On the phase structure of QCD

Jan M. Pawlowski

Universität Heidelberg & ExtreMe Matter Institute

Erice, September 17th 2021

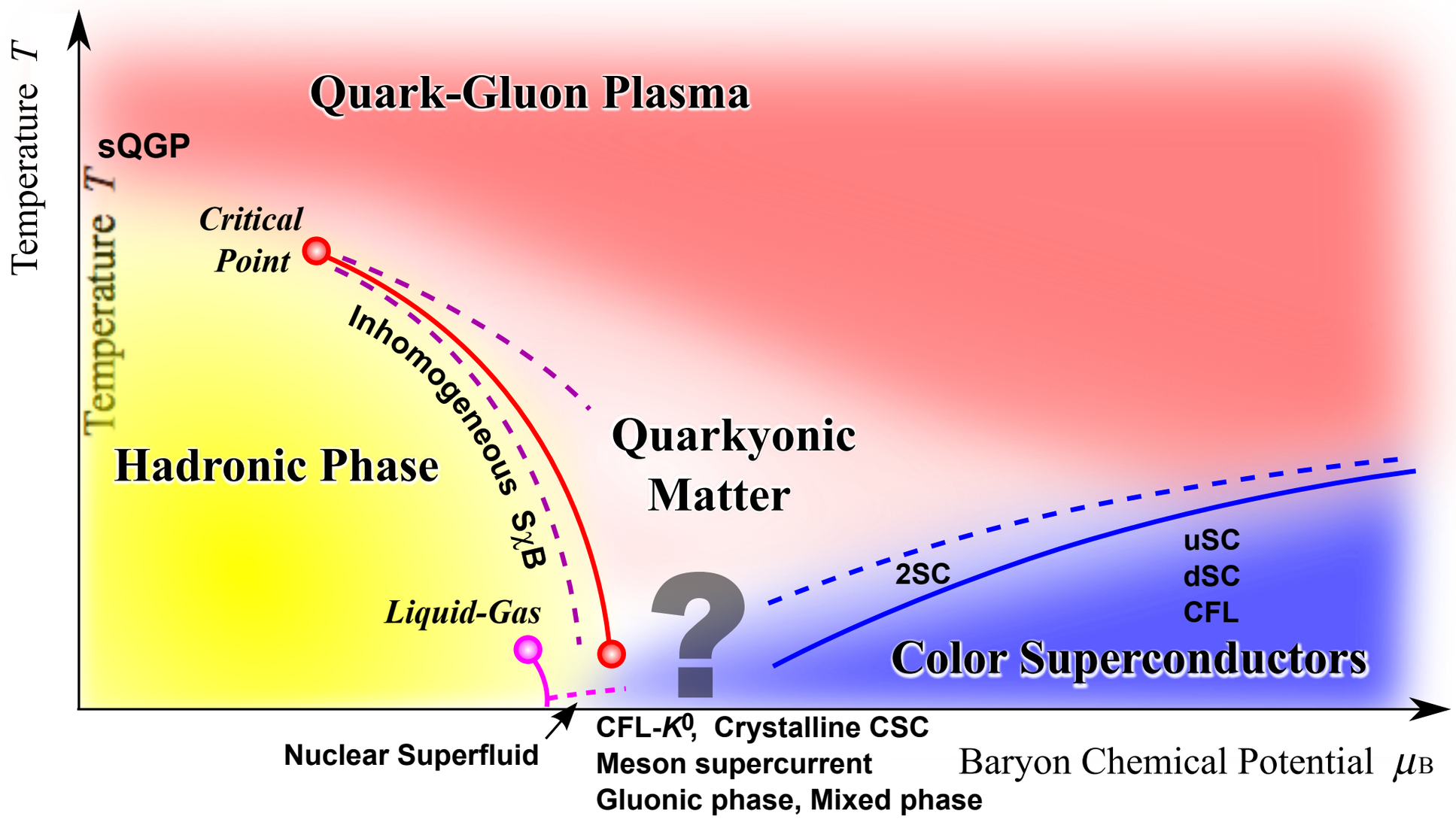
for the fQCD collaboration

GEFÖRDERT VOM
Bundesministerium für Bildung und Forschung

STRUCTURES CLUSTER OF EXCELLENCE

isoQuant SFB1225

EMMI



fQCD collaboration

Braun, Chen, Fu, Ihssen, Geissel, Horak, Huang, JMP, Rennecke, Sattler, Schallmo, Schneider, Tan, Töpfel, Wen, Wessely, Wink, Yin

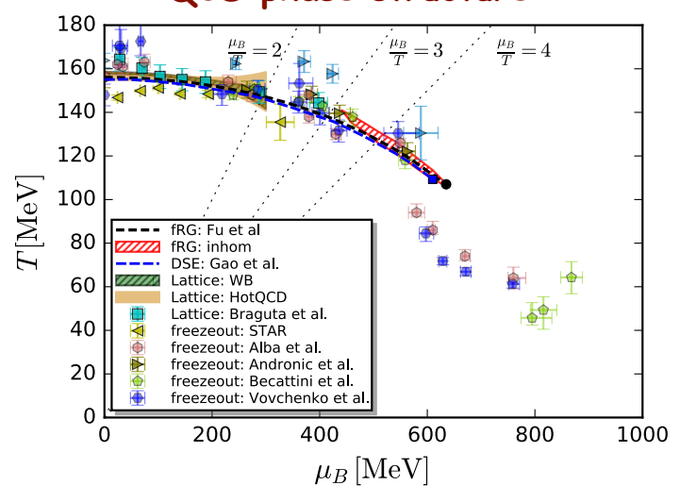
Dalian, Darmstadt, Heidelberg, Gießen

Outline

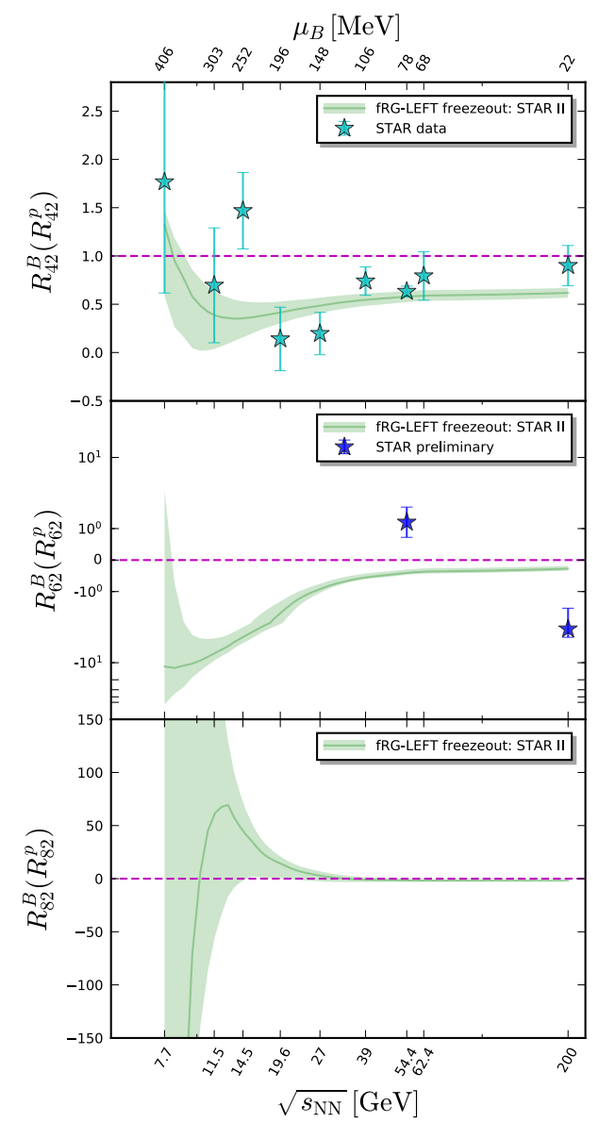
- **QCD from functional methods**

- **QCD phase structure**

QCD phase structure



Hyper-fluctuations



- **Fluctuations of conserved charges**

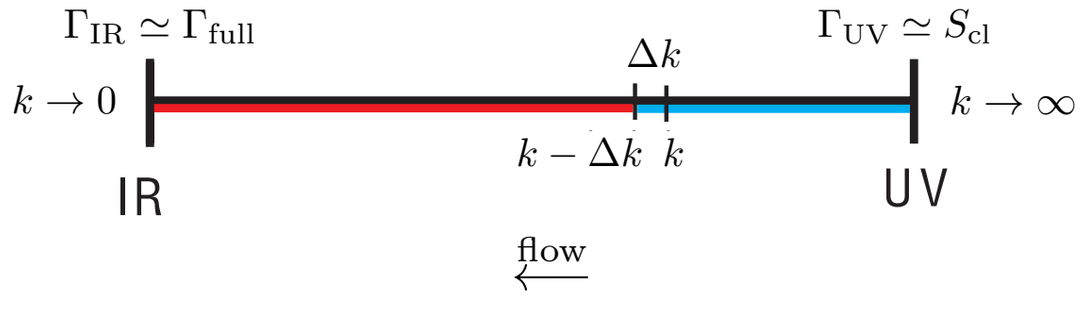
- **Summary & outlook**

Functional Methods for QCD

FRG: JMP, NPA 931 (2014) 113
Dupuis et al, Phys.Rept. 910 (2021) 1

DSE: Fischer, PPNP 105 (2019) 1

free energy at momentum scale k

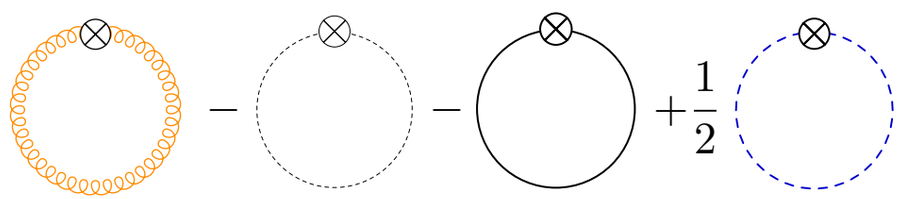


ab initio

glue quantum fluctuations | **hadronic quantum fluctuations**

functional RG:

$$\partial_t \Gamma_k[\phi] = \frac{1}{2}$$



free energy/ grand potential

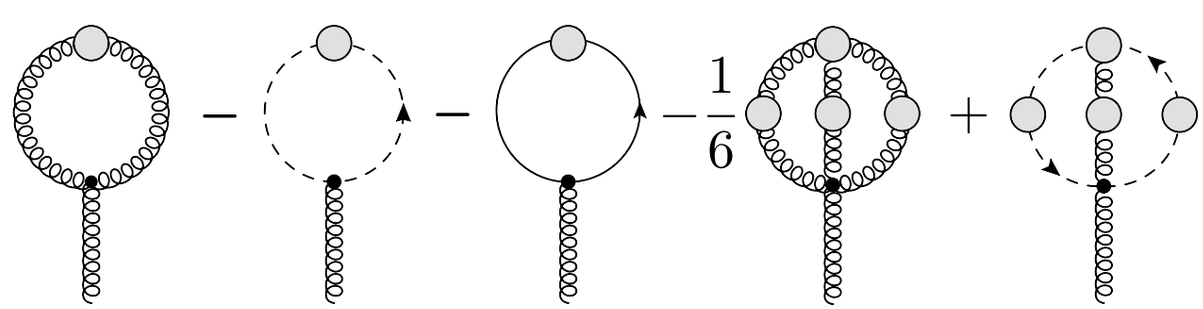
quark quantum fluctuations

$\boxed{\text{RG-scale k: } t = \ln k}$

closed form

functional DSE :

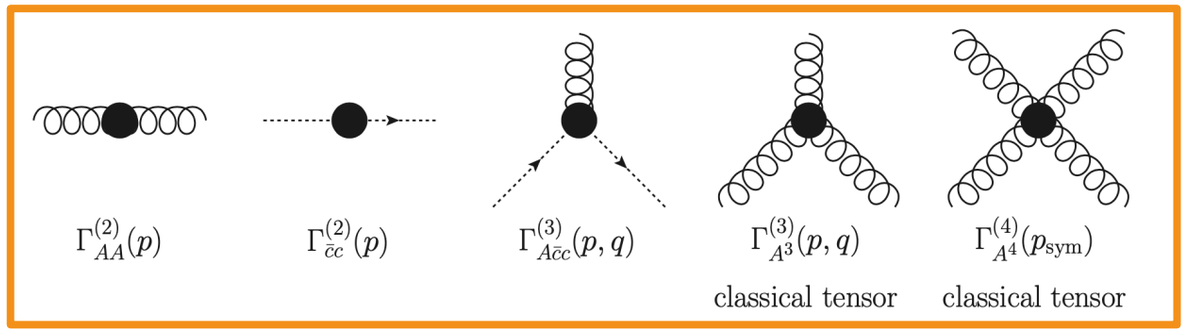
$$\frac{\delta\left(\Gamma - S\right)}{\delta A_0} = \frac{1}{2}$$



A_0 **: background field**

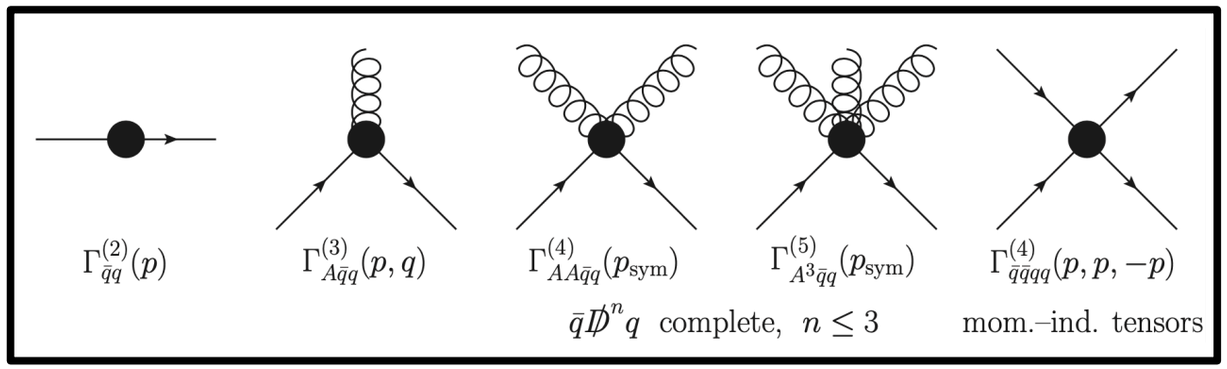
DSE: talks of Philipp Isserstedt, Julian Bernhardt, Evening Session

vacuum fQCD: current set of correlation functions

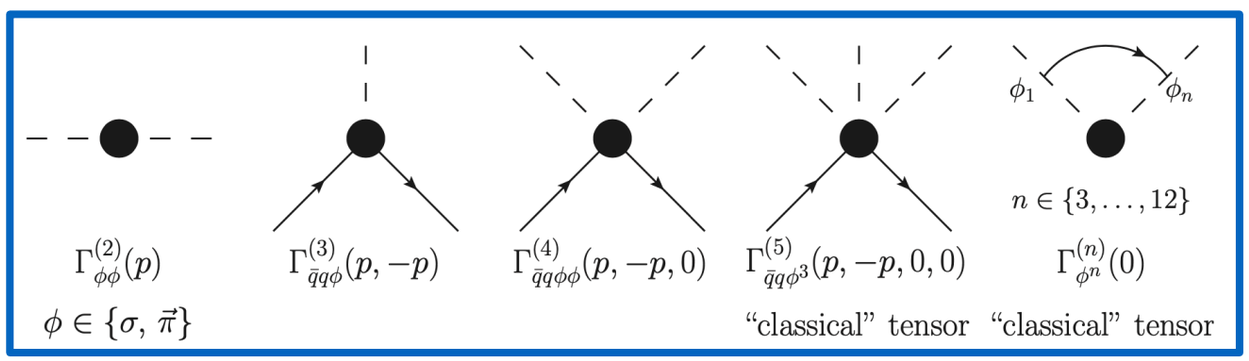


glue sector

FRG

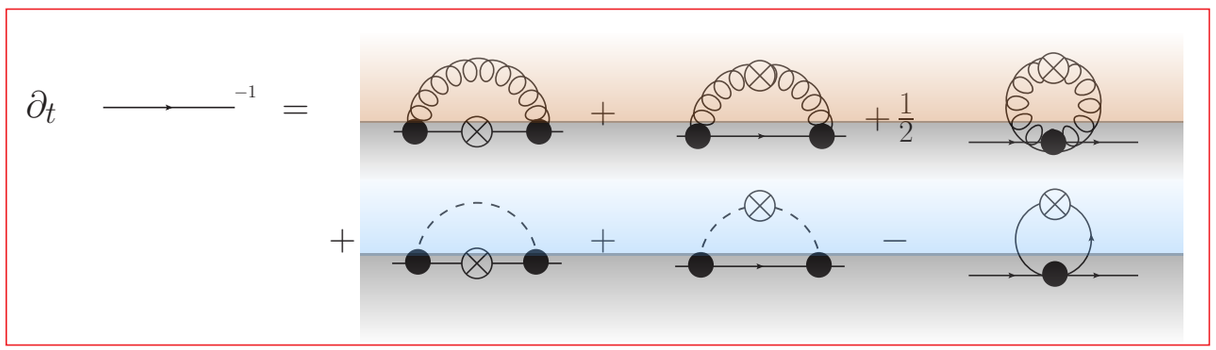


quark-glue sector



quark-meson sector

Aiming at apparent convergence



Analyticity considerations at finite density

- **Self-consistent truncations to functional relations define 'analytic' functions in μ_{B}, eg:**

$$\partial_t \langle q(x)\bar{q}(x)\rangle(\mu_B) = \mathrm{Loop}[\ \ \langle q(x)\bar{q}(x)\rangle(\mu_B),\ \ \langle q(x)A_\mu\bar{q}(x)\rangle(\mu_B), \cdots ; \mu_B]$$

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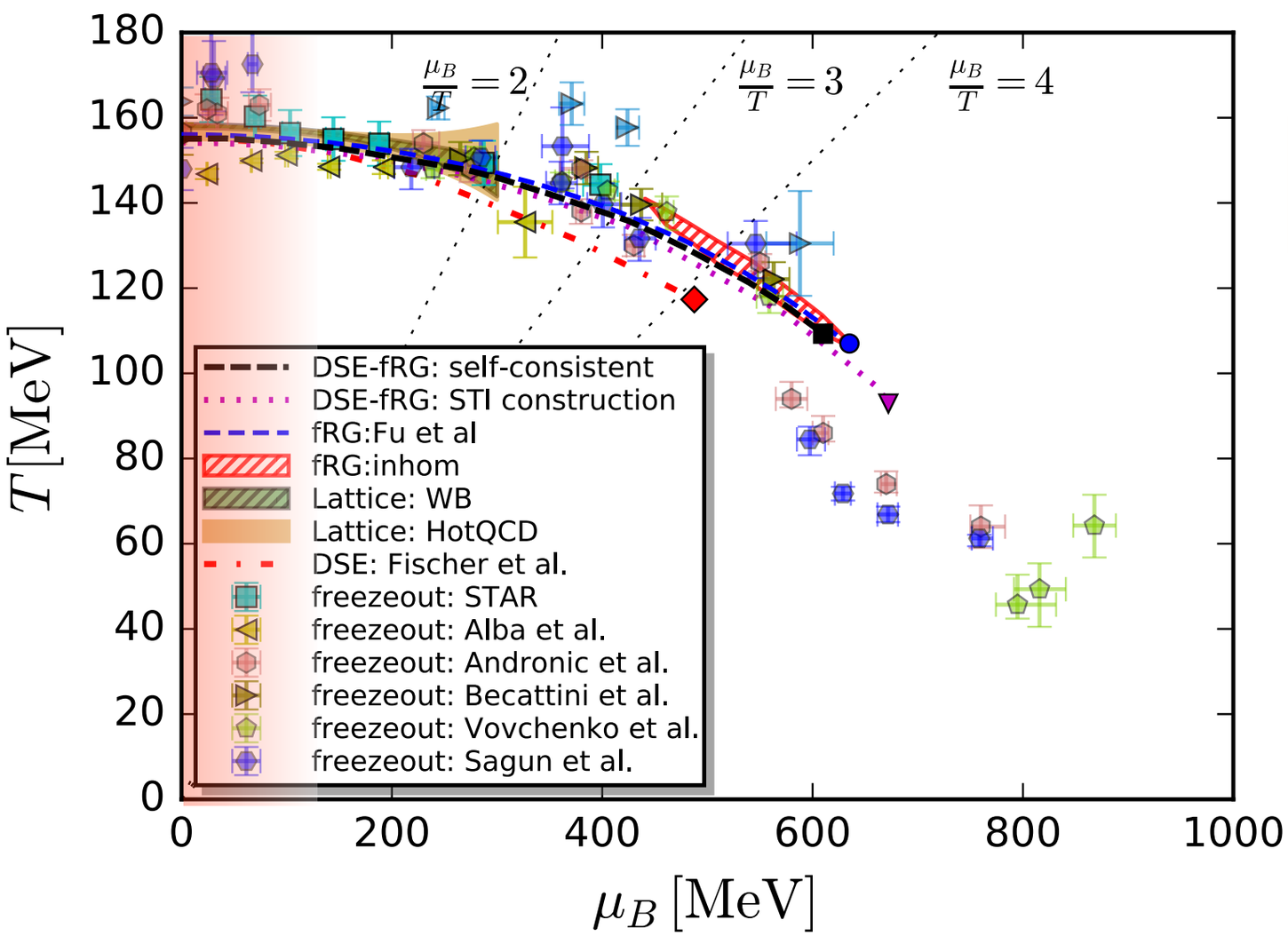
- **Consequences for functional QCD predictions at finite density**



By now the best truncations to functional QCD pass lattice benchmark tests at vanishing chemical potential

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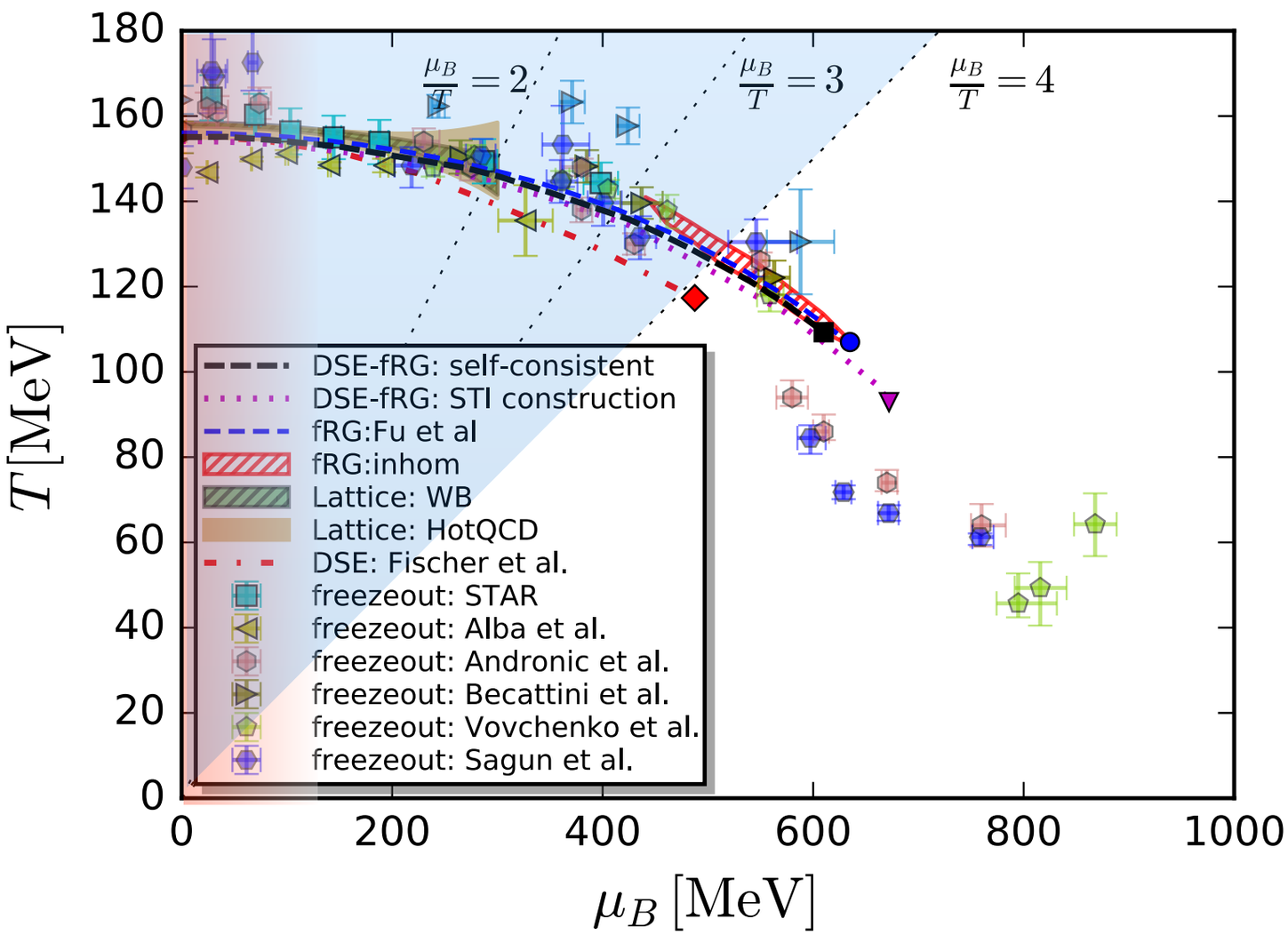

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Regime of reliability of current best truncation

Analyticity considerations at finite density

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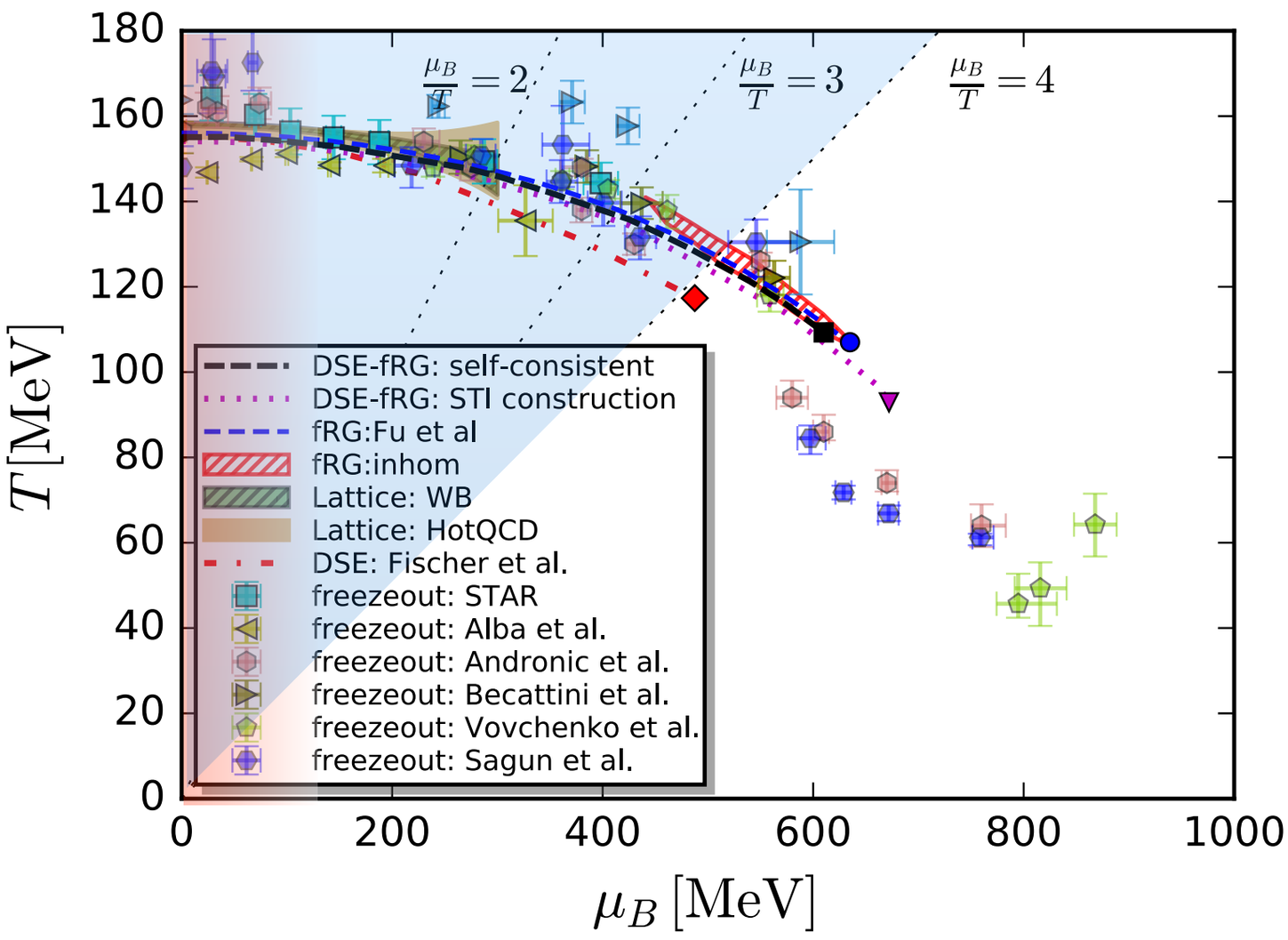


By now the best truncations to functional QCD pass lattice benchmark tests at vanishing chemical potential

Regime of reliability of current best truncation

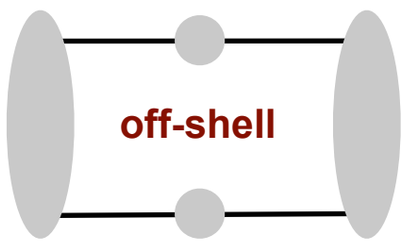
In any case: QCD-based analytic continuations that satisfy the lattice benchmarks at vanishing chemical potential.

