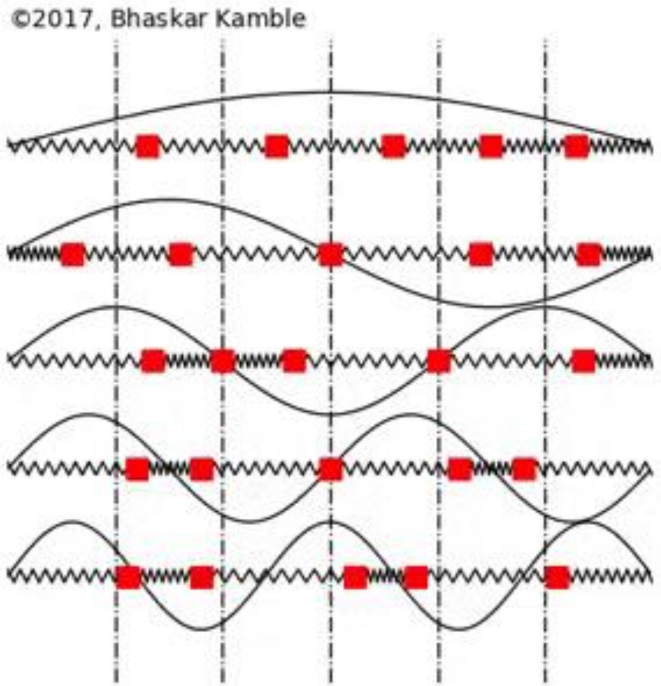
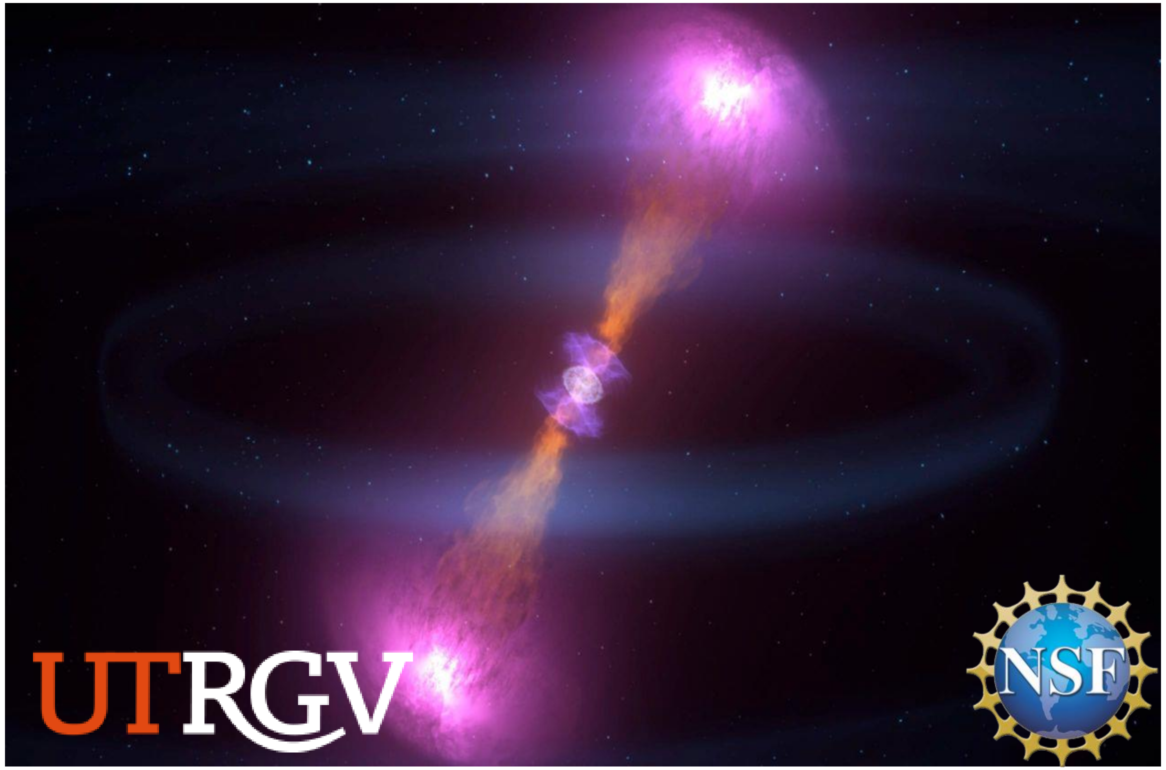
# Fluctuations in Dense Quark Matter Phases with Topology

