Machine Learning Inference of the Dense Matter EoS and Quark-Hadron Continuity

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in collaboration with: K. Fukushima, K. Murase & W. Weise

YF, K. Fukushima, K. Murase, Phys. Rev. D **98**, 023019 (2018)
YF, K. Fukushima, K. Murase, arXiv:1903.03400 [nucl-th]
YF, K. Fukushima, W. Weise, arXiv:1908.09360 [hep-ph].

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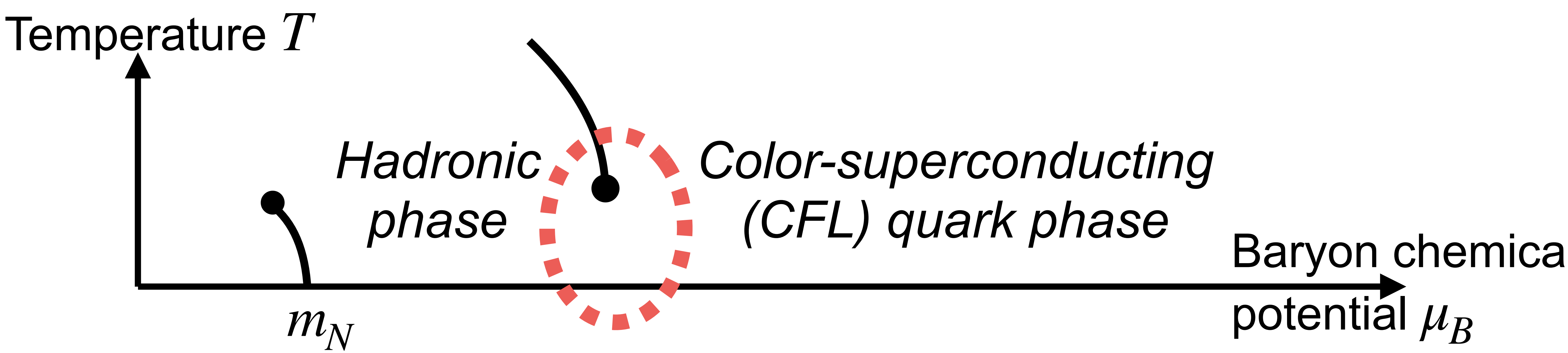
Phases of Dense Matter

- Degrees of freedom of dense matter:
  Hadron & **Quarks**

- One possibility from neutron star phenomenology:
  Quark-hadron continuity
  - both phases are indistinguishable (no order parameter)
  - connected by **crossover** transition



Schäfer-Wilczek (1999), Hatsuda et al. (2006), Masuda et al. (2013), Baym et al. (2018)

Equation of State of Dense Matter

- Dense matter properties are characterized by **Equation of State (EoS)**:
  relation between pressure p and (mass) density ρ

$$p = p(\rho)$$

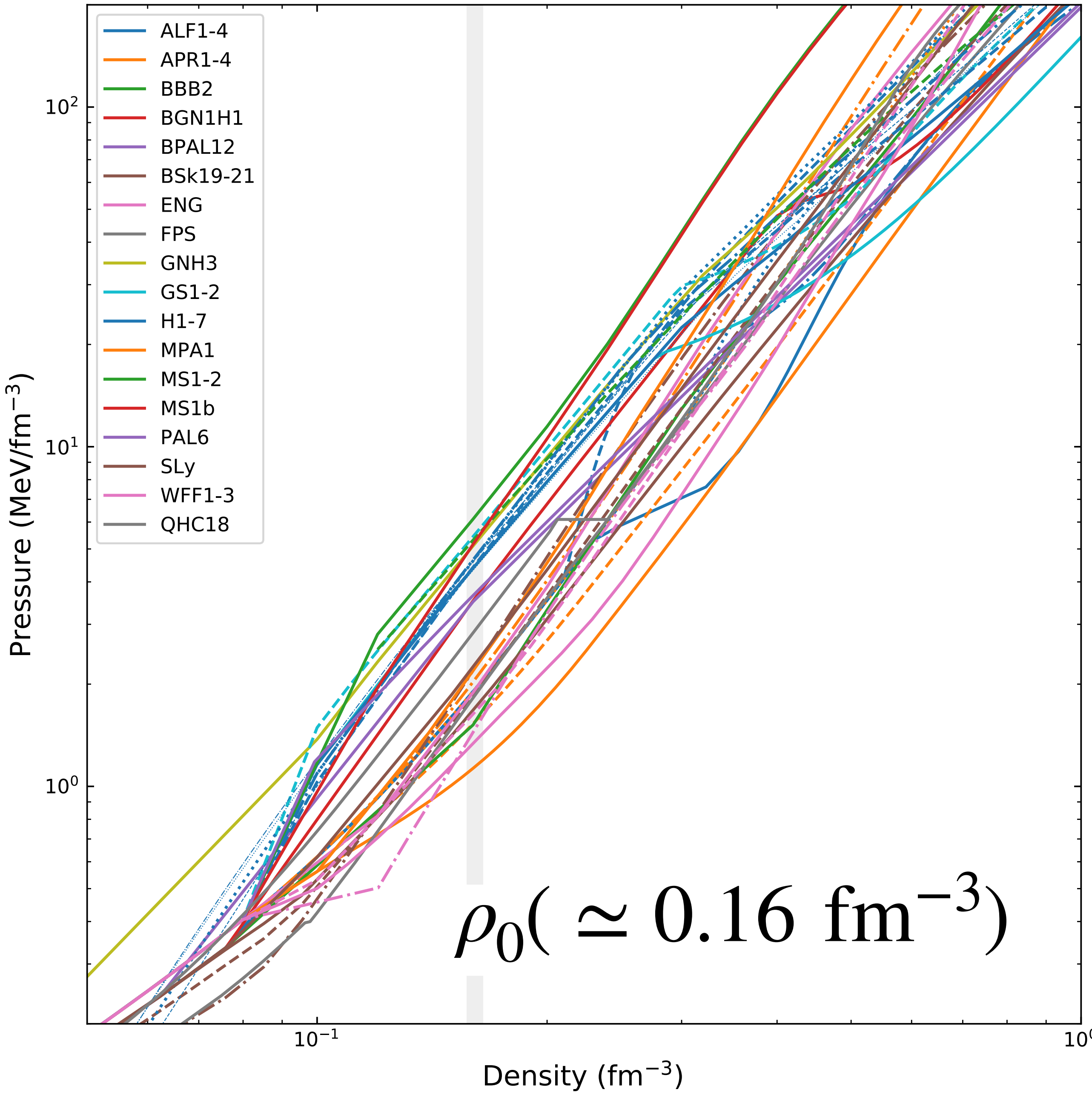
- Essential ingredients for neutron star theory;
  characterizes neutron star structure

- Should be derived from QCD in principle,
  but many difficulties such as…
  - sign problem; no lattice data
  - renormalization scale dependence in pQCD

Hester et al. (2002)

Current Status of the EoS

- Many nuclear theory calculations
  …but reliability of these models decline with growing ρ



Current Status of the EoS

- Many nuclear theory calculations
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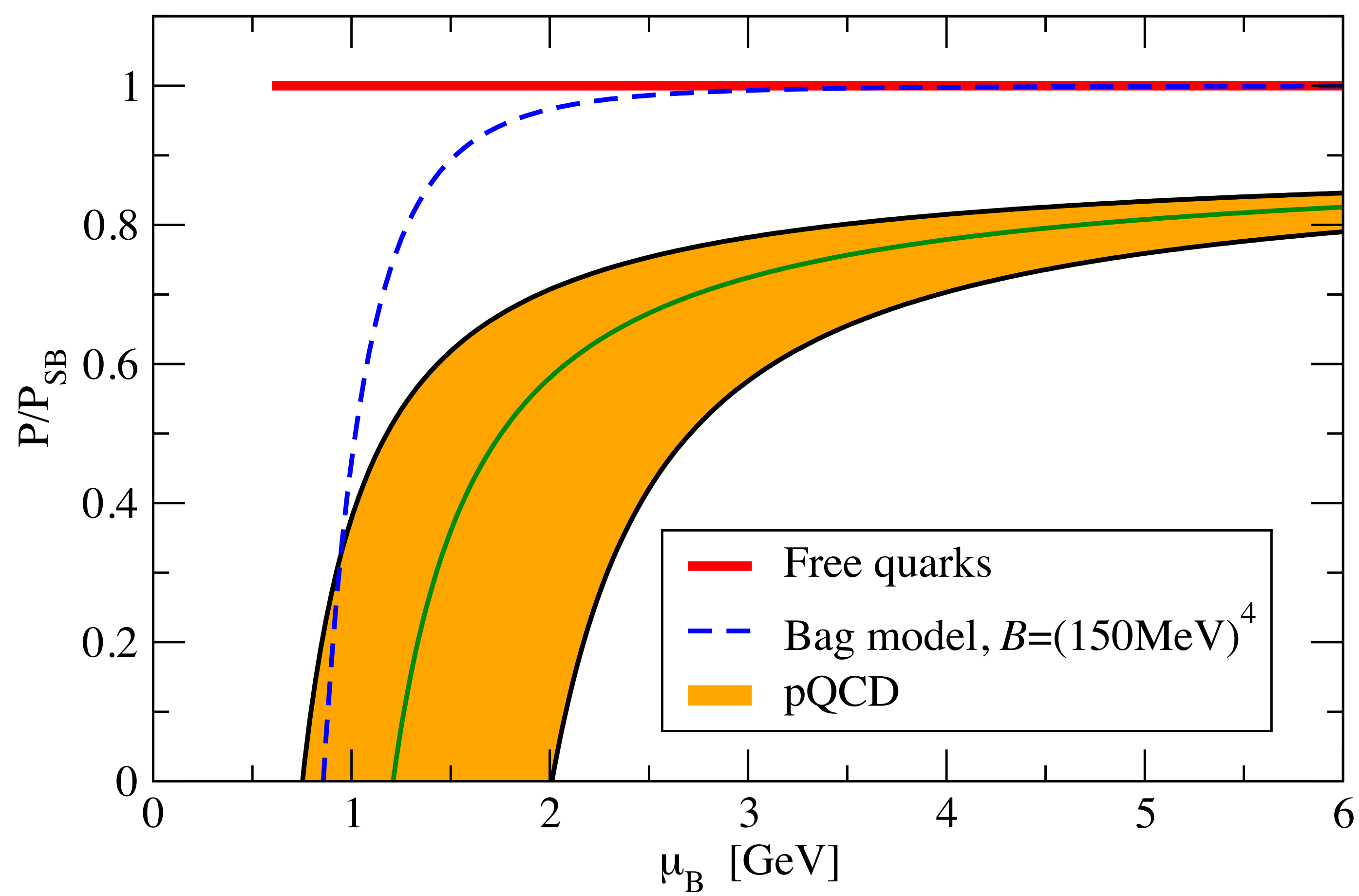
- Perturbative QCD calculation also has large uncertainty



Freedman-McLerran (1977); Baluni (1978)
Fraga-Kurkela-Vuorinen (2014)

Current Status of the EoS

- Many nuclear theory calculations
  …but reliability of these models decline with growing ρ