Great opportunity for a combined analysis of high density QCD

Three remarks on Functional Methods for QCD

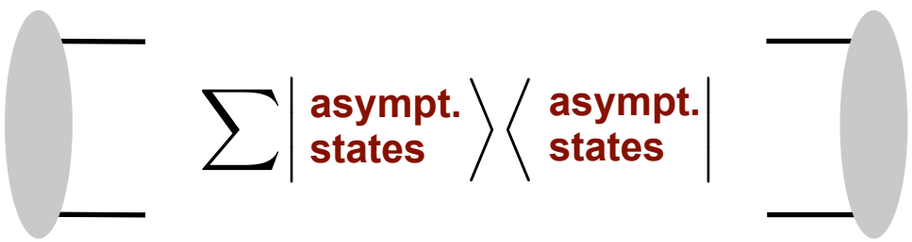
- **off-shell representation of thermodynamic observables**

e.g. $\mathrm{Tr}\,\langle q(x)\bar{q}(x)\rangle$



pressure, trace anomaly, fluctuations, volume flucs., ...

on-shell



e.g. hadron resonances

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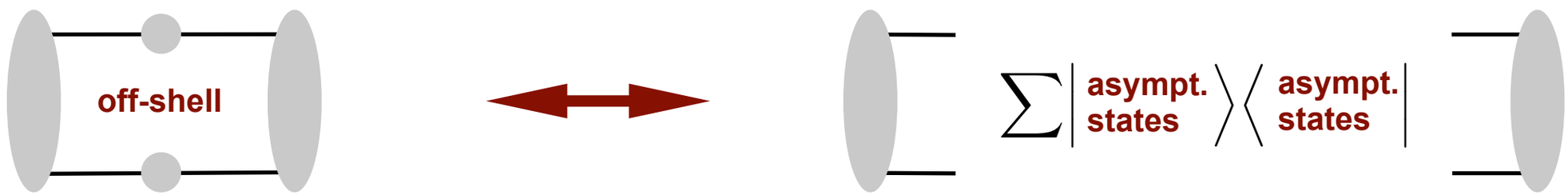


pressure, trace anomaly, fluctuations, volume flucs., ...

e.g. hadron resonances

- **gauge fixing = parameterisation**

$$\langle q(x_1)\cdots\bar{q}(x_{2n})\,A_\mu(y_1)\cdots A_\mu(y_m)\,h(z_1)\cdots h(z_l)\rangle$$

Consequences

I: simple correlations

II: Difficult access to some observables

'No free lunch theorem'

- 'Your mean field is not my mean field'

$$\left.\frac{\delta S_{\mathrm{cl}}[\phi]}{\delta\phi}\right|_{\phi=\bar{\phi}} = 0$$

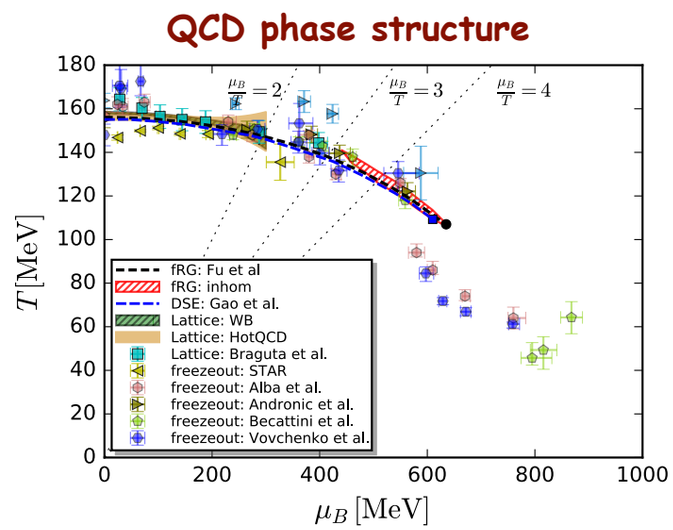
$$\left.\frac{\delta\Gamma[\phi]}{\delta\phi}\right|_{\phi=\bar{\phi}_{\mathrm{quant}}} = 0$$

full quantum equation of motion

Outline

- QCD from functional methods

- QCD phase structure

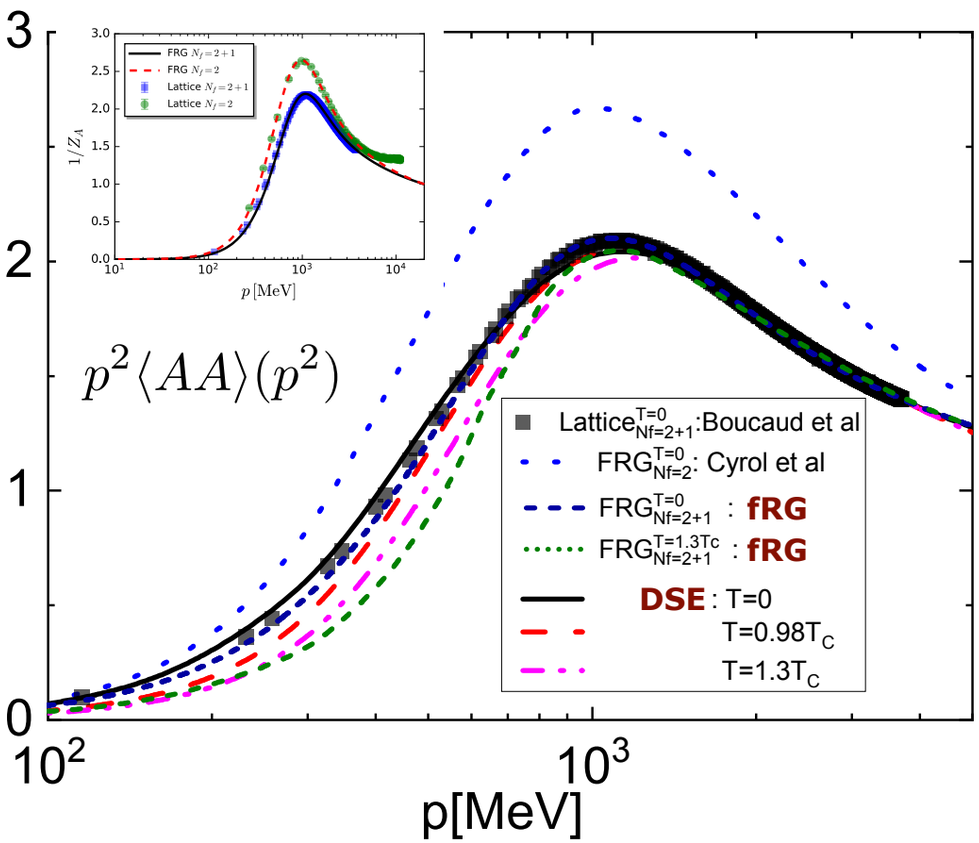


- Fluctuations of conserved charges

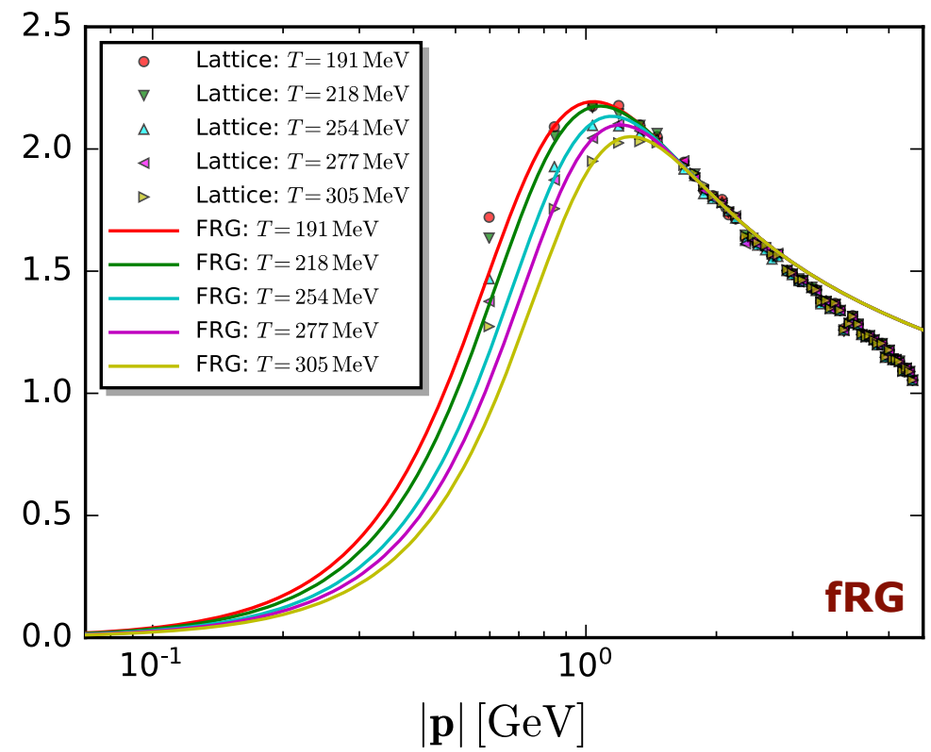
- Summary & outlook

QCD at finite density

Nf=2+1 Gluon and quark benchmark results in the vacuum and at finite T



$\vec{p}^2\langle AA\rangle(\vec{p}^2)$



fRG: Fu, JMP, Rennecke, PRD 101, (2020) 054032

DSE: Gao, JMP, PRD 102, (2020) 034027
PLB 820 (2021) 136584

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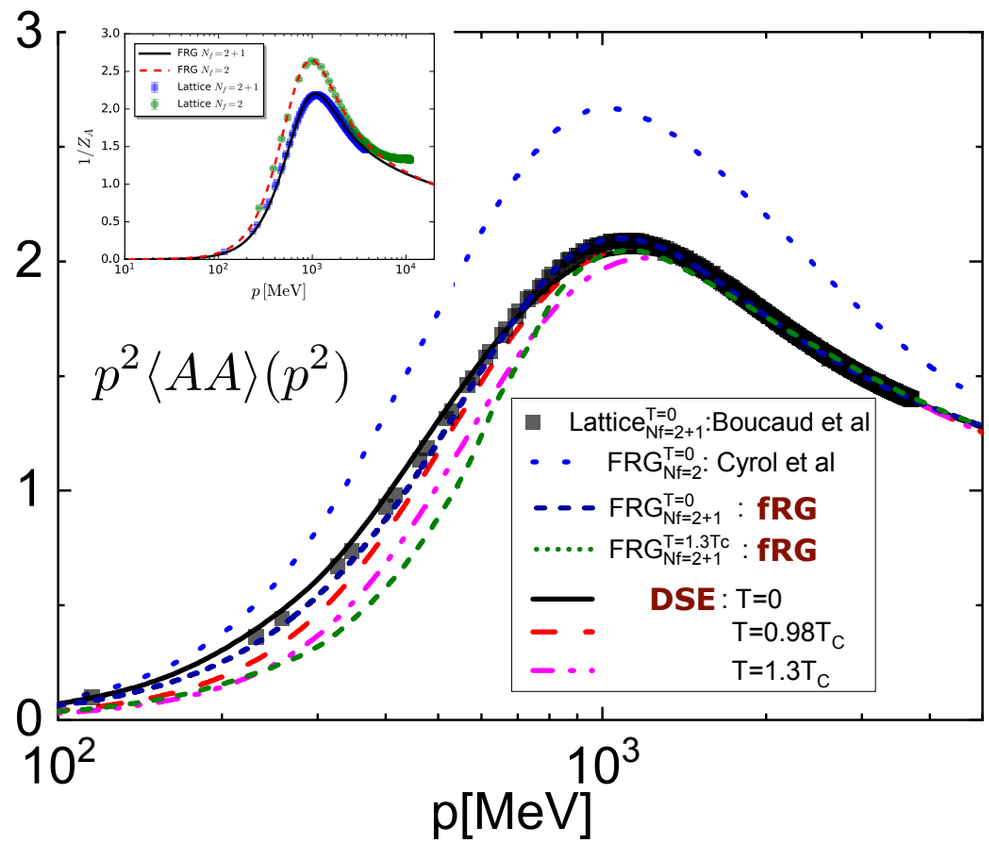

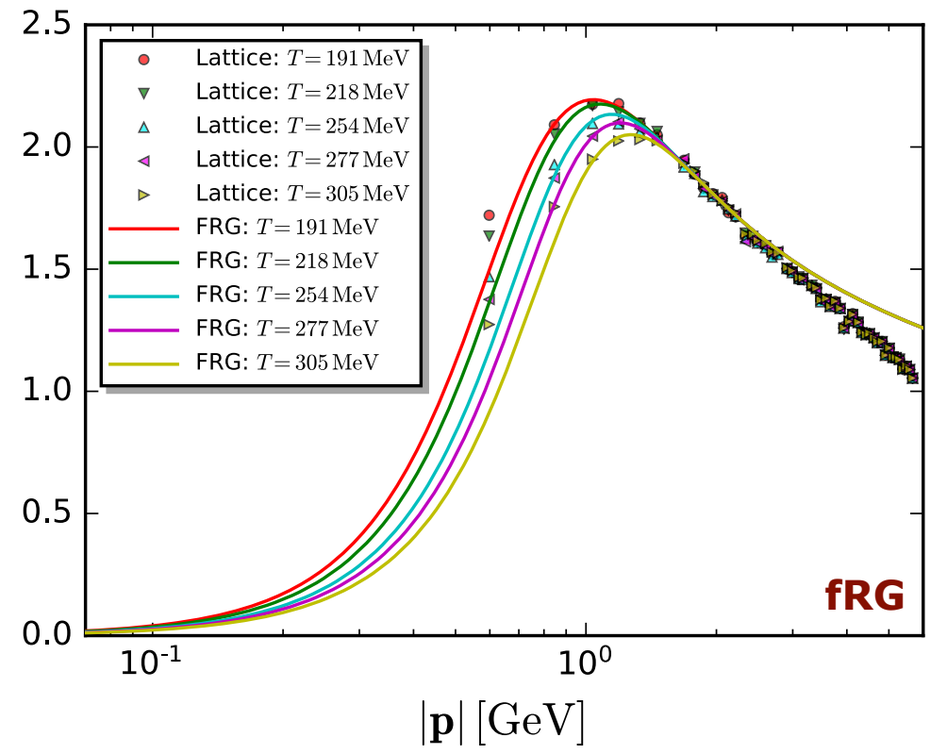

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PLB 820 (2021) 136584

DSE: vacuum & finite T

Fischer, Luecker, PLB 718 (2013) 1036
Fischer, Luecker,Welzbacher, PRD 90 (2014) 034022
Isserstedt, Buballa, Fischer, Gunkel, PRD 100 (2019) 074011
Gunkel, Fischer, 2106.08356

lattice:

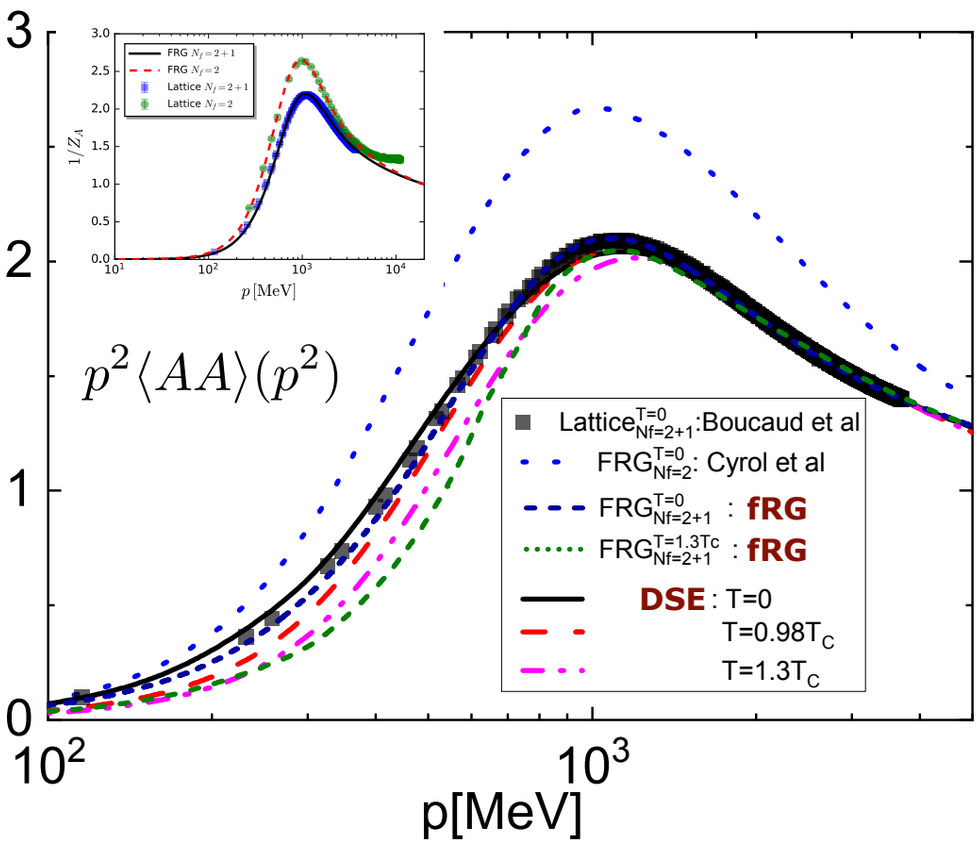
Nf=2: Sternbeck, Maltman, Müller-Preussker, von Smekal, PoS LATTICE2012, 243 (2012)

Nf=2+1: Aguilar, De Soto, Ferreira, Papavassiliou, Rodriguez-Quintero, Zafeiropoulos, EPJC 80 (2020) 2, 154,
Boucaud, De Soto, Raya, Rodriguez-Quintero, Zafeiropoulos, PRD 98, 114515 (2018)

Finite T: Ilgenfritz, JMP, Rothkopf, Trunin, EPJ C78, 127 (201) (Nf=2+1+1)

QCD at finite density

Nf=2+1 Gluon and quark benchmark results in the vacuum and at finite T



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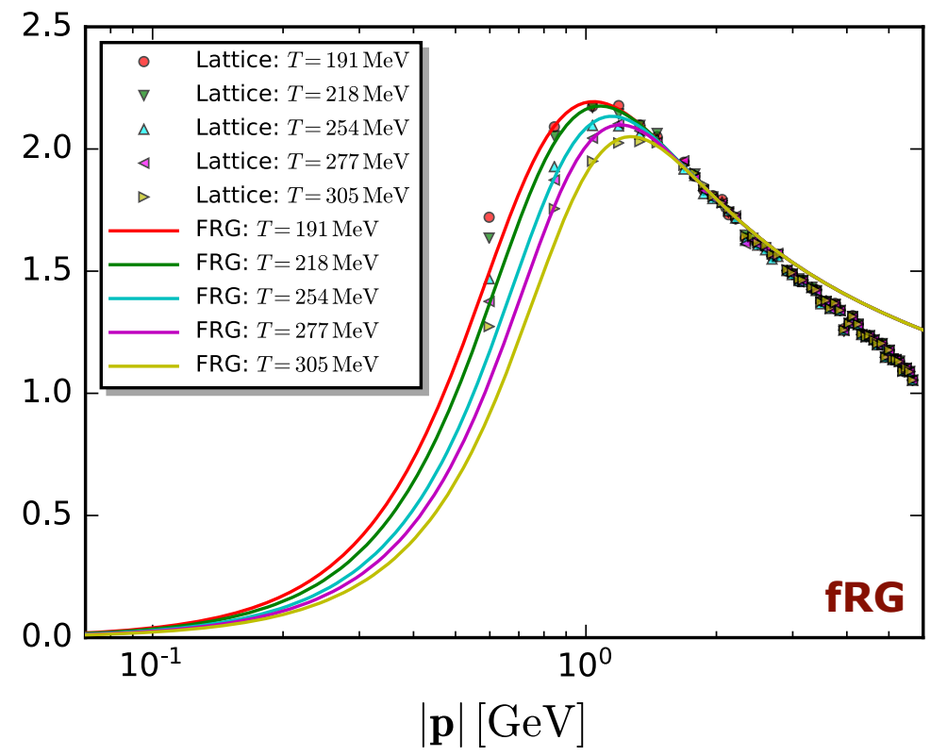

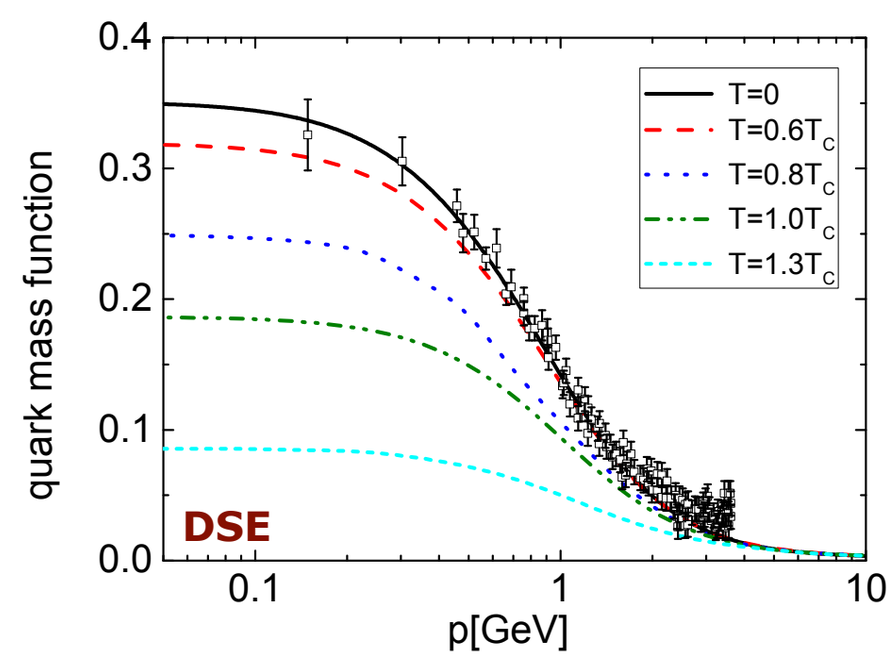

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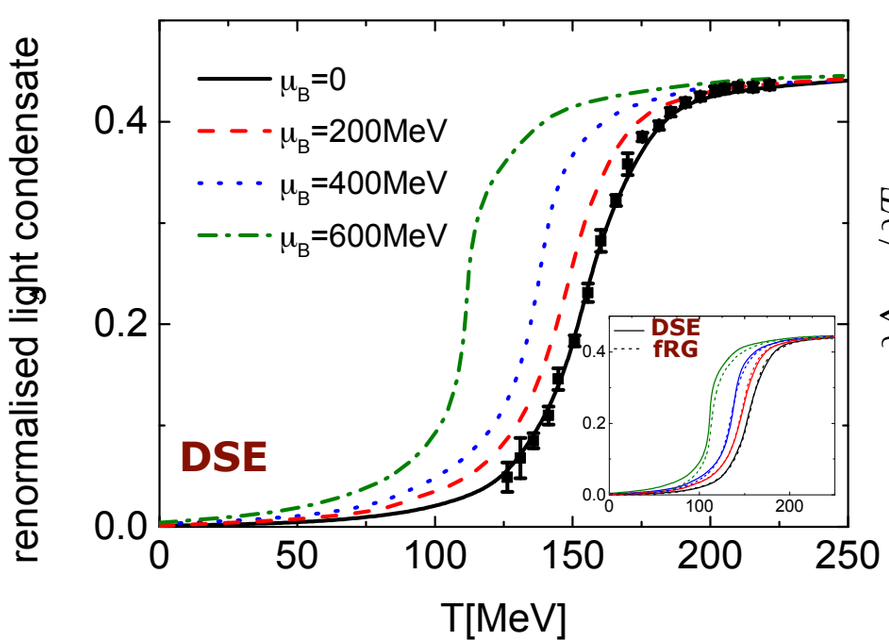
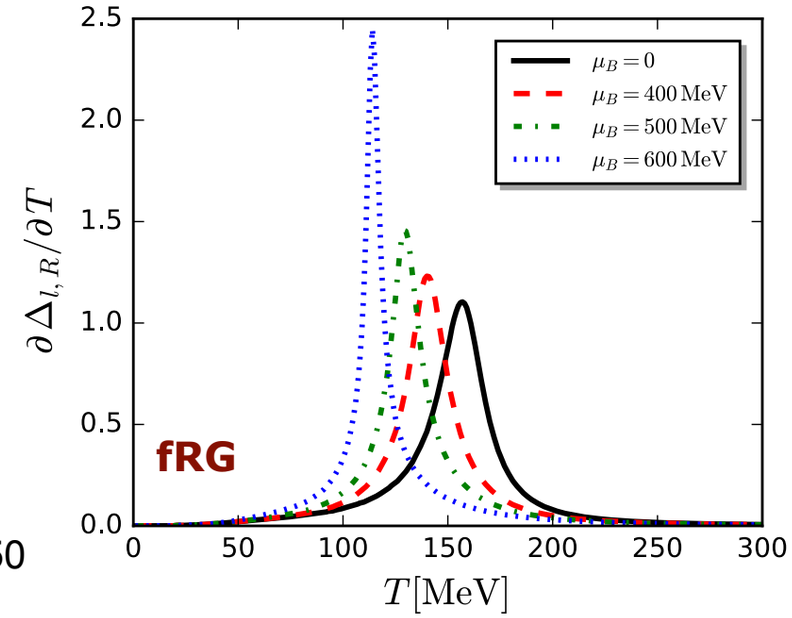
PLB 820 (2021) 136584

QCD at finite density

Chiral order parameter benchmark results at finite T

renormalised condensate





$$\Delta_{l,R}(T,\mu_B) \simeq \Delta_l(T,\mu_B) - \Delta_l(0,0)$$

$$\Delta_q(T,\mu_B) = \frac{T}{\mathcal{V}}\, m_q^0 \int_x \langle \bar{q}(x) q(x) \rangle$$

lattice: S. Borsanyi, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg, C. Ratti, and K. K. Szabo, JHEP 09, 073 (2010)

DSE: quark condensates

See also

Fischer, Luecker, PLB 718 (2013) 1036
Fischer, Luecker,Welzbacher, PRD 90 (2014) 034022
Isserstedt, Buballa, Fischer, Gunkel, PRD 100 (2019) 074011

