Investigating three-baryon interactions using femtoscopy in pp collisions with ALICE

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On behalf of the ALICE Collaboration

International School of Nuclear Physics - 42nd Course:
QCD under extreme conditions from heavy-ion collisions to the phase diagram

Erice (Italy), 16-22 September 2021

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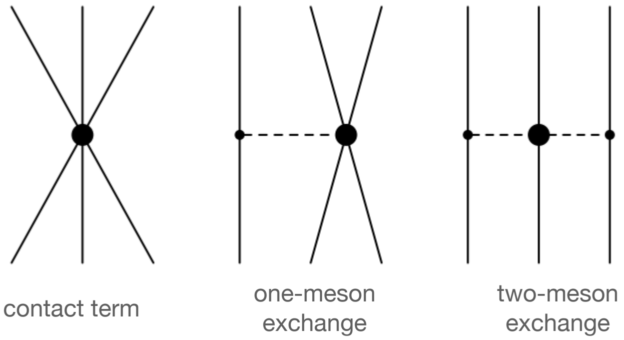
Three-body forces

- Many-body systems **cannot be described satisfactorily** with two-body forces only.

- Fundamental ingredient for the microscopic description of the **Equation of State (EoS)** of neutron stars.

Three-baryon interaction diagrams in χEFTs



- The parameters of the models are tuned using the binding energies of nuclei and hypernuclei but…

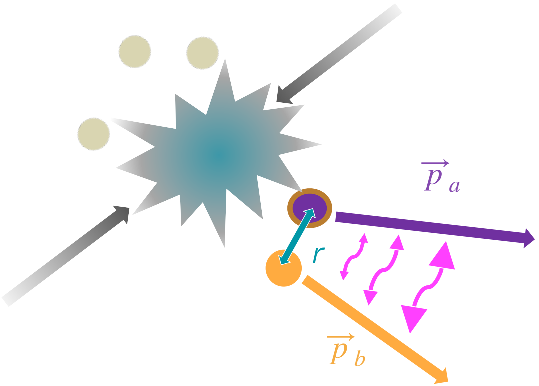
  1. such measurements yield the **superposition of two- and many-body effects**;
  2. the interaction is tested at **"large" distances.**

  ➡ In ^{12}C the average distance among nucleons is $< d > \sim 2.2$ fm

Investigating hadronic interactions at LHC

ALICE at the LHC

Hadron-hadron strong interactions

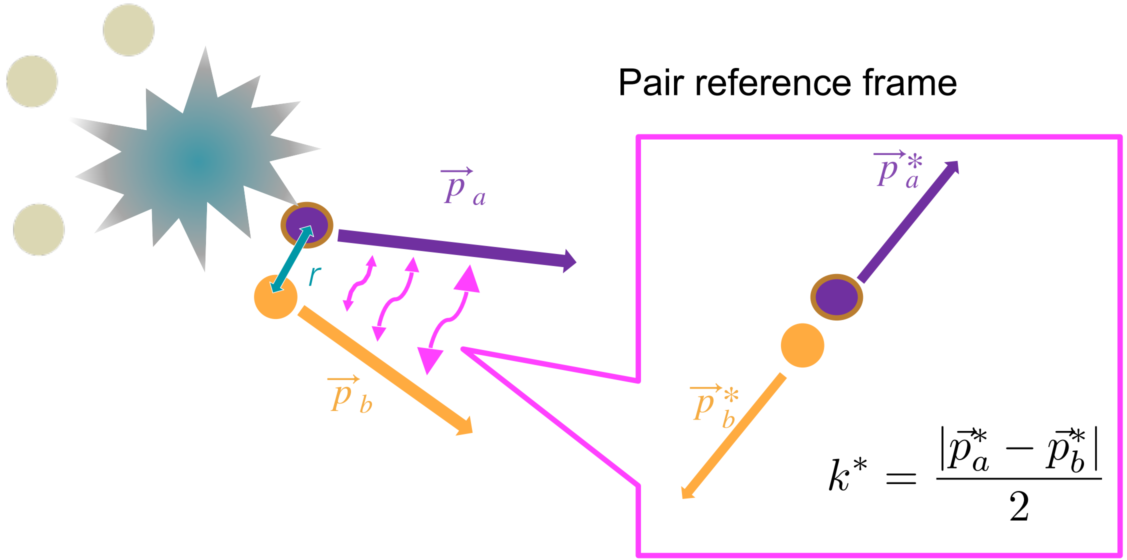


Femtoscopy technique

Two-body correlation function

$$C(\mathbf{p}_a, \mathbf{p}_b) \equiv \frac{P(\mathbf{p}_a, \mathbf{p}_b)}{P(\mathbf{p}_a) \cdot P(\mathbf{p}_b)}$$

Two-body femtoscopy



Schrödinger Equation:

V(r) -> $\left|\psi\left(k^*, r\right)\right|^2$ relative wave function for the pair

$$C\left(k^*\right) = \mathcal{N} \cdot \frac{N_{\text{same}}\left(k^*\right)}{N_{\text{mixed}}\left(k^*\right)} = \int S(r) \left|\psi\left(k^*, r\right)\right|^2 \mathrm{d}^3 r$$

Emission source Two-particle wave function

Two-body femtoscopy

$$S(r) = \left(4\pi r_{\mathrm{core}}^2\right)^{-3/2} \cdot \exp\left(-\frac{r^2}{4r_{\mathrm{core}}^2}\right) \otimes \frac{1}{s}\exp\left(-\frac{r}{s}\right), \qquad s = \beta\,\gamma\,\tau_{\mathrm{res}} = \frac{p_{\mathrm{res}}}{M_{\mathrm{res}}}\,\tau_{\mathrm{res}}$$

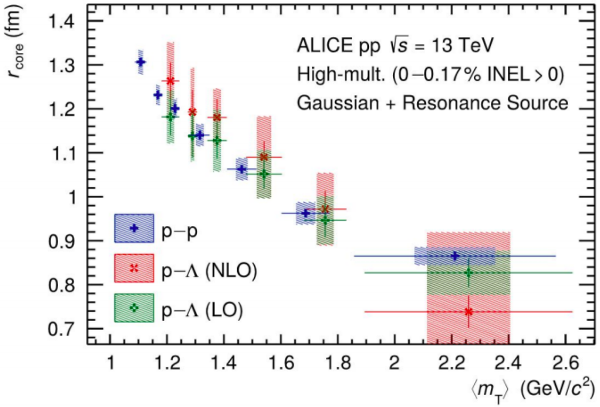
Gaussian source profile

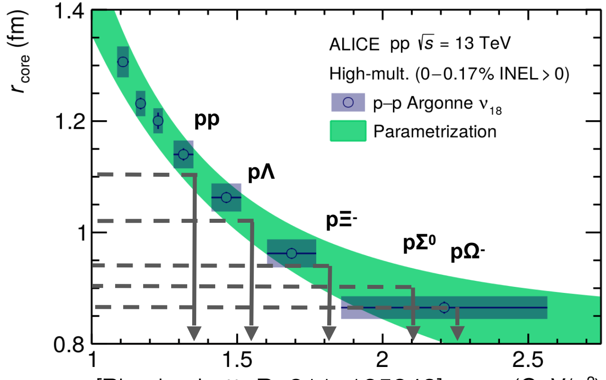
Exponential tail added to account for the effect due to strong short-lived resonances

Common Gaussian core radius to all the hadron-hadron pairs.

