Double beta decay matrix elements, QRPA vs. Shell Model

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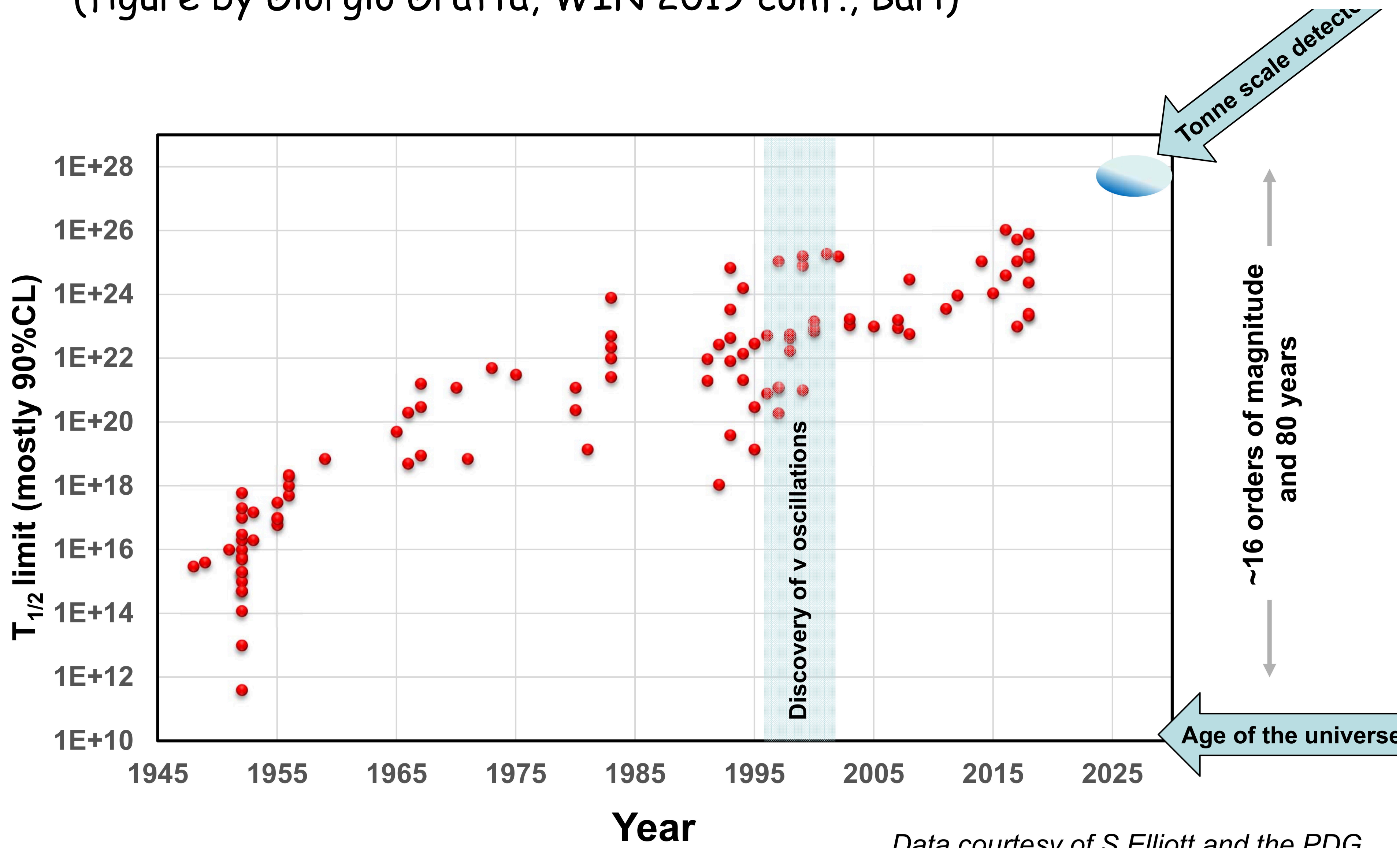
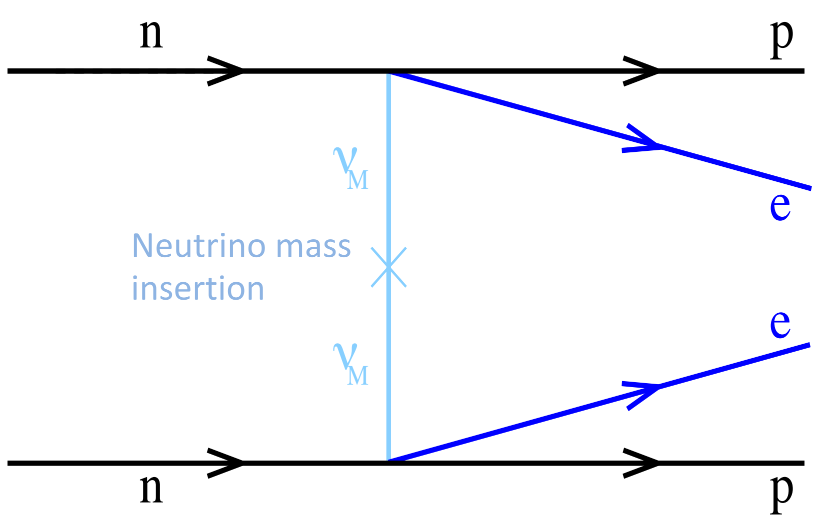
The history of 0νββ decay experiments in one slide

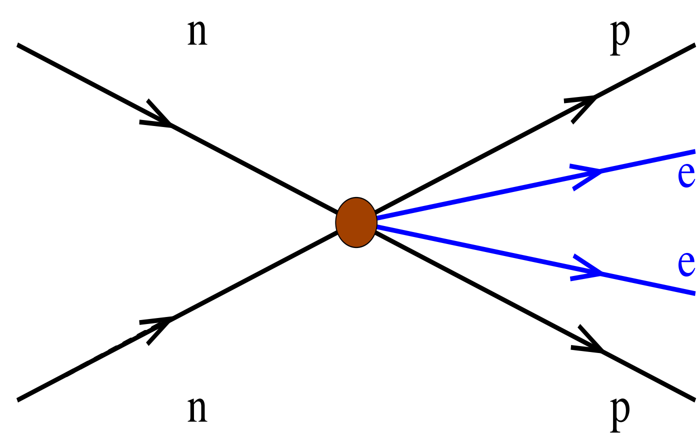
(figure by Giorgio Gratta, WIN 2019 conf., Bari)



Data courtesy of S.Elliott and the PDG

$0\nu\beta\beta$ decay involving light Majorana neutrino





A formal picture of the $0\nu\beta\beta$ decay. Since it is assumed that the exchanged neutrino is light, the corresponding range is long. Neutrino mass here is associated with the **See-saw type I** mechanism and **$m_\nu \sim v^2/M_N$,** where M_N is the very heavy sterile neutrino mass. The $0\nu\beta\beta$ decay rate is proportional to m_ν^2.

$$1/T_{1/2} = G^{0\nu}\ |M^{0\nu}|^2\ |\langle m_\nu \rangle|^2$$

Contact interaction term that, according to Cirigliano et al, based on the effective field theory of quantum chromodynamics, need to be included in the same order. Most numerical calculations thus far do not include this term.