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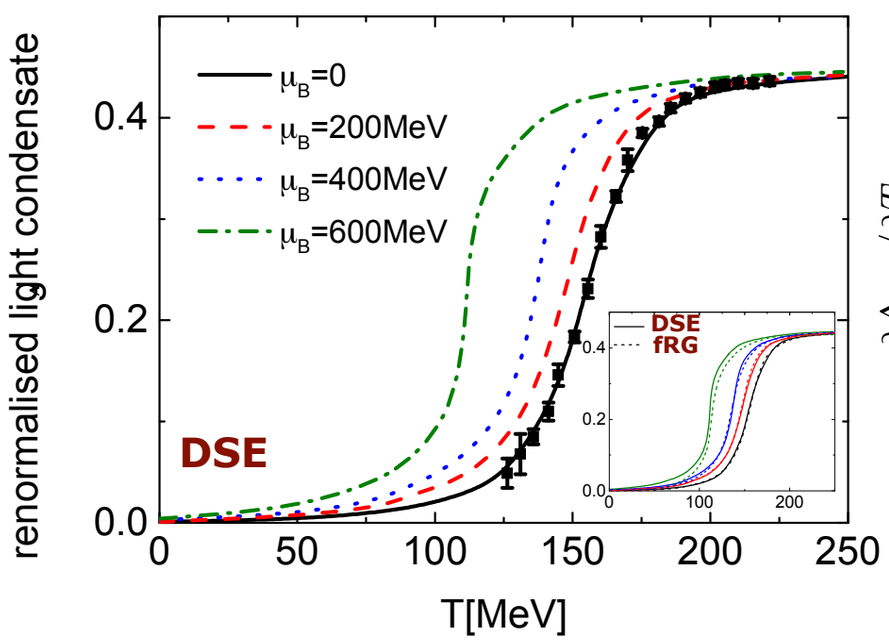

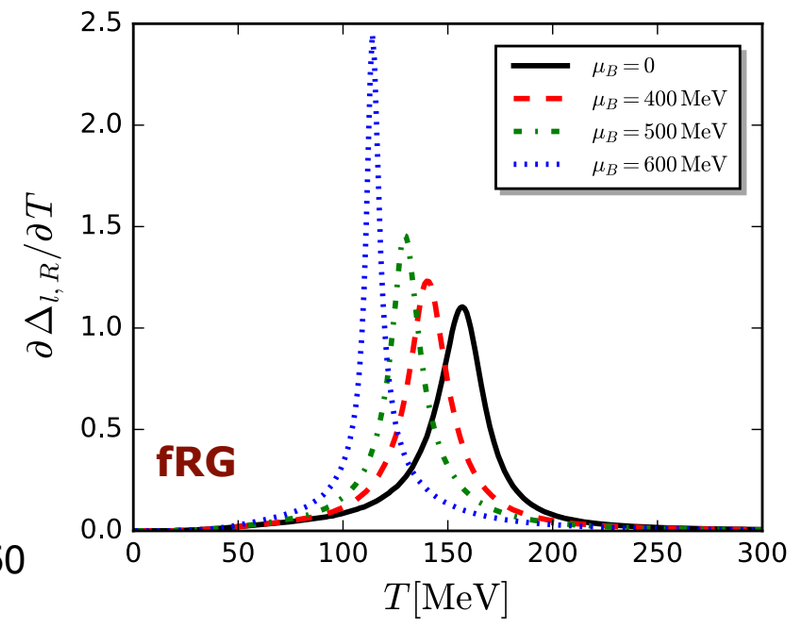

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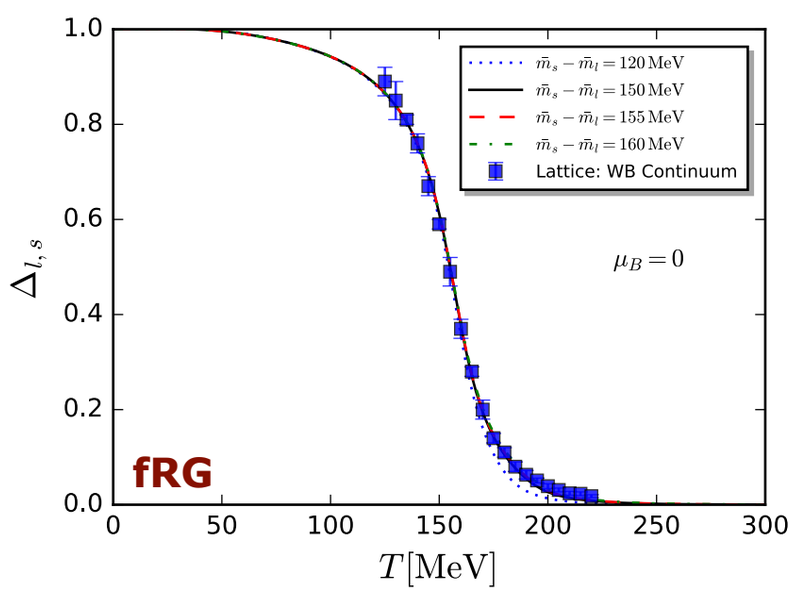
reduced condensate

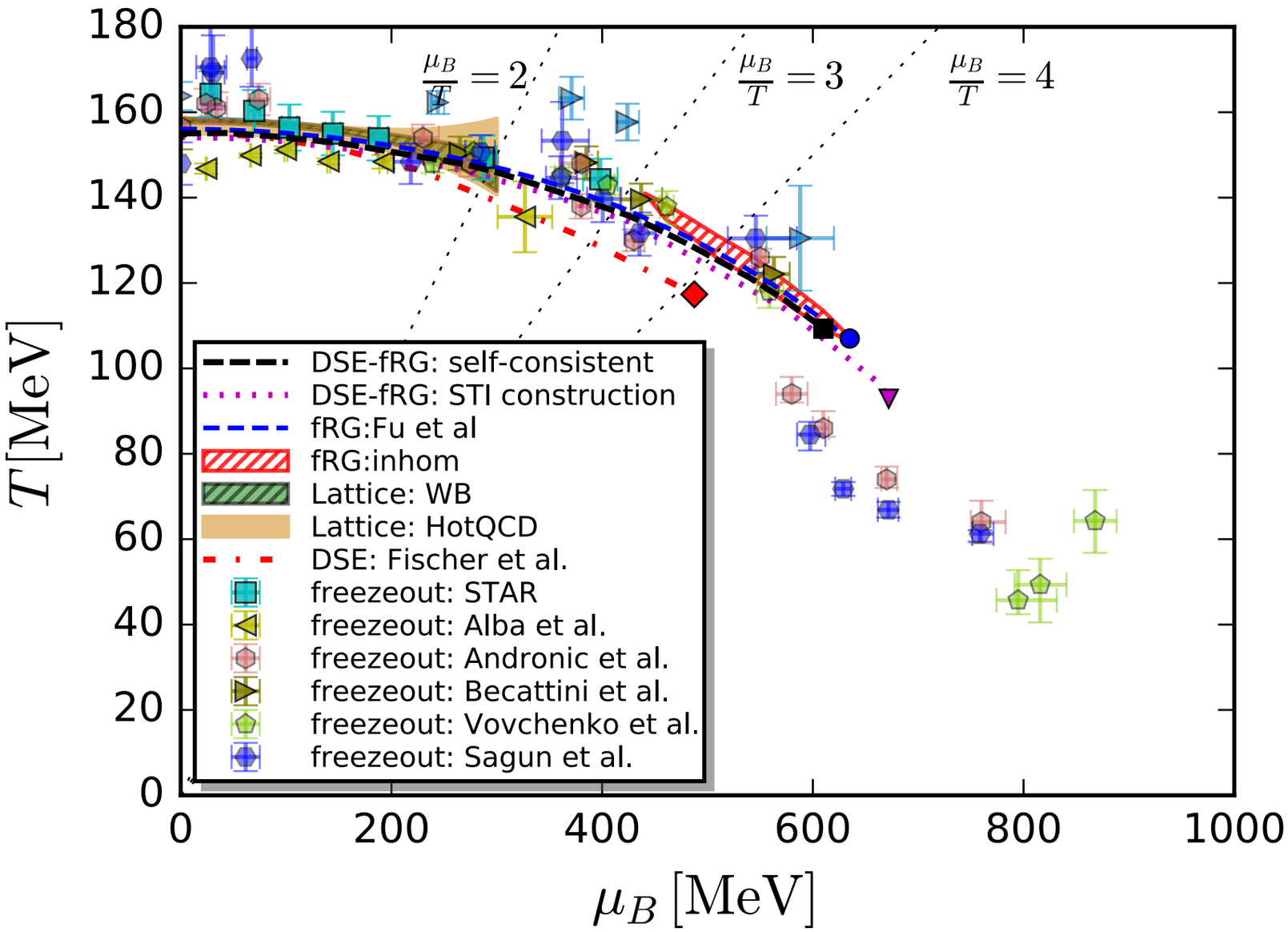


$$\Delta_{l,s}(T,\mu_B) = \frac{\Delta_l(T,\mu_B) - \left(\frac{m_l^0}{m_s^0}\right)^2 \Delta_s(T,\mu_B)}{\Delta_l(0,0) - \left(\frac{m_l^0}{m_s^0}\right)^2 \Delta_s(0,0)}$$

fRG: Fu, JMP, Rennecke, PRD 101, (2020) 054032

DSE: Gao, JMP, PLB 820 (2021) 136584

QCD phase structure



See also

Fischer, Luecker,Welzbacher, PRD 90 (2014) 034022
Isserstedt, Buballa, Fischer, Gunkel, PRD 100 (2019) 074011

talks of Philipp Isserstedt, Julian Bernhardt, Evening Session Sunday

fRG: **Fu, JMP, Rennecke, PRD 101, (2020) 054032**
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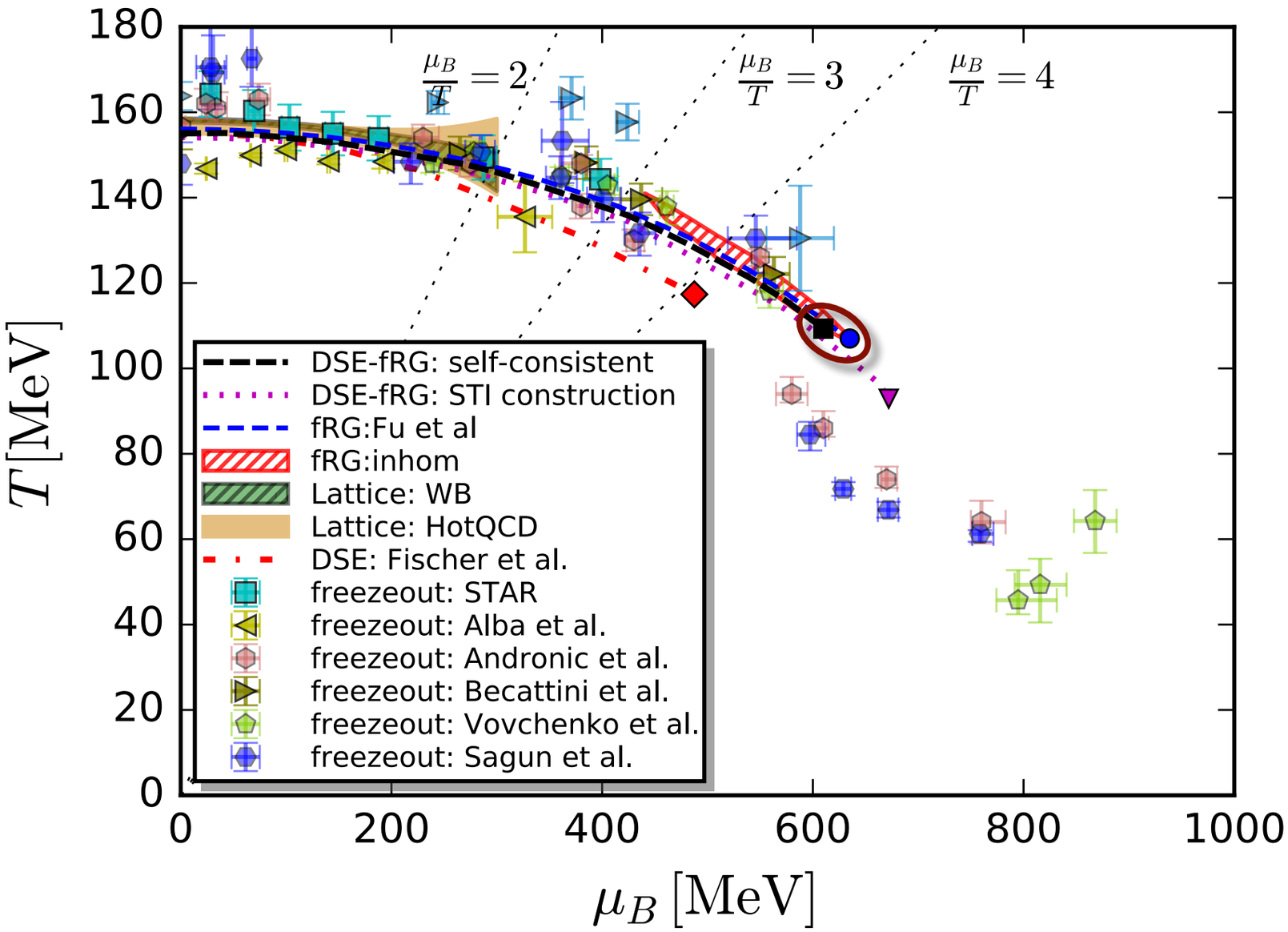

CEP fRG-DSE

$$(T, \mu_B)_{\mathrm{CEP}} = (107, 635)\,\mathrm{MeV}$$

$$(T, \mu_B)_{\mathrm{CEP}} = (109, 610)\,\mathrm{MeV}$$

fRG: Fu, JMP, Rennecke, PRD 101, (2020) 054032
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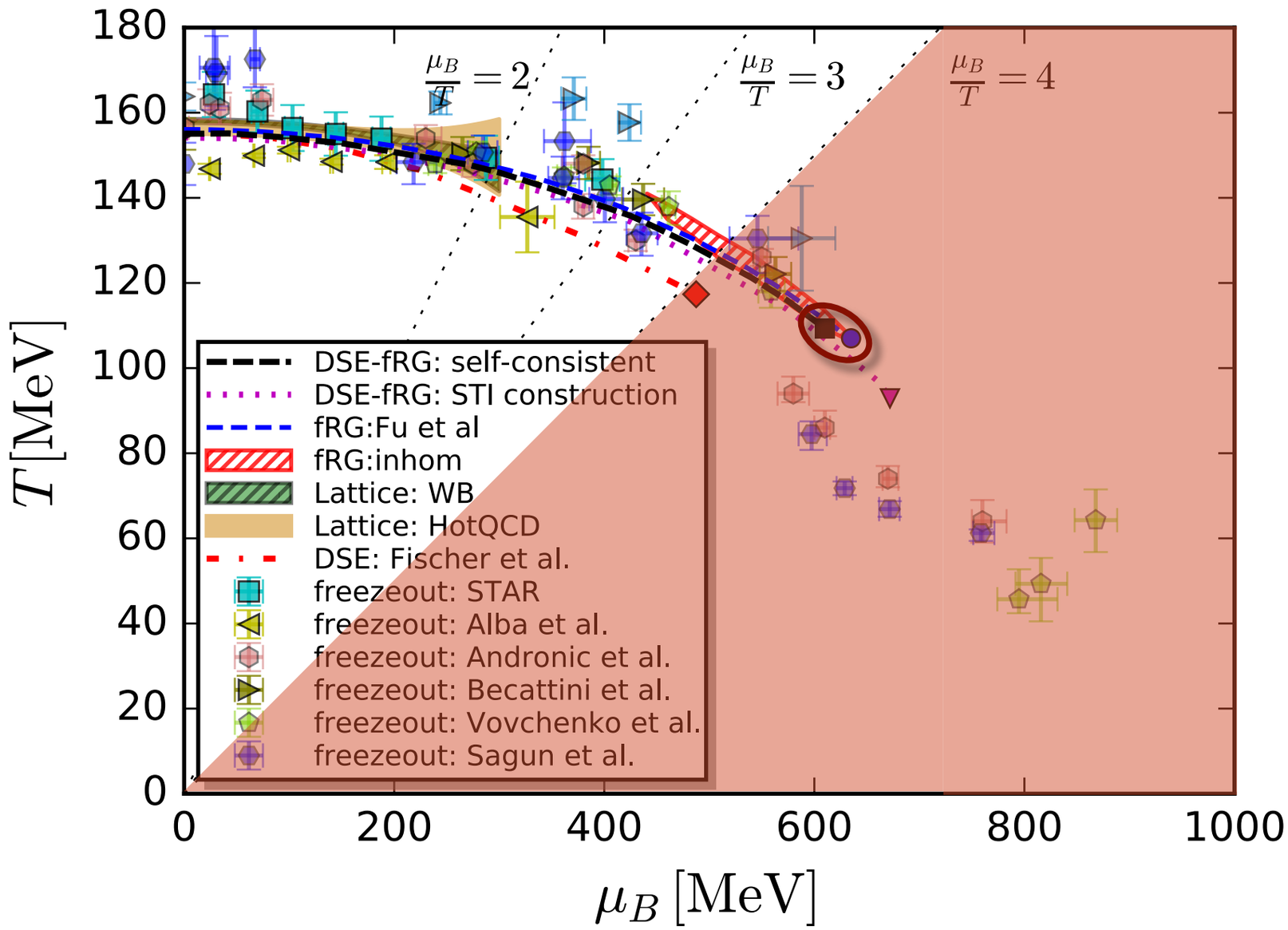


curvature fRG-DSE

$$\kappa_{\rm FRG} = 0.0142(2)$$

$$\kappa_{\rm DSE} = 0.0147(5)$$

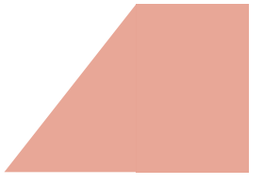
curvature lattice

$$\kappa_{\rm WB} = 0.0149(21)$$

WB, PLB 751 (2015) 559

$$\kappa_{\rm hotQCD} = 0.015(4)$$

hotQCD, PLB 795 (2019) 15



area beyond quantitative reliability bound

$$\mu_{\rm B}/{\rm T} \gtrsim 4$$

CEP fRG-DSE

$$(T, \mu_B)_{\rm CEP} = (107, 635)\,{\rm MeV}$$

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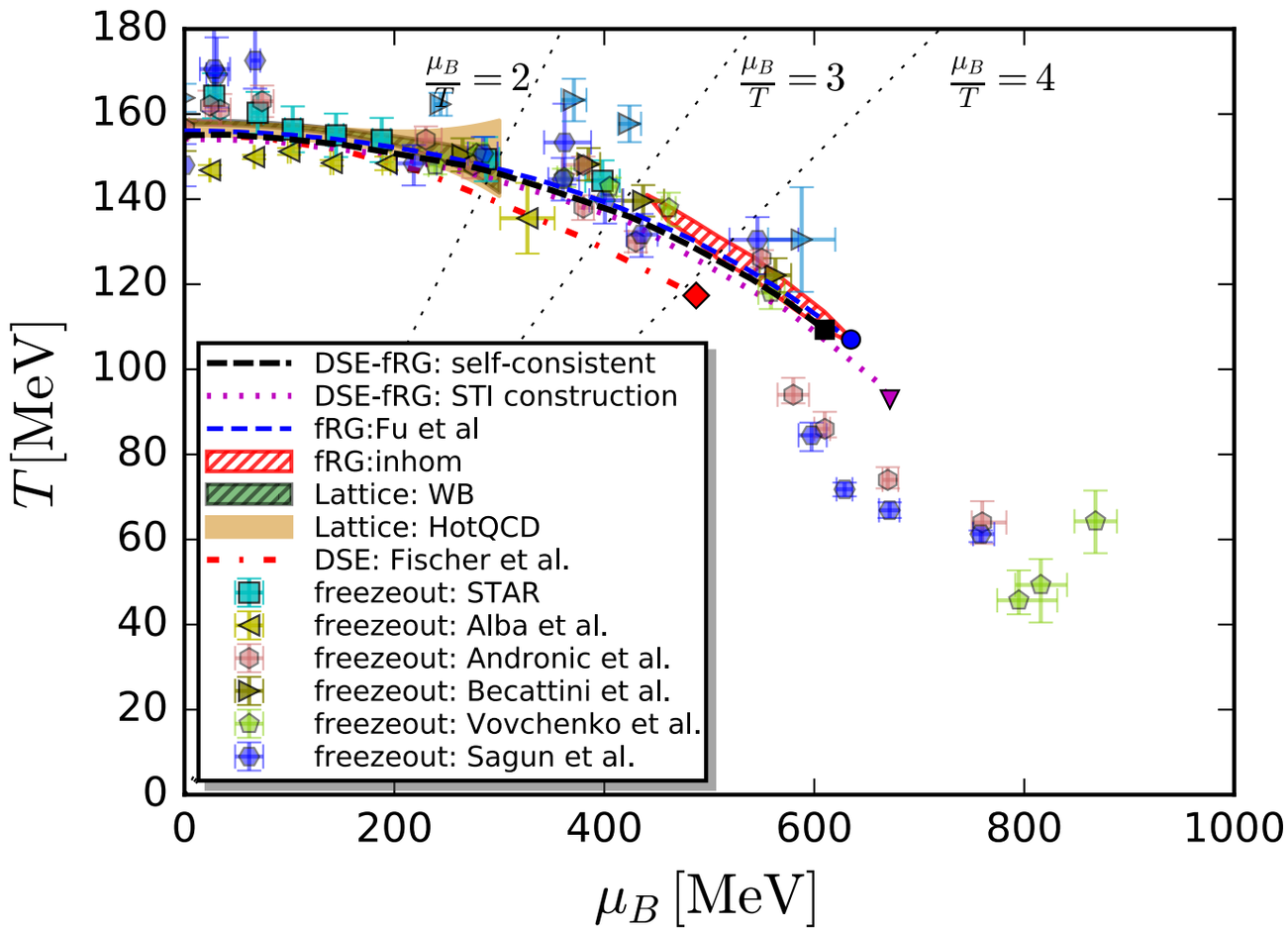
QCD phase structure

Reliability considerations



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QCD phase structure

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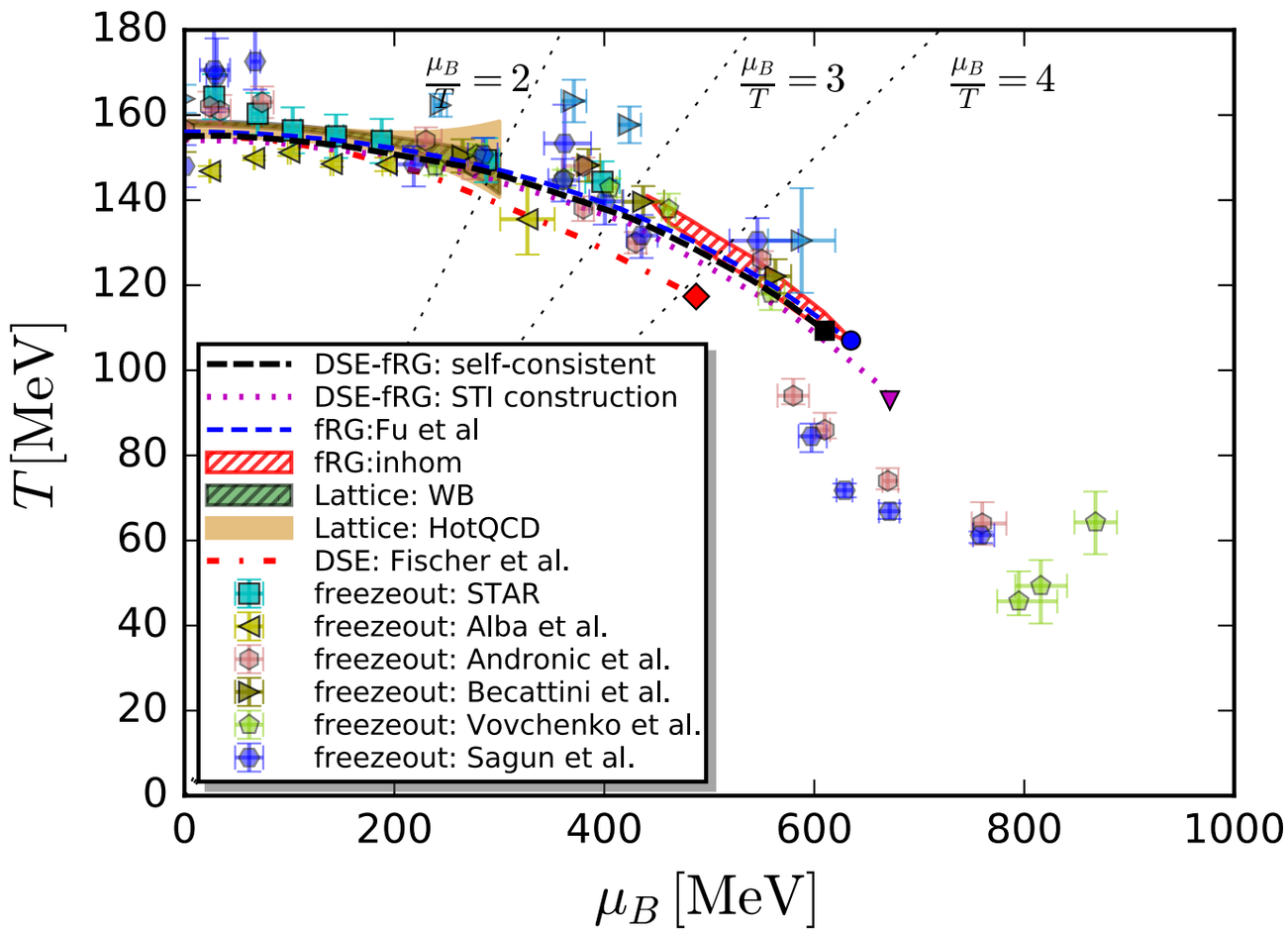
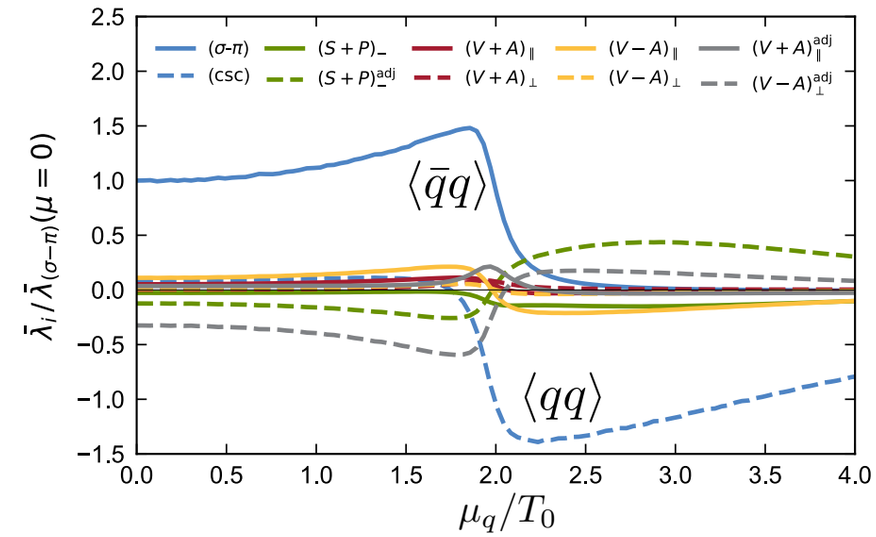


Dominant channels I (fRG)

