The deuteron as a six-quark state in QCD

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Motivation

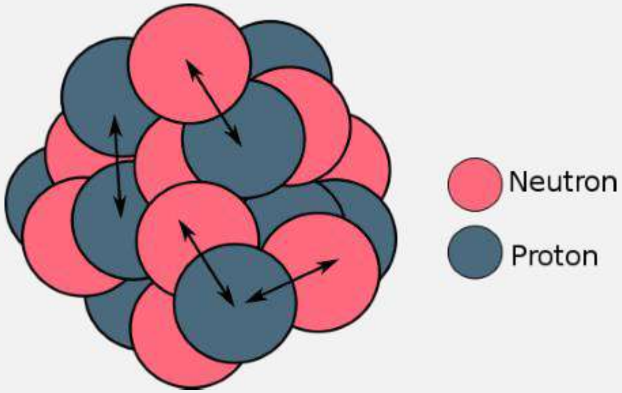


Fig.1 – Interactions between nucleons.

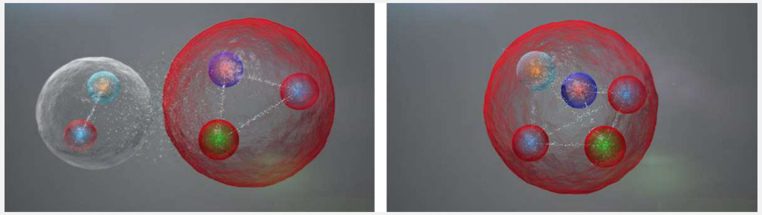


Fig.2 – From left to right: meson, baryon, pentaquark.

- **Problem:** Nuclear interaction not well understood from a fundamental level.
- In a **NR context**, short-range interaction terms are purely phenomenological ⇒ large number of fitted parameters.
- In a **R context**, hadrons with two to five valence quarks are being studied.
- **Goal:** the study of the simplest non-trivial nucleus, the deuteron, from quarks and gluons degrees of freedom.

Quantum Chromodynamics (QCD)

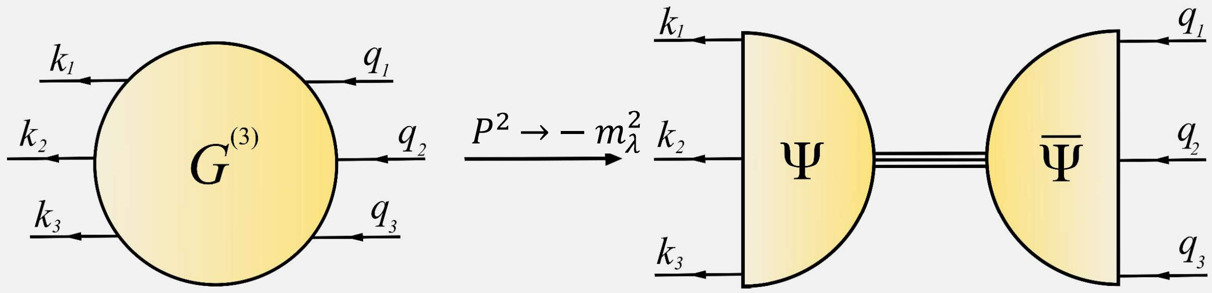


Fig.3 – Behavior of Green functions near a pole for a 6-point function.

$G^{(3)} \to$ **Green function**

$\Psi \to$ **Bethe-Salpeter Wave Function**

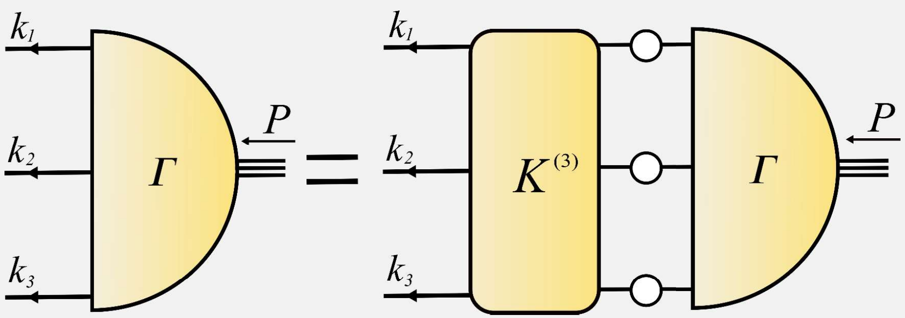
$m_\lambda \to$ **Mass of the bound state**

$$G(x_1, \ldots, x_j, y_1, \ldots, y_j) = \langle 0 | \hat{\mathrm{T}} \left(\prod_{i=1}^{j} \psi(x_i) \right) \left(\prod_{i=1}^{j} \bar{\psi}(y_i) \right) | 0 \rangle \xrightarrow{P^2 \to -m_\lambda^2} \frac{\Psi(\{k_1, \ldots, k_j\}, P) \bar{\Psi}(\{q_1, \ldots, q_j\}, P)}{P^2 + m_\lambda^2}$$

Fig.4 – Image representation of the BSE for a three-quark bound state amplitude.