<u>Determined using</u>:

- p-p interaction: Argonne v18
- crosscheck using p-Λ (χEFT)





[Physics Lett. B, 811, 135849]

$$C\left(k^*\right) = \mathcal{N} \cdot \frac{N_{\mathrm{same}}\left(k^*\right)}{N_{\mathrm{mixed}}\left(k^*\right)} = \int S(r) \left|\psi\left(k^*, r\right)\right|^2 \mathrm{d}^3 r$$

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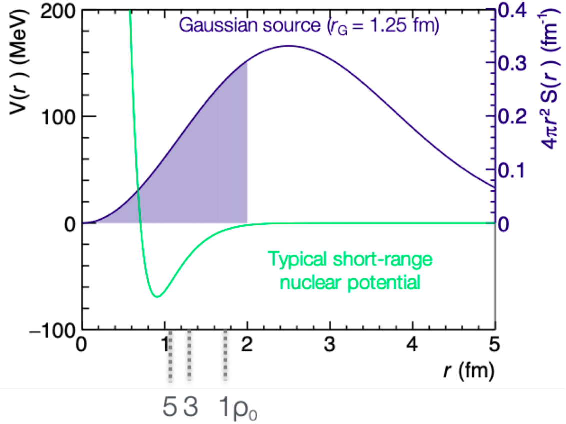
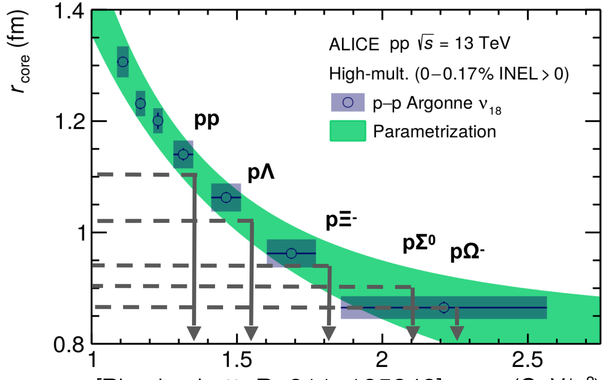
Exponential tail added to account for the effect due to strong short-lived resonances

Small particle-emitting source created in pp and p–Pb collisions at the LHC.





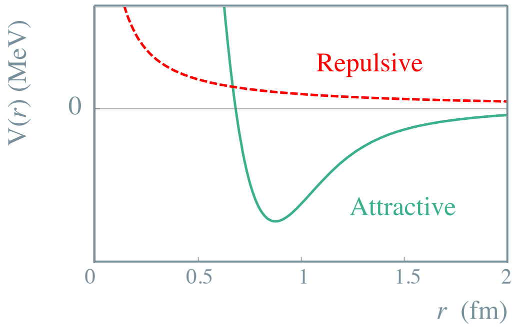
[Physics Lett. B, 811, 135849]

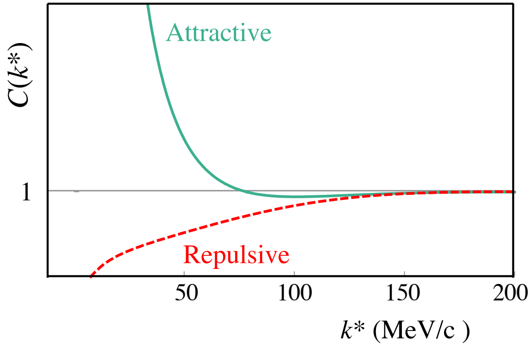
$$C\left(k^*\right) = \mathcal{N} \cdot \frac{N_{\rm same}\left(k^*\right)}{N_{\rm mixed}\left(k^*\right)} = \int S(r)\left|\psi\left(k^*, r\right)\right|^2 {\rm d}^3 r$$

Emission source Two-particle wave function

Two-body femtoscopy

Given the source function the interaction model is tested:





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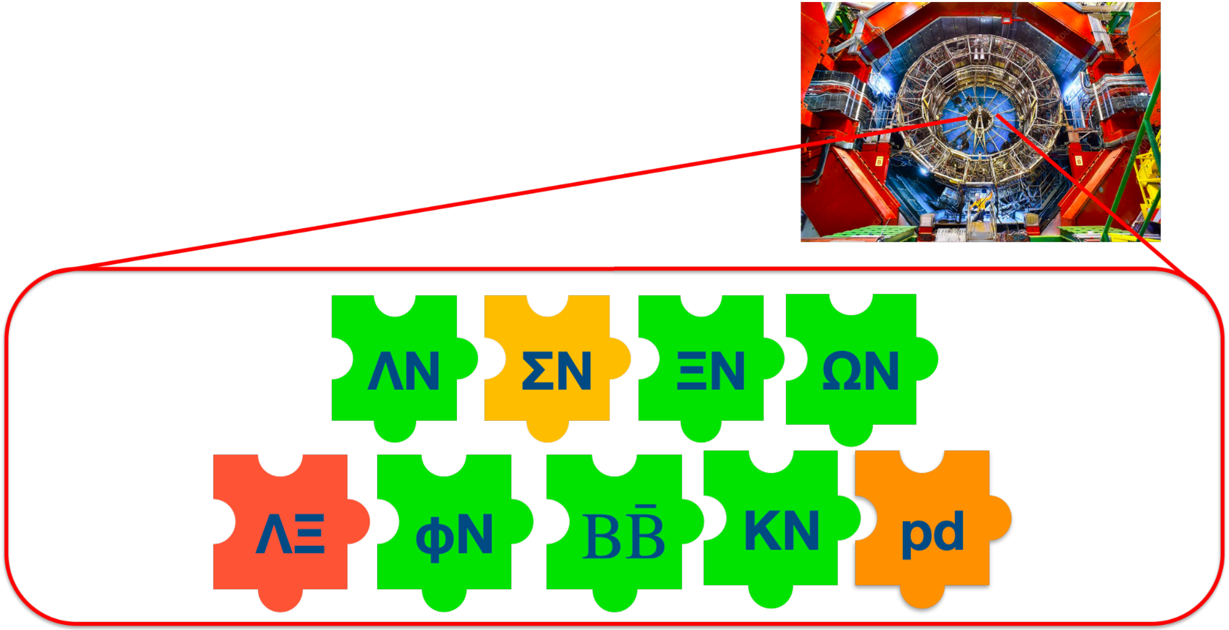
$$C\left(k^{*}\right)=\mathcal{N} \cdot \frac{N_{\text{same}}\left(k^{*}\right)}{N_{\text{mixed}}\left(k^{*}\right)}=\int S(r)\left|\psi\left(k^{*}, r\right)\right|^{2} \mathrm{d}^{3} r$$

Emission source Two-particle wave function

>1 if the interaction is attractive
= 1 if there is no interaction
<1 if the interaction is repulsive

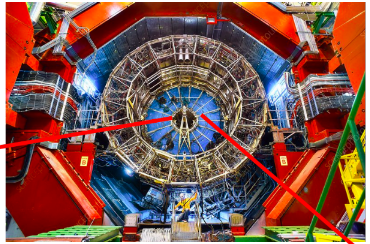
Two-body femtoscopy: achieved results

1. PRC 99 (2019) 024001
2. PLB 797 (2019) 134822
3. PRL 123 (2019) 112002
4. PRL 124 (2020) 09230
5. PLB 805 (2020) 135419
6. PLB 811 (2020) 135849
7. Nature 588 (2020) 232-238
8. ALICE Coll. arXiv:2104.04427
9. ALICE Coll. arXiv:2105.05683
10. ALICE Coll. arXiv:2105.05190
11. ALICE Coll. arXiv:2105.05578 (accepted by PRL)



see L. Fabbietti's talk
tomorrow 22nd September at 09.00 a.m.