Braun, Leonhardt, Pospiech, PRD 101 (2020) 036004



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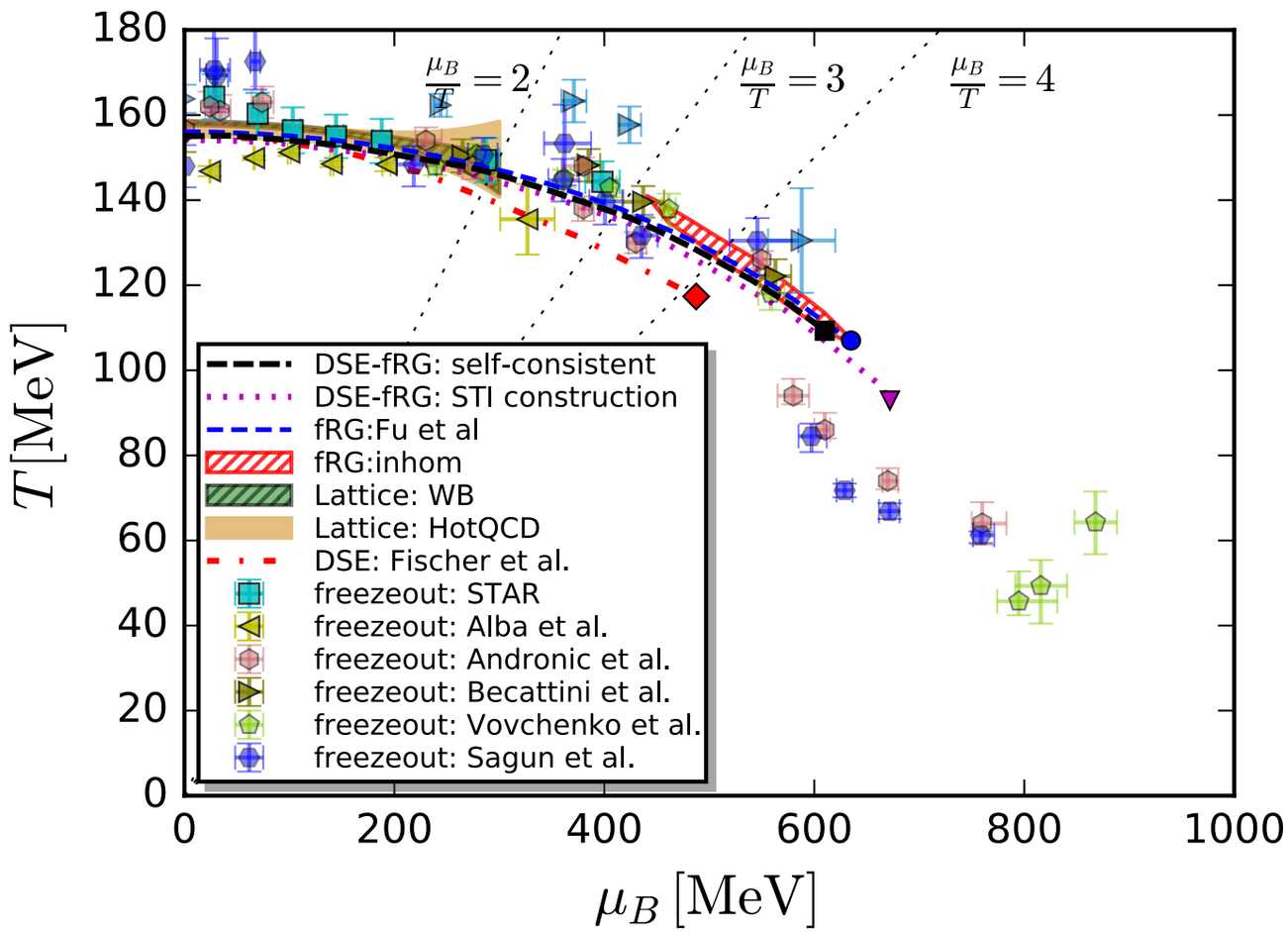

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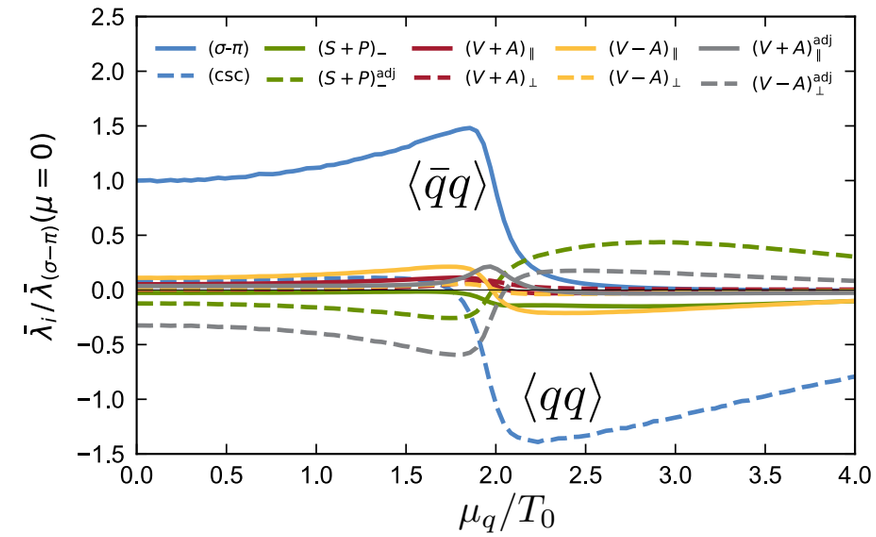
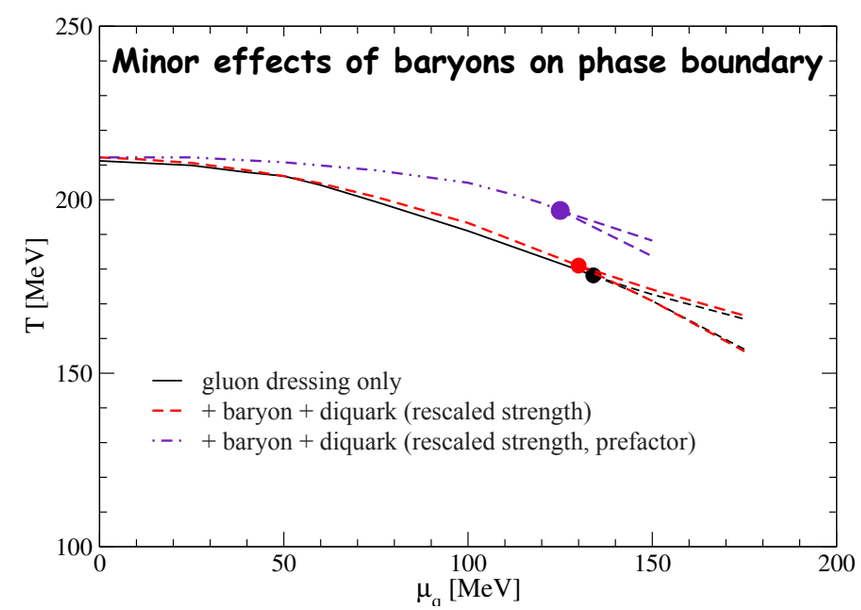


Dominant channels II (DSE)

Eichmann, Fischer, Welzbacher, PRD 93 (2016) 034013



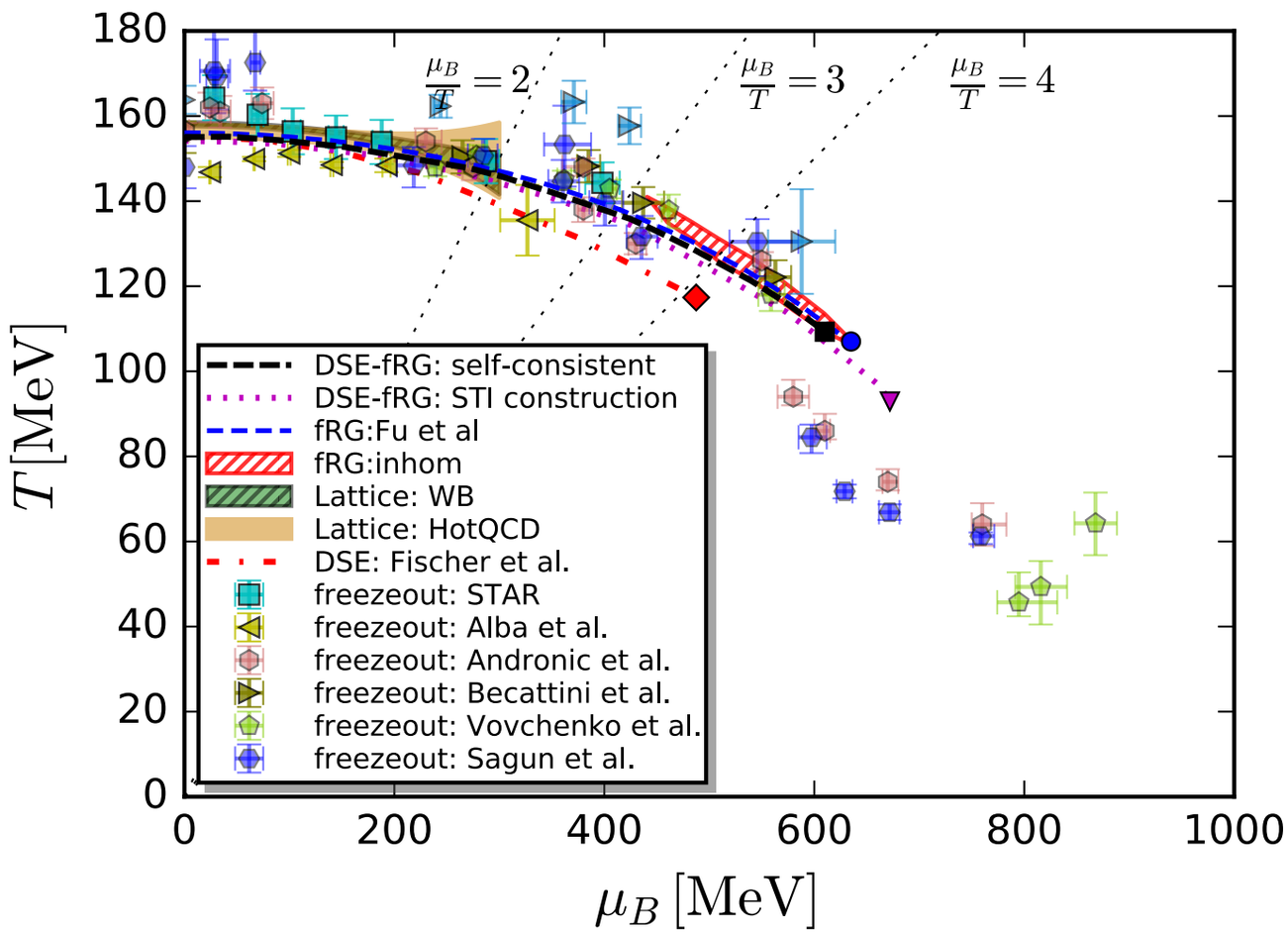
Minor effects of mesons: Gunkel, Fischer, 2106.08356

fRG: Fu, JMP, Rennecke, PRD 101, (2020) 054032

DSE: Gao, JMP, PLB 820 (2021) 136584

QCD phase structure

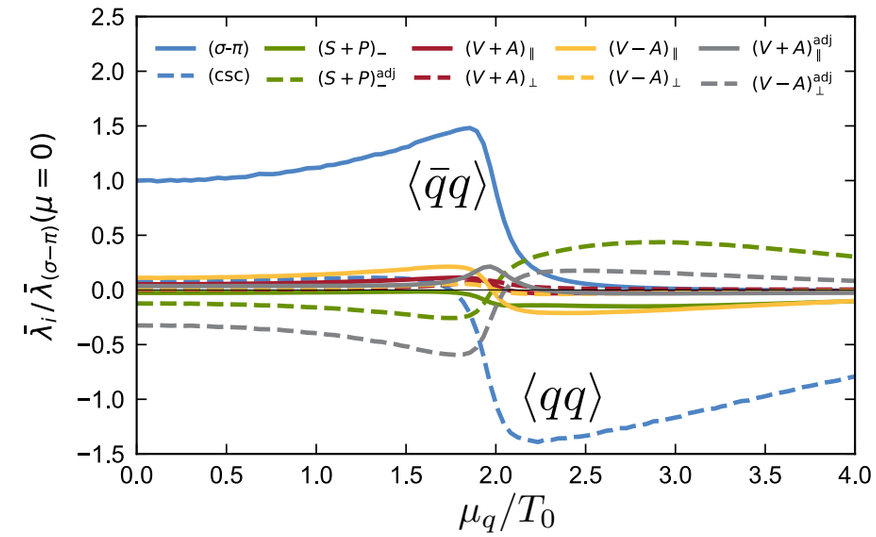
Reliability considerations



I+II → Fierz-complete computation

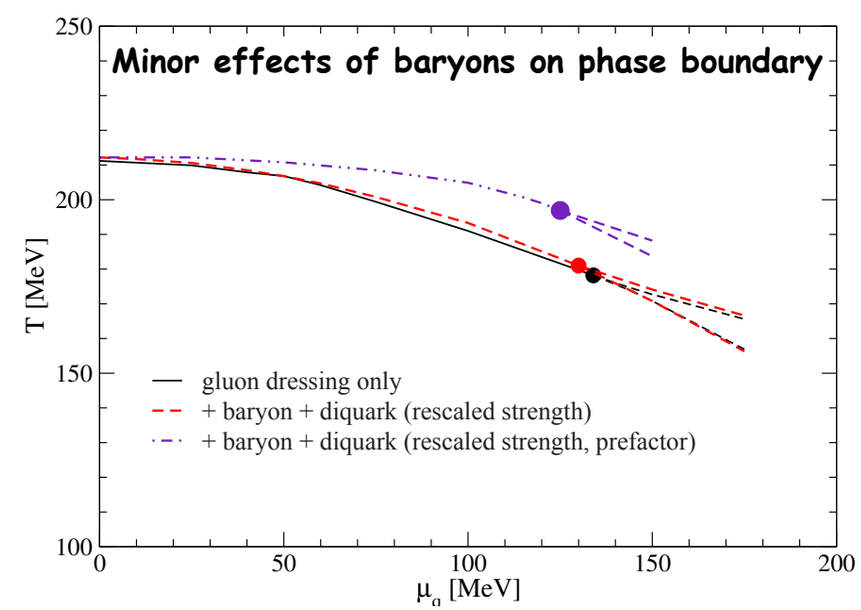
Dominant channels I (fRG)

Braun, Leonhardt, Pospiech, PRD 101 (2020) 036004



Dominant channels II (DSE)

Eichmann, Fischer, Welzbacher, PRD 93 (2016) 034013



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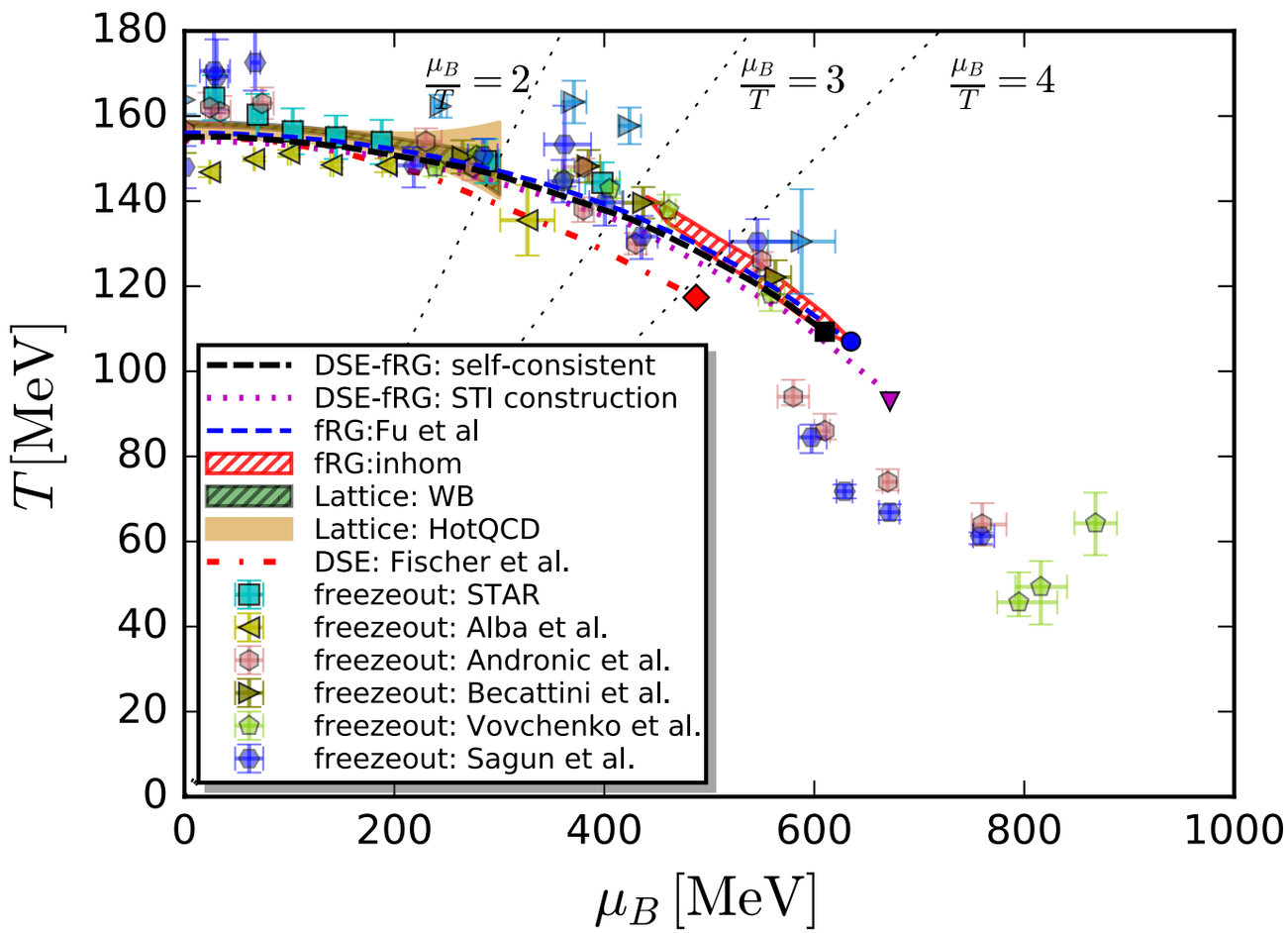
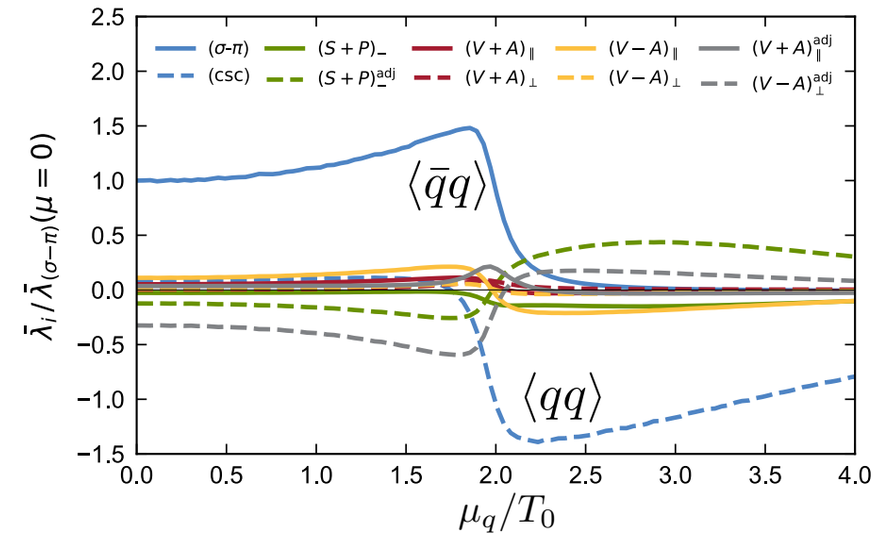


'Inhomogeneous' phase

: Pion dispersion has minimum at non-vanishing spatial momentum

Dominant channels I (fRG)

Braun, Leonhardt, Pospiech, PRD 101 (2020) 036004



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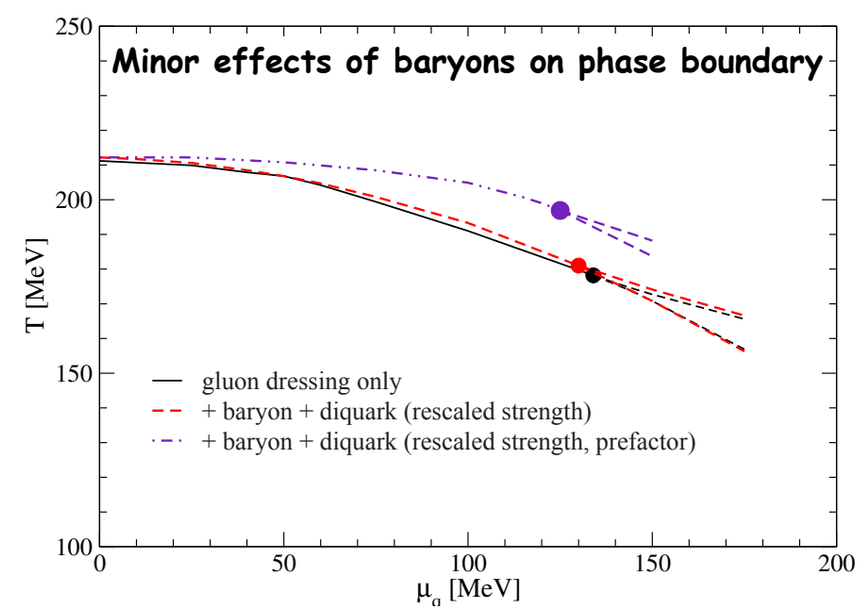
Eichmann, Fischer, Welzbacher, PRD 93 (2016) 034013



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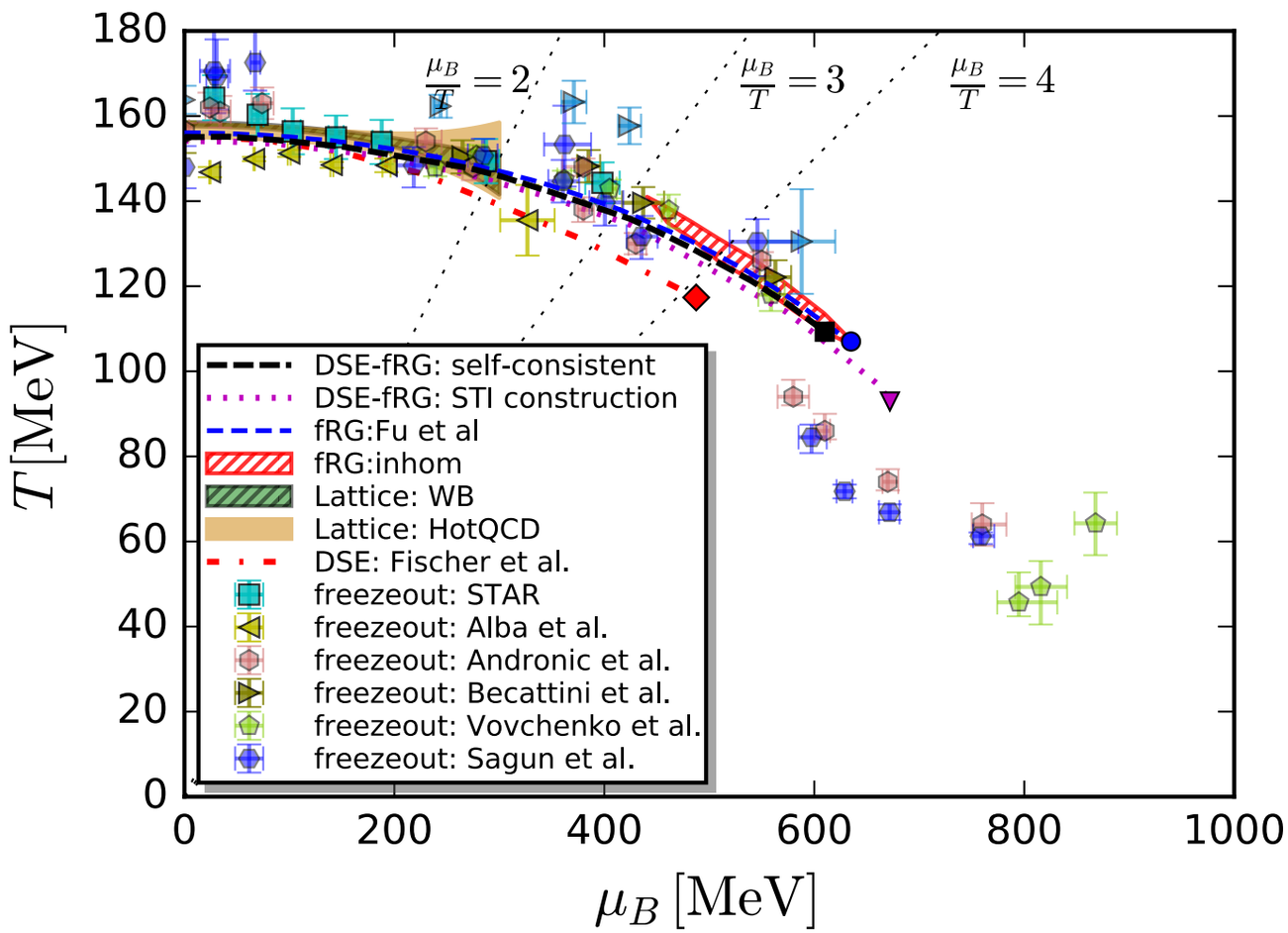

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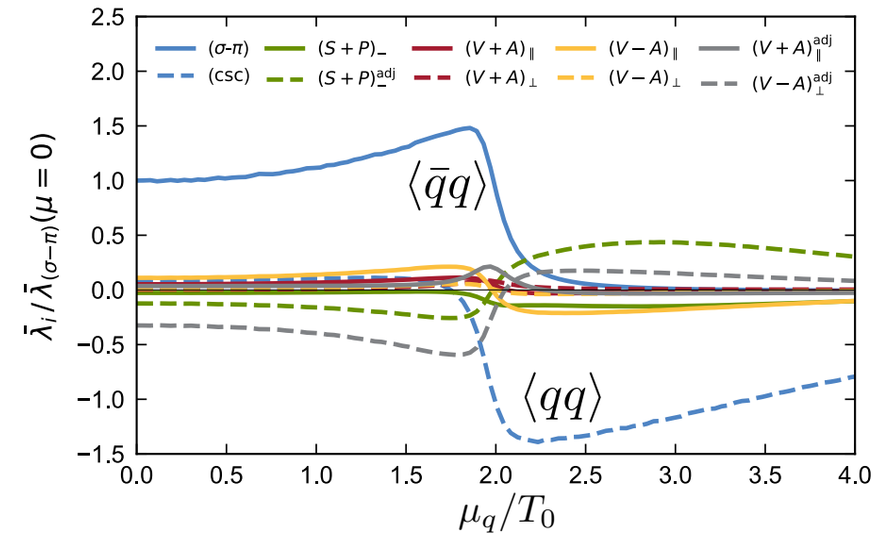

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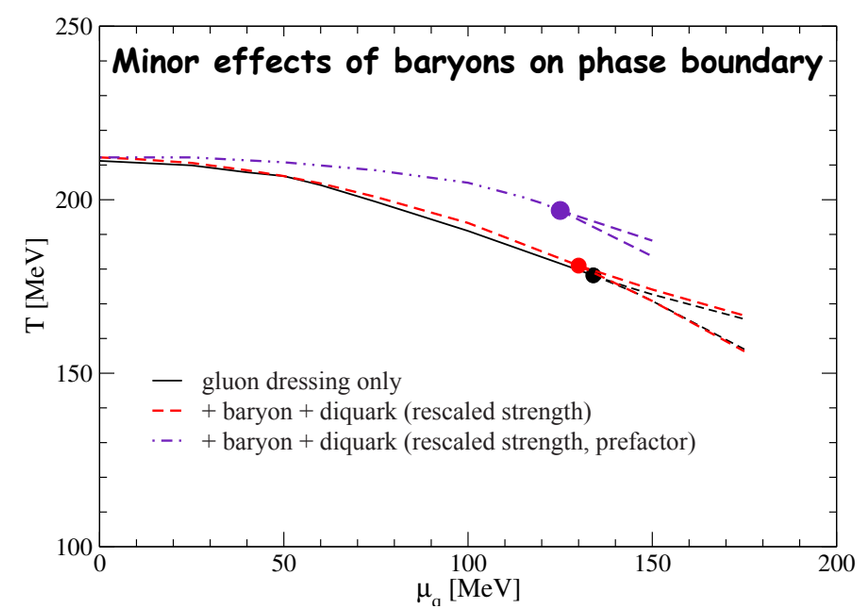