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45th International School of Nuclear Physics: Nuclei in the Lab and in Stars
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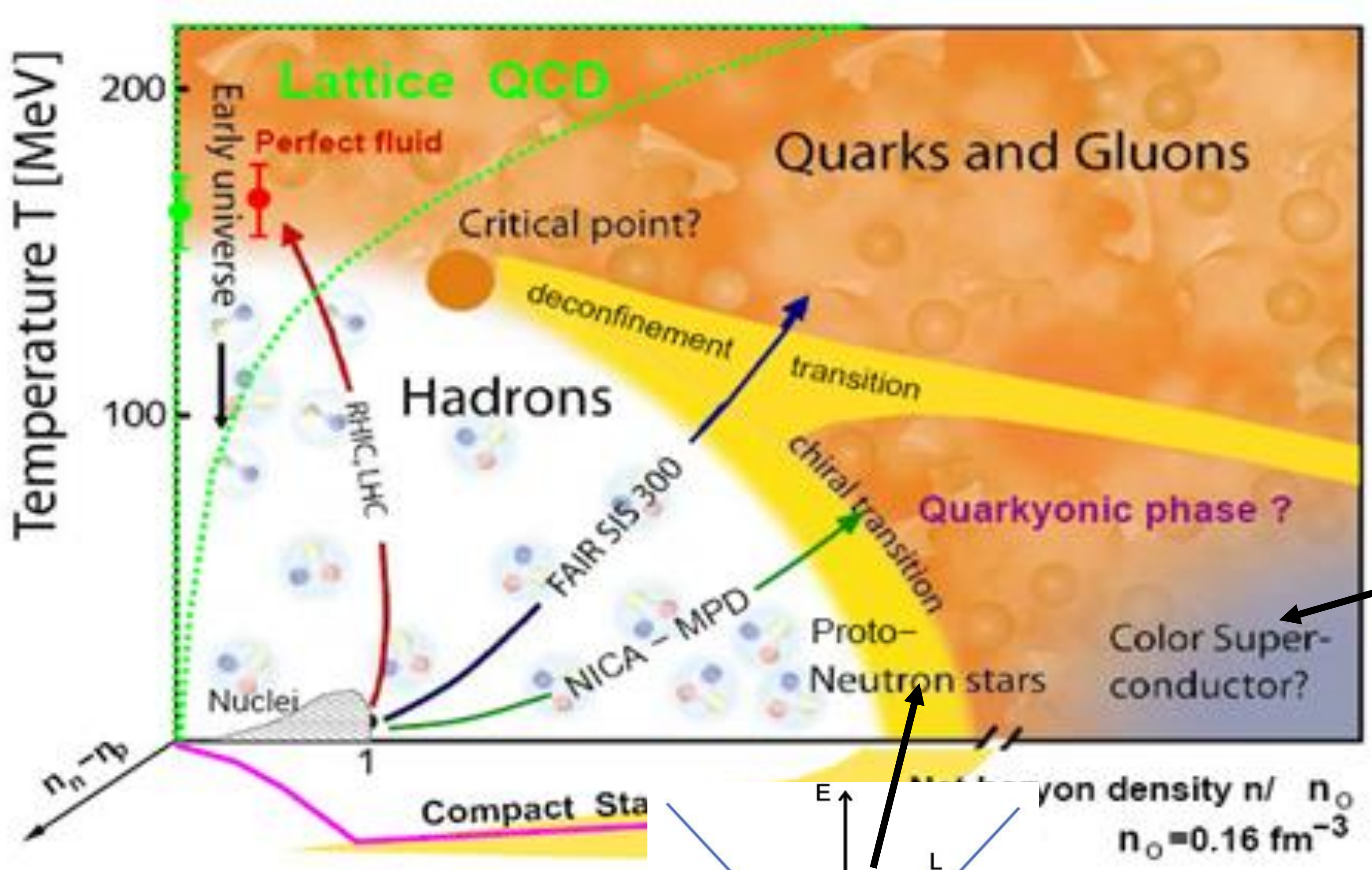
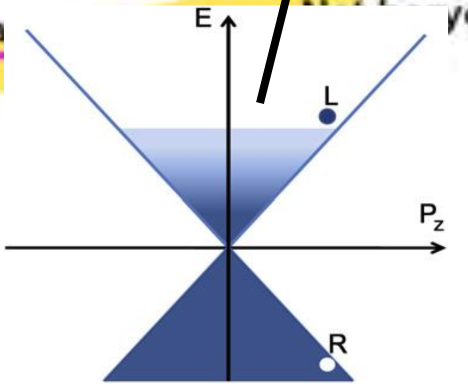
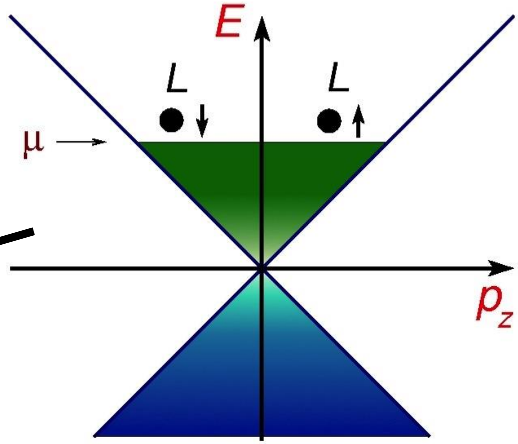
# Quark Phases in the QCD Phase Map





