Exploring the QCD phase diagram with Dyson–Schwinger equations

P.I., Buballa, Fischer, Gunkel, PRD 100 (2019) 074011, arXiv:1906.11644
P.I., Fischer, Steinert, PRD 103 (2021) 054012, arXiv:2012.04991
Bernhardt, Fischer, P.I., Schaefer, arXiv:2107.05504

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HGS-HIRe for FAIR
Helmholtz Graduate School for Hadron and Ion Research

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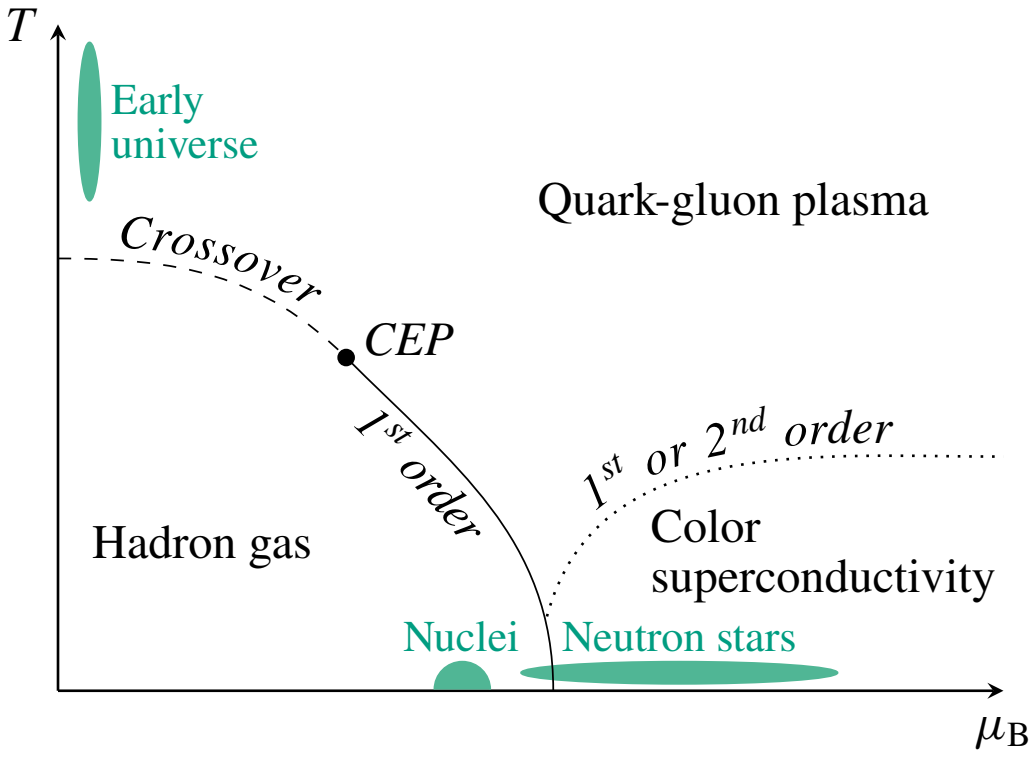
42nd course of the Erice International School of Nuclear Physics
"QCD under extreme conditions—from heavy-ion collisions to the phase diagram"
16–22 September 2021, Erice, Italy

Phase structure of QCD



- Lattice QCD ... limited due to the sign problem
- Effective models ... only effective d.o.f.; no gluons
- Functional methods ... full QCD d.o.f. & no sign problem (but truncations are necessary)

This talk: Dyson–Schwinger equations

Dyson–Schwinger equations

- Nonperturbative functional approach
- Correlation functions on quark-gluon level
- Bound states as composite objects of quarks and gluons (Bethe-Salpeter/Faddeev equations)

Working areas:

Hadron physics

- Meson and baryon properties
- Spectra
- Scattering amplitudes
- Decays
- Form factors
- Exotics (tetraquarks, glueballs, and hybrids)

Nonzero T and μ

- Phase structure of QCD
- Thermodynamics
- In-medium properties of mesons

Additionally

- Muon $g-2$ (HLbL)
- Analytic structure of propagators
- ...

Reviews: Eichmann, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016) 1
Fischer, PPNP 105 (2019) 1

How to truncate?

Quark DSE

Dressed quark-gluon vertex:

- Explicit solutions at $T = 0$

  Fischer, Williams, PRL 103 (2009) 122001
  Mitter, Pawlowski, Strodthoff, PRD 91 (2015) 054035
  Williams, EPJA 51 (2015) 53
  Williams, Fischer, Heupel, PRD 93 (2016) 034026
  Gao, Papavassiliou, Pawlowski, PRD 103 (2021) 094013

- $T \neq 0$:
  - Solve (parts of) the vertex

    Contant, Huber, Fischer, Williams, Welzbacher, Acta Phys. Polon. B Proc. Supp. 11 (2018) 483
  - Expand about FRG vacuum

    Gao, Pawlowski, PRD 102 (2020) 034027
    Gao, Pawlowski, PLB 820 (2021) 136584
  - Here: ansatz based on STI and perturbative UV

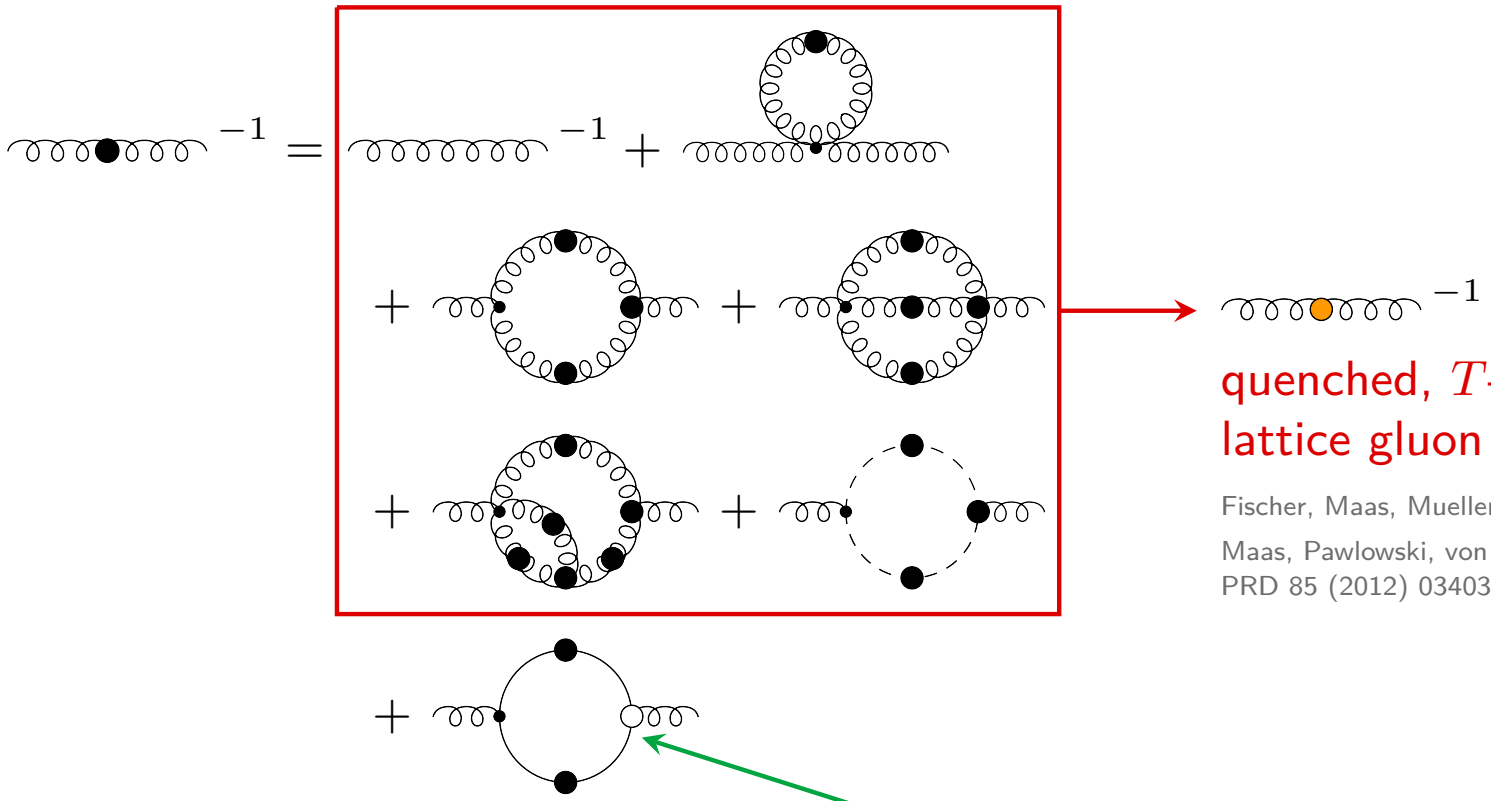
    Fischer, PPNP 105 (2019) 1 and refs. therein