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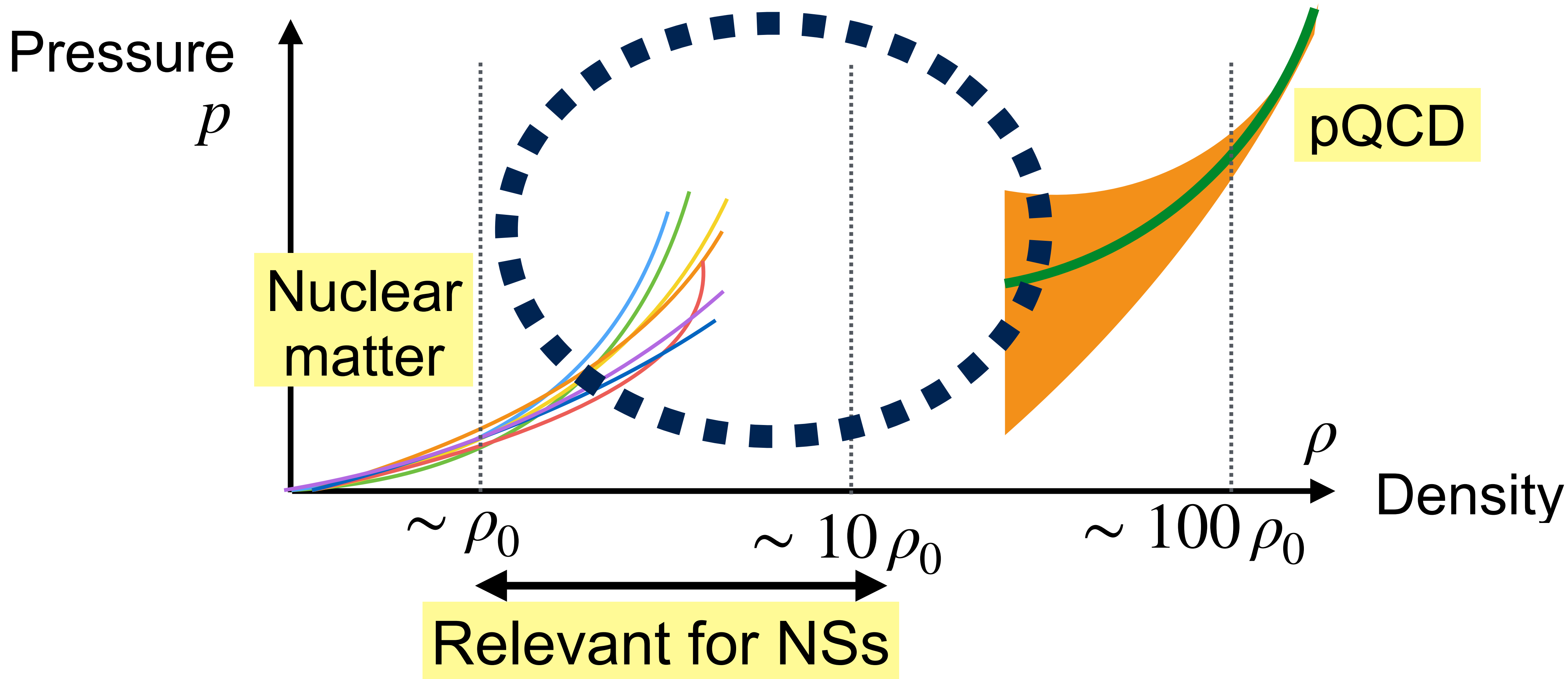
- Need systematic way to interpolate these two regions

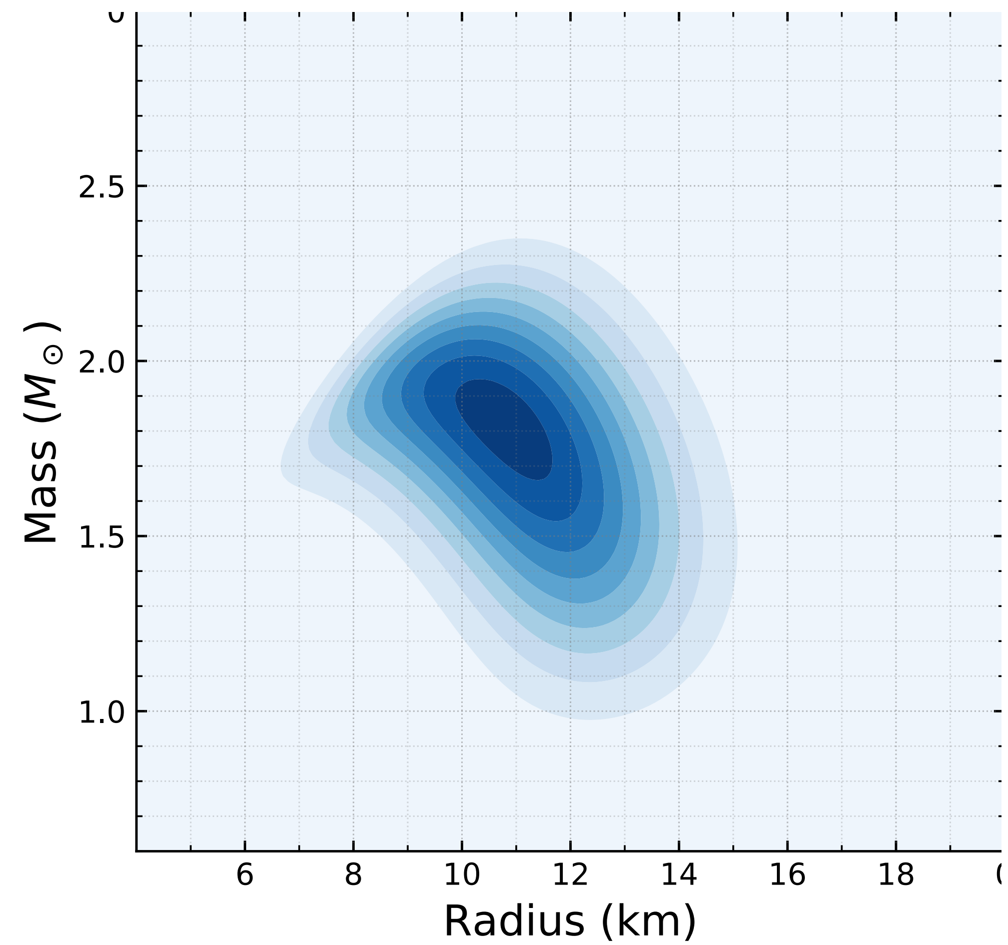


Systematic Way of EoS Inference

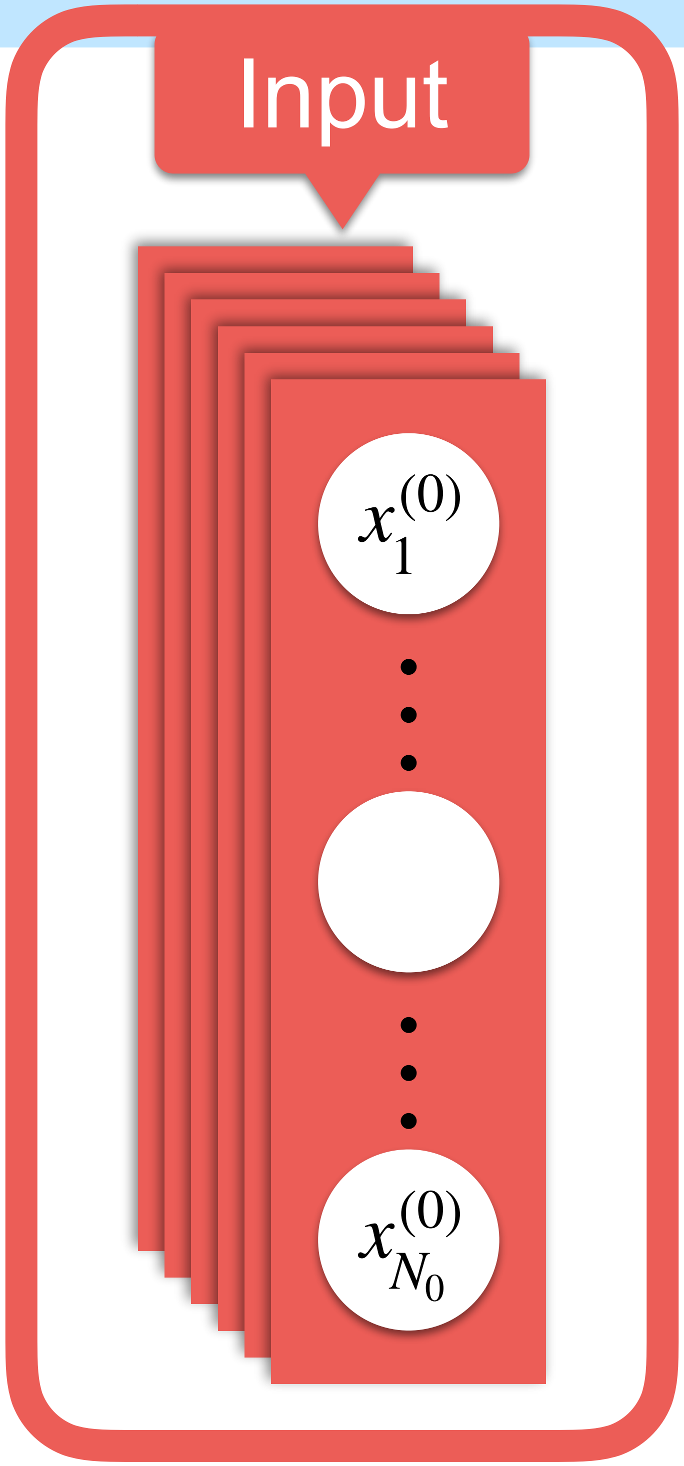
- Growing number of observables of NSs:
  e.g.) LIGO-Virgo, NICER experiment

- Utilize NS **masses** (***M***) and **radii** (***R***) Özel-Freire (2016)
  14 simultaneous measurements are publicly available;
  given in terms of likelihood distribution in (*R*, *M*) plane

- Infer EoS from *M*-*R*:
  Use **machine learning** (**ML**)
  with deep **neural network** (**NN**)



