Novel Diagrammatic Method for Computing Transport Coefficients

— Beyond the Boltzmann approximation —

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Collaboration with Teiji Kunihiro

Goal

Derive a self-consistent equation for transport coefficients.

Keywords

Beyond the Boltzmann equation,
Pinch singularity,
Eliashberg's method.

Hydrodynamics

as a Low energy effective theory

Conservation law:

$$\partial_\mu T^{\mu\nu} = 0\,, \quad \partial_\mu J^\mu = 0$$

Hydrodynamic equation is universal.

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Details of theory are reflected by

Equation of state : $P = P(\epsilon, n)$

Transport coefficients : $\eta, \zeta, \cdots$

Kubo Formula

Kubo and Tomita('54), Nakano('56), Kubo('57)

Transport coefficient ⟷ **Green function**

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Shear Viscosity:

$$\eta = \frac{1}{20} \lim_{\omega \to 0} \frac{1}{\omega} \int d^4x e^{i\omega t} \theta(t) \langle [\pi_{ij}(x), \pi^{ij}(0)] \rangle$$

Bulk Viscosity:

$$\zeta = \frac{1}{2} \lim_{\omega \to 0} \frac{1}{\omega} \int d^4x e^{i\omega t} \theta(t) \langle [\mathcal{P}(x), \mathcal{P}(0)] \rangle$$

where $\mathcal{P}(x) = -T^i{}_i(x)/3, \;\; \pi_{ij}(x) = T_{ij}(x) + g_{ij}\mathcal{P}(x)$

Why diagram?

Can start with **FUNDAMENTAL THEORY**.

In principle, **EXACT**.

Can apply **FIELD THEORETICAL TECHNICS**.

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Can start with **FUNDAMENTAL THEORY**.

In principle, **EXACT**.

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Our aim

Develop **SYSTEMATIC METHOD**
for calculating transport coefficients.

Apply **ELIASHBERG's METHOD** ('62)
to relativistic quantum field theory.

Diagrammatic Method

PREVIOUS WORKS

Fermi liquid: Eliashberg('62), ...

Leading order:

Scalar theory: Jeon('95), Jeon and Yaffe('96), Carrington, Hou and R. Kobes ('00), Basagoiti ('02), Wang and Heinz ('03)

Chiral perturbation theory: Fernandez-Fraile and Nicola('06)

O(N) model, 2PI expansion: Aarts and Martinez Resco ('03), ('04), ('05)

QED: Gagnon and Jeon ('07)

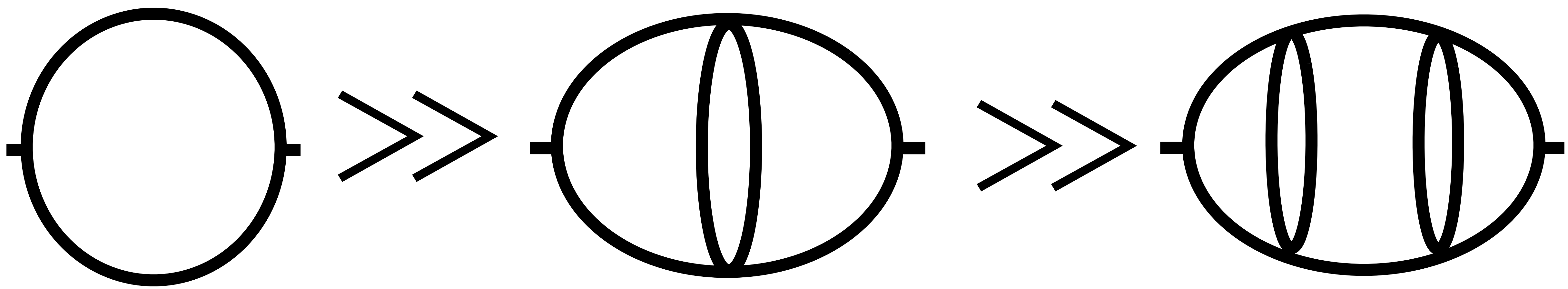
QCD, 3PI expansion: Carrington and Kovalchuk ('07), ('08), ('09)

Next to leading order:

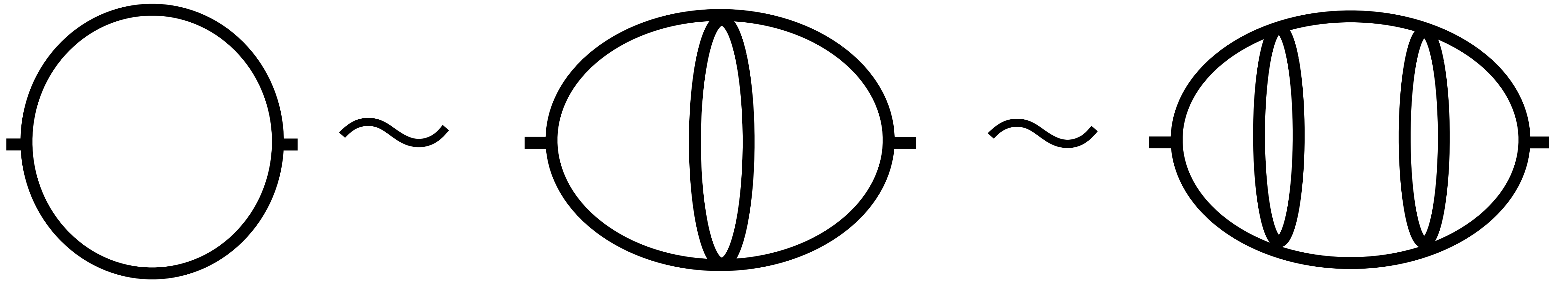
Scalar theory: Moore ('07)

4PI expansion: Carrington and Kovalchuk ('09)

Perturbation theory in vacuum



Perturbation theory in medium

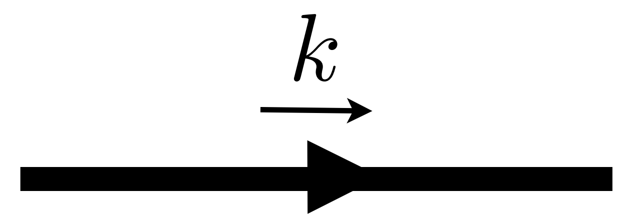


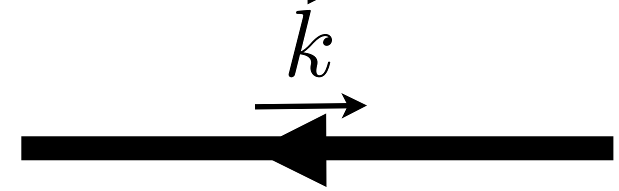
Real time formalism in R/A basis

Aurenche and Becherraw('92)

R/A basis

Propagator: $D^{\alpha\beta}(k) = \begin{pmatrix} 0 & -iD_R(k) \\ -iD_A(k) & 0 \end{pmatrix}$

Retarded propagator: $D_R(k)$ 

Advanced propagator: $D_A(k)$ 

$= D_R(-k)$

Real time formalism in R/A basis

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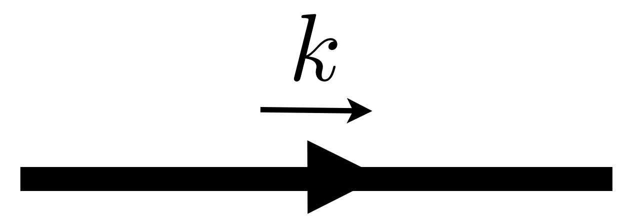
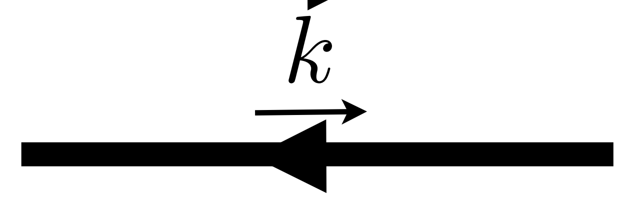
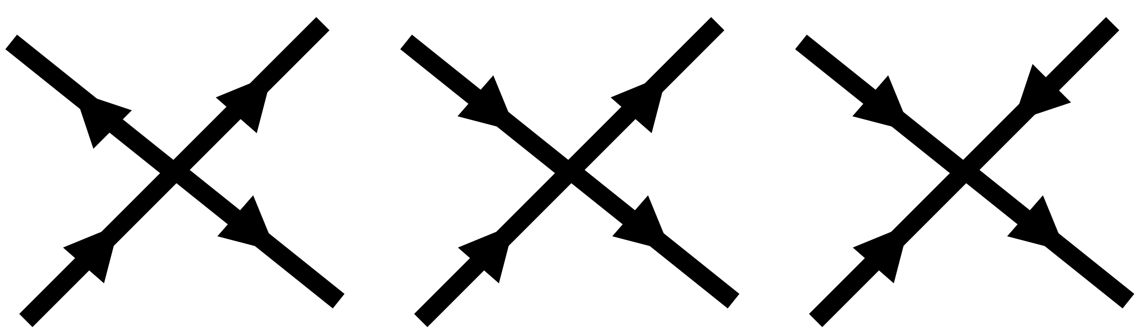
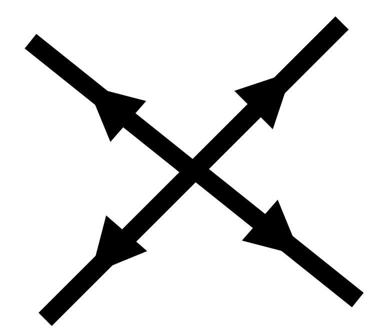
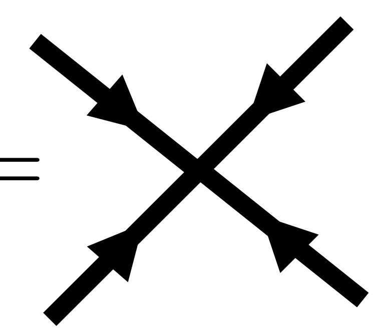
Retarded propagator: $D_R(k)$ 