Dressed gluon propagator:

- Two strategies:
  - Model gluon propagator

    Qin, Chang, Chen, Liu, Roberts, PRL 106 (2011) 172301
    Gao, Liu, PRD 94 (2016) 076009
  - Explicit treatment of gluonic sector
- Here: use the latter
  - Consistent flavor dependencies
  - Gluon becomes sensitive to chiral dynamics
  - Generally: backcoupling (unquenching) effects important

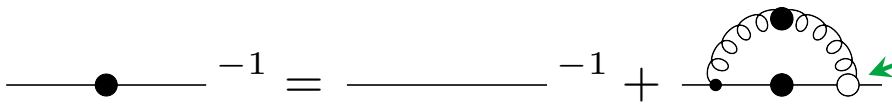
How to truncate?



quenched, T-dependent lattice gluon propagator

Fischer, Maas, Mueller, EPJC 68 (2010) 165

Maas, Pawlowski, von Smekal, Spielmann, PRD 85 (2012) 034037



(T, μ)-dependent ansatz for quark-gluon vertex

Fischer, Luecker, Welzbacher, PRD 90 (2014) 034022 (and references therein)

Benchmark at $\mu = 0$

P.I., Buballa, Fischer, Gunkel, PRD 100 (2019) 074011

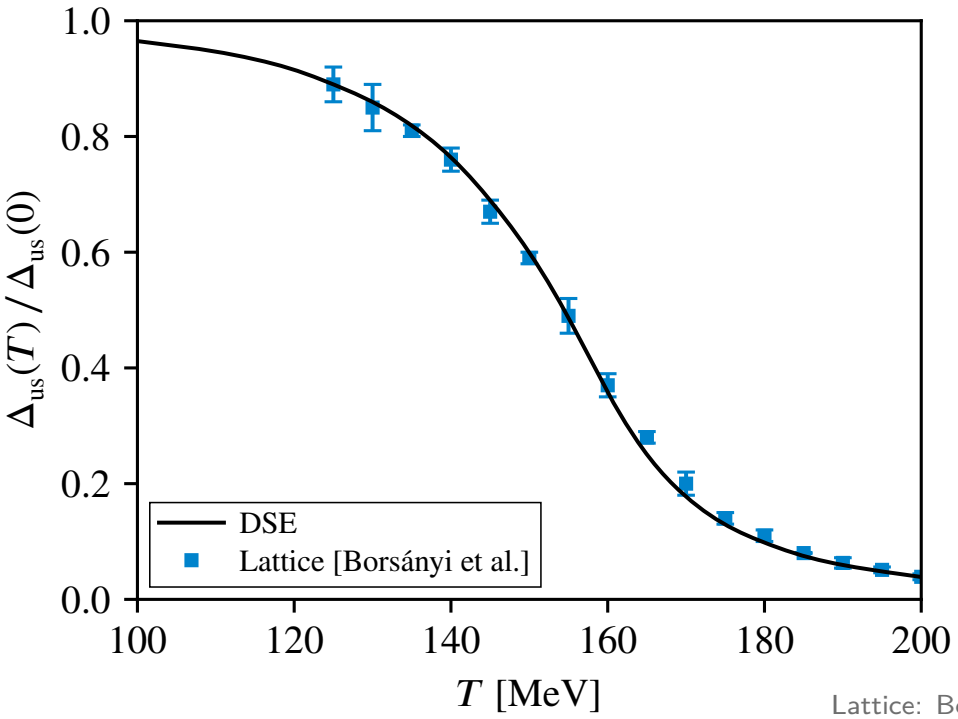
Chiral order parameter:

Quark condensate

$$\langle\bar{\psi}\psi\rangle_f \sim \mathrm{Tr}[S_f]$$

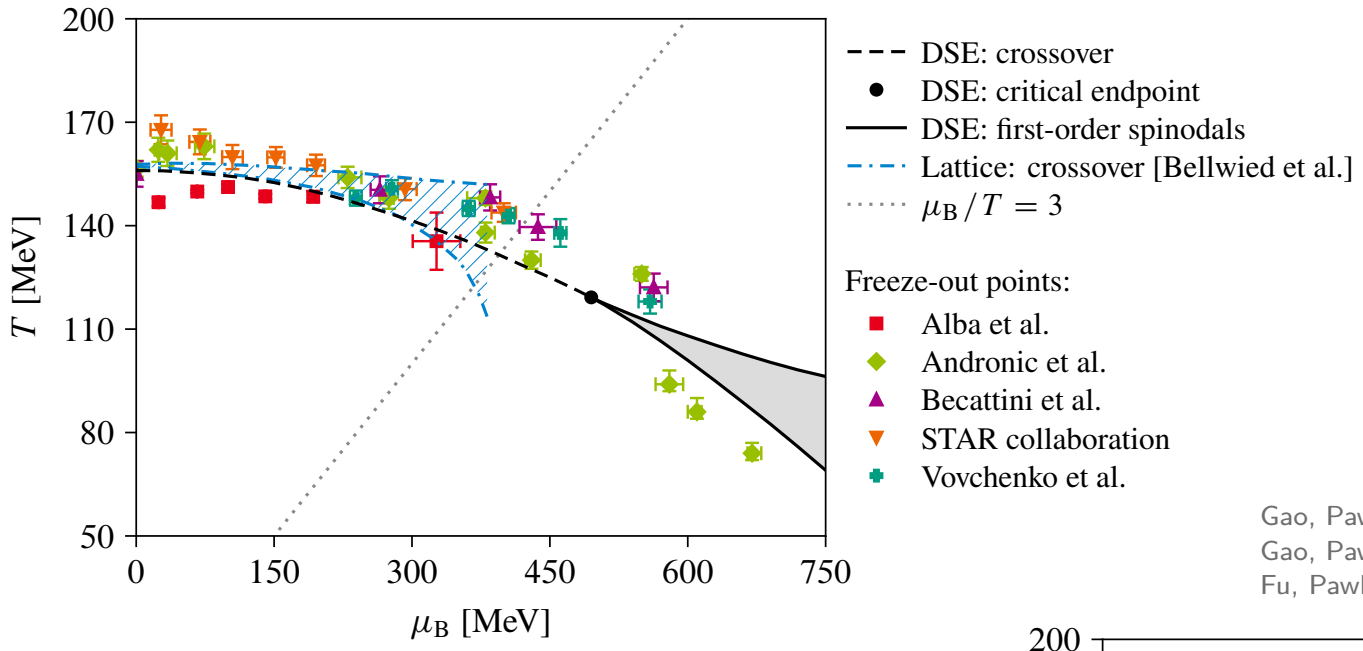
Subtracted condensate

$$\Delta_{ff'} = \langle\bar{\psi}\psi\rangle_f - \frac{m_f}{m_{f'}}\langle\bar{\psi}\psi\rangle_{f'}$$