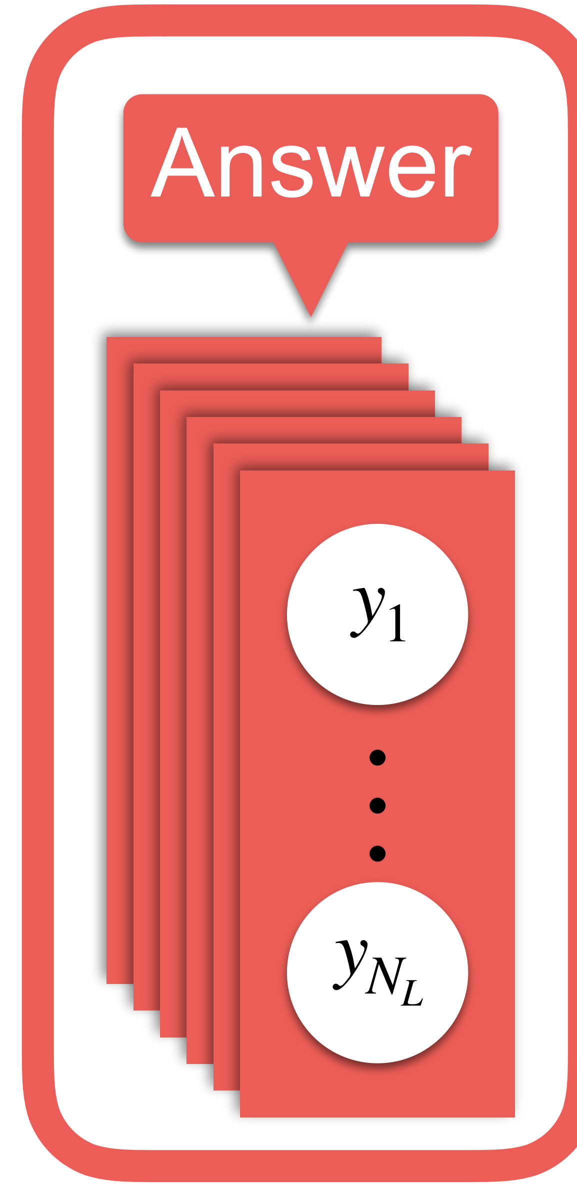
Method of Machine Learning



Training data

↓

**By "training"
the neural network,
we extract the relation
between input and answer**



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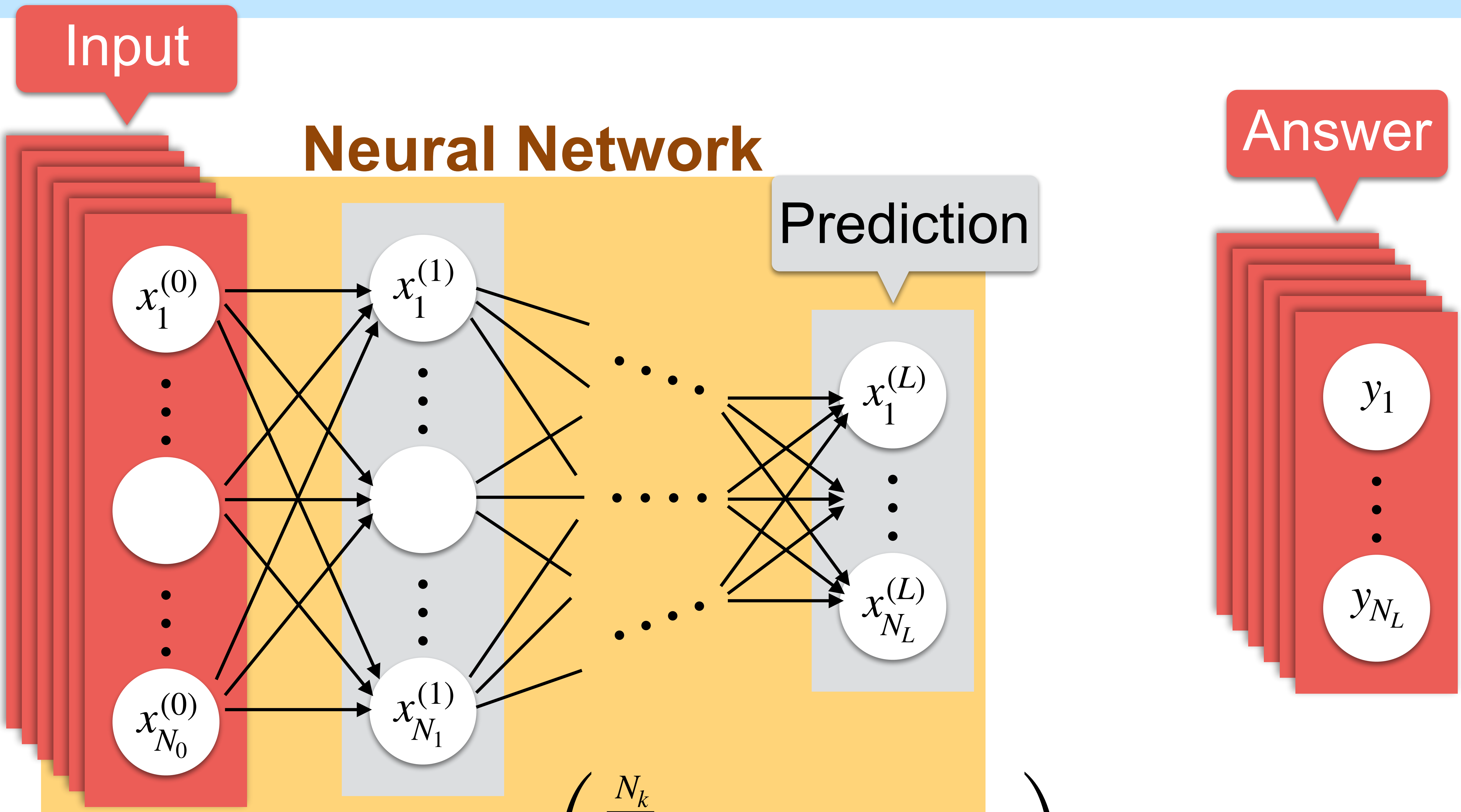
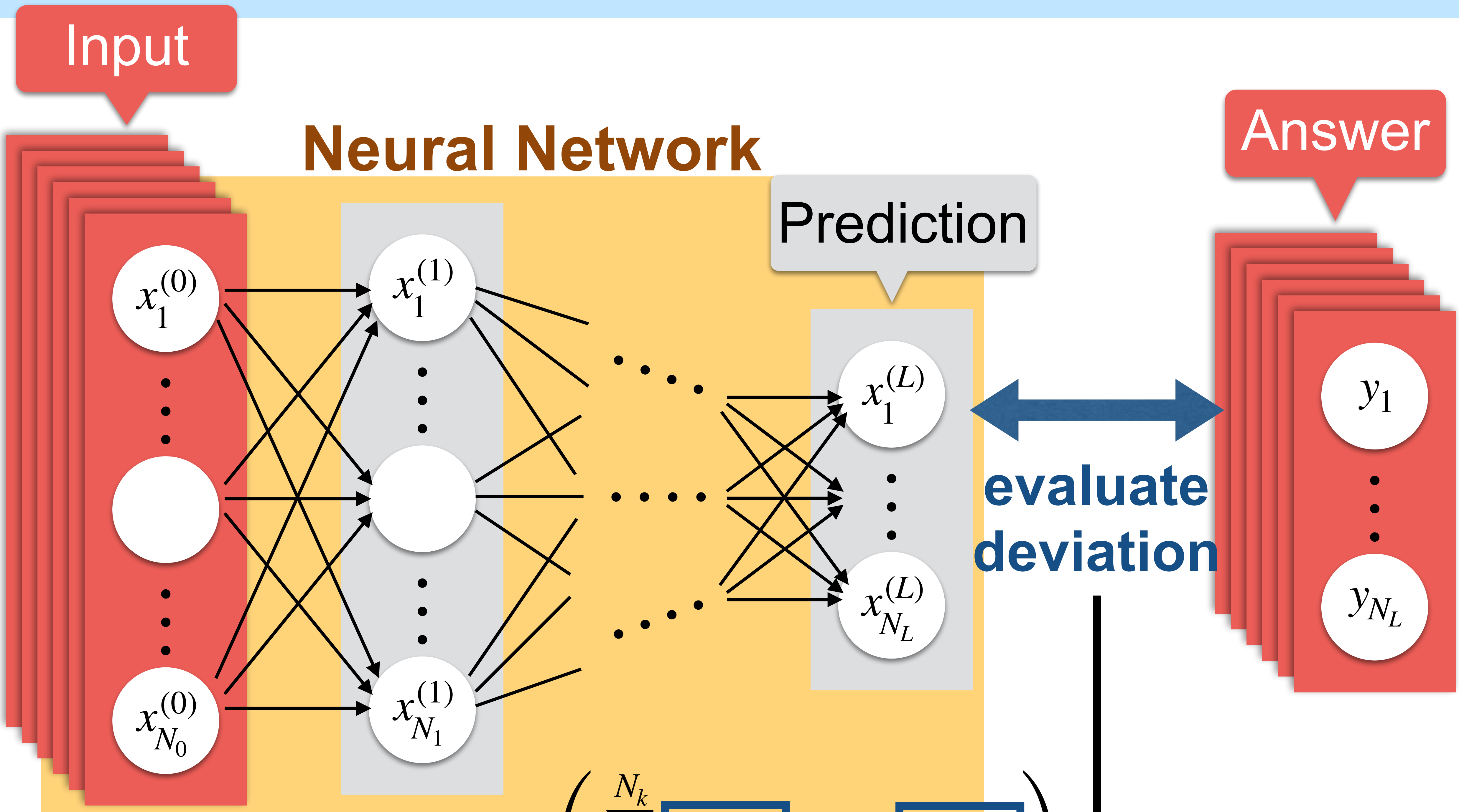


$$x_i^{(k+1)} = a^{(k+1)}\left(\sum_{i=1}^{N_k} W_{ij}^{(k+1)} x_j^{(k)} + b_i^{(k+1)}\right)$$

Method of Machine Learning



$$x_i^{(k+1)} = a^{(k+1)}\left(\sum_{i=1}^{N_k} W_{ij}^{(k+1)} x_j^{(k)} + b_i^{(k+1)}\right)$$

Tune these parameters inside network to minimize the deviation

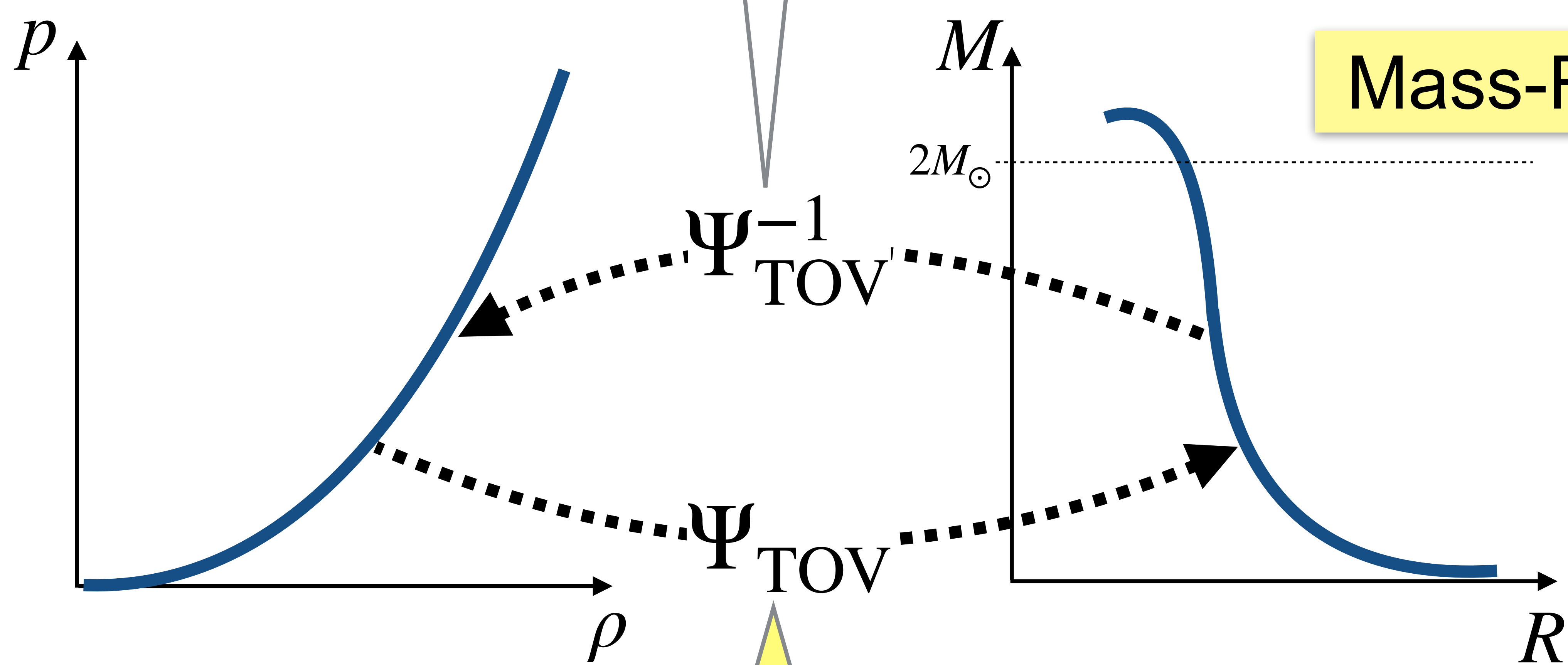
EoS & *M-R*: TOV Mapping Ψ_{TOV}

If *M-R* curve is given,
there is **one-to-one correspondence**

Lindblom (1992)

