Lifetimes of three-body resonances: dimensionality and mass ratio

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„Few-body systems in physics“ Laboratory

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INTERNATIONAL SCHOOL OF NUCLEAR PHYSICS
16-22.09.2024

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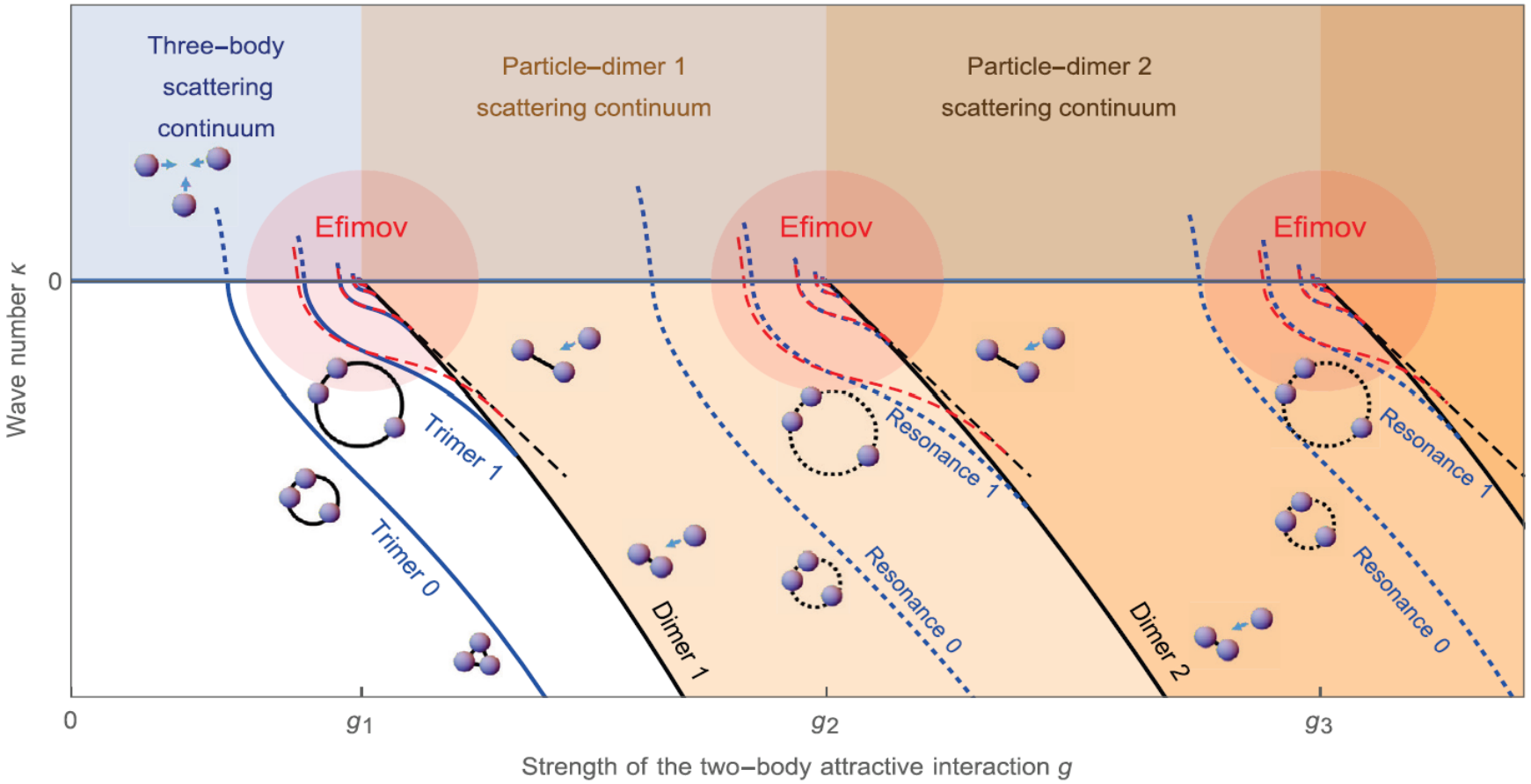
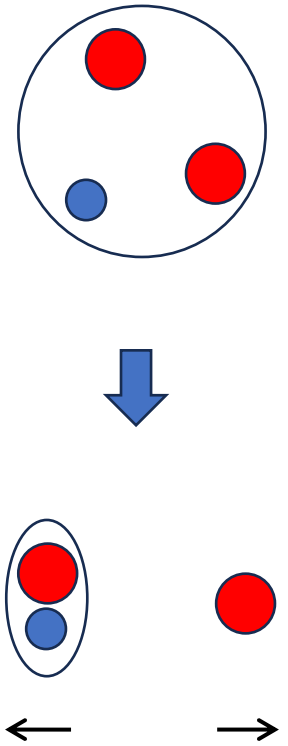
Three-body resonances