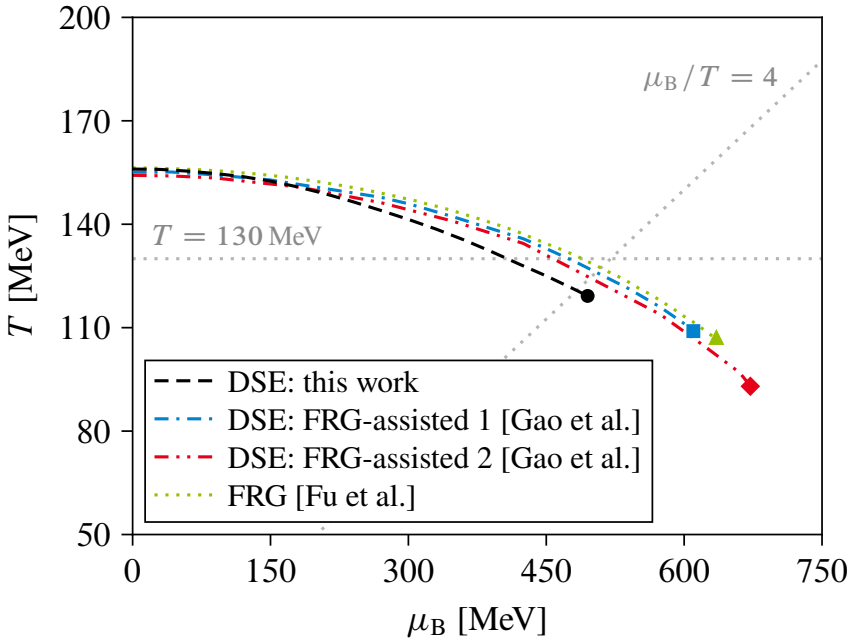
Lattice: Borsányi et al., JHEP 09 (2010) 073

QCD's phase diagram



P.I., Buballa, Fischer, Gunkel, PRD 100 (2019) 074011

Gao, Pawlowski, PLB 820 (2021) 136584
Gao, Pawlowski, PRD 102 (2020) 034027
Fu, Pawlowski, Rennecke, PRD 101 (2020) 054032



- CEPs cluster around

$$490\,\mathrm{MeV} \lesssim \mu_B \lesssim 680\,\mathrm{MeV}$$
$$90\,\mathrm{MeV} \lesssim T \lesssim 120\,\mathrm{MeV}$$

- No CEP for $\mu_B/T \lesssim 4$

Fluctuations

Fluctuations from QCD's grand-canonical potential

$$\chi_{ijk}^{\mathrm{BQS}} = -\frac{1}{T^{4-(i+j+k)}} \frac{\partial^{i+j+k}\,\Omega}{\partial\mu_{\mathrm{B}}^{i}\,\partial\mu_{\mathrm{Q}}^{j}\,\partial\mu_{\mathrm{S}}^{k}}$$

- Relation to cumulants of probability distribution:

$$C_n^X = V T^3 \chi_n^X$$

- Statistical quantities:

$$\sigma_X^2 = C_2^X\,, \qquad S_X = C_3^X (C_2^X)^{-3/2}\,, \qquad \kappa_X = C_4^X (C_2^X)^{-2}$$

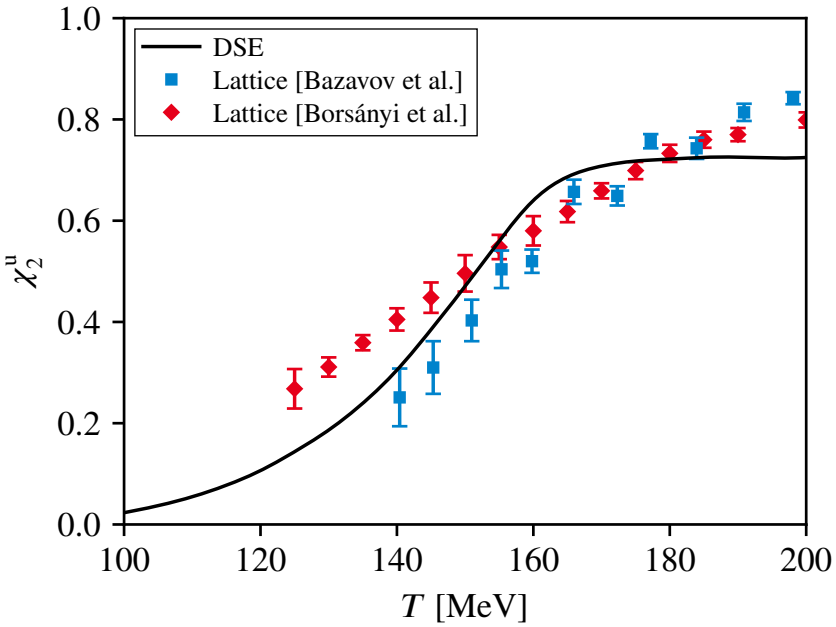
- Prominent quantities are ratios:

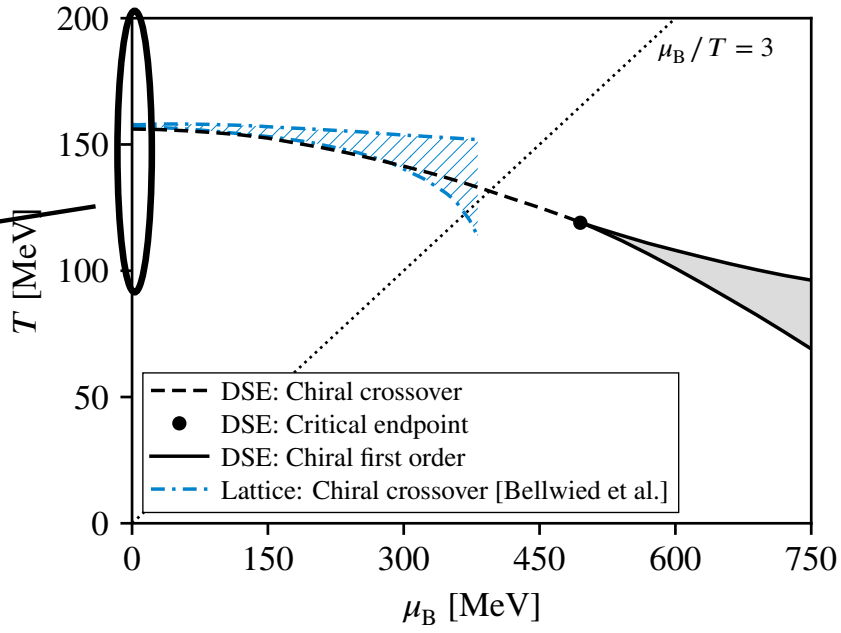
$$\frac{\chi_3^{\mathrm{B}}}{\chi_2^{\mathrm{B}}} = S_{\mathrm{B}}\,\sigma_{\mathrm{B}}\,, \qquad \frac{\chi_4^{\mathrm{B}}}{\chi_2^{\mathrm{B}}} = \kappa_{\mathrm{B}}\,\sigma_{\mathrm{B}}^2$$

Reviews: Luo, Xu, Nucl. Sci. Tech. 28 (2017) 112
Bzdak, Esumi, Koch, Liao, Stephanov, Xu, Phys. Rep. 853 (2020) 1

