kämpfer, Soff: PRC'00, PRD'02

- quasilinear PDE for $G^2(T, \mu \neq 0)$:
  $\rightarrow$ T-μ-plane accessible

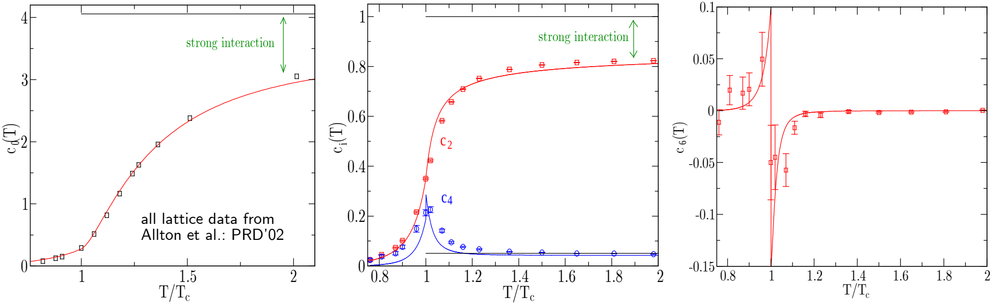
caveat: for perfect solution collective excitations and damping terms necessary

RS, Bluhm, Kämpfer: EPJ ST'08

Small chemical potential

- test with $p(T,\mu \gtrsim 0)$ lattice data

$$p = T^4 \sum_n c_n(T) \left(\frac{\mu}{T}\right)^n \qquad c_n(T) = \frac{1}{n!} \frac{\partial^n (p/T^4)}{\partial(\mu/T)^n}$$



Bluhm, Kämpfer, Soff: PLB'05

- application in hydro @ RHIC successful

Bluhm, Kämpfer, RS, Seipt, Heinz: PRC'07

Isospin asymmetric QPM

- five chemical potentials

$$\mu_u,\ \mu_d,\ \mu_s \quad + \quad \mu_e,\ \mu_\mu$$

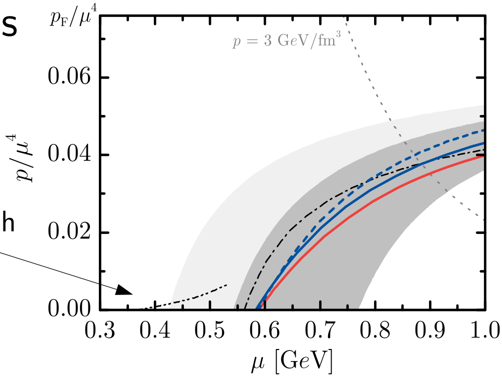
- four side conditions
  - β equilibrium (e.g. $n \leftrightarrow p^+ + e^- + \bar{\nu}_e;\ \mu_d = \mu_u + \mu_e$)
  - equilibrium in strangeness changing decays (e.g. $\Lambda \leftrightarrow p^+ + \pi^-;\ \mu_s = \mu_d$)
  - muon decay (e.g. $\mu^- \leftrightarrow e^- + \bar{\nu}_e + \nu_\mu;\ \mu_\mu = \mu_e$)
  - electric neutrality

$\longrightarrow$ only one independent chemical potential $\mu = \mu_u$

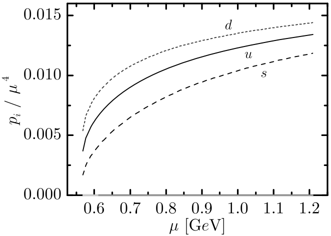
At $T=0$

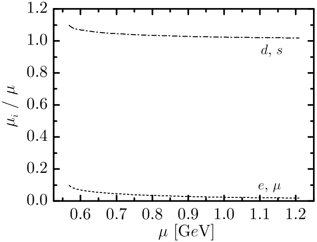
- thermodynamic quantities well within perturbative predictions
  (Andersen, Strickland: PRD'02
  Fraga et al.: NPA'02)