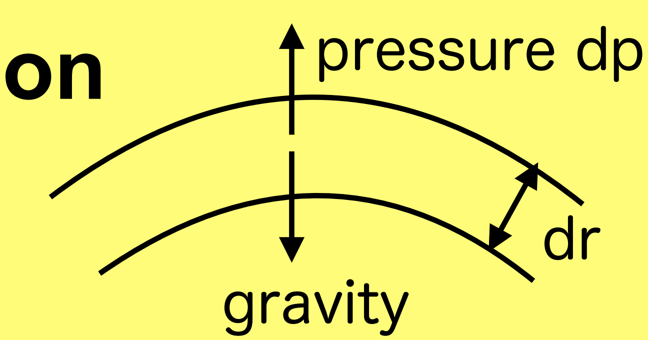
EoS

Mass-Radius



Tolman-Oppenheimer-Volkoff (TOV) equation

General relativistic structural equation
for stars at hydrostatic equilibrium



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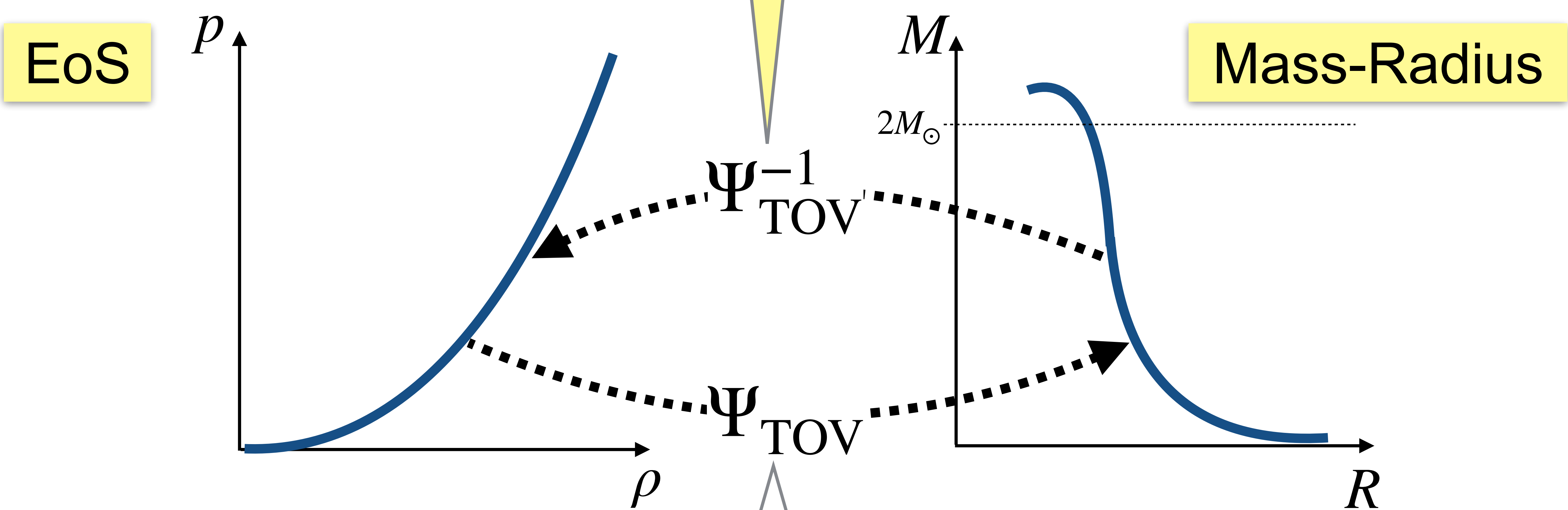

Tolman-Oppenheimer-Volkoff (TOV) equation

General relativistic equation for
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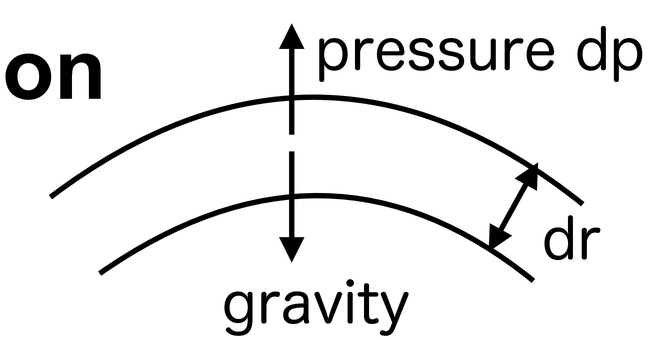
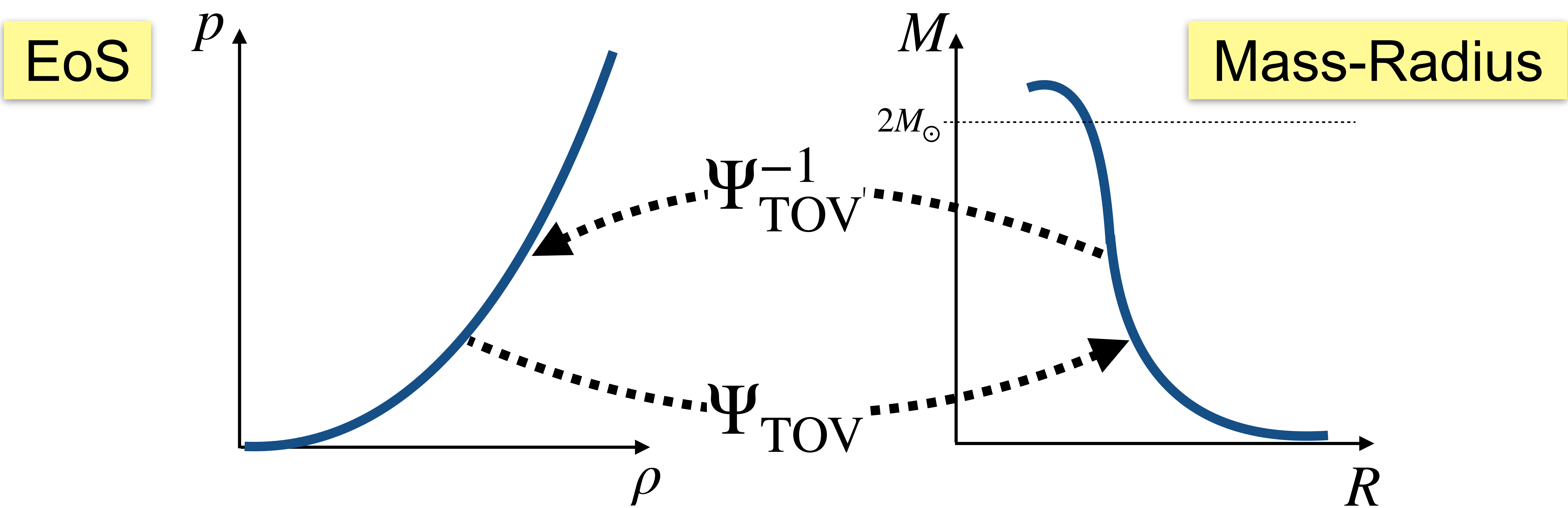


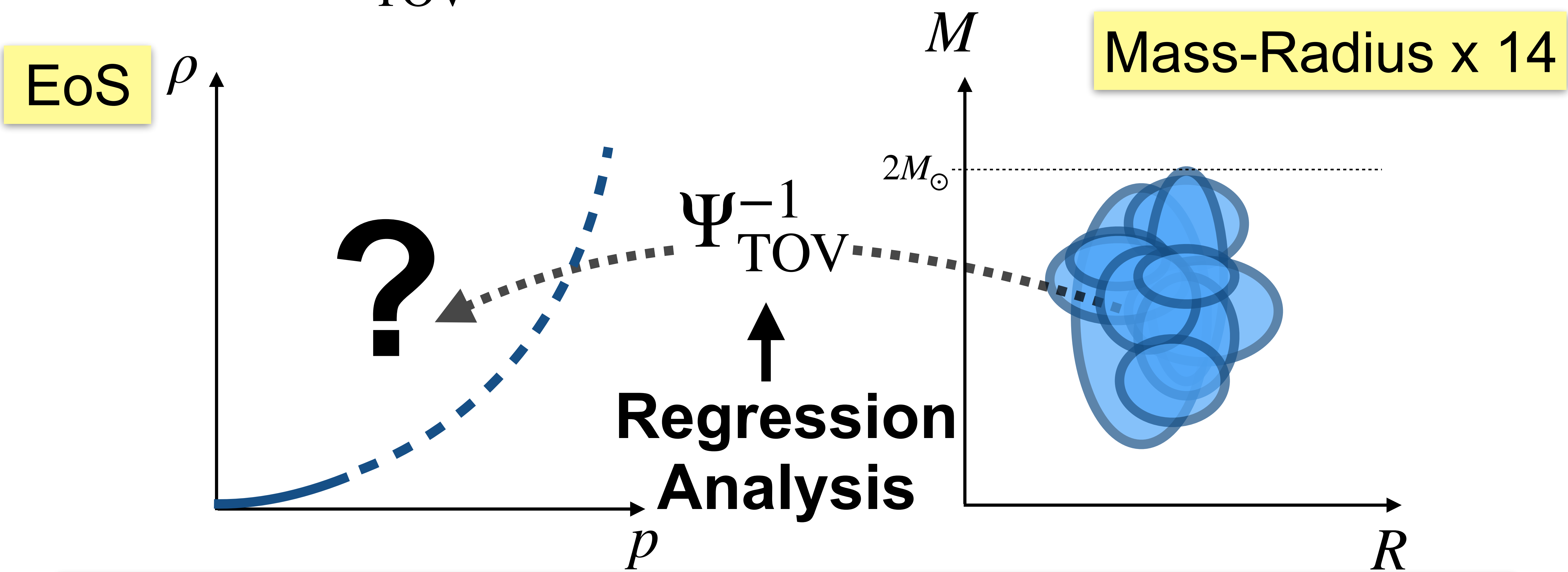
EoS & *M-R*: TOV Mapping Ψ_{TOV}

The operation of the TOV equation can be regarded as one-to-one mapping: **TOV mapping** Ψ_{TOV}



Regression Analysis of TOV Mapping

In reality… *M-R* **is point-like** and **has uncertainties**
→ finding Ψ_{TOV}^{-1} becomes non-trivial!