Calculated $M^{0\nu}$ by different methods (color coded)

The spread of the M^{0n} values for each nucleus is ~ 3. On the other hand, there is relatively little variation from one nucleus to the next. (Remember the ``provocative'' paper by Bahcall, Muryama, and Pena-Garay (2003))

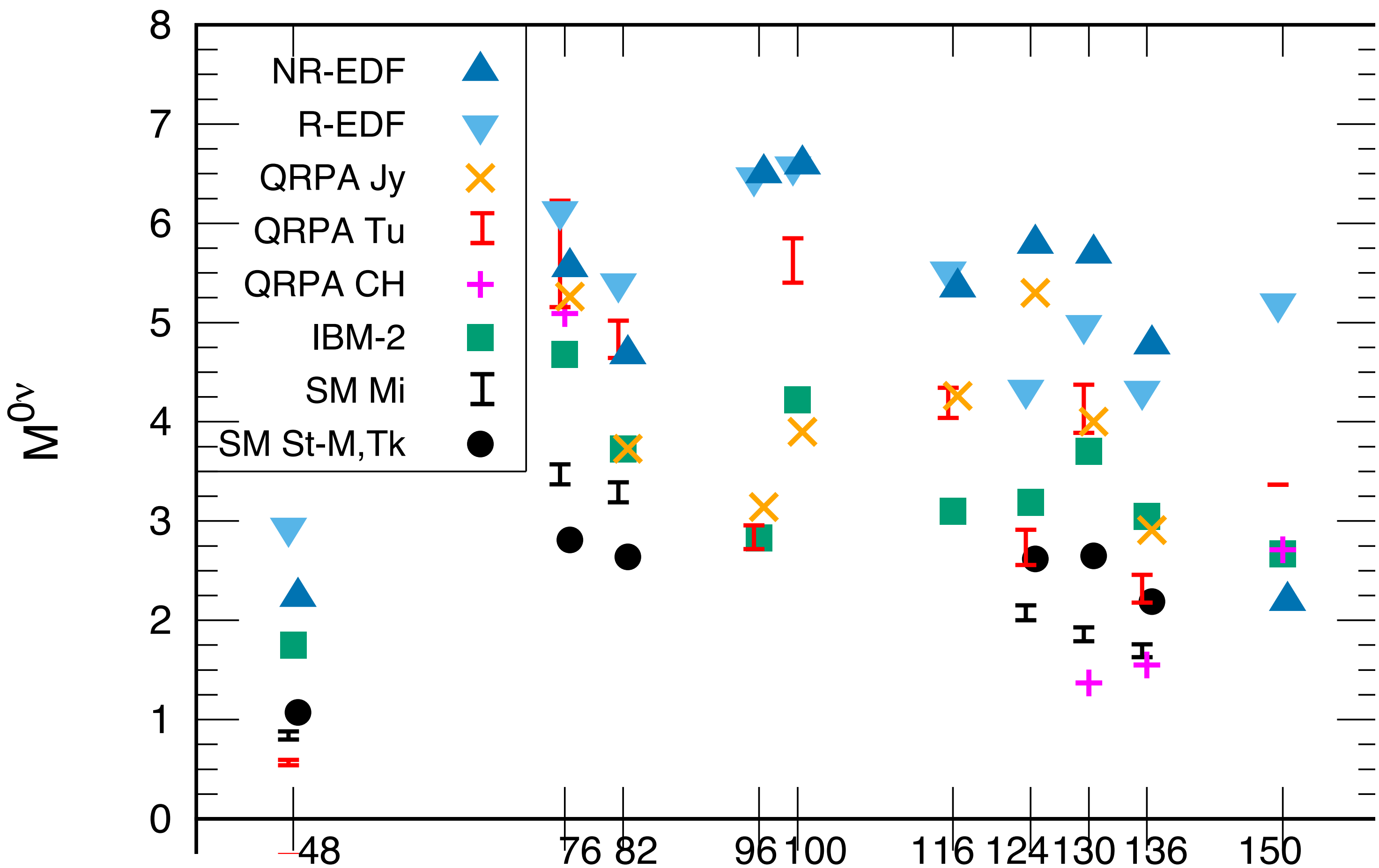


Figure from review by Engel and Menendez (2017)

Quite generally the double beta decay nuclear matrix element consists of three parts:

$$M^{0\nu} = M^{0\nu}_{\rm GT} - \frac{M^{0\nu}_F}{g_A^2} + M^{0\nu}_T \equiv M^{0\nu}_{\rm GT}(1 + \chi_F + \chi_T),$$

The Gamow-Teller part M_{GT} is the dominant one. When treated in the closure approximation it is

$$M^{0\nu}_{\rm GT} = \langle f | \Sigma_{lk} \sigma_l \cdot \sigma_k \tau_l^+ \tau_k^+ H(r_{lk}, \bar{E}) | i \rangle,$$

How does the matrix element $M^{0\nu}{}_{GT}$ depend on the distance between the two neutrons that are transformed into two protons ? This is determined by the function $C^{0\nu}{}_{GT}(r)$

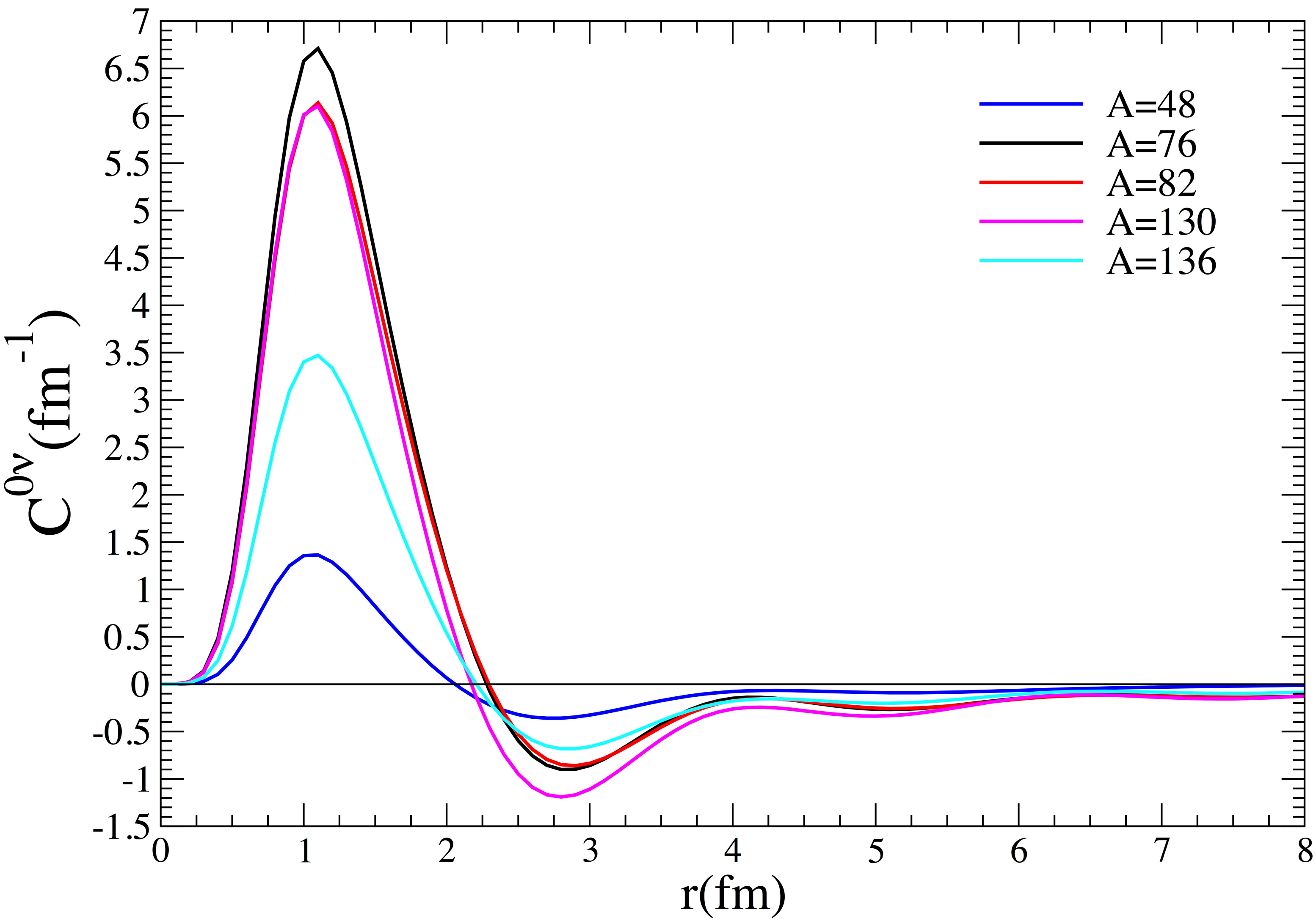
$$C^{0\nu}_{\rm GT}(r) = \langle f | \Sigma_{lk} \sigma_l \cdot \sigma_k \tau_l^+ \tau_k^+ \delta(r - r_{lk}) H(r_{lk}, \bar{E}) | i \rangle,$$

