Nuclear Astrophysics in Relativistic Plasmas

Uncertainties at High T, ρ, and B


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17 September 2019
Relativistic Screening

Goals

- Improved screening in current stellar nucleosynthesis reaction rates.
- Addition of magnetic fields.
- Results from QTFT treatment of screening.

Outline

1. Introduction
2. Significance of Relativistic Effects
3. Some Results
4. Conclusions

NOTE: Will mostly concentrate on reaction rate screening, but mention some other effects.
Review: Screening in Nuclear Reactions
A One-Slide Summary

Coulomb Barrier
Astrophysics: WKB Barrier Penetration
Nuclei in Boltzmann Distribution

Nuclear Potential: Bare Nucleus

Coulomb Potential. Reaction rates determined from WKB Penetrability.

\[ \langle \sigma v \rangle = \frac{1}{\pi m_1 m_2} \left( \frac{2}{T} \right)^{3/2} \int_0^\infty e^{-E/kT} E \sigma(E) dE \]

\[ \nabla^2 \phi(r) = -4\pi Ze \delta(r^3) \]

\[ \phi(r) = \frac{Ze^2}{r} \]
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\phi(r) = \frac{Ze^2}{r}
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But the electrons and other nuclei provide a "background" potential.

- Coulomb Barrier
- Astrophysics: WKB Barrier Penetration
- Nuclei in Boltzmann Distribution
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Nuclear Potential: Bare Nucleus

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\[ \langle \sigma v \rangle = \frac{1}{\pi m_{12}} \left( \frac{2}{T} \right)^{3/2} \int_0^\infty e^{-E/kT} E \sigma(E) dE \]

\[ \nabla^2 \phi(r) = -4\pi Ze \delta(r^3) \]

\[ \phi(r) = \frac{Ze^2}{r} \]

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Classical Thermal Nuclear Potential: Electron Background

\[ \nabla^2 \phi(r) = -4\pi Ze \delta(r^3) - 4\pi Zn e \exp \left[ \frac{Ze\phi}{kT} \right] \\
+ 4\pi Zn e \exp \left[ \frac{-e\phi}{kT} \right] \]

\[ e\phi \ll kT \rightarrow \text{First Order in Potential: Mod. Helm. Eqn.} \]

\[ \phi(r) = \frac{Ze^2}{r} e^{-r/\lambda_D} \quad \text{Smaller } \lambda_D \rightarrow \text{lower barrier.} \]

\[ \lambda_D \equiv \left( \frac{T}{4\pi e^2 \sum_i (Z_i + Z_i^2) Y_i} \right)^{1/2} \]
Review: Nuclear Screening

Small shift in potential could be big shift in $r_{tp}$:

Salpeter Approx.:

\[
\langle \sigma v \rangle_{scr} = \exp \left[ \frac{Z_1 Z_2 e^2}{\lambda T} \right] \langle \sigma v \rangle_0
\]

More charge means more enhanced screening, regardless of the sign of the charge.
Review: Nuclear Screening

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Exponential in $\lambda_D^{-1}$

More charge means more enhanced screening, regardless of the sign of the charge.
However, For High Temperature in Plasma....

... Fermi-Dirac Statistics

**Screening With FD Statistics: Poisson Equation With Pair Production**

\[
\nabla^2 \phi = -4\pi Z e \delta(r^3) - 4\pi Z n_z e \exp \left(\frac{Ze\phi}{kT}\right)
\]

\[+
Ze \int_0^{\infty} d^3 p \left[ \frac{1}{e^{(E-\mu-e\phi)/T} + 1} - \frac{1}{e^{(E+\mu+e\phi)/T} + 1} \right]
\]

\[
\frac{\pi^2}{\lambda^2} = e \frac{\partial n}{\partial \phi} = \frac{\partial n}{\partial \mu} = e^2 \frac{\partial}{\partial \mu} \int_0^{\infty} dp p^2 \left[ \frac{1}{e^{(E-\mu-e\phi)/T} + 1} - \frac{1}{e^{(E+\mu+e\phi)/T} + 1} \right]
\]

**NOTE:** At high $T$, this solves the Schwinger-Dyson equation for the photon propagator to arbitrary order. [Kapusta (2006), Famiano et al. (2016)]
Including Magnetic Fields

Magnetic Fields

Hamiltonian for electron results in a 2D harmonic oscillator.

\[ p_n^2 = p_1^2 + p_\parallel^2 = neB + p_\parallel^2 \]

\[ n_e = \frac{eB}{2\pi^2} \sum_{\nu=0}^{\infty} g_\nu \]

\[ \int_0^\infty dp_z \left( \exp \left( \frac{\sqrt{p_z^2 + m_e^2 + 2\nu eB} - \mu - e\phi}{T} \right) + 1 \right)^{-1} \]

\[ - \left[ \exp \left( \frac{\sqrt{p_z^2 + m_e^2 + 2\nu eB} + \mu + e\phi}{T} \right) + 1 \right]^{-1} \]

B=4.41 \times 10^{11} G

0.05
0
0.1
0.15
0.2
5
fFD
0.25
0.3
0.35
Landau level
10
0.4
10^2
10^1
pz (MeV)
15
10^0
20
10^{-1}

Yudong Lou

Individual terms in number density sum.
Including Magnetic Fields

Magnetic Fields

Hamiltonian for electron results in a 2D harmonic oscillator.