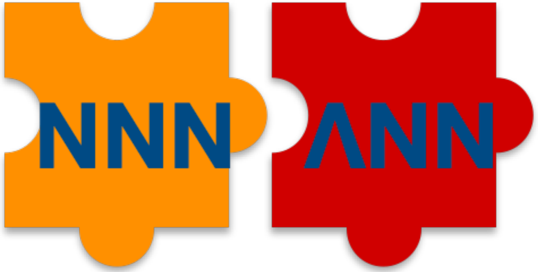
Investigating three-body interactions at the LHC



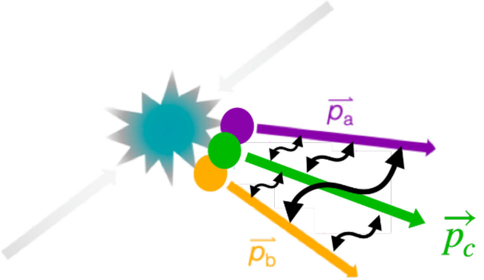
First femtoscopic measurement of p-p-Λ and p-p-p

Colliding system: pp @ $\sqrt{s}$ = 13 TeV
Data set: High Multiplicity events

- purity of p: 98.3%
- purity of Λ: 95.6%



Three-body femtoscopy at the LHC

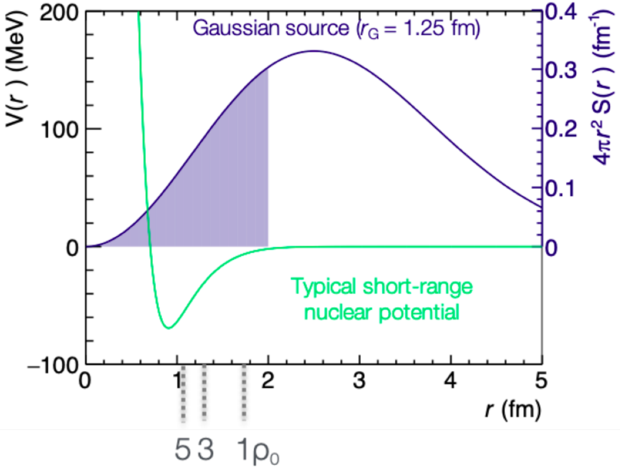


Three particles are emitted from a same common source and may undergo *final-state interactions* before the detection.

Advantages:

- **Not affected by nuclear medium effects** which are instead present in bound objects;

- The typical source radii in two-body femtoscopy is ~1.25 fm
  → **test of the interaction at short distances**



Three-body correlation function

Two-body correlation function

$$C(\mathbf{p}_1, \mathbf{p}_2) \equiv \frac{P(\mathbf{p}_1, \mathbf{p}_2)}{P(\mathbf{p}_1) \cdot P(\mathbf{p}_2)} = \mathcal{N} \cdot \frac{N_{\mathrm{same}}(k^*)}{N_{\mathrm{mixed}}(k^*)}$$

$\Rightarrow$

Three-body correlation function

$$C(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) \equiv \frac{P(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3)}{P(\mathbf{p}_1) \cdot P(\mathbf{p}_2) \cdot P(\mathbf{p}_3)} = \mathcal{N} \cdot \frac{N_{\mathrm{same}}(Q_3)}{N_{\mathrm{mixed}}(Q_3)}$$

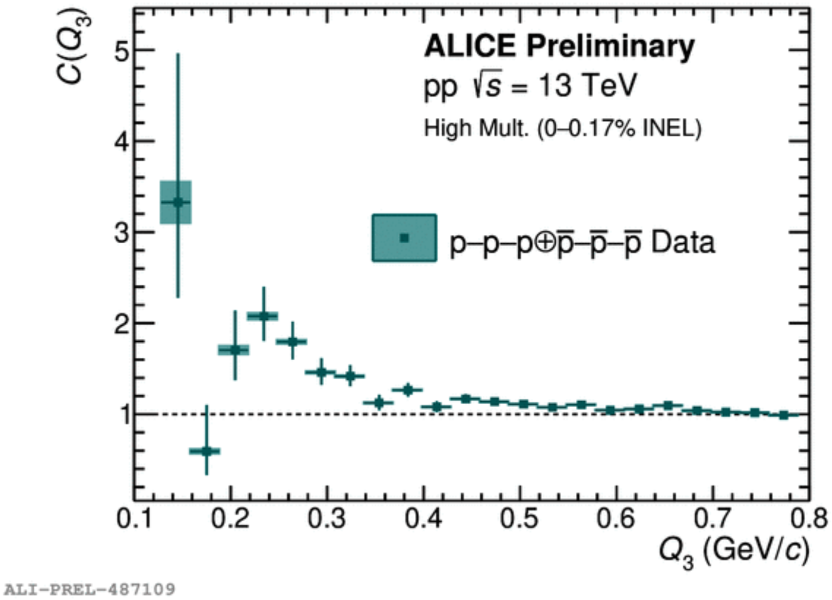
The small statistics requires to project the correlation function on **1-dimensional** observable. The Lorentz invariant Q_3 is defined as:

$$Q_3 = \sqrt{-q_{12}^2 - q_{23}^2 - q_{31}^2}$$

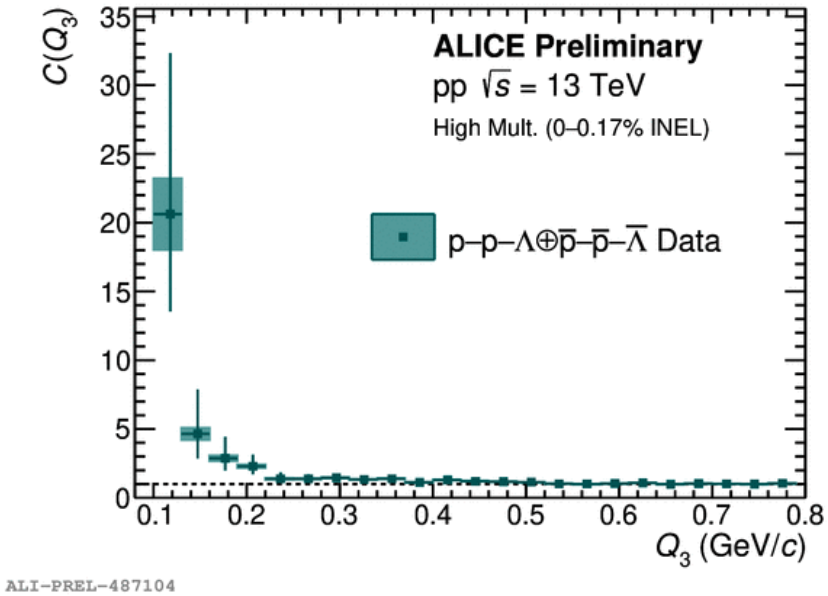
$$q_{ij}^{\mu} = \left(p_i - p_j\right)^{\mu} - \frac{\left(p_i - p_j\right) \cdot P_{ij}}{P_{ij}^2} P_{ij}^{\mu} \qquad\qquad P_{ij} \equiv p_i + p_j$$

p-p-p and p-p-Λ correlation functions

Measured triplets
at $Q_3 < 0.4$ GeV/c ----> 1011 triplets



Measured triplets
at $Q_3 < 0.4$ GeV/c ----> 496 triplets

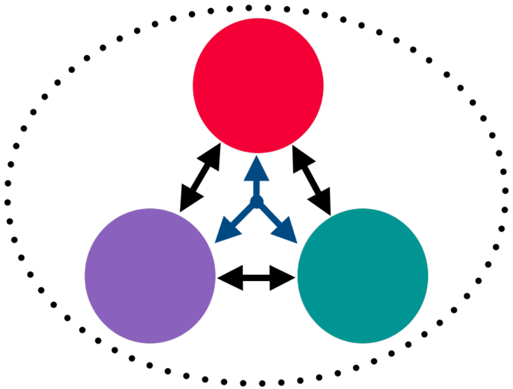


<u>These are not genuine three-body correlation functions</u>

Accessing the genuine three-body correlation

- Measured three-particle correlation function includes both <u>two-body</u> and genuine <u>three-body</u> interactions.

- Kubo's cumulant expansion method is used to access genuine three-body correlation.