Fluctuations from DSEs

P.I., Buballa, Fischer, Gunkel, PRD 100 (2019) 074011





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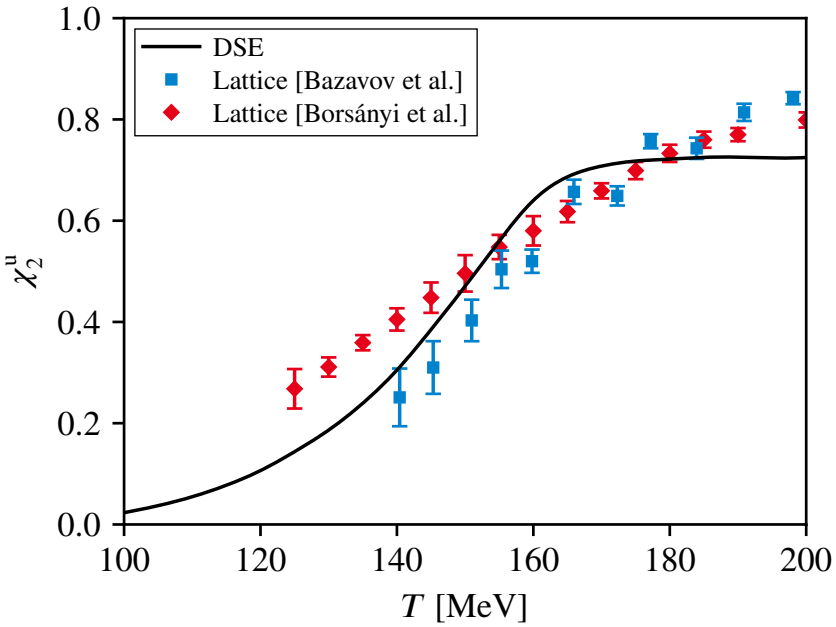

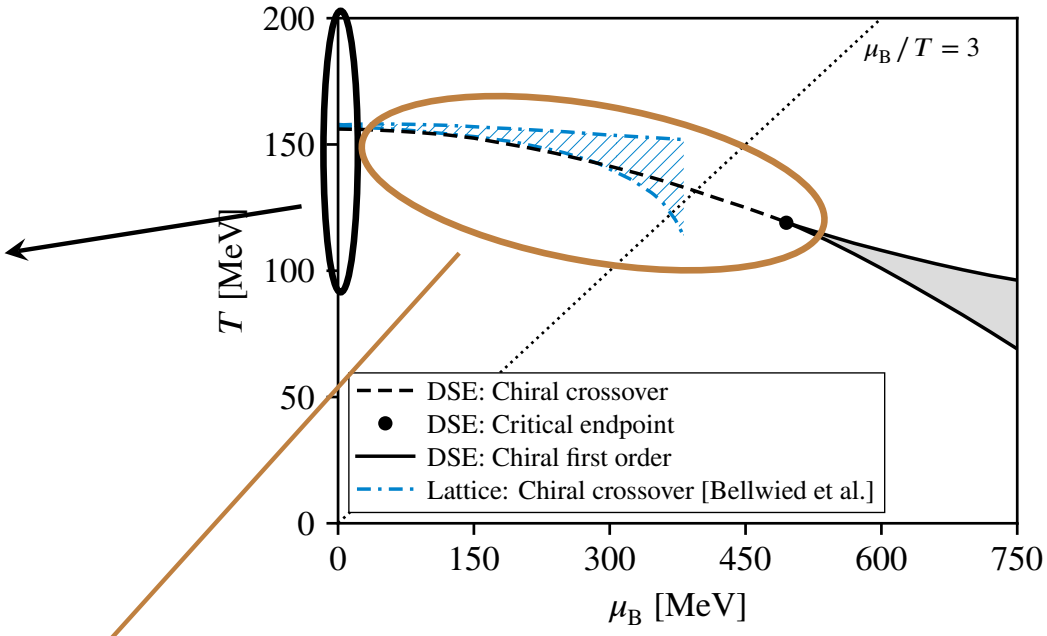

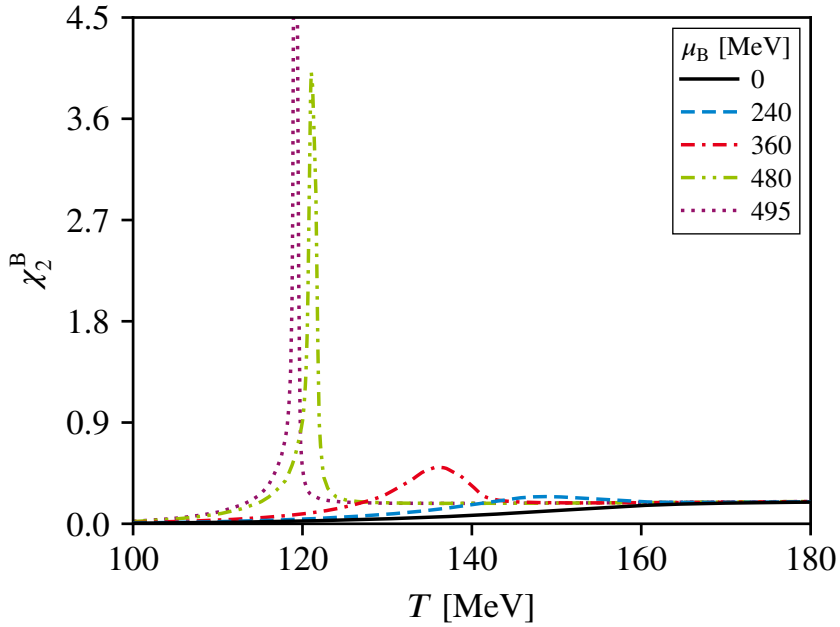

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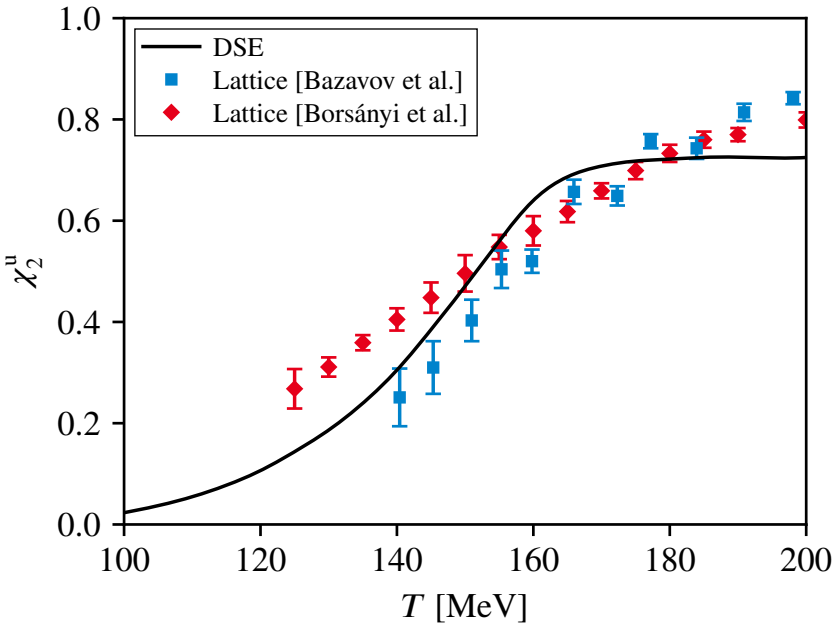