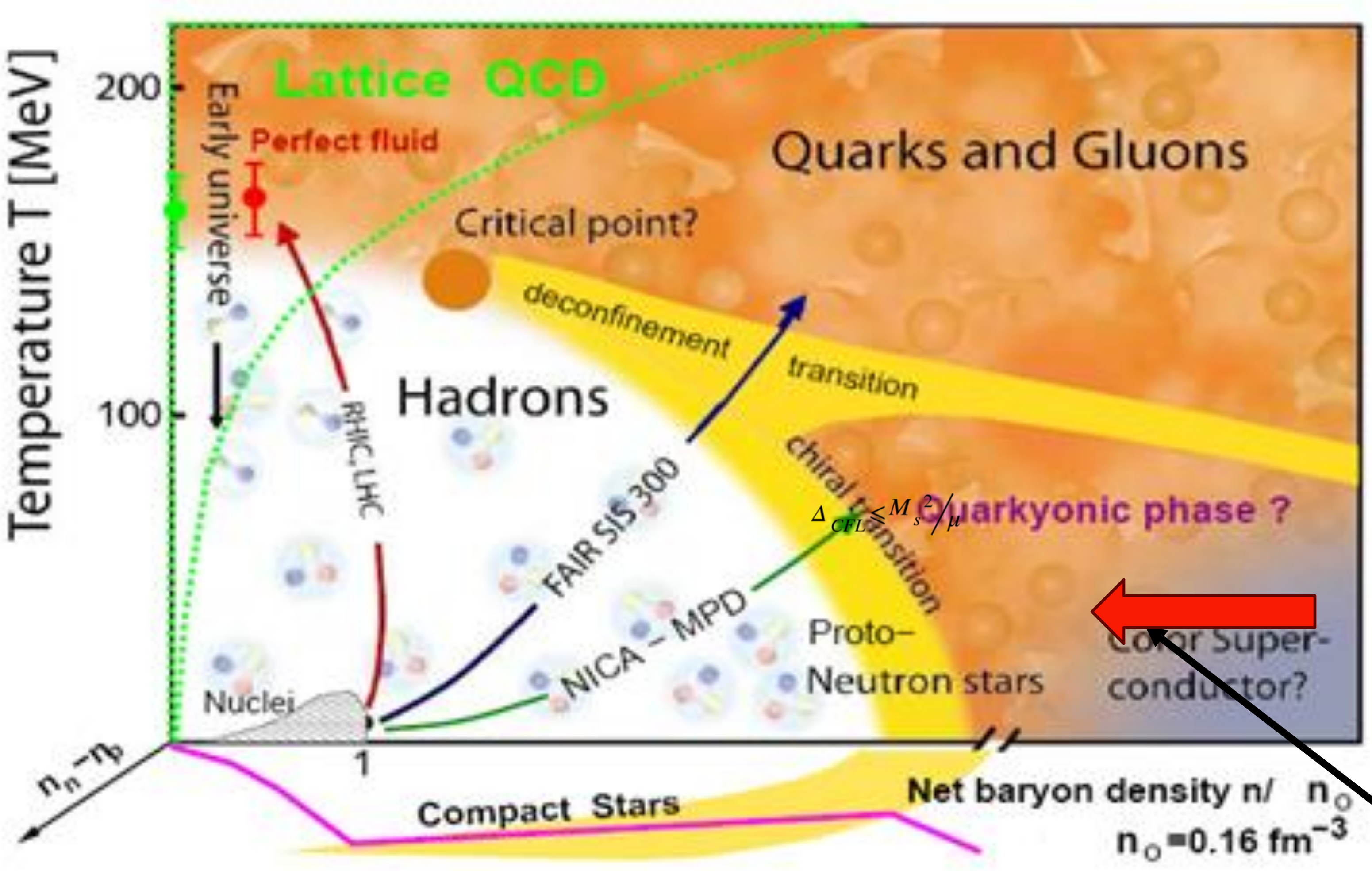

CS Cooper Pairing: Antisymmetric in flavor and color. Pair has zero net momentum.



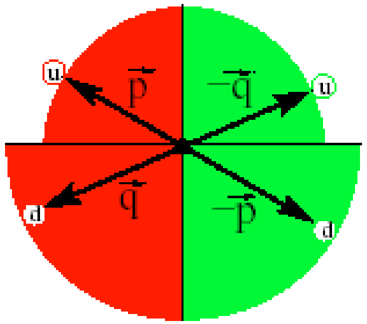

CFL Phase is favored at asymptotically large density

At chemical potentials smaller than the quark mass quark and antiquark pair with equal and opposite momentum and spin:
Homogeneous Chiral Condensate

# Coming from high densities




With decreasing density, the energy cost of forcing different species to pair with equal and opposite Fermi momentum eventually leads to gapless phases and chromomagnetic instabilities.