hybrid approach needed



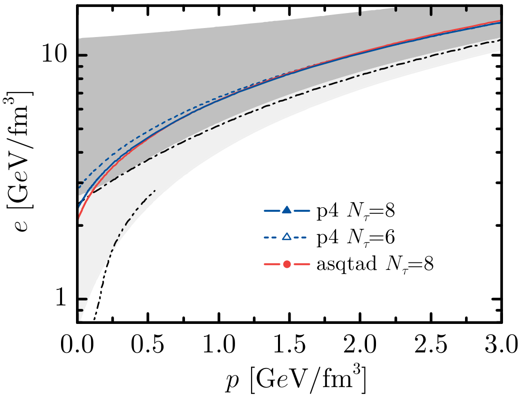
- individual contributions





At T=0

- EOS: narrow range for all actions
  - vacuum energy density dep. on lattice spacing
  - asymptotics governed by lattice action



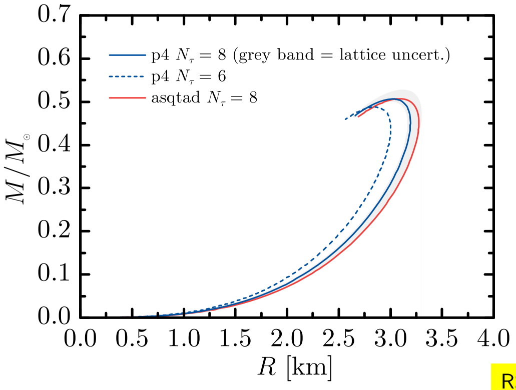
- good approximation

$$e = v_s^{-2} p + e_0$$

v_s^{-2}: $3.3 - 3.6$, e_0: $(375 - 395\text{MeV})^4$

Pure quark stars

- solutions of TOV equations



RS, Kämpfer: arXiv:0912.2827
submitted to PRC

→ rather small and light ($M, R \sim e_0^{-1/2}$)

→ no twin candidates

Summary & Outlook

- ℓQCD results mapped to large μ, even $T{=}0$
- EOS for quark stars similar for all actions
- quark stars with rather smaller radii $+$ masses

- outlook: hybrid stars
  full HTL quasiparticle model with
  Landau damping and collective modes
  EOS for FAIR/CBM

Quarks stars

- static, spherical stellar objects

$$\frac{dp}{dr} = -G\frac{(e+p)(m+4\pi r^3 p)}{r^2(1-\frac{2m}{r}G)}$$

$$\frac{dm}{dr} = 4\pi r^2 e,$$

TOV equations

$$e = e(p)$$

- EOS of the quark-gluon plasma
  $\rightarrow$ from where?

CJT formalism

- effective action

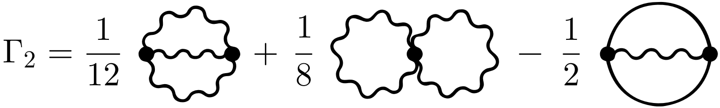
$$\Gamma[D,S] = I - \frac{1}{2}\left\{\mathrm{Tr}\left[\ln D^{-1}\right] + \mathrm{Tr}\left[D_0^{-1}D - 1\right]\right\} + \left\{\mathrm{Tr}\left[\ln S^{-1}\right] + \mathrm{Tr}\left[S_0^{-1}S - 1\right]\right\} + \Gamma_2[D,S]$$

- translation-invariant systems, no broken symmetries

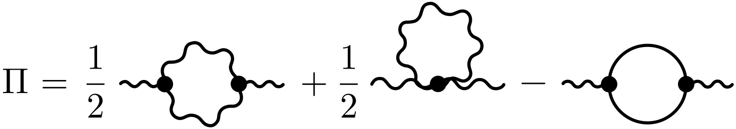
$$\frac{\Omega}{V} = \mathrm{tr}\int\frac{\mathrm{d}^4k}{(2\pi)^4} n_\mathrm{B}(\omega)\,\mathrm{Im}\left(\ln D^{-1} - \Pi D\right) + 2\,\mathrm{tr}\int\frac{\mathrm{d}^4k}{(2\pi)^4} n_\mathrm{F}(\omega)\,\mathrm{Im}\left(\ln S^{-1} - \Sigma S\right) - \frac{T}{V}\Gamma_2$$