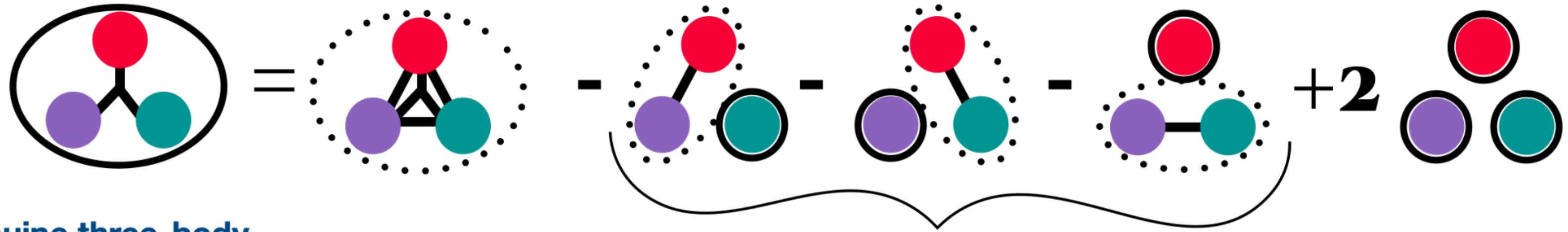
JOURNAL OF THE PHYSICAL SOCIETY OF JAPAN, Vol. 17, No. 7, JULY 1962

Generalized Cumulant Expansion Method*

Ryogo KUBO
Department of Physics, University of Tokyo

(Received April 11, 1962)

Kubo’s cumulant expansion method



In terms of correlation functions:

$$\underbrace{c_3\left(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3\right)}_{\text{Genuine three-body (Cumulant)}} = \underbrace{C([\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3])}_{\text{Measured triplets}} - \underbrace{C([\mathbf{p}_1, \mathbf{p}_2], \mathbf{p}_3) - C(\mathbf{p}_1, [\mathbf{p}_2, \mathbf{p}_3]) - C([\mathbf{p}_1, \mathbf{p}_3], \mathbf{p}_2)}_{\text{Two-body correlations}} + \underbrace{2}_{\text{Uncorrelated}}$$

The pairs in the square brackets are correlated, the particle outside is not correlated.

Lower order contributions evaluation

Data-driven approach

Using the **same** and **mixed events** distributions:

$$C([\mathbf{p}_1, \mathbf{p}_2], \mathbf{p}_3) = \frac{N_2(\mathbf{p}_1, \mathbf{p}_2)\, N_1(\mathbf{p}_3)}{N_1(\mathbf{p}_1)\, N_1(\mathbf{p}_2)\, N_1(\mathbf{p}_3)}$$

The scalar Q_3 is calculated from the measured single-particle momenta

$$(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) \rightarrow Q_3$$

Projector method

[R. Del Grande, L. Šerkšnytė et al, arXiv:2107.10227v1 (2021)]

Using the **two-body correlation function** of the pair (1,2).

A kinematic transformation from

k^*_{12} (pair) → Q_3 (triplet)

$C(k^*_{12})$ → $C(Q_3)$

is performed.

For the pair i-j we have

$$C_3^{ij}(Q_3) = \int \underbrace{\boxed{C_2(k^*_{ij})}}_{\text{two-body correlation function}} \underbrace{\boxed{W_{ij}(k^*_{ij}, Q_3)}}_{\text{projector}}\, dk^*_{ij}$$

Lower order contributions evaluation

Data-driven approach

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$$C([\mathbf{p}_1,\mathbf{p}_2],\mathbf{p}_3)=\frac{N_2\left(\mathbf{p}_1,\mathbf{p}_2\right)N_1\left(\mathbf{p}_3\right)}{N_1\left(\mathbf{p}_1\right)N_1\left(\mathbf{p}_2\right)N_1\left(\mathbf{p}_3\right)}$$

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Two-body correlations

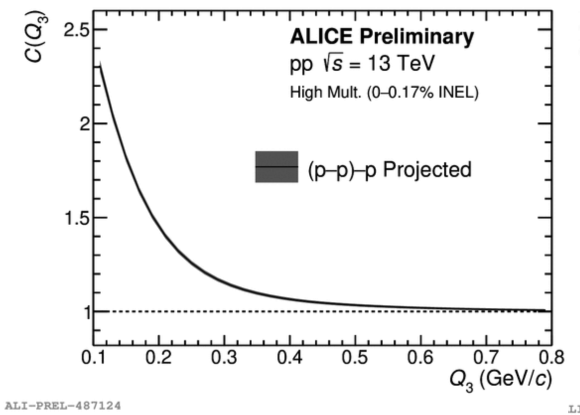
$$\boxed{C_3^{ij}(Q_3)} = \int \boxed{C_2(k^*_{ij})}\, W_{ij}(k^*_{ij}, Q_3)\; dk^*_{ij}$$

Outputs:

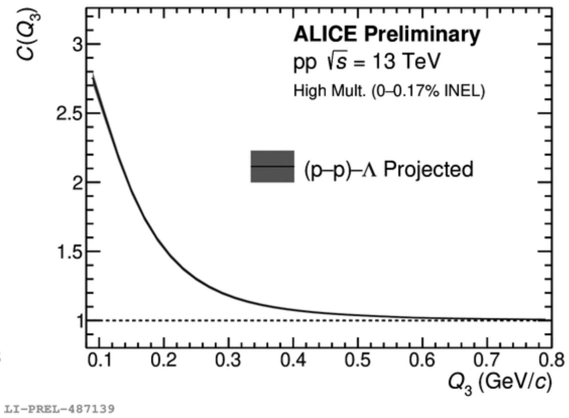
Input:

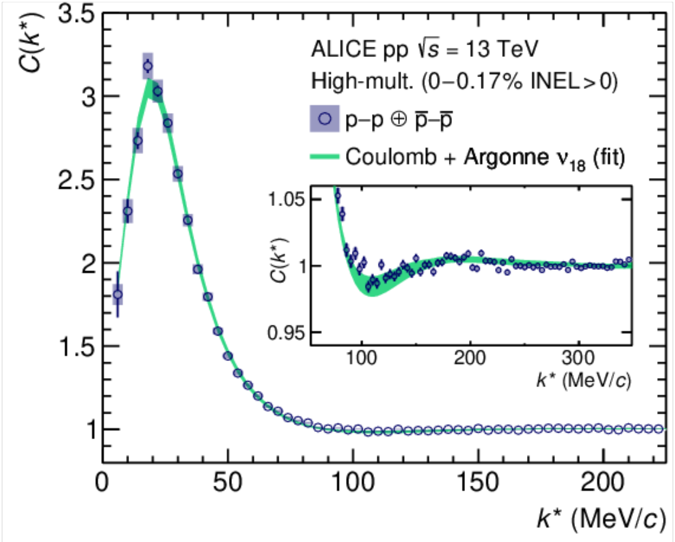
proton-proton

(proton-proton)-proton



(proton-proton)-Λ





[ALICE Collaboration / Physics Letters B 805 (2020) 135419]

