i) Sterile Neutrinos, ii) Quantum Entanglement in Collective Neutrino Oscillations

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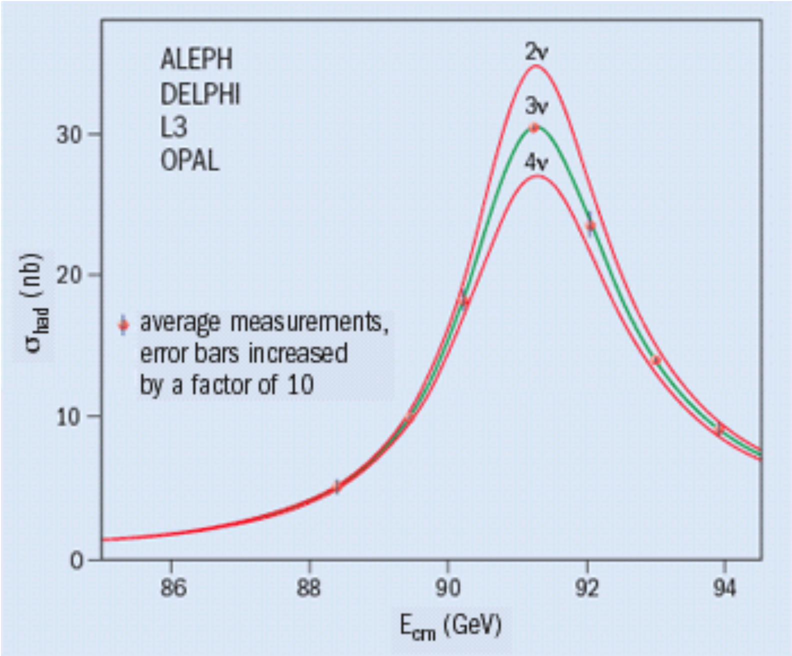


- From Z decays (invisible decay width) we know that there are only three flavors of neutrinos which couple to Z.
- There is overwhelming experimental evidence that neutrinos change flavor after traveling a finite distance.
- The only consistent explanation of all those experiments is that neutrino flavor states are mixtures of mass eigenstates.
- The probability of observing a given flavor depends on the mixing angles and mass-square differences.
- Solar, atmospheric, and reactor neutrino experiments measured the two mass differences of three neutrinos to be 7.5×10^{-5} eV^2 and 2.5×10^{-3} eV^2.

- There are experimental neutrino anomalies that are not resolved. Among those are

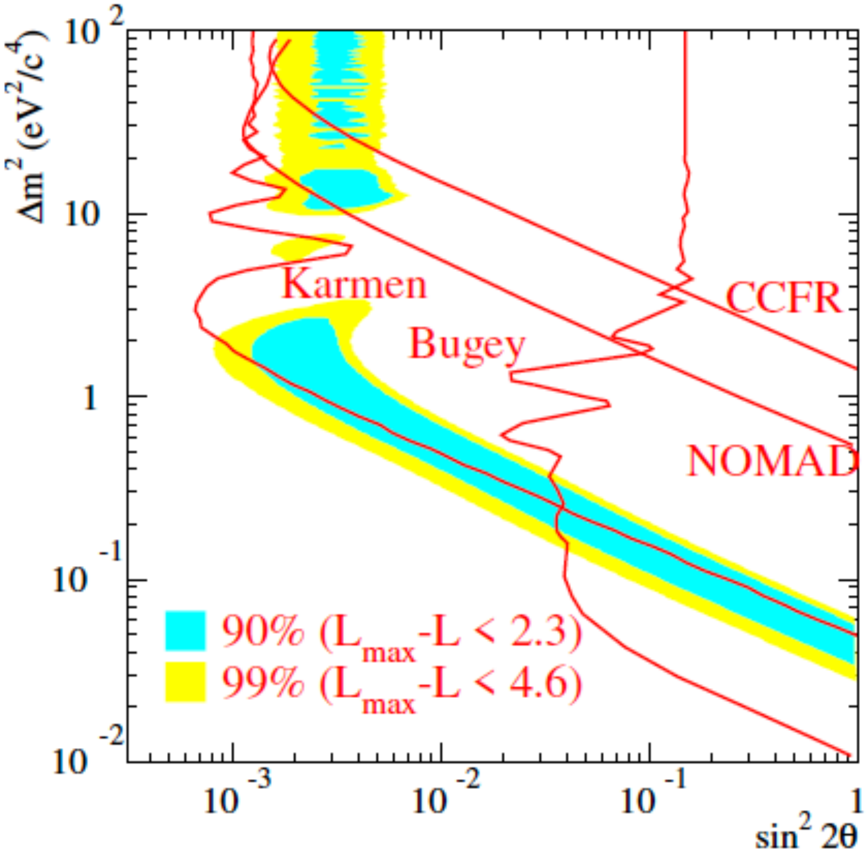
1. Oscillatory appearance of electron (anti)neutrinos in muon (anti)neutrino beams: the LSND anomaly and related MiniBooNE low-energy excess.
2. Normalization discrepancy of electron antineutrinos from commercial power reactors: the reactor neutrino anomaly.
3. Normalization deficit in ^{71}Ga decays: the gallium anomaly.

- These anomalies can be interpreted as oscillations of light sterile neutrinos with oscillation frequency $\Delta m^2 > 1$ eV^2 that mix with three Standard Model neutrinos.

- Here I will first briefly review the current status. For more detailed information I refer to the recent Snowmass sterile neutrino report, arXiv:2203.07323 [hep-ex].

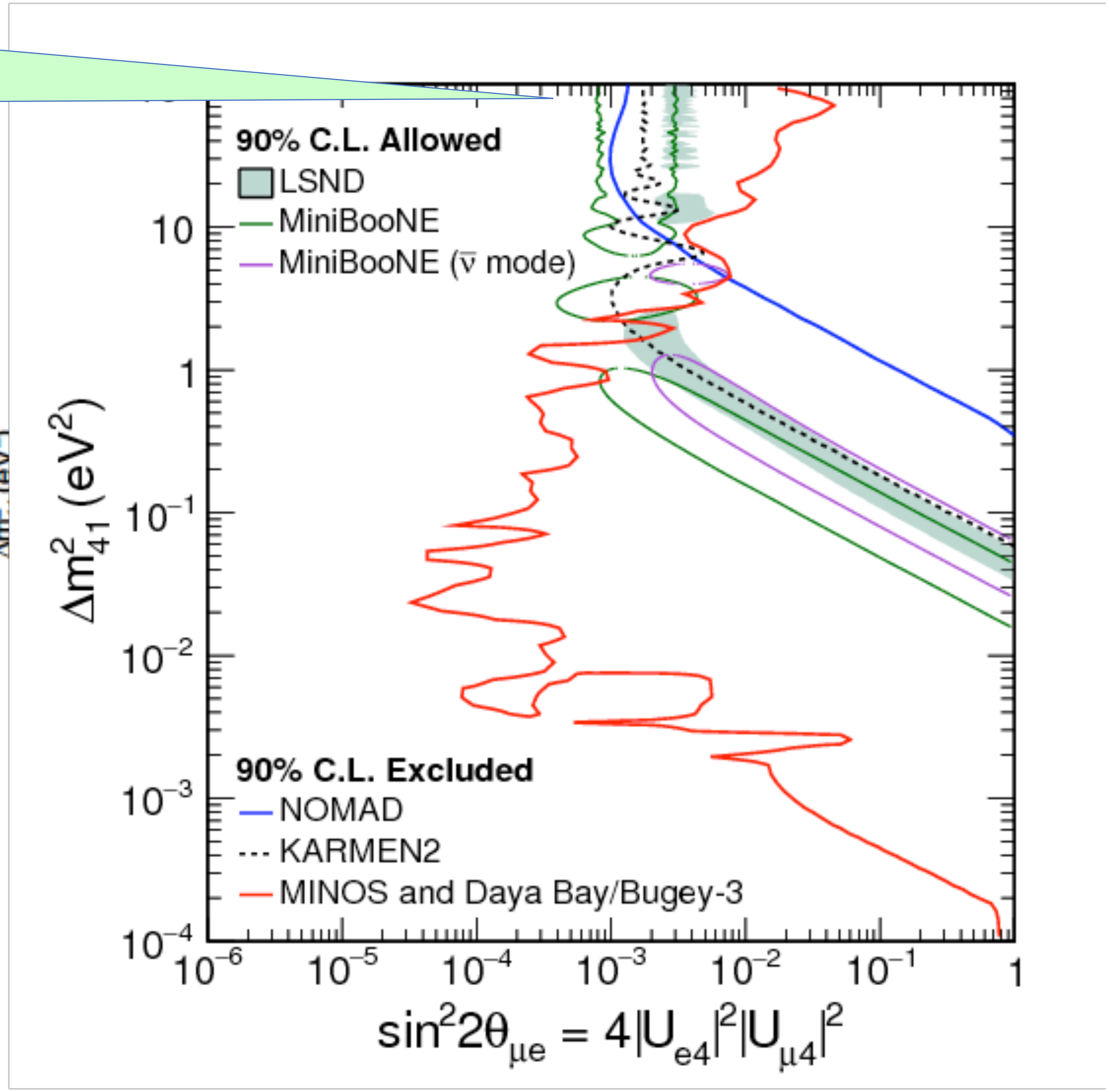
- In the second part of these lectures I will present some recent interesting developments in collective neutrino oscillations.

LSND



arXiv:hep-ex/0104049

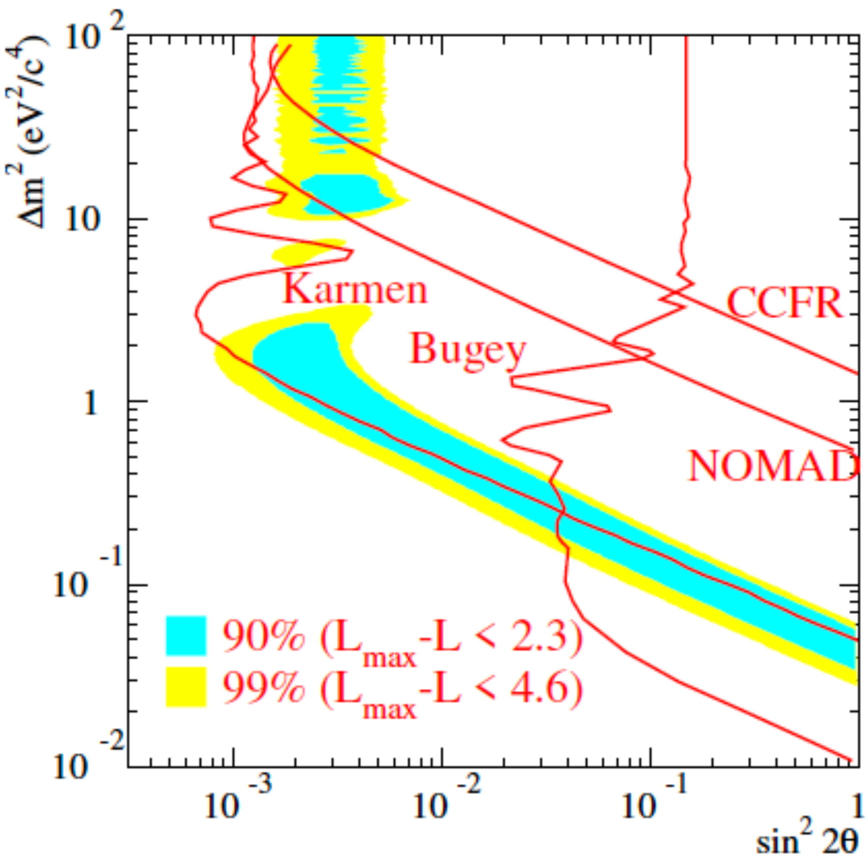
This region is still allowed for LSND/MiniBooNE