Naidon, Endo, *Rep. Prog. Phys.* **80** 056001 (2017)

complex energy

$$E = E_R - \frac{i}{2}\Gamma$$

Lifetime

$$\tau = \frac{\hbar}{\Gamma} = \frac{-\hbar}{2\,Im(E)}$$

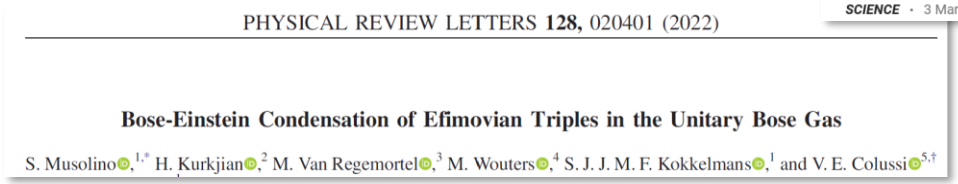
Three-body resonances

- **Ultracold atoms**


Control of reactive collisions by quantum interference

HYUNGMOK SON, JULIANA J. PARK, YU-KUN LU, ALAN O. JAMISON, TIJS KARMAN, AND WOLFGANG KETTERLE Authors Info & Affiliations

SCIENCE · 3 Mar 2022 · Vol 375, Issue 6584 · pp. 1006-1010 · DOI: 10.1126/science.abl7257

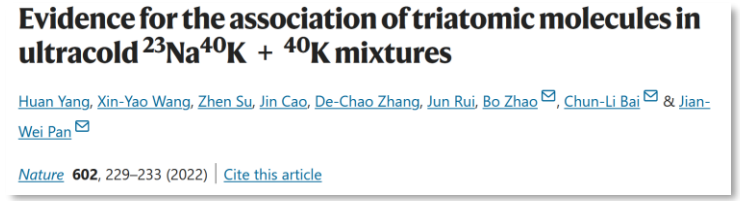

PHYSICAL REVIEW LETTERS 128, 020401 (2022)

Bose-Einstein Condensation of Efimovian Triples in the Unitary Bose Gas

S. Musolino, H. Kurkjian, M. Van Regemortel, M. Wouters, S. J. J. M. F. Kokkelmans, and V. E. Colussi


Reshaped three-body interactions and the observation of an Efimov state in the continuum

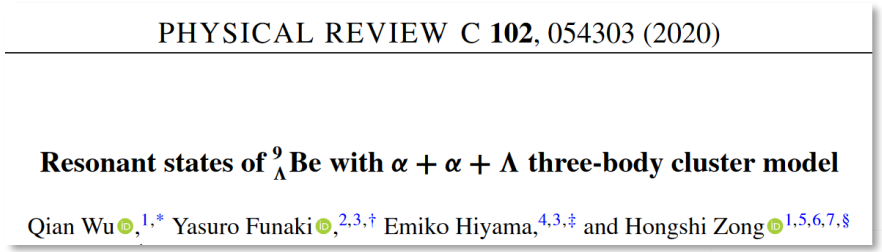
Yaakov Yudkin, Roy Elbaz, José P. D'Incao, Paul S. Julienne & Lev Khaykovich