Advanced propagator: $D_A(k)$  $= D_R(-k)$

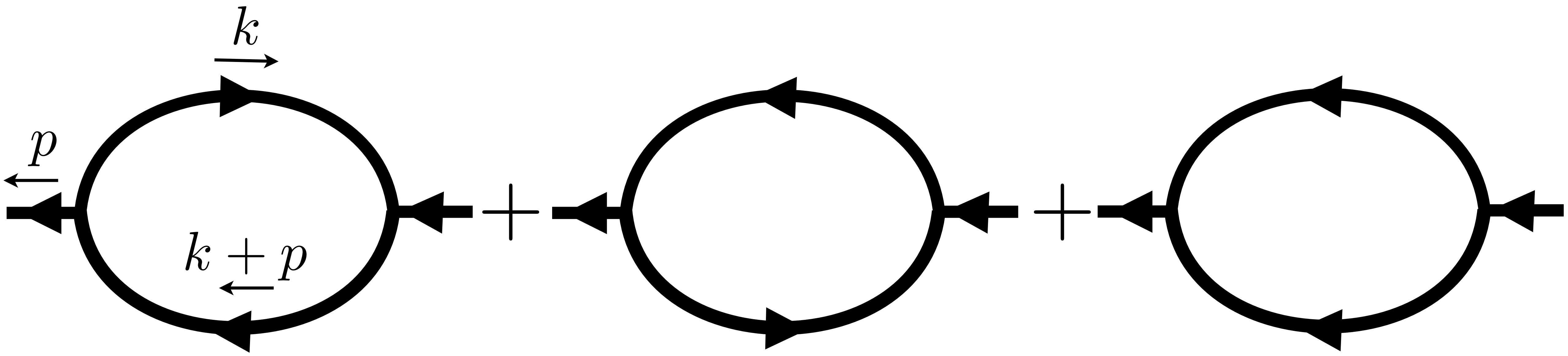
Vertex: Φ^4 theory for simplicity



 $=$  $= 0$

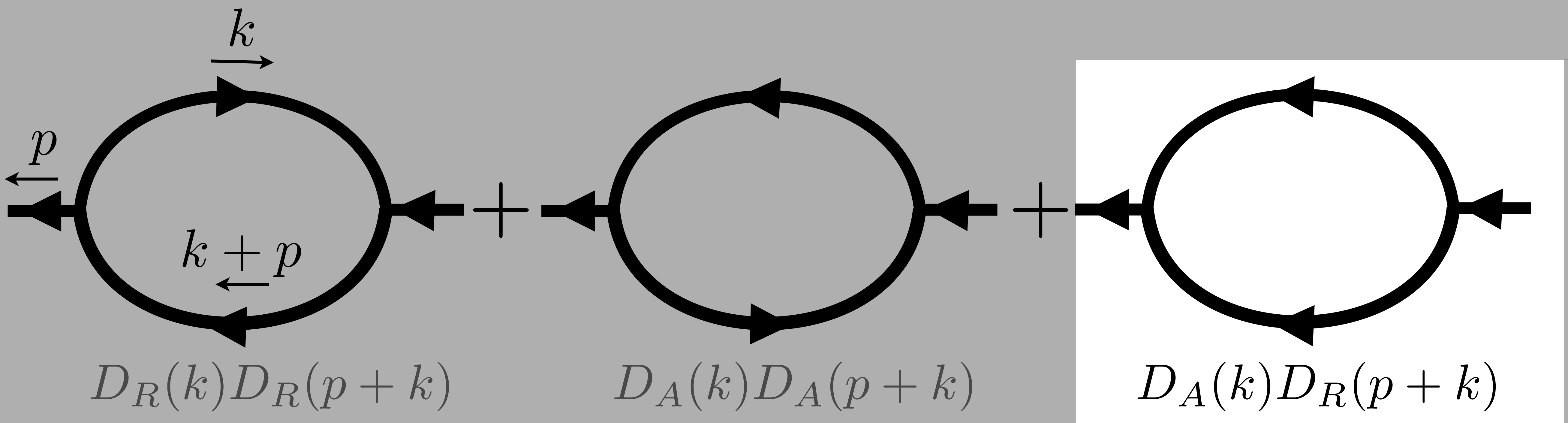
because of causality.

One loop diagram



$D_R(k)D_R(p+k)$ $D_A(k)D_A(p+k)$ $D_A(k)D_R(p+k)$

One loop diagram



Pinch singularity

Quasi particle approximation

$$D_R(p+k)D_A(k) = \frac{2\pi i z_k^2}{p_0 - \boldsymbol{p}\cdot\boldsymbol{v}_k + 2i\gamma_k}\delta(k_0 - \epsilon_k) + \cdots$$

for small p

$$(\partial_t - \boldsymbol{v_k}\cdot\boldsymbol{\partial})f(x,p) = -C[f]$$

Inverse of LHS in Boltzmann equation

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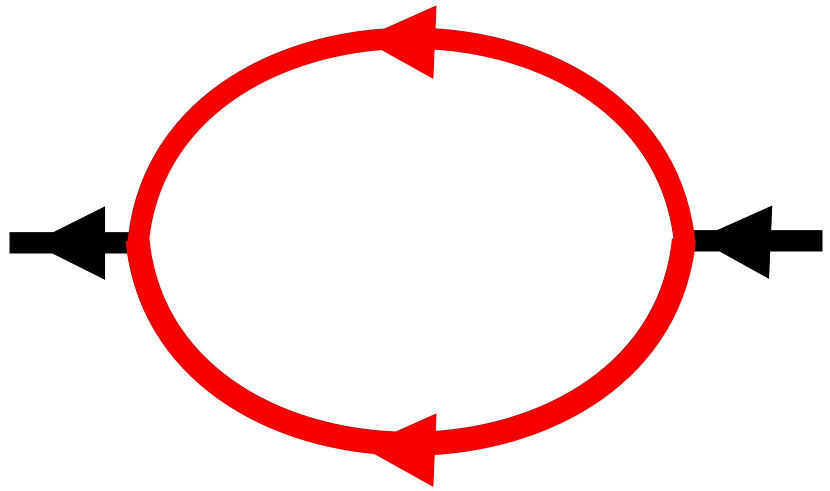
Inverse of LHS in Boltzmann equation

At $p=0$,

$$D_R(k)D_A(k) = \frac{1}{|k^2 - m^2 - \Pi(k)|^2} = \frac{1}{2\mathrm{Im}\,\Pi(k)}\rho(k)$$

$\rho(k)$:spectral function

Shear viscosity at one-loop



$$\sim \frac{1}{\mathrm{Im}\,\Pi(k)} \sim \frac{1}{\lambda^2} \sim \lambda_{\mathrm{mfp}}$$

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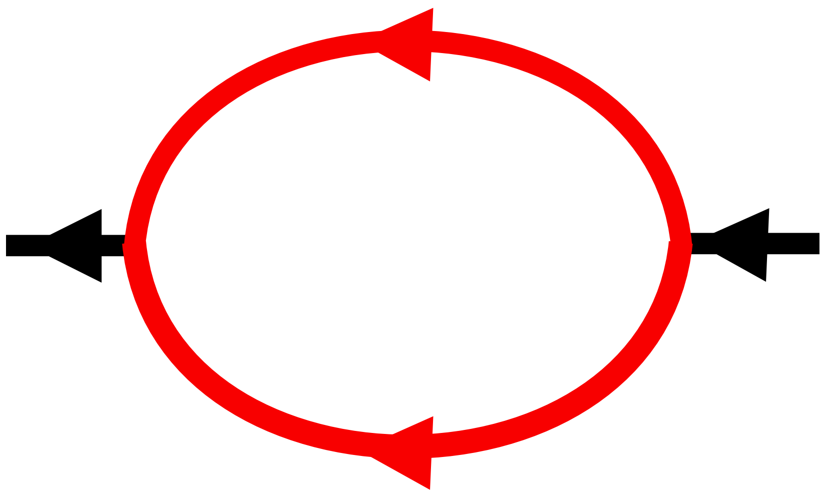


$$\sim \frac{1}{\mathrm{Im}\,\Pi(k)} \sim \frac{1}{\lambda^2} \sim \lambda_{\mathrm{mfp}}$$

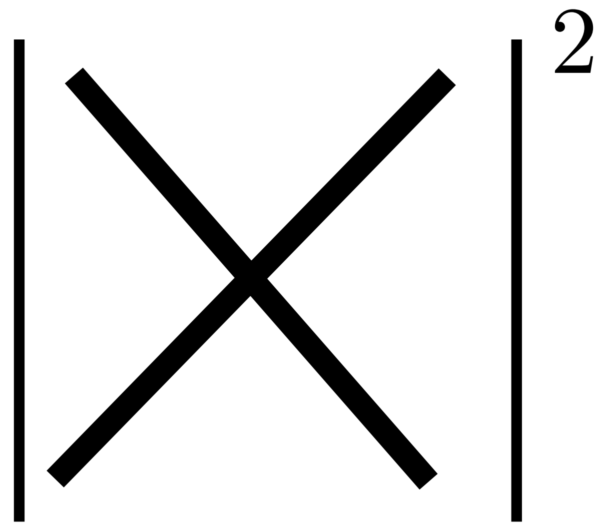
From transport theory

$$\Rightarrow \eta \approx \frac{1}{3}\bar{p}n\lambda_{\mathrm{mfp}}$$

Maxwell(1860)

