

Nuclear Astrophysics in Relativistic Plasmas

Uncertainties at High T , ρ , and B

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Relativistic Screening

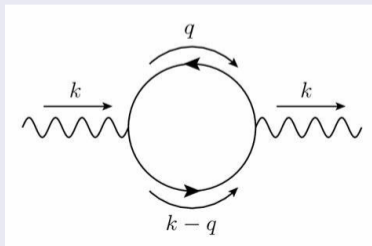
Outline

- 1 Introduction
- 2 Significance of Relativistic Effects
- 3 Some Results
- 4 Conclusions

NOTE: Will mostly concentrate on reaction rate screening, but mention some other effects.

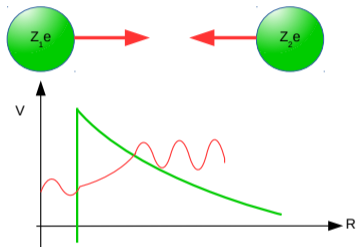
Goals

- Improved screening in current stellar nucleosynthesis reaction rates.
- Addition of magnetic fields.
- Results from QTFT treatment of screening.



Review: Screening in Nuclear Reactions

A One-Slide Summary



- Coulomb Barrier
- Astrophysics: WKB Barrier Penetration
- Nuclei in Boltzmann Distribution

Nuclear Potential: Bare Nucleus

Coulomb Potential. Reaction rates determined from WKB Penetrability.

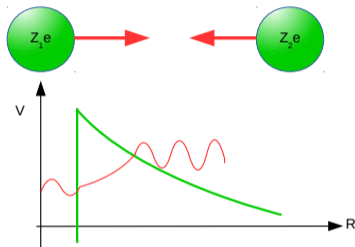
$$\langle \sigma v \rangle = \frac{1}{\pi m_{12}} \left(\frac{2}{T} \right)^{3/2} \int_0^\infty e^{-E/kT} E \sigma(E) dE$$

$$\nabla^2 \phi(r) = -4\pi Z e \delta(r^3)$$

$$\phi(r) = \frac{Z e^2}{r}$$

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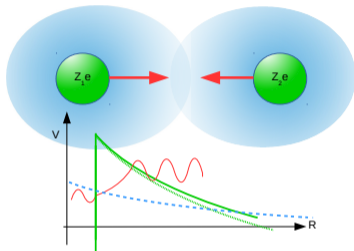
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But the electrons and other nuclei provide a “background” potential.

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- Nuclear Potential Perturbation
- Electrons in Boltzmann Distribution
- Poisson-Boltzmann Equation

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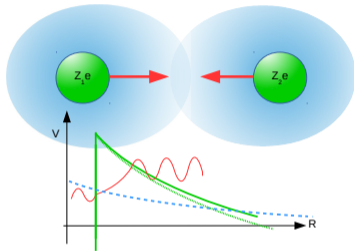
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Classical Thermal Nuclear Potential: Electron Background

$$\nabla^2 \phi(r) = -4\pi Z e \delta(r^3) - 4\pi Z n_z e \exp\left[\frac{Ze\phi}{kT}\right] + 4\pi Z e n_z \exp\left[\frac{-e\phi}{kT}\right]$$

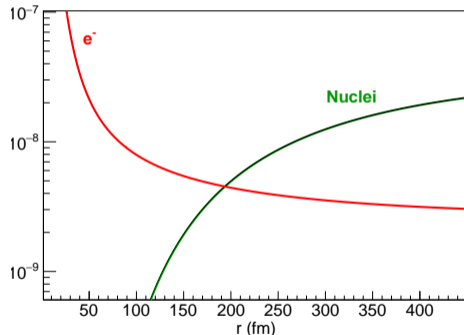
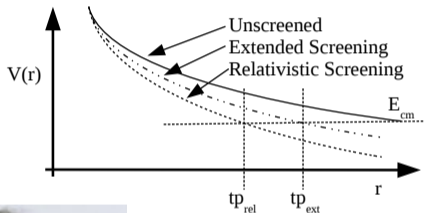
$e\phi \ll kT \rightarrow$ First Order in Potential: Mod. Helm. Eqn.

$$\phi(r) = \frac{Ze^2}{r} e^{-r/\lambda_D} \quad \text{Smaller } \lambda_D \rightarrow \text{lower barrier.}$$

$$\lambda_D \equiv \left(\frac{T}{4\pi e^2 \sum_i (Z_i + Z_i^2) Y_i} \right)^{1/2}$$

Review: Nuclear Screening

Small shift in potential could be big shift in r_{tp} :