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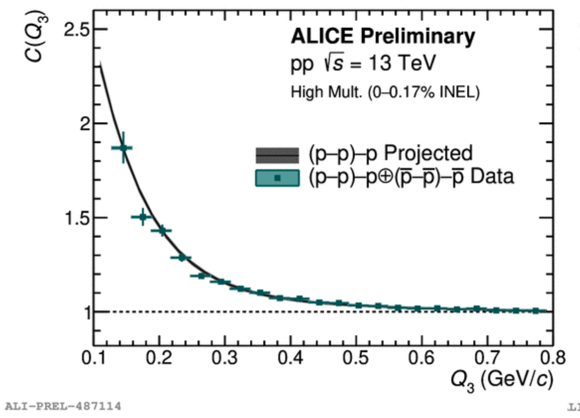
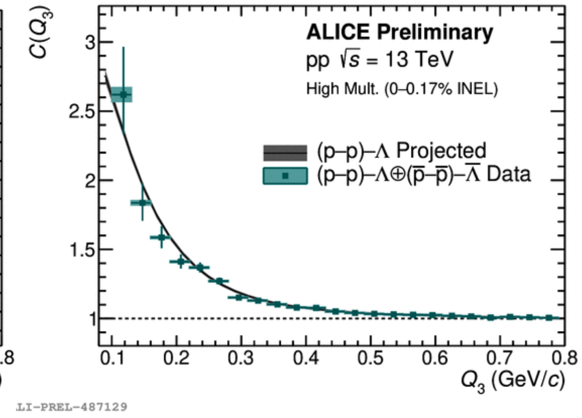
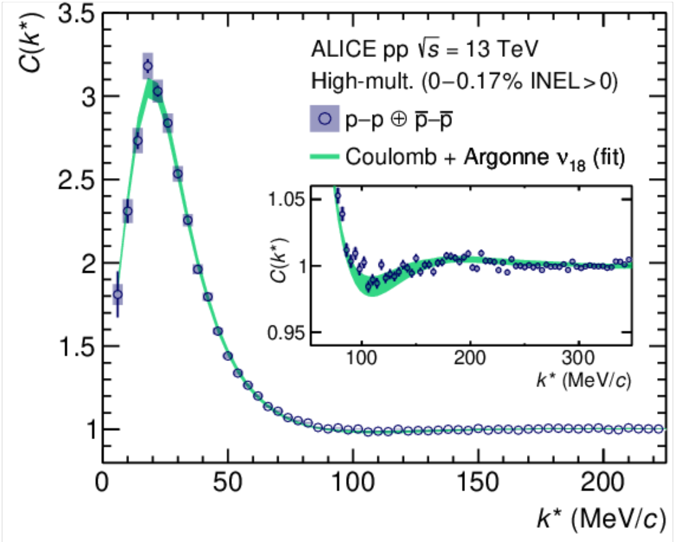


(proton-proton)-Λ





Data-driven approach VS **Projector method**

[ALICE Collaboration / Physics Letters B 805 (2020) 135419]

Two-body correlations

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Outputs:

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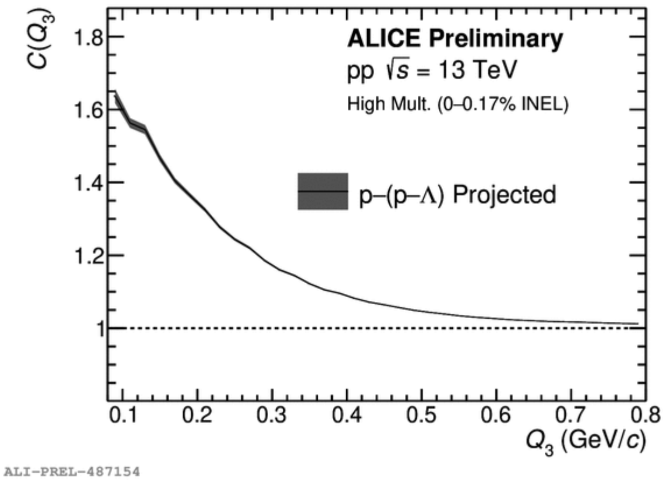
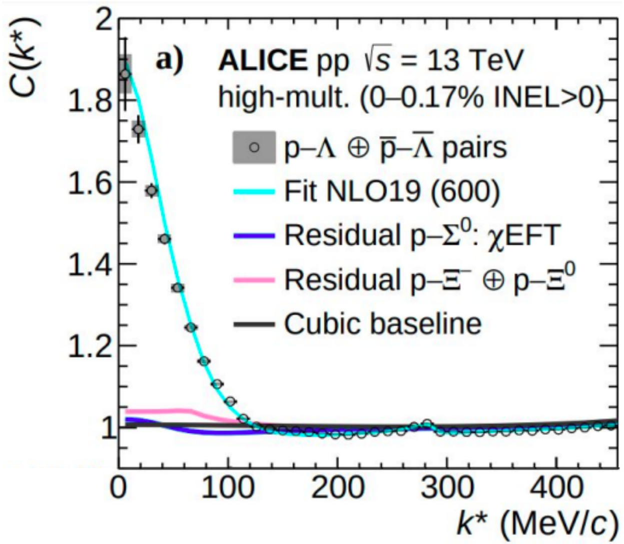


Input:

proton-Λ



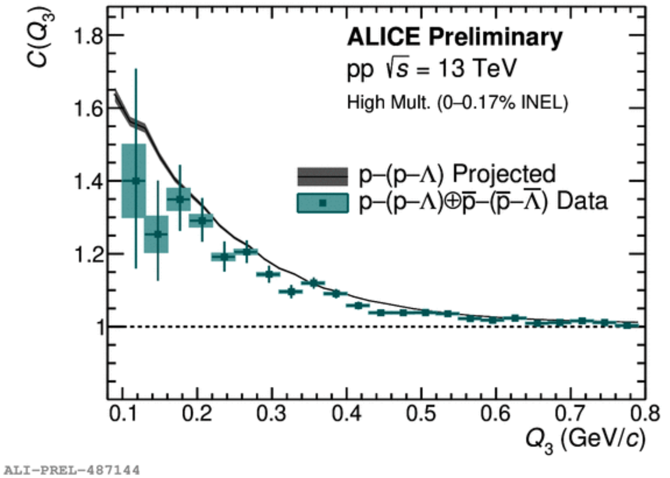
[ALICE Collaboration / arXiv:2104.04427]

Two-body correlations

$$\boxed{C_3^{ij}(Q_3)} = \int \boxed{C_2(k_{ij}^*)}\, W_{ij}(k_{ij}^*, Q_3)\; dk_{ij}^*$$

Outputs:

(proton-Λ)-proton



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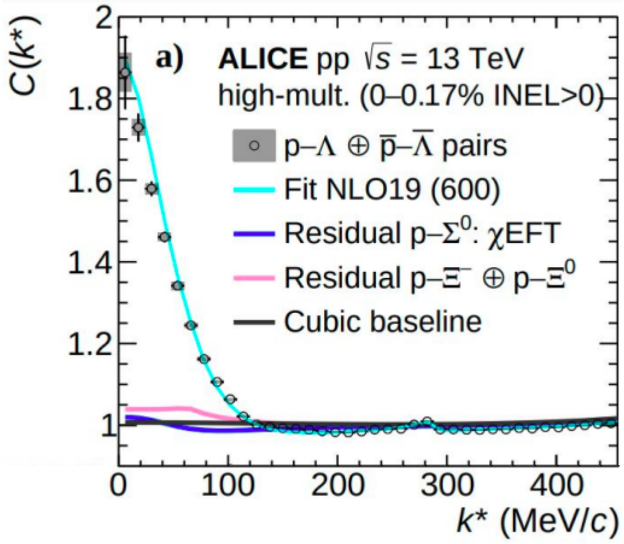

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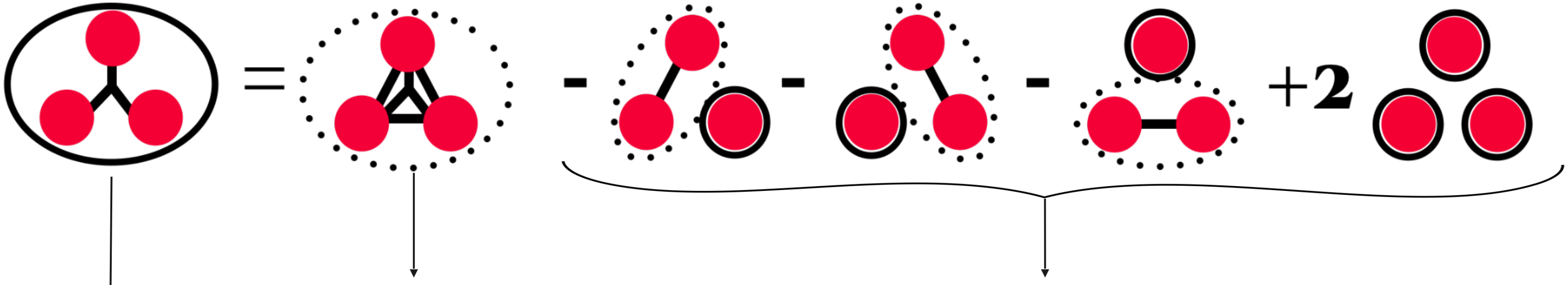
Kubo’s cumulant expansion method