2-loop QCD thermodynamics

- truncate Γ_2 at 2-loop order



$\longrightarrow$ self-energies of 1-loop order



- gauge invariance: hard thermal loops (HTL)

Pressure

$$s \sim \left(\frac{\partial \Omega}{\partial T}\Big|_{\mathrm{expl.}} + \underbrace{\frac{\delta \Omega}{\delta D}}_{0} \frac{\partial D}{\partial T} \right)_{\mu}$$

$$s_i \sim \int_{\mathrm{d}^4k} \frac{\partial n_{\mathrm{B/F}}}{\partial T} \left\{ \mathrm{qp} + \mathrm{damping} \right\}$$

- self-consistent formulation of the pressure

$$p = -\frac{\Omega}{V} := \sum_i p_i - B \qquad p_i \sim \int_{\mathrm{d}^4k} n_{\mathrm{B/F}} \left\{ \mathrm{qp} + \mathrm{damping} \right\}$$

$$\frac{\partial B}{\partial T} := \sum_i \frac{\partial p_i}{\partial \Pi_i} \frac{\partial \Pi_i}{\partial T} \qquad \left(\frac{\partial B}{\partial \mu} = \sum \frac{..}{..} \frac{..}{\partial \mu} \right)$$

- entropy density

$$s = \frac{\partial p}{\partial T} = \sum_i s_i + \frac{\partial p_i}{\partial \Pi_i} \frac{\partial \Pi_i}{\partial T} - \frac{\partial B}{\partial T} = \sum_i s_i$$

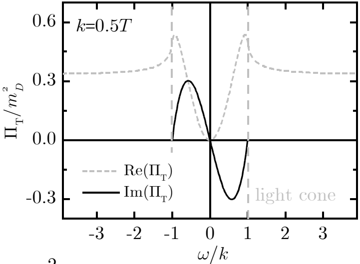
- net quark density

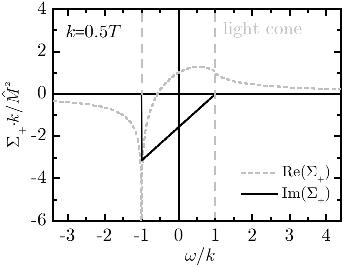
$$n \sim \frac{\partial \Omega}{\partial \mu}\Big|_{\mathrm{expl.}} + \underbrace{\frac{\delta \Omega}{\delta D}}_{0} \frac{\partial D}{\partial \mu}$$

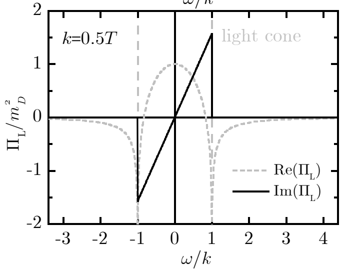
$$n_q = \frac{\partial p}{\partial \mu} \sim \int_{\mathrm{d}^4k} \left(\frac{\partial n_{\mathrm{F}}}{\partial \mu} + \frac{\partial n_{\mathrm{F}}^{A}}{\partial \mu} \right) \left\{ \mathrm{qp} + \mathrm{damping} \right\}$$

HTL self-energies

- $\mathrm{Im}\,\Pi \neq 0$ below the lightcone (solid lines)







$\longrightarrow$ Landau damping

Effective coupling

- fundamental parameter

$$g^2(x^2) = \frac{16\pi^2}{\beta_0 \ln(x^2)} \left(1 - \frac{4\beta_1}{\beta_0^2} \frac{\ln[\ln(x^2)]}{\ln(x^2)}\right)$$

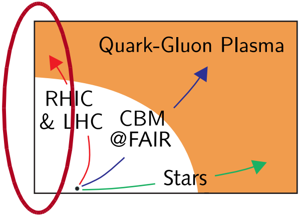
- running coupling g^2 $\quad x = \frac{\bar{\mu}}{\Lambda_{\mathrm{QCD}}} \quad \bar{\mu} \sim T$

$$T > T_c,\ \mu = 0$$

- effective coupling G^2 $\quad x = \frac{(T\text{-}T_s)}{\lambda_{\mathrm{QCD}}}$