Salpeter Approx.:

$$\langle \sigma v \rangle_{scr} = \exp \left[\frac{Z_1 Z_2 e^2}{\lambda T} \right] \langle \sigma v \rangle_0$$

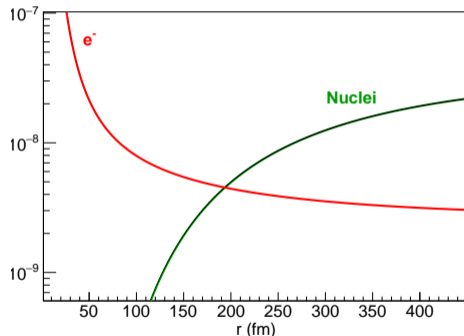
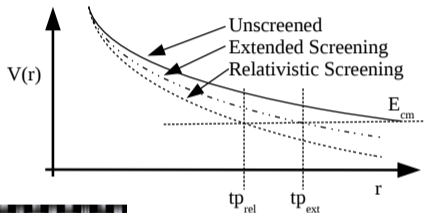
Exponential in λ_D^{-1}

More charge means more enhanced screening, regardless of the sign of the charge.



Review: Nuclear Screening

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However, For High Temperature in Plasma....

... Fermi-Dirac Statistics

Screening With FD Statistics: Poisson Equation With Pair Production

$$\nabla^2 \phi = -4\pi Ze\delta(r^3) - 4\pi Zn_z e \exp\left[\frac{Ze\phi}{kT}\right]$$

$$+ Ze \int_0^\infty d^3p \left[\frac{1}{e^{(E-\mu-e\phi)/T} + 1} - \frac{1}{e^{(E+\mu+e\phi)/T} + 1} \right]$$

$$\frac{\pi^2}{\lambda^2} = e \frac{\partial n}{\partial \phi} = \frac{\partial n}{\partial \mu} = e^2 \frac{\partial}{\partial \mu} \int_0^\infty dp p^2 \left[\frac{1}{e^{(E-\mu-e\phi)/T} + 1} - \frac{1}{e^{(E+\mu+e\phi)/T} + 1} \right]$$

NOTE: At high T, this solves the Schwinger-Dyson equation for the photon propagator to arbitrary order. [Kapusta (2006), Famiano et al. (2016)]

Including Magnetic Fields

Magnetic Fields

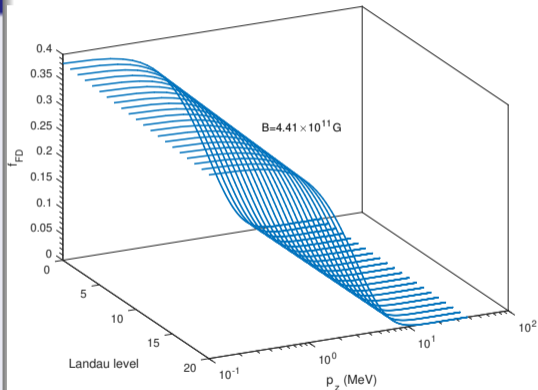
Hamiltonian for electron results in a 2D harmonic oscillator.

$$p_n^2 = p_{\perp}^2 + p_{\parallel}^2 = neB + p_{\parallel}^2$$

$$n_e =$$

$$\frac{eB}{2\pi^2} \sum_{\nu=0}^{\infty} g_{\nu}$$

$$\int_0^{\infty} dp_z \left(\left[\exp \left(\frac{\sqrt{p_z^2 + m_e^2 + 2\nu eB} - \mu - e\phi}{T} \right) + 1 \right]^{-1} - \left[\exp \left(\frac{\sqrt{p_z^2 + m_e^2 + 2\nu eB} + \mu + e\phi}{T} \right) + 1 \right]^{-1} \right)$$



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Individual terms in number density sum.