Evidence for the association of triatomic molecules in ultracold $^{23}Na^{40}K + {}^{40}K$ mixtures

Huan Yang, Xin-Yao Wang, Zhen Su, Jin Cao, De-Chao Zhang, Jun Rui, Bo Zhao, Chun-Li Bai & Jian-Wei Pan

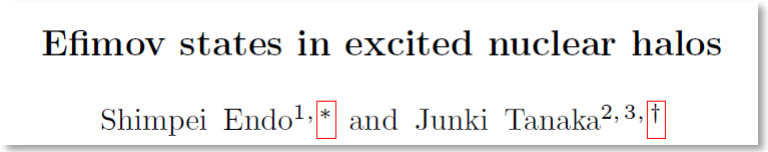
Nature **602**, 229–233 (2022) | Cite this article

- **Nuclear physics**


PHYSICAL REVIEW C 102, 054303 (2020)

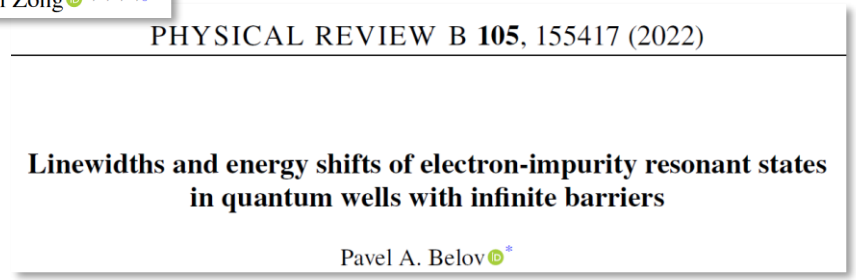
Resonant states of $^{9}_{\Lambda}$Be with $\alpha + \alpha + \Lambda$ three-body cluster model

Qian Wu, Yasuro Funaki, Emiko Hiyama, and Hongshi Zong


Efimov states in excited nuclear halos

Shimpei Endo[1,*] and Junki Tanaka[2,3,†]

- **Excitons in semiconductors, …**

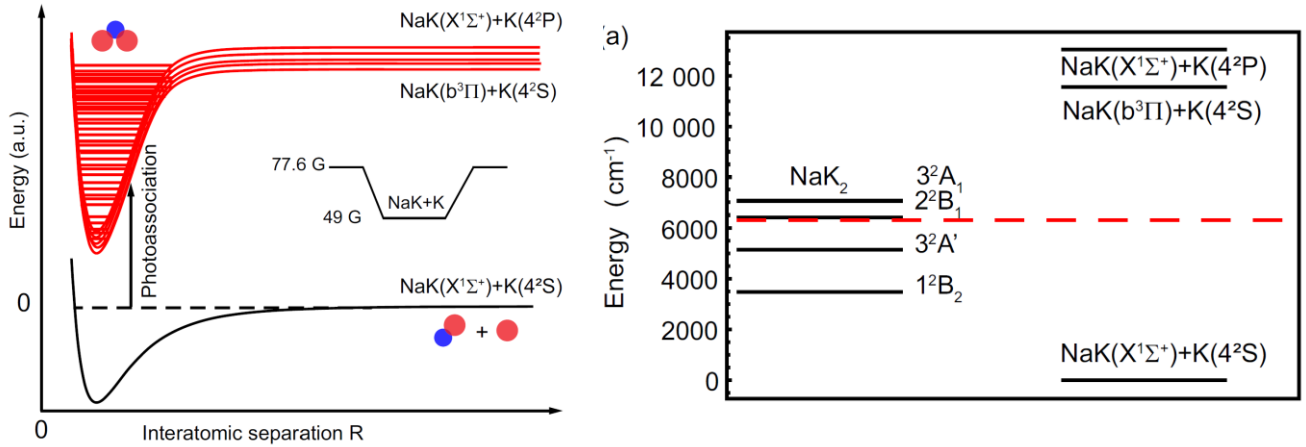

PHYSICAL REVIEW B 105, 155417 (2022)