It is normalized by the obvious relation $M^{0\nu}_{\rm GT} = \int_0^\infty C^{0\nu}_{\rm GT}(r)\, dr,$

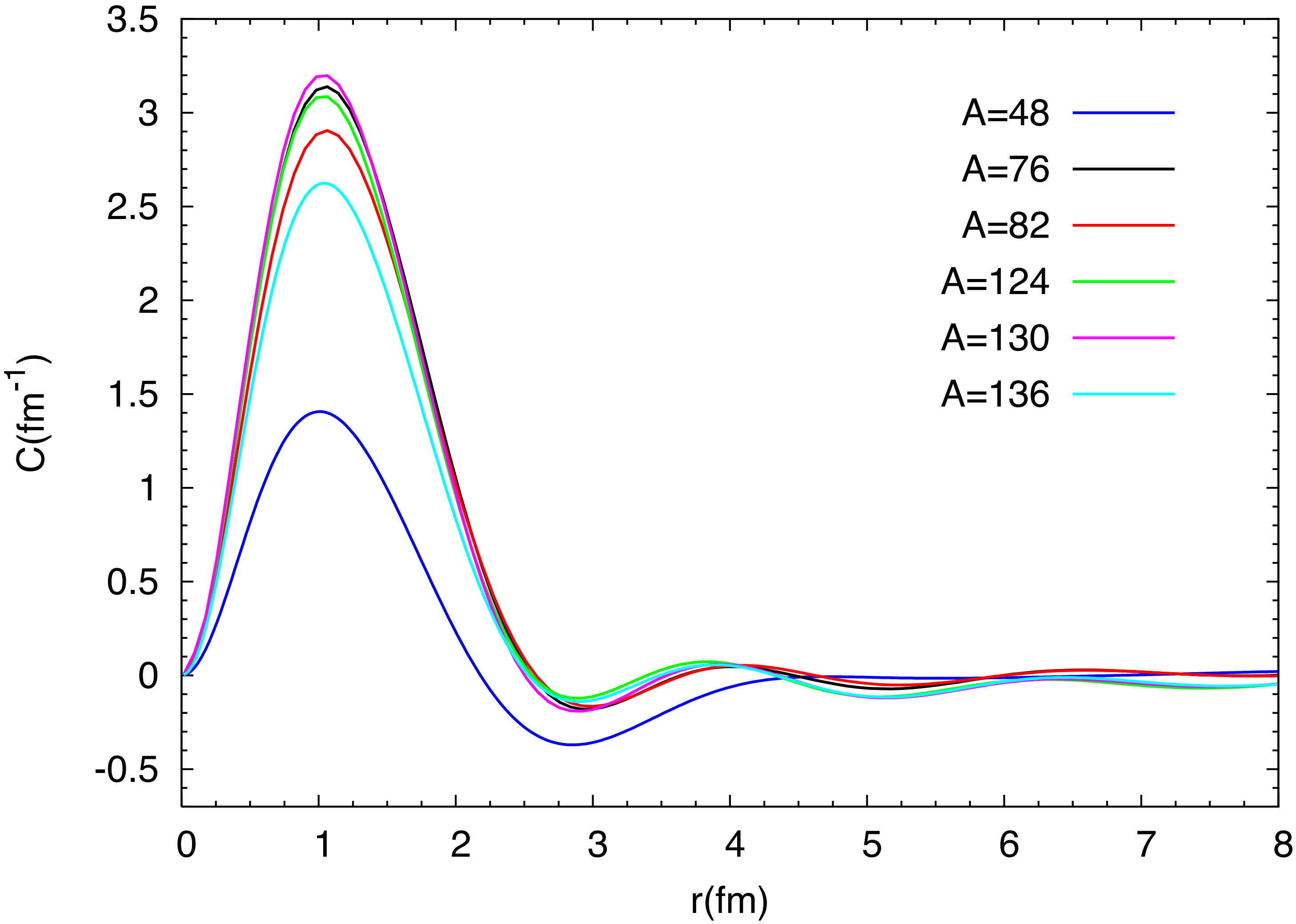
Thus, if we could somehow determine C(r) we could simply obtain $M^{0\nu}$.

Simkovic et al. Phys. Rev. *C77*, 045503 (2008)

Function $C^{0\nu}(r)$ evaluated in QRPA in the ``standard scenario".
Note the peak at ~ 1fm. There is little contribution from r > 2-3 fm.
The functions for different nuclei look very similar, essentially universal.
The magnitude of $M^{0\nu}$ is determined basically by the height of the peak.