$\Gamma \to$ **Bethe-Salpeter Amplitude**

$K \to$ **Kernel**



$$G^{(j)} = G_0^{(j)} + G_0^{(j)} T^{(j)} G_0^{(j)}$$

$$\Psi = \prod_{i=1}^{j} S_i \Gamma$$

$$T^{(j)} \xrightarrow{P^2 \to -m_\lambda^2} \frac{\Gamma \bar{\Gamma}}{P^2 + m_\lambda^2}$$

Model

1. SU(2) flavor symmetry;

2. The six quarks are divided into two nucleons:

Deuteron amplitude　　　Nucleon-nucleon amplitude

$$\psi_{aa'} = \Psi_a S_a^N \Psi_{a'} S_{a'}^N \Gamma_{aa'}$$

3. The nucleon is approximated as a quark-diquark bound state because:

    - **the two-body force is dominant**: color trace for the leading three body irreducible interaction vanishes;

    - to form a **color singlet** (the nucleon), two quarks must belong to an attractive color anti-triplet.

Model

- The six-body kernel takes the form:

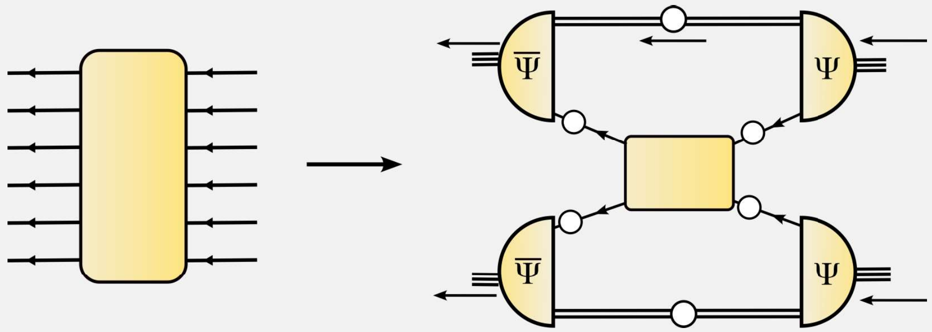


Fig.5 – Approximation of the six-body interaction kernel.

- The four-point function is approximated as:

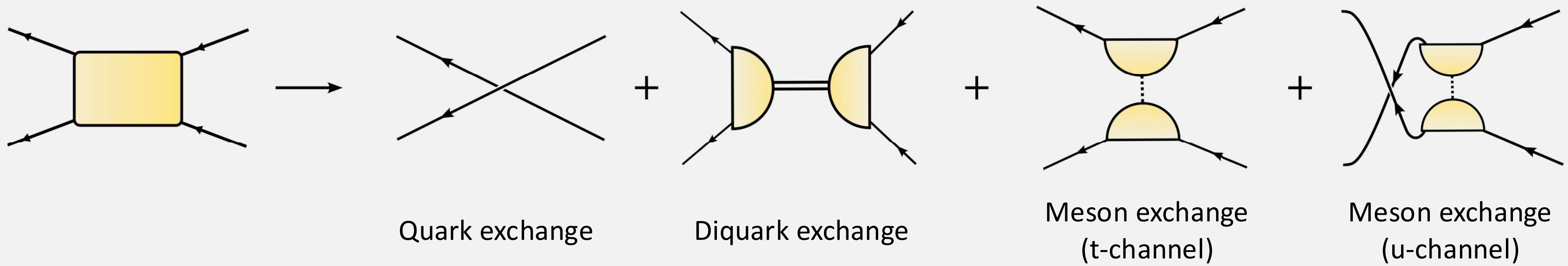


Fig.6 – Approximation of the four-point function.