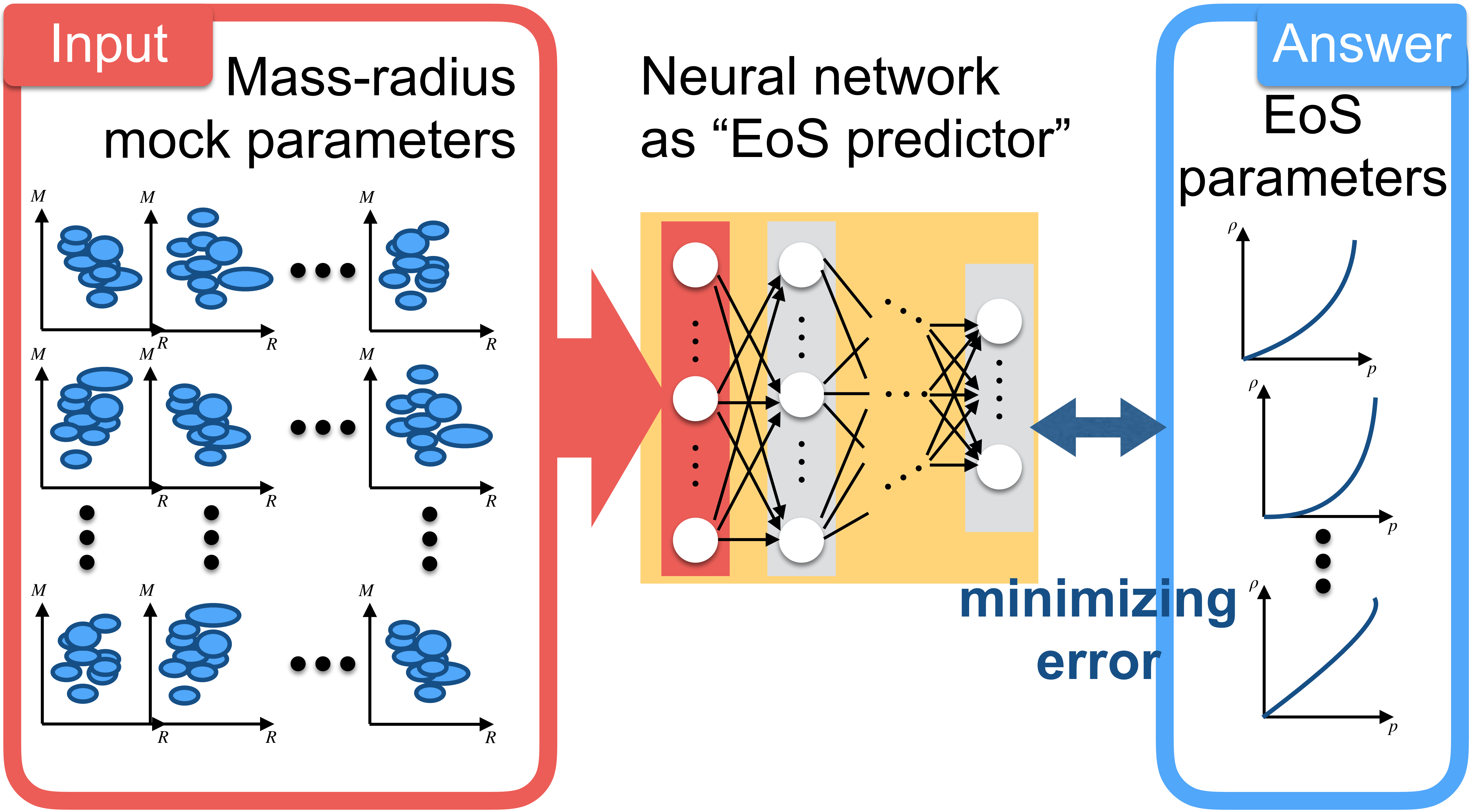


Express Ψ_{TOV}^{-1} in terms of Neural Network
→ problem becomes regression analysis finding

$$\mathrm{EoS} = \Psi_{\mathrm{TOV}}^{-1}(\mathrm{M\text{-}R})$$

Expressing TOV by Neural Network

Find the “EoS predictor” using machine learning

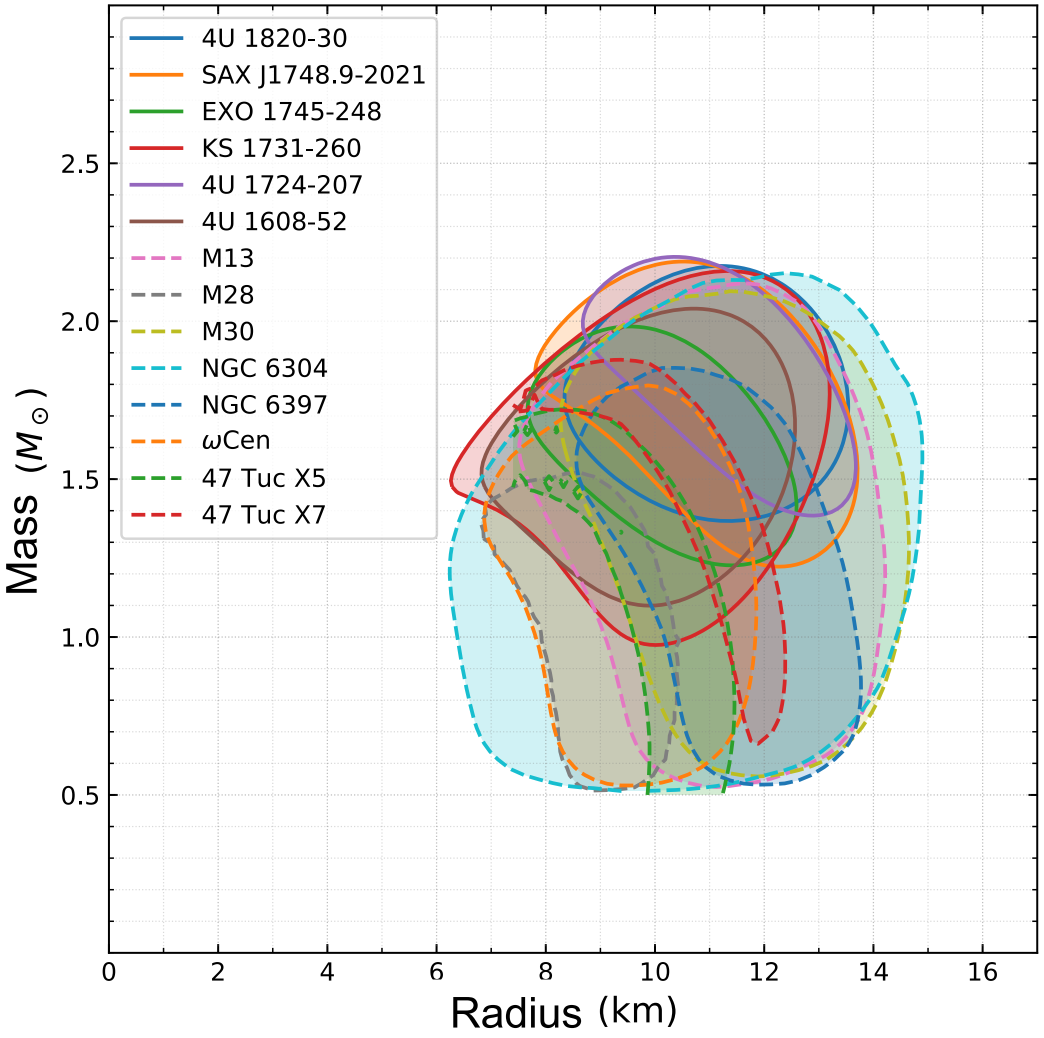


Result: Inferred EoS

YF-Fukushima-Murase (2019)