Lattice QCD

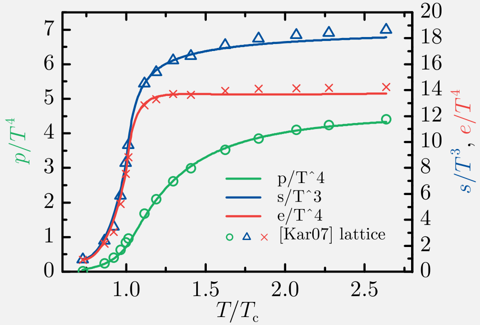
- lattice results: availability limited



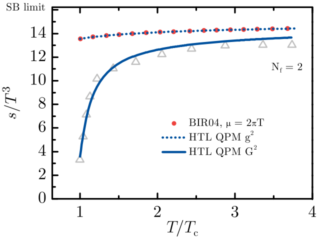
- one answer: quasiparticle model
  - self-consistency allows mapping to $T = 0$
  - ensure β stability and charge neutrality

Adjustment @ $\mu = 0$

- $\mu = 0$: adjust to ℓQCD
  - $\rightarrow$ T_s, λ fixed
  - $\rightarrow$ $G^2(T, \mu = 0)$

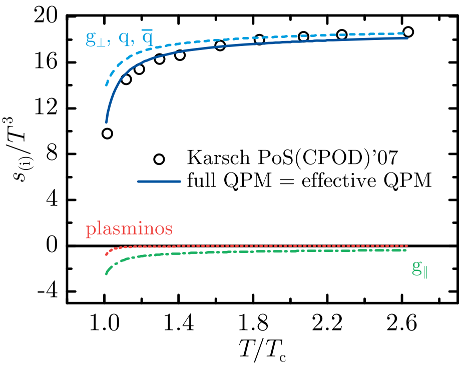


- $T_s = 0$:



Influence of coll. modes + LD @ $\mu = 0$

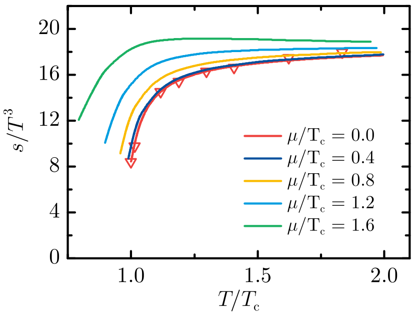
- individual entropy contributions

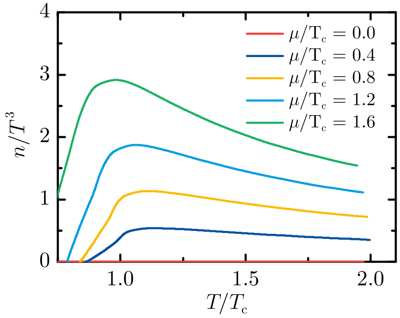


- Landau damping large close to T_c, decreases for higher temperatures

Thermodynamic bulk variables

- entropy density and net quark density



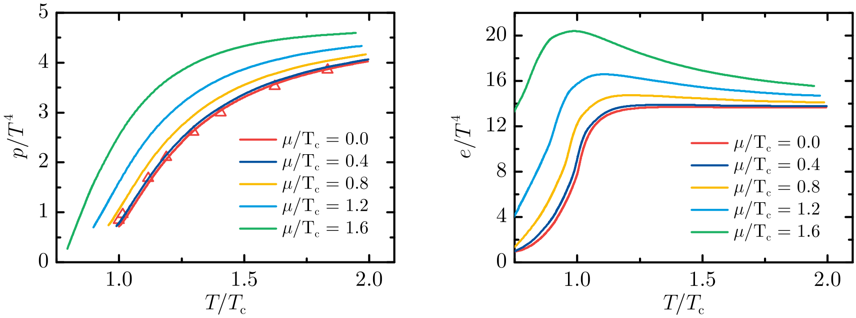


RS, Bluhm, Kämpfer: PPNP'09

- increase with chemical potential

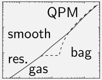
Thermodynamic bulk variables

- pressure and energy density



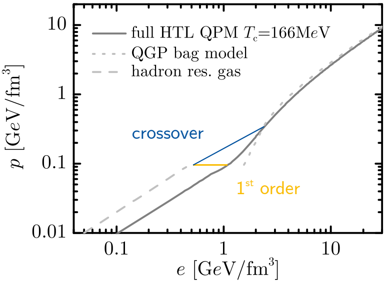
- small area of negative pressure
  $\rightarrow$ no problems for EOS @ RHIC, LHC, SPS, FAIR
  $\rightarrow$ natural limit of stability for quark stars (CFL?)