Including Magnetic Fields

Magnetic Fields

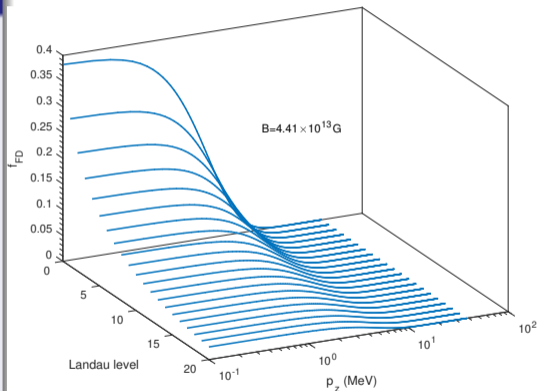
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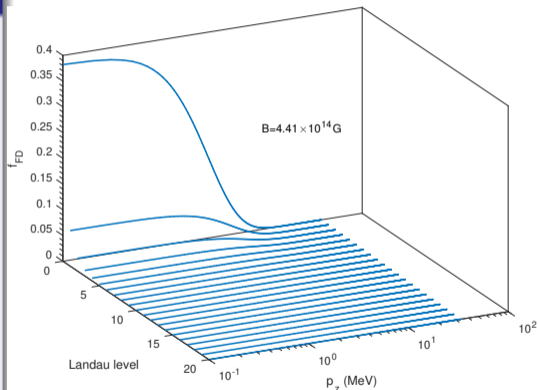
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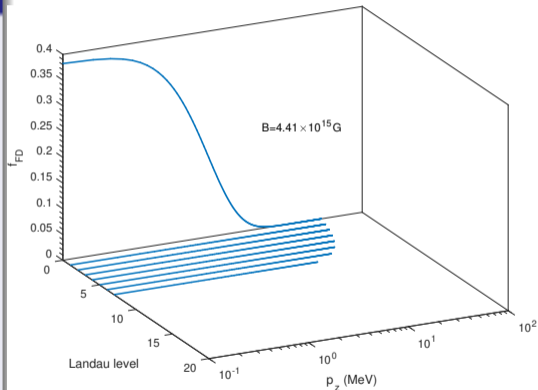
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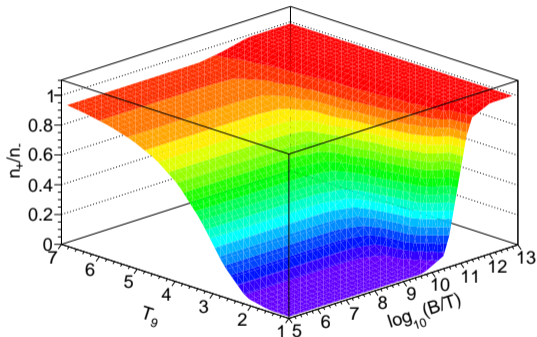


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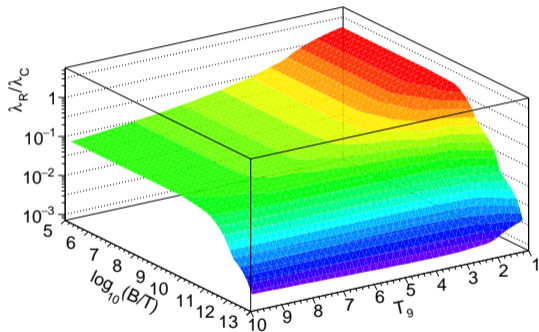
Individual terms in number density sum.

Are Relativistic Effects Important?

Positron-Electron Ratios and Screening Length Ratios



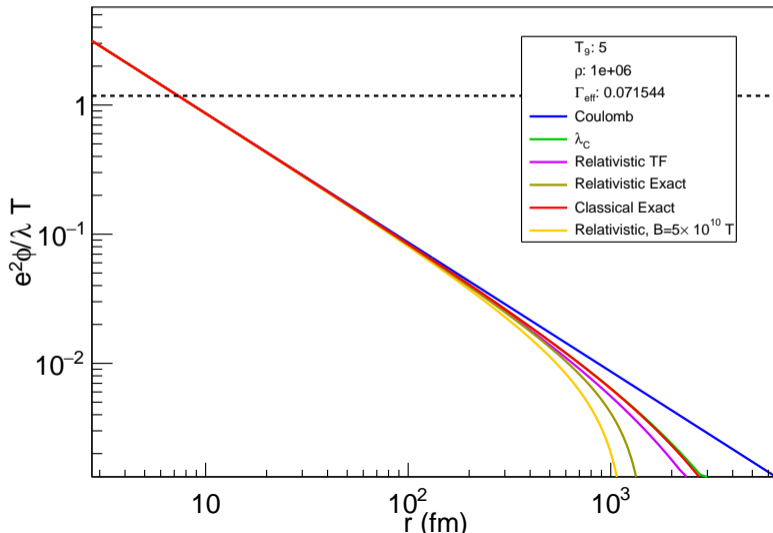
Positron/electron ratio vs. T_9 and B.
 $\rho = 10^6 \text{ g cm}^3$, $Y_e = 0.5$.
Effects from chemical potential and low T.