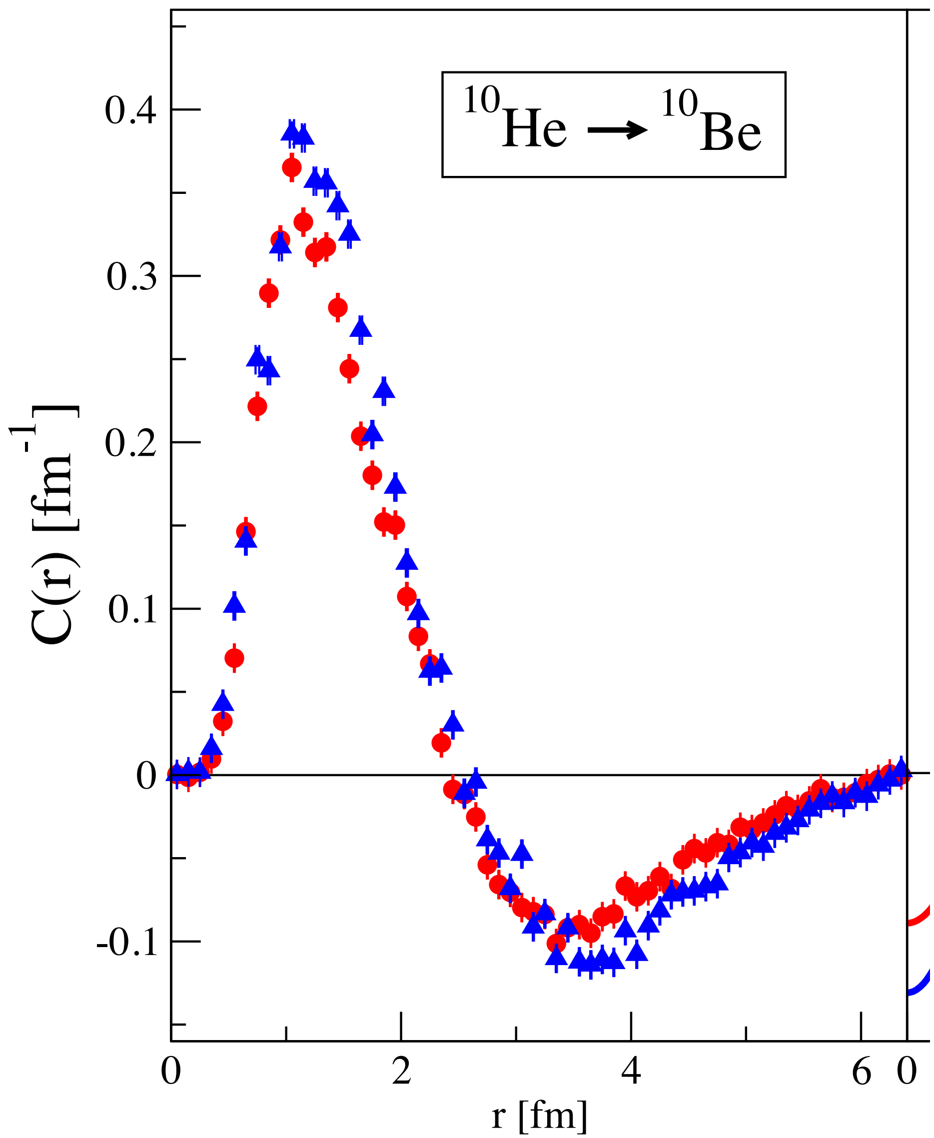


Now C(r) evaluated in the nuclear shell model. All relevant features look the same as in QRPA despite the very different way the equations of motion are formulated and solved.
The peak heights are, naturally, different given the different values of the matrix elements in NSM and QRPA.



From Menendez et al, Nucl. Phys. *A* **818**, 130 (2009)

C(r) for the hypothetical $0\nu\beta\beta$ decay of ^{10}He.



The calculation was performed using the *ab initio* variational Monte-Carlo method. So most of the approximations inherent in NSM or QRPA are avoided. Yet the C(r) function looks, at least qualitatively, very similar to the results shown before.

We can conclude, therefore, that the shape of C(r) is ``universal", independent of the way the nuclear wave functions are evaluated, thus it is very likely ``correct".

Figure from Pastore et al.Phys. Rev. **C97**,014606(2018)

Lets consider once more the GT m.e. for $0\nu\beta\beta$

$$M^{0\nu}_{\rm GT} = \langle f | \Sigma_{lk} \sigma_l \cdot \sigma_k \tau_l^+ \tau_k^+ H(r_{lk}, \bar{E}) | i \rangle,$$

If we remove from the operator the neutrino potential H(r,E) we obtain the matrix element of the double GT operator connecting the ground states of the initial and final nuclei. The same operator would be responsible for the $2\nu\beta\beta$ decay if it would be OK to treat it in the closure approximation. It is also a component of the ``double GT" strength function for the initial nucleus |i>.

$$M^{2\nu}_{\rm cl} \equiv \langle f | \Sigma_{lk} \sigma_l \cdot \sigma_k \tau_l^+ \tau_k^+ | i \rangle,$$

In reality, the closure approximation is not good for the $2\nu\beta\beta$ decay, but we can still consider the corresponding value if we somehow can guess the correct average energy denominator.

The correct expression for $M^{2\nu}$ includes energy denominators

$$M^{2\nu} = \Sigma_m \frac{\langle f || \sigma\tau^+ || m \rangle \langle m || \sigma\tau^+ || i \rangle}{E_m - (M_i + M_f)/2},$$

We can define the radial function $C^{2\nu}{}_{cl}(r)$ for the $M^{2\nu}{}_{cl}$ same way as for the genuine $M^{0\nu}$ matrix element, thus

$$C^{2\nu}_{\mathrm{cl}}(r) = \langle f | \Sigma_{lk} \sigma_l \cdot \sigma_k \delta(r - r_{lk}) \tau_l^+ \tau_k^+ | i \rangle,$$

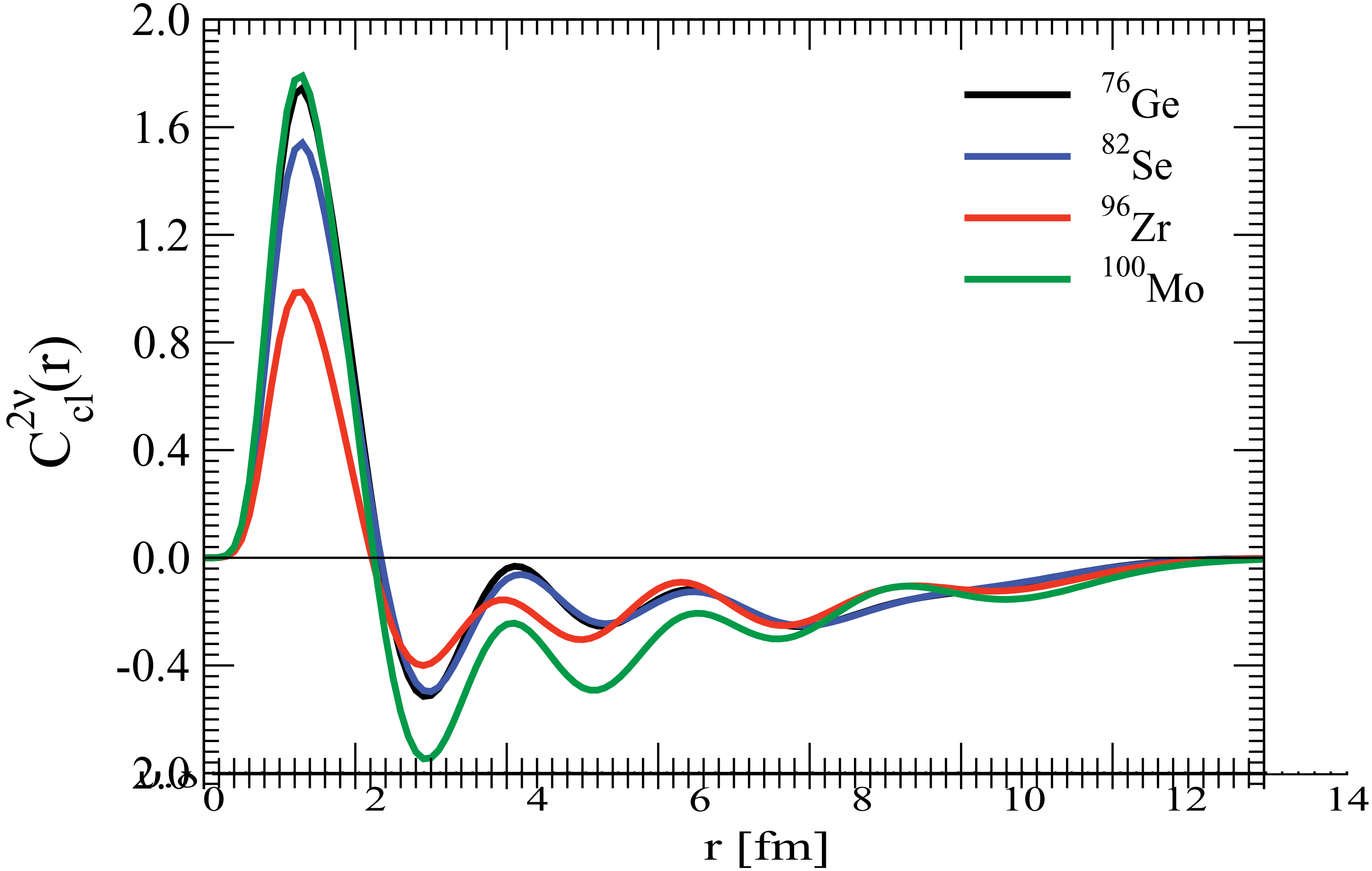
$$M^{2\nu}_{\mathrm{cl}} = \int_0^\infty C^{2\nu}_{\mathrm{cl}}(r)\, dr.$$