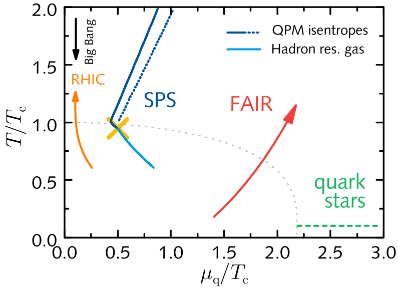
EOS for RHIC and LHC



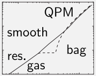
- EOS for LHC, RHIC

$$n_b/s \approx 0$$

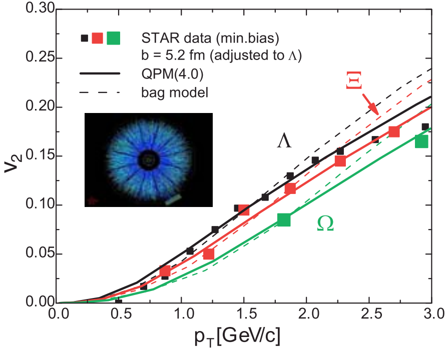




Comparison with the experiment



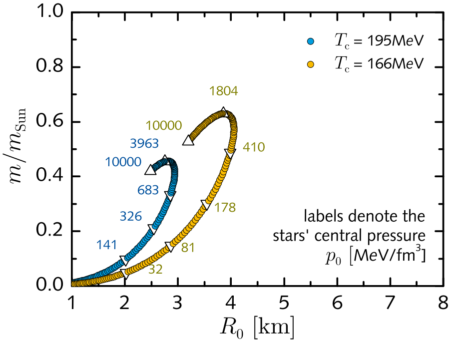
- calculate elliptic flow using relativistic hydro code
- compare with experimental data (RHIC)



Bluhm, Kämpfer, RS, Seipt, Heinz: PRC'07

Compact stellar matter

- Tolman-Oppenheimer-Volkov equations
- β-equilibrium by $d, s \leftrightarrow u, l, \nu_l$
  $\rightarrow \mu_l$ from charge neutrality
- compare with bag-like EOS $e = \alpha p + 4\tilde{B}$
  $\rightarrow \alpha \approx 4$
- strong dependence on critical temperature



Summary & Outlook

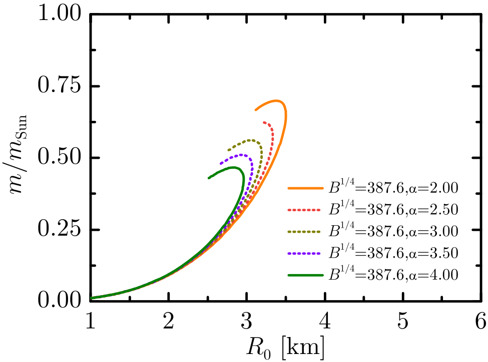
- 2-loop Γ_2 + eff. coupling G^2 $\rightarrow$ HTL QPM
- ℓQCD results describable; used as input
  $\rightarrow$ large μ accessible due to self-consistency
- EOS for heavy ion collision experiments available
- quark stars with even smaller radii than bag model