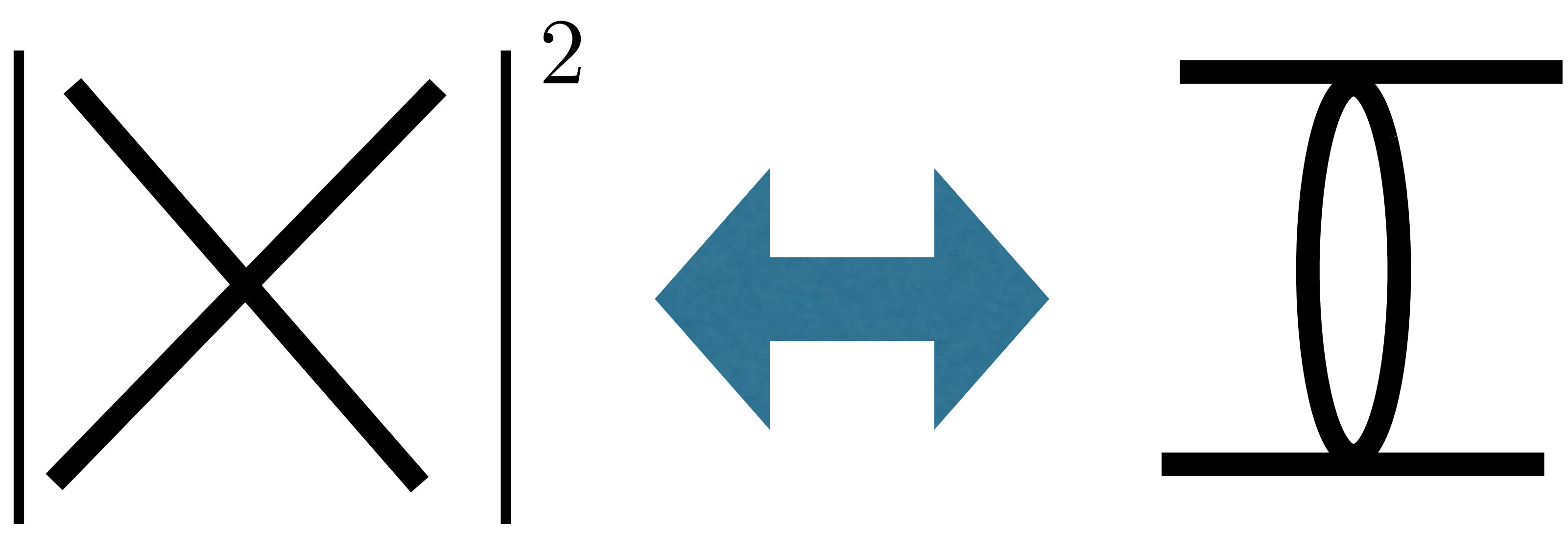
This is the same order as the one-loop diagram.

Optical theorem



Squared scattering
amplitude

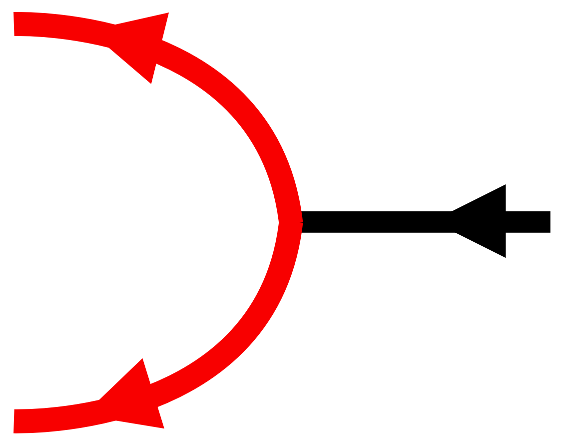
Optical theorem



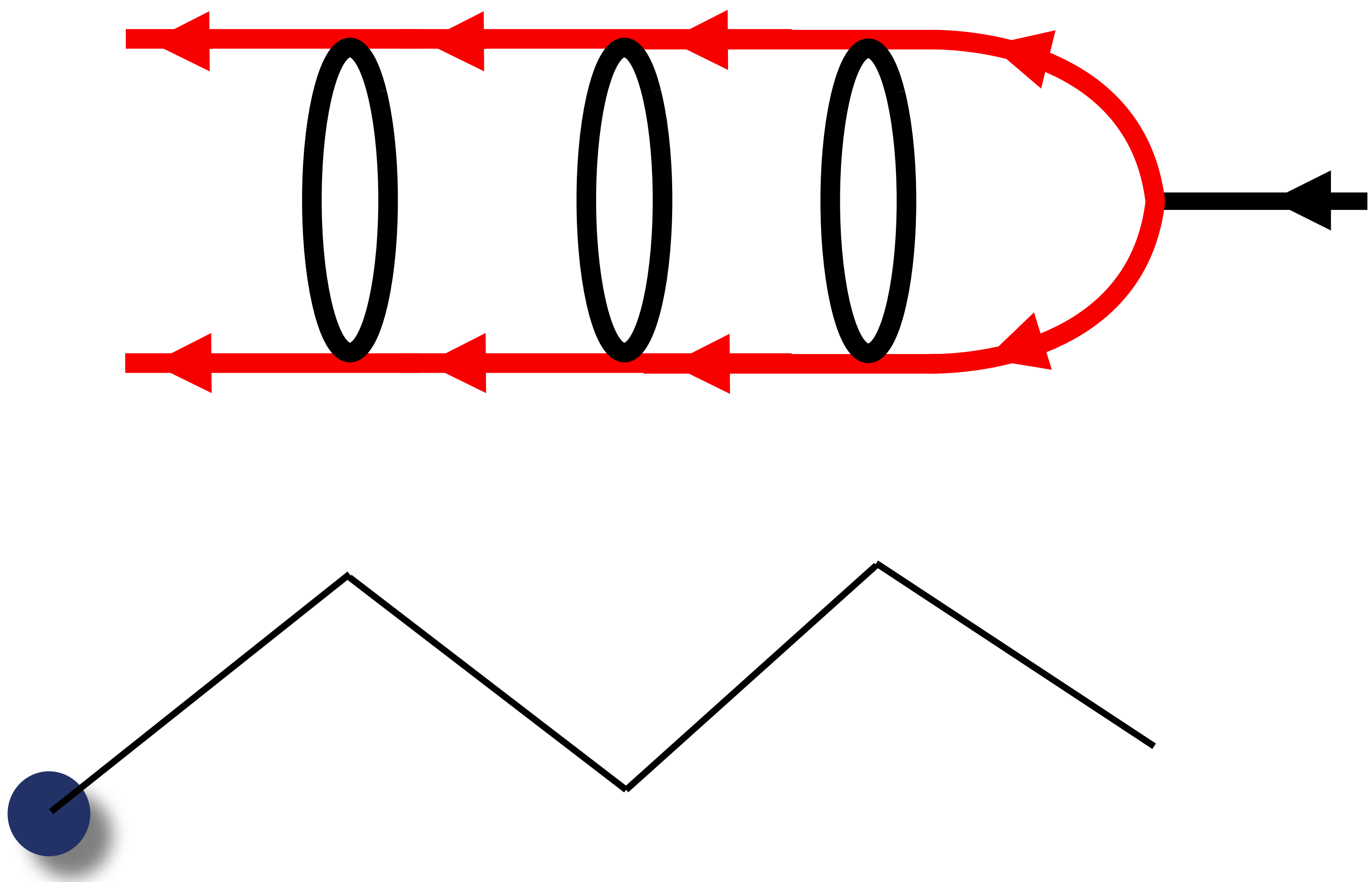
Squared scattering amplitude

Imaginary part of loop diagram

Resummation



Resummation



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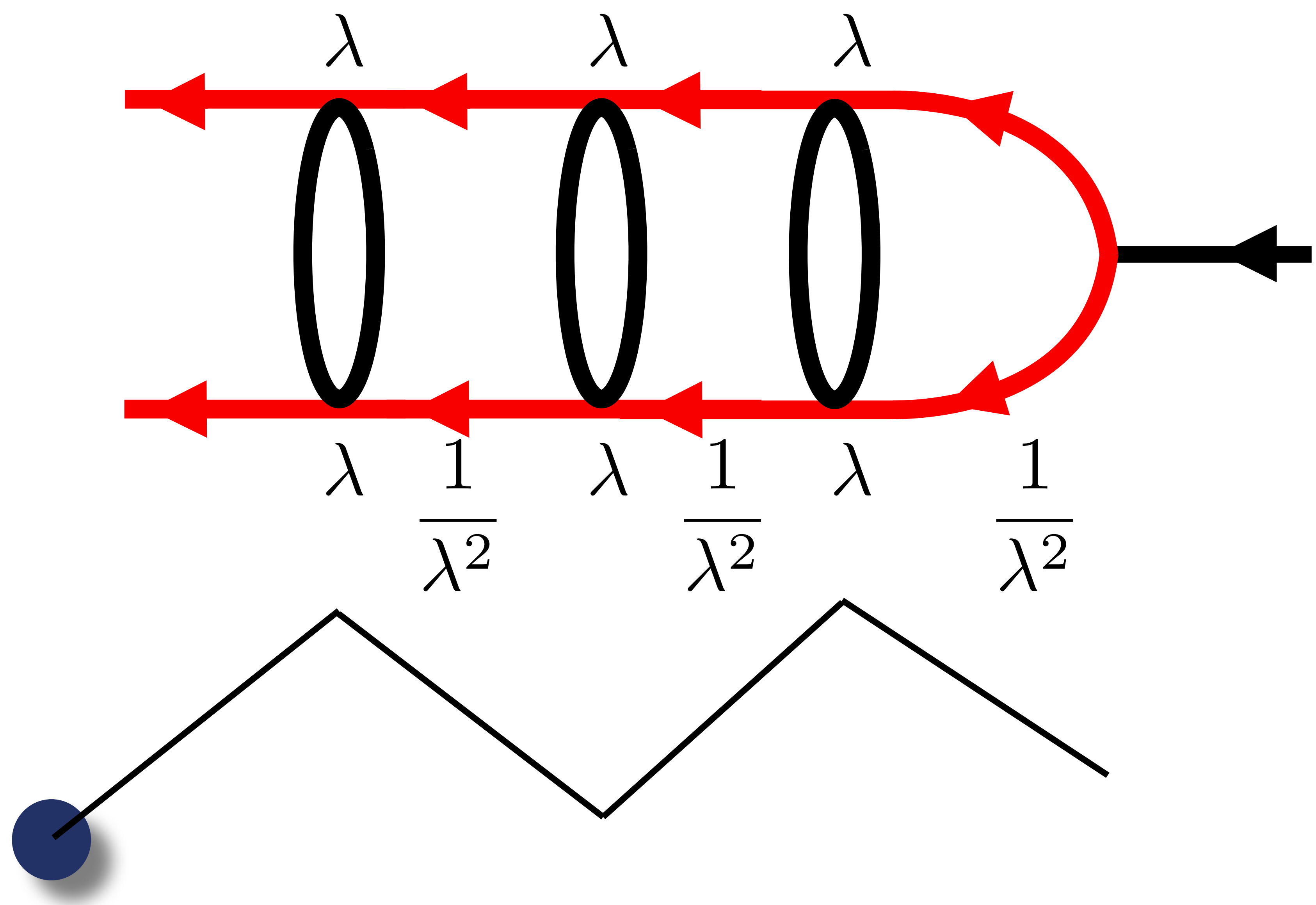