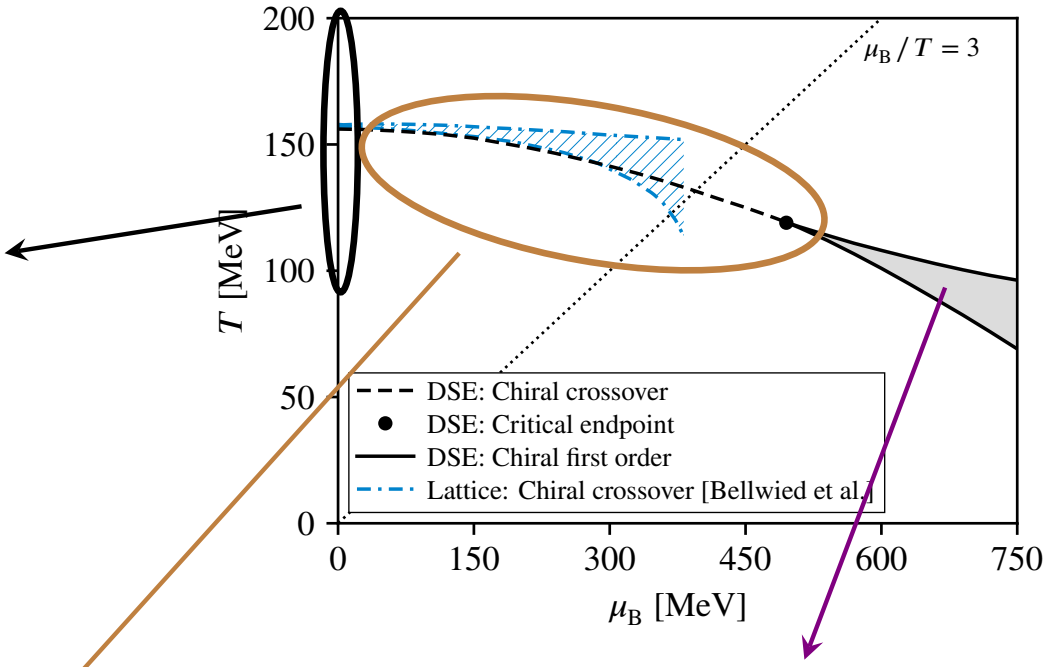

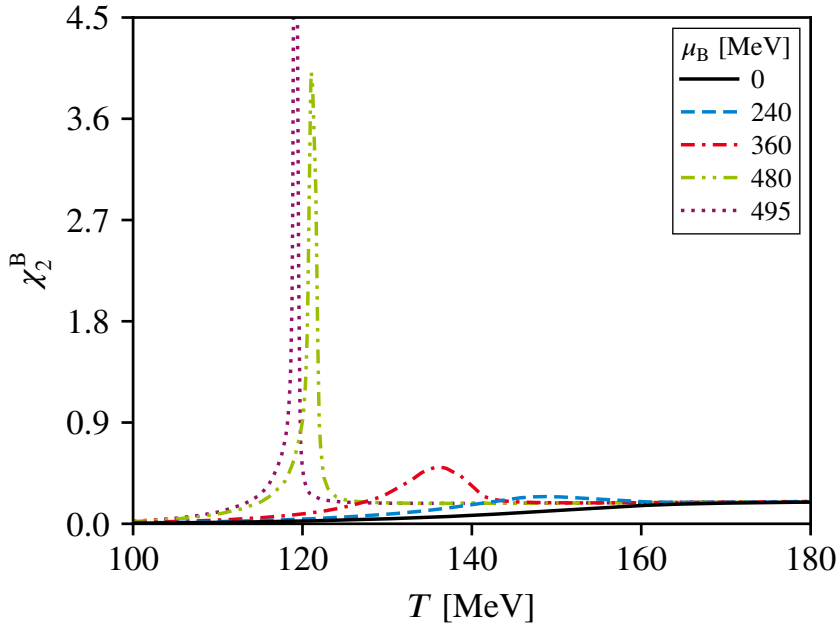

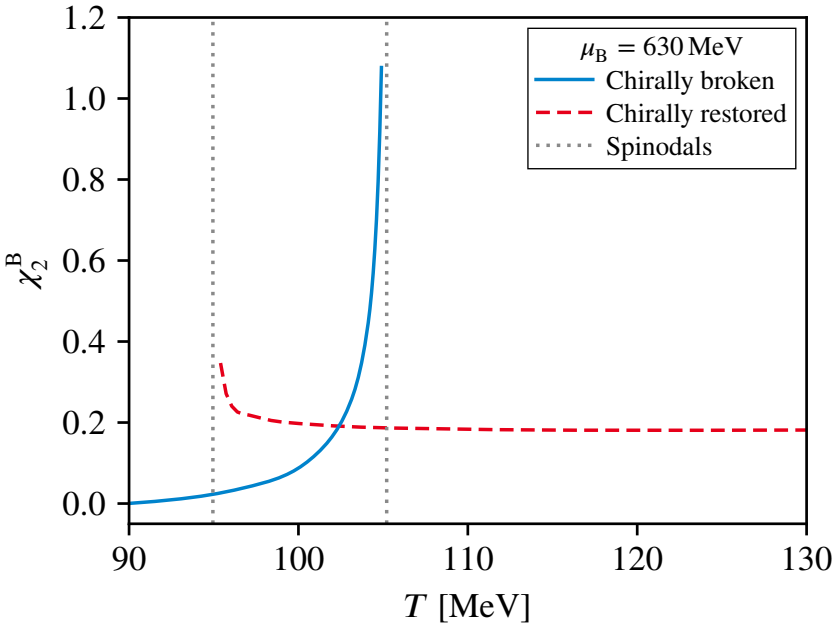

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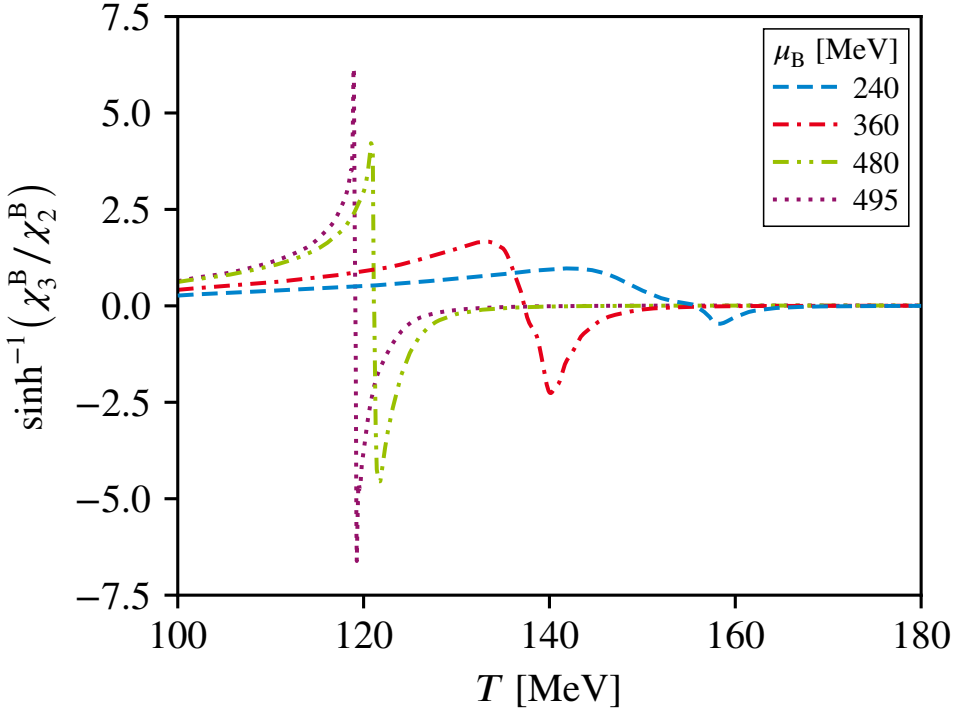
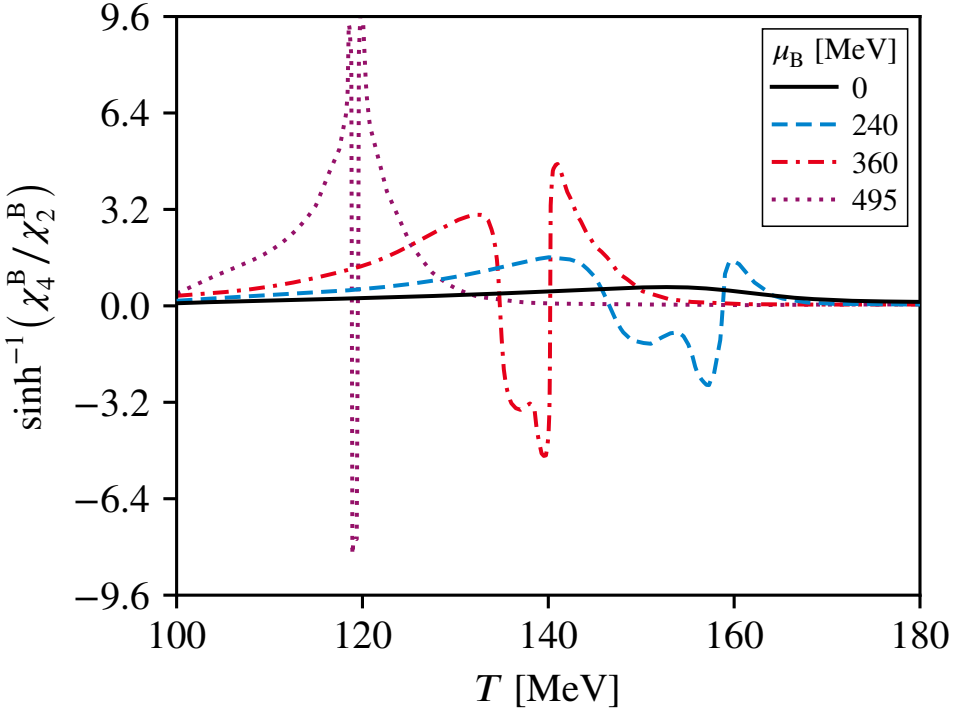
P.I., Buballa, Fischer, Gunkel, PRD 100 (2019) 074011