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\[ \int_{0}^{\infty} dp_z \left[ \exp \left( \frac{\sqrt{p_z^2 + m_e^2 + 2\nu eB}}{T} - \mu - e\phi \right) + 1 \right]^{-1} \]

\[ - \left[ \exp \left( \frac{\sqrt{p_z^2 + m_e^2 + 2\nu eB} + \mu + e\phi}{T} \right) + 1 \right]^{-1} \]

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\[ \left. - \left[ \exp \left( \frac{\sqrt{p_z^2 + m_e^2 + 2\nu eB} + \mu + e\phi}{T} \right) + 1 \right]^{-1} \right) \]

B = 4.41 \times 10^{14} \text{G}

Individual terms in number density sum.
Including Magnetic Fields

Magnetic Fields

Hamiltonian for electron results in a 2D harmonic oscillator.

\[
\begin{align*}
    p_n^2 &= p_1^2 + p_\parallel^2 = neB + p_\parallel^2 \\
    n_e &= \\
    \frac{eB}{2\pi^2} \sum_{\nu=0}^{\infty} g_{\nu} \\
    \int_0^\infty dp_z \left( \exp \left( \frac{\sqrt{p_z^2 + m_e^2 + 2\nu eB} - \mu - e\phi}{T} \right) + 1 \right)^{-1} \\
    &- \left[ \exp \left( \frac{\sqrt{p_z^2 + m_e^2 + 2\nu eB} + \mu + e\phi}{T} \right) + 1 \right]^{-1} 
\end{align*}
\]

Yudong Lou

Individual terms in number density sum.
Are Relativistic Effects Important?  
Positron-Electron Ratios and Screening Length Ratios

Positron/electron ratio vs. $T_9$ and B.

$\rho = 10^6 \text{ g cm}^{-3}$, $Y_e = 0.5$.

Effects from chemical potential and low $T$.

$\lambda_R/\lambda_C$ vs. $T_9$ and B

Shorter screening length $\rightarrow$ Stronger Screening
Screened Nuclear Potential Comparison

Weak Screening: High $T$, Low $\rho$

![Graph showing the screened nuclear potential comparison with various curves for different potentials including Coulomb, $\lambda_C$, Relativistic TF, Relativistic Exact, Classical Exact, and a curve for Relativistic with $B=5 \times 10^{10}$ T. The graph plots $e^2 \phi / \lambda T$ against $r$ (fm). The parameters used are $T_9 = 5$, $\rho = 1 \times 10^6$, $\Gamma_{\text{eff}} = 0.071544$.](image-url)
Solution Comparison
Gamow Windows

Gamow Windows: C-C Plasma
- Exact solution for intermediate SEF.
- Shift to lower energy.
Preliminary Results
Where Do Thermal and Field Effects Become Important?

We are currently examining:

- SNell: High T - shock heated mantle
- Massive Stars: High T - core temp
- Pair production SNe: High core T - nucleosynthesis
- BBN - Magnetic Fields, High T, EC - Yudong Luo
- SNela - High T, EC - Kanji Mori
- X-Ray bursts - High B, High T
- Neutron Star Cores - “Effective” $\mu$
- Dynamic Effects, e.g., Alfven Wave effects
Preliminary Results

X-ray Bursts: Maybe B-field is Important? Maybe Not Temperature?

- Rapid Proton Capture
- Accretion onto magnetized neutron-star surface.
- \( B = 5 \times 10^9 \) T.
- Increased reaction rates \( \rightarrow \) increased heating.
- Did not examine phase-space/Pauli blocking in electron captures.
Preliminary Results

r-Process Nucleosynthesis: No B-Field, But High T

r-Process in SNeII

- Expanding neutrino-heated bubble in SNeII.
- $B = 0$, but $T_9 \lesssim 2.5$
- Does not affect $(n, \gamma)$, but could change $(\alpha, n)$ early on.
- Additional screening from thermal pair production.
Preliminary Results

p-Nuclei: Hot Shock Heating in SNe

Shock Heating in s-Process Shell - $^{196}$Hg

- VERY PRELIMINARY!
- B = 0, but $T_9 \sim 3$
- Only charged-particle (e.g., $(p,\gamma)$) affected.
- Even small increase in reaction rates can dramatically change low Y nuclei.
Weak Interactions
Yudong Luo

Electron Captures and Decays in Magnetized Plasmas

\[
\lambda_{n \rightarrow p e^- \bar{\nu}_e} = \frac{G_F^2 \tilde{B} m_e (g_V^2 + 3g_A^2)}{2\pi^3 T} \sum_{n=0}^{n_m} (2 - \delta_{n0}) \int_0^{p_m} dp_z E_{\nu}^2 g(E_e) g(E_\nu)
\]

\[
\lambda_{n(\bar{\nu}_e / e^+) \rightarrow p(e^- / \bar{\nu}_e)} = \frac{G_F^2 \tilde{B} m_e (g_V^2 + 3g_A^2)}{2\pi^3 T} \sum_{n=0}^{\infty} (2 - \delta_{n0}) \int_0^{\infty} dp_z E_{\nu}^2 g(E_{e/\nu}) f_{FD}(E_{\nu/e})
\]

Momentum terms, FD distribution, Pauli blocking factor changes.
Neutronization of proto-neutron star.
Possible effects in BBN fields? Possible effects prior to weak decoupling?
Conclusions

- Effective Screening Length: Potentially dramatic shifts at high B/T.
  - Screening enhancement factor for relativistic environments changes
  - Effective reduction in chemical potential.

- Possible change in stellar core burning.

- Future Work: NS Crust Effects, Pair Production SNe, BBN, NS Cores?, Experiment?

- Extending our TF screening model accurate at lower-T/higher-\(\rho\).

- Magnetized plasmas could be dramatically different!
  - Field evolution in NS merger events?

Work supported by NSF PHY-1204486 and PHY-1712832, an NAOJ Visiting Professorship, and the Fulbright Program