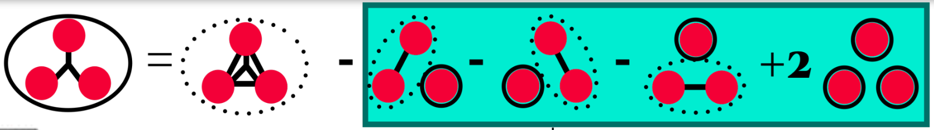


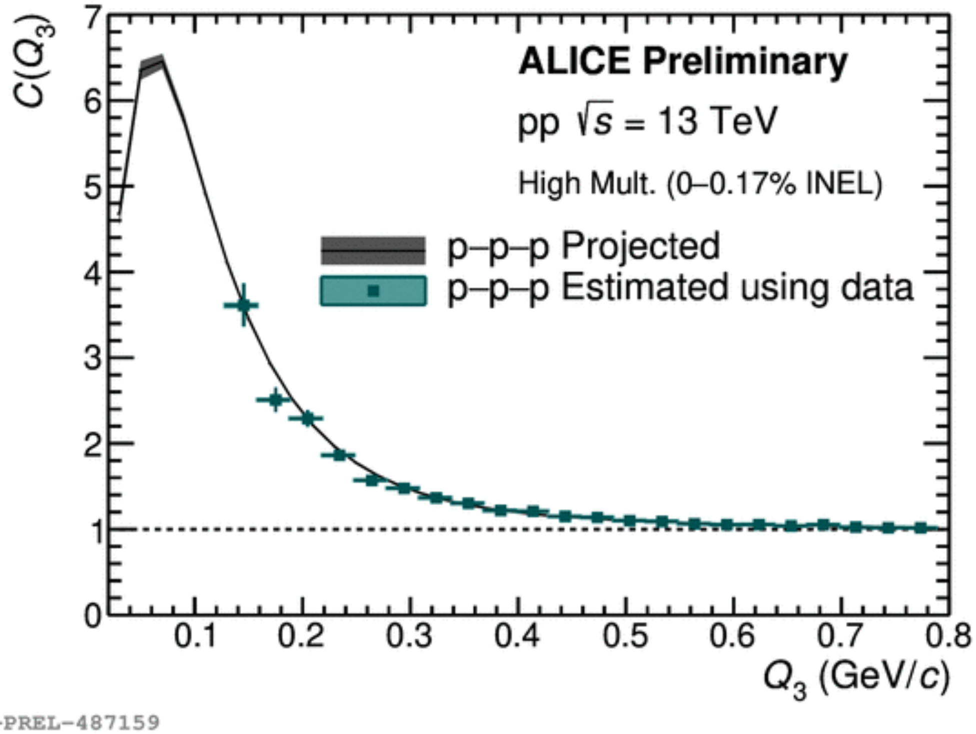
Measured correlation function

Lower-order correlations

Three-body cumulant
(to be extracted)

p-p-p: two-body correlation functions projected onto Q_3





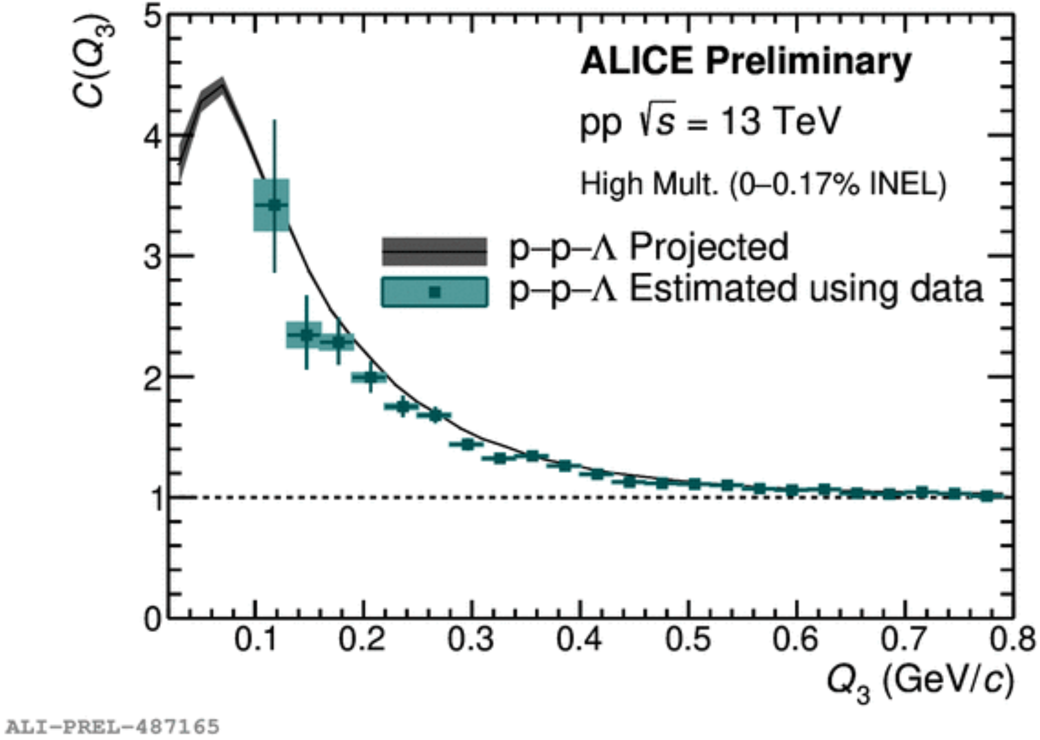
Lower-order contributions to the three-body cumulant

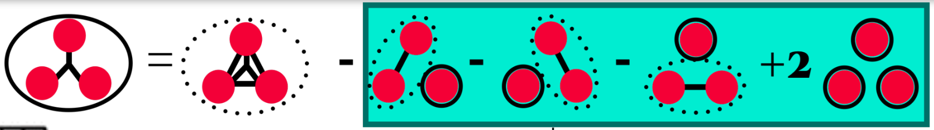
$$C_{\mathrm{ppp}}^{\mathrm{two-body}}\left(Q_3\right) = 3C_3^{\mathrm{pp}}\left(Q_3\right) - 2$$

Comparison:

- **Data-driven approach**
- **Projector method**

p-p-Λ: two-body correlation functions projected onto Q_3





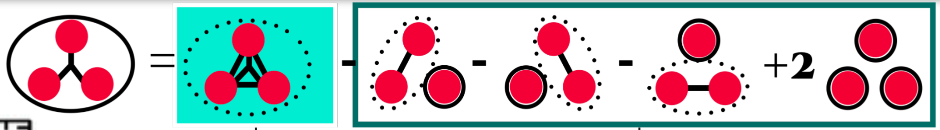
Lower-order contributions to the three-body cumulant