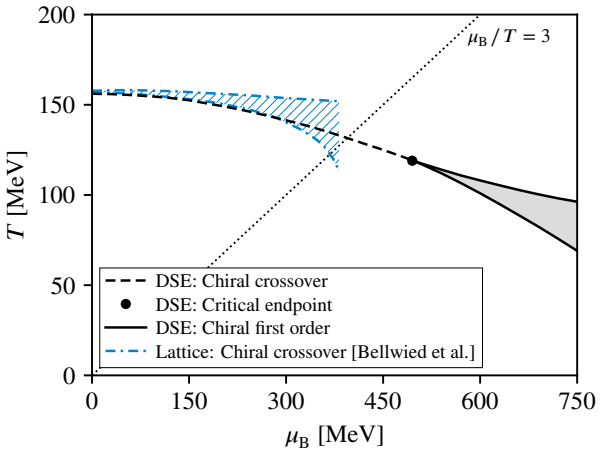
Very sensitive to phase structure & clear signals for CEP

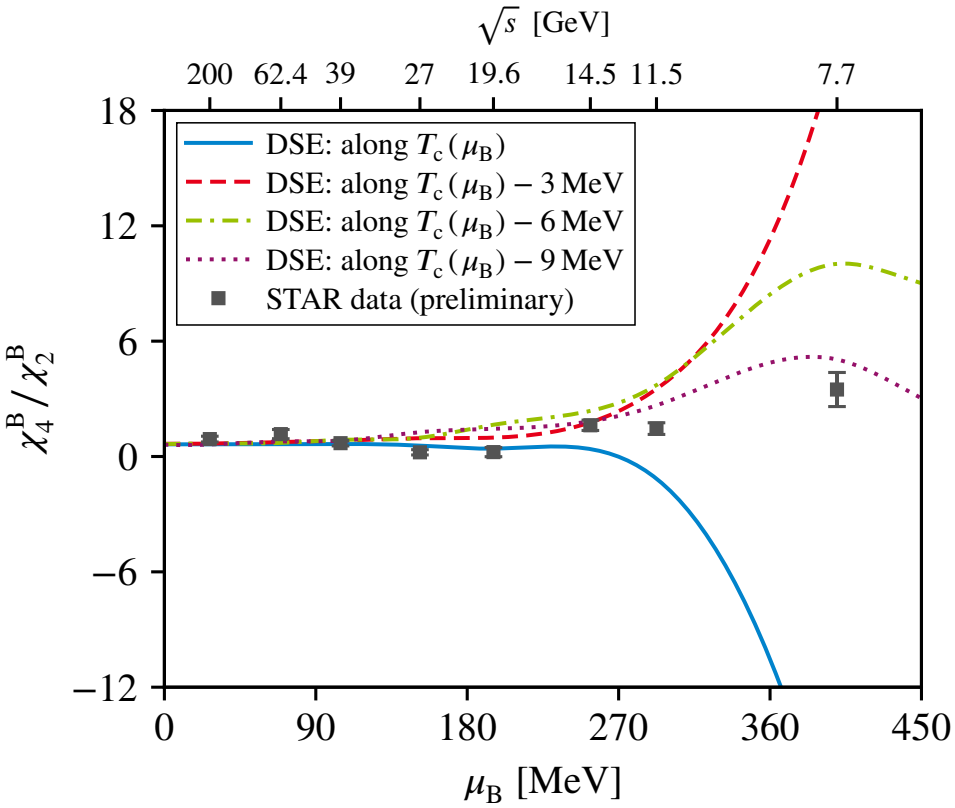
Caveats when comparing with experiment:

- No off-diagonal fluctuations yet
- Only "naive" strangeness neutrality
- Critical region may be too large
- Ordering of freeze-out points

Fluctuations from DSEs

P.I., Buballa, Fischer, Gunkel, PRD 100 (2019) 074011





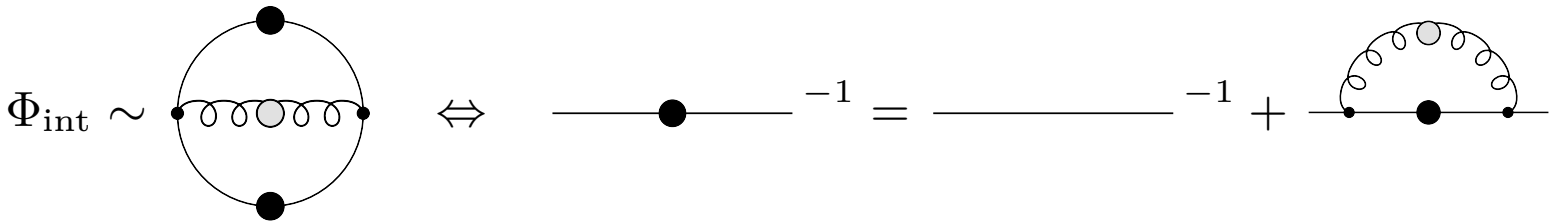
- $\sqrt{s} > 14.5\,\mathrm{GeV}$: Good agreement; variation of $T_c(\mu_B)$ has only mild impact
- $\sqrt{s} = 14.5\,\mathrm{GeV}$: Trend ok; freeze-out close to crossover favored
- $\sqrt{s} \leq 11.5\,\mathrm{GeV}$: freeze-out line $\neq$ crossover line

Thermodynamics from DSEs

- 2PI formalism (quark-only):
  Cornwall, Jackiw, Tomboulis, PRD 10 (1974) 2428

$$\Omega[S] = -\frac{T}{V}\left(\mathrm{Tr}\log\frac{S^{-1}}{T} - \mathrm{Tr}\left[\mathbb{1} - S_0^{-1}S\right] + \Phi_{\mathrm{int}}[S]\right)$$

- Physical propagator from stationary condition: $\delta\Omega/\delta S = 0$
  $\Rightarrow S^{-1} = S_0^{-1} + \Sigma$ with $\Sigma \sim \delta\Phi_{\mathrm{int}}/\delta S$
- Closed form for Φ_{int}: quark-gluon vertex must not depend on quark