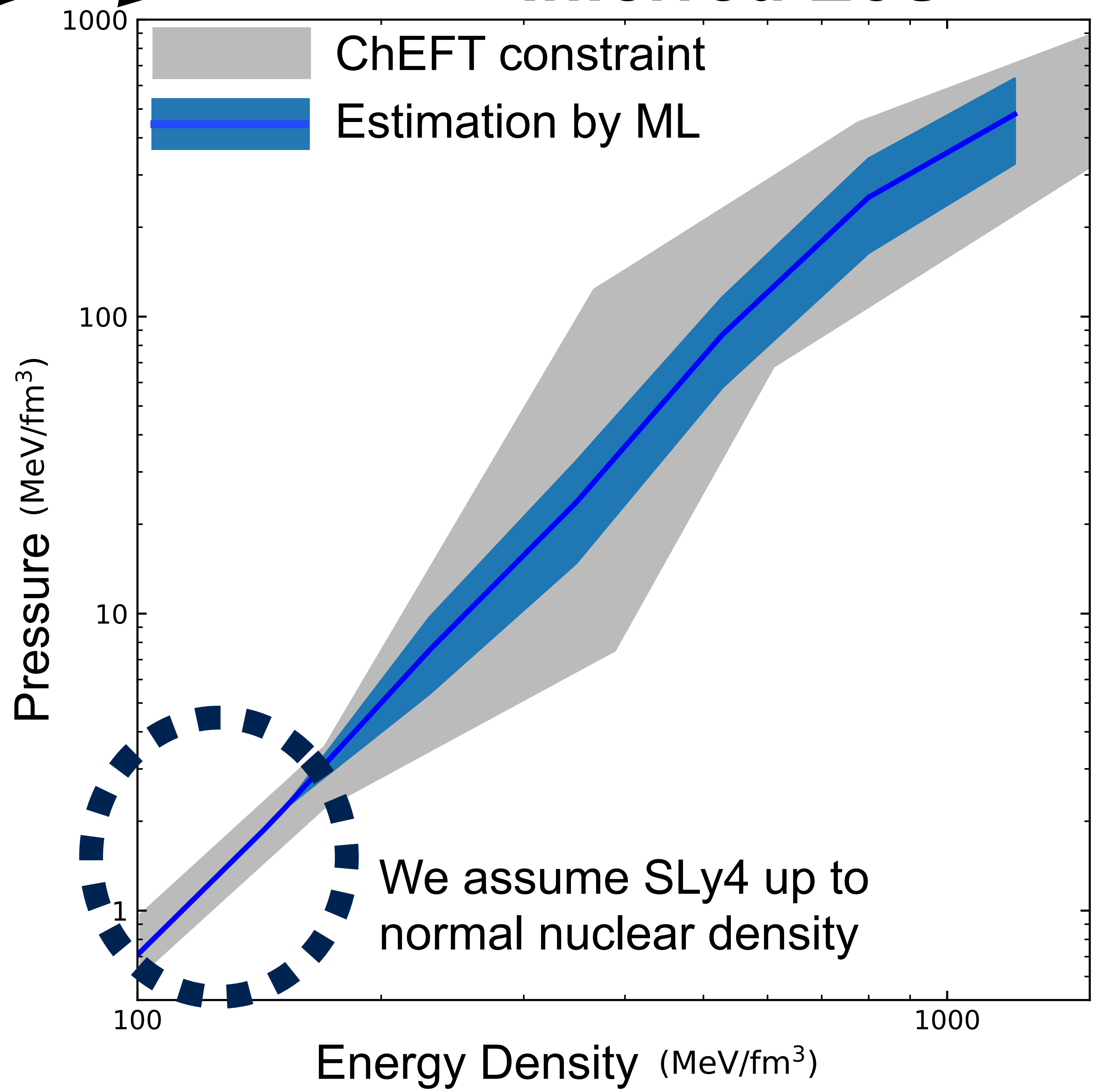
Inferred by machine learning

X-ray observation



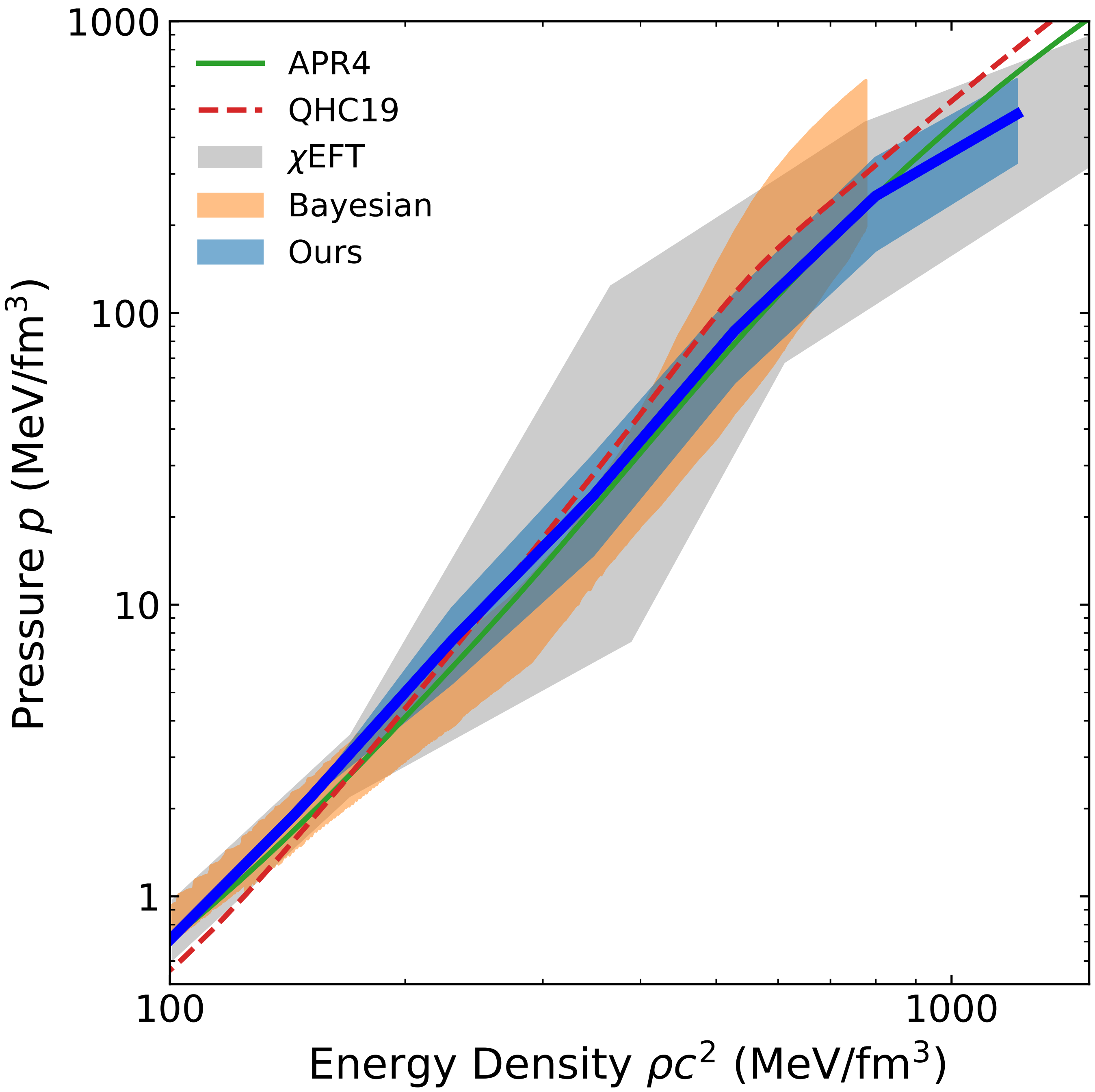
Özel-Freire (2016)

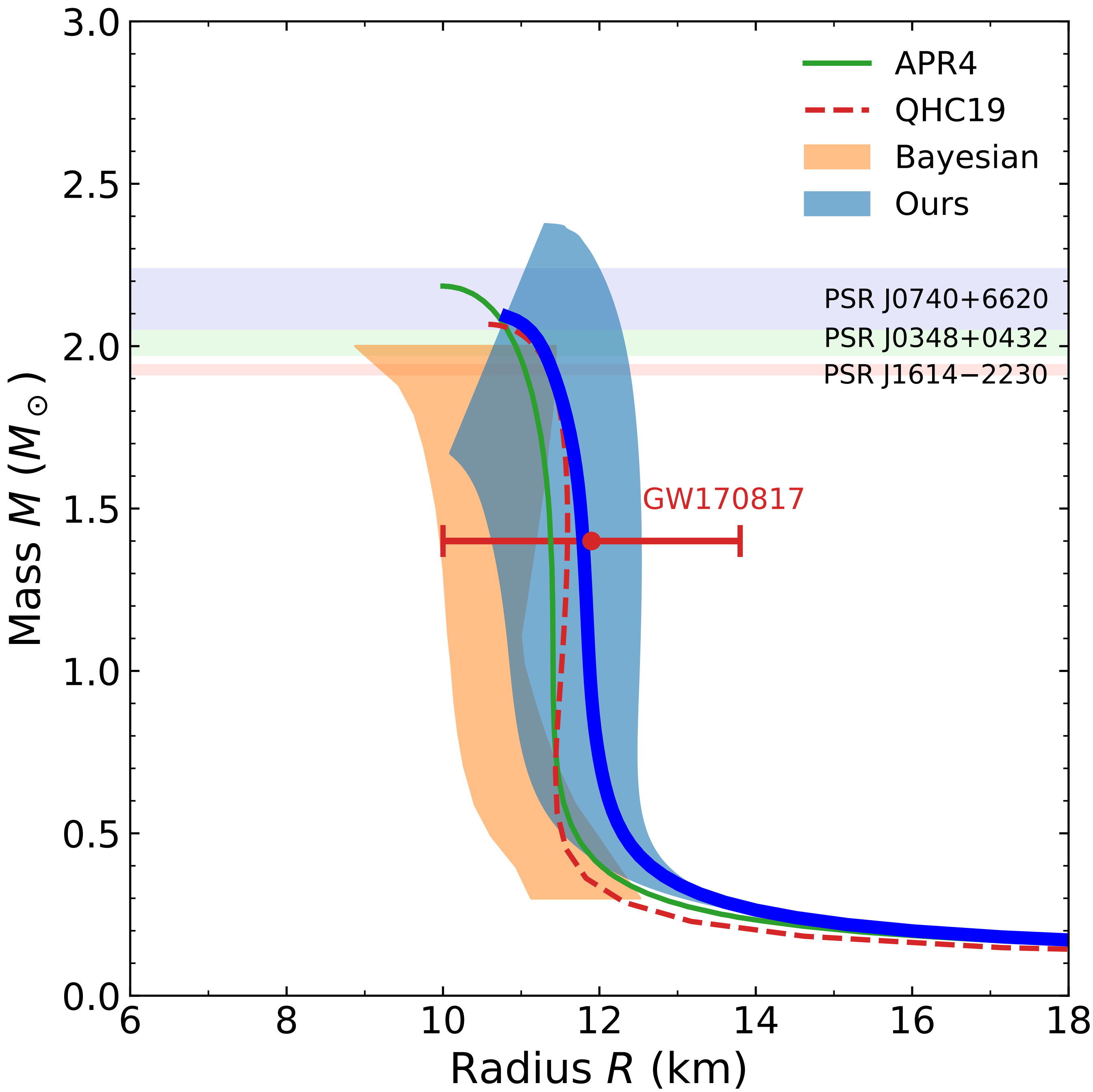
Inferred EoS



Hebeler-Lattimer-Pethick-Schwenk (2013)

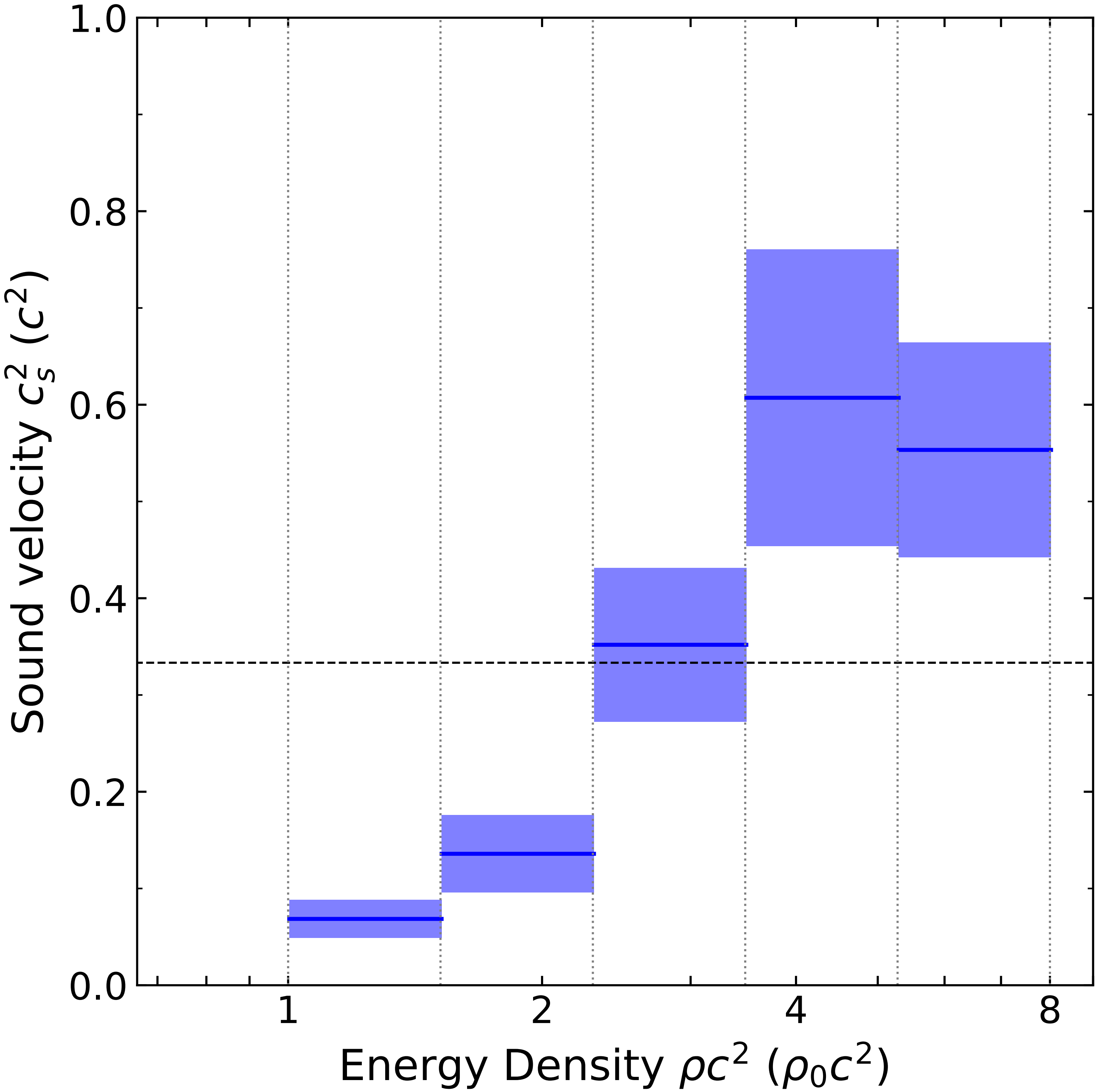
EoS with M-R relation





Result: Sound velocity

Average sound velocity:



$$c_s^2 = \frac{\partial p}{\partial \rho}$$

- Measure of **stiffness** for EoS
- Our inferred result shows **Soft-to-Stiff** behavior
- This property may imply **Quarkyonic** picture; Nuclear matter seems as if it is quark matter

Fukushima-Kojo (2015), McLerran-Reddy (2018)