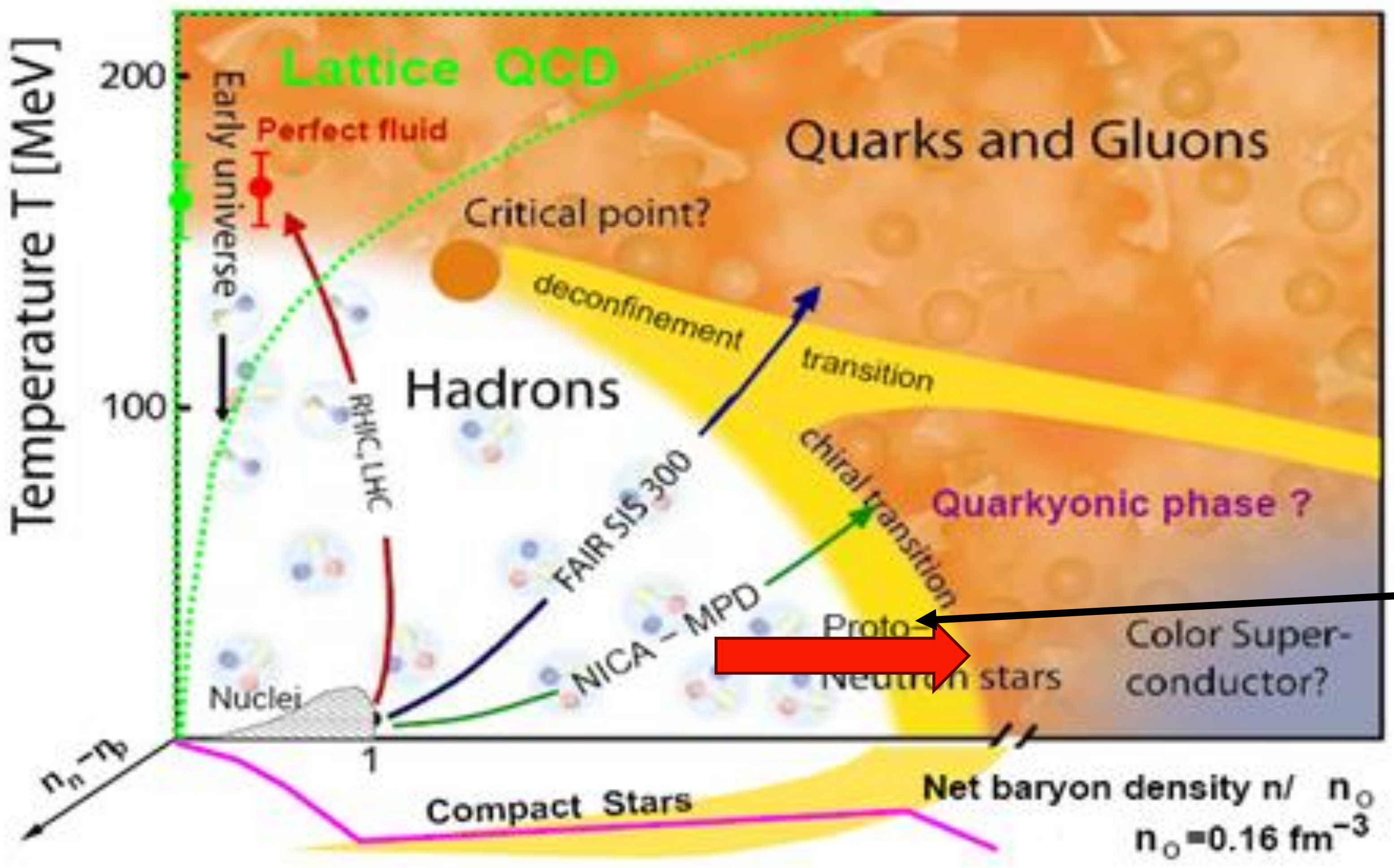


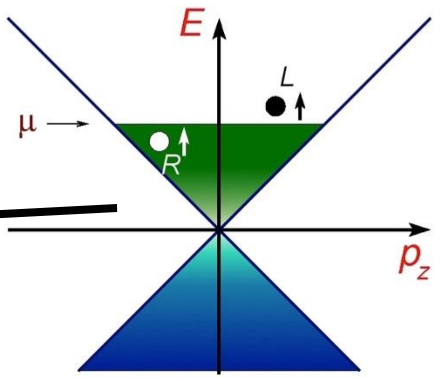

*Alford, Bowers, and Rajagopal, Phys. Rev. D 63, 2001; Bowers, and Rajagopal, Phys. Rev. D 66, 2002; Casalbuoni and Nardulli, Rev. Mod. Phys. 76, 2004*

Solution: Pairs with nonzero net momentum: Inhomogeneous Color Superconductivity

# or from lower densities





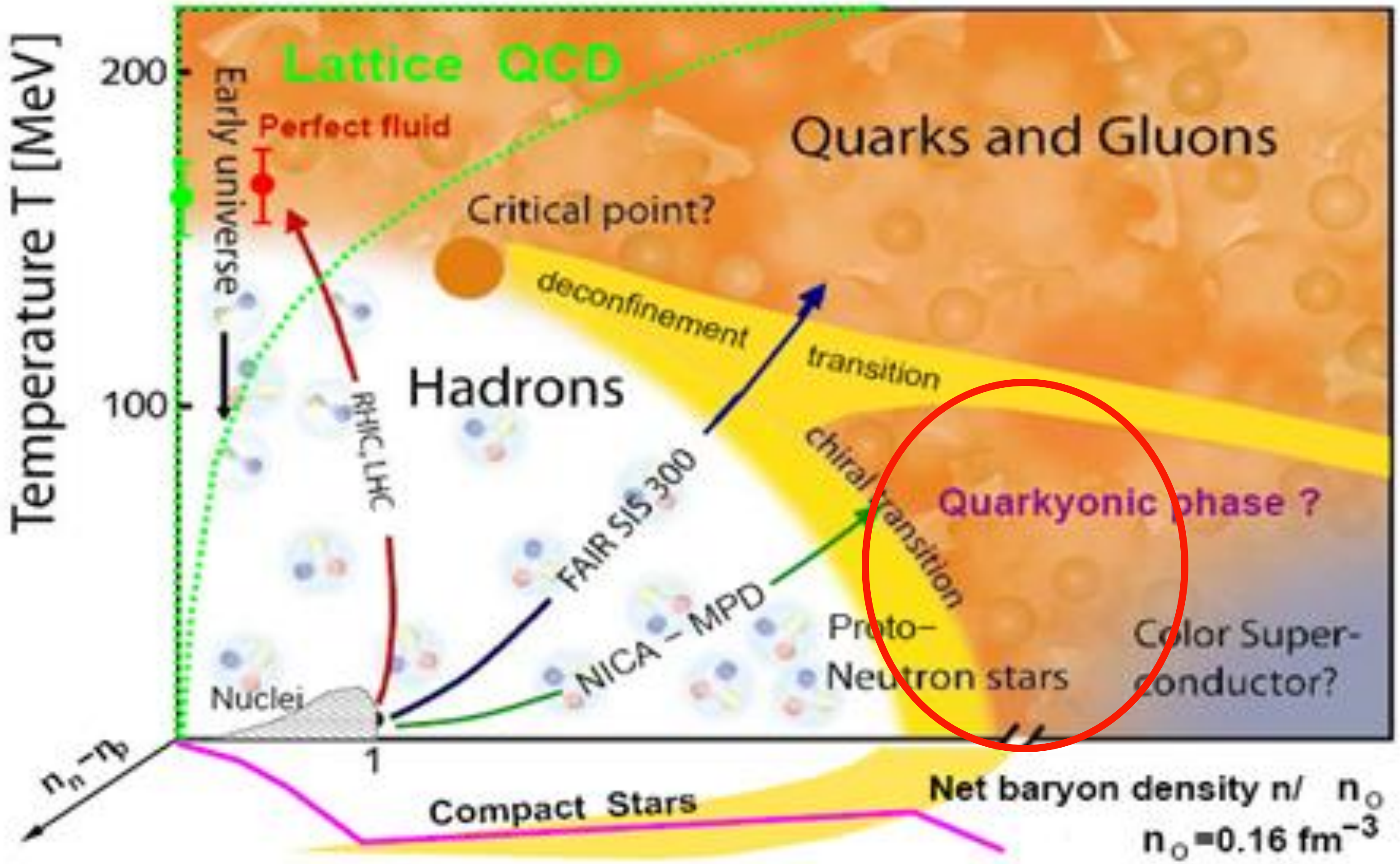

Particle-Antiparticle pairing becomes unfavored with increasing density but particles and holes with parallel momenta can pair with low energy cost:

Inhomogeneous Chiral Condensate

See references at
Buballa & Carignano, Prog. Part Nucl Phys 81, 2015

# I-Phases seem unavoidable at intermediate densities



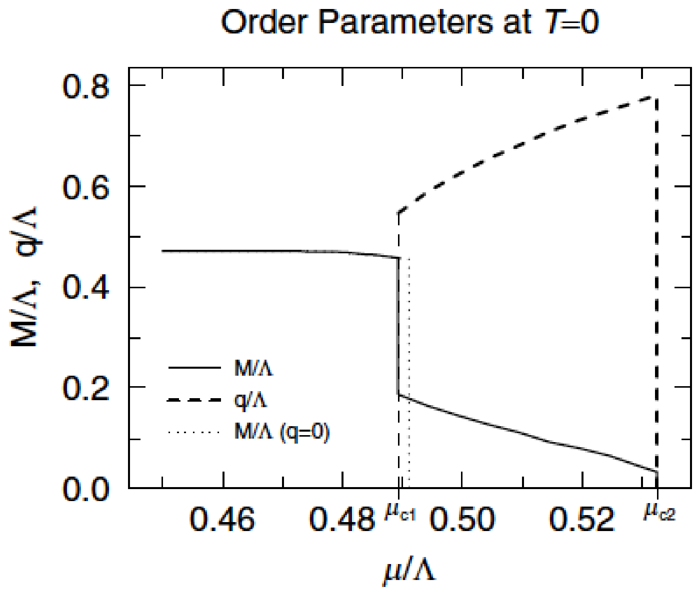


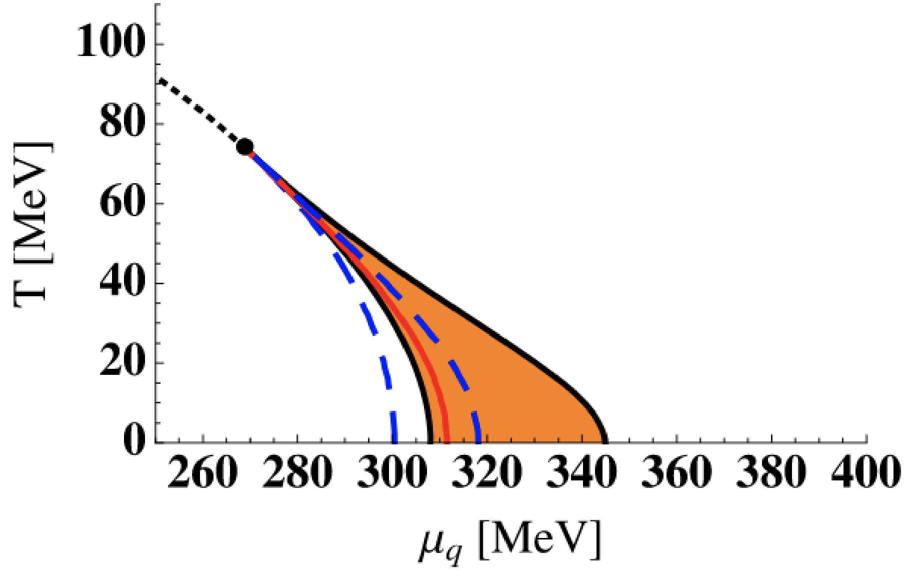

For reviews see:

Alford, Schmitt, Rajagopal, and Schafer, Rev. Mod. Phys. 80, 2008

Buballa & Carignano, Prog. Part Nucl Phys 81, 2015

# Single-Modulated Spatially Inhomogeneous Chiral Phases are favored over the restored phase at intermediate densities




## DCDW Condensate

$$\langle\bar\psi\psi\rangle = \Delta\cos q_\mu x^\mu, \qquad \langle\bar\psi i\tau_3\gamma_5\psi\rangle = \Delta\sin q_\mu x^\mu \qquad q^\mu = (0,0,0,q)$$

Nakano and Tatsumi, PRD 2005

## Kink Crystal Condensate

$$\langle\bar\psi\psi\rangle = M_{1\mathrm{D}}(z) = \sqrt{\nu} q \ \mathrm{sn}(qz,\nu)$$

Nickel, PRL, PRD 2009

# However, single-modulated phases in 3+1 d are unstable against thermal phonon fluctuations.

$$\langle u^2 \rangle = \frac{T}{4\pi v_z \zeta} \ln\left(\frac{v_z l_\perp}{\zeta}\right)$$

The phonon free energy has a soft mode in the transverse direction that leads to IR divergent fluctuations

$$\langle M \rangle \sim e^{-\langle u^2 \rangle / 2} \longrightarrow 0$$

The fluctuations wash out the long-range order at any finite T: Landau-Peierls Instability

True for any single-modulated phase (kink crystal, DCDW, etc.)

Still, a quasi-long-range order remains, characterized by a power-law decreasing of the order parameter correlation function.

$$\langle M(\boldsymbol{x}) M(0) \rangle \approx \sum_{n \geq 1} \frac{|\mathcal{M}_n|^2}{L} \begin{cases} 2\cos(nQz) \times |z|^{-n^2 \eta_c} & (\boldsymbol{x}_\perp = \boldsymbol{0}) \\ |x_\perp|^{-2n^2 \eta_c} & (z = 0) \end{cases} .$$

Similar to smectic liquid crystals.