Daya Bay, Bugey, MINOS joint analysis

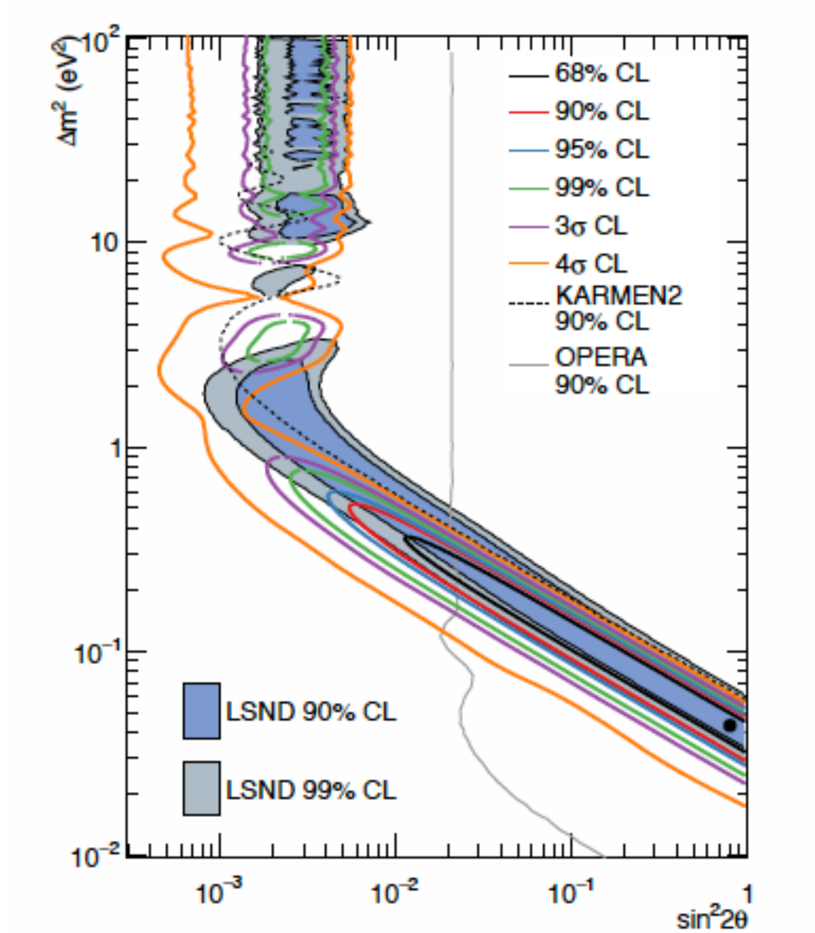
arXiv:1607.01177

LSND



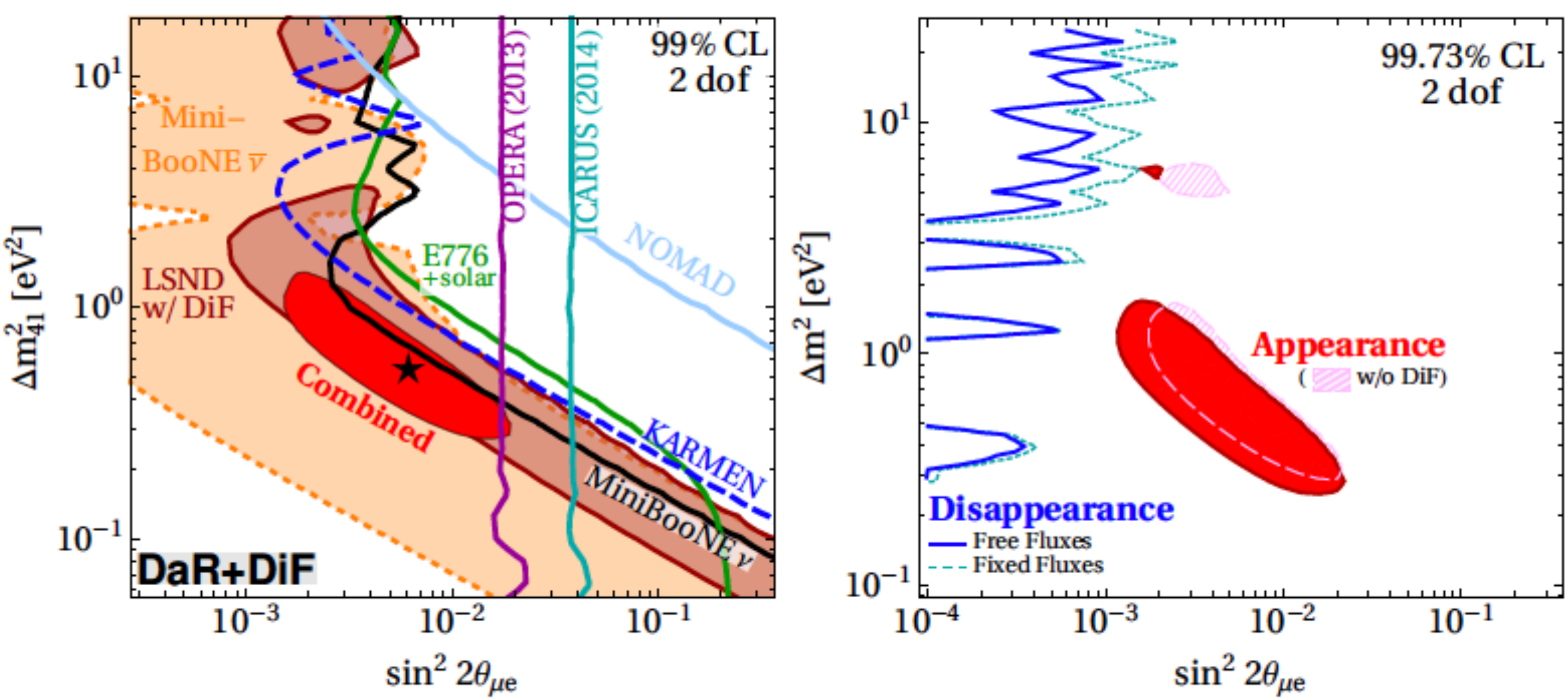
arXiv:hep-ex/0104049

MiniBooNE



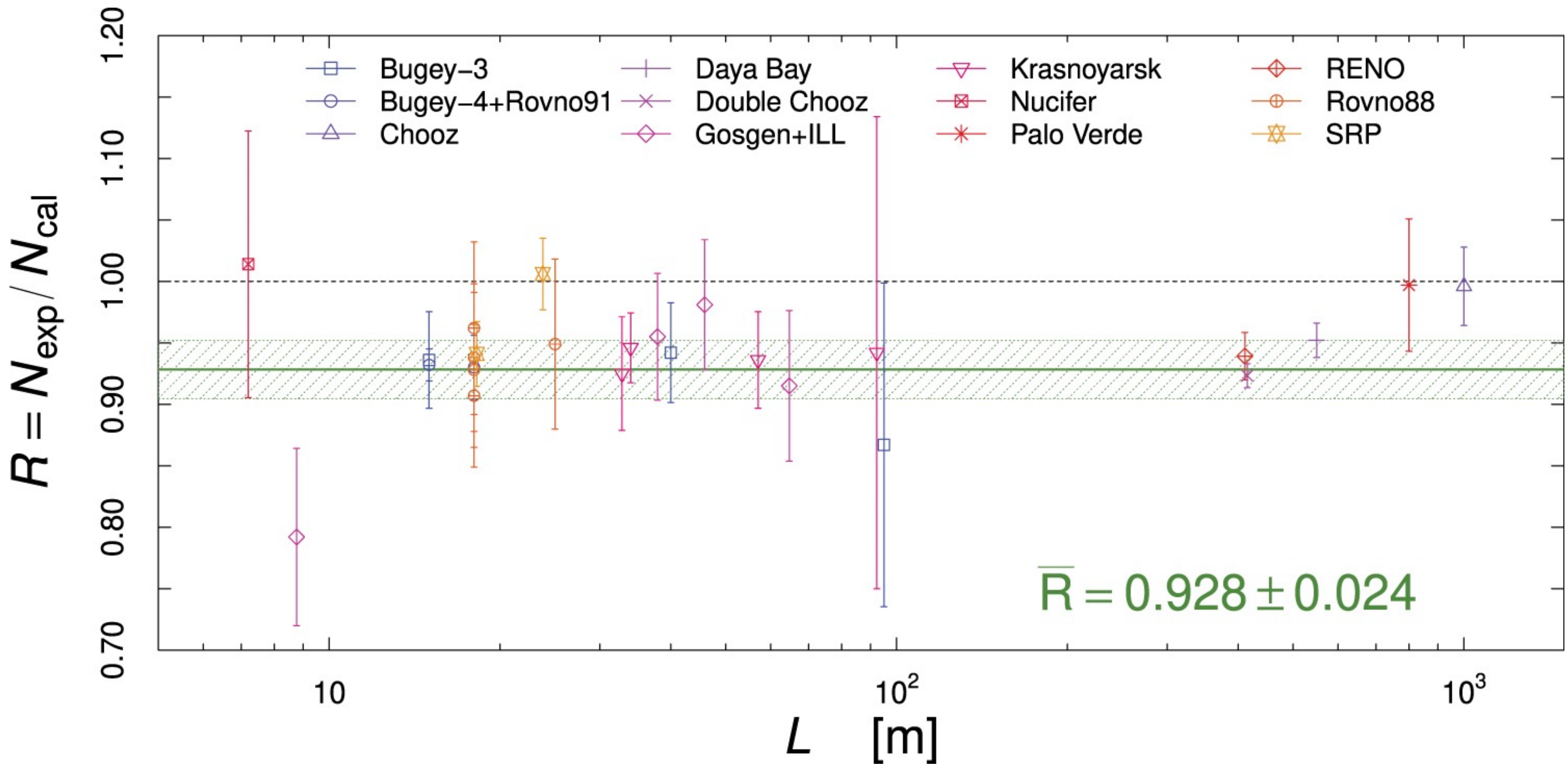
arXiv:2006.16883 [hep-ex]

arXiv:1803.10661 [hep-ph]

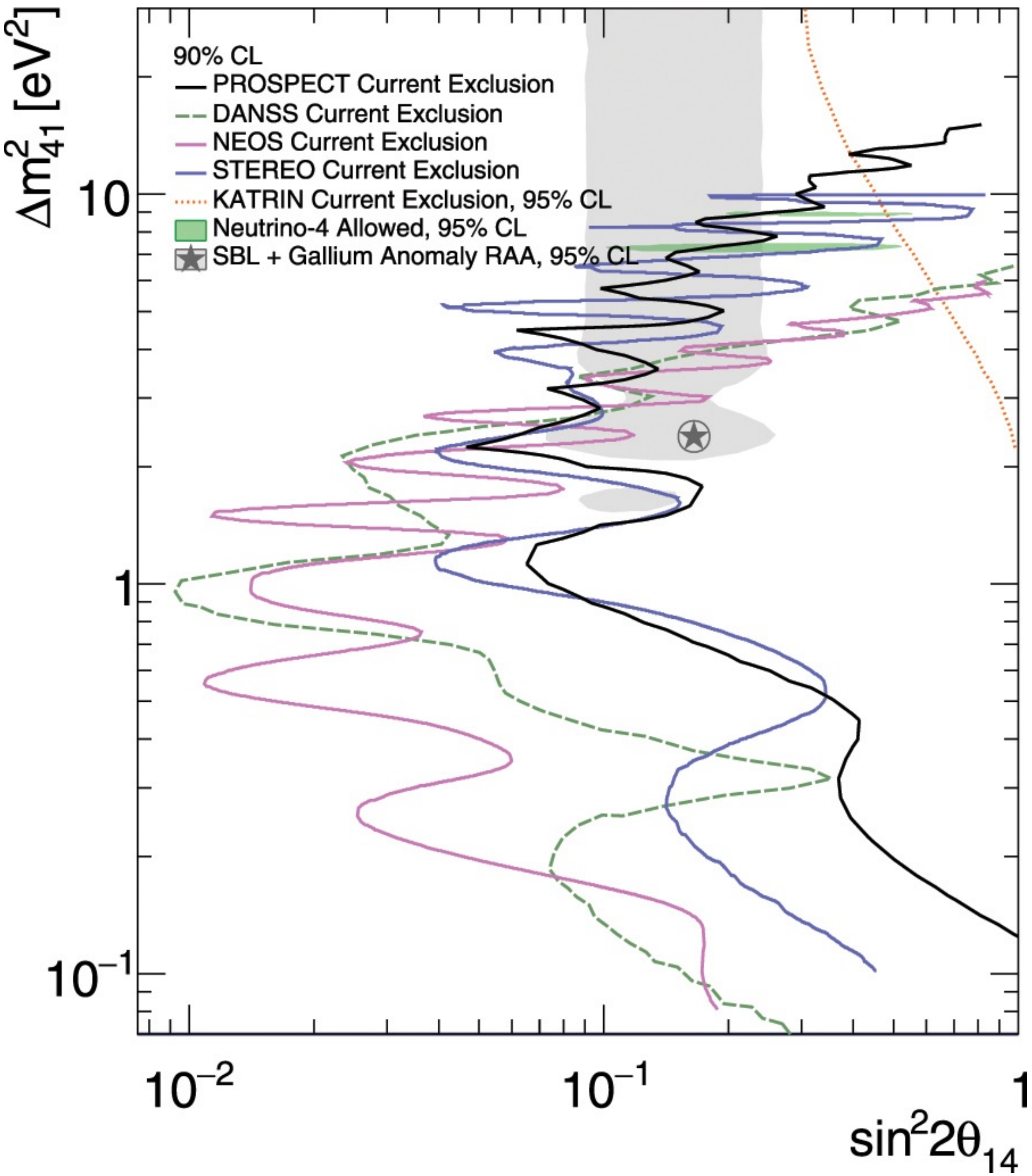


$$\sin^2 2\theta_{\mu e} = 4|U_{e4}|^2 |U_{\mu 4}|^2$$

Reactor Anomaly



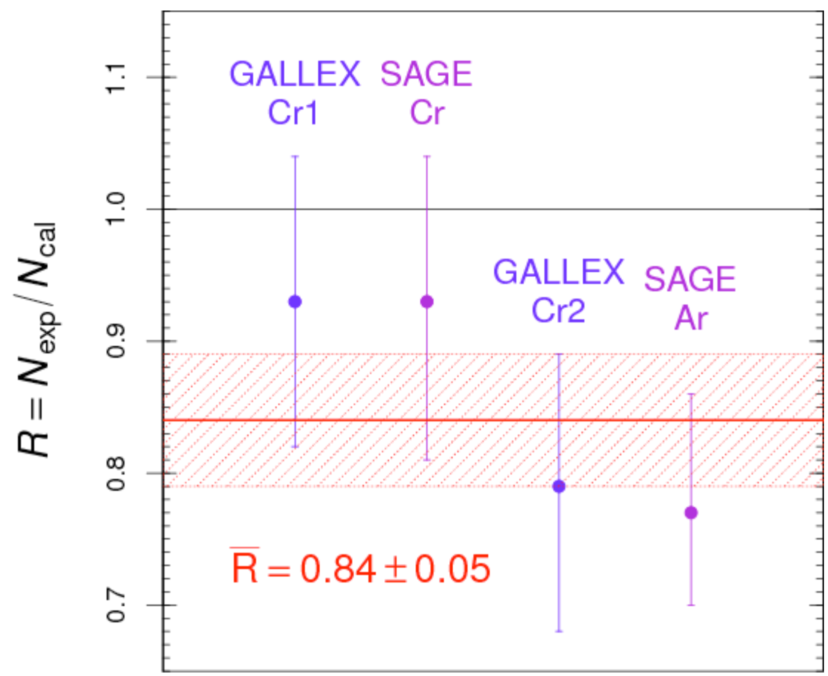
arXiv:2107.03934 [hep-ex]



PROSPECT

arXiv:2107.03934 [hep-ex]