It is now clear that, at least formally, the following equality holds:

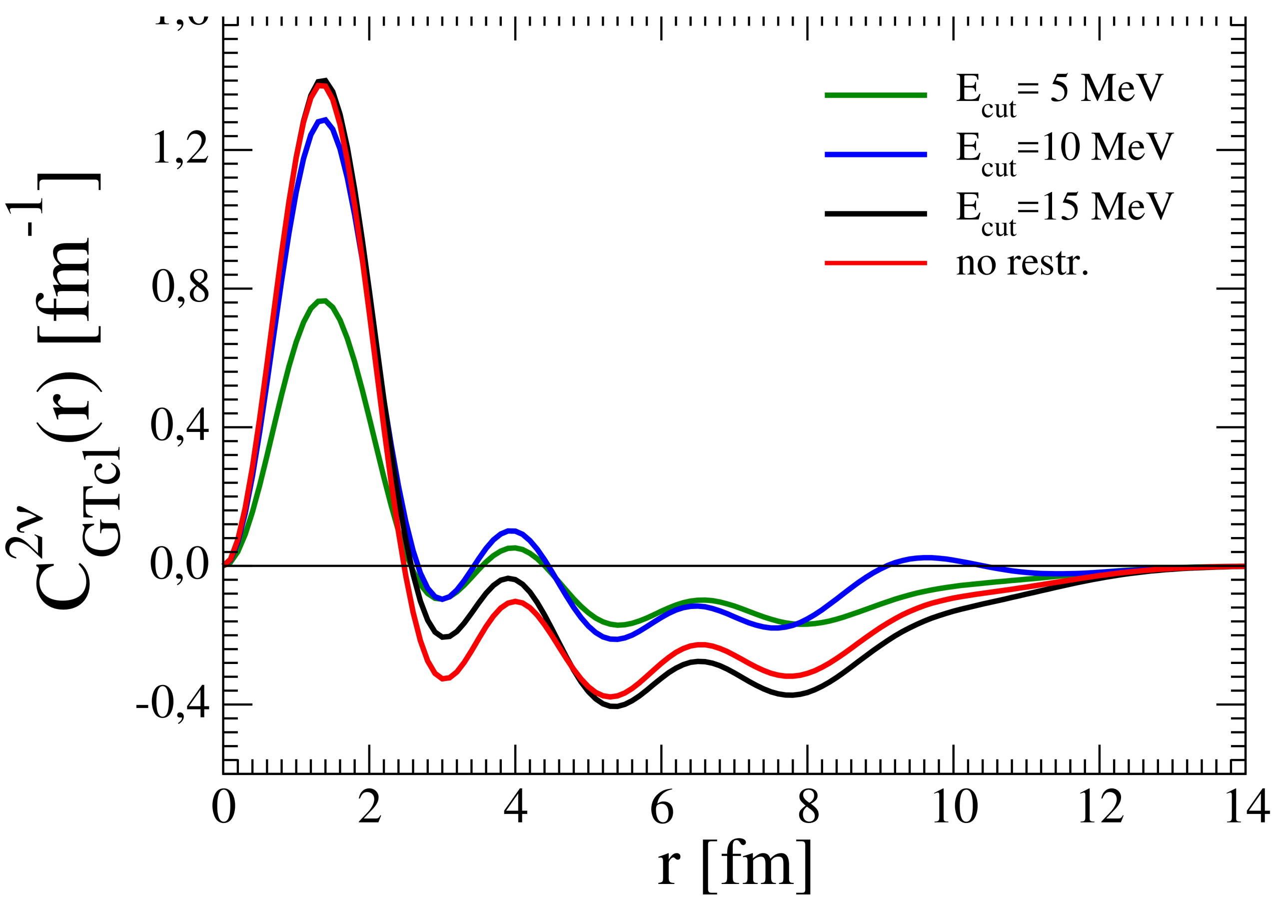
$C^{0\nu}(r) = H(r,E_0)\ C^{2\nu}{}_{cl}(r)$, while $M^{0\nu}_{\mathrm{GT}} = \int_0^\infty C^{0\nu}_{\mathrm{GT}}(r)\, dr,$