Hidaka, Kamikado, Kanazawa & Noumi, PRD 92, 2015, 034003

Lee et al. PRD 92, 2015, 0304024

# Can a Magnetic Field Change this Outcome?



Magnetars:

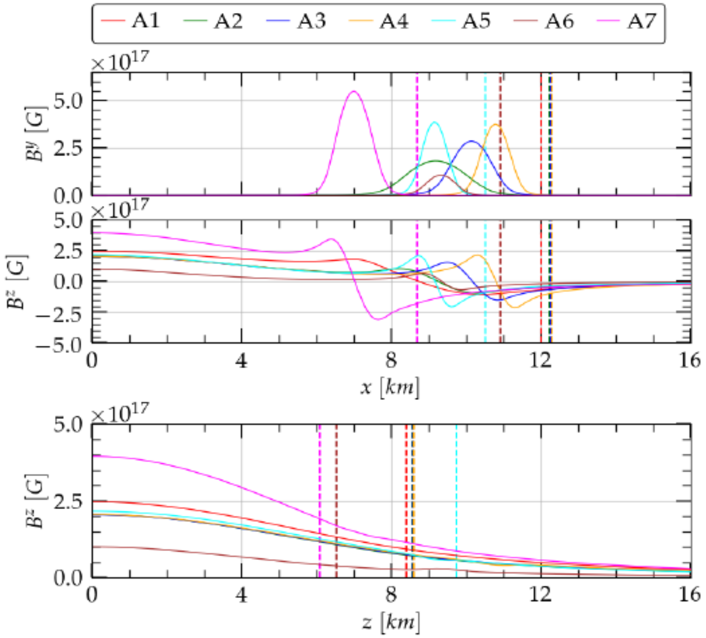
Core: B < 8 x$10^{18}$ G

Magnetars

surface: B ~ $10^{15}$G

core B ~ $10^{17}$- $10^{18}$ G

*Cardall, Prakash, and Lattimer, ApJ 554, 2001*




GRMHD simulations of magnetars' field evolution leads to several times $10^{17}$ G for $B_z$ at the core

*Tsokaros, Ruiz, Shapiro, & Uryu, PRL 128, 2022*

# Magnetic Dual Chiral Density Wave Model

2-flavor NJL model at finite baryon density and with magnetic field B|| $z$

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}[i\gamma^{\mu}(\partial_{\mu} + iQA_{\mu}) + \gamma_0\mu]\psi + G[(\bar{\psi}\psi)^2 + (\bar{\psi}i\tau\gamma_5\psi)^2].$$

It favors the formation of an inhomogeneous chiral condensate

$$\langle\bar{\psi}\psi\rangle = \Delta\cos q_{\mu}x^{\mu}, \qquad \langle\bar{\psi}i\tau_3\gamma_5\psi\rangle = \Delta\sin q_{\mu}x^{\mu} \qquad q^{\mu} = (0,0,0,q)$$

Mean-field Lagrangian

$$\mathcal{L}_{MF} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}[i\gamma^{\mu}(\partial_{\mu} - i\mu\delta_{\mu 0} + iQA_{\mu} - i\tau_3\gamma_5\delta_{\mu 3}\frac{q}{2}) - m]\psi - \frac{m^2}{4G}$$

$E_0 = \epsilon\sqrt{m^2 + k_3^2} + b, \quad \epsilon = \pm,$ $b=q/2$, LLL mode is Asymmetric!

$$E_k^{l>0} = \epsilon\sqrt{(\xi\sqrt{\Delta^2 + k_3^2} + q/2)^2 + 2e|B|l}, \quad \epsilon = \pm, \xi = \pm, l = 1,2,3,\ldots$$

Frolov, et al PRD82,'10
Tatsumi et al PLB743,'15

# Low-energy theory in MDCDW



The LLL antisymmetric spectrum leads to nontrivial topology that manifest through properties like an anomalous baryon number (proportional to an Atiyah-Singer invariant), axial electrodynamics, anomalous electric transport, etc.

MDCDW ansatz

$$M(z) = \sigma + i\pi = -2G\Delta e^{iqz} = me^{iqz}$$

$$\sigma = -2G\bar{\psi}\psi \qquad \pi = -2G\bar{\psi}i\gamma^5\tau_3\psi,$$

The low-energy theory for this order parameter is invariant under $U_V(1)xU_A(1)xSO(2)xR^3$

$$\mathcal{F} = a_{2,0}m^2 + b_{3,1}qm^2 + a_{4,0}m^4 + a_{4,2}q^2m^2 + b_{5,1}qm^4 + b_{5,3}q^3m^2 + a_{6,0}m^6 + a_{6,2}q^2m^4 + a_{6,4}q^4m^2,$$

Odd in b=q/2 terms come from the antisymmetric part of the LLL asymmetric modes

Ferrer & VI, NPB’2019, PRD’2020

# Spontaneous Breaking of Chiral and Translational Symmetries



$\bar{M}(z) = m e^{iqz}$ with m and q solutions of the stationary equations:

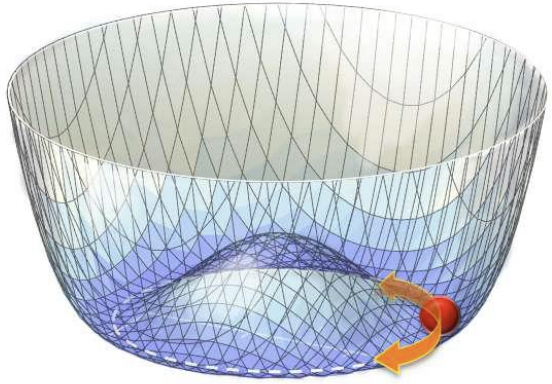
$$\partial\mathcal{F}/\partial m = 2m\{a_{2,0} + 2a_{4,0}m^2 + 3a_{6,0}m^4 + q^2[a_{4,2} + 2a_{6,2}m^2 + a_{6,4}q^2] + q[b_{3,1} + 2b_{5,1}m^2 + b_{5,3}q^2]\} = 0,$$

$$\partial\mathcal{F}/\partial q = m^2\{2q[a_{4,2} + a_{6,2}m^2 + 2a_{6,4}q^2] + b_{3,1} + b_{5,1}m^2 + 3b_{5,3}q^2\} = 0.$$