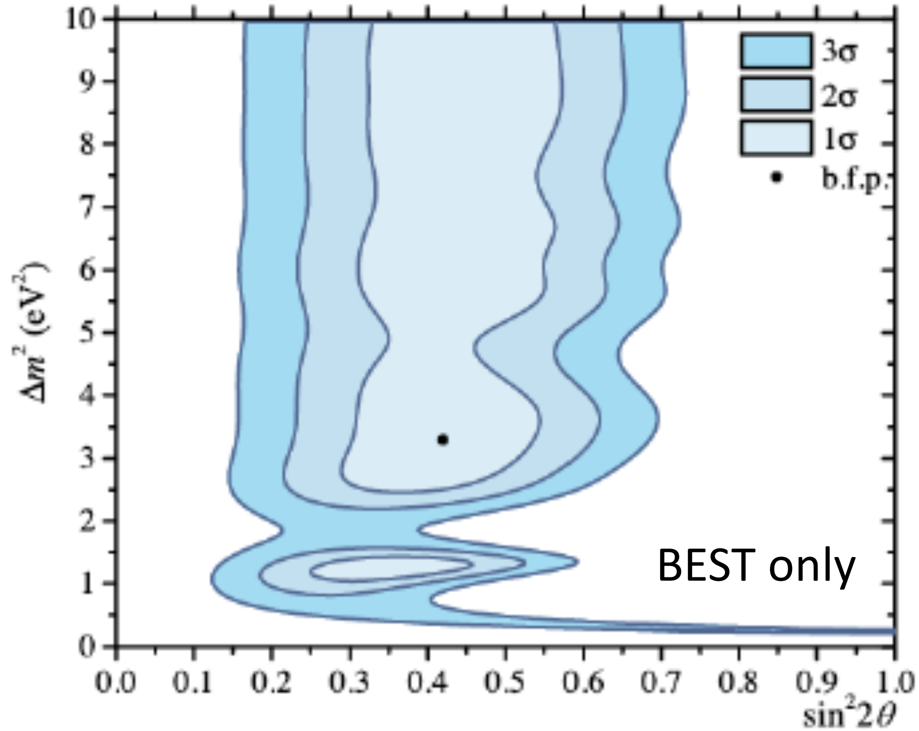
The Gallium Anomaly

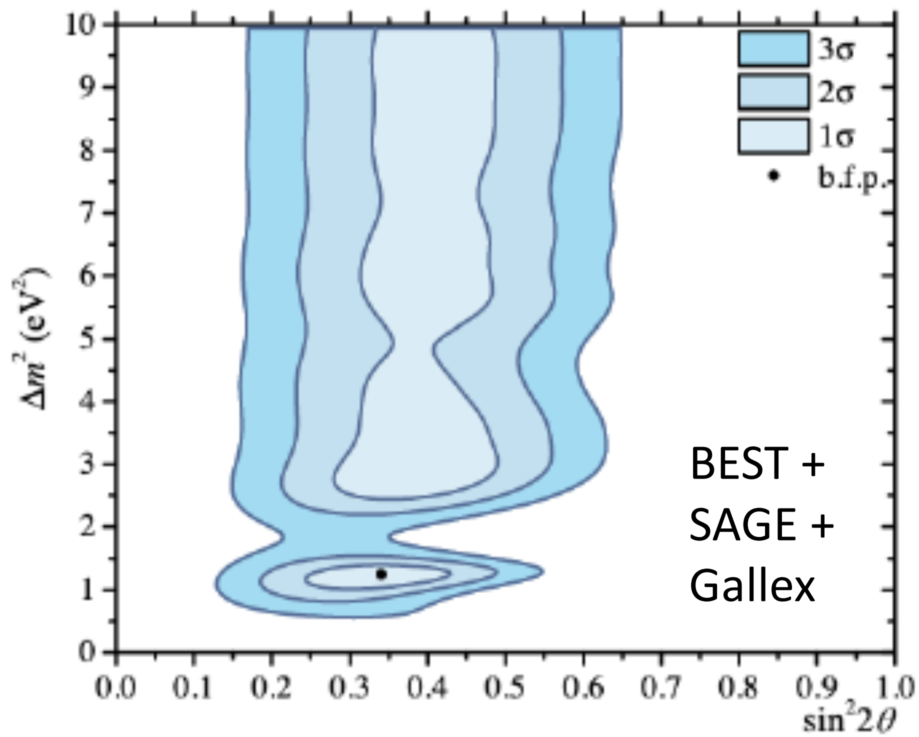


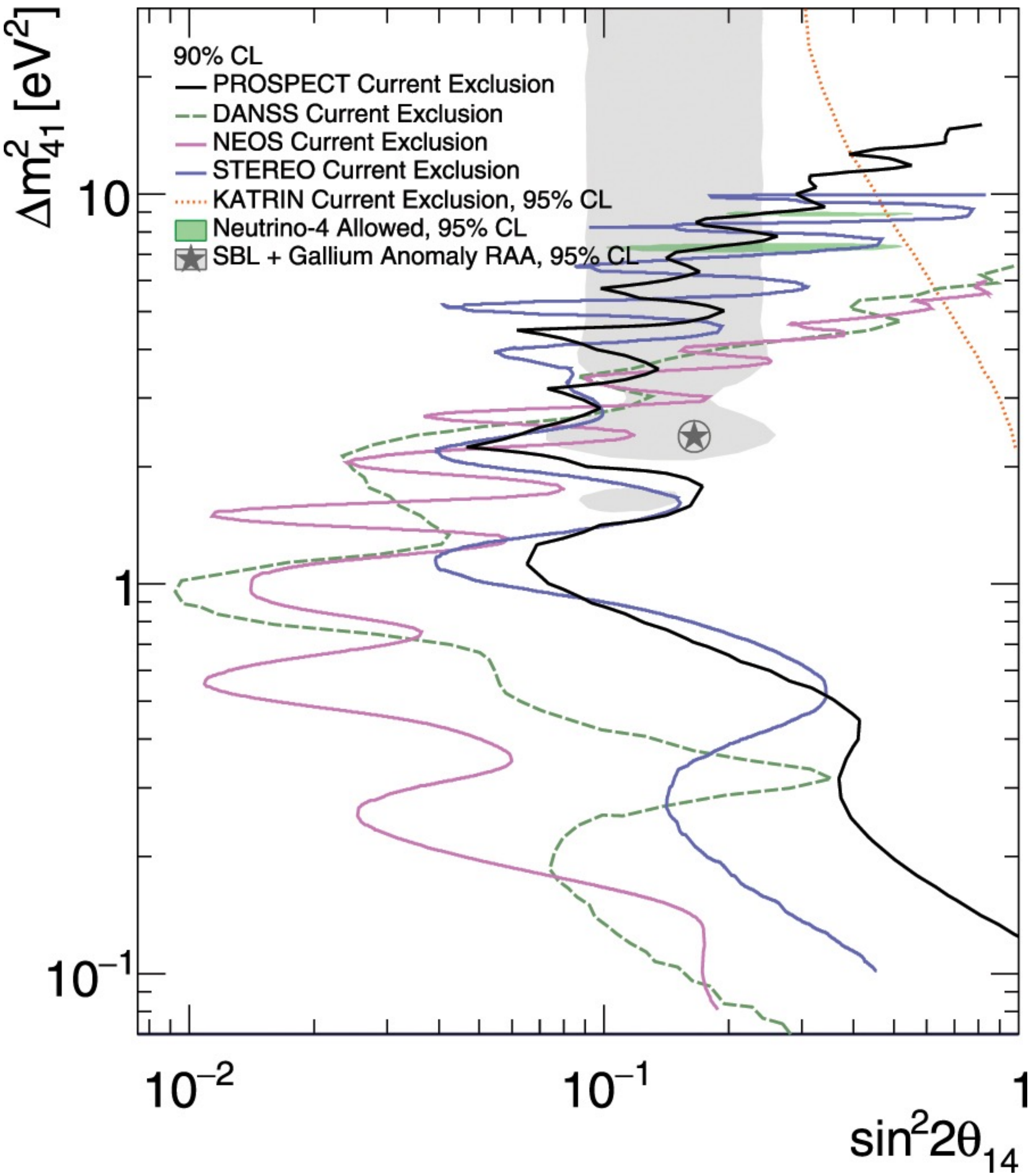
BEST: Baksan Experiment on Sterile Transitions

These results are consistent with $\nu_e \rightarrow \nu_s$ oscillations with a relatively large Δm^2 (>1 eV^2) and mixing $\sin^2 2\theta$ (≈0.4).

arXiv:2109.11482

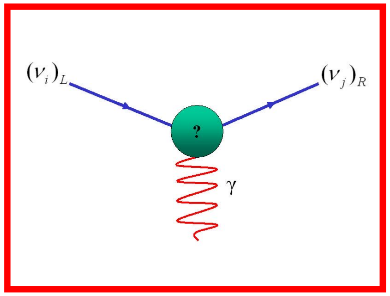






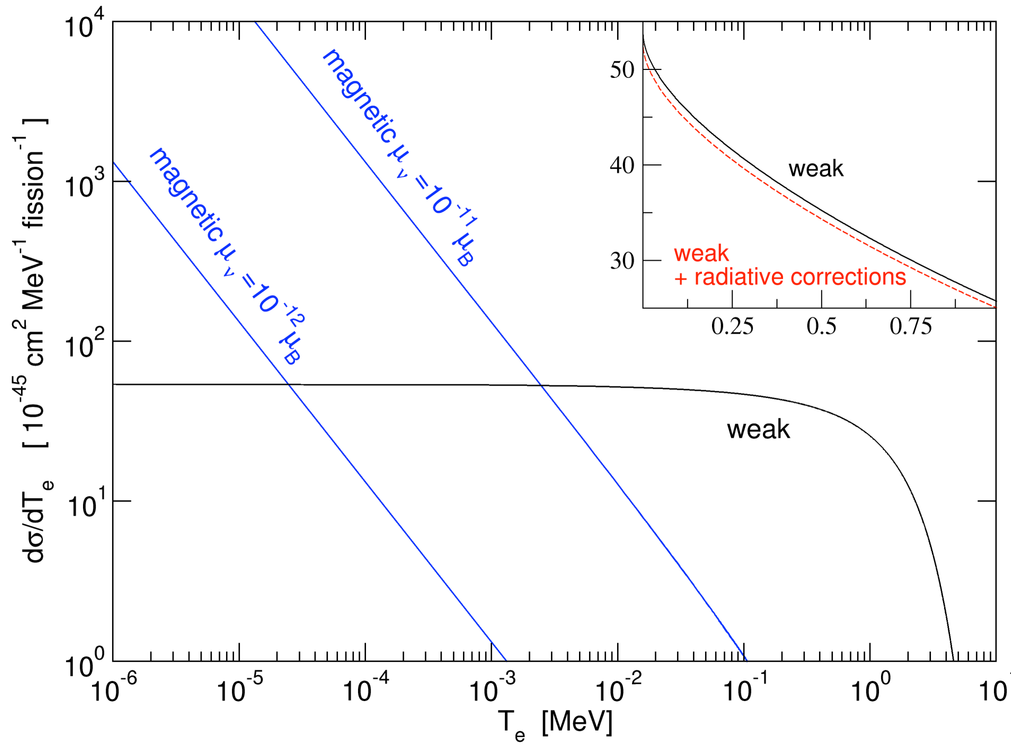
arXiv:2107.03934 [hep-ex]

Neutrino Magnetic Moment



$$\frac{d\sigma}{dT_e} = \frac{\alpha^2\pi}{m_e^2}\mu_{\text{eff}}^2\left[\frac{1}{T_e} - \frac{1}{E_\nu}\right]$$

$$\mu_{\text{eff}}^2 = \sum_i \left|\sum_j U_{ej} e^{-iE_jL}\mu_{ji}\right|^2$$



At lower energies, beyond Standard Model physics is described by local operators

$$L = L_{SM} + \frac{C^{(5)}}{\Lambda}O^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2}O_i^{(6)} + \sum_i \frac{C_i^{(7)}}{\Lambda^3}O_i^{(7)} + \ldots$$

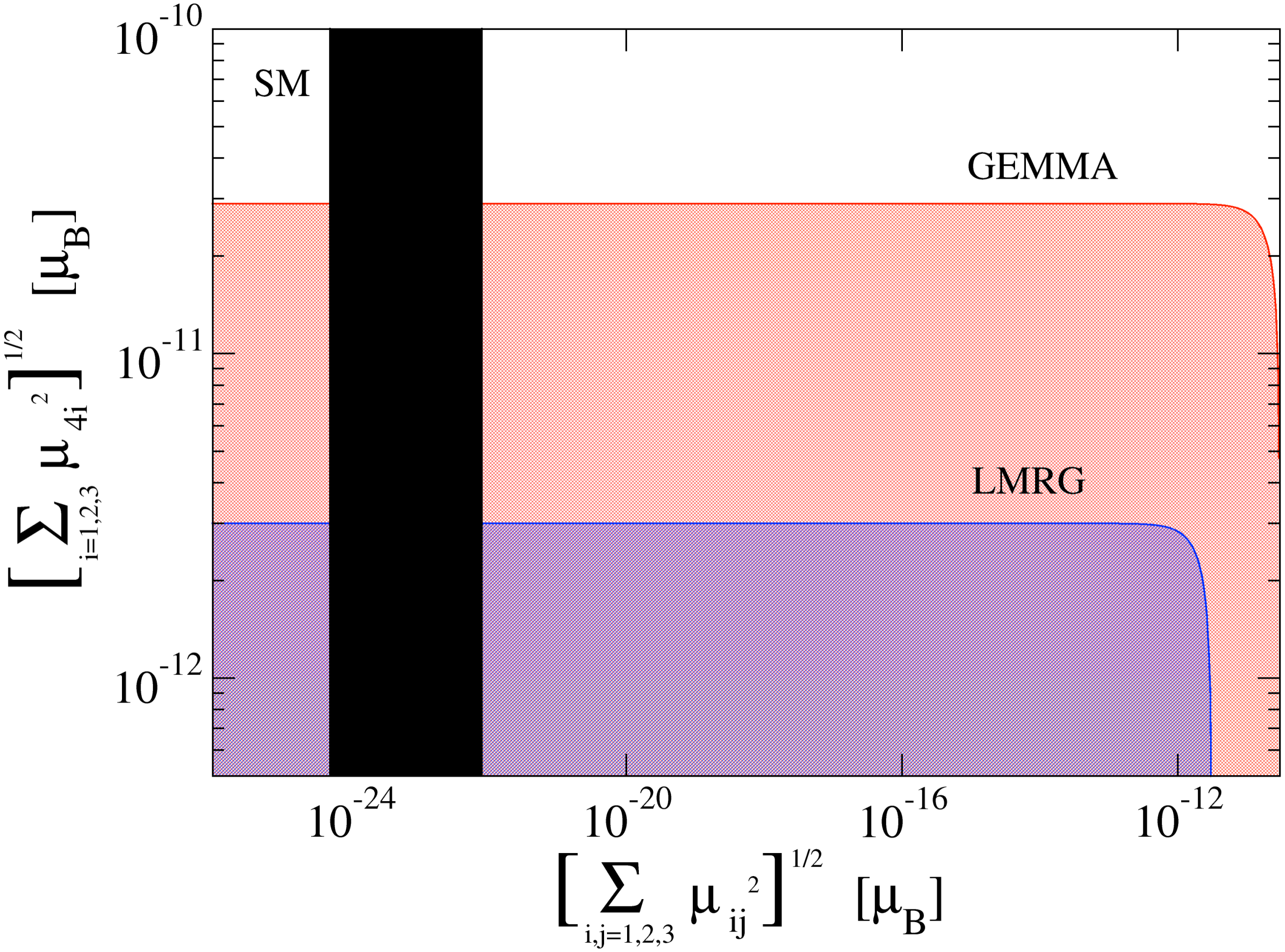
Majorana neutrino mass (unique)

Includes Majorana neutrino magnetic moment

For a sufficiently heavy sterile neutrino the phases with $(E_4 - E_i)L$ average to zero

$$\mu_{eff}^2 = \sum_{i,j=1}^{3}\left[U_{ei}\left(\mu\mu^+\right)_{ij}U_{je}^+\right] + U_{e4}\left(\mu\mu^+\right)_{44}U_{4e}^+$$

$$\Rightarrow \mu_{eff}^2 \le \sum_{i=1}^{3}\mu_{i4}^2 + \left(1-\left|U_{e4}\right|^2\right)\sum_{i,j=1}^{3}\mu_{ij}^2$$



Sterile Neutrino Decay (assuming CPT invariance)

$$\frac{d\Gamma(N \rightarrow \nu_\ell + X)}{d(\cos\theta)} = \frac{\Gamma}{2}(1 + \alpha\cos\theta)$$

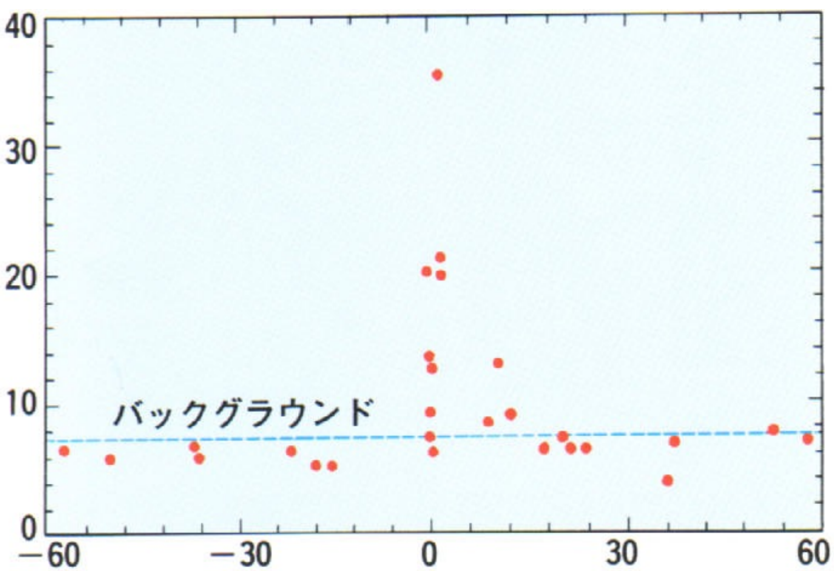
$$\frac{d\Gamma(\bar{N} \rightarrow \bar{\nu}_\ell + X)}{d(\cos\theta)} = \frac{\Gamma}{2}(1 - \alpha\cos\theta)$$

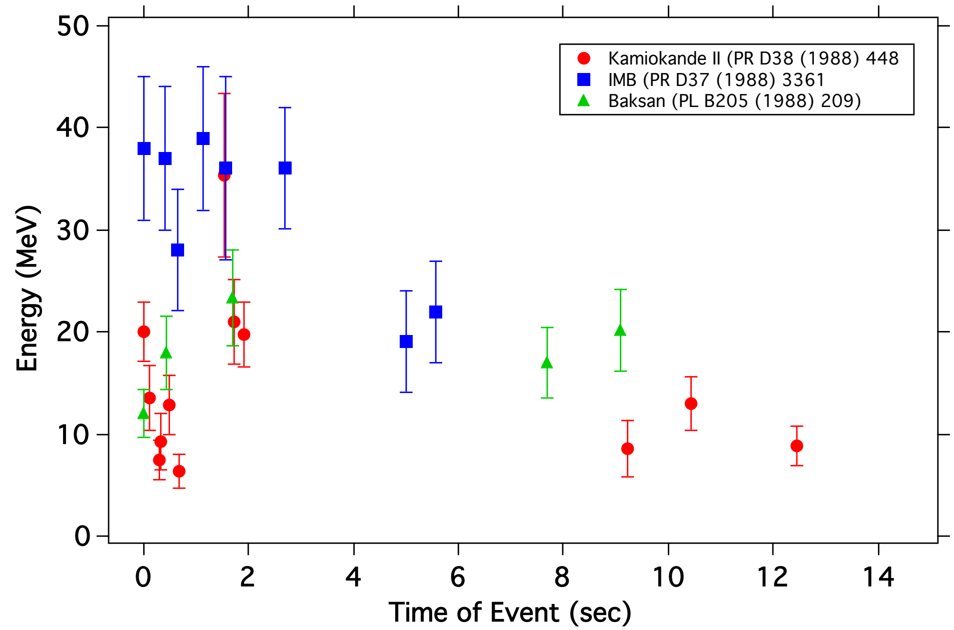
Since $\alpha = -\bar{\alpha}$, for Majorana neutrinos we get $\alpha = 0$. This result holds for any self-conjugate boson X.