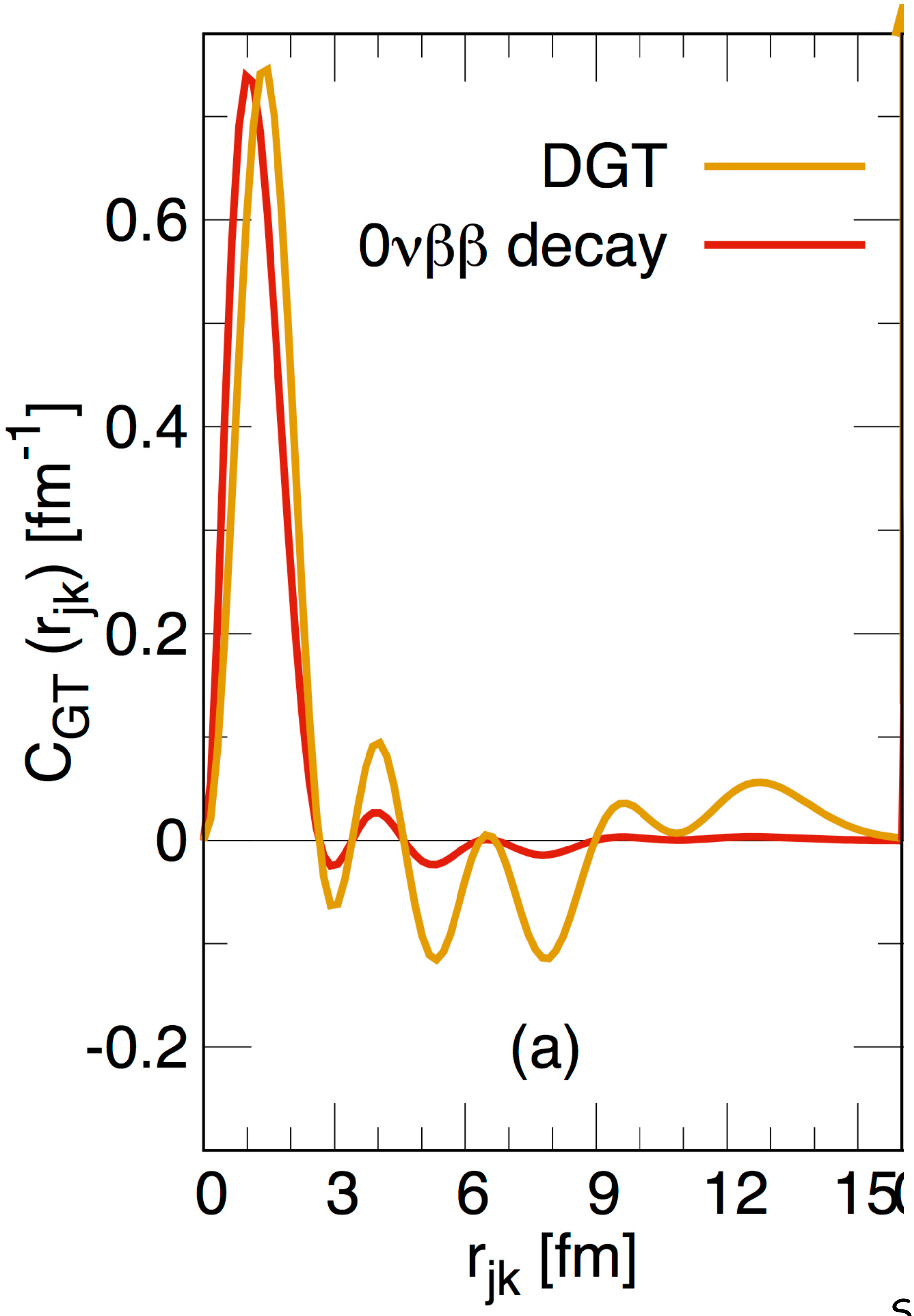
So, if we can somehow determine the function $C^{2\nu}{}_{cl}(r)$ we will be able to determine $C^{0\nu}(r)$ and thus also the ultimate goal, the $M^{0\nu}$. And, moreover, this is so for **any neutrino potential**. Thus, evaluation of $M^{0\nu}$ is reduced to a simple integral, provided any one of the functions $C(r)$ is known. All of such $M^{0\nu}{}_i$ are then consistent and easily evaluated.

Here are the functions $C^{2\nu}{}_{cl}(r)$ evaluated with QRPA for several nuclei. The peak at small r is essentially compensated by the substantial tail at larger r, thus $M^{2\nu}{}_{GTcl}$ is very small. Besides, the $C^{2\nu}{}_{cl}(r)$ depends strongly on the nuclear parameters used, thus it is rather uncertain, particularly its tail at r > 2fm.



When evaluating $M^{2\nu}_{cl}$, and the function $C^{2\nu}_{cl}(r)$ it is crucial to include all intermediate states. The depth of the tail, and hence the magnitude of the $M^{2\nu}_{cl}$ sensitively depends on the possible energy cutoff.
(The figure is for the ^{76}Ge decay, evaluated in QRPA)





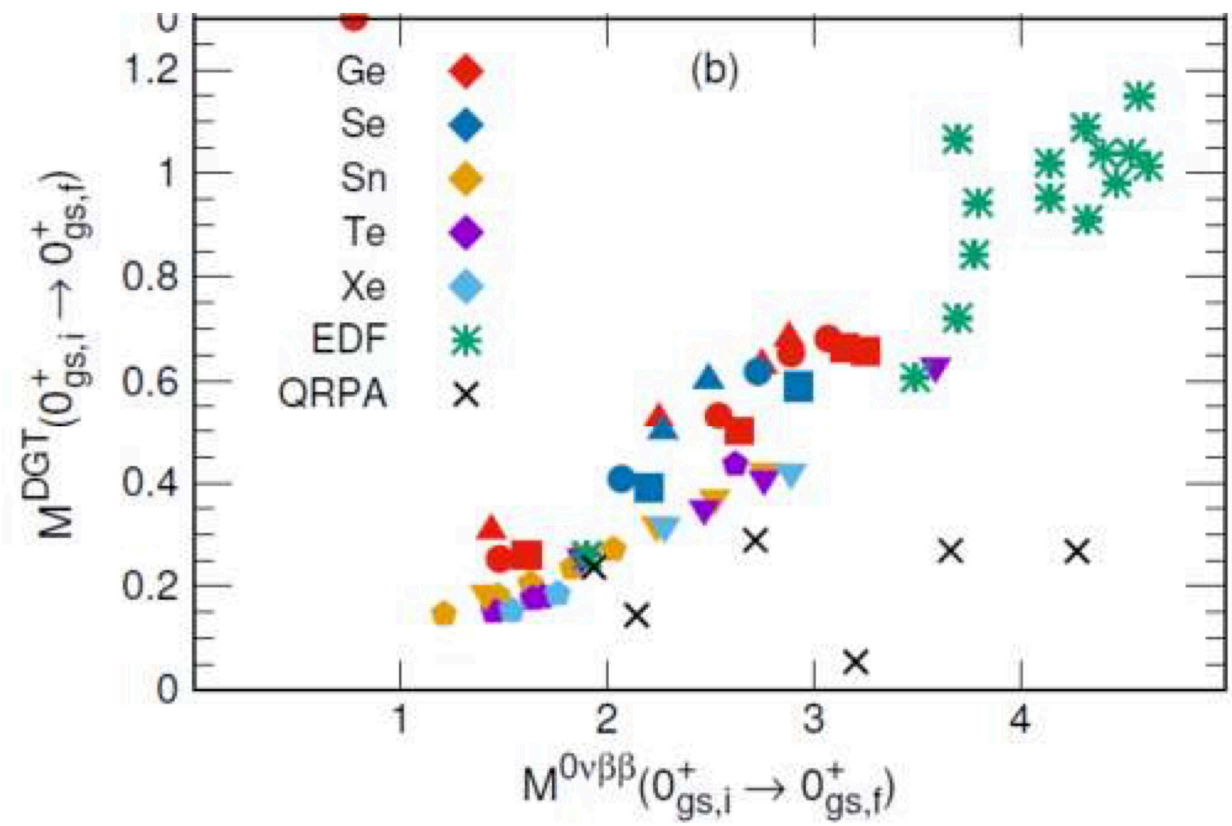
For comparison, the C(r) function for ^{136}Xe evaluated in the NSM by Shimizu et al. The yellow line corresponds to the $C^{2\nu}{}_{cl}(r)$. It is somewhat similar to the corresponding QRPA curve. However, differences are be expected due to the absence of the giant GT state in the NSM in this case.

In particular, the area under the tail at r > 2 fm is less and does not compensate for the peak area.

Shimizu et al, Phys. Rev. Lett. **120**, 142502(2018)

There is a fundamental difference between the $M^{2\nu}_{cl}$ evaluated in the NSM and QRPA. The NSM results are substantially larger then the QRPA ones. The figure is from Shimizu et al, Phys. Rev. Lett. **120**, 142502(2018), we believe now, see Simkovic et al., Phys. Rev. C**98**, 064325 (2018), that the ``natural " value of $M^{2\nu}_{cl}$ should be $M^{2\nu}_{cl}$ = 0.