- So far: thermodynamics from DSEs only in rainbow-ladder truncation
  Blaschke, Roberts, Schmidt, PLB 425 (1998) 232
  Xu, Yan, Cui, Zong, IJMPA 30 (2015) 1550217
  Gao et al., PRD 93 (2016) 094019
- Needed: Ω from a **truncation-independent** method

Thermodynamics from DSEs

P.I., Fischer, Steinert, PRD 103 (2021) 054012

- Consider: $\Omega = \Omega(T, \mu; m)$
- Current-quark mass: external source for bilinear $\bar{\psi}\psi$
  $\Rightarrow \langle\bar{\psi}\psi\rangle(T, \mu; m) = \partial\Omega(T, \mu; m)/\partial m$
- Integrate:

$$\Omega(T, \mu; m_2) - \Omega(T, \mu; m_1) = \int_{m_1}^{m_2} \mathrm{d}m' \, \langle\bar{\psi}\psi\rangle(T, \mu; m')$$

- Ω and $\langle\bar{\psi}\psi\rangle$ are divergent but **suitable derivatives are finite!**
- Derivative w.r.t. T:

$$s(T, \mu; m_2) - s(T, \mu; m_1) = -\int_{m_1}^{m_2} \mathrm{d}m' \, \frac{\partial\langle\bar{\psi}\psi\rangle}{\partial T}(T, \mu; m')$$

- Integral limits: let $m_1 = m$ and $m_2 \to \infty$

General relation between s and $\langle\bar{\psi}\psi\rangle$

$$s(T, \mu; m) = s_{\mathrm{YM}}(T) + \int_{m}^{\infty} \mathrm{d}m' \, \frac{\partial\langle\bar{\psi}\psi\rangle}{\partial T}(T, \mu; m')$$

Thermodynamics from DSEs

P.I., Fischer, Steinert, PRD 103 (2021) 054012

Maxwell relation

$$\frac{\partial^2 \Omega}{\partial m \partial T} = \frac{\partial^2 \Omega}{\partial T \partial m} \quad \Rightarrow \quad -\frac{\partial s}{\partial m} = \frac{\partial \langle \bar{\psi}\psi \rangle}{\partial T}$$

- Pressure at vanishing chemical potential:

$$p(T,0) = p(T_0,0) + \int_{T_0}^{T} \mathrm{d}T' \, s(T',0)$$

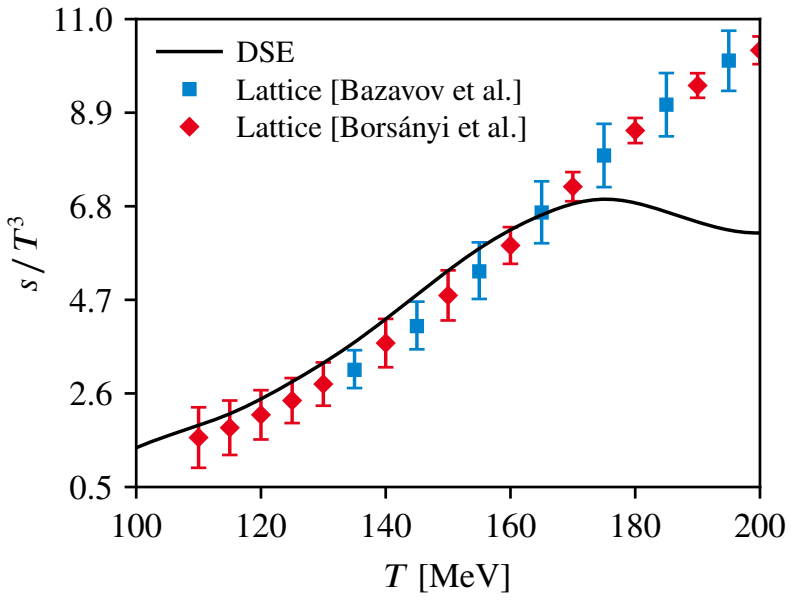
- Nonzero chemical potential:

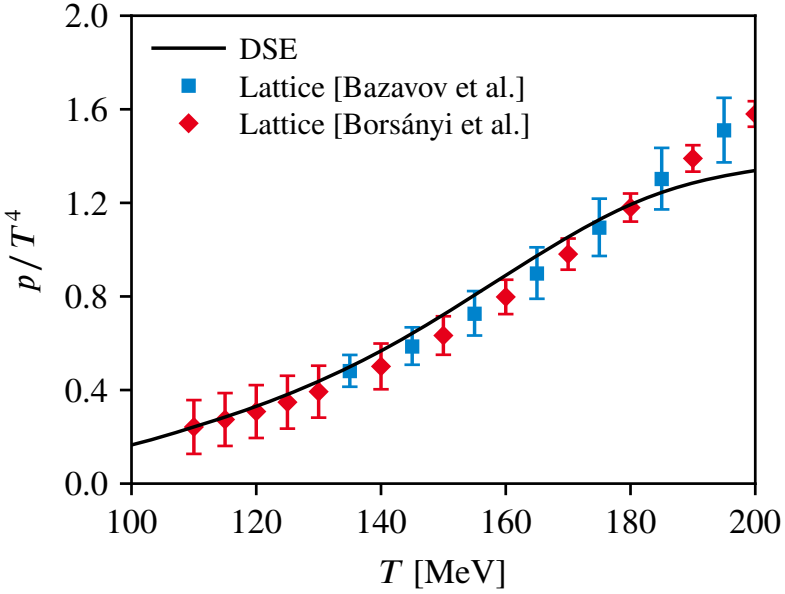
$$p(T,\mu) = p(T_0,0) + \int_{T_0}^{T} \mathrm{d}T' \, s(T',0) + \int_{0}^{\mu} \mathrm{d}\mu' \, n(T,\mu')$$

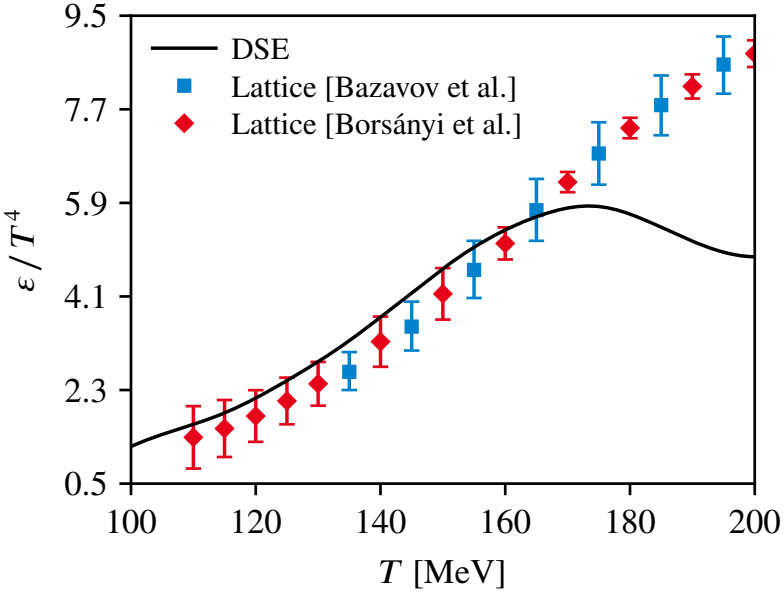
- **Applicable as soon as the quark condensate is available!**

Thermodynamics from DSEs ($\mu = 0$)