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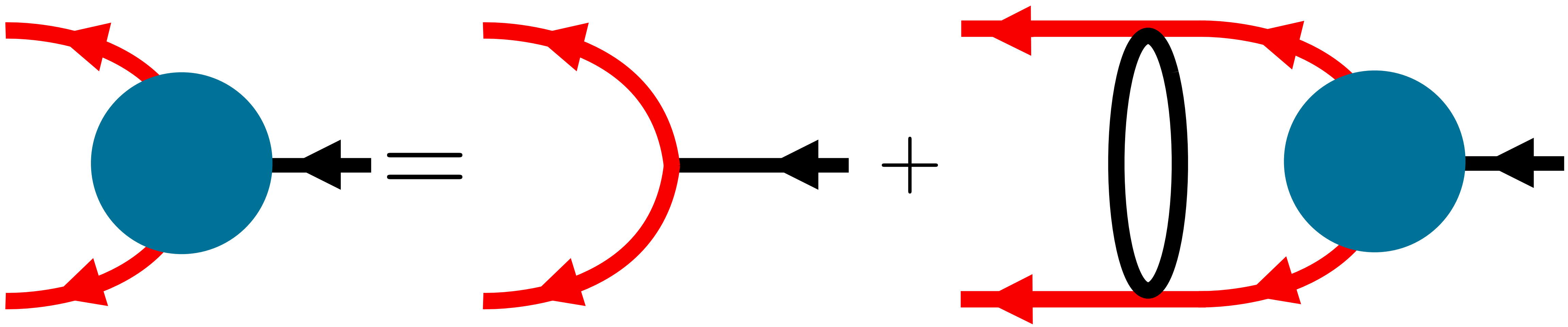

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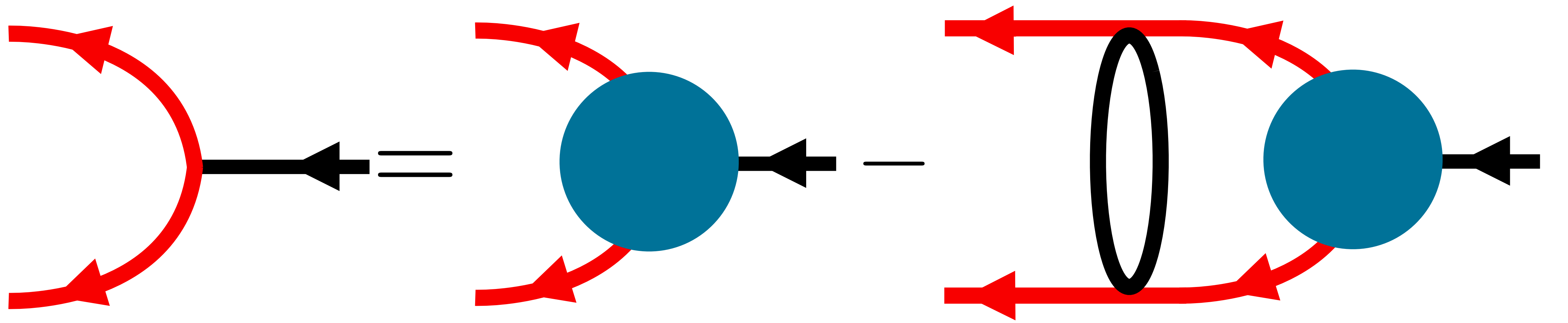

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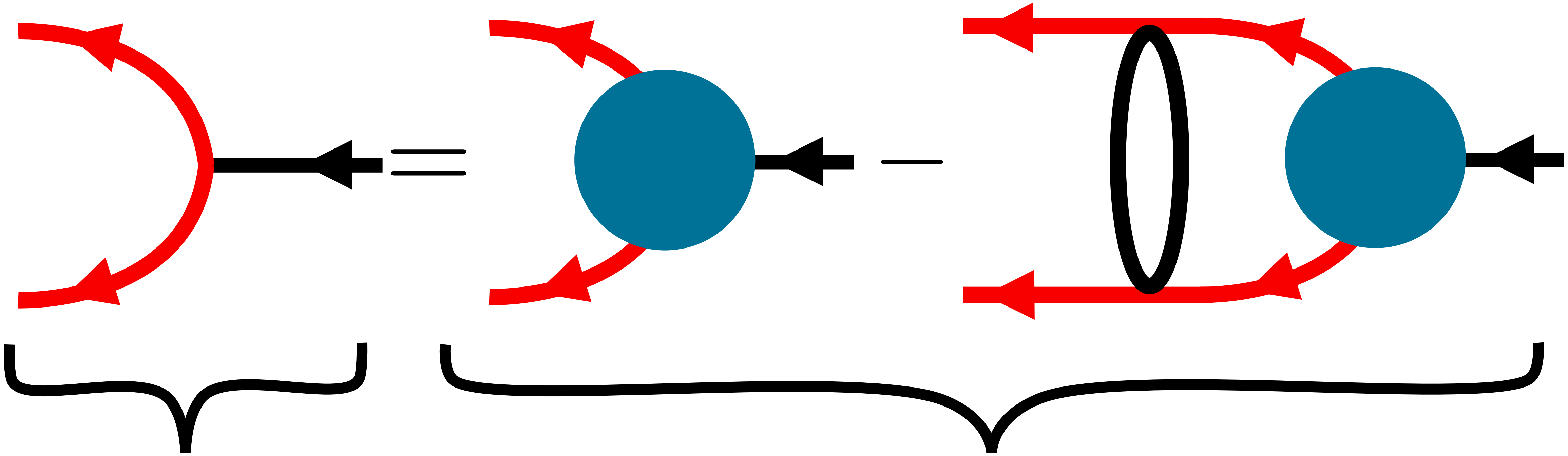