Quark Exchange

$$\Gamma^{\lambda}(p,P)=\int\frac{d^4q}{(2\pi)^4}\int\frac{d^4k}{(2\pi)^4}\bar{\Psi}^{\mu'}(r_1',p_1)S(l_1)\Psi^{\nu}(r_2,q_2)\left[\Phi^{\lambda}(q,P)\right]^T \times D^{\mu'\mu}(k_1)\left[\bar{\Psi}^{\nu'}(r_2',p_2)S(l_2)\Psi^{\mu}(r_1,q_1)\right]^T D^{\nu'\nu}(k_2)$$

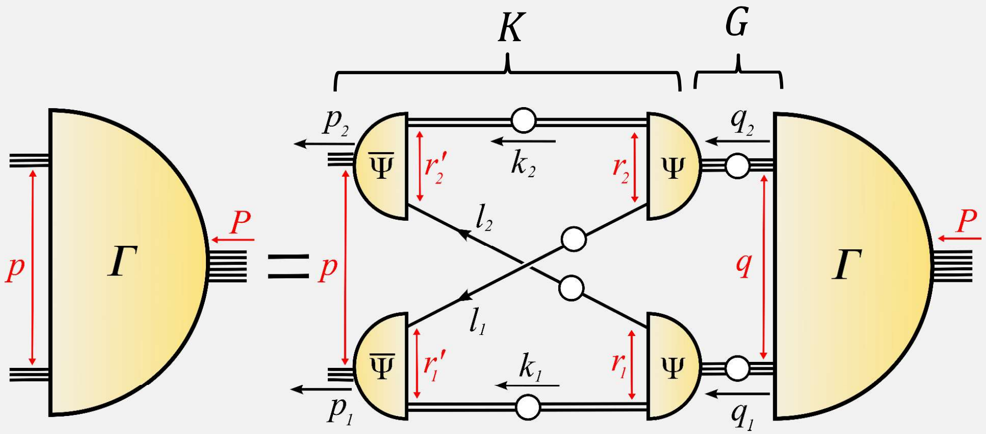


Fig.7 – Feynman diagram representing the deuteron BSE in the quark exchange model.

$\Phi^{\lambda}(q,P)=S^N(q_1)\Gamma^{\lambda}(q,P)[S^N(q_2)]^T$ ⟶ BSWF

$S^N(q_i)$ ⟶ Nucleon Propagator

$\Psi^{\mu}(r_i,q_i)$ ⟶ Nucleon amplitude

$D^{\mu'\mu}(k_i)$ ⟶ Diquark Propagator

$S(l_i)$ ⟶ Quark Propagator

Solution Strategy

- To obtain the deuteron mass we solve an eigenvalue problem:

$$KG\Gamma^{\mu} = \lambda(P)\Gamma^{\mu} \qquad P = (0,0,0,iM_D)$$

- The BSA is divided in three components:

$$\Gamma^{\mu} = \Gamma^{\mu}_{Dirac} \otimes \Gamma_{Flavor} \otimes \Gamma_{Color}$$

$J^{\pi} = 1^{+}$

Expansion using a Dirac basis

$$\Gamma^{\mu}_{Dirac}(p,P) = \sum_{k=1}^{8} a_k\left(p^2, z, P^2\right)\left\{i\tau^{\mu}_{k}(p,P)C\right\}$$

Dressing functions $\quad z = \hat{p}\cdot\hat{P}$

$\mathrm{I} = 0$

Color trace from BSE $\rightarrow 1/3$

- The propagators and amplitudes in the kernel were calculated using the **AWW interaction** with **rainbow ladder truncation**.

Meson Exchange

$$\Gamma^{\lambda}(p,P) = \int_q \int_k \int_r \bar{\Psi}^{\mu'}(r'_1, p_1) S(l_3) \Theta(r_3, k_3) S(l_1) \Psi^{\mu}(r_1, q_1) \Phi^{\lambda}(q,P)$$
$$\left[\bar{\Psi}^{\nu'}(r'_2, p_2) S(l_4) \bar{\Theta}(r_4, k_3) S(l_2) \Psi^{\nu}(r_2, q_2)\right]^T D^{\mu'\mu}(k_1) D^{\nu'\nu}(k_2) H(k_3)$$