P.I., Fischer, Steinert, PRD 103 (2021) 054012

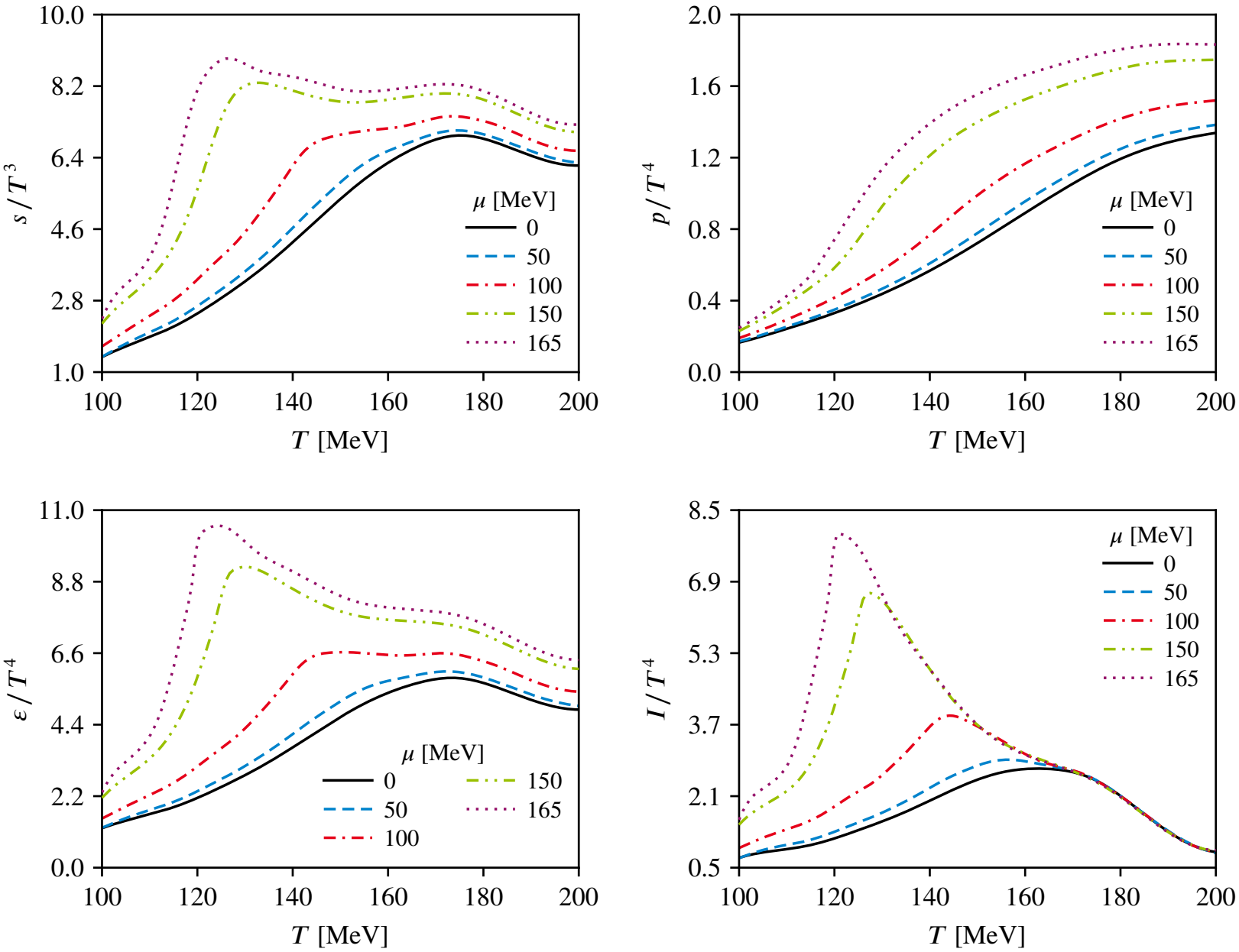






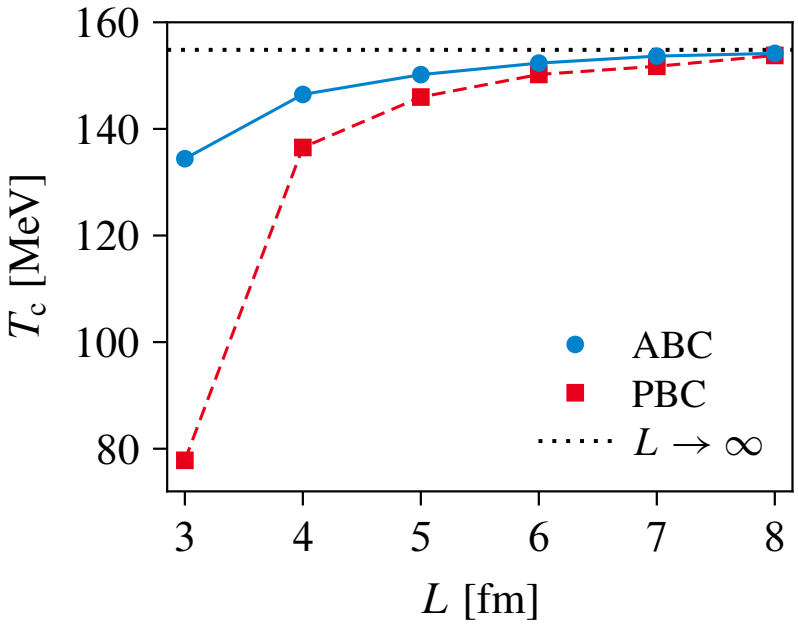
Thermodynamics from DSEs ($\mu \neq 0$)

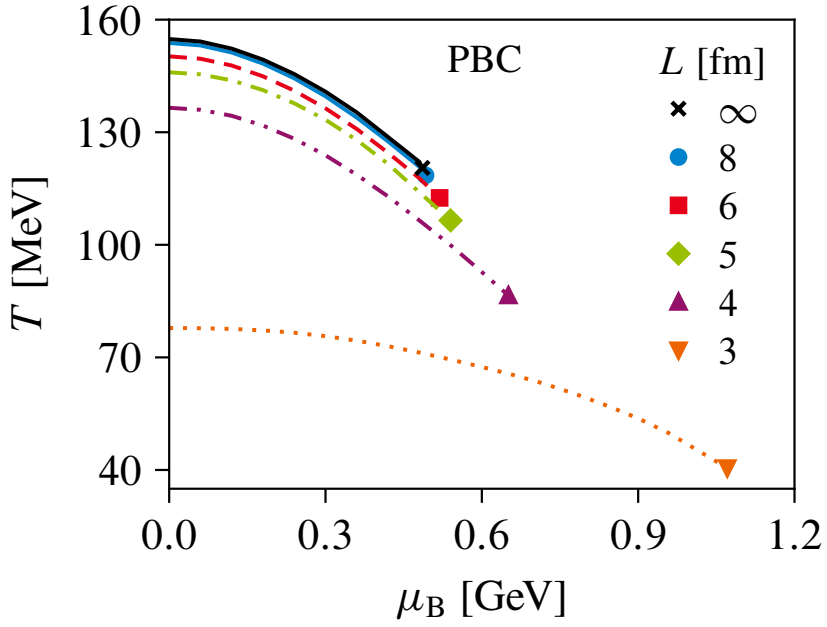
P.I., Fischer, Steinert, PRD 103 (2021) 054012



Finite-volume effects

Bernhardt, Fischer, P.I., Schaefer, arXiv:2107:05504





$\to$ talk by Julian Bernhardt
(in a few minutes)

Epilogue

Phase diagram with functional methods:

- No CEP for $\mu_\mathrm{B}/T \lesssim 4$
- CEPs seem to cluster around $490\,\mathrm{MeV} \lesssim \mu_\mathrm{B} \lesssim 680\,\mathrm{MeV}$ and $90\,\mathrm{MeV} \lesssim T \lesssim 120\,\mathrm{MeV}$

Fluctuations:

- First results within DSEs beyond simple rainbow-ladder truncation
- Results support a freeze-out line that bends below CEP

Thermodynamics:

- Truncation-independent way to compute thermodynamic quantities using DSEs
- High-quality quark-gluon vertex needed at high T and/or μ_B

Finite-volume effects:

$\rightarrow$ next talk by Julian Bernhardt

Outlook:

- Working toward unified statement regarding CEP location
- Fluctuations in a finite volume (wip)