λ_R/λ_C vs. T_9 and B
Shorter screening length → Stronger Screening

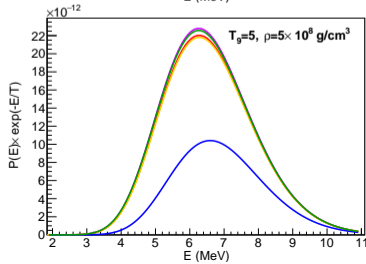
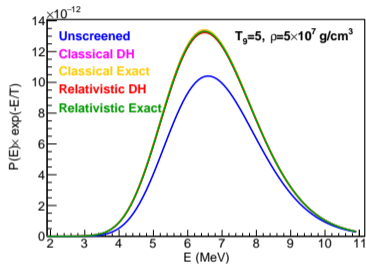
Screened Nuclear Potential Comparison

Weak Screening: High T, Low ρ



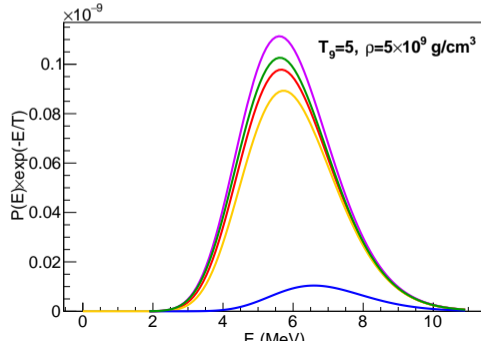
Solution Comparison

Gamow Windows



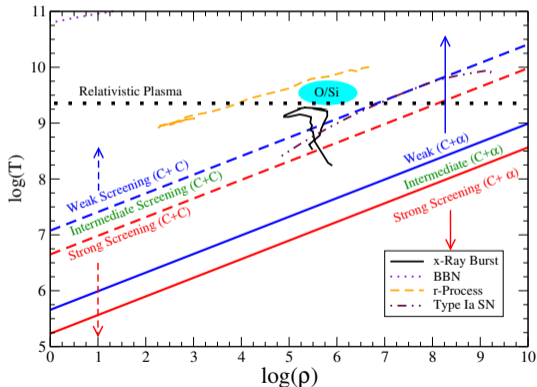
Gamow Windows: C-C Plasma

- Exact solution for intermediate SEF.
- Shift to lower energy.



Preliminary Results

Where Do Thermal and Field Effects Become Important?

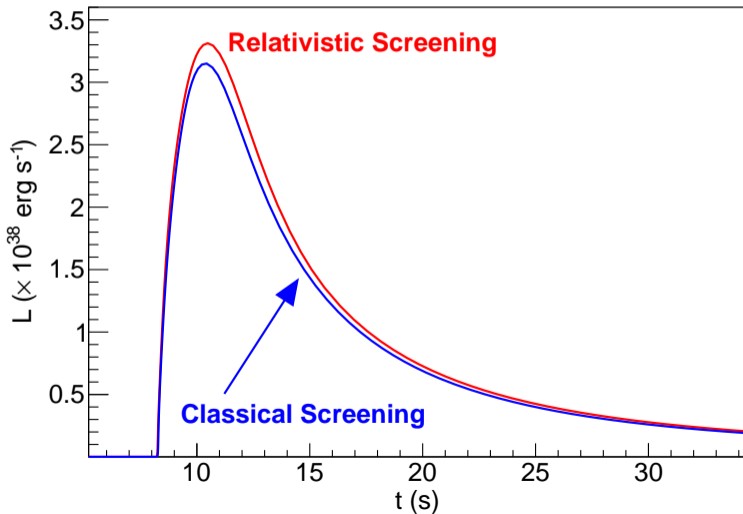


We are currently examining:

- SNell: High T - shock heated mantle
- Massive Stars: High T - core temp
- Pair production SNe: High core T - nucleosynthesis
- BBN - Magnetic Fields, High T, EC-
Yudong Luo
- SNela - High T, EC - **Kanji Mori**
- X-Ray bursts - High B, High T
- Neutron Star Cores - "Effective" μ
- Dynamic Effects, e.g., Alfvén Wave effects

Preliminary Results

X-ray Bursts: Maybe B-field is Important? Maybe Not Temperature?