Meson amplitude

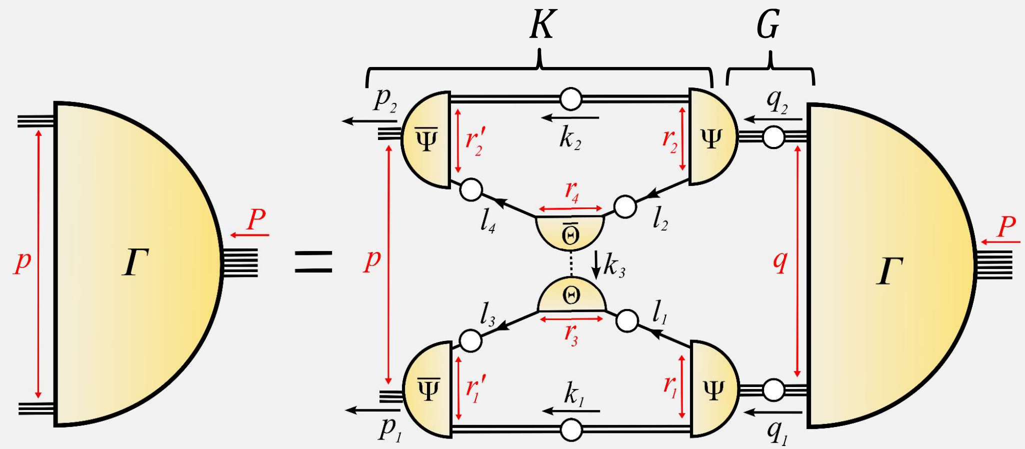


Fig.8 – Feynman diagram representing the deuteron BSE in the meson exchange model from the quark level.

- **Flavor component:** different flavor factor for charged and neutral pions;
- **Color component**;
- Meson amplitudes calculated using the **AWW interaction** with **rainbow ladder truncation**.

Pion-Nucleon Vertex

- The pion-nucleon vertex is approximated as:

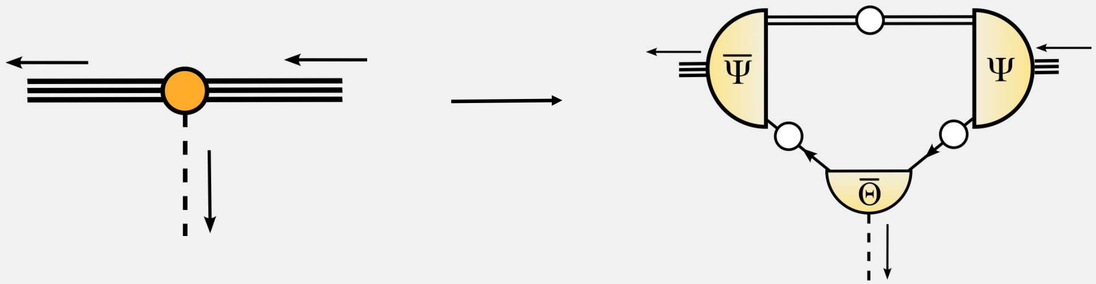


Fig.9 – Approximation of the pion-nucleon interaction vertex.

- The nucleon amplitudes are normalized to reproduce the pion-nucleon coupling constant on-shell:

$$\Psi^\mu \longrightarrow \Psi^\mu/\sqrt{N}\,, \qquad N = 0.61$$

Results: Quark Exchange

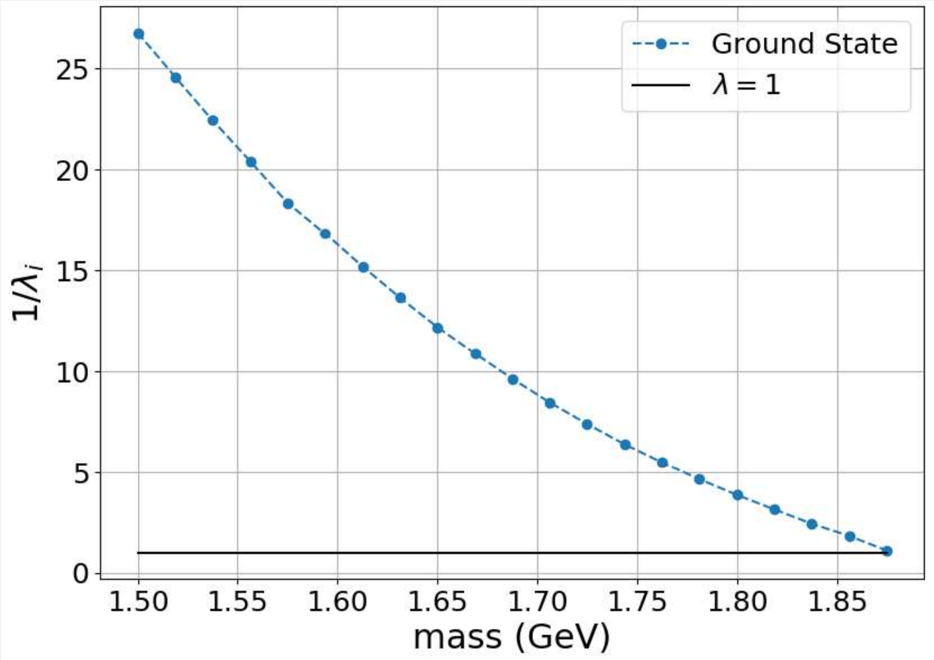


Fig.10 – Inverse of the ground state eigenvalue as a function of the deuteron mass for the quark exchange.