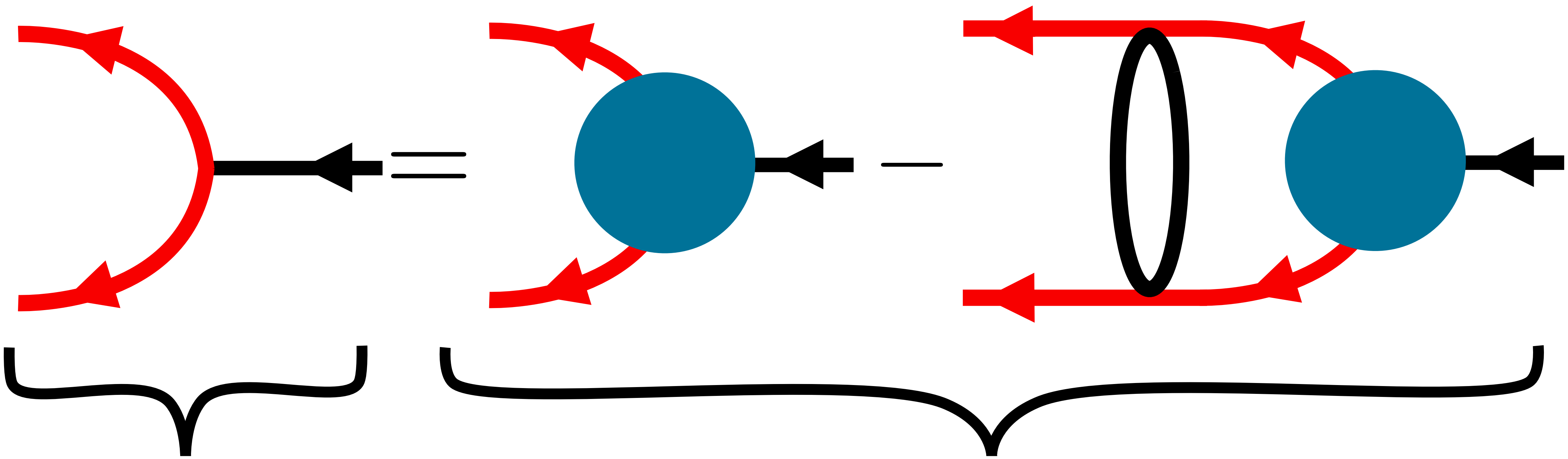
Left hand side

$2p^\mu \partial_\mu f$

Collision Term

$C[f]$

Resummation



Left hand side

$$2p^\mu \partial_\mu f$$

Collision Term

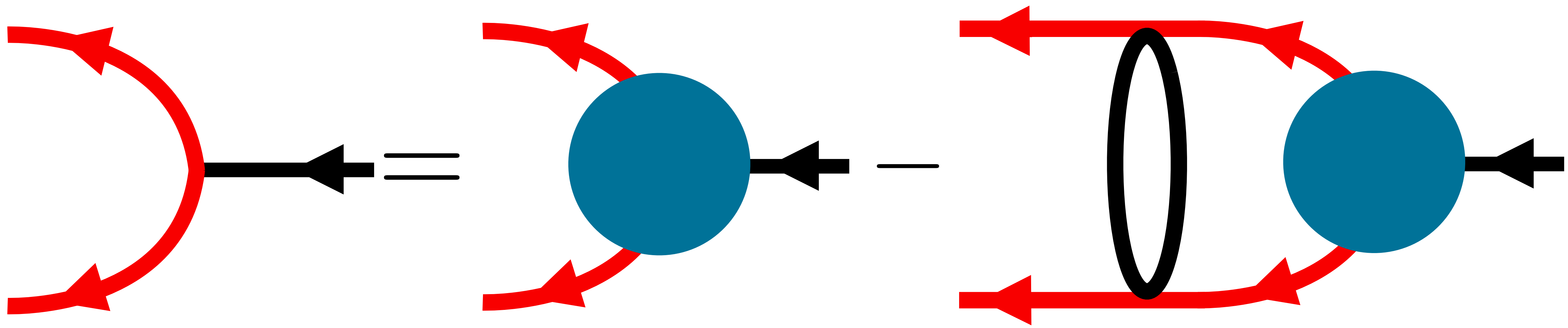
$$C[f]$$

+ quasi-particle approximation

=Linearized Boltzmann Equation

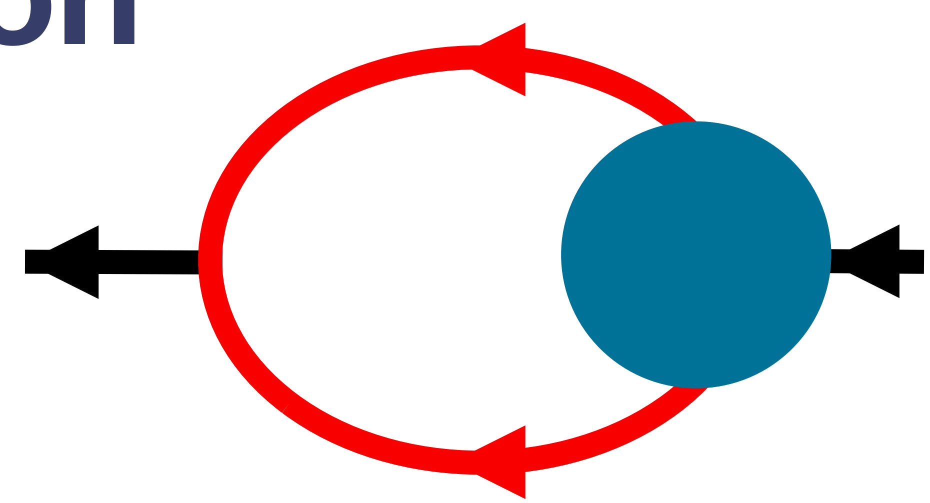
Jeon(1995)

Resummation



Correlation function

$$\langle [T_{xy}(x), T_{xy}(0)] \rangle =$$



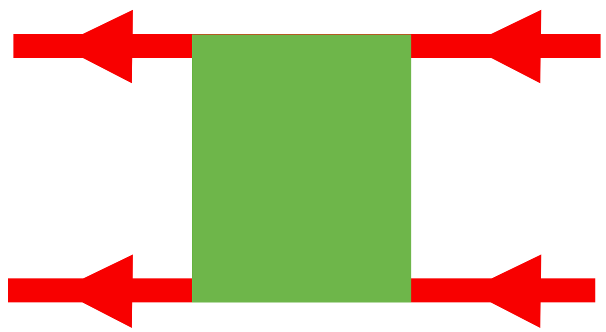
Beyond the Boltzmann Equation

YH, Kunihiro

Eliashberg's method

Eliashberg('62)

Decompose four point function to them with the pinch singularities and the others.



Both sides connect to pinch diagrams.

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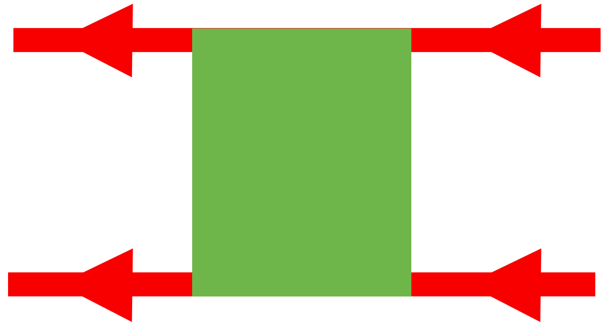

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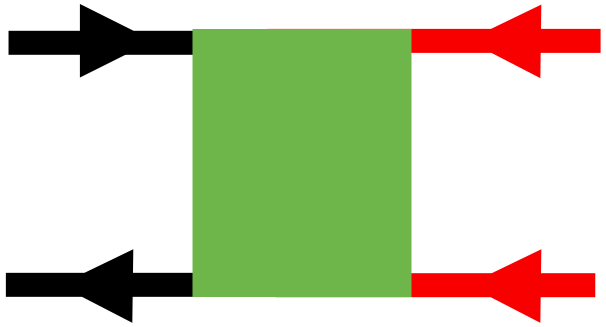

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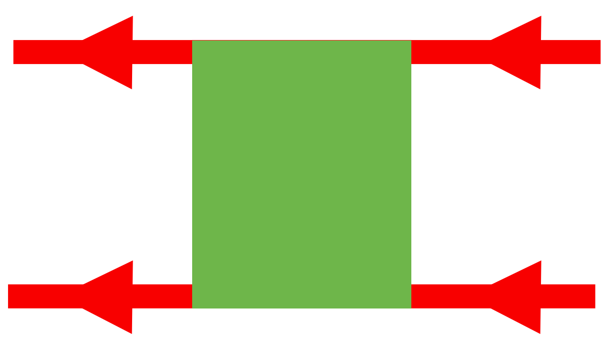

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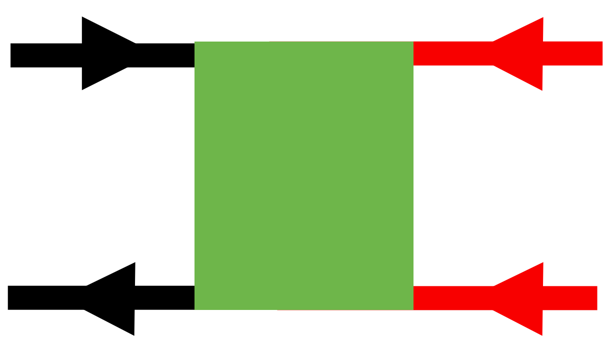
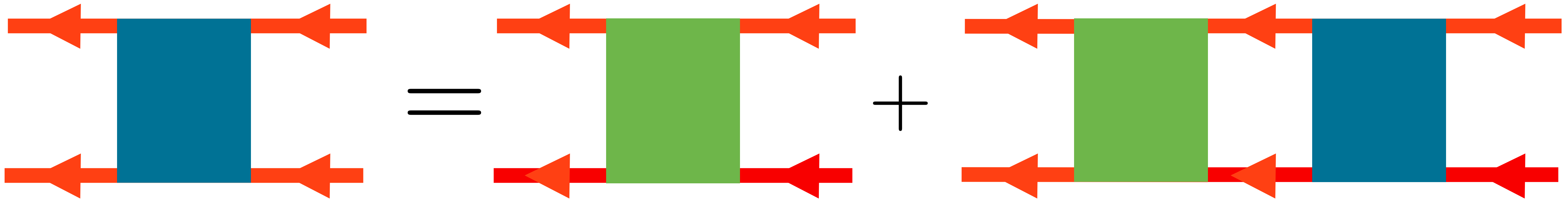


One side connects to pinch diagrams.

Both sides do not connect to pinch diagrams.

Resummation of Pinch Singularities



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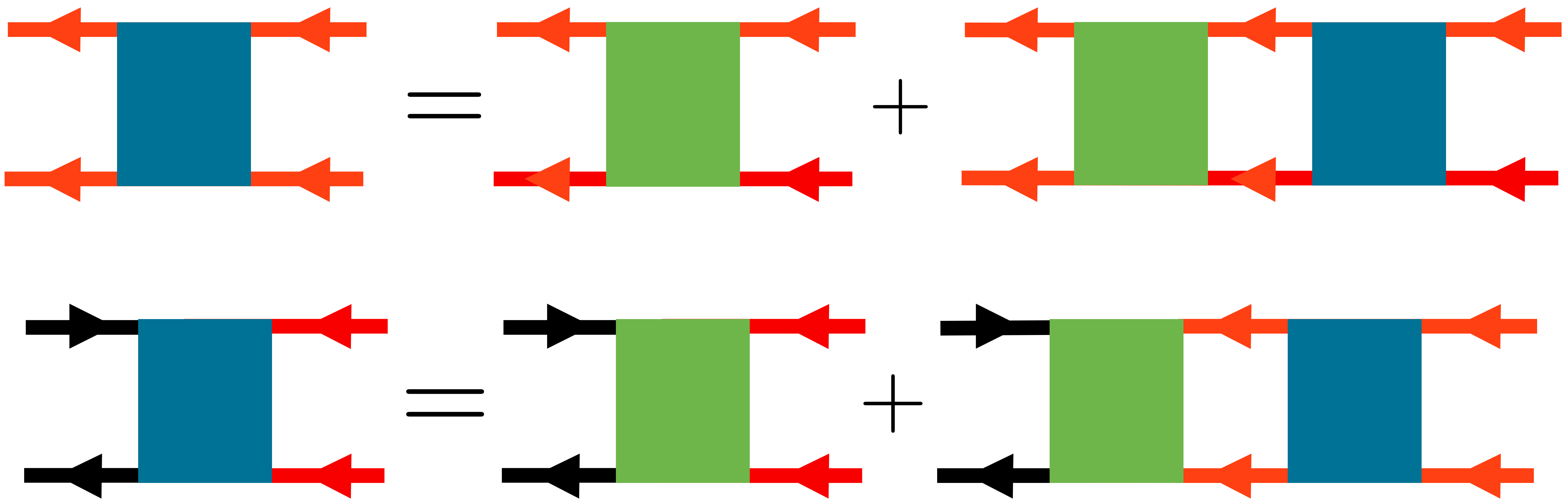