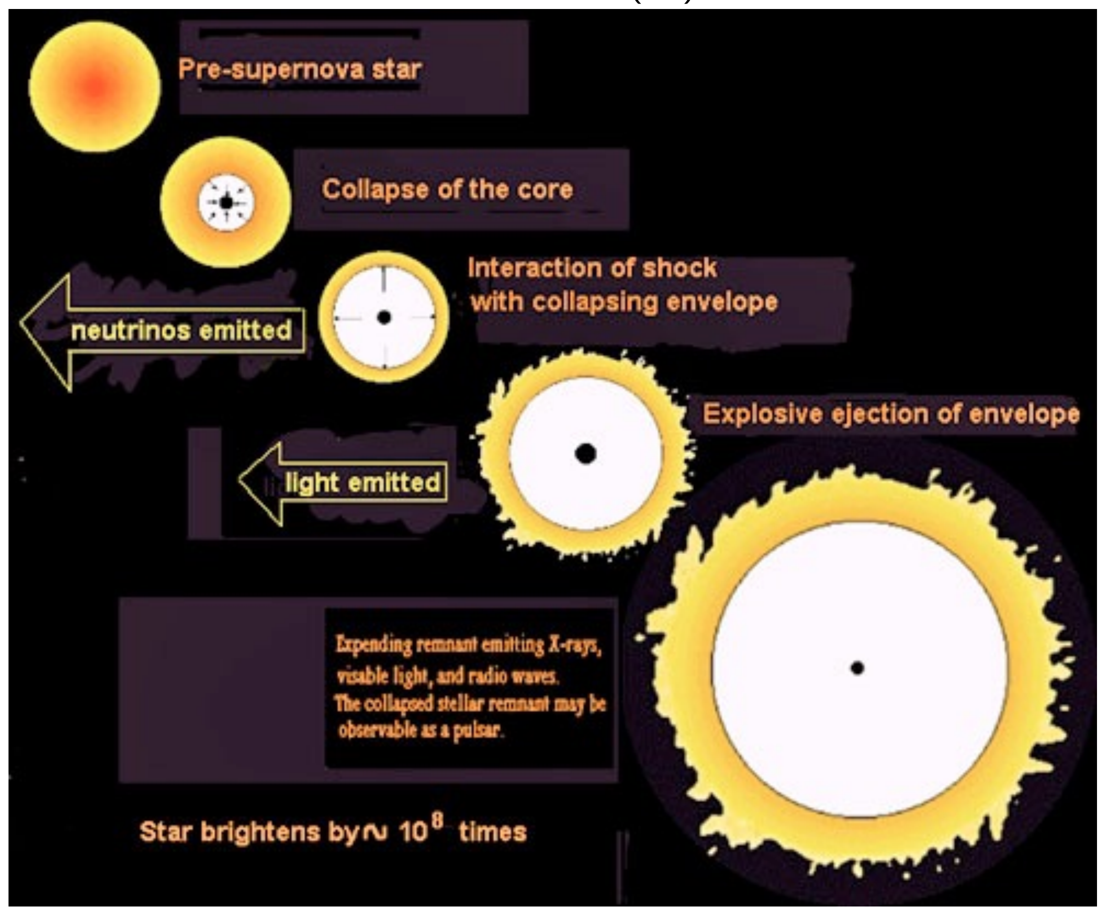
A.B. Balantekin and B. Kayser, Ann. Rev. Nucl. Part. Sci. **68,** 313 (2018)

A.B. Balantekin, A. de Gouvêa, and B. Kayser, Phys. Lett. B **789**, 488 (2019)

Neutrinos from core-collapse supernovae 1987A

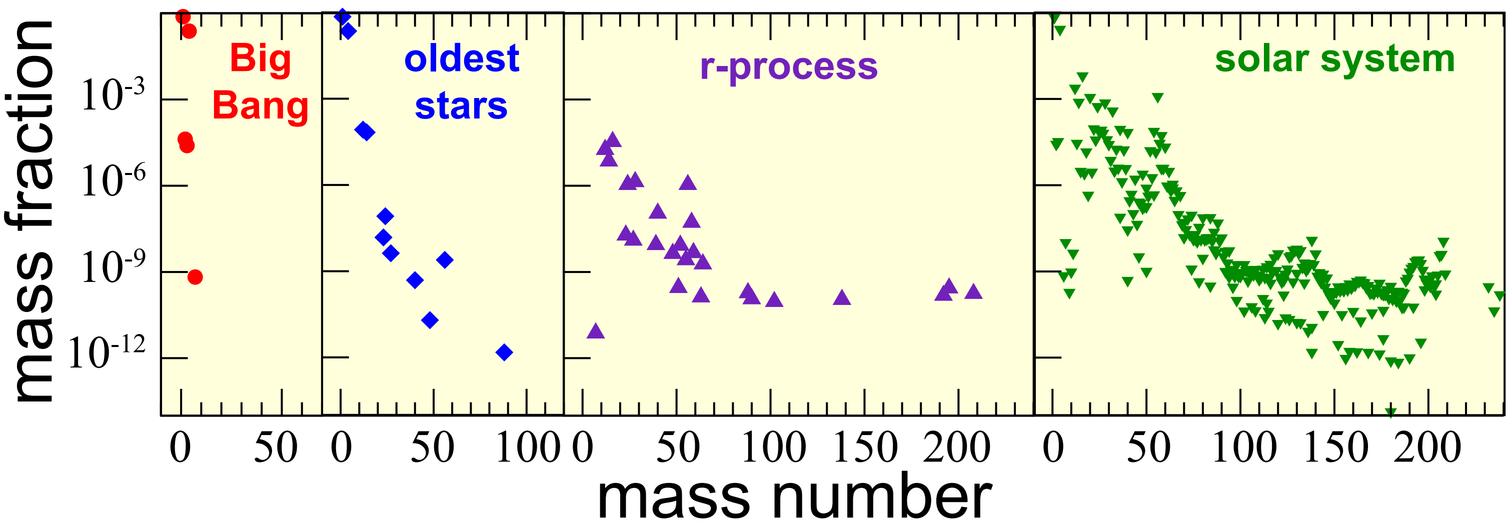


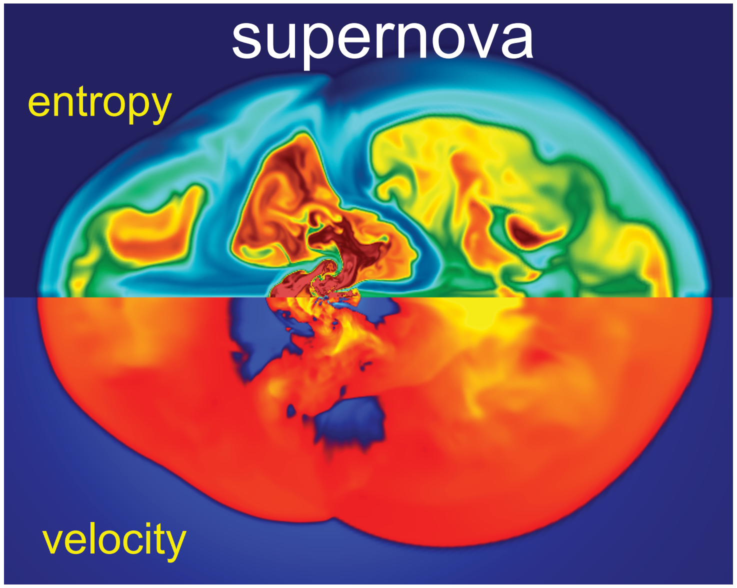


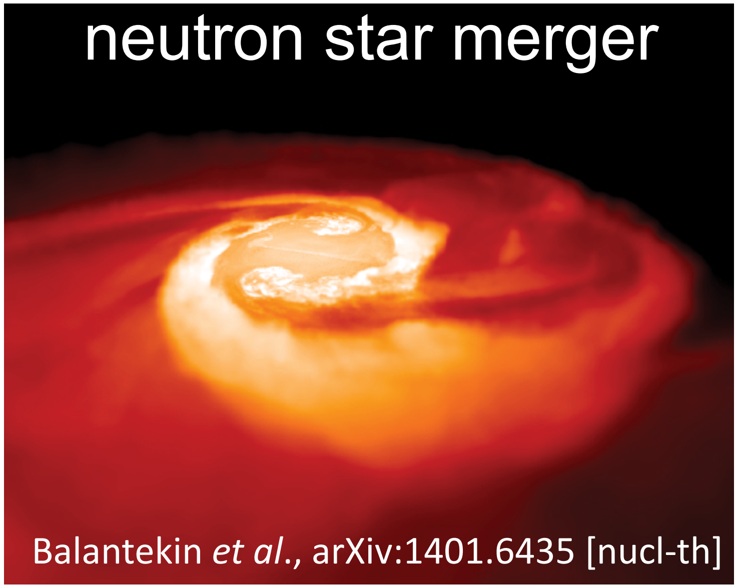


- $M_{prog} \geq 8\ M_{sun} \Rightarrow \Delta E \approx 10^{53}$ ergs $\approx 10^{59}$ MeV
- 99% of the energy is carried away by neutrinos and antineutrinos with $10 \leq E_\nu \leq 30$ MeV $\Rightarrow 10^{58}$ neutrinos

The origin of elements





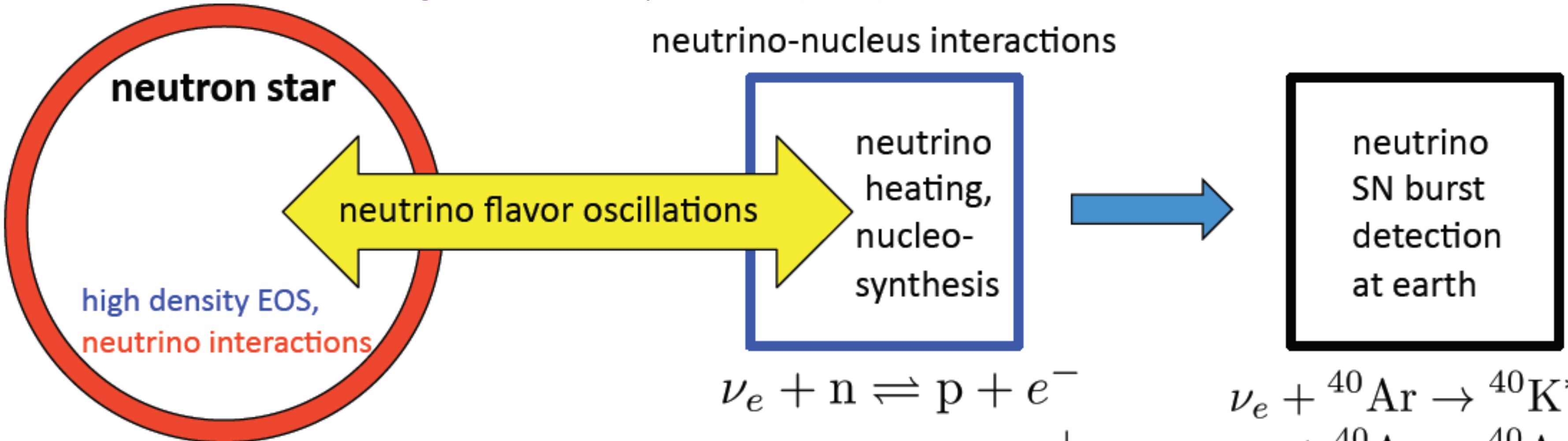


Possible sites for the r-process

Neutrinos not only play a crucial role in the dynamics of these sites, but they also control the value of the electron fraction, the parameter determining the yields of the r-process.

Understanding a core-collapse supernova requires answers to a variety of questions some of which need to be answered, both theoretically and experimentally.

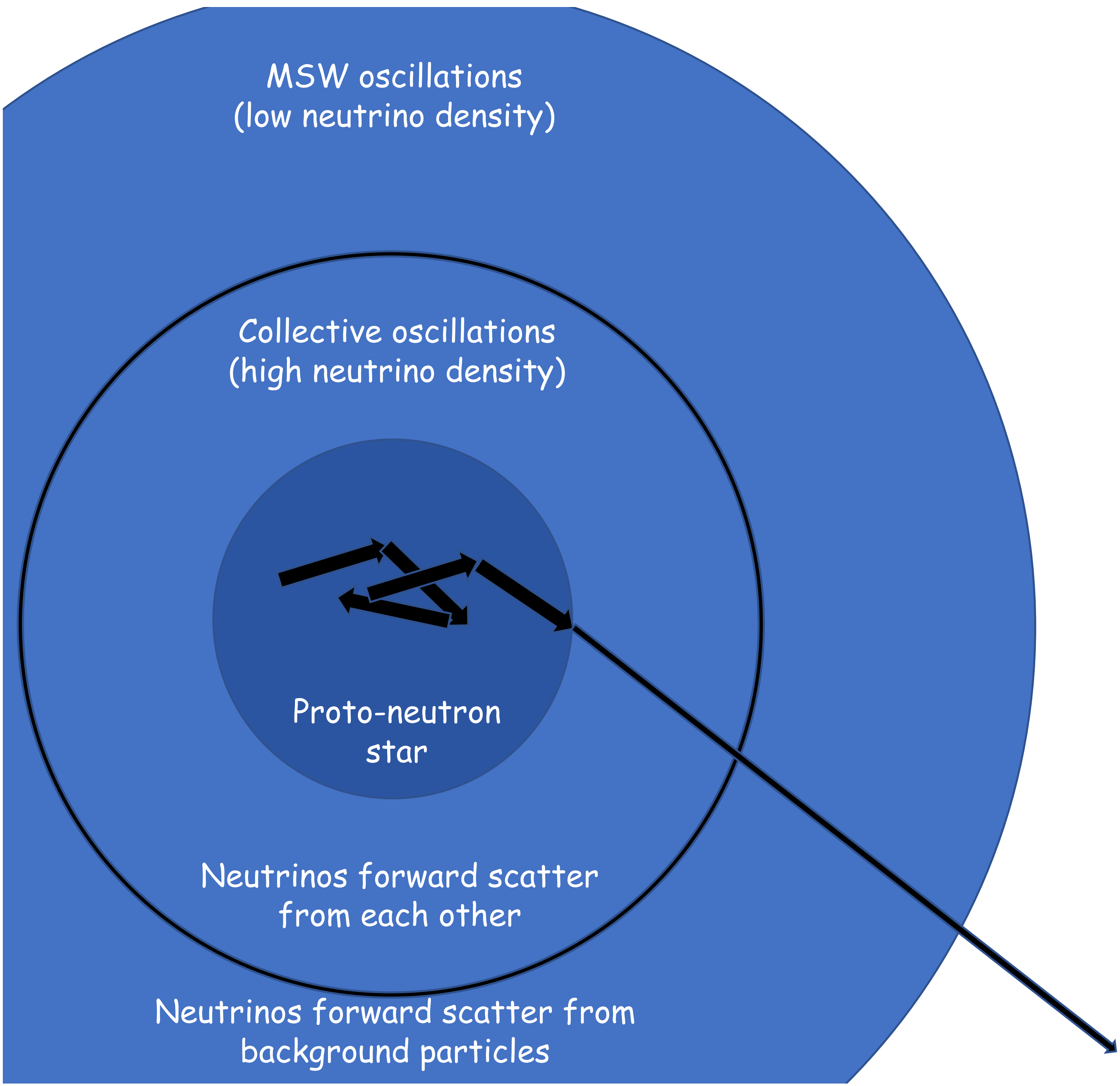
Balantekin and Fuller, Prog. Part. Nucl. Phys. **71** 162 (2013)



$$\nu_e + \mathrm{n} \rightleftharpoons \mathrm{p} + e^-$$
$$\bar{\nu}_e + \mathrm{p} \rightleftharpoons \mathrm{n} + e^+$$

$$\nu_e + {}^{40}\mathrm{Ar} \to {}^{40}\mathrm{K}^* + e^-$$
$$\nu_x + {}^{40}\mathrm{Ar} \to {}^{40}\mathrm{Ar}^* + \nu_x$$

Neutron-to-proton ratio depends on relative intensities of electron neutrinos and electron antineutrinos, which in turn depend on neutrino oscillations


MSW oscillations
(low neutrino density)
Collective oscillations
(high neutrino density)
Proto-neutron
star
Neutrinos forward scatter
from each other
Neutrinos forward scatter from
background particles