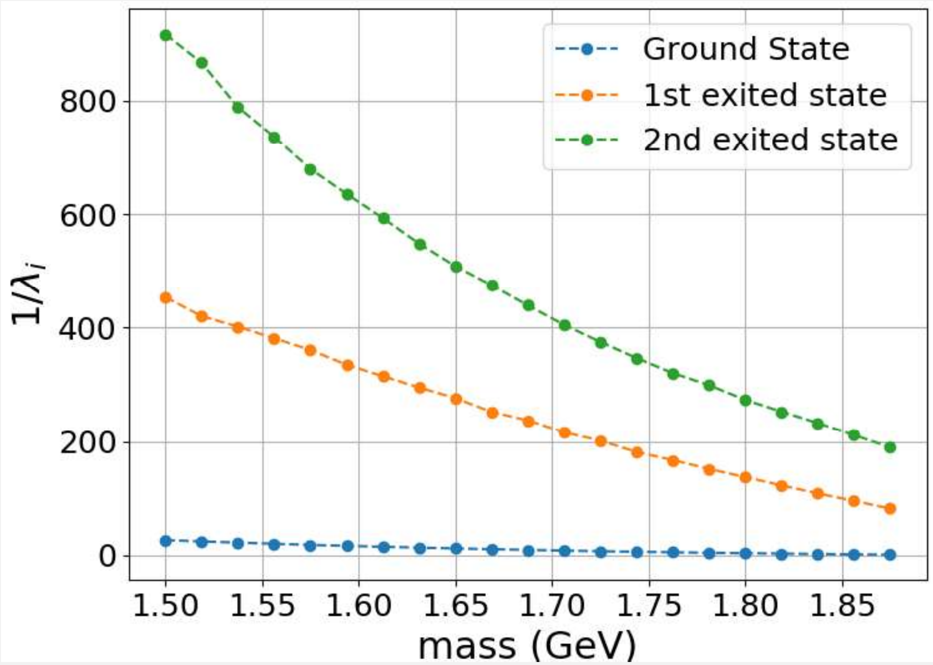


Fig.11 – Inverse of the eigenvalue as a function of the deuteron mass for the quark exchange. Ground, first exited and second excited states.