So, who is right?



$^{74-82}$Ge, ^{74,76}Se, $^{124-132}$Sn, $^{128-130}$Te, ^{134,136}Xe

SM: shell model
GCN2850, jj44b,
JUN45,
GCN5082,QX

EDF: Gogny+GCM
Rodriguez *et al.*,
PLB719 174 (2013)

QRPA: AV18+G-matrix
F. Simkovic *et al.*,
PRC83, 015502 (2011).

The assumption that the ``natural" value of $M^{2\nu}{}_{cl}$ = 0 is based on expressing the matrix element in the LS coupling scheme.

In that case the closure Fermi and GT matrix element are related, and so are the corresponding C(r) functions:

$M^{2\nu}{}_{GT,\,S=0} = -3\, M^{2\nu}{}_{F,\,S=0}$, and $M^{2\nu}{}_{GT,\,S=1} = M^{2\nu}{}_{F,\,S=1}$

Our numerical evaluation in QRPA suggests that $M^{2\nu}{}_{GT,\,S=1}$ and $M^{2\nu}{}_{F,\,S=1}$ are not only very small by themselves but, that the corresponding $C(r)^{S=1}$ fimctions are negligibly small at all r values.

Since $M^{2\nu}{}_{F}$ must vanish if isospin is conserved, $M^{2\nu}{}_{F,\,S=0}$ must also vanish provided $M^{2\nu}{}_{F,\,S=1}$ is negligible. Hence $M^{2\nu}{}_{GT,\,cl}$ should vanish as well.

That requirement represents partial restoration of the SU(4) symmetry just as $M^{2\nu}{}_{F,cl}$ = 0 is following from the isospin symmetry restoration.

$M^{2\nu}_{GT}$ and $M^{2\nu}_{GT-cl}$ evaluated in QRPA as functions of the excitation energy

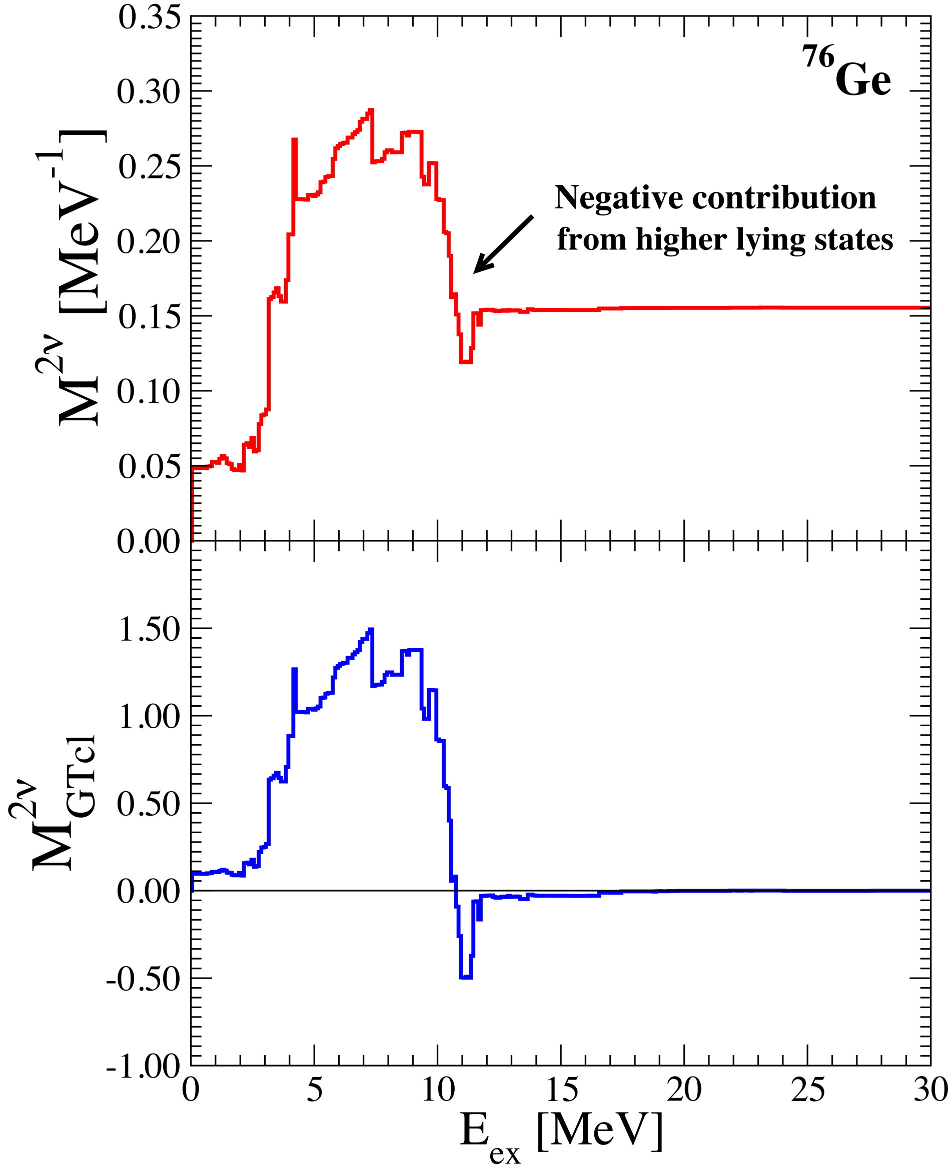
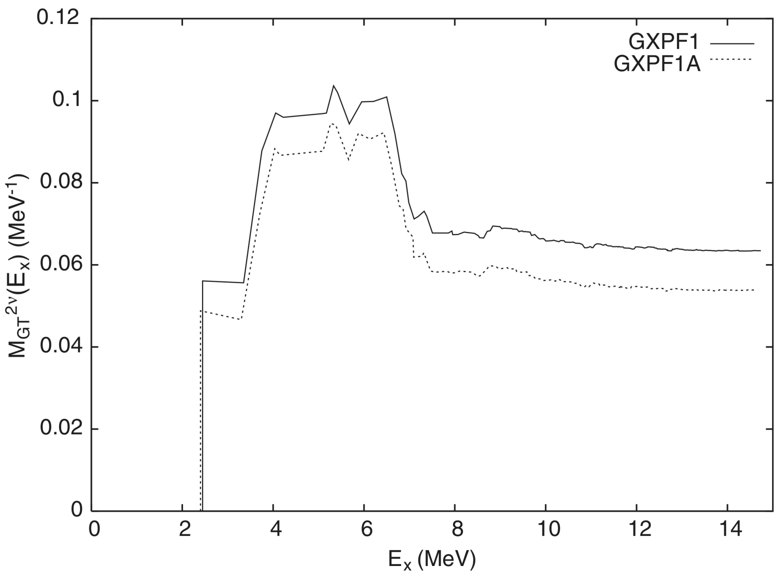
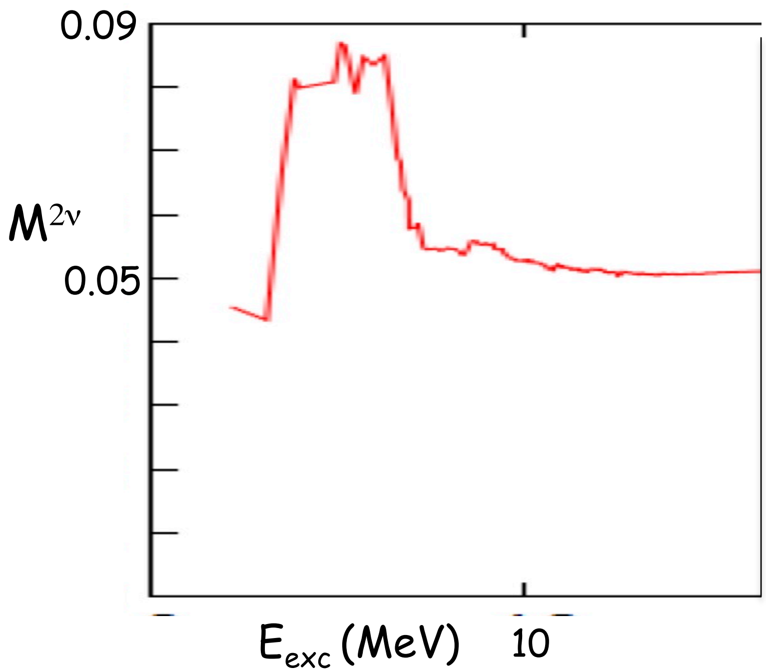