Linewidths and energy shifts of electron-impurity resonant states in quantum wells with infinite barriers

Pavel A. Belov

Realization in ultracold atoms

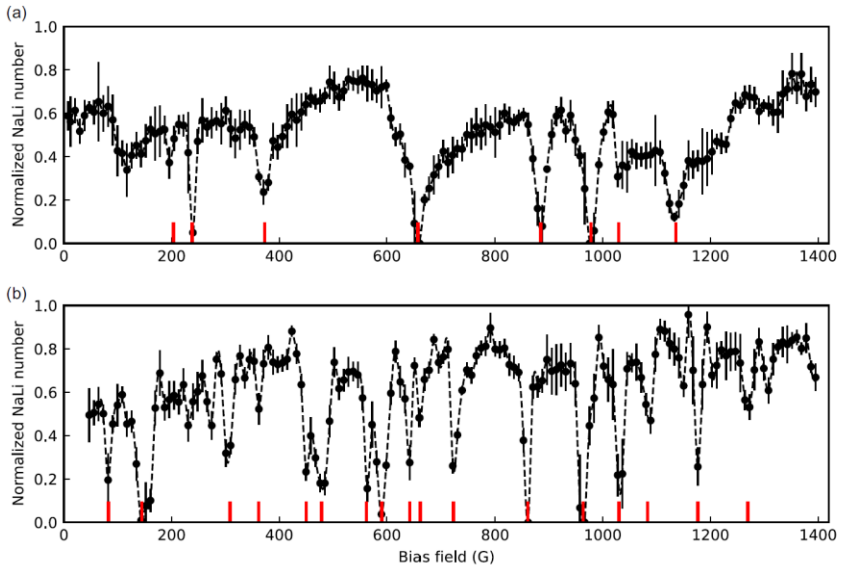
Photoassociation of three-body resonances



Cao et al., PRL **132** 093403 (2024)

ultracold: µK … nK

Controlled collision of cold atoms and cold molecules



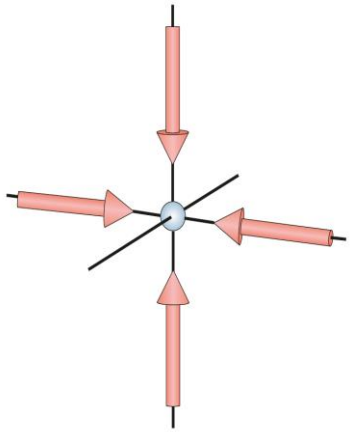
Park et al., **13** 031018 (2023)

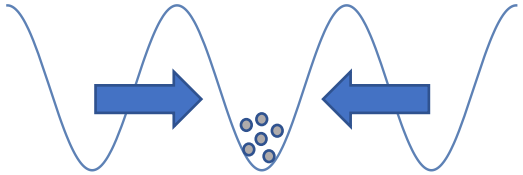
Dimensionality and mass ratio

Lifetime of three-body resonances

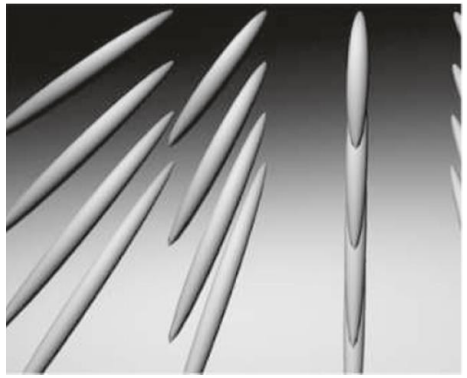
Dimensionality

Realization of low-dimensional geometries (quasi-1D)