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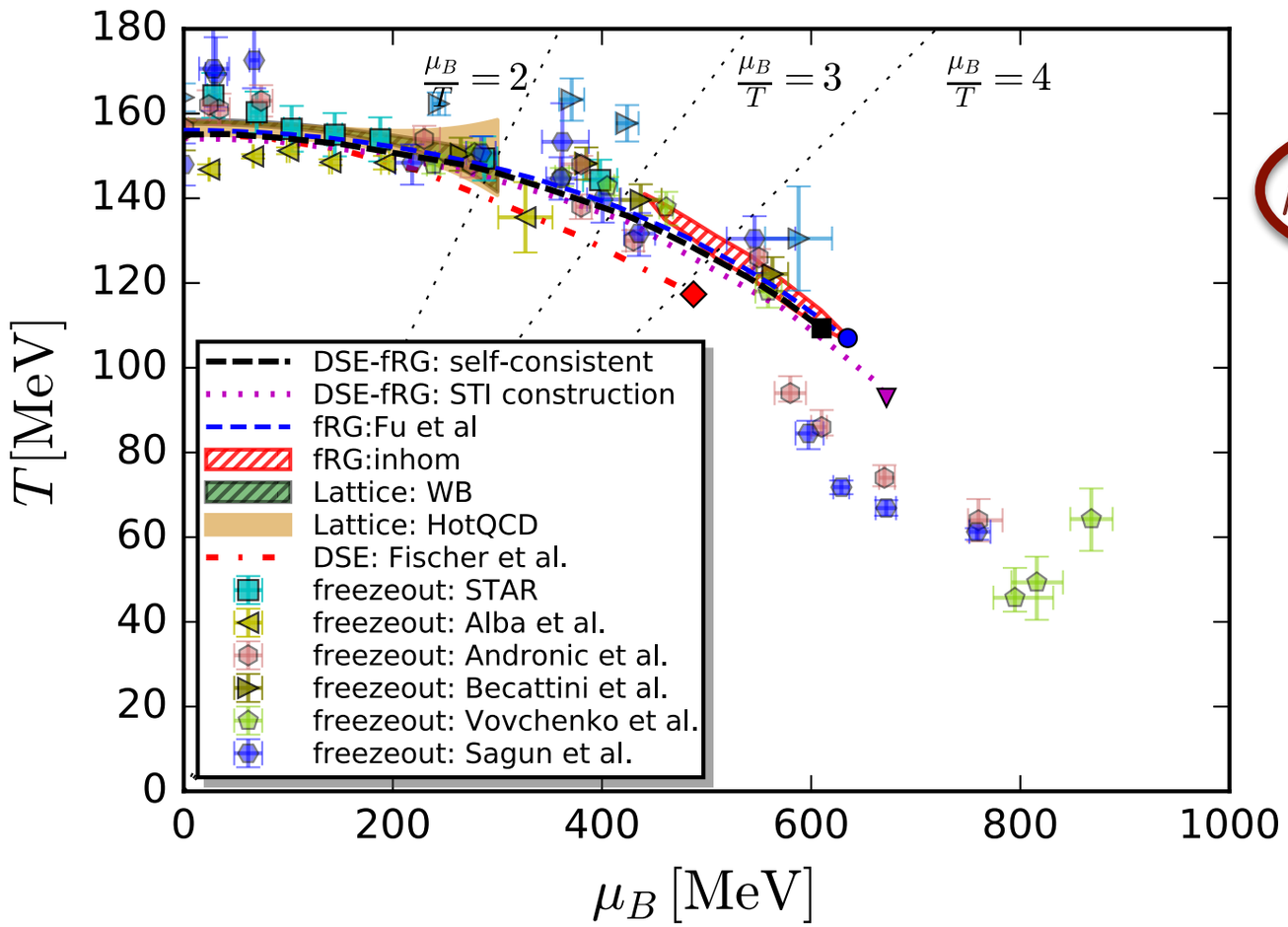
Non-trivial background

fRG: Fu, JMP, Rennecke, PRD 101, (2020) 054032

DSE: Gao, JMP, PLB 820 (2021) 136584

QCD phase structure

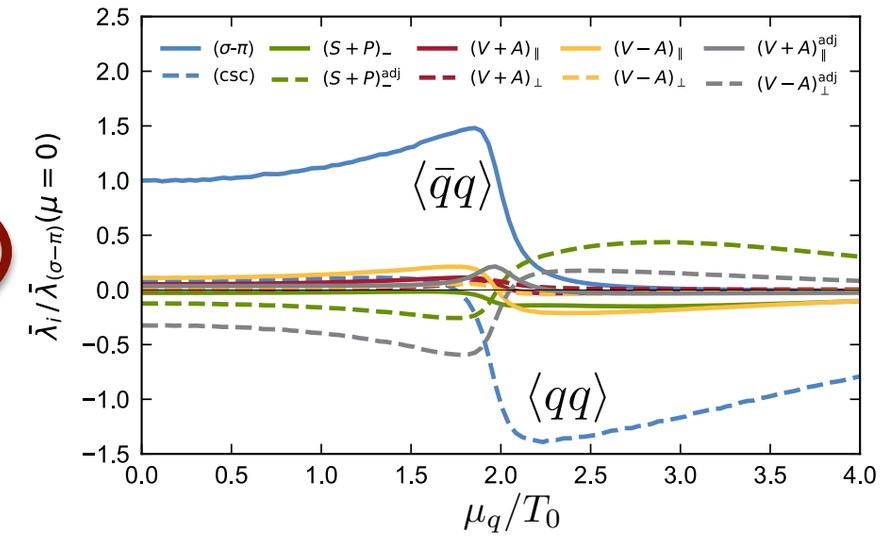
Reliability considerations



$\mu_B/T \lesssim 4$

Dominant channels I (fRG)

Braun, Leonhardt, Pospiech, PRD 101 (2020) 036004



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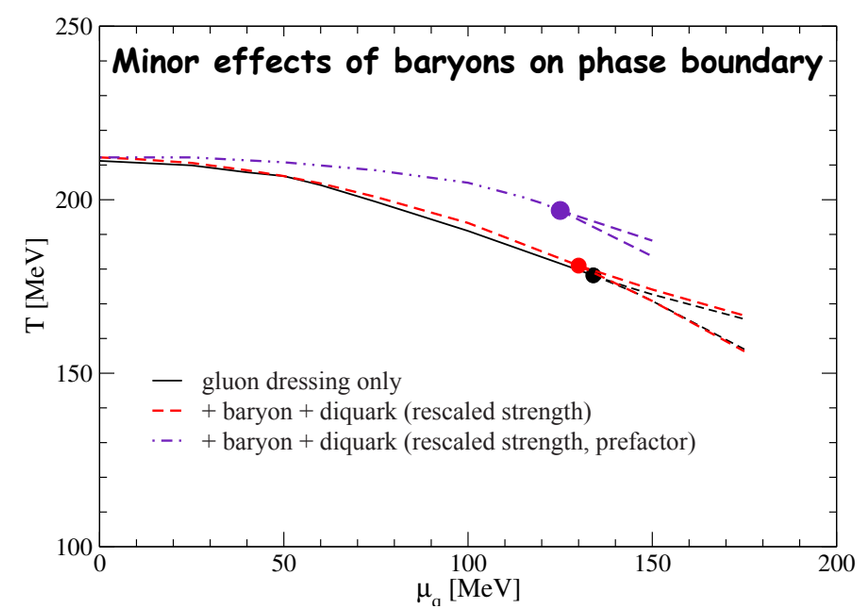

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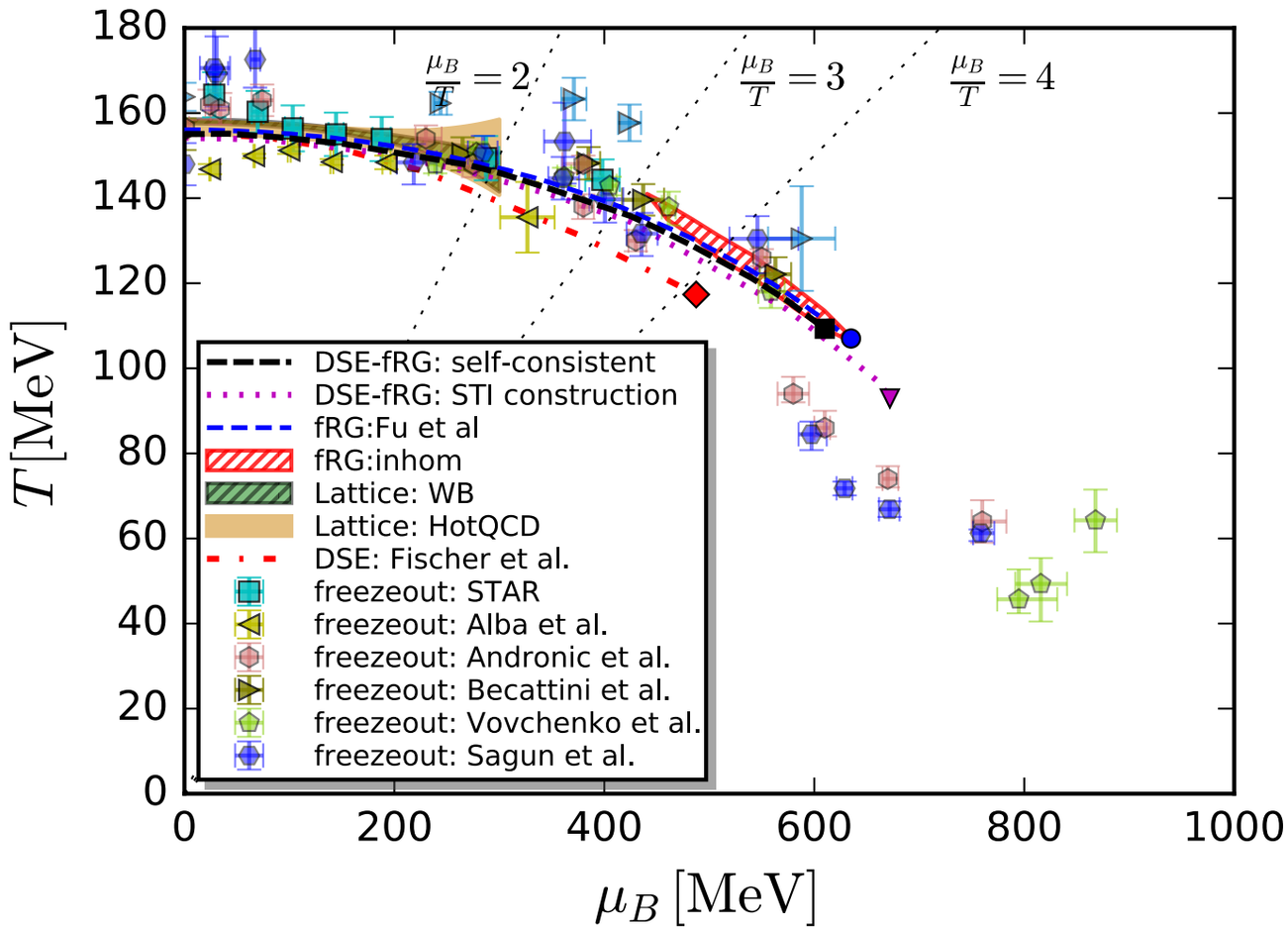
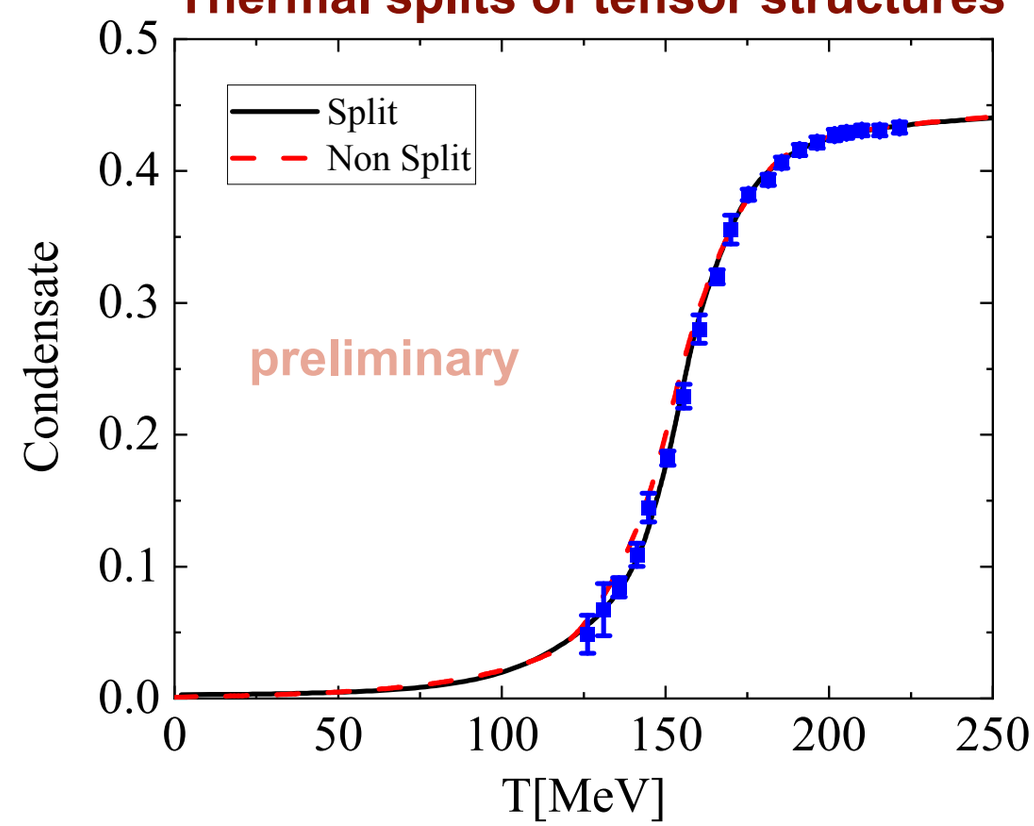


Thermal splits of tensor structures

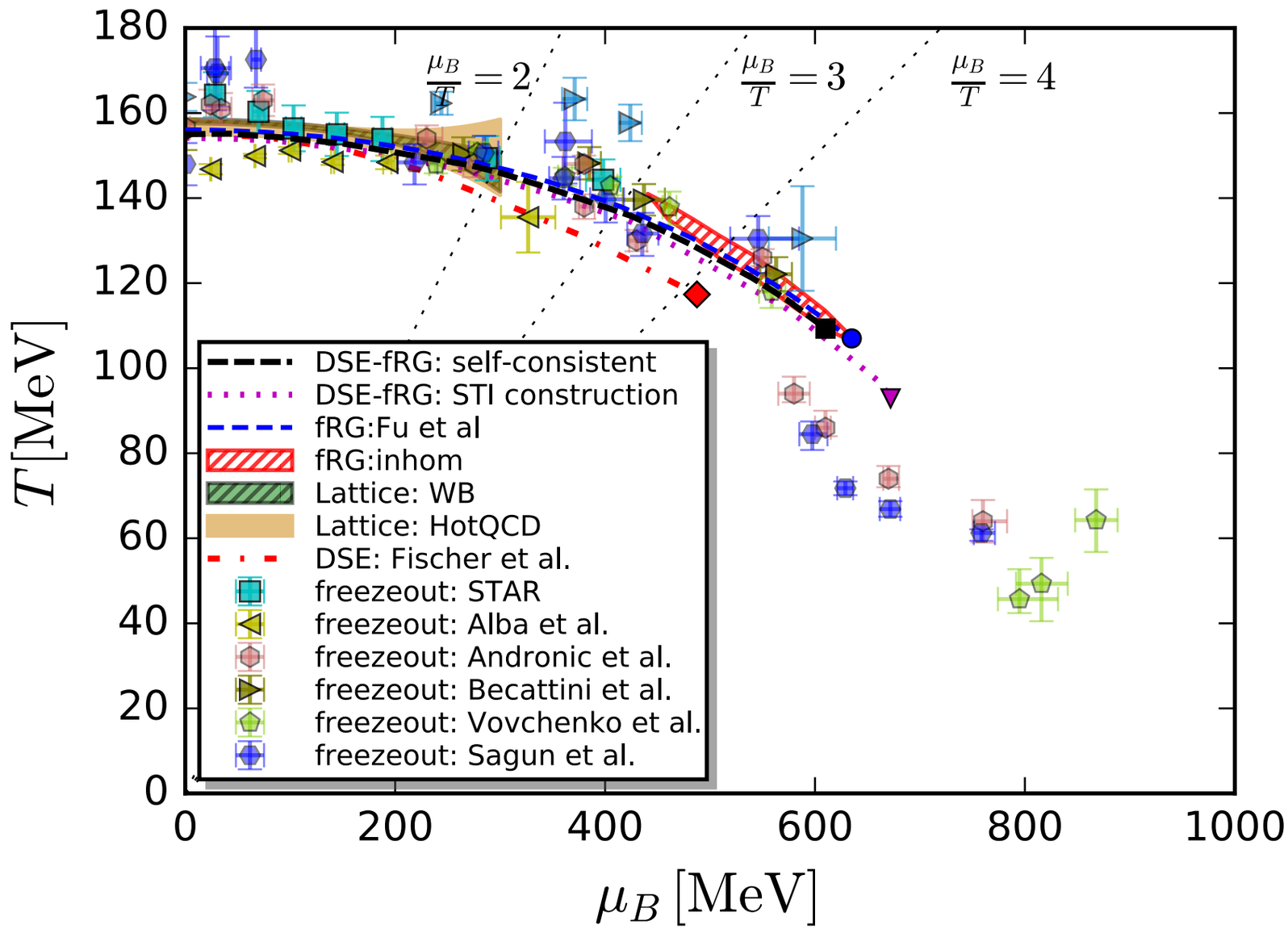


Gao, JMP, Schneider, in preparation

fRG: Fu, JMP, Rennecke, PRD 101, (2020) 054032
DSE: Gao, JMP, PLB 820 (2021) 136584

QCD phase structure

Estimate for CEP



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QCD phase structure

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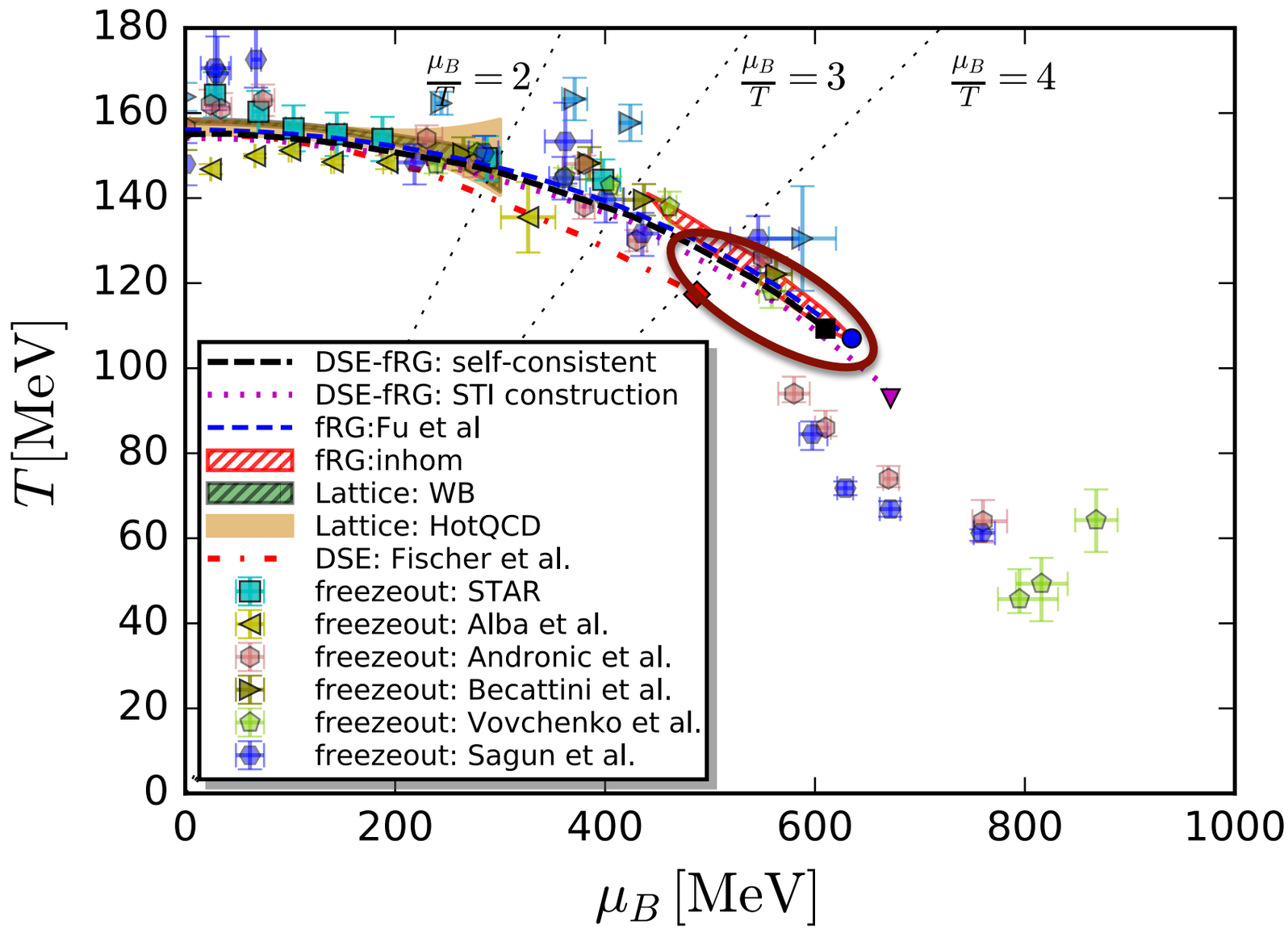


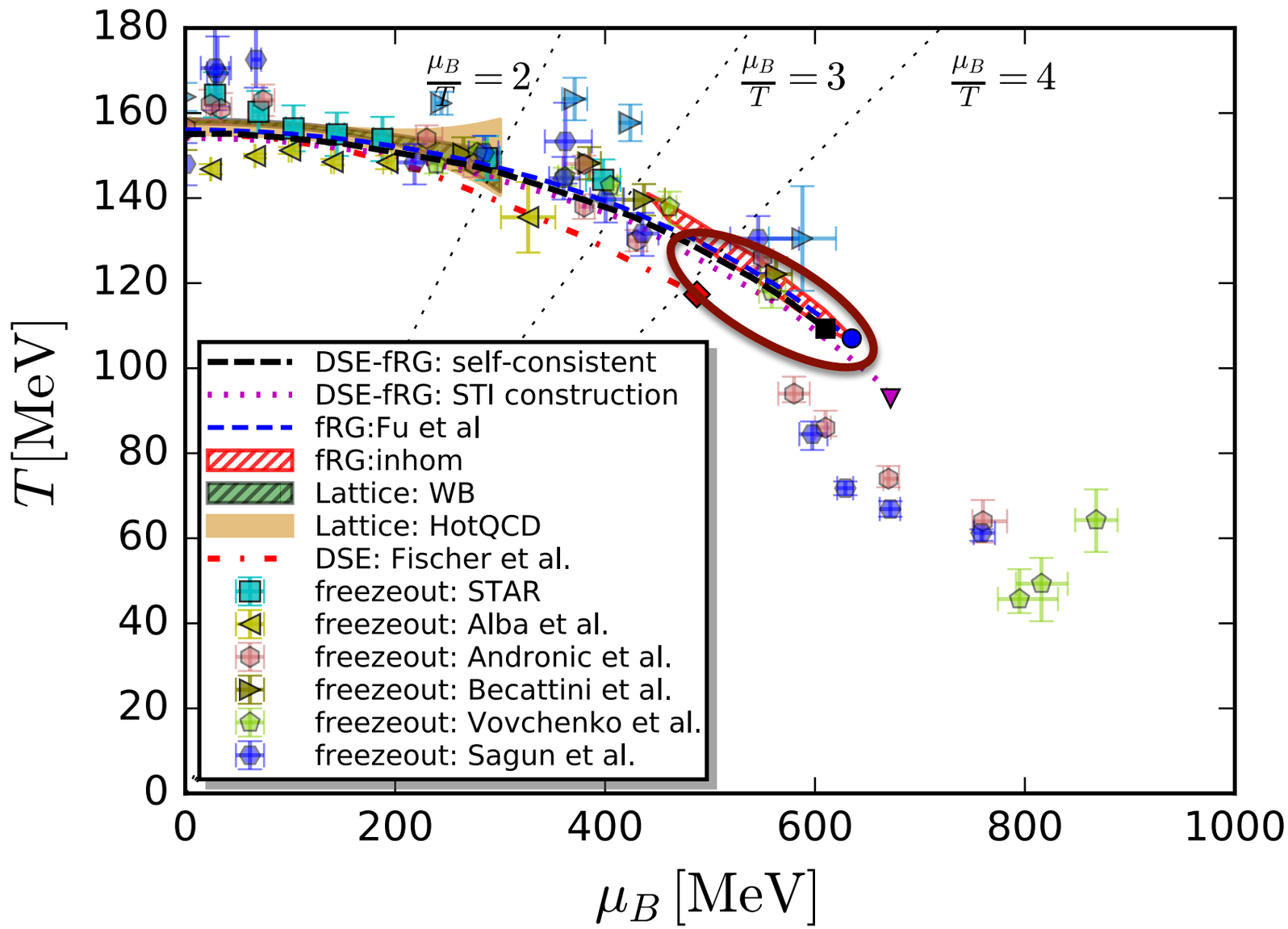
CEP-estimate fRG-DSE

$$(135\,,\,450\,)\,\mathrm{MeV} \lesssim (T_{\mathrm{CEP}}\,,\,\mu_{B_{\mathrm{CEP}}}) \lesssim (100\,,\,650)\,\mathrm{MeV}$$

fRG: Fu, JMP, Rennecke, PRD 101, (2020) 054032
DSE: Gao, JMP, PLB 820 (2021) 136584

QCD phase structure

Estimate for CEP



Stay tuned!

CEP-estimate fRG-DSE

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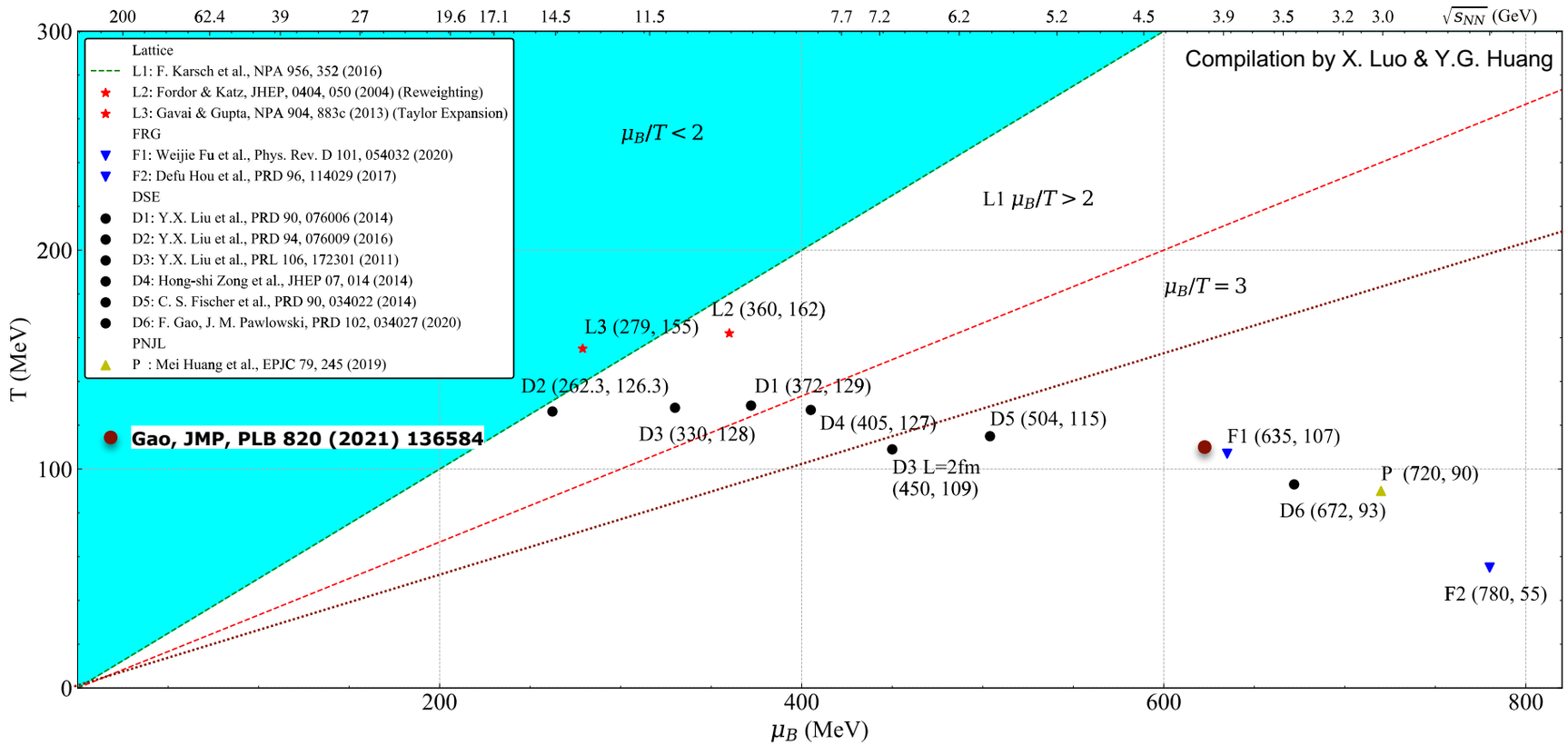
fRG: Fu, JMP, Rennecke, PRD 101, (2020) 054032
DSE: Gao, JMP, PLB 820 (2021) 136584

Location of CP : Theoretical Prediction

Preliminary collection from Lattice, DSE, FRG and PNJL (2004-2020)



Large uncertainties for the estimation of CP location.