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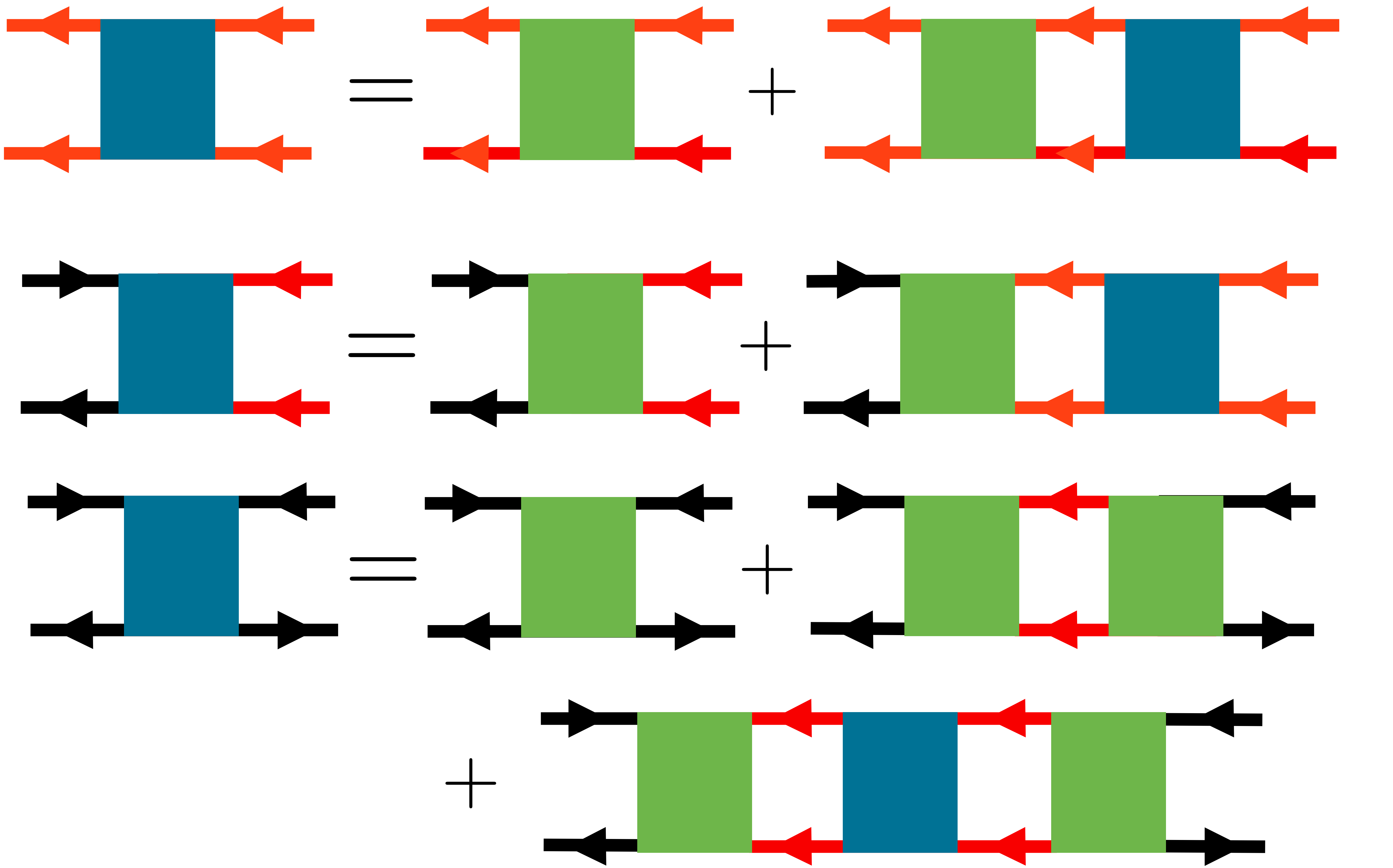
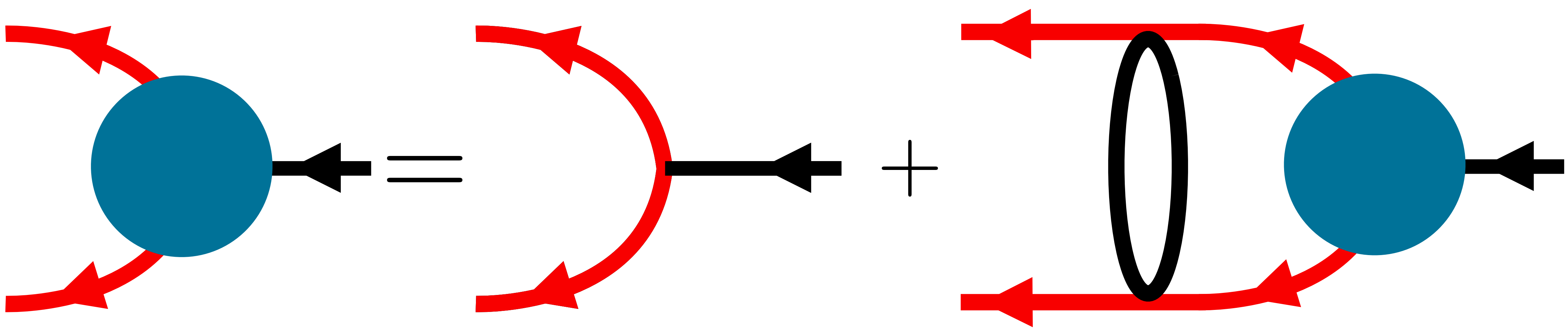


Beyond the Boltzmann Eq.

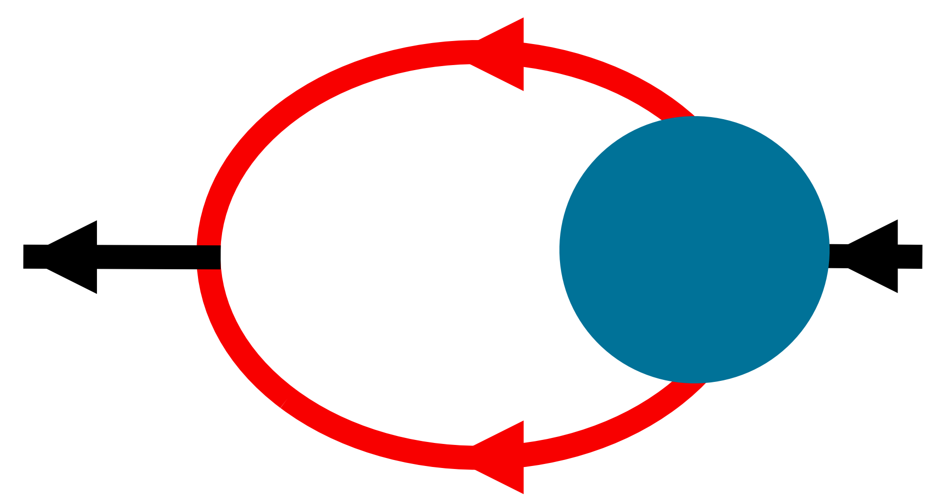
YH, Kunihiro

Leading order



Energy Momentum Tensor

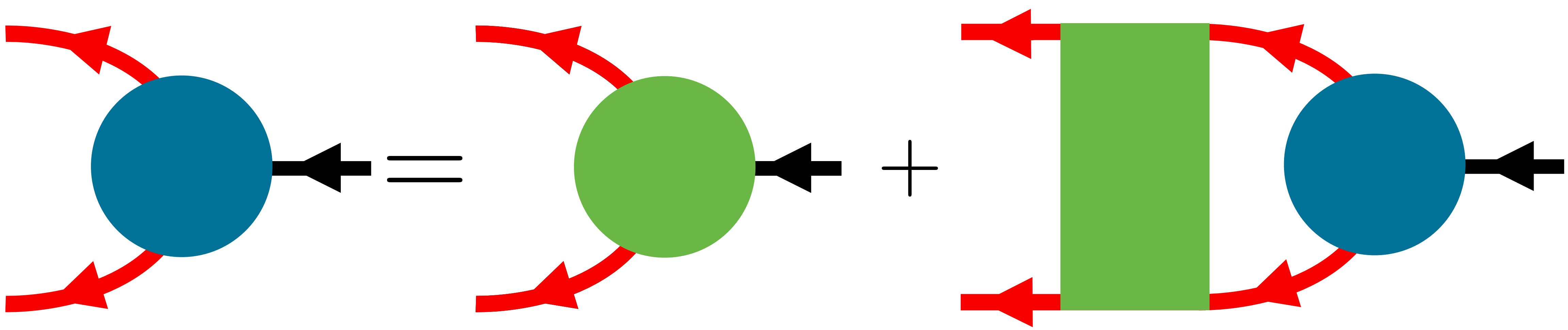
$$\langle [T_{xy}(x), T_{xy}(0)] \rangle =$$



Beyond the Boltzmann Eq.