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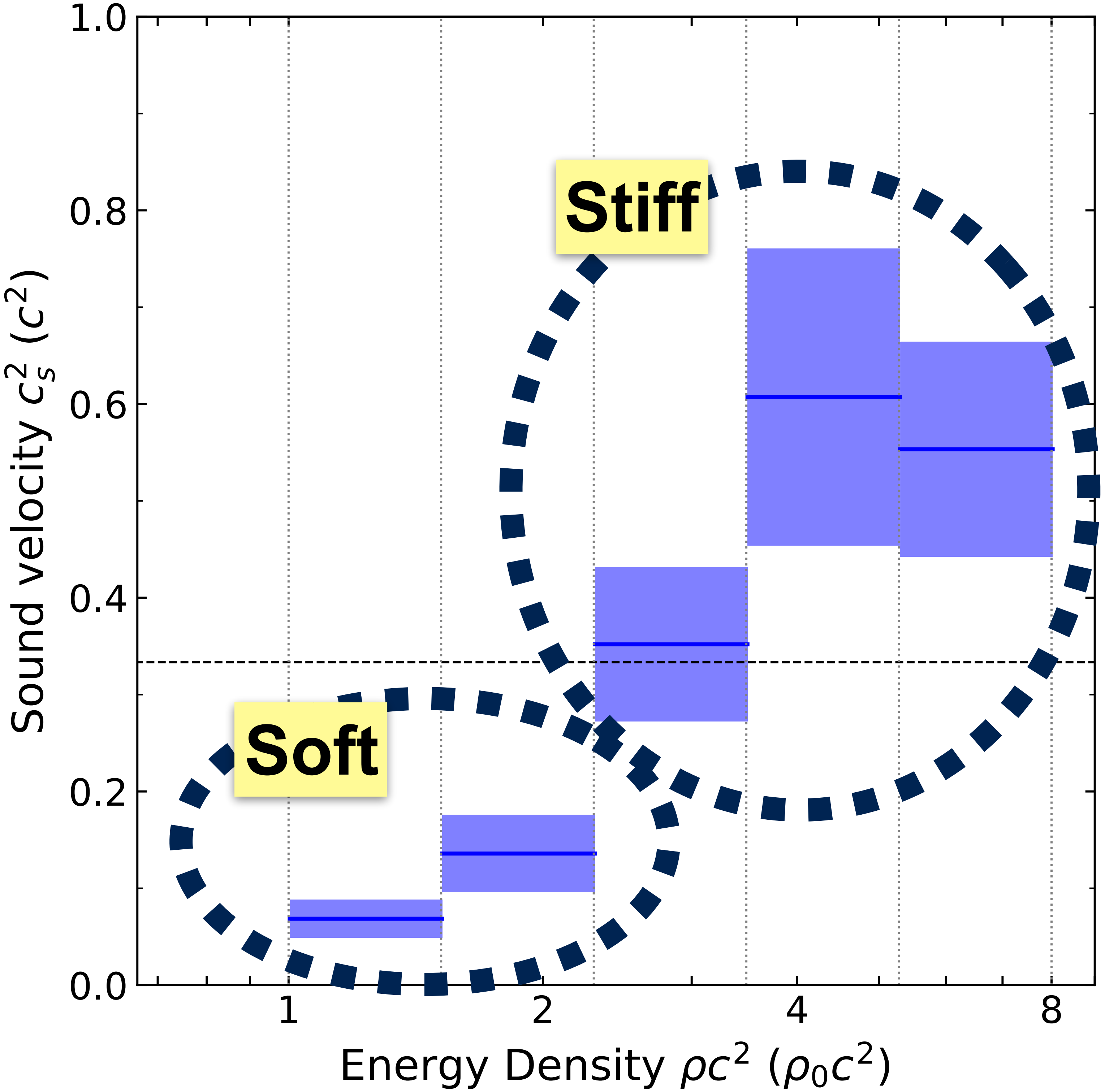

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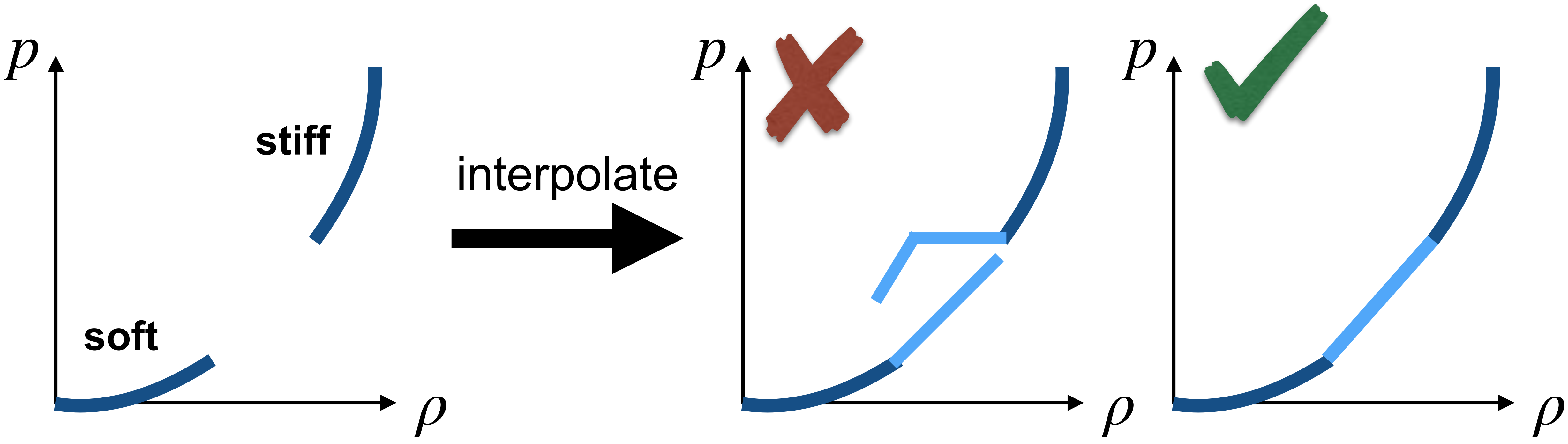
Soft-to-Stiff EoS

- Our inferred EoS shows soft-to-stiff behavior

- Model study of an EoS shows: Han et al. (2019)
  If EoS is soft-to-stiff, strong 1st-order QCD transition cannot exist… Only the **crossover** transition is allowed!

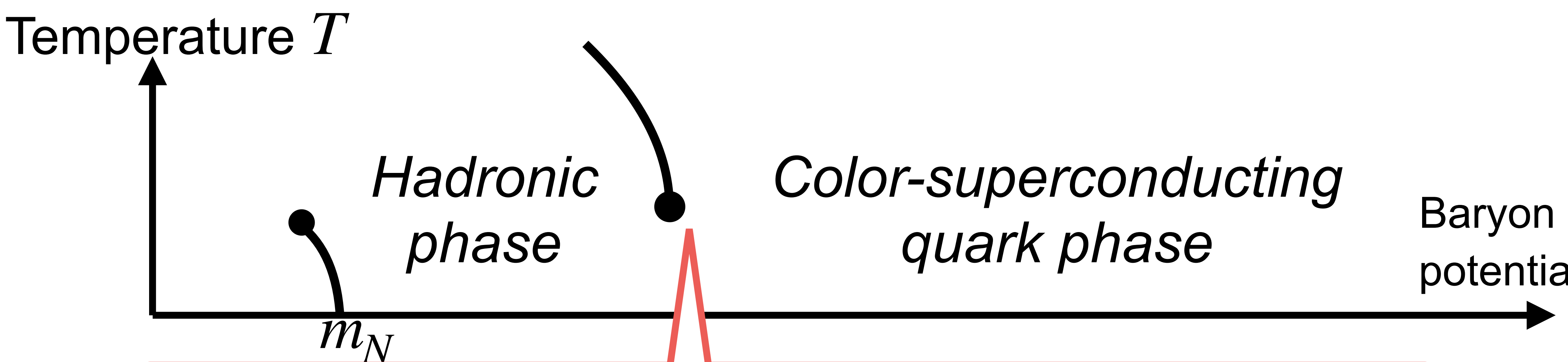


- What is the physical foundation for the crossover?:
  Quark-Hadron Continuity

Quark-Hadron Continuity

Hatsuda et al. (2006)

Phase diagram of QCD at low temperature:



Both phases are continuously connected
(**Quark-Hadron Continuity**)

Schäfer-Wilczek (1999)

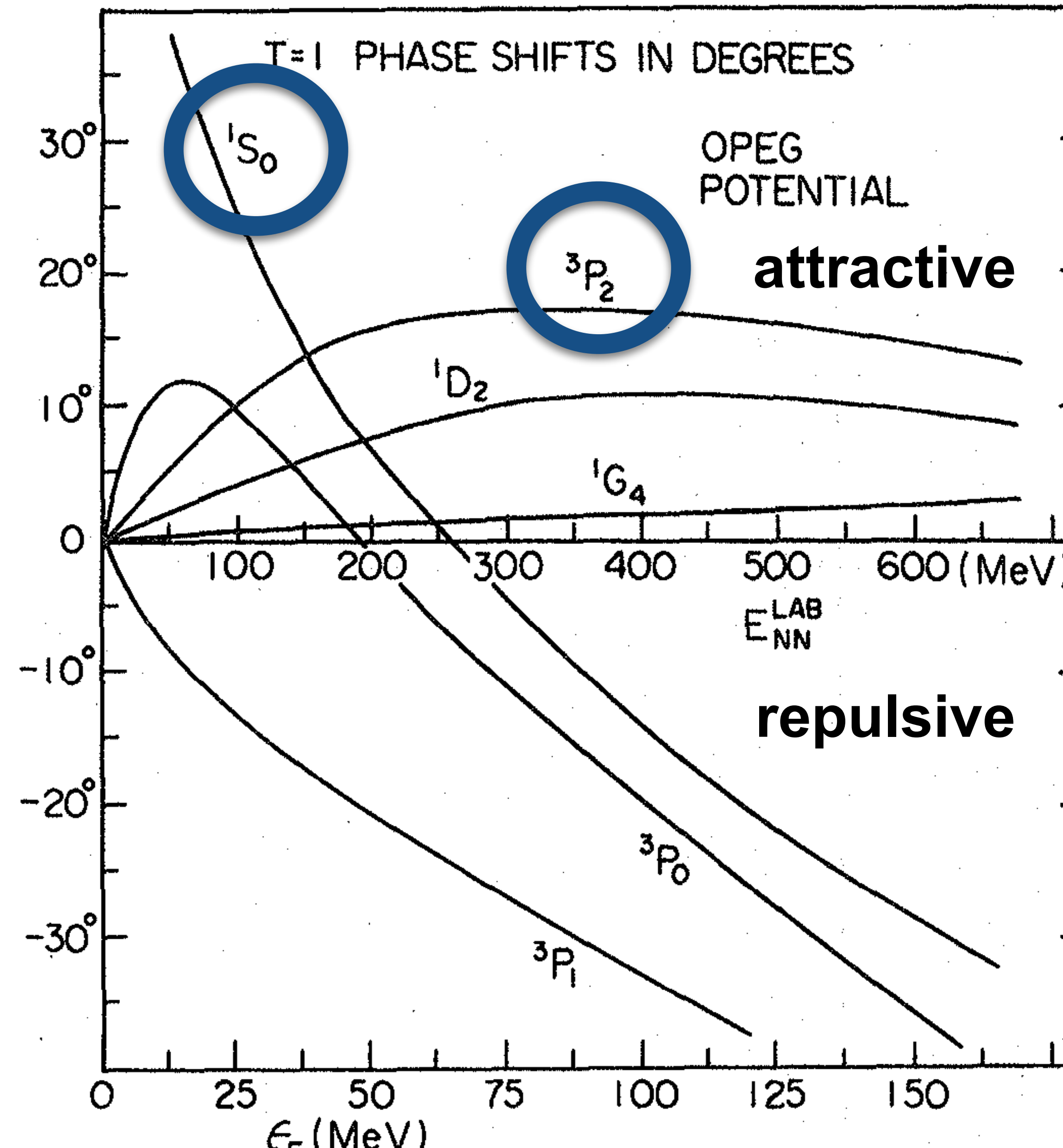
- Preceding analysis has been made to **three-flavor symmetric** matter with masses $m_u = m_d = m_s$