Symmetry breaks spontaneously to

$U_V(1) x SO(2) x R^2$

Fluctuations of the condensate come from two Goldstone Bosons: pions and phonons



# Low Energy Theory of Fluctuations in the MDCDW Phase

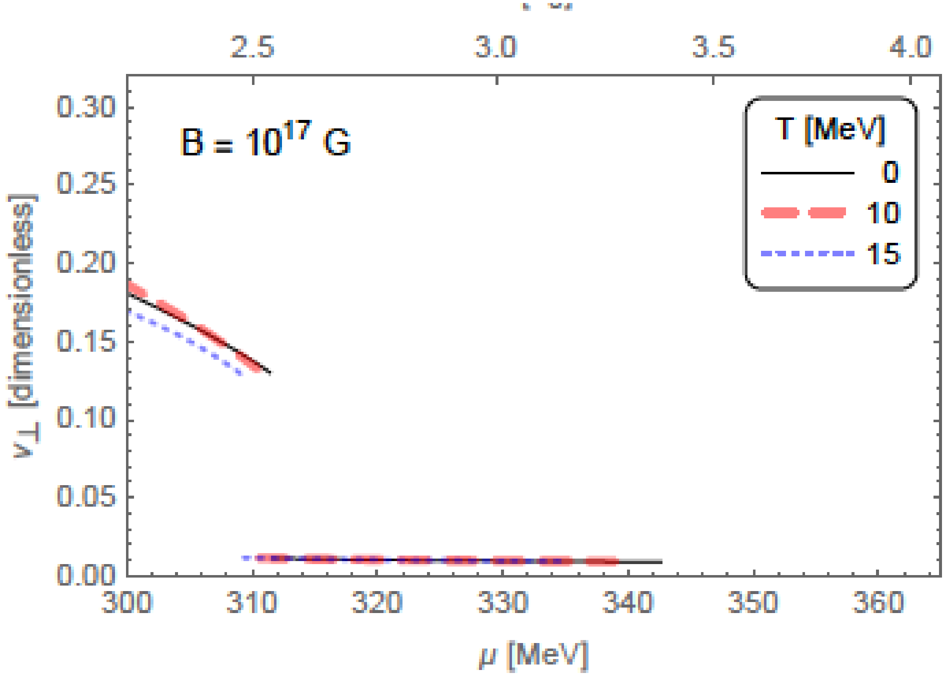


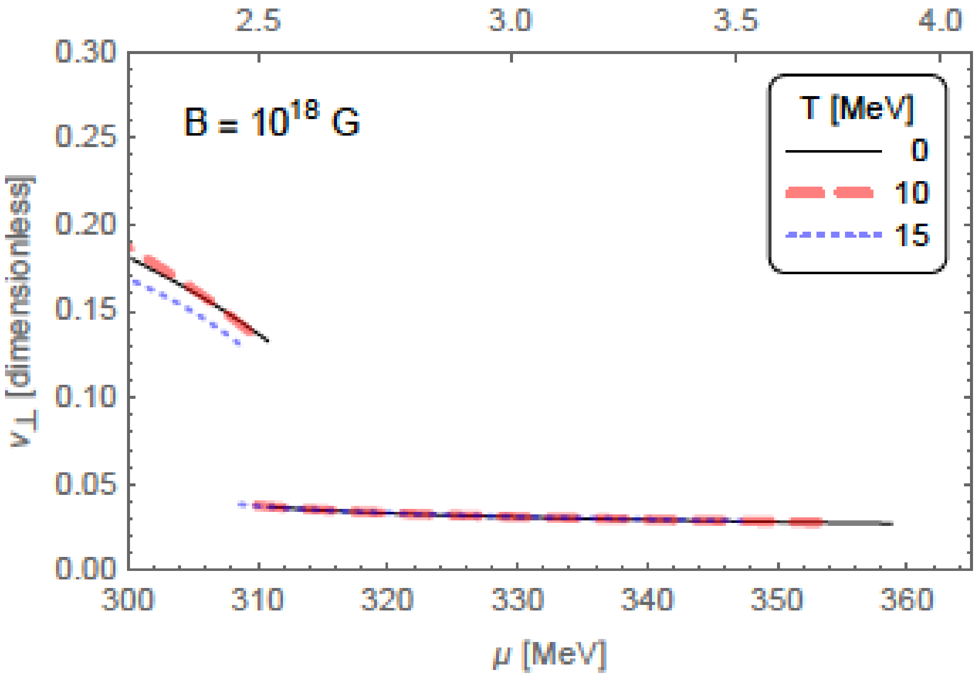
Chiral rotations and translations are locked, reducing the problem to a single Goldstone boson with low-energy theory:

$$\mathcal{F}[M(x)] = \mathcal{F}_0 + v_z^2(\partial_z\theta)^2 + v_\perp^2(\partial_\perp\theta)^2 + \zeta^2(\partial_z^2\theta + \partial_\perp^2\theta)^2 \qquad \theta = qmu$$

$$v_\perp^2 = -\frac{b_{3,1}}{2q} - b_{5,1}\frac{m^2}{2q} - \frac{1}{2}qb_{5,3} \longrightarrow \boxed{\langle q^2u^2\rangle/2 \simeq \frac{T}{8\pi m\sqrt{v_z^2 v_\perp^2}}.} \longrightarrow$$

No Landau-Peierls instability!