YH, Kunihiro

Including higher orders



Energy Momentum Tensor

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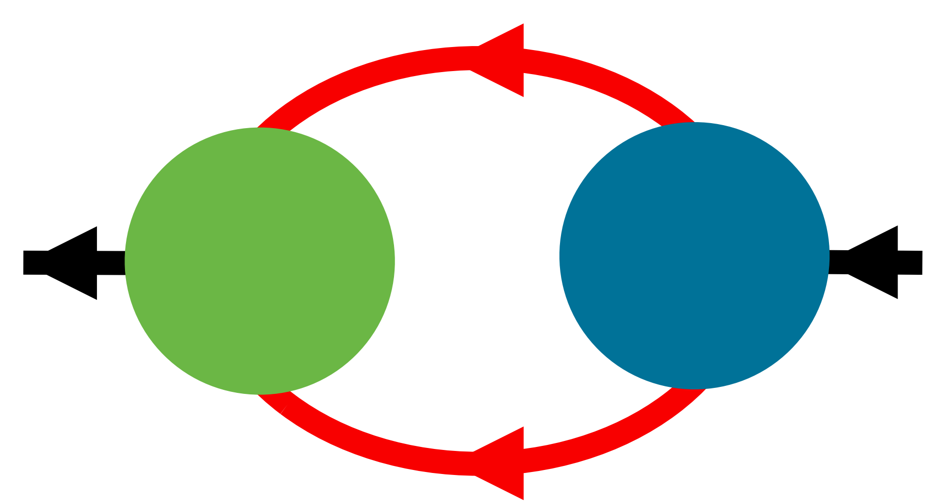
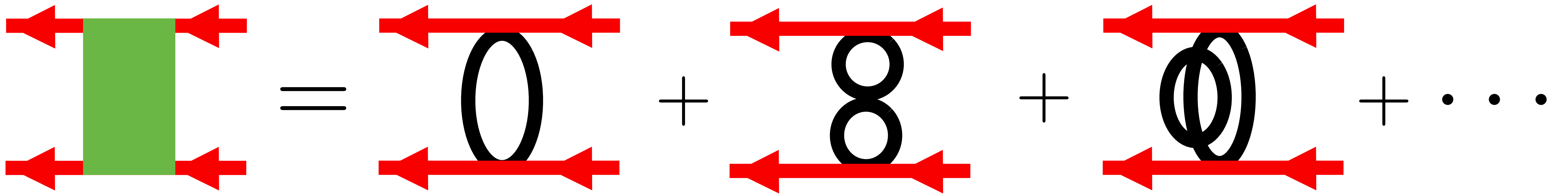


+ no pinch singular diagrams

Beyond the Boltzmann Eq.

YH, Kunihiro

Correction to scattering amplitude



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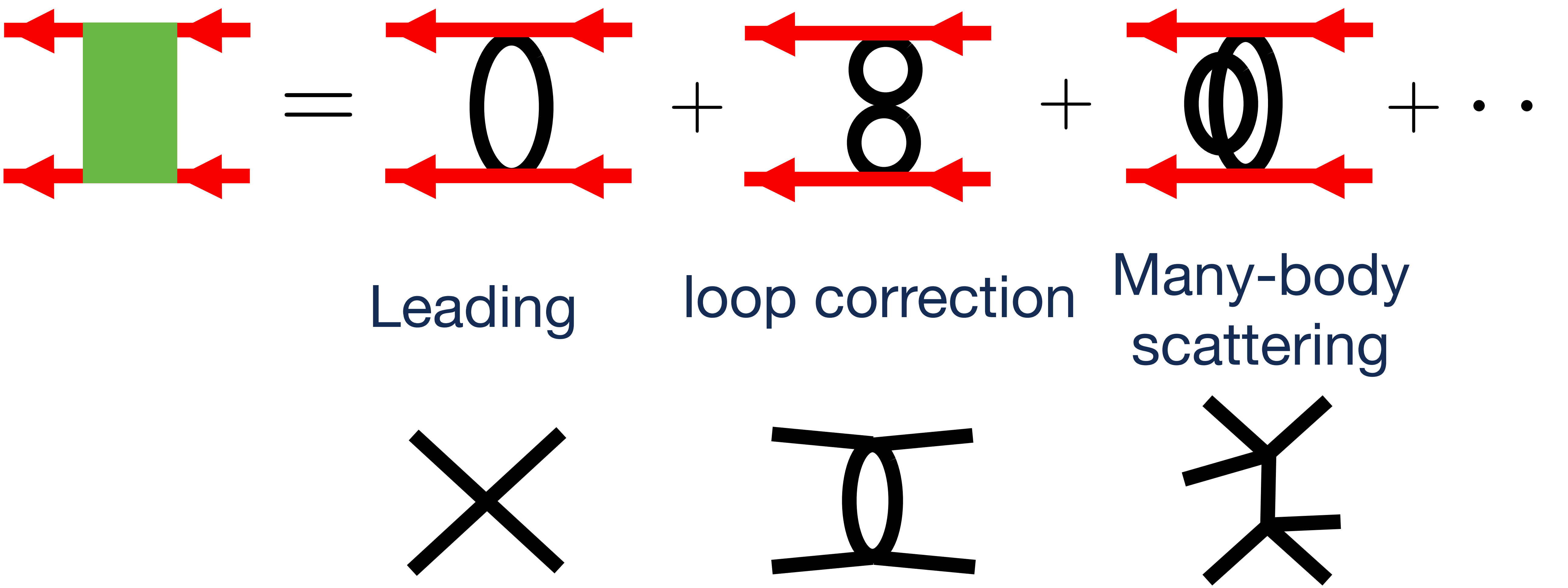

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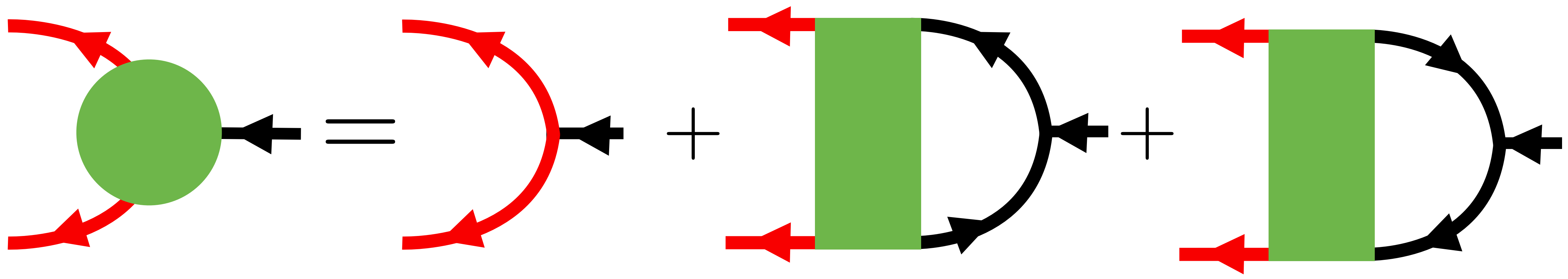
Vertex correction



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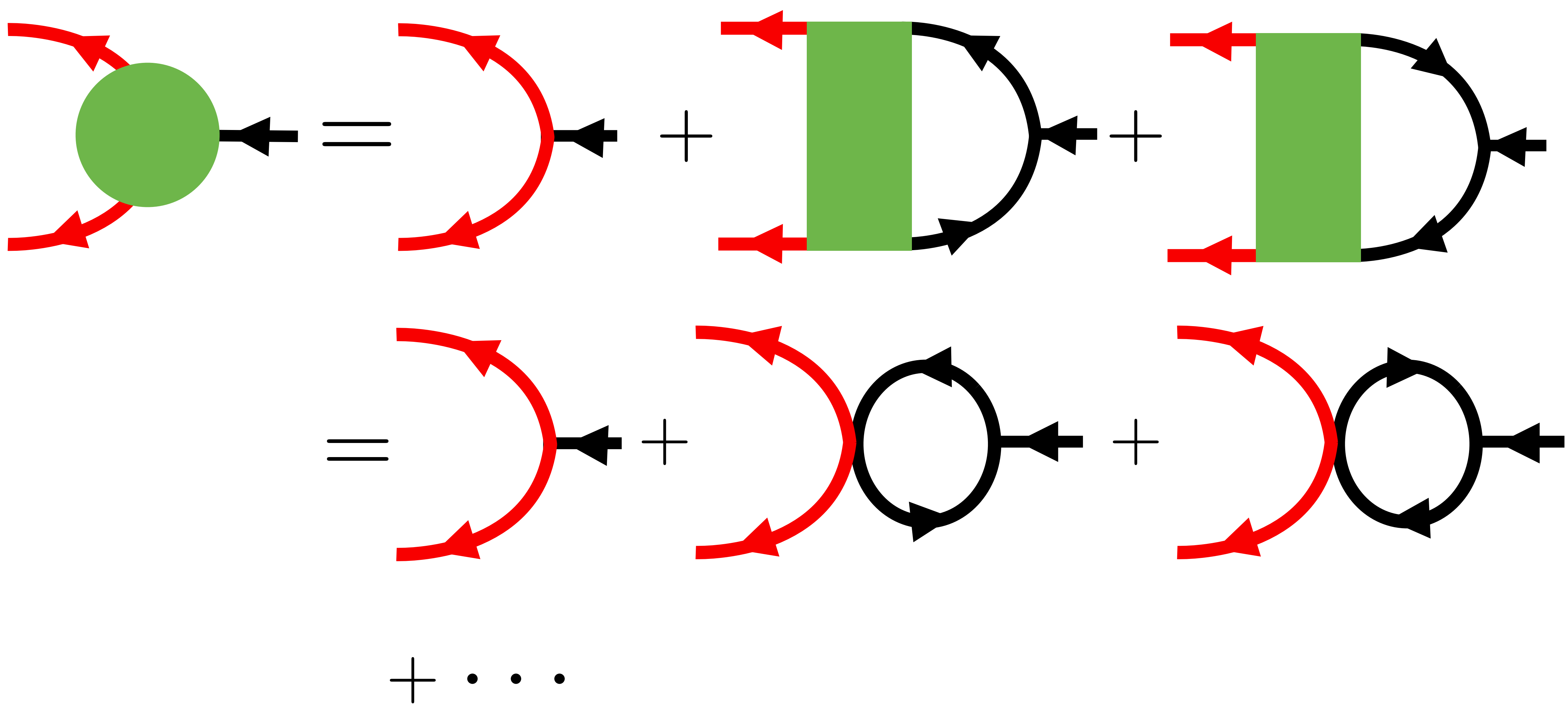