Disclaimer

Most functional computations (LEFT or QCD) have not been set-up for CEP-predictions!

Lack of predictive power for CEP-predictions is no quality measure!

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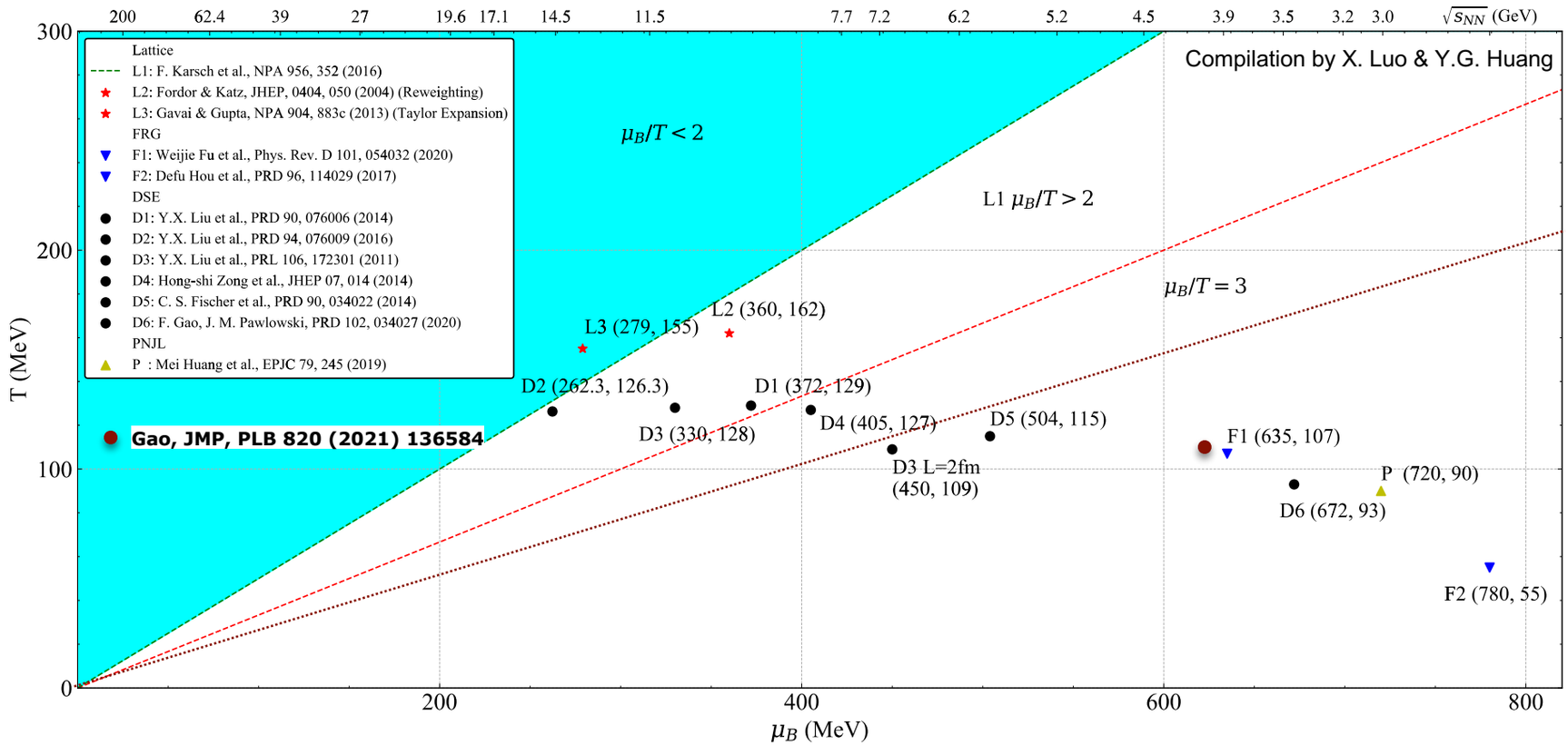

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Remove CEP-predictions:

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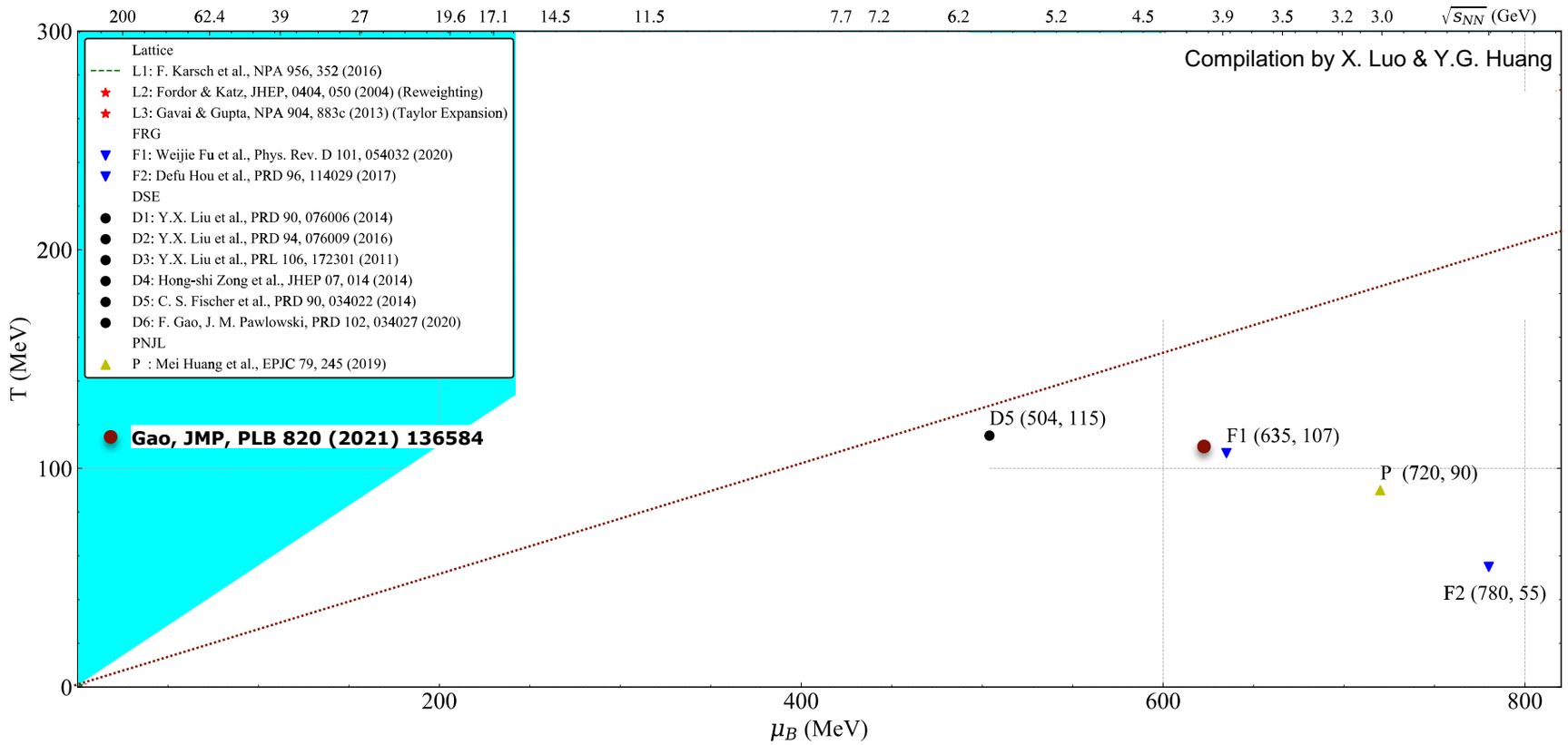

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Remove CEP-predictions:

(i) 'old' CEPs: lattice, Functional QCD approaches, LEFTS (updated computations available)

(ii) LEFTs & Functional Results (qualitative approximations) that miss lattice benchmarks at μ_B=0

Location of CP : Theoretical Prediction

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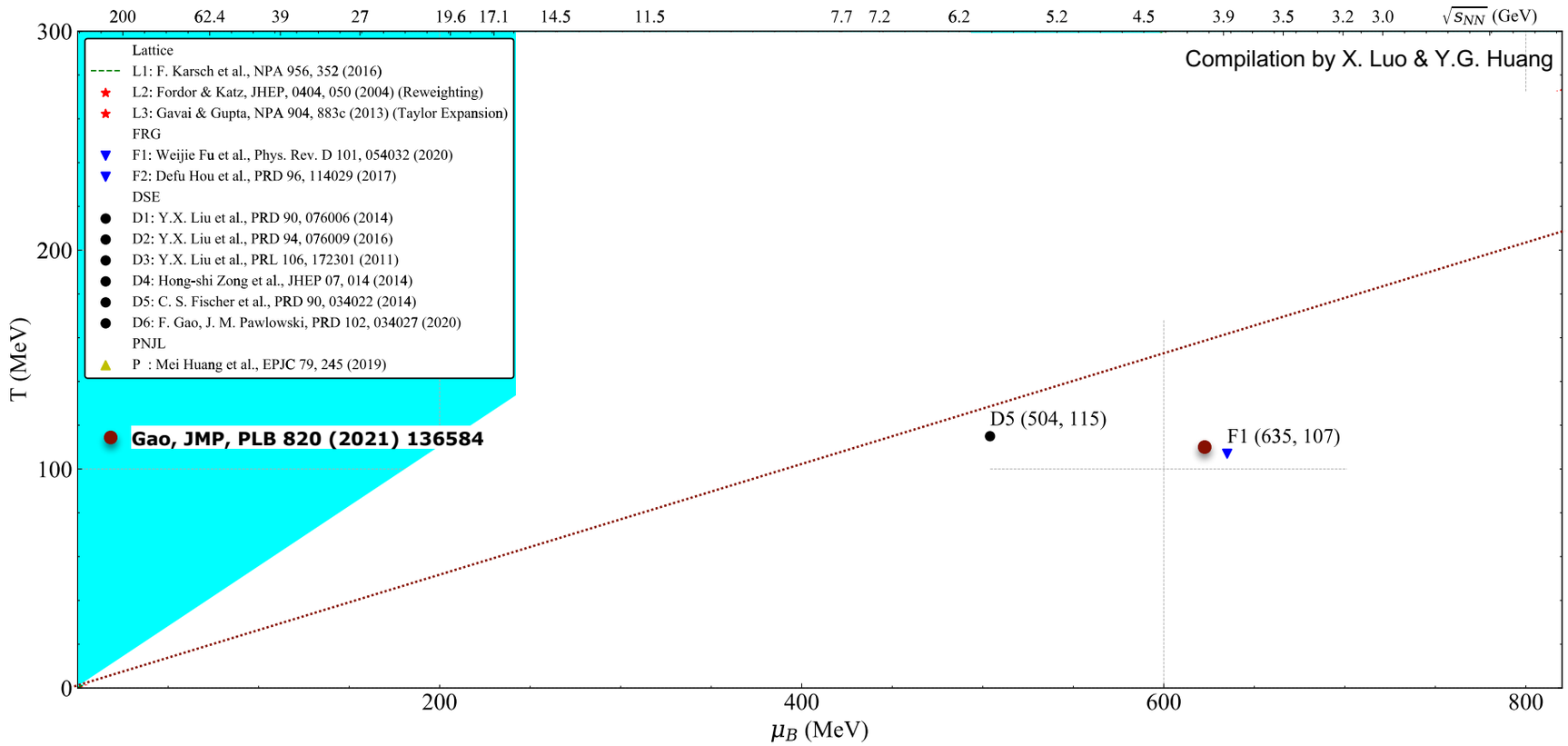

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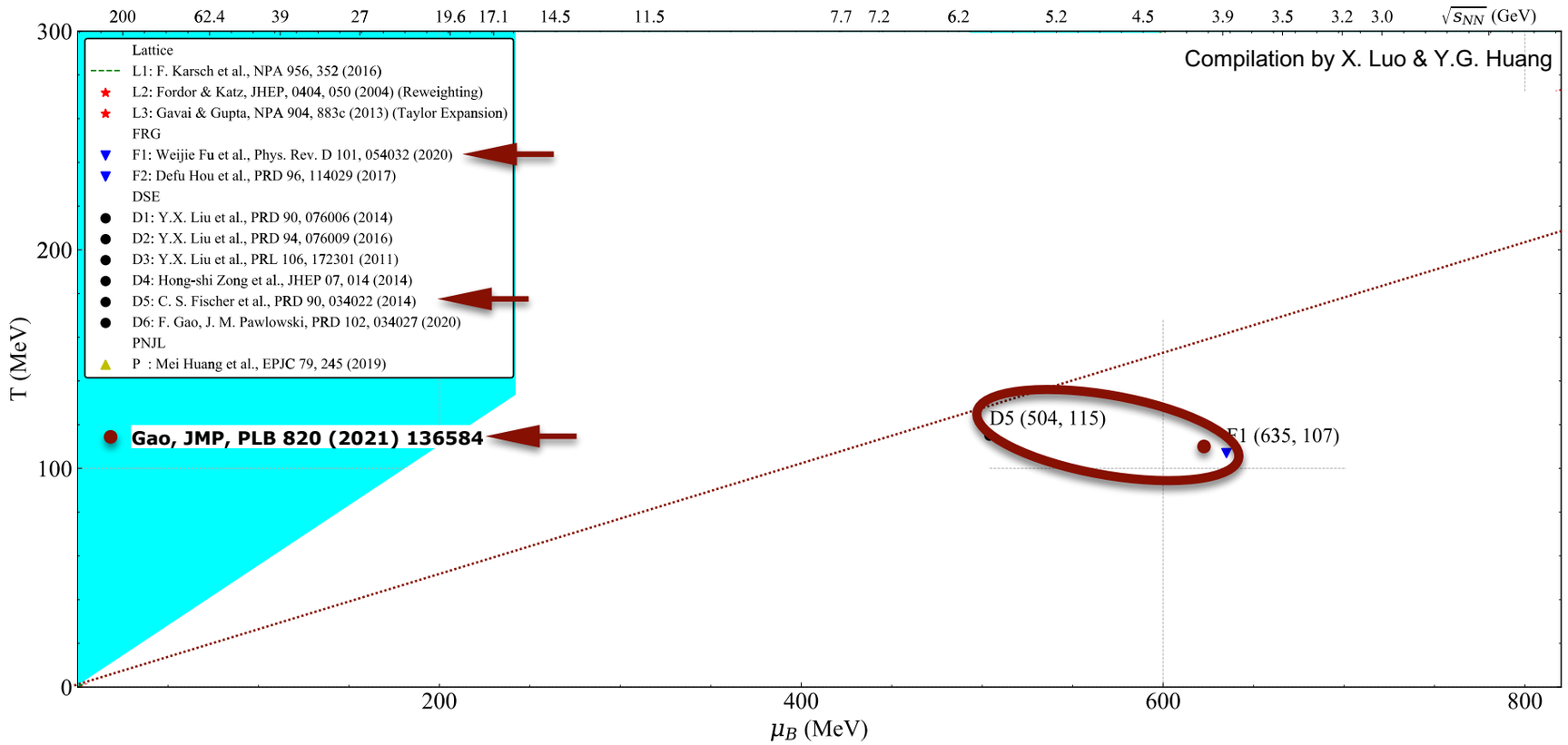

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Still uncertainties for the estimation of CP location.

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Outline

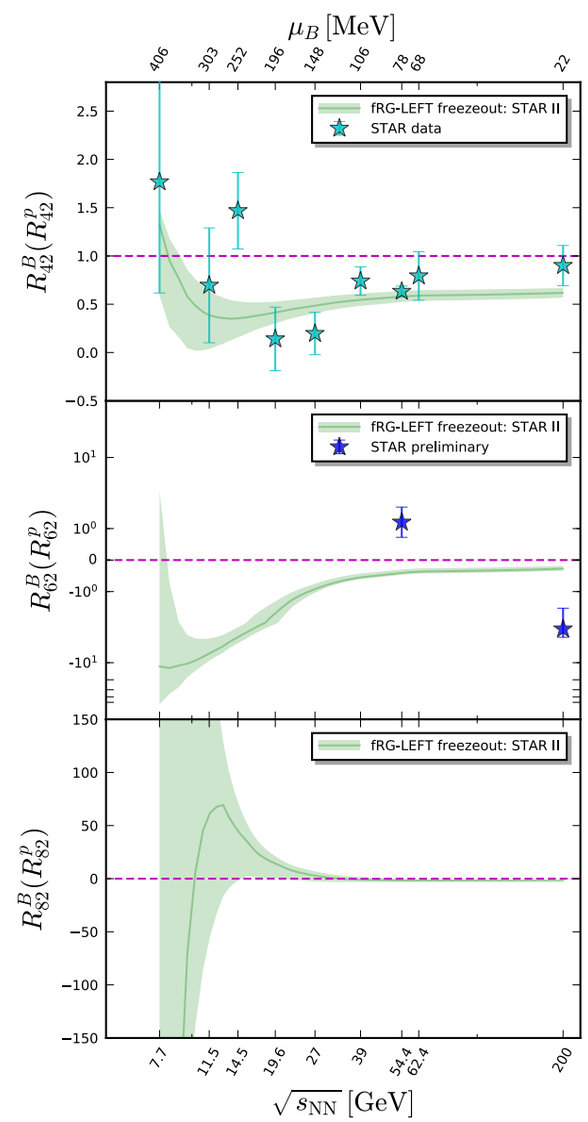
- QCD from functional methods

- QCD phase structure

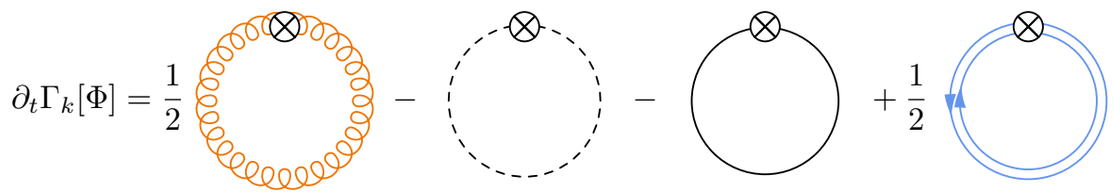
- Fluctuations of conserved charges

- Summary & outlook

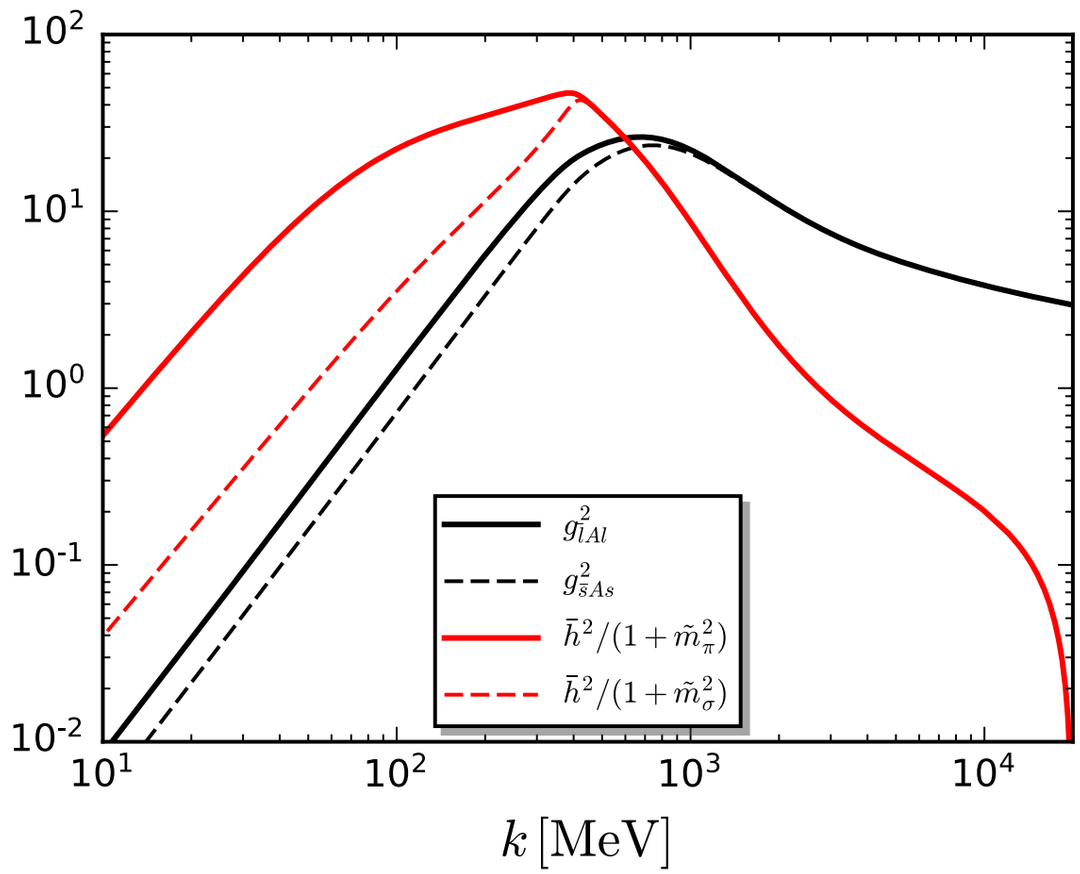
Hyper-fluctuations



On the unreasonable effectiveness of low energy effective theories



Sequential decoupling of gluon, quark, sigma, pion fluctuations



Fu, JMP, Rennecke, PRD 101, (2020) 054032

Based on:

Braun, Fister, Haas, JMP, Rennecke, PRD 94 (2016) 034016

Rennecke, PRD 92 (2015) 076012

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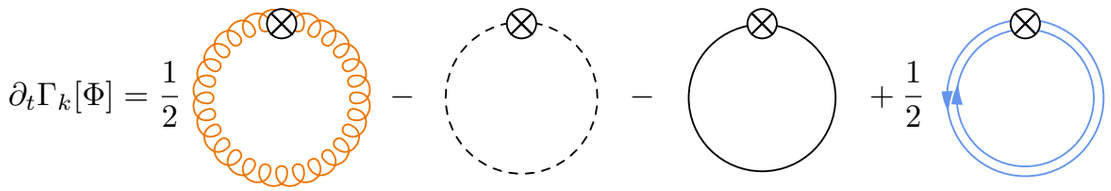

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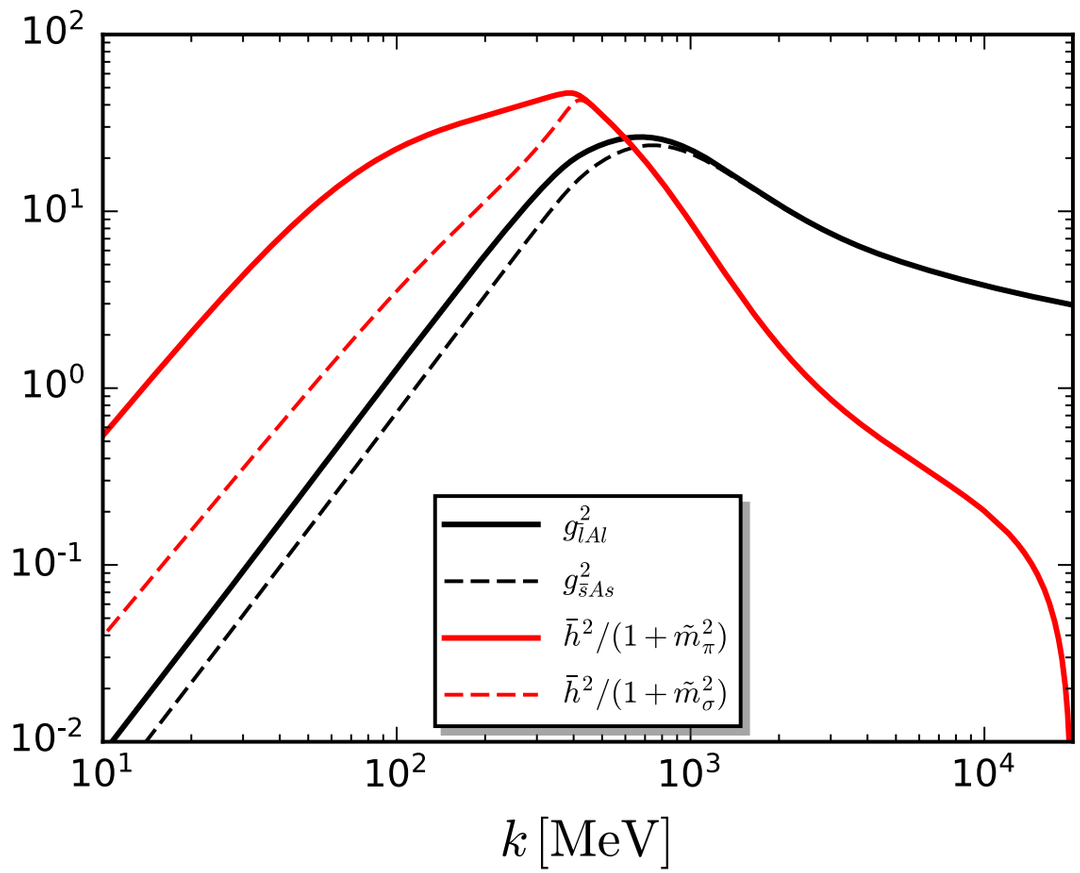
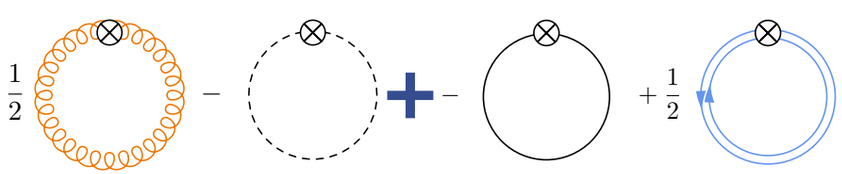
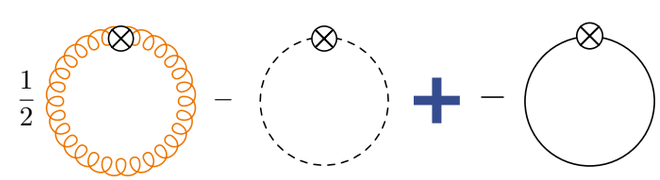


Fu, JMP, Rennecke, PRD 101, (2020) 054032

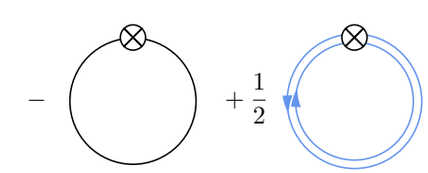
PQM-model



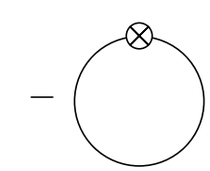
PNJL-model



QM-model



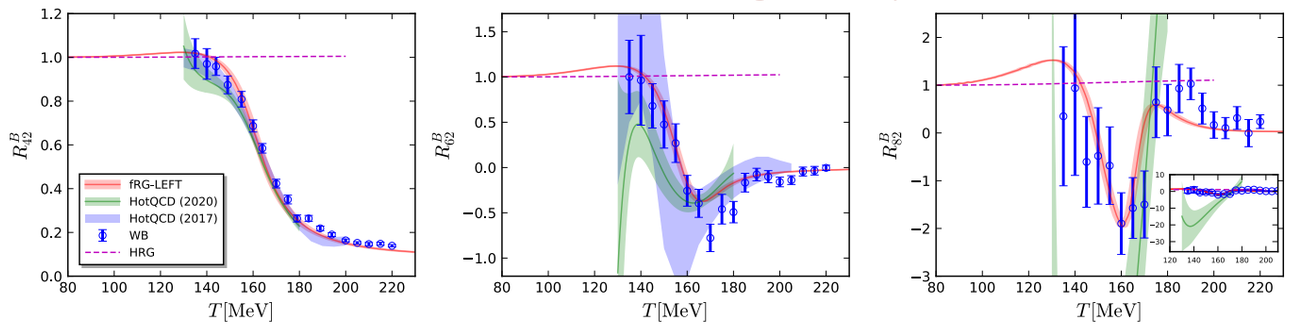
NJL-model



QCD-assisted low energy effective theories

Fluctuations of conserved charges

Benchmark at vanishing density



QCD-assisted LEFT

Builds on

Fu, JMP, PRD 93 (2016) 091501
Fu, JMP, Schaefer, Rennecke, PRD 94 (2016) 116020
Fu, JMP, Rennecke, PRD 101, (2020) 054032