Matter-enhanced oscillations with active-sterile mixing

$$i\frac{\partial}{\partial r}\begin{pmatrix}\Psi_{e,x}(r)\\ \Psi_s(r)\end{pmatrix}=\begin{pmatrix}\varphi_{e,x}(r) & \Lambda_{e,x}\\ \Lambda_{e,x} & -\varphi_{e,x}(r)\end{pmatrix}\begin{pmatrix}\Psi_{e,x}(r)\\ \Psi_s(r)\end{pmatrix}$$

$$\Lambda_{e,x}=\frac{\delta m^2}{4E}\sin 2\theta_{es,ex}$$

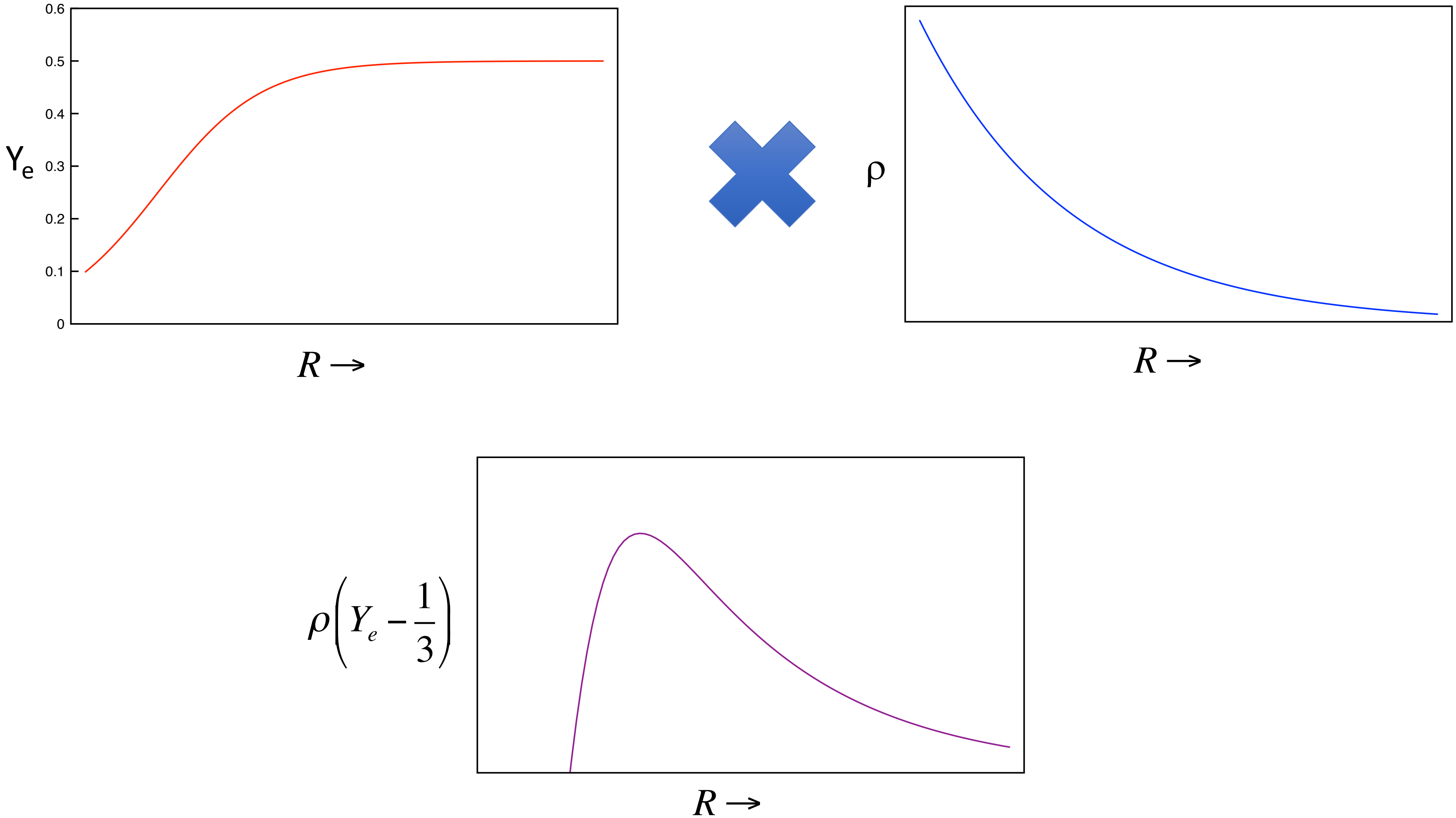
$$\varphi_e(r)=\frac{1}{4E}\left(\pm 2\sqrt{2}G_F\left[N_e^-(r)-N_e^+(r)-\frac{N_n(r)}{2}\right]E-\delta m^2\cos 2\theta_s\right)$$

$$\varphi_e(r)=\pm\frac{3G_F\rho(r)}{2\sqrt{2}m_N}\left(Y_e-\frac{1}{3}\right)-\frac{\delta m^2}{4E}\cos 2\theta_{es}$$

$$\varphi_{\mu,\tau}(r)=\pm\frac{G_F\rho(r)}{2\sqrt{2}m_N}\left(Y_e-1\right)-\frac{\delta m^2}{4E}\cos 2\theta_{\mu s,\tau x}$$

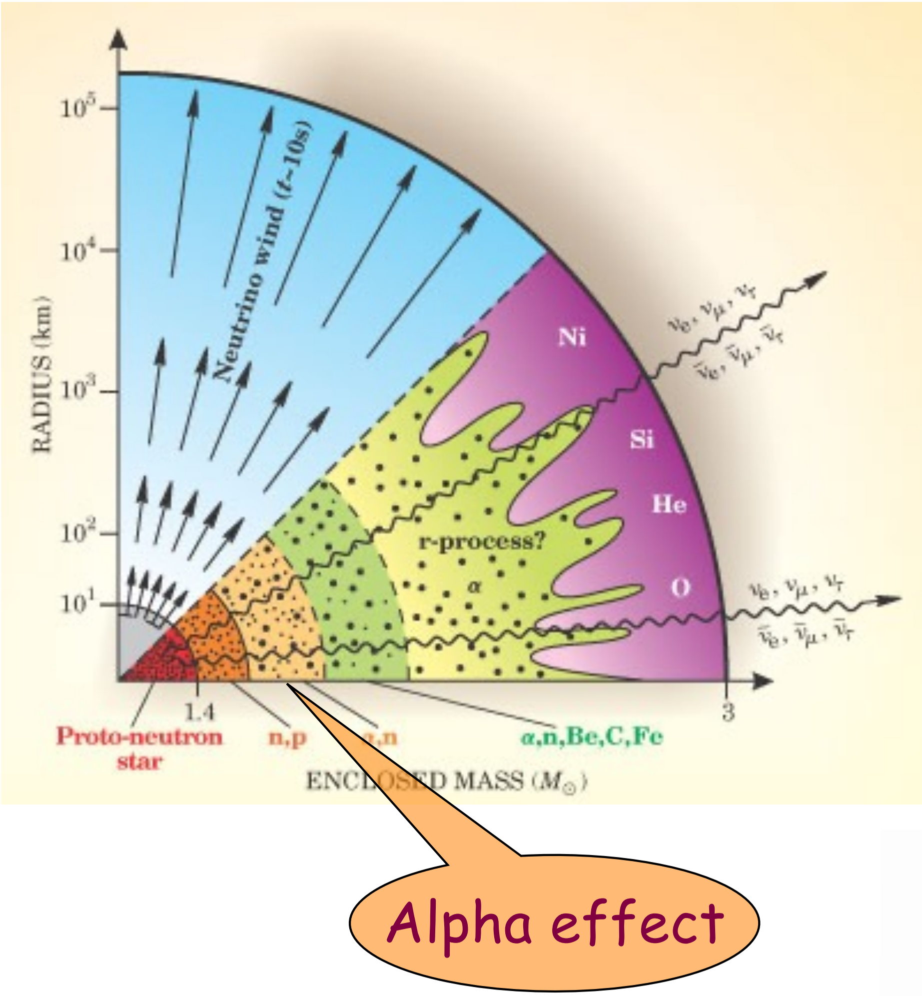
Neutrinos: + sign

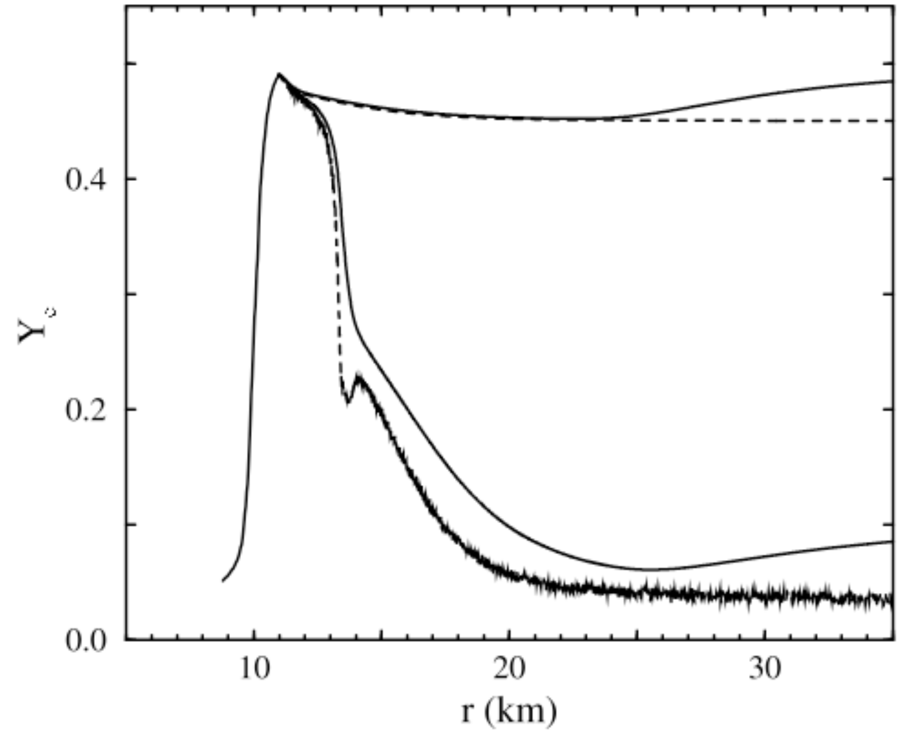
Antineutrinos: - sign

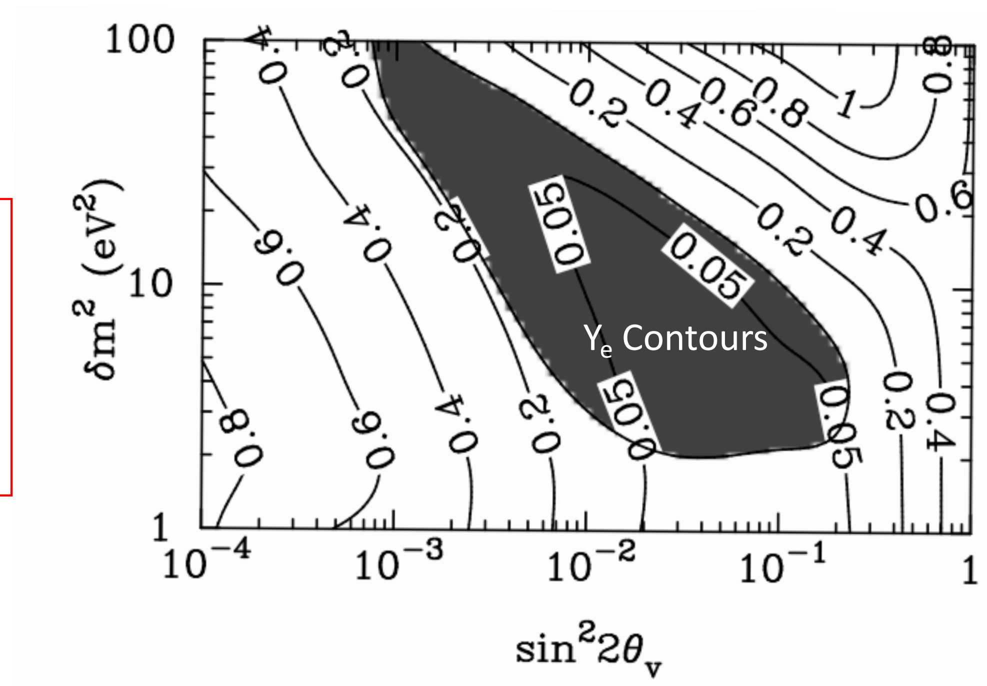


MSW Resonance Condition:

$$\varphi_e(r) = \pm \frac{3G_F\rho(r)}{2\sqrt{2}m_N}\left(Y_e - \frac{1}{3}\right) - \frac{\delta m^2}{4E}\cos 2\theta_{es} = 0$$

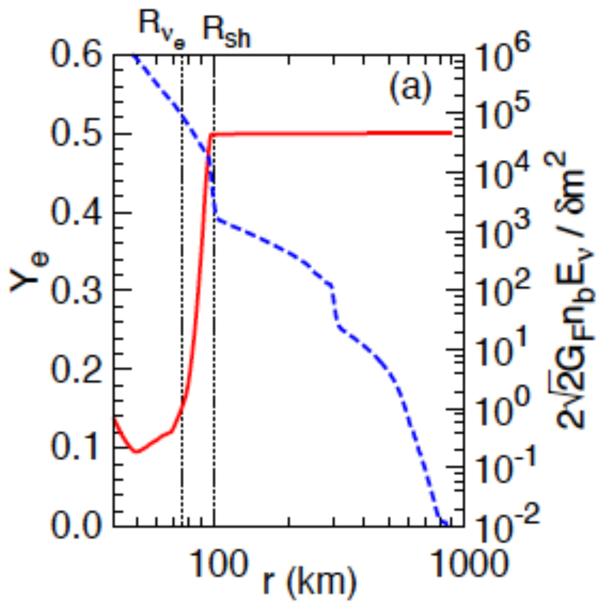


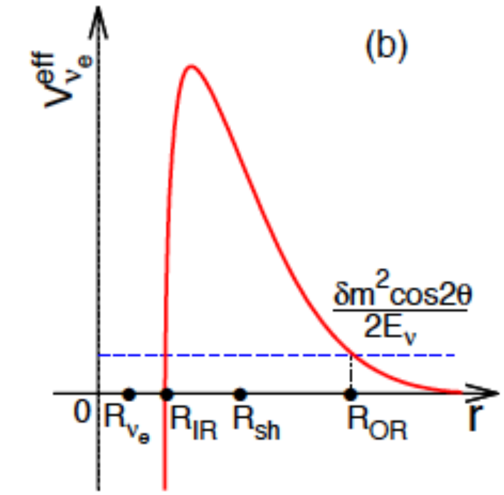


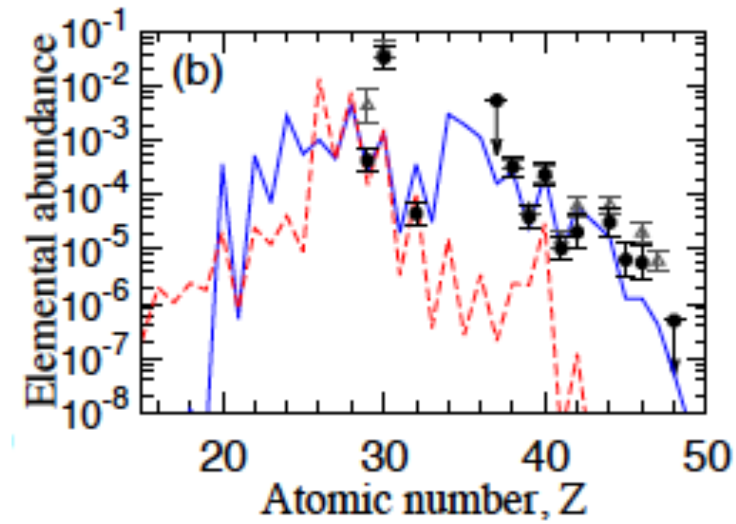


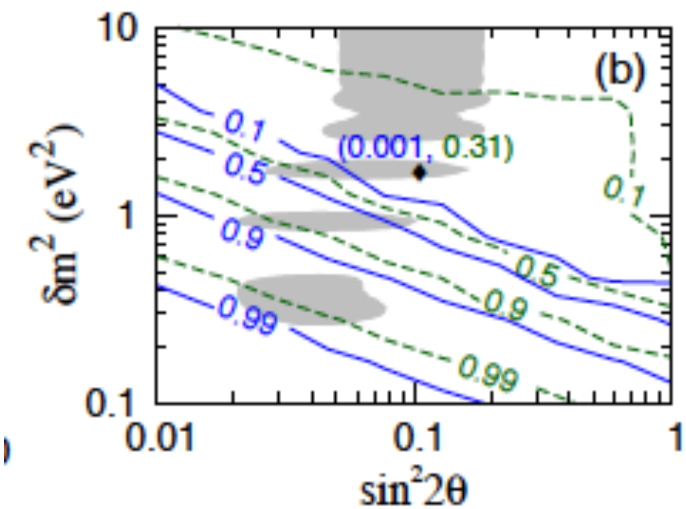
Active-sterile mixing could yield very low values of Y_e, which is crucial for r-process nucleosynthesis