- outlook: hydro for SPS,FAIR
  critical endpoint

Kämpfer, Bluhm, RS, Seipt: NPA'06

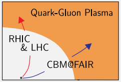
Backup

- influence of α

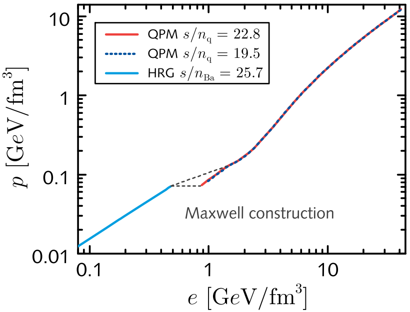


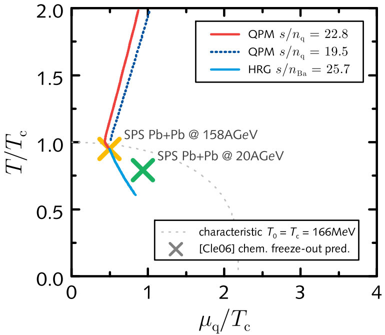
EOS for SPS

PRELIMINARY



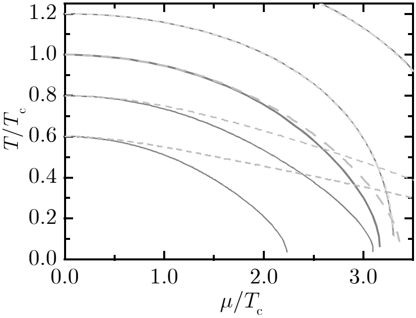
- SPS $s/n_{\mathrm{q}} \approx 25.5\text{-}8.5$





More effects of collective excitations

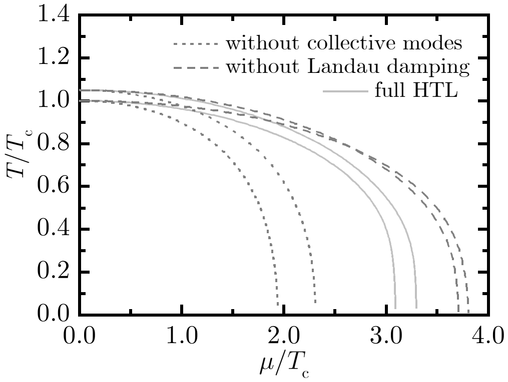
- collective modes
  $\longrightarrow$ neg. entropy contrib.



situation improves

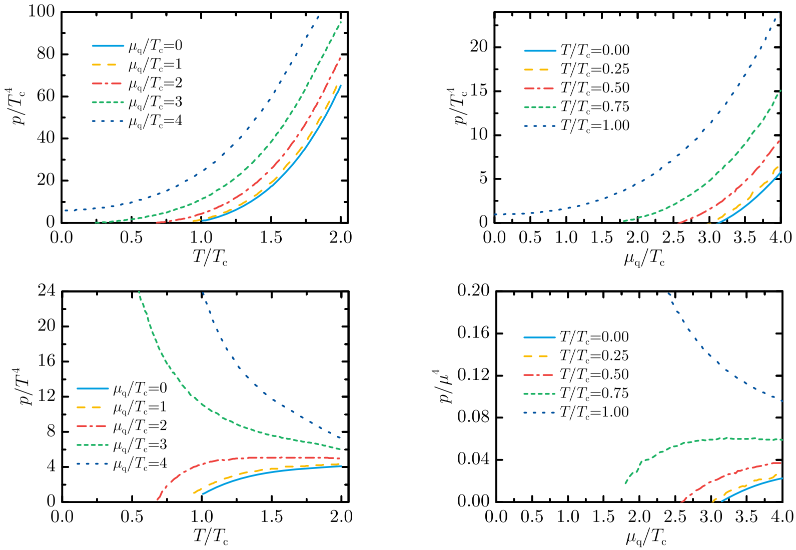
More effects of Landau damping

- only minor contribution at $\mu = 0$
- essential for $\mu > 0$



Results for the pressure (2)