Results: Pion Exchange

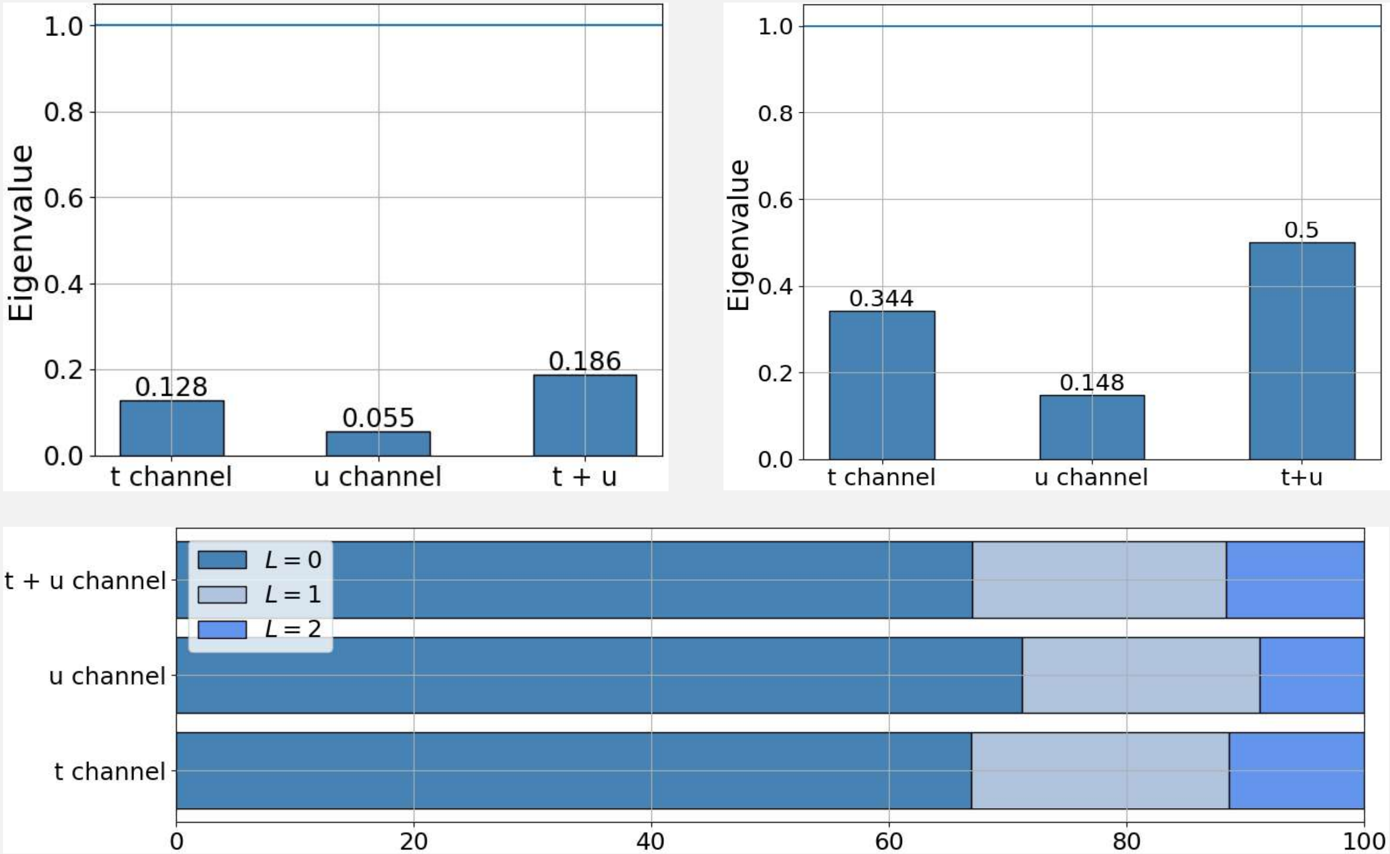


Fig.12 – Eigenvalue with (upper right) and without (upper left) normalization and orbital angular momentum contributions (bottom) of the pion exchange.