Strangeness

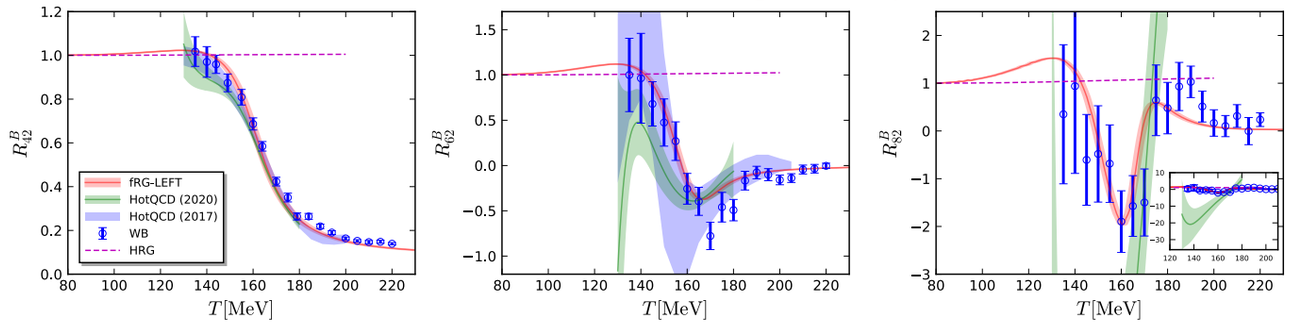
Fu, JMP, Rennecke, SciPost Phys. Core 2, 002 (2020)
PRD 100 (2019) 11, 111501
Wen, Huang, Fu, PRD 99 (2019) 094019

Fu, Luo, JMP, Rennecke, Wen, Yin, 2101.06035

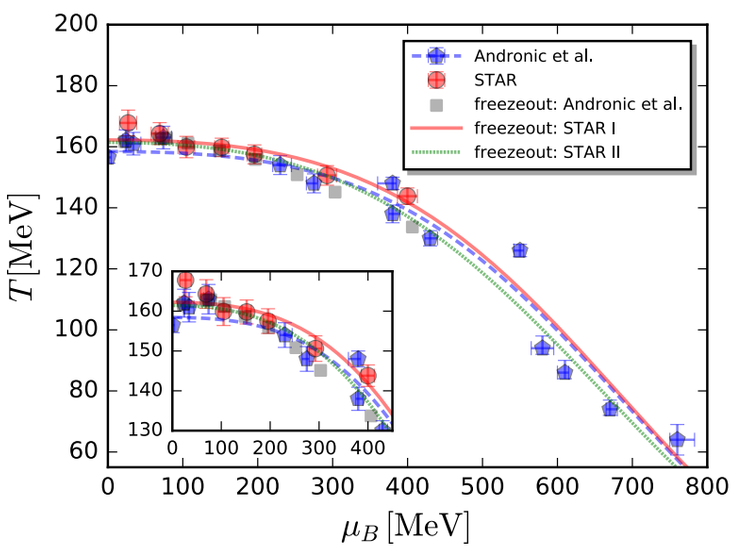
Fluctuations of conserved charges

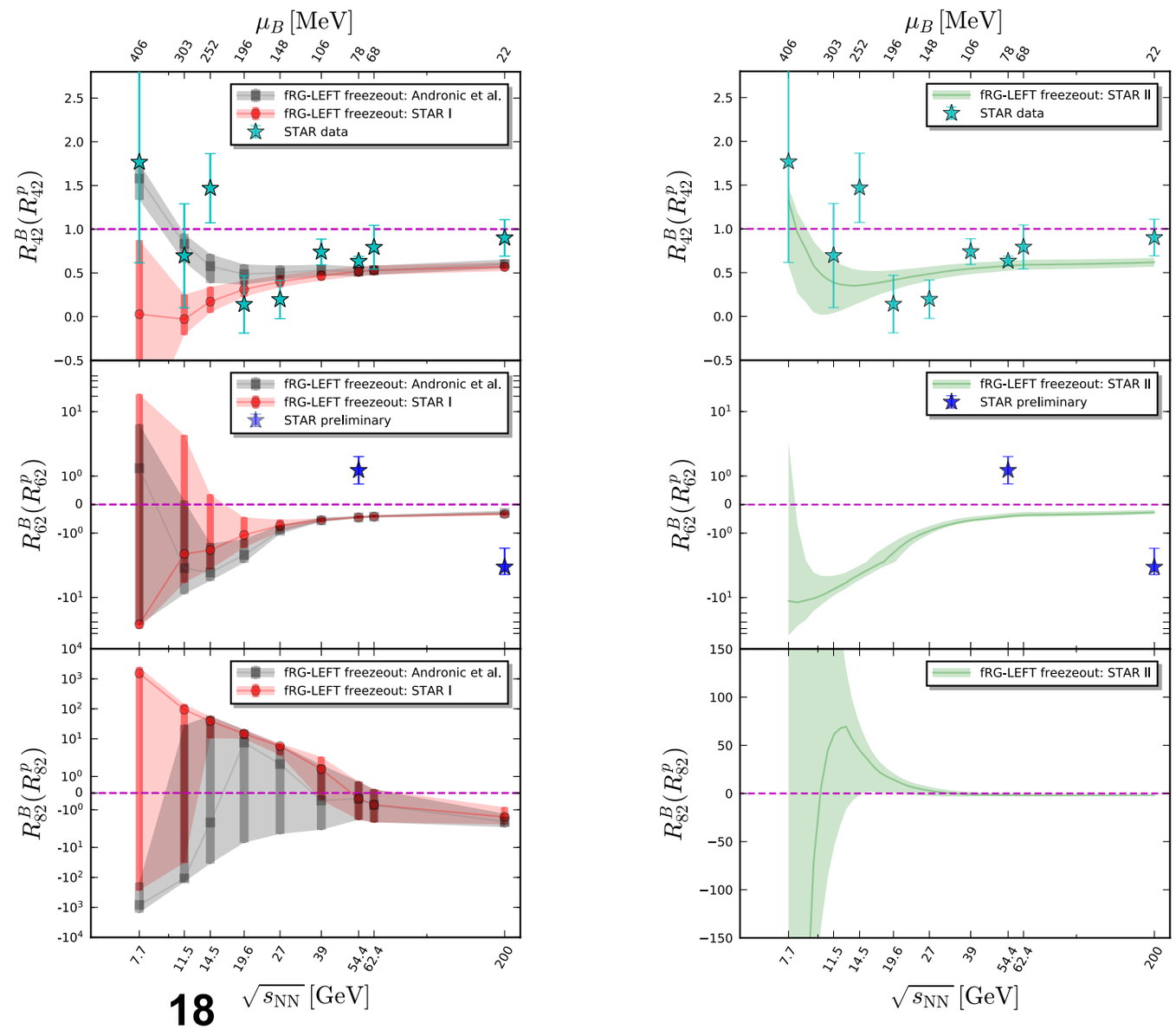
QCD-assisted LEFT

Benchmark at vanishing density



Freezeout curve





Fu, Luo, JMP, Rennecke, Wen, Yin, 2101.06035

Fluctuations of conserved charges

QCD-assisted LEFT

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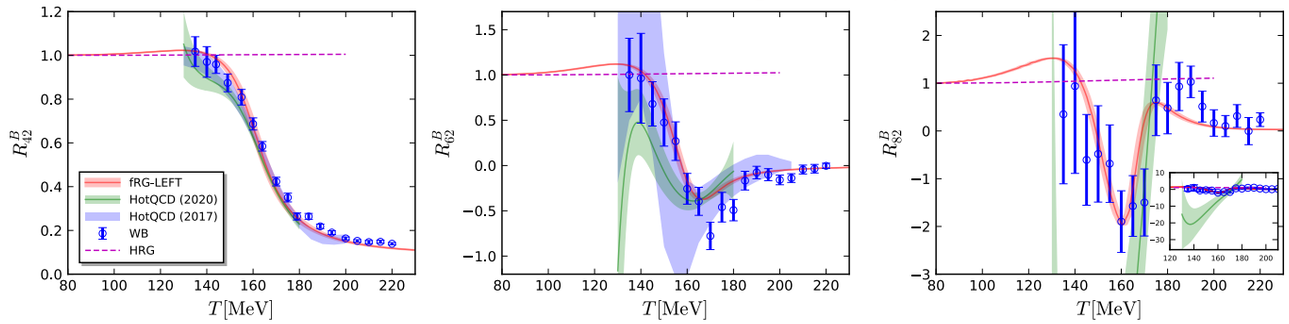

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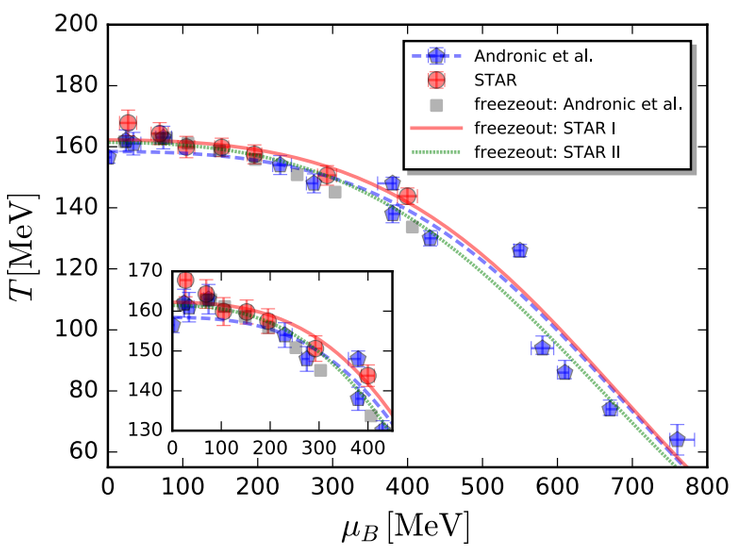

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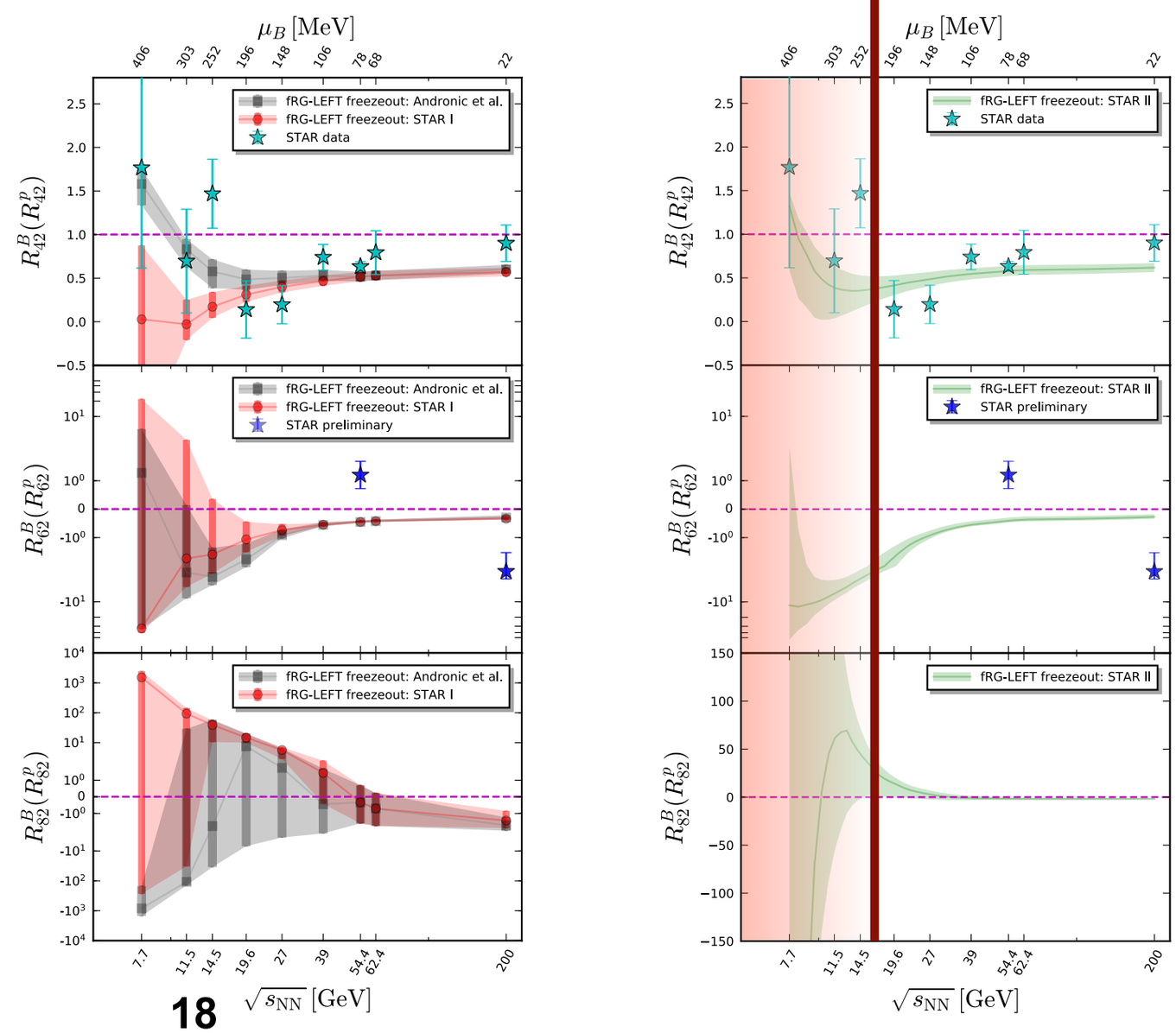

Fu, Luo, JMP, Rennecke, Wen, Yin, 2101.06035

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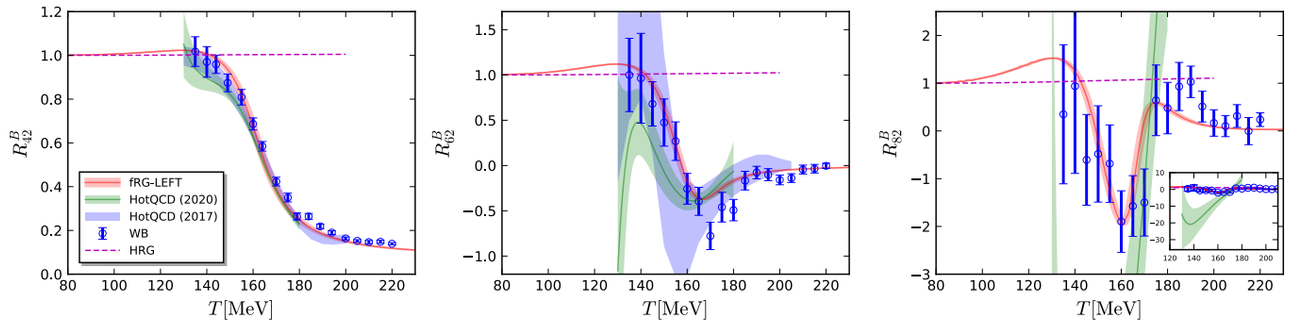

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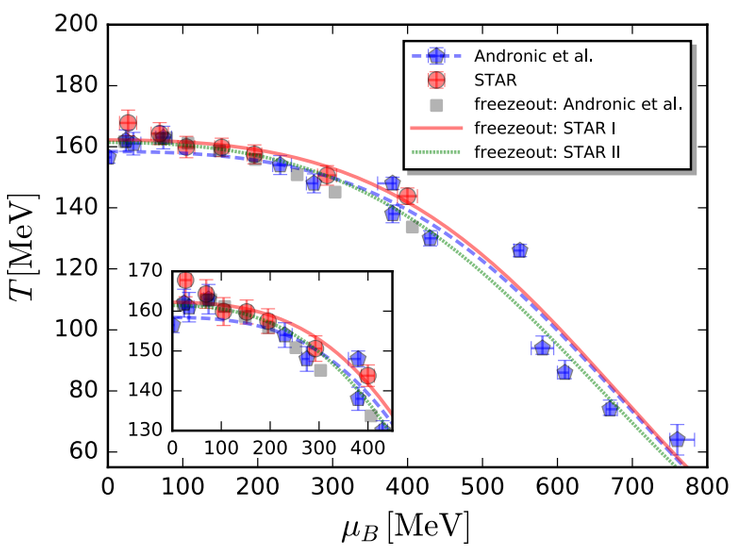
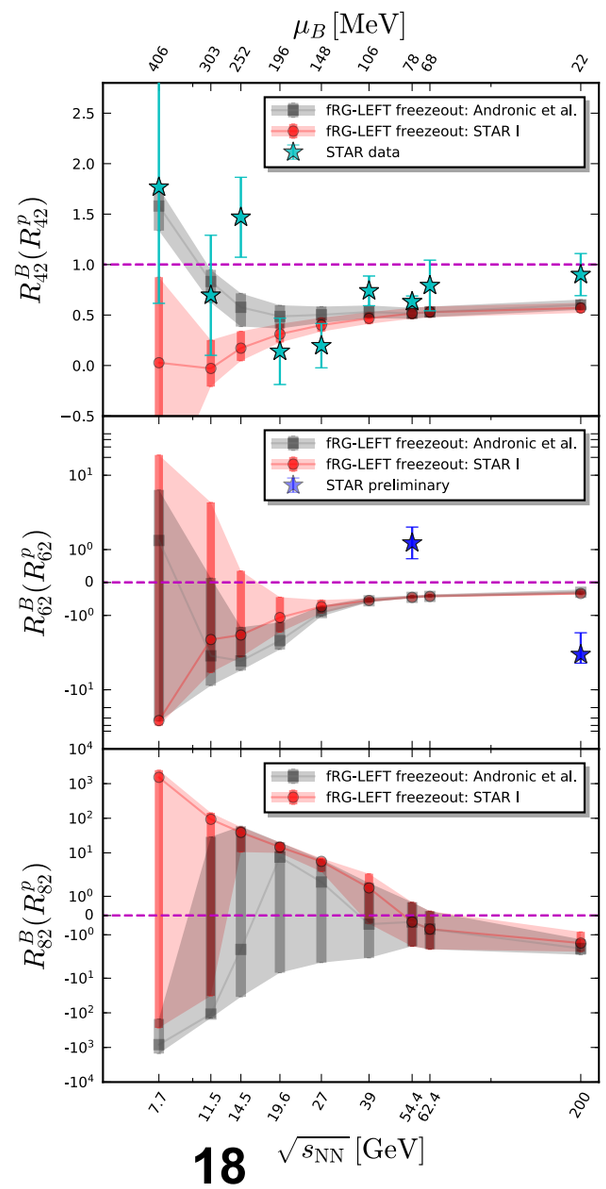
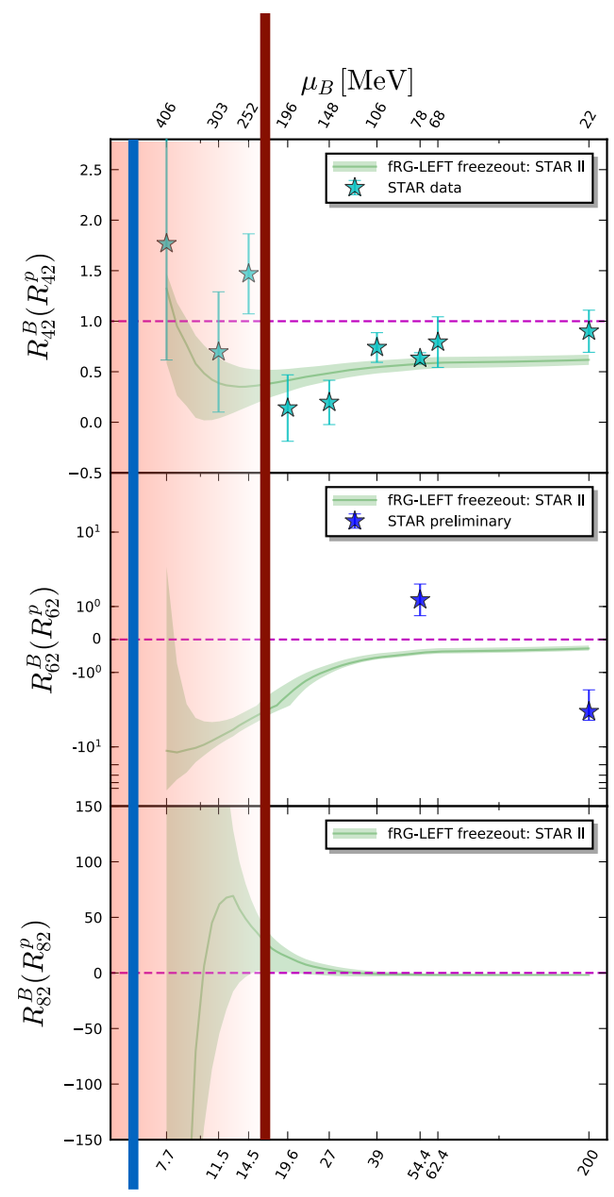




QCD-assisted LEFT



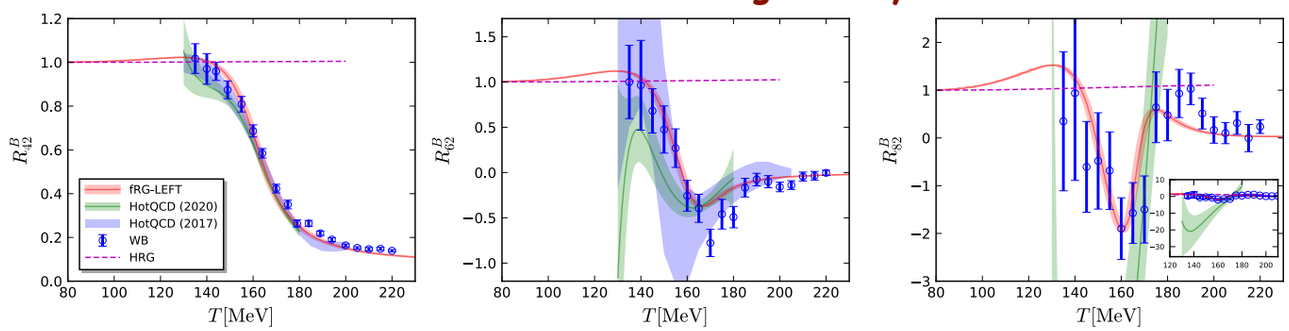
Fun QCD, in progress

Fu, Luo, JMP, Rennecke, Wen, Yin, 2101.06035

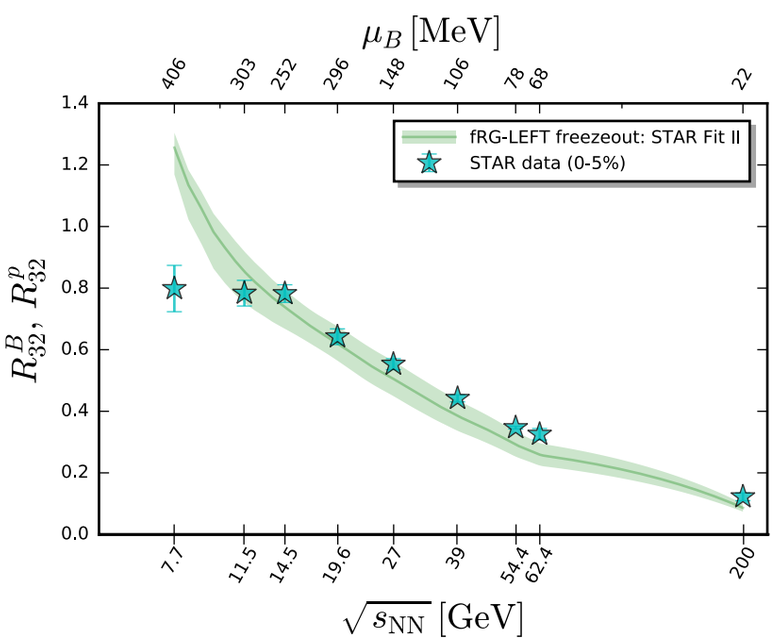
Fluctuations of conserved charges

QCD-assisted LEFT

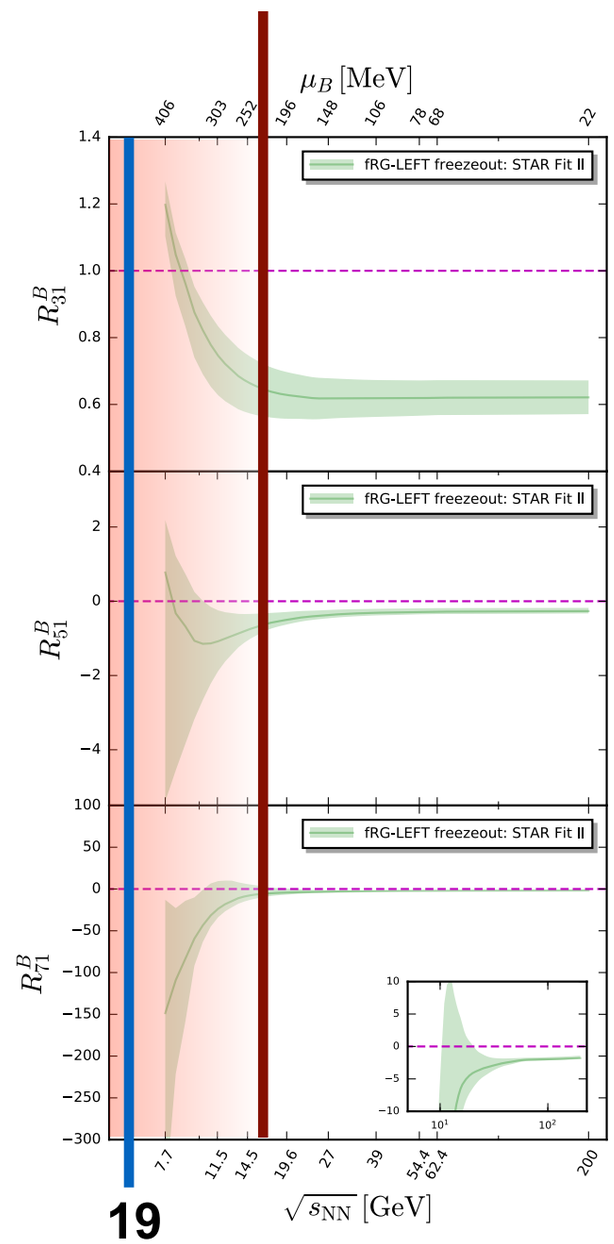
Benchmark at vanishing density

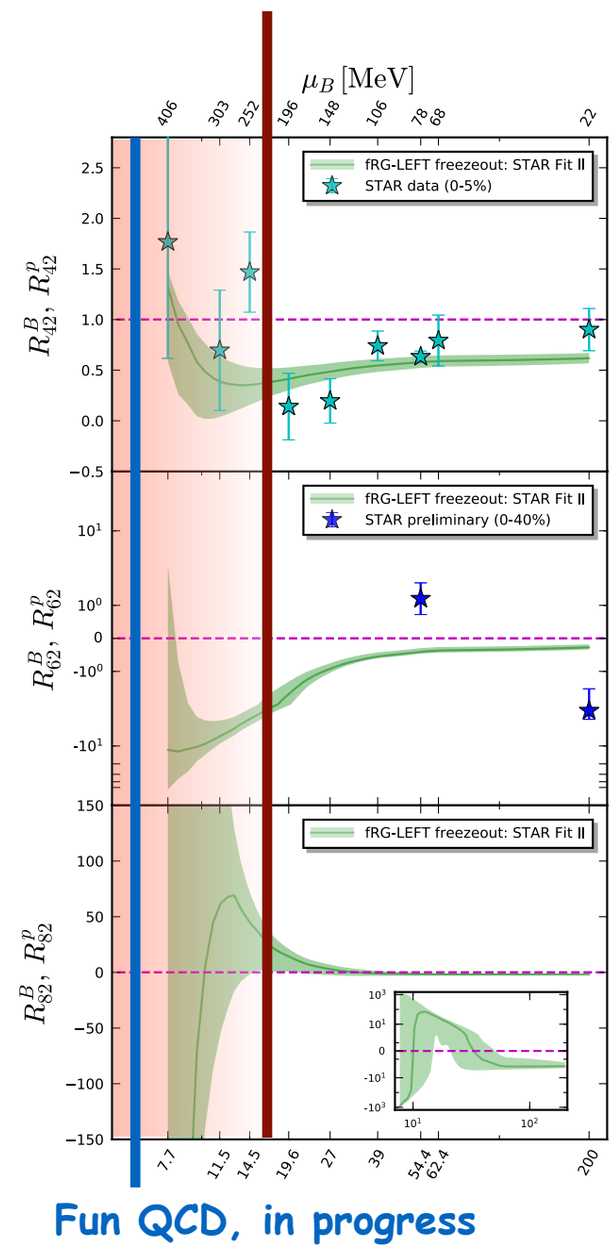


Baryon number conservation?