$V_{dipole} \propto |E(r)|^2$



Bloch, *Nat. Phys.* **1**, 23 (2005)

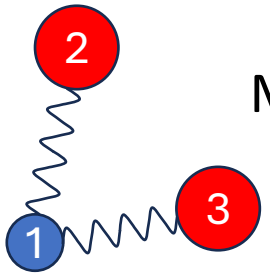
Mass ratio

Different atomic species:
- Li-6
- Na-23
- K-40
- Rb-87
- Cs-133
- …

Using two atomic species:
Mass ratios between 1/22 … 22

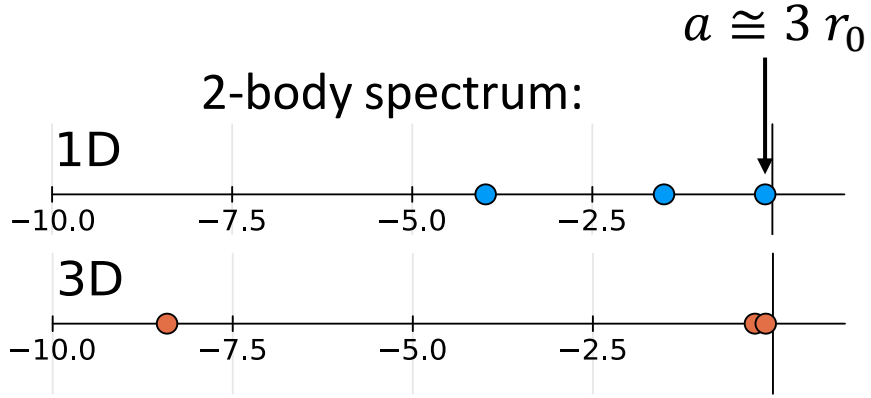
The three-body system

- Two identical bosons (2,3), one distinguishable (1)



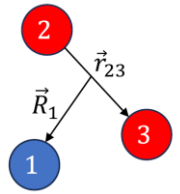
Mass ratio $\beta = \frac{m_B}{m_X}$

- No interaction between identical particles



- Gaussian pair-interactions $V_{ij}(r) = v_0 e^{-(r_{ij}/r_0)^2}$

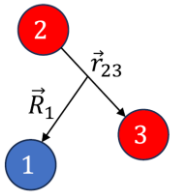
$$\left[-\frac{\hbar^2}{2\mu_{ij}}\nabla^2_{\vec{r}_{ij}} - \frac{\hbar^2}{2\mu_k}\nabla^2_{\vec{R}_k} + V(r_{12}) + V(r_{31})\right]\Psi(\vec{r}_{ij}, \vec{R}_k) = E\Psi(\vec{r}_{ij}, \vec{R}_k)$$



Method

Gaussian Expansion Method (GEM)

$$|\Psi\rangle = |\Phi^{(1)}\rangle + |\Phi^{(2)}\rangle + |\Phi^{(3)}\rangle$$



$$\Phi^{(c)}(\vec{r}_c, \vec{R}_c) = \sum_i \phi^{(c)}_{n_i,l_i}(r_c)\psi^{(c)}_{N_i,L_i}(R_c) \times \left[Y_{l_i,m_i}(\hat{\vec{r}}_c)Y_{L_i,M_i}(\hat{\vec{R}}_c)\right]_{J\mathcal{M}}$$

$$\phi_{n,l}(r) = N_{n,l} r^l e^{-\nu_n r^2}$$

$$\psi_{N,L}(R) = N_{N,L} R^L e^{-\lambda_N R^2}$$

Hiyama et al, *Prog. Partcl. Nucl. Phys.* **351**, 223 (2003)

3D: only s-wave ($l = L = 0$)