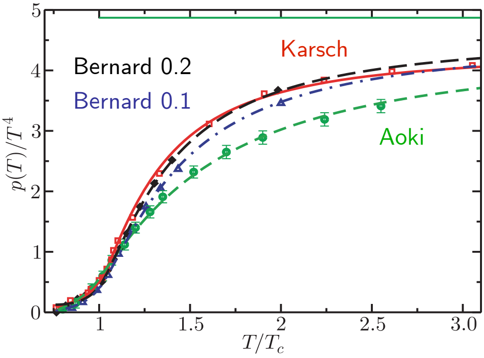
- pressure cuts

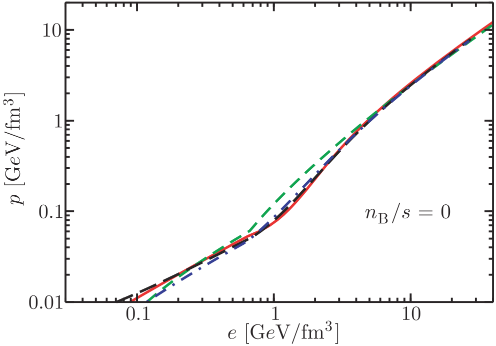


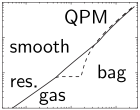
EOS for $N_f = 2+1$

- RHIC, LHC:

$$\mu = 0$$







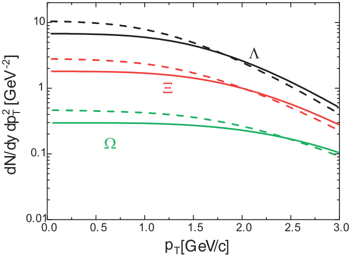
Kämpfer, Bluhm, RS, Seipt, Heinz: NPA'05
Bluhm, Kämpfer, RS, Seipt, Heinz: PRC'07

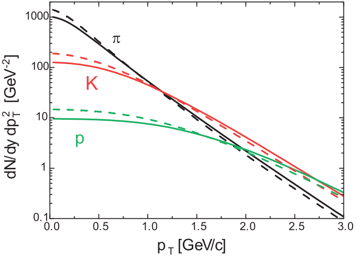
Predictions for LHC

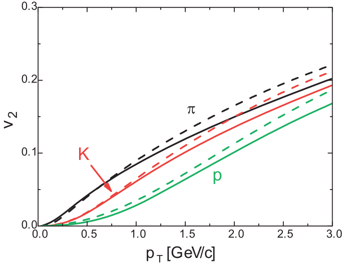
- LHC Pb+Pb collisions - conservative guess:

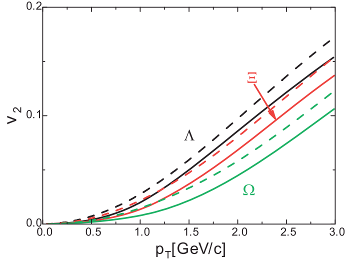
$$s_0 = 330\,\mathrm{fm}^{-3}, \quad \tau_0 = 0.6\,\mathrm{fm}/c$$
$$b = 5.2\,\mathrm{fm} \qquad T_0 = 515\,\mathrm{MeV}$$

- higher initial temperature
  $\rightarrow$ flatter p_T spectra
  $\rightarrow$ smaller v_2









More LHC predictions

- initial parameters translate to
$$e_0 = 127\,\mathrm{GeV},\ p_0 = 42\,\frac{\mathrm{GeV}}{\mathrm{fm}^3},\ T_0 = 515\,\mathrm{MeV}$$
- LHC: higher initial temperature $\rightarrow$ longer fireball lifetime $\rightarrow$ stronger radial flow $\rightarrow$ p_T spectra flat

More full HTL quasiparticle model

- now: $\boxed{\mathrm{Im}\,\Pi \neq 0}$ $+$ collective excitations

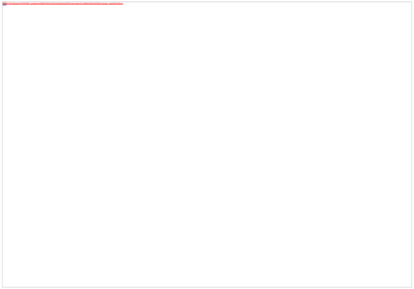
$$s = s_{qp} + s_{damp} = s_{qp} + (\tilde{s} - s_{qp})$$

$$\tilde{s} = \int d\omega \underbrace{\int dk\,\sigma(\omega, k)}_{\hat{=} s_{qp}(\omega)} \cdot F(\mathrm{Im}\Pi(\omega, k)) \qquad \xi := \frac{\mathrm{Im}\Pi}{\mathrm{Re}D^{-1}}$$

$$F := -\frac{1}{\pi}\left(\frac{\xi^2(\omega)}{(1+\xi^2(\omega))^2} + \frac{\xi^2(-\omega)}{(1+\xi^2(-\omega))^2}\right)\frac{\partial \xi}{\partial \omega}$$