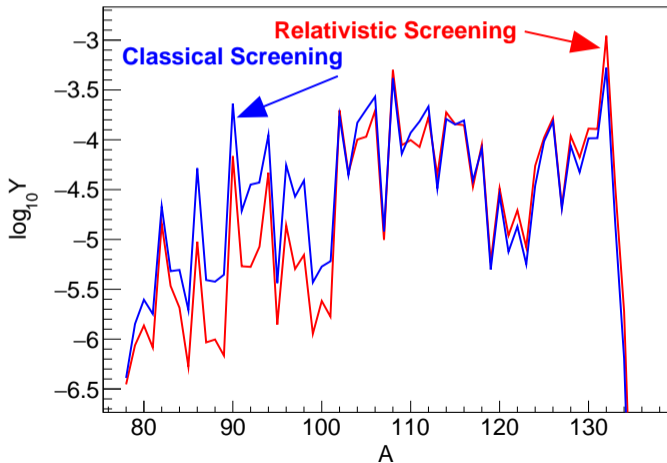


Rapid Proton Capture

- Accretion onto magnetized neutron-star surface.
- $B = 5 \times 10^9 \text{ T}$.
- Increased reaction rates \rightarrow increased heating.
- Did not examine phase-space/Pauli blocking in electron captures.

Preliminary Results

r-Process Nucleosynthesis: No B-Field, But High T

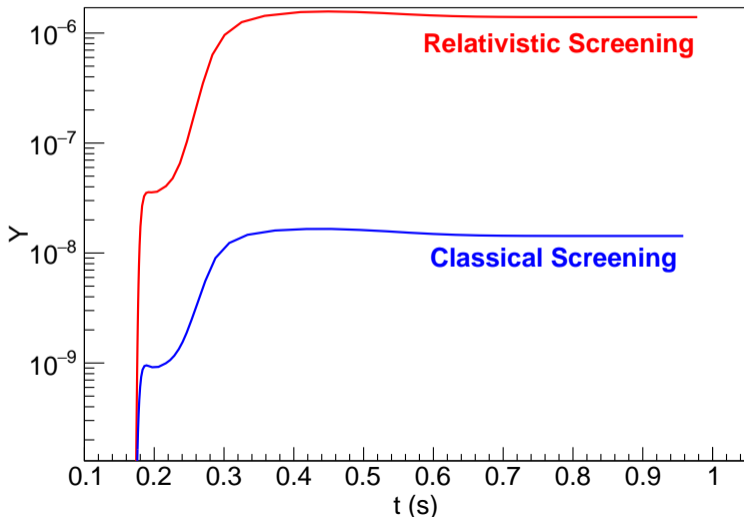


r-Process in SNeII

- Expanding neutrino-heated bubble in SNeII.
- $B = 0$, but $T_9 \lesssim 2.5$
- Does not affect (n, γ) , but could change (α, n) early on.
- Additional screening from thermal pair production.

Preliminary Results

p-Nuclei: Hot Shock Heating in SNe



Shock Heating in s-Process Shell - ^{196}Hg

- **VERY PRELIMINARY!**
- $B = 0$, but $T_9 \sim 3$
- Only charged-particle (e.g., (p, γ)) affected.
- Even small increase in reaction rates can dramatically change low Y nuclei.

Weak Interactions

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Electron Captures and Decays in Magnetized Plasmas

$$\lambda_{n \rightarrow p e^{-} \bar{\nu}_e} = \frac{G_F^2 \tilde{B} m_e (g_V^2 + 3g_A^2)}{2\pi^3 T} \sum_{n=0}^{n_m} (2 - \delta_{n0}) \int_0^{p_m} dp_z E_\nu^2 g(E_e) g(E_\nu)$$

$$\lambda_{n(\bar{\nu}_e/e^+) \rightarrow p(e^-/\bar{\nu}_e)} = \frac{G_F^2 \tilde{B} m_e (g_V^2 + 3g_A^2)}{2\pi^3 T} \sum_{n=0}^{\infty} (2 - \delta_{n0}) \int_0^{\infty} dp_z E_\nu^2 g(E_{e/\nu}) f_{FD}(E_{\nu/e})$$

Momentum terms, FD distribution, Pauli blocking factor changes.

Electron captures and NS matter Lai & Shapiro, ApJ (1991); Gao et al., Astroph. Space Sci. (2011)

Neutronization of proto-neutron star.

Possible effects in BBN fields? Possible effects prior to weak decoupling?

Conclusions

Conclusions

- Effective Screening Length: Potentially dramatic shifts at high B/T .
 - Screening enhancement factor for relativistic environments changes
 - Effective reduction in chemical potential.
- Possible change in stellar core burning.
- Future Work: NS Crust Effects, Pair Production SNe, BBN, NS Cores?, Experiment?
- Extending our TF screening model accurate at lower- T /higher- ρ .
- **Magnetized plasmas could be dramatically different!**
 - Field evolution in NS merger events?

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