McLaughlin, Fetter, Balantekin, Fuller, Astropart. Phys., 18, 433 (2003)



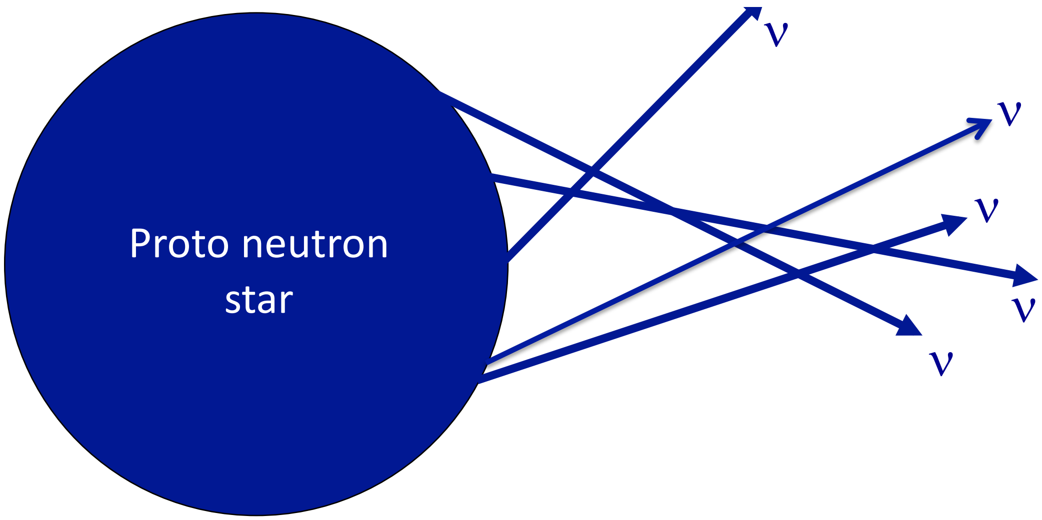






Active-sterile mixing with the parameters inferred from reactor anomaly enables nucleosynthesis, but seems to suppress shock reheating by neutrinos.

Wu, Fischer, Martinez-Pinedo, Qian, 2013



Energy released in a core-collapse SN: $\Delta E \approx 10^{53}$ ergs $\approx 10^{59}$ MeV

99% of this energy is carried away by neutrinos and antineutrinos!

~ 10^{58} Neutrinos!

This necessitates including the effects of νν interactions!

$$H = \underbrace{\sum a^\dagger a}_{\nu \text{ oscillations, MSW effect}} + \underbrace{\sum (1-\cos\varphi) a^\dagger a^\dagger a a}_{\text{neutrino-neutrino interactions}}$$

ν oscillations
MSW effect

neutrino-neutrino interactions

The second term makes the physics of a neutrino gas in a core-collapse supernova a very interesting many-body problem, driven by weak interactions.

Neutrino-neutrino interactions lead to novel collective and emergent effects, such as conserved quantities and interesting features in the neutrino energy spectra (spectral "swaps" or "splits").

Many neutrino system

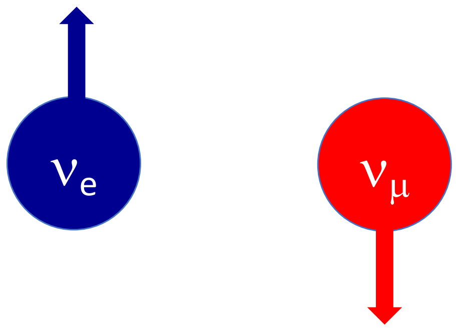
This is the only many-body system driven by the weak interactions:

Table: Many-body systems

Nuclei	Strong	at most ∼250 particles
Condensed matter	E&M	at most N_A particles
ν**'s in SN**	Weak	$\sim 10^{58}$particles

Astrophysical extremes allow us to test physics that cannot be tested elsewhere!

Neutrino flavor isospin



$$\hat{J}_+ = a_e^\dagger a_\mu \qquad \hat{J}_- = a_\mu^\dagger a_e$$

$$\hat{J}_0 = \frac{1}{2}\left(a_e^\dagger a_e - a_\mu^\dagger a_\mu\right)$$

These operators can be written in either mass or flavor basis

Free neutrinos (only mixing)

$$\hat{H} = \frac{m_1^2}{2E} a_1^\dagger a_1 + \frac{m_2^2}{2E} a_2^\dagger a_2 + (\cdot\cdot)\hat{1}$$

$$= \frac{\delta m^2}{4E} \cos 2\theta \left(-2\hat{J}_0\right) + \frac{\delta m^2}{4E} \sin 2\theta \left(\hat{J}_+ + \hat{J}_-\right) + (\cdot\cdot)' \hat{1}$$

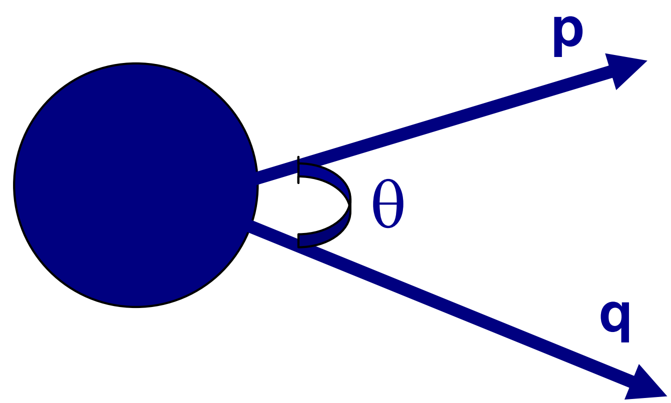
Interacting with background electrons

$$\hat{H} = \left[\frac{\delta m^2}{4E} \cos 2\theta - \frac{1}{\sqrt{2}} G_F N_e\right]\left(-2\hat{J}_0\right) + \frac{\delta m^2}{4E} \sin 2\theta \left(\hat{J}_+ + \hat{J}_-\right) + (\cdot\cdot)'' \hat{1}$$

Neutrino-Neutrino Interactions

Smirnov, Fuller and Qian, Pantaleone, McKellar, Friedland, Lunardini, Raffelt, Duan Balantekin, Kajino, Pehlivan …

$$\hat{H}_{\nu\nu} = \frac{\sqrt{2}G_F}{V}\int dp\, dq \left(1-\cos\theta_{pq}\right)\vec{\mathbf{J}}_p \cdot \vec{\mathbf{J}}_q$$



This term makes the physics of a neutrino gas in a core-collapse supernova a genuine many-body problem

$$\hat{H} = \int dp \left(\frac{\delta m^2}{2E}\vec{\mathbf{B}}\cdot\vec{\mathbf{J}}_p - \sqrt{2}G_F N_e \mathbf{J}_p^0\right) + \frac{\sqrt{2}G_F}{V}\int dp\, dq \left(1-\cos\theta_{pq}\right)\vec{\mathbf{J}}_p \cdot \vec{\mathbf{J}}_q$$

$$\vec{\mathbf{B}} = \left(\sin 2\theta,\ 0,\ -\cos 2\theta\right)$$

Neutrino-neutrino interactions lead to novel collective and emergent effects, such as conserved quantities and interesting features in the neutrino energy spectra (spectral “swaps” or “splits”).

Including antineutrinos

$$H = H_\nu + H_{\bar{\nu}} + H_{\nu\nu} + H_{\bar{\nu}\bar{\nu}} + H_{\nu\bar{\nu}}$$

Requires introduction of a second set of SU(2) algebras!

Including three flavors

Requires introduction of SU(3) algebras.

Both extensions are straightforward, but tedious!
Balantekin and Pehlivan, J. Phys. G **34**, 1783 (2007).

This problem is “exactly solvable” in the single-angle approximation

$$H = \sum_p \frac{\delta m^2}{2p} \hat{B} \cdot \vec{J}_p + \frac{\sqrt{2} G_F}{V} \sum_{\mathbf{p},\mathbf{q}} (1 - \cos \vartheta_{\mathbf{pq}}) \, \vec{J}_{\mathbf{p}} \cdot \vec{J}_{\mathbf{q}}$$

$$H = \sum_p \omega_p \vec{B} \cdot \vec{J}_p + \mu(r) \vec{J} \cdot \vec{J}$$

Two of the adiabatic eigenstates of this equation are easy to find in the single-angle approximation:

$$H = \sum_p \omega_p \vec{B} \cdot \vec{J}_p + \mu(r) \vec{J} \cdot \vec{J}$$

$$|j, +j\rangle = |N/2, N/2\rangle = |\nu_1, \ldots, \nu_1\rangle$$

$$|j, -j\rangle = |N/2, -N/2\rangle = |\nu_2, \ldots, \nu_2\rangle$$

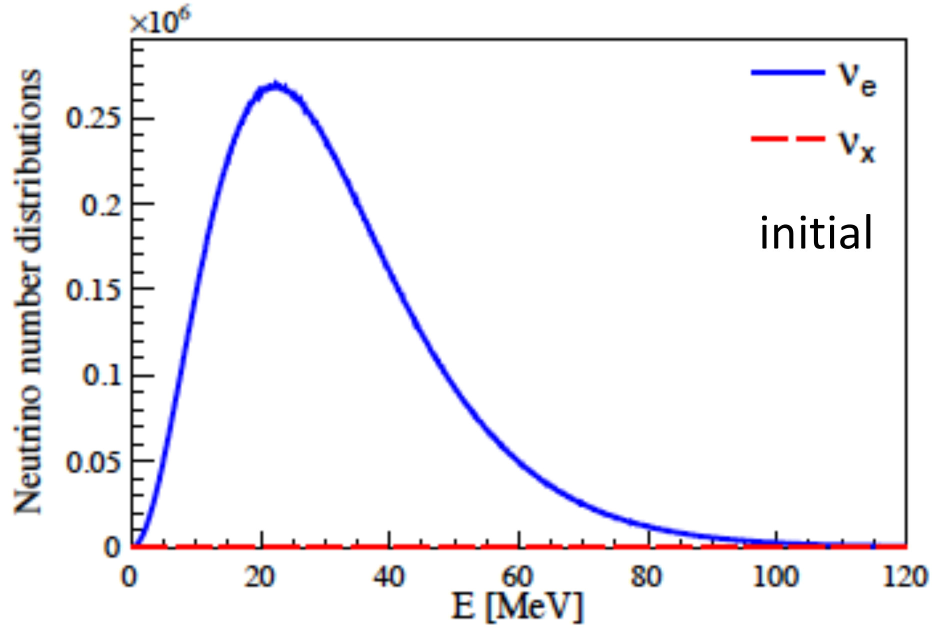
$$E_{\pm N/2} = \mp \sum_p \omega_p \frac{N_p}{2} + \mu \frac{N}{2} \left(\frac{N}{2} + 1 \right)$$

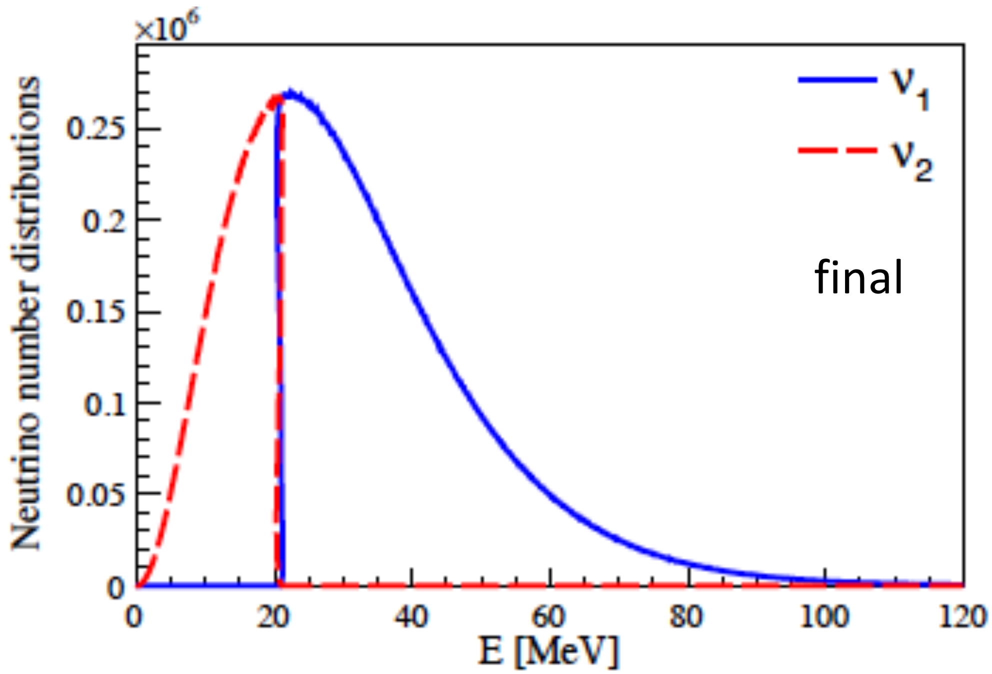
To find the others will take a lot more work

Away from the mean-field: Adiabatic solution of the *exact* many-body Hamiltonian for extremal states

Adiabatic evolution of an initial thermal distribution (T = 10 MeV) of electron neutrinos. 10^8 neutrinos distributed over 1200 energy bins with solar neutrino parameters and normal hierarchy.

Birol, Pehlivan, Balantekin, Kajino
arXiv:1805.11767
PRD98 (2018) 083002





A system of N particles each of which can occupy k states (k = number of flavors)

Exact Solution → **Mean-field approximation**

Entangled and unentangled states → Only unentangled states

Dimension of Hilbert space: k^N

Dimension of the diagonalizing space: kN

von Neumann entropy $S = -\,\mathrm{Tr}\,(\rho \log \rho)$

	Pure State	Mixed State
Density matrix	$\rho^2 = \rho$	$\rho^2 \neq \rho$
Entropy	$S = 0$	$S \neq 0$

Polarization vector for a two-level system

$$\rho = \frac{1}{2}\left(\mathbb{I} + \vec{\sigma}\cdot\vec{P}\right)$$

Pick one of the neutrinos and introduce the reduced density matrix for this neutrino (with label "b")

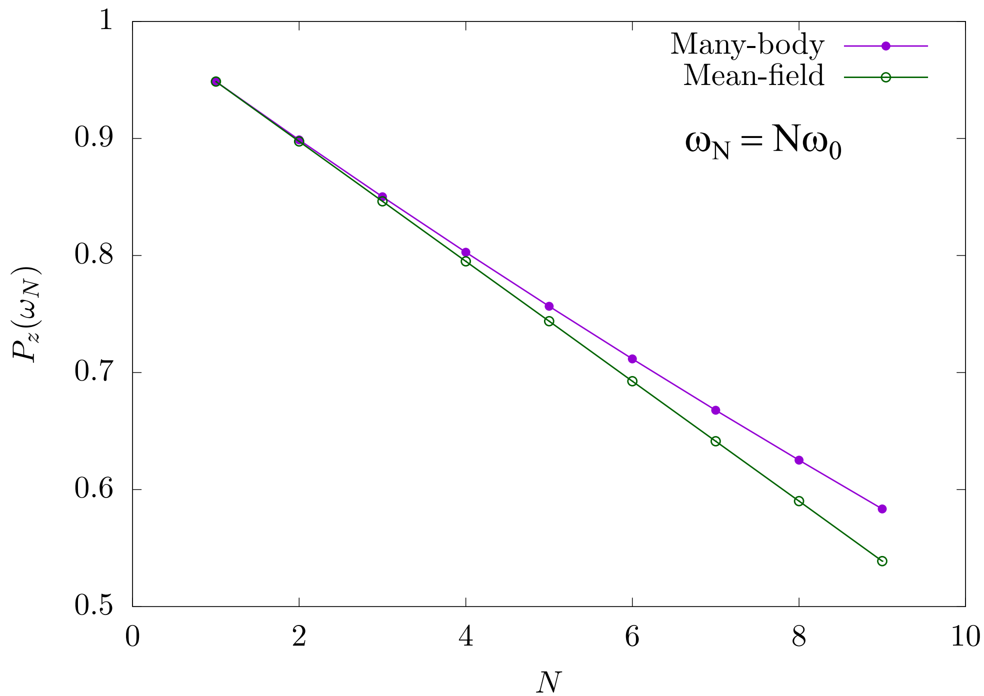
$$\tilde{\rho} = \rho_b = \sum_{a,c,d,\dots} \langle \nu_a, \nu_c, \nu_d, \cdots |\rho| \nu_a, \nu_c, \nu_d, \cdots \rangle$$