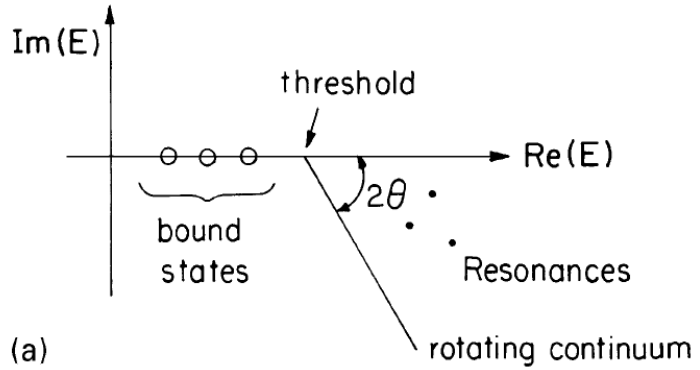
1D: no $Y_{l,m}$; $l, L \in \{0,1\}$

Complex Rotation Method (CSM)

$$r \to r\exp(i\theta)$$

$$H(0) = T + V(r)$$
$$H(\theta) = T\exp(-2i\theta) + V(r\exp(i\theta))$$

→ Resonances can be found via bound-state methods

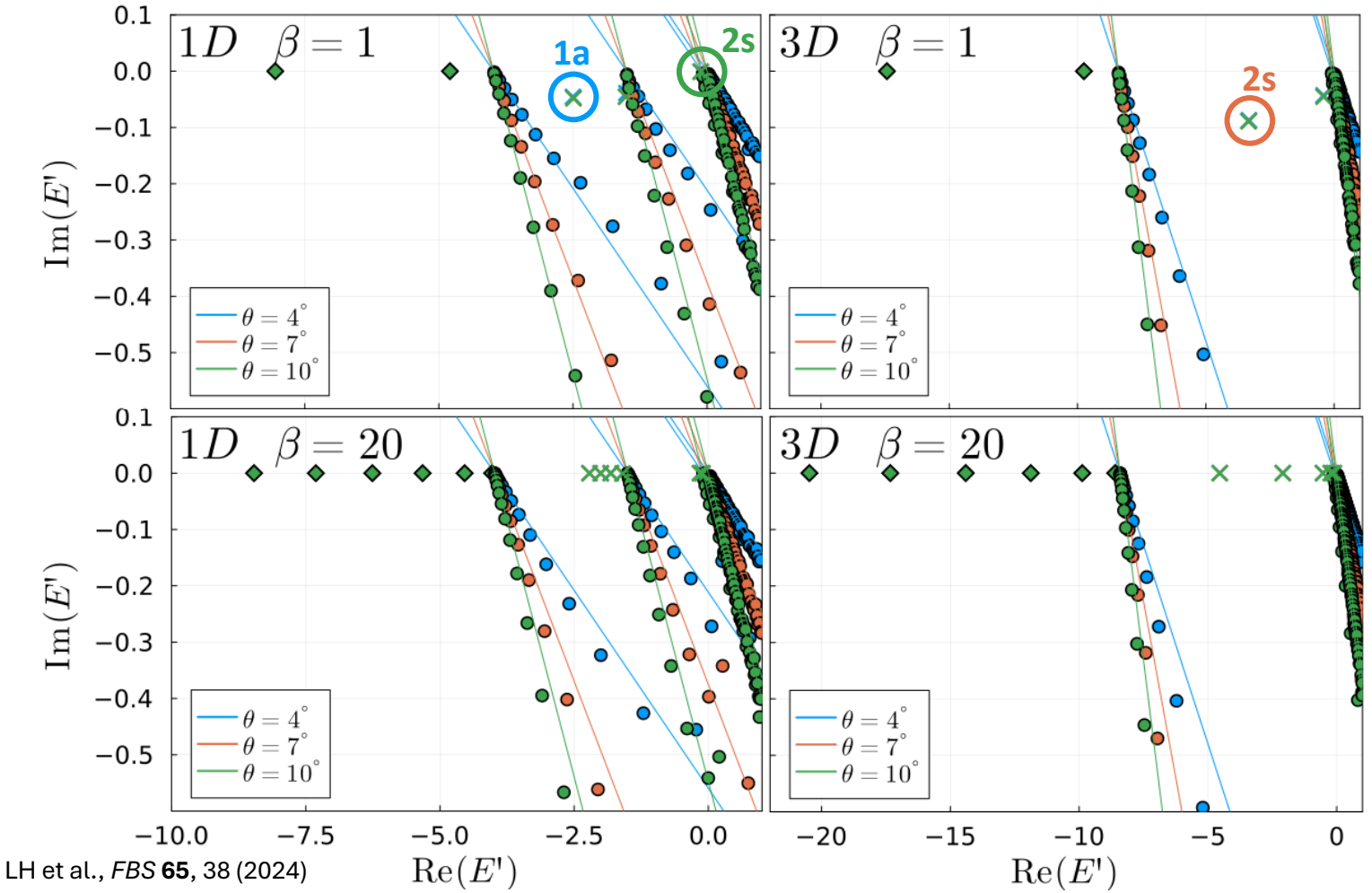


Moiseyev, *Phys. Rep.* **302**, 211 (1998)

Complex-rotated spectra

$\frac{M}{m} = 1$

$\frac{M}{m} = 20$



LH et al., *FBS* **65**, 38 (2024)