**Long-range order is robust even at high T**

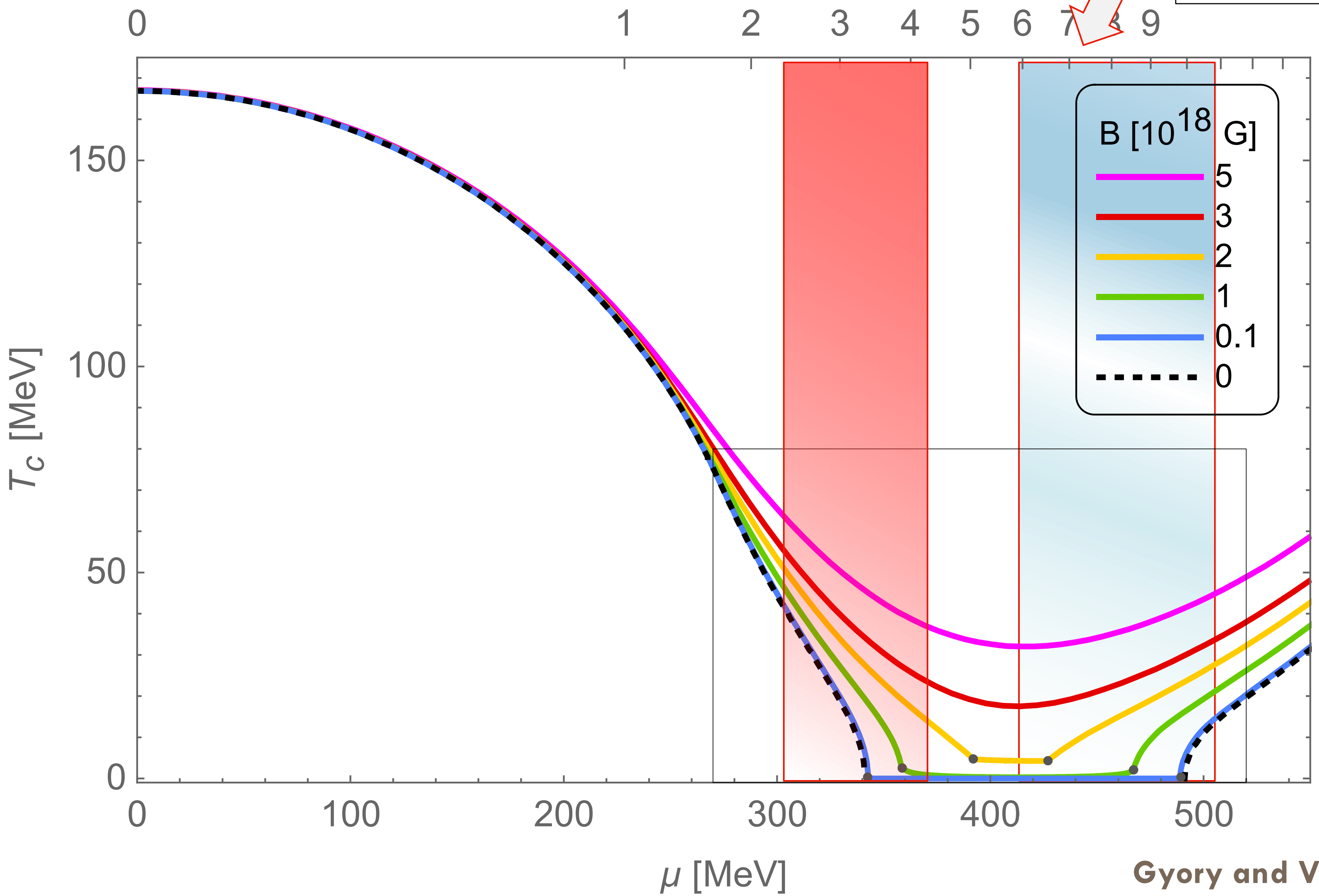
Ferrer & VI, PRD 102, 2020;
Gyory, Ferrer, & VI, PRD' 2024

# MDCDW Robustness and Relevance for NS



**Relevant for old Magnetars**

**Can survive large T. Relevant for NSB mergers?**




Gyory and VI, PRD 106, 016011, 2022

# Does a magnetic field cure the LP instability in all single-modulated phases?

$$
\begin{aligned}
\Omega^{(6)} &= a_{2,0}|M|^2 - i\frac{b_{3,1}}{2}[M^*(\hat{B}\cdot\nabla M) - (\hat{B}\cdot\nabla M^*)M] + a_{4,0}|M|^4 + a_{4,2}|\nabla M|^2 \\
&- i\frac{b_{5,1}}{2}|M|^2[M^*(\hat{B}\cdot\nabla M) - (\hat{B}\cdot\nabla M^*)M] + \frac{ib_{5,3}}{2}[(\nabla^2 M^*)\hat{B}\cdot\nabla M - \hat{B}\cdot\nabla M^*(\nabla^2 M)] \\
&+ a_{6,0}|M|^6 + a_{6,2}|M|^2|\nabla M|^2 + a_{6,4}|\nabla^2 M|^2,
\end{aligned}
$$

$$M(x) = \sigma(x) + i\pi(x)$$

1. One can show that without the b-terms, $v_1^2$ vanishes. Hence, B is needed to cure the LP instability.
2. Omega must be invariant under all the original symmetries of the theory, including time-inversion. However, for the b-structures to be T-invariant, the order parameter has to break T.
3. The b-terms are forbidden if the fermion spectrum is symmetric. They come only from the asymmetric spectrum of the LLL, which, in turn, signals the nontrivial topology of the LLL fermion dynamics.

Gyory, Ferrer, & VI, PRD' 2024

# Conclusions:

1. A magnetic field is necessary but not sufficient for the LP instability to be avoided. The system needs B and a T-breaking chiral condensate.
2. One can use these symmetry arguments to identify which chiral single-modulated phases are robust against phonon fluctuations. For instance, a phase with just will exhibit LP instabili $\sigma = -2G\bar{\psi}\psi$ agnetic field because σ does not break T (kink crystal phase will remain quasi-long range only)
3. The nontrivial topology in the MDCDW is a consequence of the asymmetric spectrum of the LLL, which in turn leads to the b-terms. On the other hand, these terms can only come from the coexistence of B and a T-breaking condensate. Does this reflect a deeper, more general connection valid beyond the MDCDW model?

## Final Remarks



1. Single-modulated chiral condensates found within NJL models are sensible to regularization. A new approach uses SDE calculations to investigate the stability of homogenous phases in QCD against inhomogeneous fluctuations. These analyses need to be expanded to explore inhomogeneous phases in a magnetic field with T-symmetry breaking.

2. We note that the MDCDW has been investigated using two independent regularizations that produce the exact same physics.