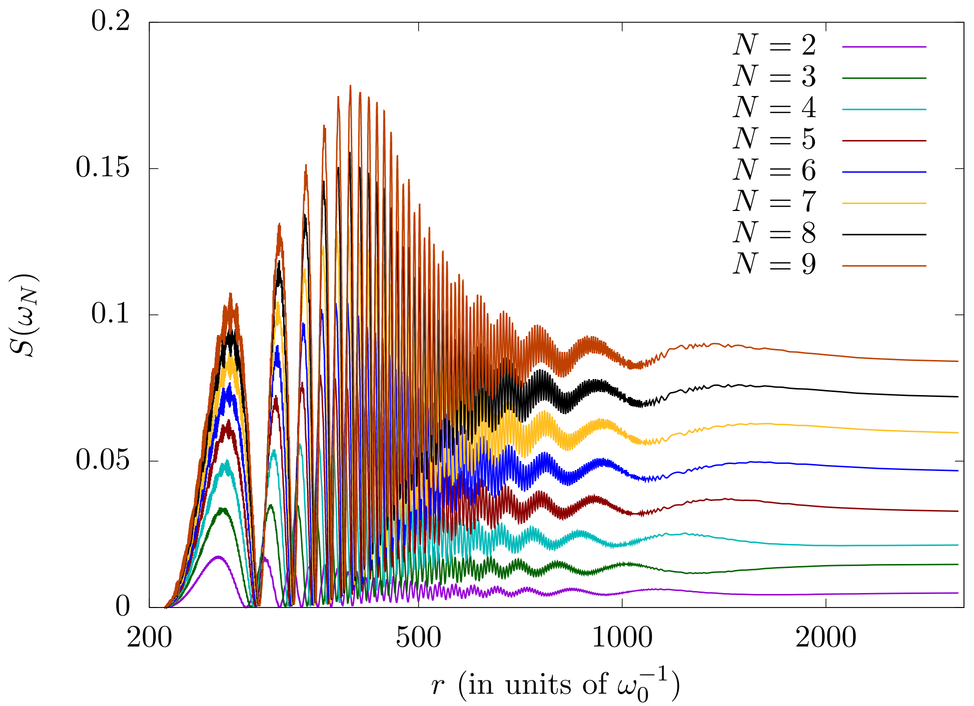
Entanglement entropy

$$S = -\mathrm{Tr}\left(\tilde{\rho}\ \log \tilde{\rho}\right)$$

$$\tilde{\rho} = \frac{1}{2}(\mathbb{I} + \vec{\sigma} \cdot \vec{P})$$

$$S = -\frac{1-|\vec{P}|}{2} \log\left(\frac{1-|\vec{P}|}{2}\right) - \frac{1+|\vec{P}|}{2} \log\left(\frac{1+|\vec{P}|}{2}\right)$$



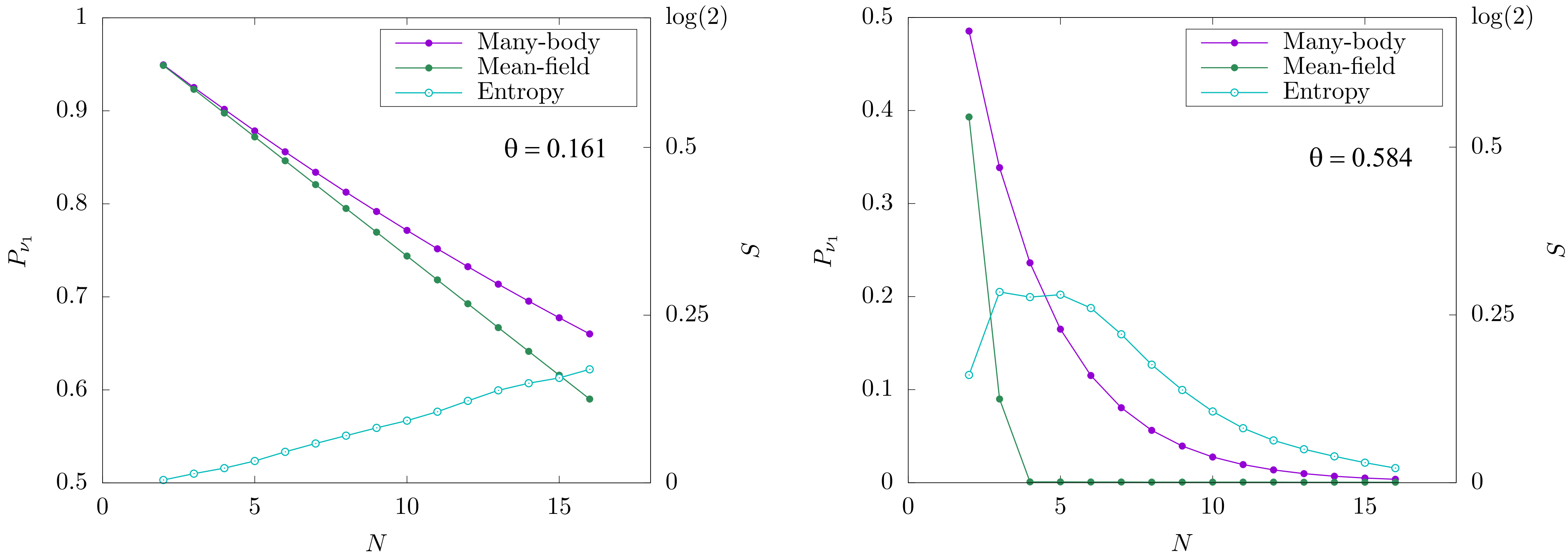


Initial state:
all electron neutrinos

Note: S = 0 for mean-field approximation

Cervia, Patwardhan, Balantekin, Coppersmith, Johnson, arXiv:1908.03511
PRD, **100**, 083001 (2019)

Comparing many-body and mean-field results



Initial state: all electron neutrinos. Probability of detecting the neutrino with the highest oscillation frequency ω_N in the first mass eigenstate and the entanglement entropy of that neutrino

Patwardhan, Cervia, Balantekin, arXiv:2109.08995

Mean Field: $\rho = \rho_1 \otimes \rho_2 \otimes \cdots \otimes \rho_N$

$$\omega_A = \frac{\delta m^2}{2E_A}$$

$\mathbf{P} = \mathrm{Tr}\,(\rho \mathbf{J})$

Mean-field evolution

$$\frac{\partial}{\partial t}\mathsf{P}^{(A)} = (\omega_A \mathcal{B} + \mu \mathsf{P}) \times \mathsf{P}^{(A)}$$

$$\mathsf{P} = \sum_A \mathsf{P}^{(A)}.$$

$$\frac{\partial}{\partial t}\mathsf{P} = \mathcal{B} \times \left(\sum_A \omega_A \mathsf{P}^{(A)} \right)$$

$\mathcal{B} \cdot \mathsf{P}$ is a constant of motion.

$$\frac{\partial}{\partial t}\mathsf{P}^{(A)} = (\omega_A \mathcal{B} + \mu \mathsf{P}) \times \mathsf{P}^{(A)}$$

$$\mathsf{P} = \sum_A \mathsf{P}^{(A)}.$$

$$\mathsf{P}^{(A)} = \alpha_A \mathcal{B} + \beta_A \mathsf{P} + \gamma_A (\mathcal{B} \times \mathsf{P}),$$

$$\sum_A \alpha_A = 0, \quad \sum_A \beta_A = 1, \quad \sum_A \gamma_A = 0.$$

If initially all N neutrinos have the same flavor, then in the mass basis would be $\alpha_0 = 0, \beta_0 = 1/N$, and $\gamma_0 = 0$.

$$\frac{\partial}{\partial t}\mathsf{P} = \left(\sum_A \beta_A \omega_A\right)(\mathcal{B} \times \mathsf{P}) + \left(\sum_A \gamma_A \omega_A\right)[(\mathcal{B} \cdot \mathsf{P})\mathcal{B} - \mathsf{P}]$$

Adopt for the mass basis and define $\Gamma = \left(\sum_A \gamma_A \omega_A\right)$. Unless Γ is positive the solutions for P_x and P_y exponentially grow.

$$P_{x,y} = \Pi_{x,y} \exp\left(-\int \Gamma(t) dt\right)$$

$$\frac{\partial}{\partial t}\Pi_x = \left(\sum_A \beta_A \omega_A\right)\Pi_y, \quad \frac{\partial}{\partial t}\Pi_y = -\left(\sum_A \beta_A \omega_A\right)\Pi_x.$$

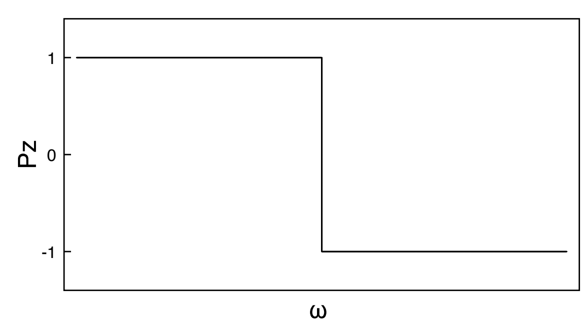
$$P_{x,y} = \Pi_{x,y} \exp\left(-\int \Gamma(t) dt\right)$$

$$\frac{\partial}{\partial t}\Pi_x = \left(\sum_A \beta_A \omega_A\right)\Pi_y, \quad \frac{\partial}{\partial t}\Pi_y = -\left(\sum_A \beta_A \omega_A\right)\Pi_x.$$

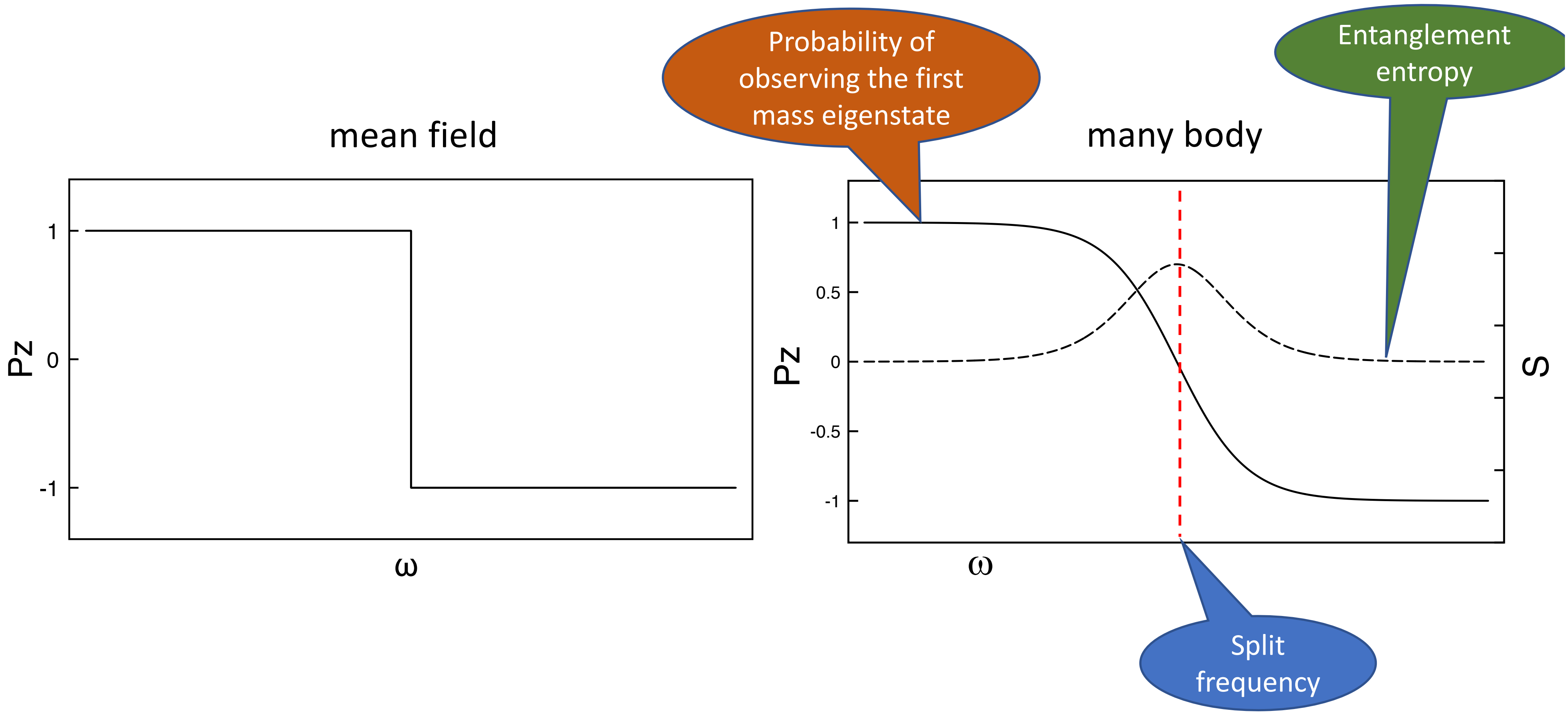
In the mean-field approximation Π_x and Π_y precess around $\mathcal{B}$ with a time-dependent frequency (through the time-dependence of β_As). Then P_x and P_y also precess similarly while decaying due to the exponential terms. Hence asymptotically P_x and P_y tend to be very small. Then x and y components of each $\mathrm{P}^{(A)}$ are asymptotically very small. Since $|\mathrm{P}^{(A)}|^2 = 1$ for uncorrelated neutrinos, it then follows that

$$\left(\mathrm{P}_z^{(A)}\right)^2 \sim 1$$

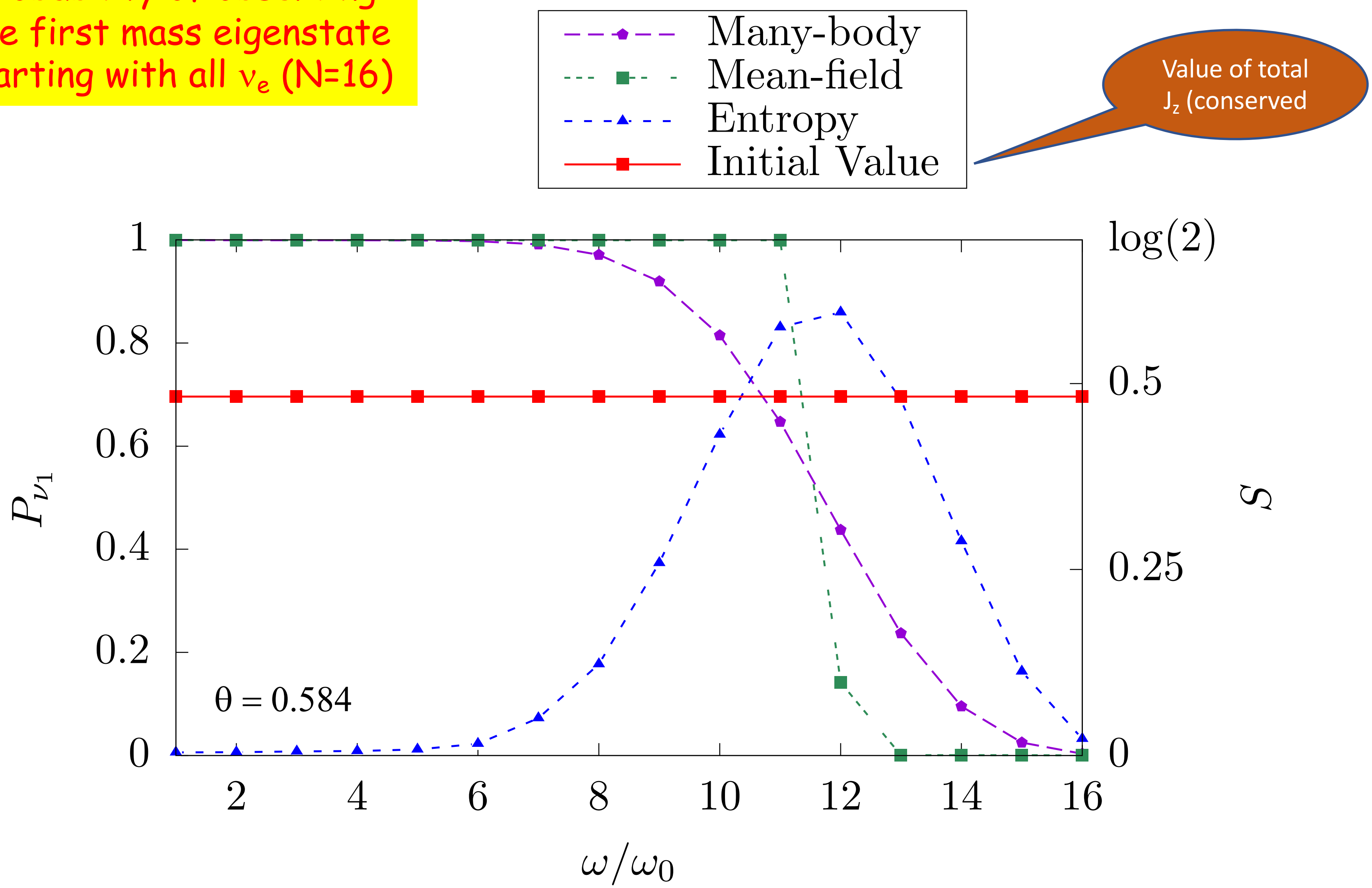
asymptotically. Consequently allowed asymptotic values of $\mathrm{P}_z^{(A)}$ are $\sim \pm 1$. Since the constant of motion $\sum_A \mathrm{P}_z^{(A)}$ (in the mass basis) is fixed by the initial conditions, some of the final $\mathrm{P}_z^{(A)}$ values will be $+1$ and some of them will be -1. This is the "spectral split" phenomenon. Depending on the initial conditions, there may exist one or more spectral splits.