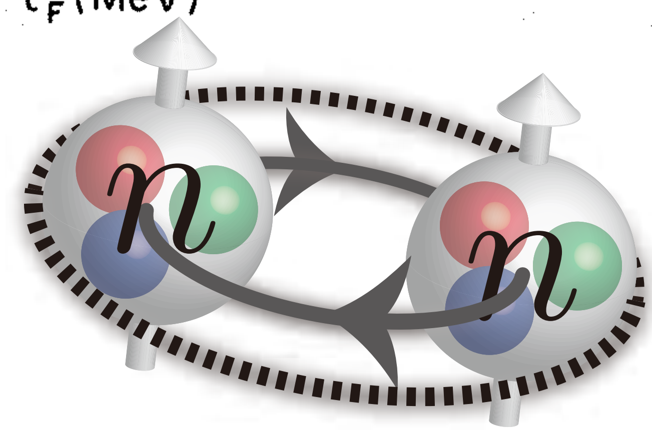
- Natural starting point: **two-flavor symmetric matter** because in reality $m_s/m_{u,d} \sim 30$

Neutron Superfluidity

- Neutrons show superfluidity because effective *nn*-interaction is attractive in some channel
- At higher densities, pairing in ${}^1S_0$ state gets weakened by repulsive core; $\rightarrow {}^3P_2$ state takes over
- Is this neutron superfluid continuously connected to quark phase? → YES!

Hoffberg et al. (1970), Tamagaki (1970)





Order Parameter Rearrangement

YF-Fukushima-Weise (2019)

Order parameter of neutron superfluid: $\langle \hat{n}\,\gamma^i\,\nabla^j \hat{n}\rangle$

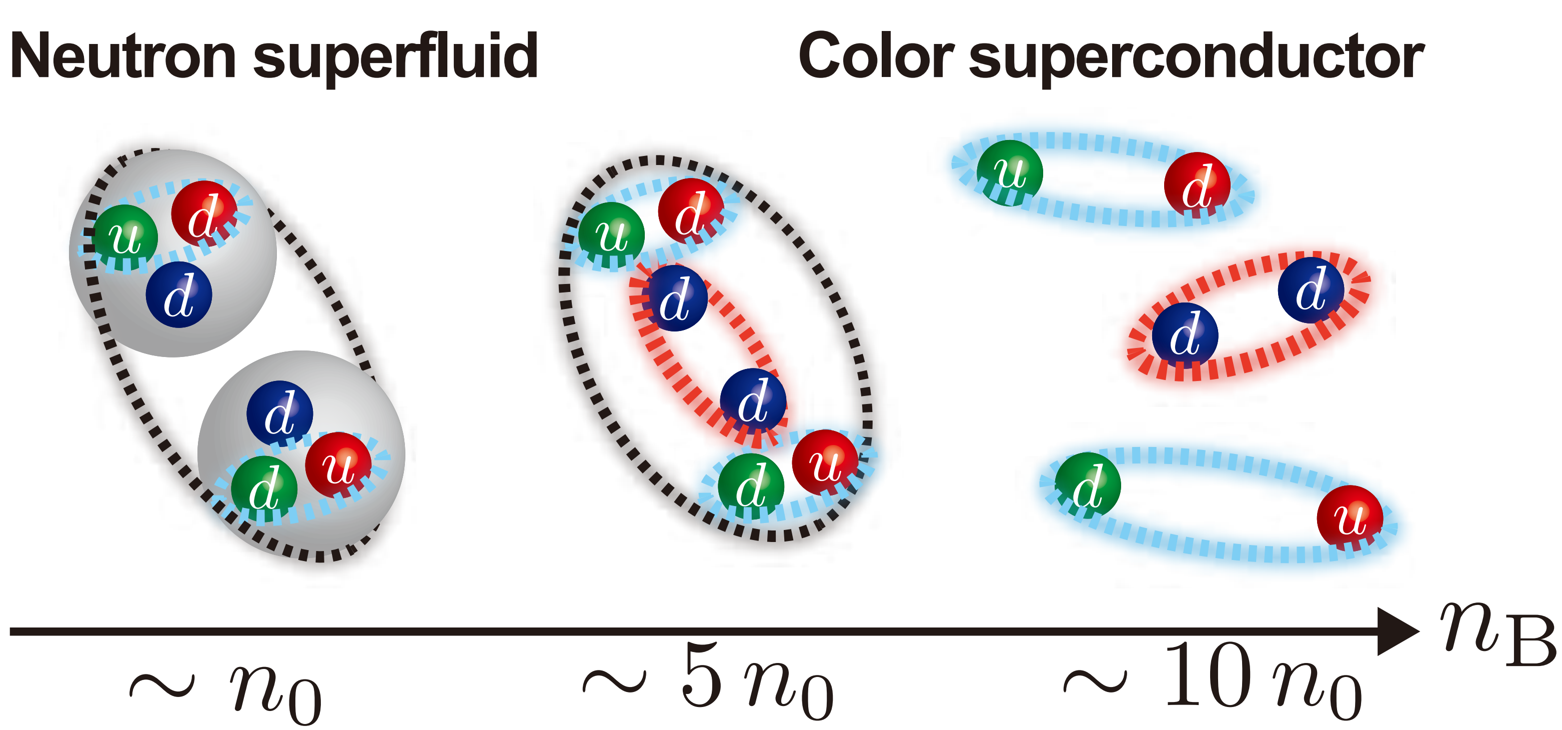
Neutron operator: $\hat{n} = (\hat{u}\,\gamma_5\,\hat{d})\,\hat{d}$

Order parameter can be rearranged (in MFA) as …

$$\langle \hat{n}\,\gamma^i\,\nabla^j \hat{n}\rangle \simeq \underline{\langle \hat{u}\gamma_5\hat{d}\rangle\langle \hat{u}\gamma_5\hat{d}\rangle}\,\langle \underline{\hat{d}\gamma^i\,\nabla^j\,\hat{d}}\rangle$$

2SC diquark condensate

Additional <dd> arises!



Two-flavor Continuity: 2SC+<dd>

YF-Fukushima-Weise (2019)

- Two diquark condensates:

$$\langle ud \rangle + \langle dd \rangle$$

- **2SC + <dd>** induces…
  - Chiral symmetry breaking
  - Baryon $U(1)_B$ breaking (leads baryon superfluidity)

  → Same as the neutron superfluid, **continuity set in!**

- For dynamical aspects and supportive evidences, please refer to:

  YF, K. Fukushima & W. Weise, arXiv:1908.09360

Summary

- Established the method of machine learning to infer EoS from mass and radius observations

- Our EoS result is consistent with quark-hadron continuity scenario

- Established the quark-hadron continuity in more realistic context of two-flavor than the previous one;
    neutron superfluid → 2SC + <dd> condensate

Supplementary materials

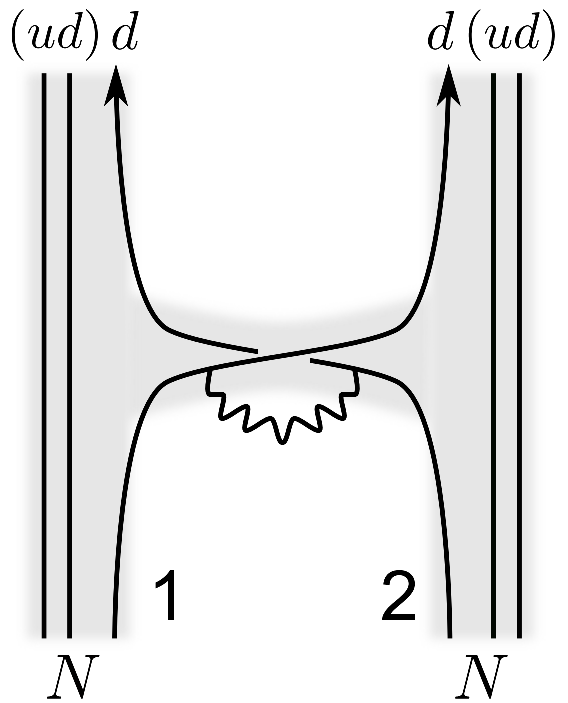
Dynamical Aspects of 2SC+<dd>

To have $^3P_2$ pairing at high density, one needs:

- **short-range repulsive core**
  $\rightarrow$ disfavors pairing in $^1S_0$ state

- **attractive spin-orbit potential**
  $\rightarrow$ favors $^3P_2$ state among $^3P_{J=0,1,2}$ states

Dynamical Aspects - Repulsive Core



Dynamics that disfavors $^1S_0$ pairing

- Short-range interaction between d-quarks can be understood with this diagram

- Potential between d-quarks reads:

$$V_{12}^{\mathrm{OGE}} = \left(\sum_A T_1^A T_2^A\right) \frac{\alpha_s}{4} \left[\frac{1}{r_{12}} - \frac{2\pi}{3m_d^2}(\boldsymbol{s_1}\cdot\boldsymbol{s_2})\delta(\boldsymbol{r}_{12})\right]$$

- <dd> condensate is in color-sextet channel
  → Color factor above becomes repulsive!

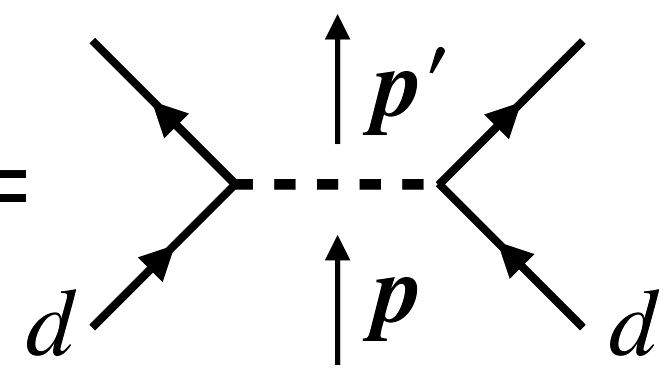
Dynamical Aspects - LS Potential

Dynamics that favors ${}^3P_2$ pairing

- NJL-type model with scalar and vector couplings + diquark correlations:

$$\mathscr{L}_{\rm int} = G(\bar{\psi}\psi)^2 + H(\bar{\psi}\bar{\psi})(\psi\psi) - G_V(\bar{\psi}\gamma^\mu\psi)^2$$

$$\langle \boldsymbol{p}' | V_{LS} | \boldsymbol{p} \rangle =$$



→ Fourier transform $V_{LS}(\boldsymbol{r}) \propto -(1+mr)\dfrac{e^{-mr}}{r^2}\,\boldsymbol{L}\cdot\boldsymbol{S}$

$(m > 0)$

$$\boldsymbol{L}\cdot\boldsymbol{S} = \begin{cases} -2 & ({}^3P_{J=0}) \\ -1 & ({}^3P_{J=1}) \\ 1 & ({}^3P_{J=2}) \end{cases}$$

→ **Spin-orbit potential is attractive only in ${}^3P_2$ state**