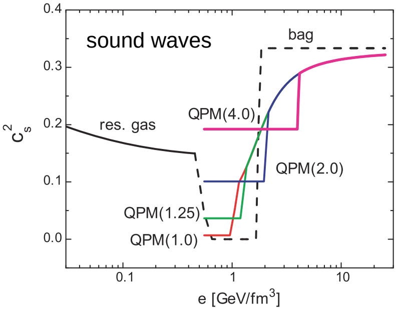
Backup

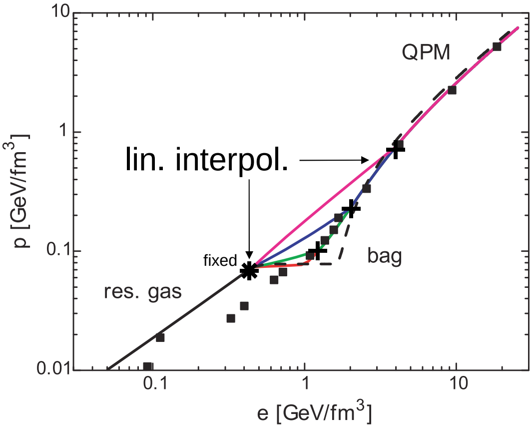
- model describes all available quantities:



A family of EOS's $\mu_B \ll T$

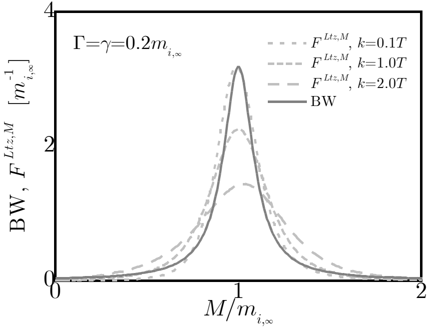
- interpolate between hadron gas and QPM description





Backup: Inclusion of widths

- Peshier: $\mathrm{Im}\,\Pi = 2\gamma\omega$

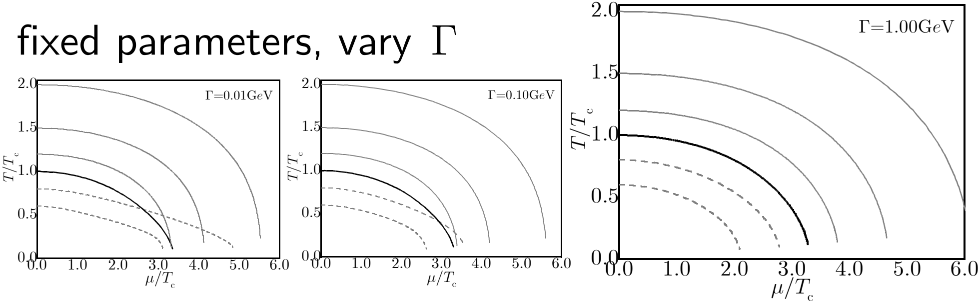


- ansatz $F(\omega,k) \rightarrow \mathrm{BW}(m)$

$$s(T) = \int \mathrm{d}M\, s_{qp}(T, M)\mathrm{BW}(m, M, \Gamma)$$

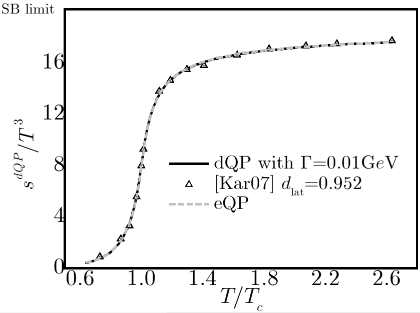
Backup: Distributed quasiparticle model

- fixed parameters, vary Γ



- adjustment to lattice

$$\Gamma = 0.01\,\mathrm{GeV}$$



Backup: Distributed quasiparticle model II

- bias adjustment $\Gamma \overset{!}{=} 1\,\mathrm{GeV}$