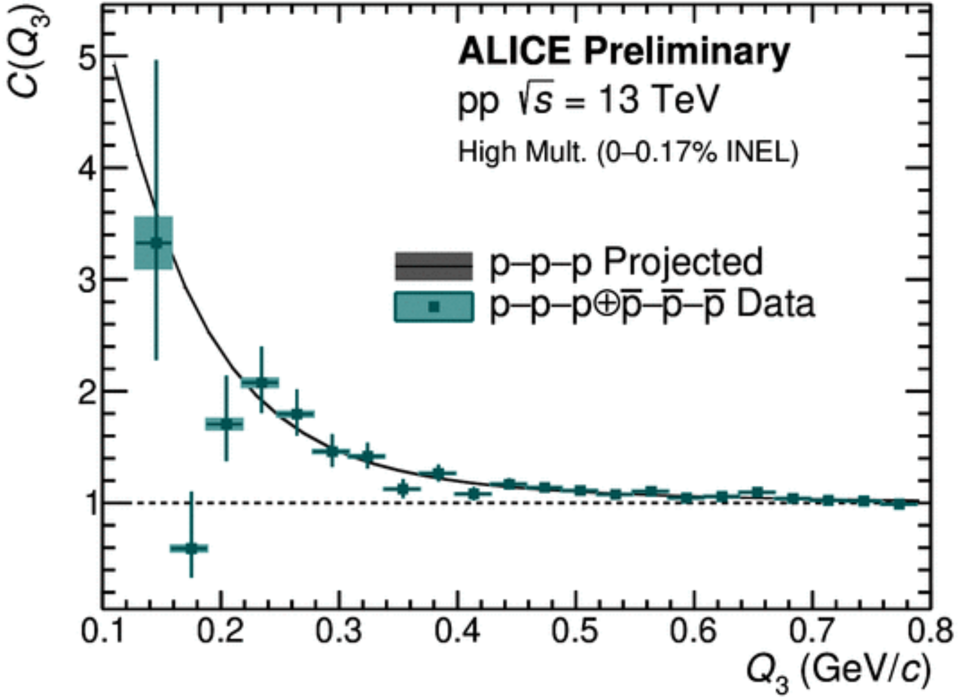
$$C_{\mathrm{pp}\Lambda}^{\mathrm{two-body}}\left(Q_3\right) = C_3^{\mathrm{pp}}\left(Q_3\right) + 2\ C_3^{\mathrm{p}\Lambda}\left(Q_3\right) - 2$$

Comparison:

- **Data-driven approach**
- **Projector method**

p-p-p correlation function





ALI-PREL-487054

The measured p-p-p correlation function

Lower-order contributions calculated with the projector method

Significant deviation of the measured correlation function from the two-body correlations projected onto Q_3 (gray curve).

λ parameters

The measured correlation function includes also misidentified particles and and feed-down particles coming from decays of resonances. Total **measured** function thus is:

$$C(XYZ) = \sum_{i,j,k} \lambda_{i,j,k}(XYZ) C_{i,j,k}(XYZ) = \boxed{\lambda_{X_0,Y_0,Z_0}(XYZ) C_{X_0,Y_0,Z_0}(XYZ)} + \sum_{ijk!=X_0Y_0Z_0} \lambda_{i,j,k}(XYZ) C_{i,j,k}(XYZ)$$

Correctly identified primary particles

- The cumulant is calculated with the measured correlation functions not accounting for the λ parameters.

$$\lambda_{i,j,k}(XYZ) = \mathscr{P}(X_i) f(X_i) \mathscr{P}(Y_j) f(Y_j) \mathscr{P}(Z_k) f(Z_k)$$

Extracted from measurement

What we are interested in

Feed-down and misidentified particle contribution

$$\boxed{c(XYZ)} = \sum_{i,j,k} \lambda_{i,j,k}(XYZ) c(X_i Y_j Z_k) = \boxed{\lambda_{X_0Y_0Z_0}(XYZ) c(X_0Y_0Z_0)} + \boxed{\sum_{i,j,k \neq (X_0Y_0Z_0)} \lambda_{i,j,k}(XYZ) c(X_i Y_j Z_k)}$$

- The genuine three body interaction for the feed-down and misidentified particle contributions is currently not known.

λ parameters

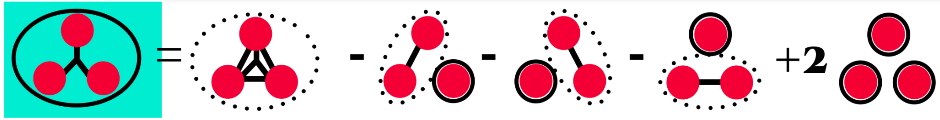
- The λ parameters requires purity and the secondary fraction evaluation.
- The average Λ purity is 95.57% and for protons the purity is 98.34%.
- The fractions of secondaries are estimated using Monte Carlo simulations.

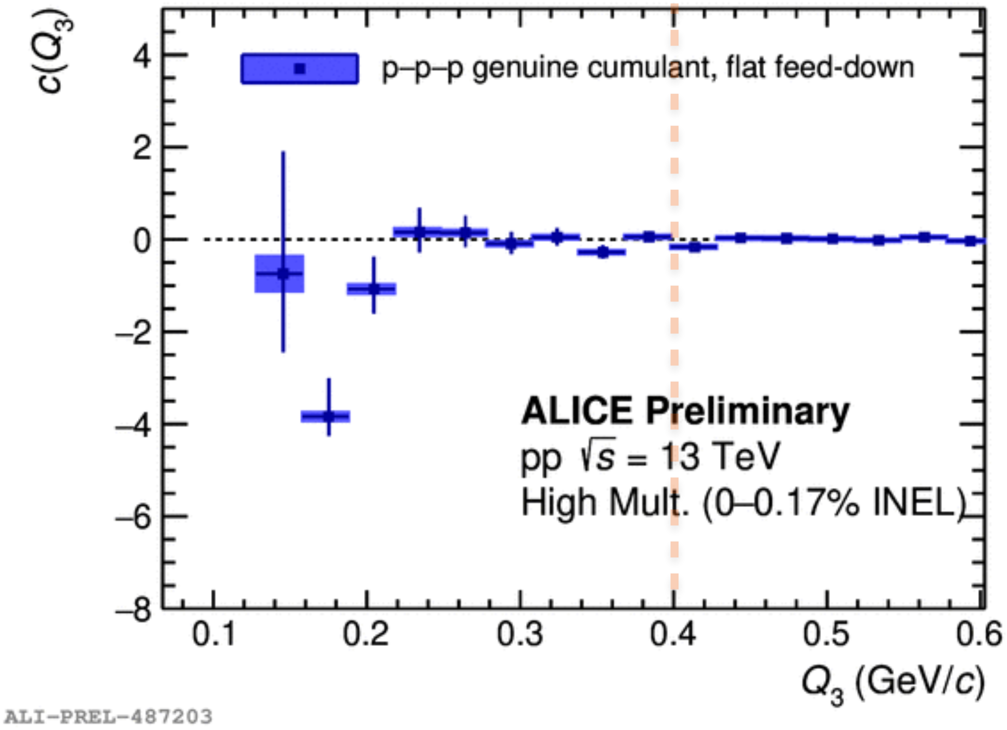
Some of the contributions with highest lambda parameters:

p-p-p	61.8%
p-p-p_{Λ} x3	19.6%
p-p-p_{Σ^+} x3	8.5%
p-p_{Λ}-p_{Λ} x3	0.69%
p-p_{Λ}-p_{Σ^+} x3	0.3 %
p-p_{Σ^+}-p_{Σ^+} x3	0.13%

p-p-Λ	40.5%
p-p-Λ_{Σ^0}	13.5%
p-p-Λ_{Ξ^0}	7.56%
p-p-Λ_{Ξ^-}	7.56%
p-p_{Λ}-Λ x2	8.56%
p-p_{Σ^+}-Λ x2	3.7%

p-p-p cumulant





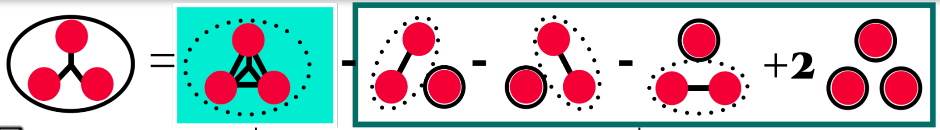
Cumulant extracted. Flat feed-down from the resonances is considered.