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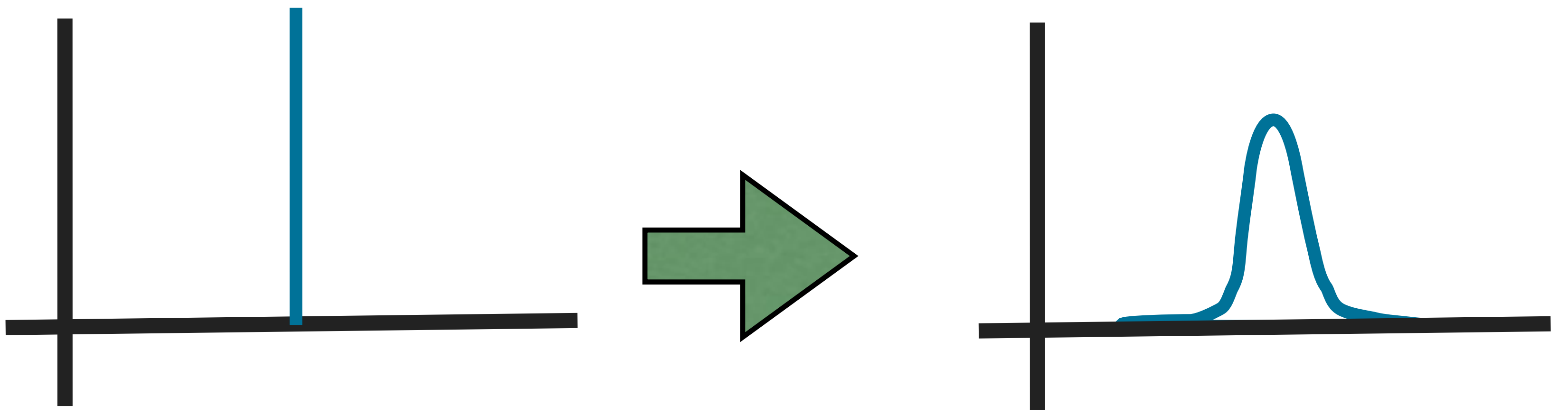
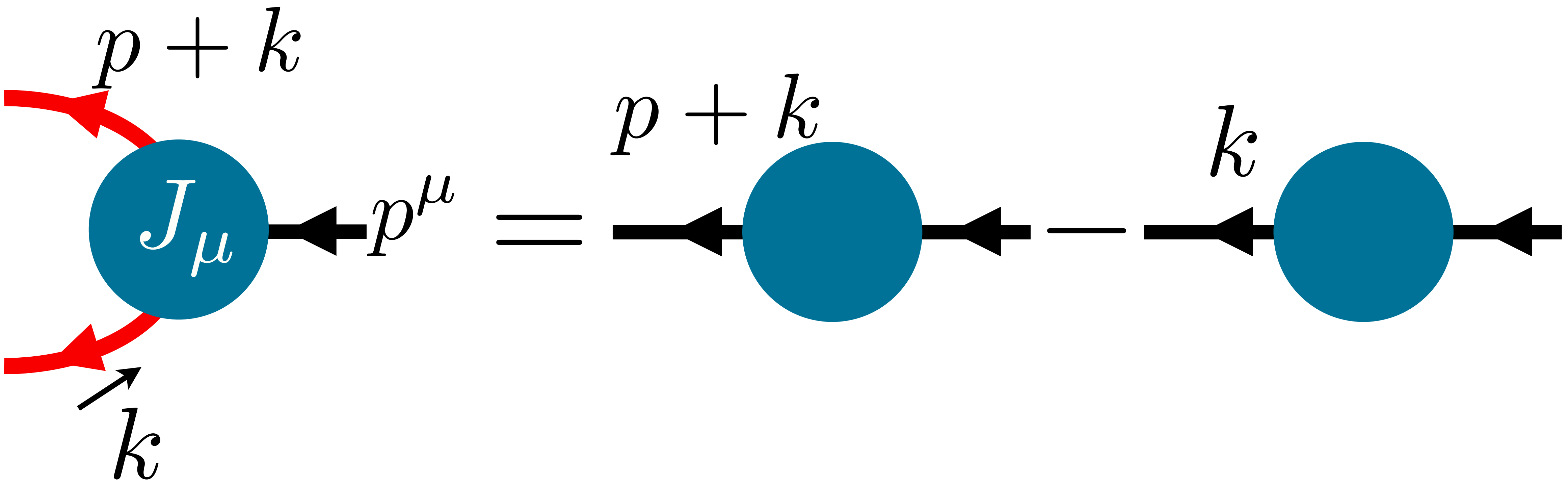
Spectrum

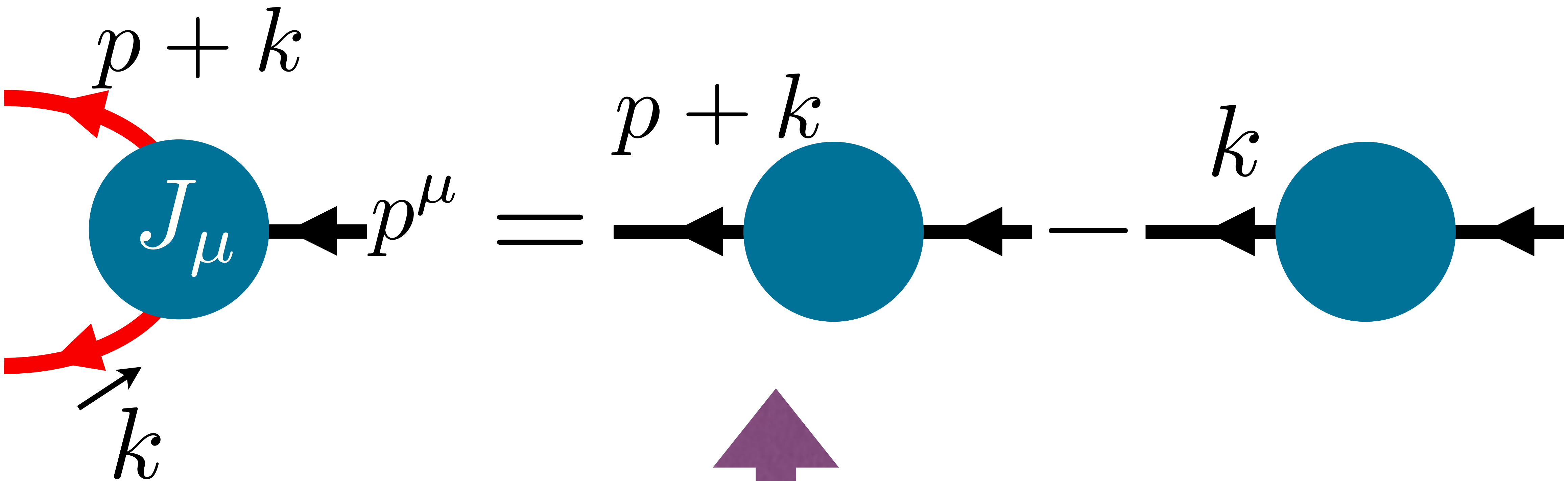


Quasi-particle spectrum Full spectrum

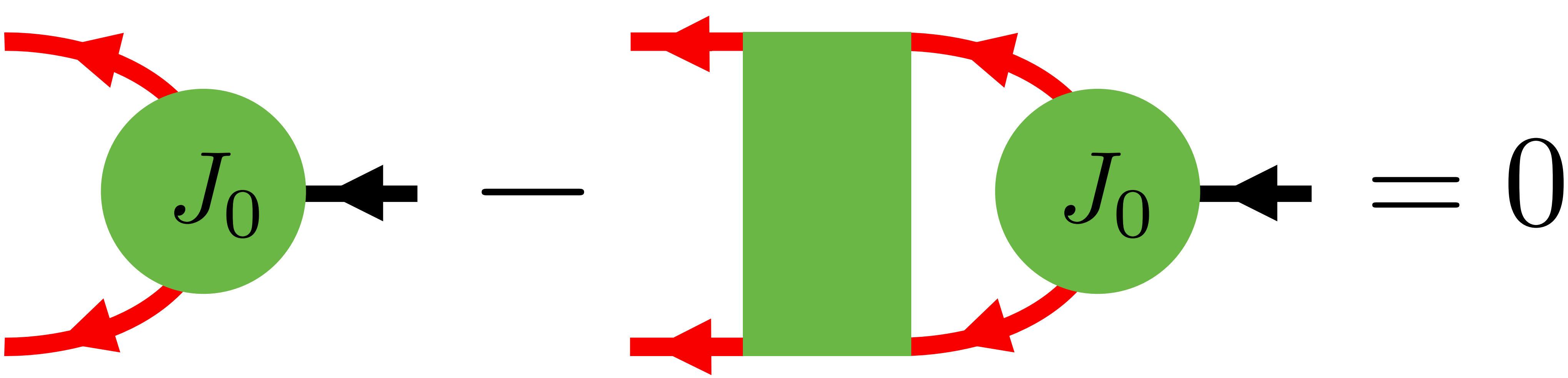
Ward-Takahashi identity



Ward-Takahashi identity



Collision invariant



Summary

- Applied Eliasberg's method to QFT.
- Leading order: Boltzmann Equation.
- Higher order: Modification of spectra, vertex renormalization, and many-body scatterings.
- Ward-Takahashi identity
  ⇔ collision invariant

Future Plan

- Apply to QCD.
  - Not only pinch singularity but also collinear singularity must be summed (Landau-Pomeranchuk-Migdal effect).
- Apply to critical phenomena.
  - Can’t neglect hydrodynamic modes.

    Relate our formalism to mode coupling theory.