Results: Scalar Exchange

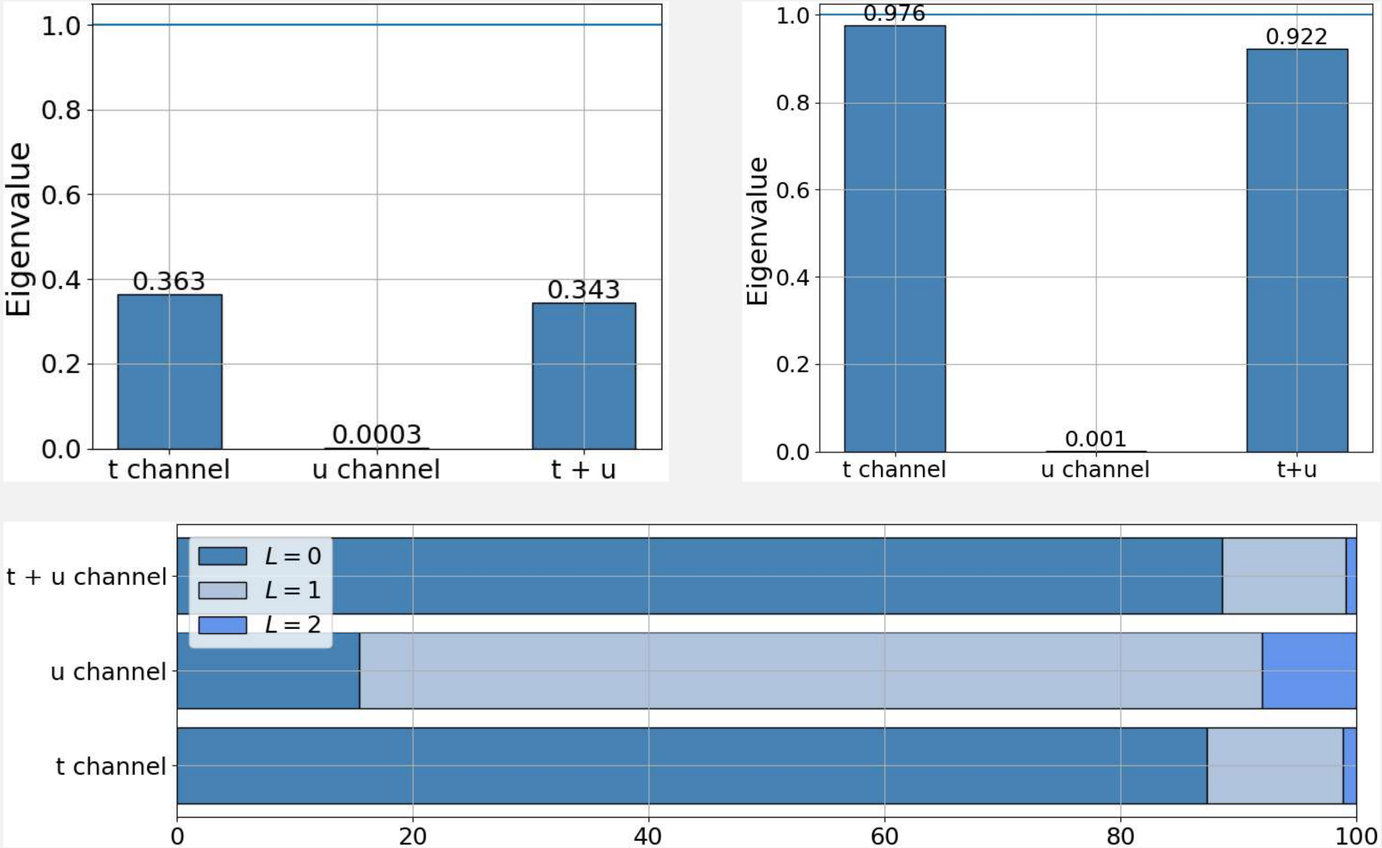


Fig.13 – Eigenvalue with (upper right) and without (upper left) normalization and orbital angular momentum contributions (bottom) of the scalar exchange.

Results: Individual Contributions

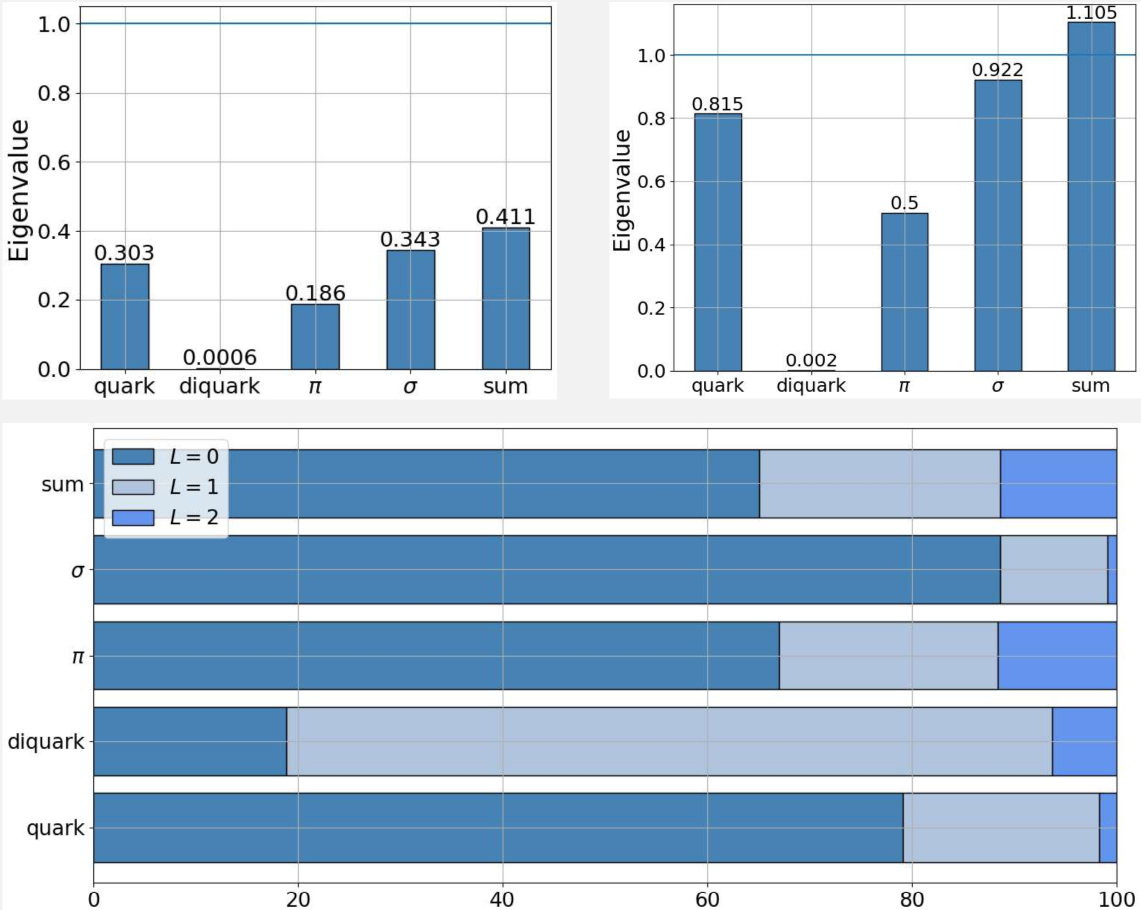


Fig.14 – Eigenvalue with (upper right) and without (upper left) normalization and orbital angular momentum contributions (bottom) of the scalar exchange.