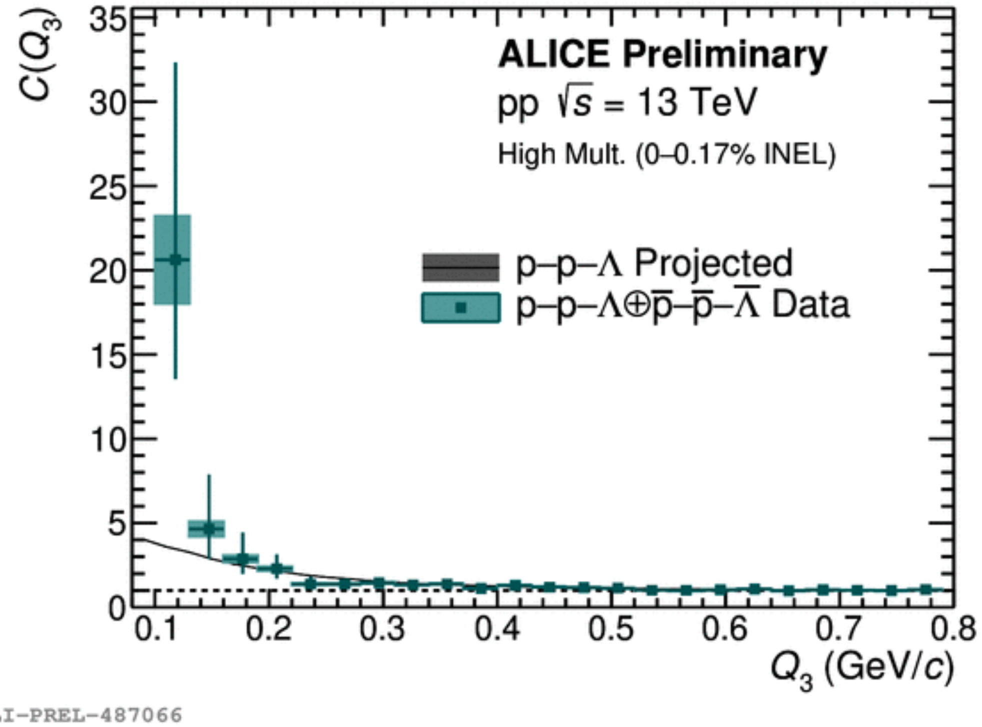
The statistical significance for the measured deviation is:

$$n_\sigma = 2.9 \quad \text{for } Q_3 < 0.4 \text{ GeV/}c$$

Theoretical calculations of the three-body scattering are needed to interpret the data

p-p-Λ correlation function

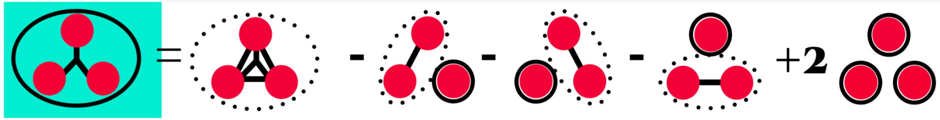


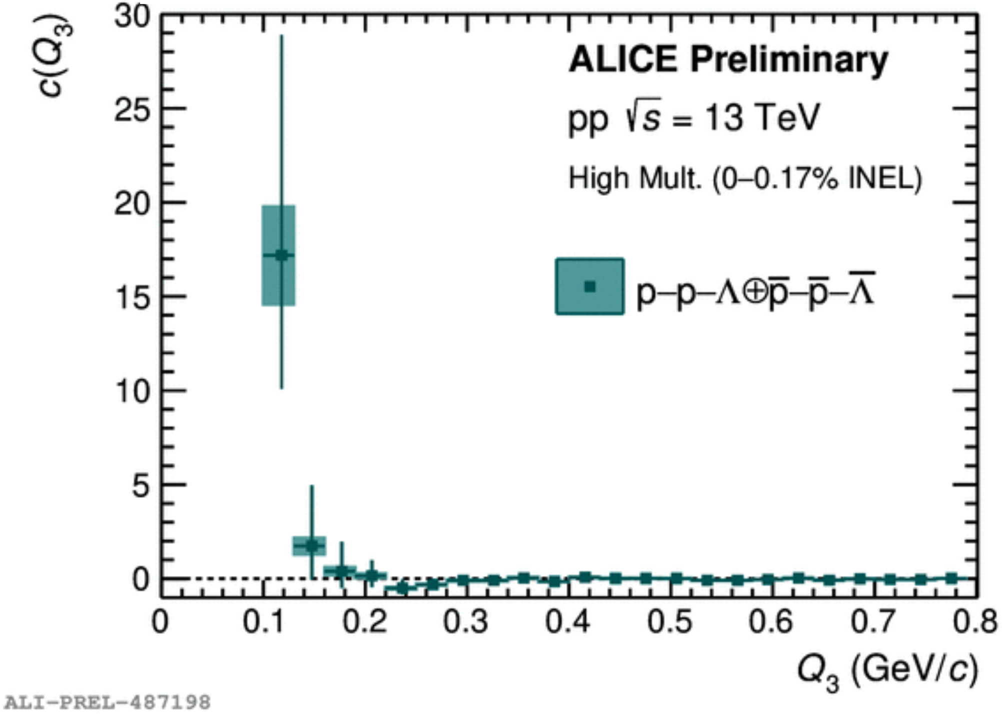


The measured p-p-Λ correlation function

Lower-order contributions calculated with the projector method

The shape deviates from the two-body correlations projected onto Q_3 (gray curve).

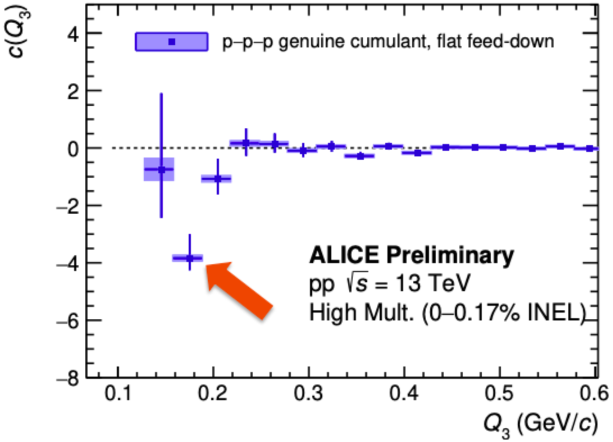


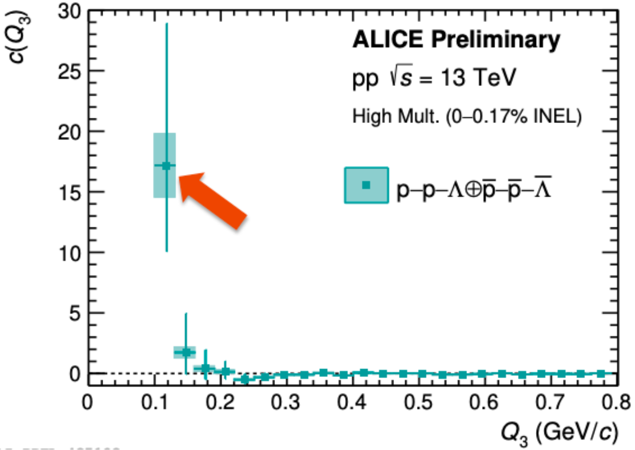


Positive cumulant for p-p-Λ

Summary

- First measurement of three-baryon correlation functions
- Cumulants for p-p-p and p-p-Λ extracted with the Kubo's method





- **p-p-p:** significant deviation (**n_σ = 2.9** for $Q_3 < 0.4$ GeV/c)

  → FIRST HINT of genuine p-p-p correlation
- **p-p-Λ:** positive cumulant → ALICE Run 3 data should provide statistically significant result
- Calculations for the three-body scattering are needed