QCD-assisted LEFT





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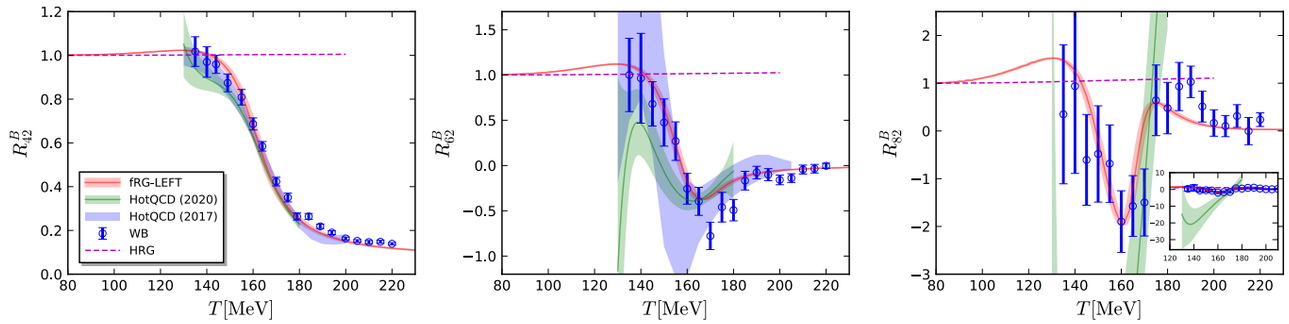
Fun QCD, in progress

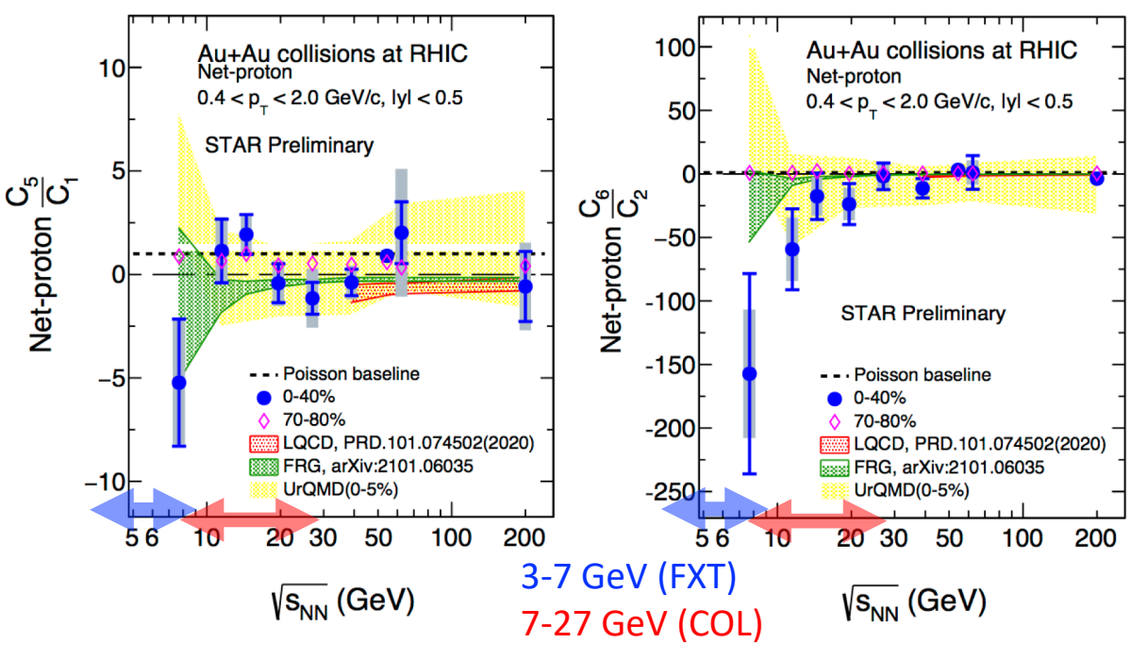
Fluctuations of conserved charges

QCD-assisted LEFT

Benchmark at vanishing density



STAR measurements

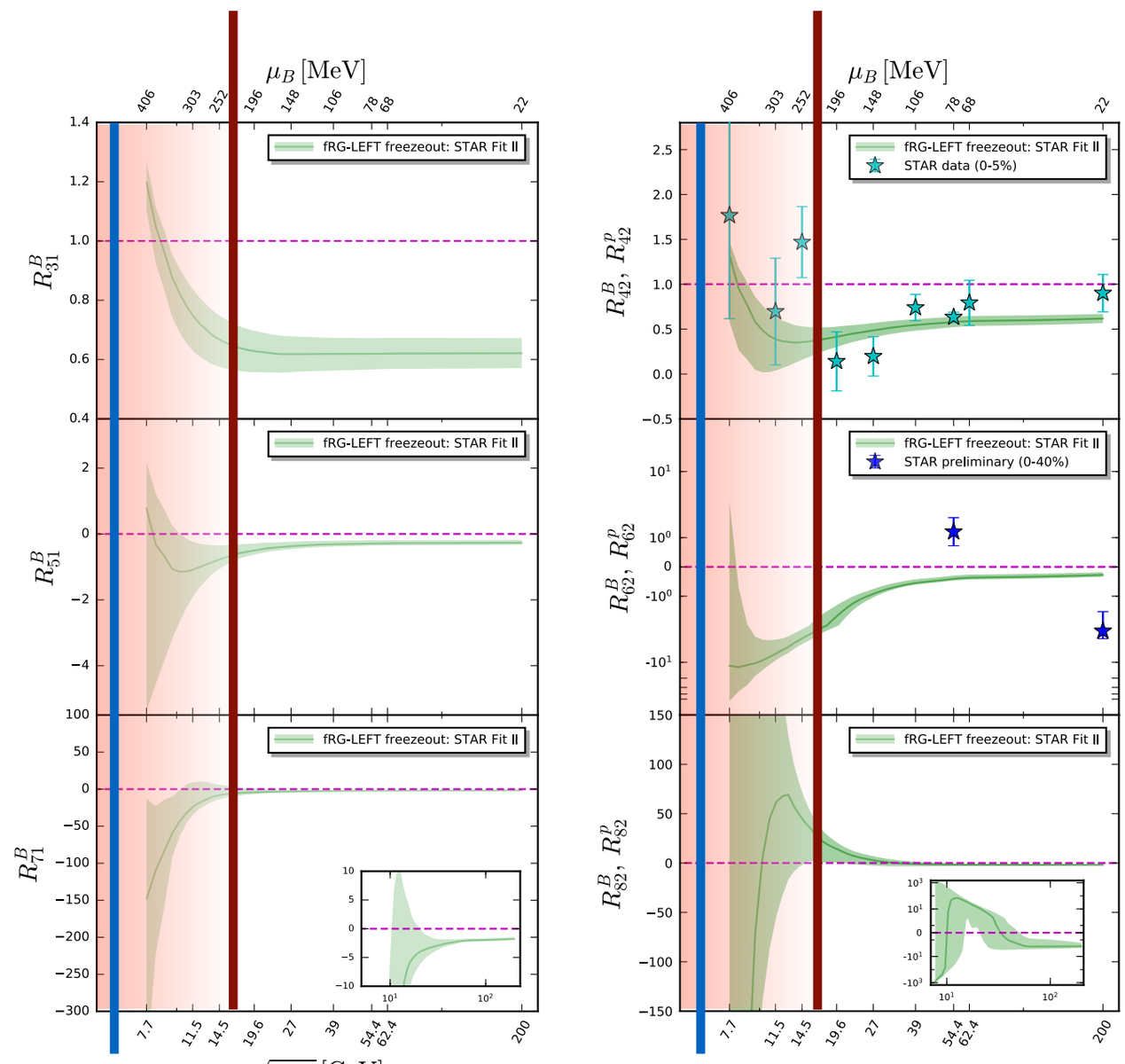


3-7 GeV (FXT)
7-27 GeV (COL)

S Results from RHIC-STAR, CPOD2021, 15-19/Mar, Online

S. Esumi, CPOD

QCD-assisted LEFT



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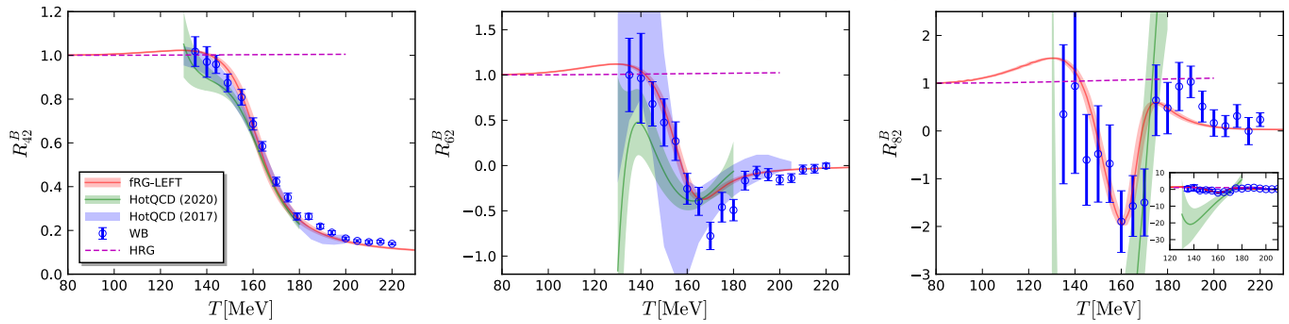
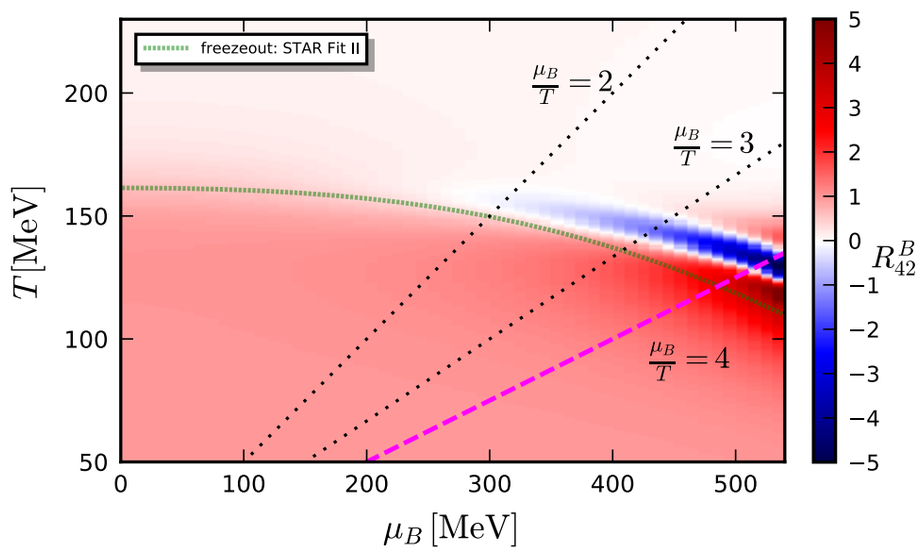


Freezeout curve



Great opportunity for a combined analysis
-Exp. data, lattice QCD, functional QCD-
of high density QCD

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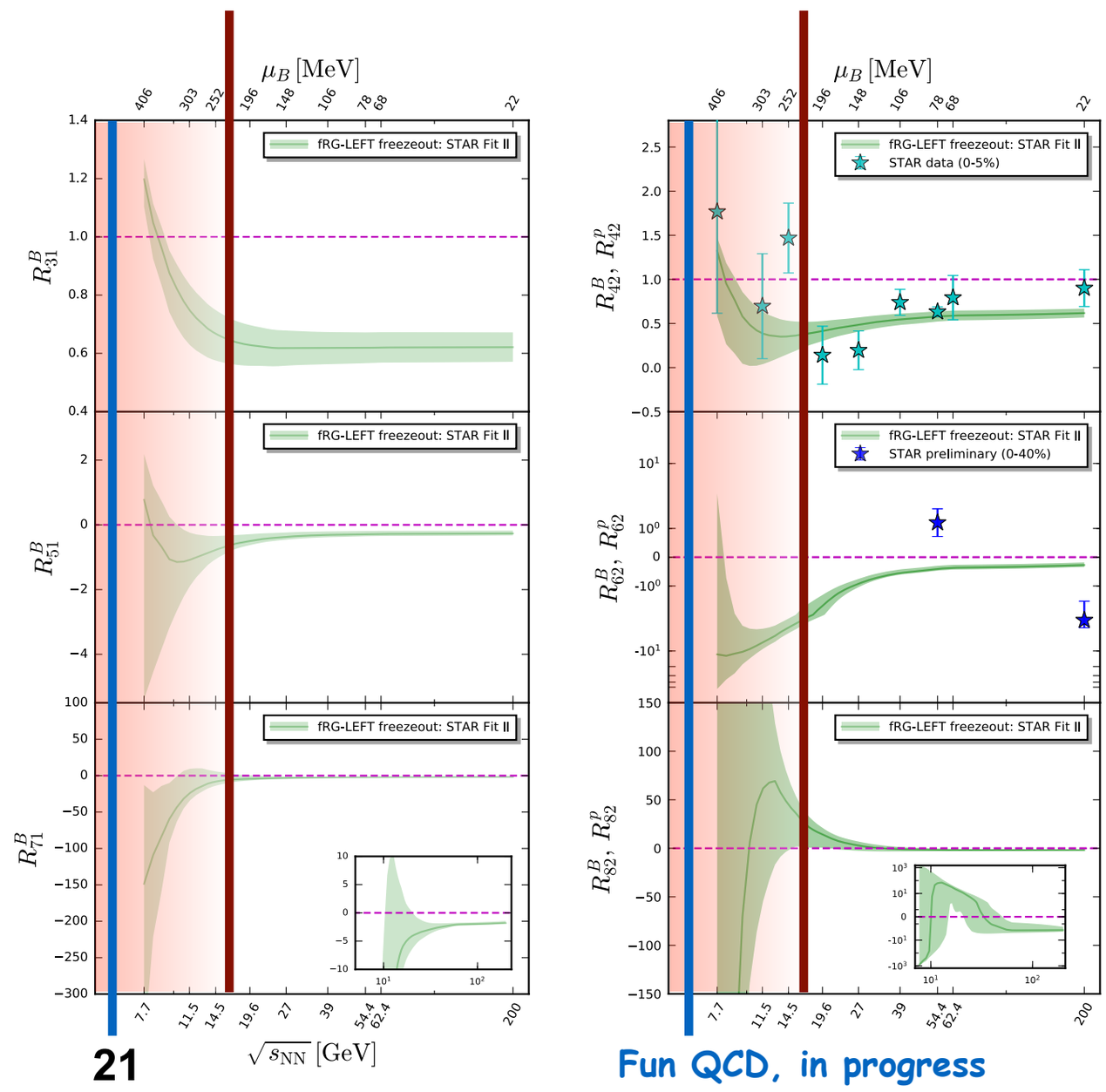
QCD-assisted LEFT

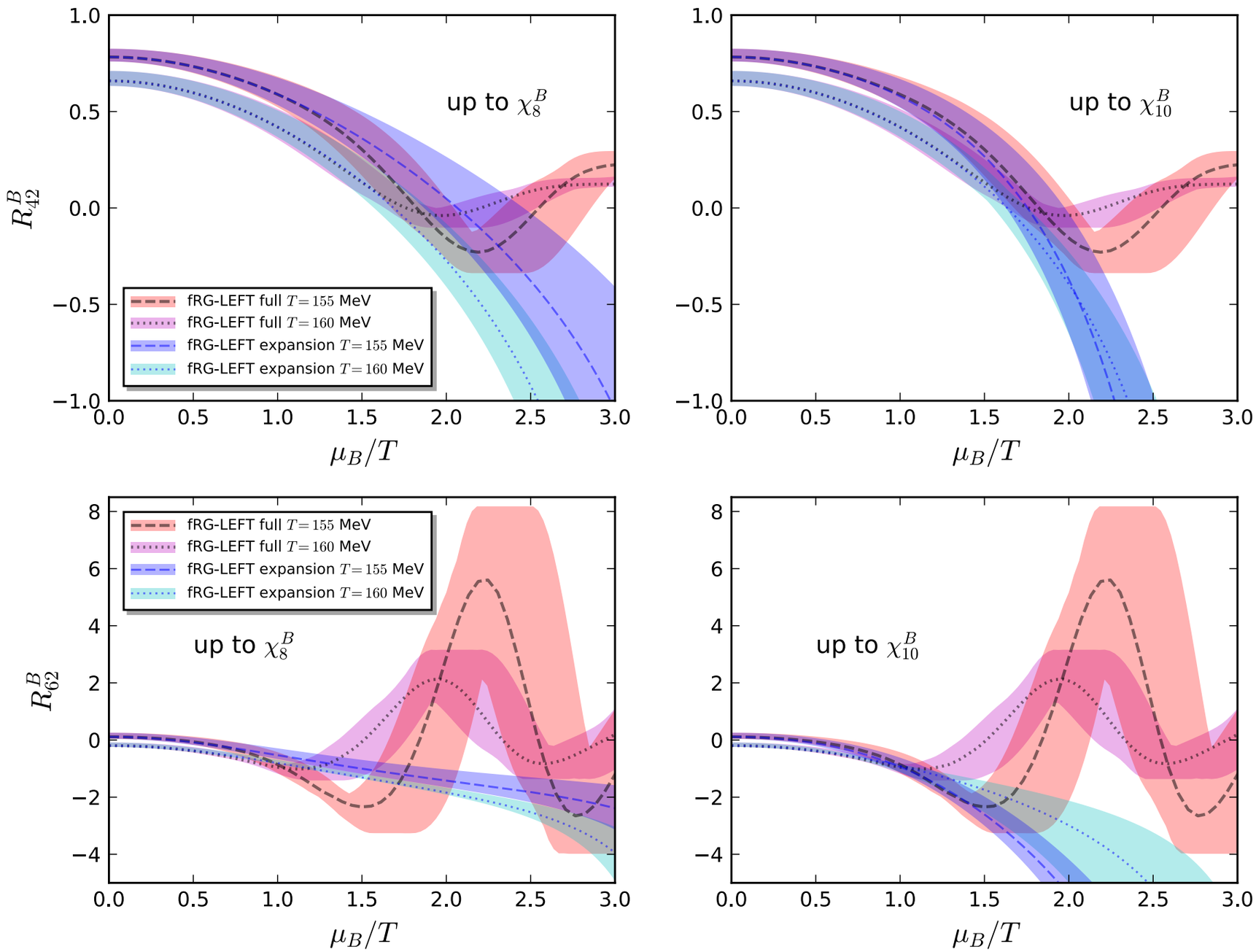


Fun QCD, in progress

Fluctuations of conserved charges

Fluctuation of conserved charges

QCD-assisted LEFT: Taylor expansion vs full results



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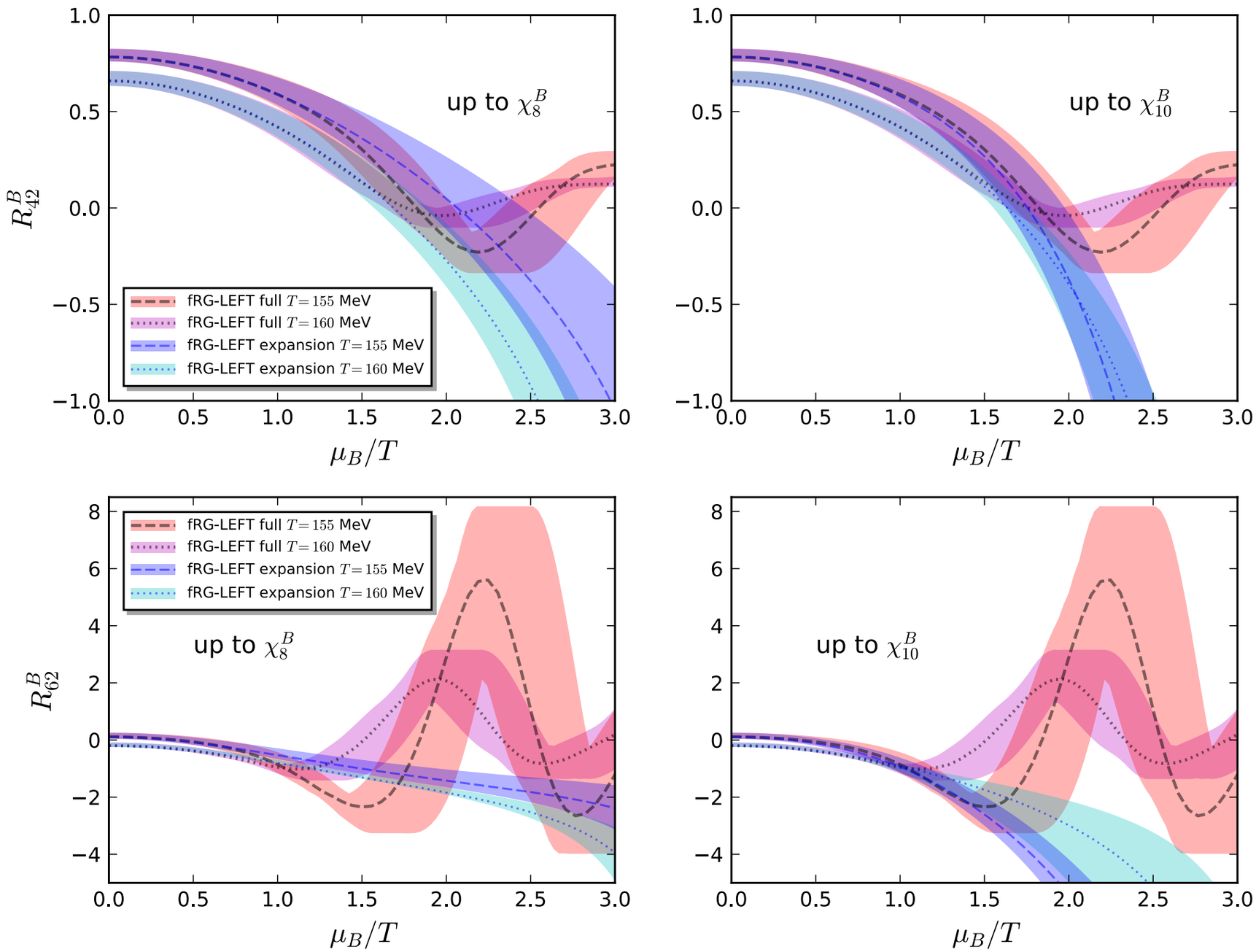
Fluctuations of conserved charges

Fluctuation of conserved charges

QCD-assisted LEFT: Taylor expansion vs full results



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Outline

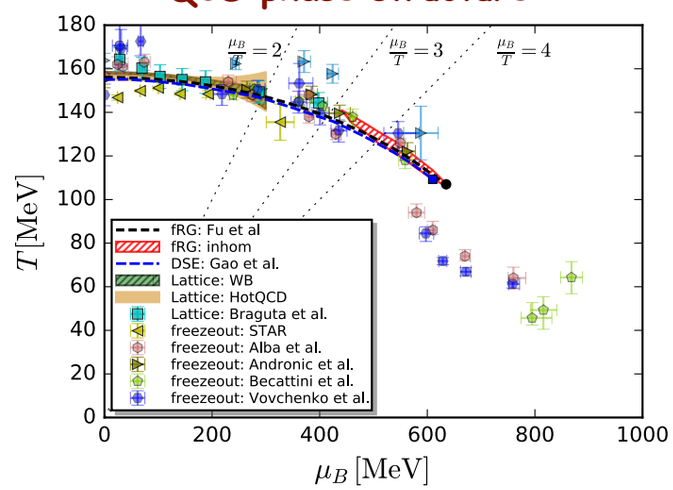
- **QCD from functional methods**

- **QCD phase structure**

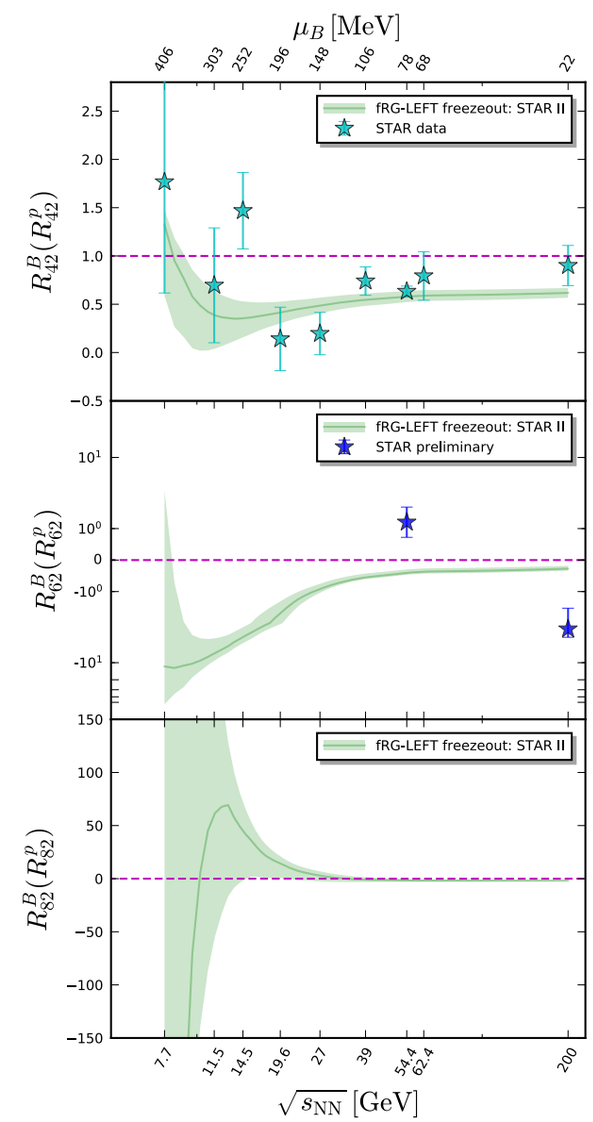
- **Fluctuations of conserved charges**

- **Summary & outlook**

QCD phase structure



Hyper-fluctuations



Summary & Outlook

- Towards apparent convergence in functional approaches to QCD
- Results & predictive power for the phase structure of QCD
  - Observables: quark condensates, fluctuations of conserved charges
- Towards quantitative precision at high densities
  - Systematic improvements under way for $\mu_B/T \gtrsim 4$

Summary & Outlook

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- Results & predictive power for the phase structure of QCD
  - Observables: quark condensates, fluctuations of conserved charges
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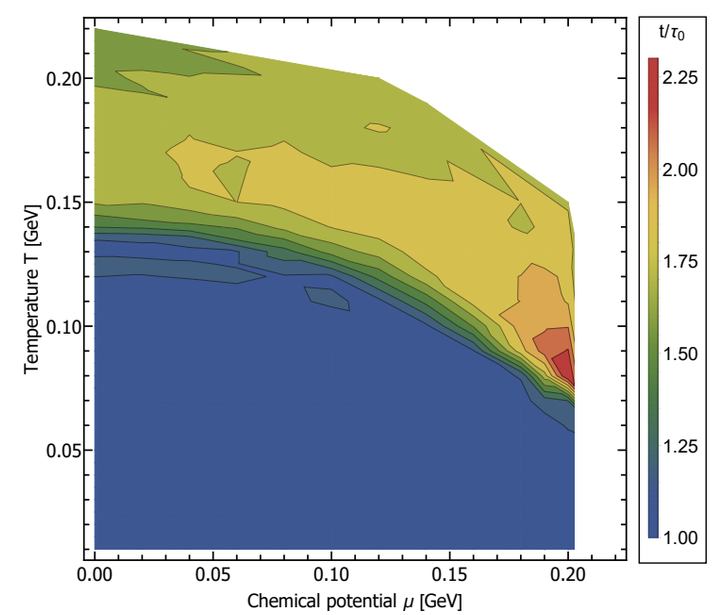
  - Systematic improvements under way for $\mu_B/T \gtrsim 4$
- Transport, hydro, and critical region
  - Real-time correlation functions
  - Transport at finite mu & T
  - Transport coefficients



Blum, Jiang, Nahrgang, JMP, Rennecke, Wink, NPA 982 (2019) 871