Results: Different Sums

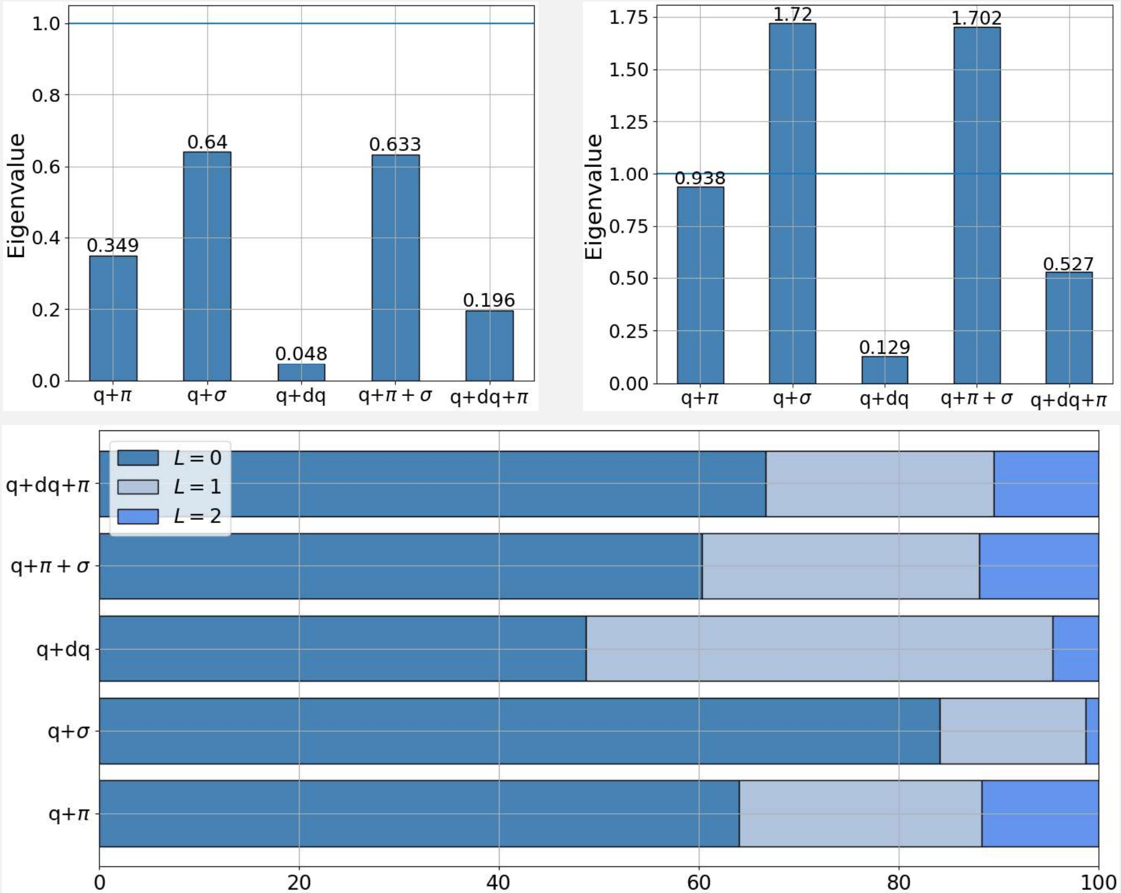


Fig.15 – Eigenvalue with (upper right) and without (upper left) normalization and orbital angular momentum contributions (bottom) of the scalar exchange.

Conclusions and Future Work

- **Conclusions:**

  1. P-wave contribution is significant in relativistic calculations, although it is forbidden in NR calculations;
  2. No exited states predicted;
  3. Scalar exchange is the dominant interaction (pion exchange dominant in NR calculations);
  4. Diquark exchange might be the origin of short-range repulsion.

- **Future work:**

  1. Substitute AWW interaction with Maris-Tandy interaction;
  2. Calculate scattering amplitudes and interaction potentials;
  3. Scalar meson might be a tetraquark.