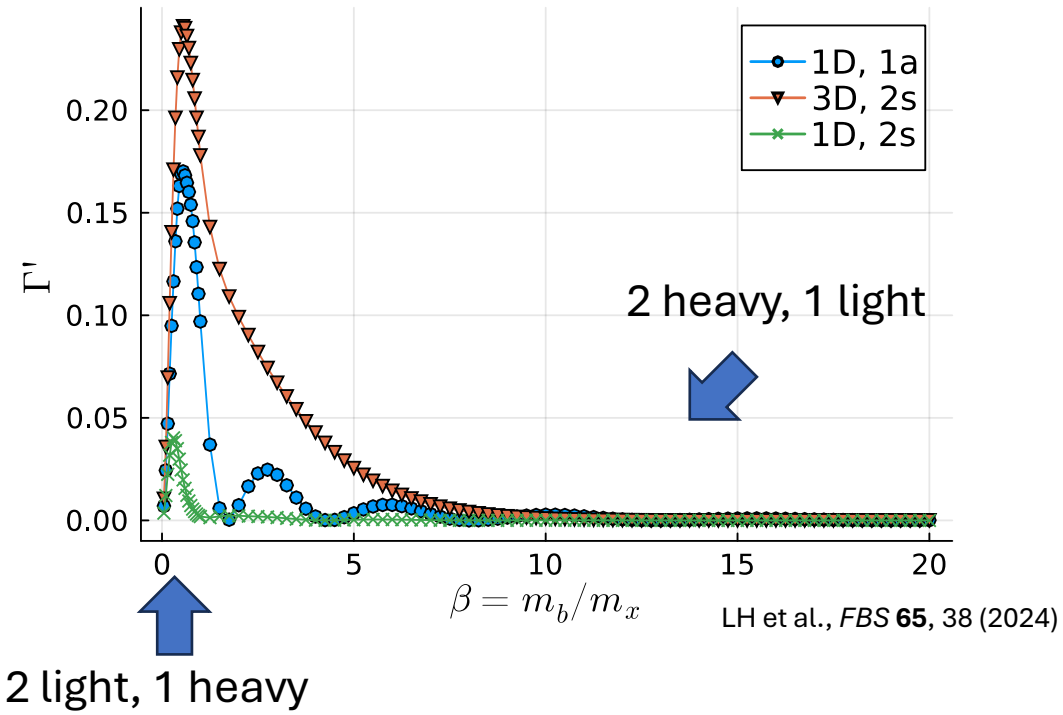
- Bound state
- Resonance
- Continuum

$Im(E') \simeq 0 \pm 10^{-5}$

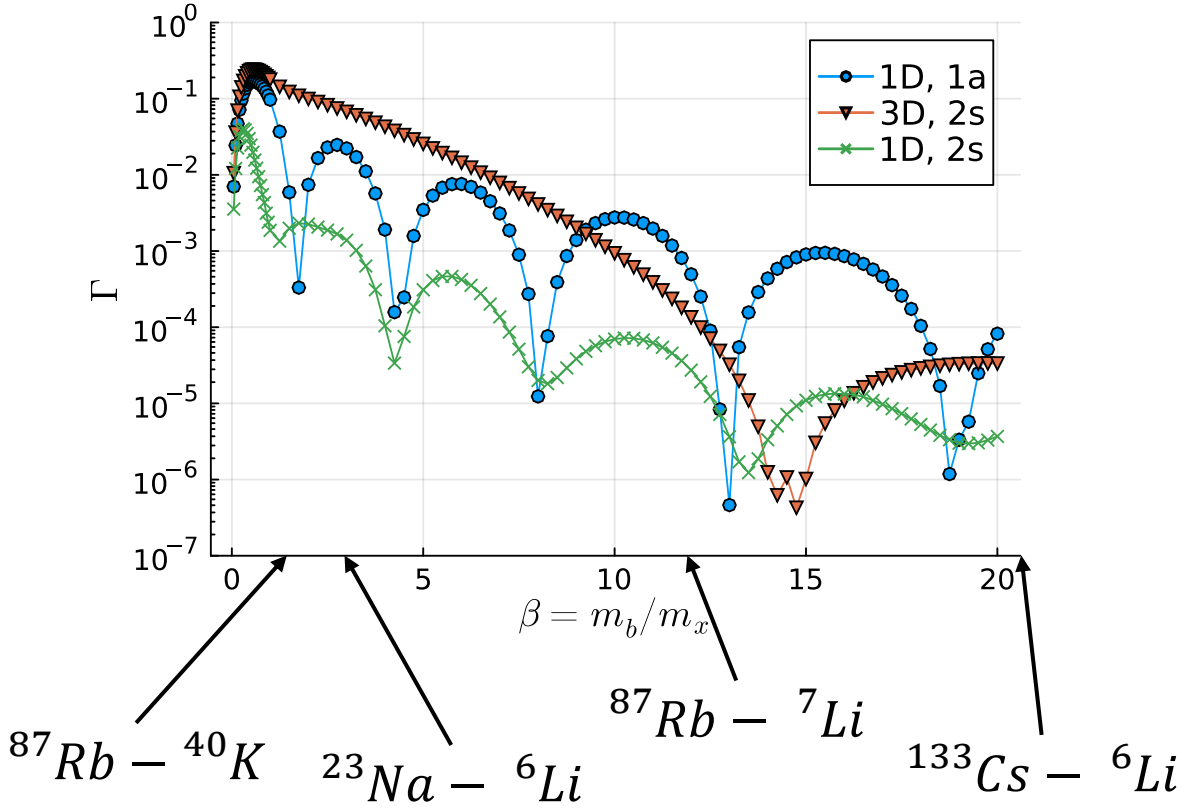
➢ Similar result for both 1D & 3D

➢ Γ decreases with β

Width vs mass ratio



LH et al., *FBS* **65**, 38 (2024)



- Damped-oscillatory behavior
- Specific points of stability (BIC)

Summary & Outlook

Summary:

- Γ shows damped-oscillatory dependence on the mass-ratio
- Specific mass-ratios with exceptional stability (BIC)

Outlook:

- Validity of theory: deep resonances (Cao et al., *PRL* **132**, 093403 (2024))?
- Universality? Other systems?

Few-Body Syst **65**, 38 (2024)
arXiv:2312.04080

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