We find that the presence of spectral splits is a good proxy for deviations from the mean-field results

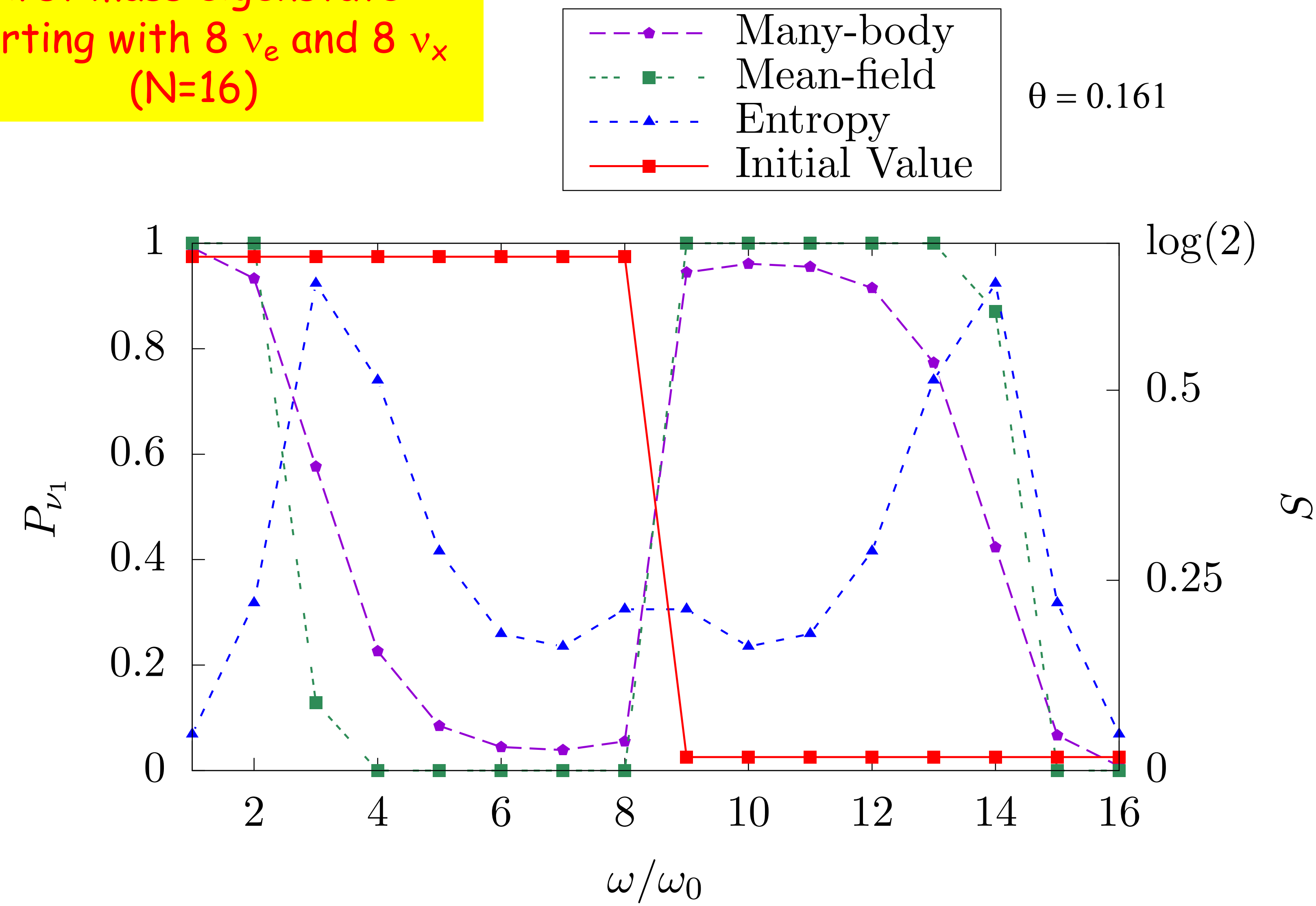


Probability of observing the first mass eigenstate starting with all ν_e (N=16)



Patwardhan, Cervia, Balantekin, arXiv:2109.08995
Phys. Rev. D **104**, 123035 (2021)

Probability of observing the first mass eigenstate starting with 8 ν_e and 8 ν_x (N=16)

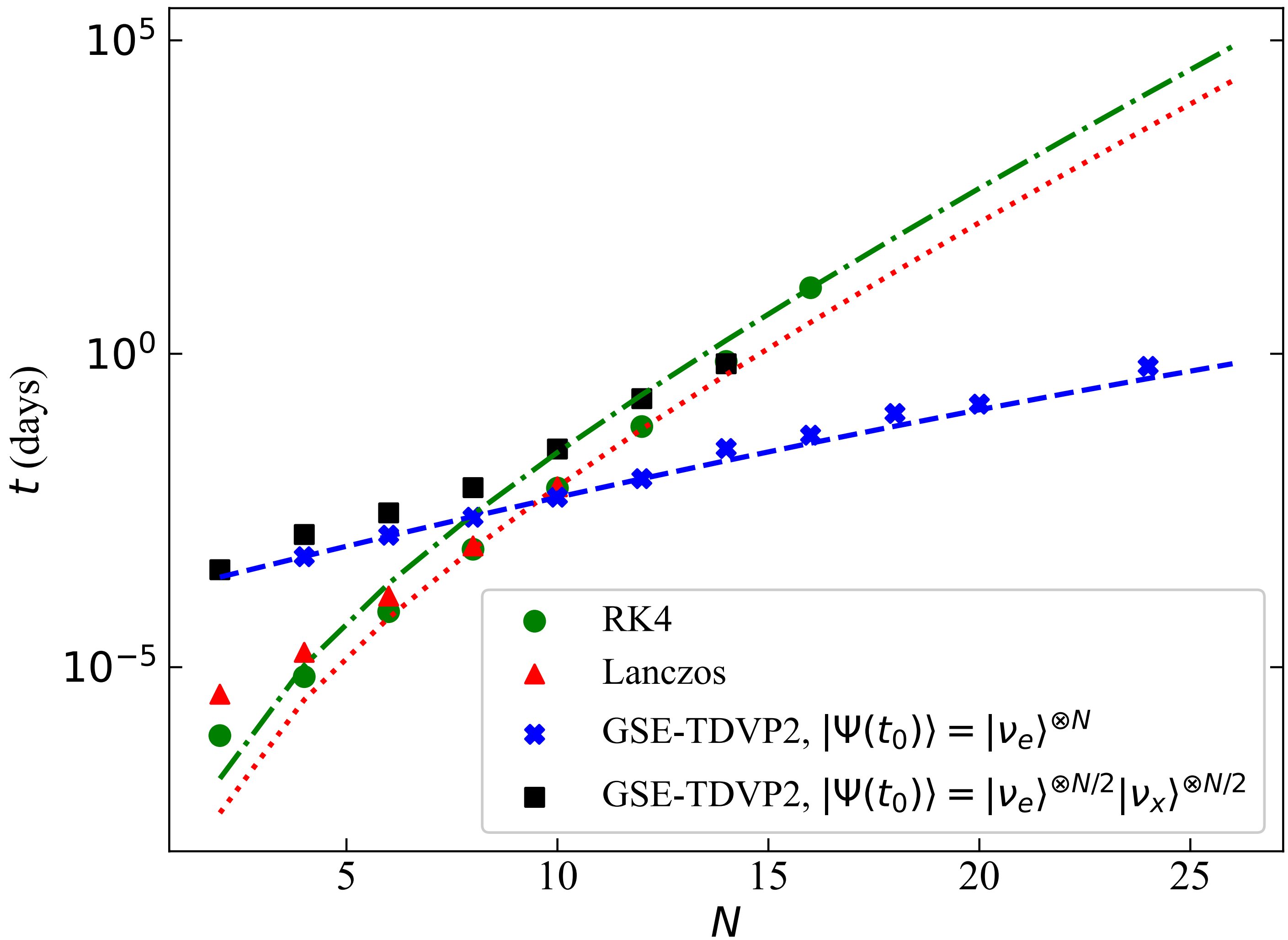


Patwardhan, Cervia, Balantekin, arXiv:2109.08995

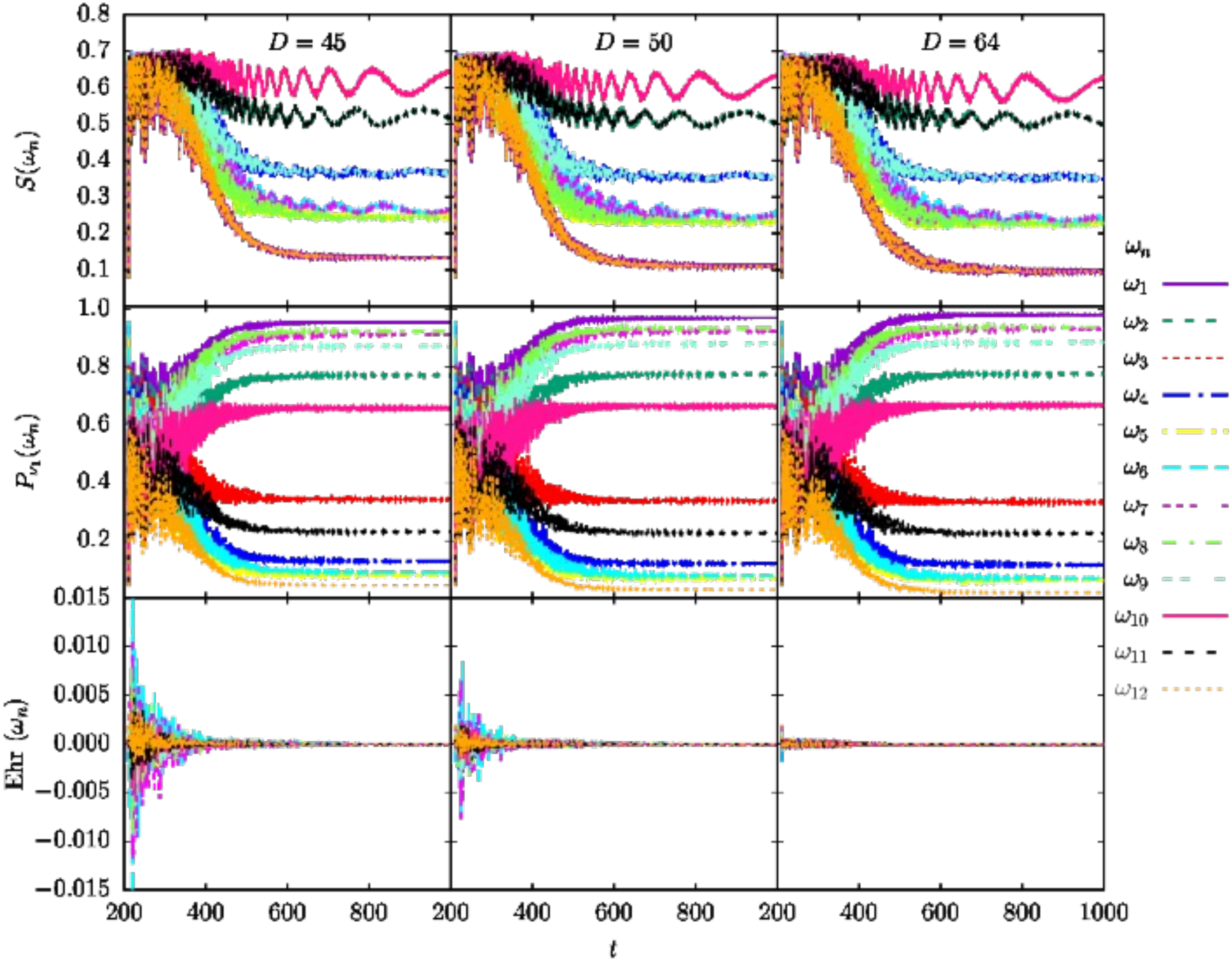
Where do we go from here?

- Explore the efficacy of tensor methods utilizing invariants obtained in the Bethe ansatz approach. ✅
  Cervia, Siwach, Patwardhan, Balantekin, Coppersmith, Johnson
- Use tensor methods to explore scaling behavior (Can you get away with smaller bond dimensions?)
  Cervia, Siwach, Patwardhan, Balantekin, Coppersmith, Johnson
- Explore the impact of using many-body solution instead of the mean-field solution in calculating element synthesis (especially r- and rp-process).
  X. Wang, Patwardhan, Cervia, Surman, Balantekin

Computation times:



Cervia, Siwach, Patwardhan, Balantekin, Coppersmith, Johnson, arXiv:2202.01865



Time evolution for 12 neutrinos (initially six ν_e and six ν_x). D is the bond dimension. The largest possible value of D is $2^6=64$.

CONCLUSIONS

- There are several anomalies at lower energy neutrino experiments which are consistent with one or more sterile neutrinos with δm^2 of the order of a few eV^2 mixing with active flavors. The signals are only 2 to 3 sigma and there are tensions between different experiments.
- However, if such sterile states exist there are interesting consequences for astrophysics and cosmology.
- The decay of a heavy sterile state into an active neutrino state is isotropic in the rest frame of the heavy state if the neutrinos are Majorana, but anisotropic if they are Dirac. It may be possible to distinguish those two cases by studying the energy distribution of the final state products in the laboratory frame.

CONCLUSIONS

- Calculations performed using the mean-field approximation have revealed a lot of interesting physics about collective behavior of neutrinos in astrophysical environments. Here we have explored possible scenarios where further interesting features can arise by going beyond this approximation.
- We found that the deviation of the adiabatic many-body results from the mean field results is largest for neutrinos with energies around the spectral split energies. In our single-angle calculations we observe a broadening of the spectral split region. This broadening does not appear in single-angle mean-field calculations and seems to be larger than that was observed in multi-angle mean-field calculations (or with BSM physics).
- This suggests hybrid calculations may be efficient: many-body calculations near the spectral split and mean-field elsewhere.
- There is a strong dependence on the initial conditions.