Illustration of the difficulties. In the upper panel are the contributions to the $M^{2\nu}$ from states up to E. Even though the correct value is reached (by design), it is also crossed at lower energies, followed by a drop at ~ 10 MeV.

In the lower panel the same calculation is done for $M^{2\nu}_{cl}$. In this case the high energy drop is much larger because it is not reduced by the energy denominator present in the true $M^{2\nu}$.

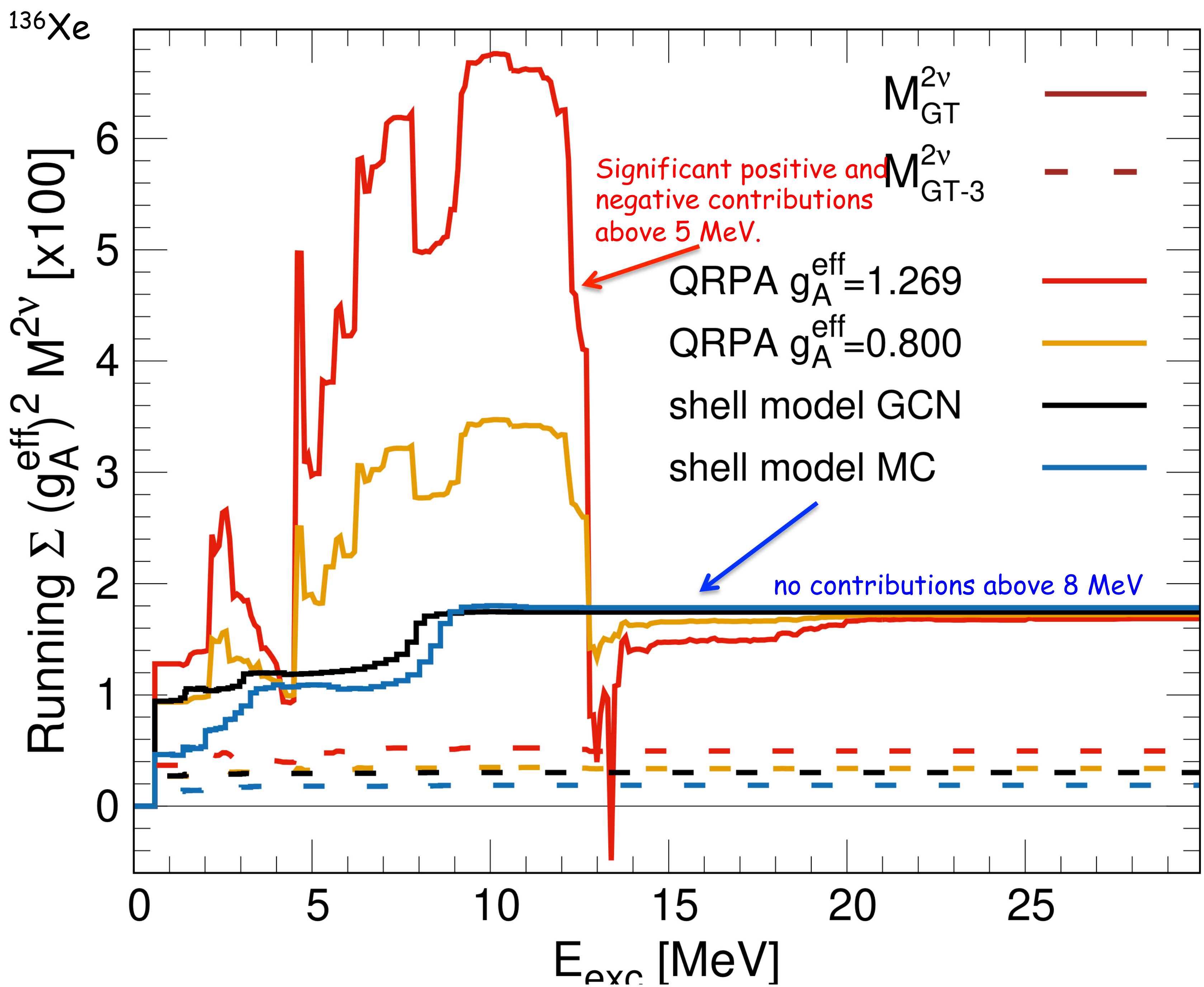
While the states up to ~5 MeV can be studied experimentally, the ~ 10 MeV can not. It is not clear whether they exist or not.





This feature, i.e. first an increase of $M^{2\nu}$ followed by decrease at higher energies appears to be present in other nuclear models as well. Here are the shell model results for $M^{2\nu}$ in ^{48}Ca (upper panel, Horoi et al, Phys. Rev.C75,034303(2007)).

And lower panel, Kontensalo and Suhonen, 1908.07911.The drop at ~ 10 MeV is again clearly visible. It is not clear to what extend the $M^{2\nu}_{cl}$ is suppressed. The inherent uncertainty in $M^{2\nu}_{cl}$ is substantial.

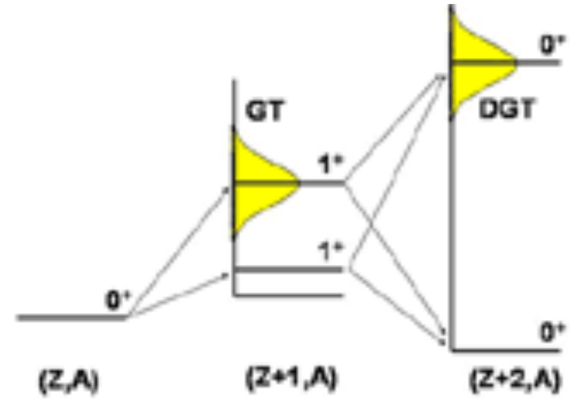


136Xe
Running Σ (gA eff)2 M2ν [x100]
Eexc [MeV]
M2ν GT
M2ν GT-3
QRPA gA eff=1.269
QRPA gA eff=0.800
shell model GCN
shell model MC
Significant positive and negative contributions above 5 MeV.
no contributions above 8 MeV
0
1
2
3
4
5
6
0
5
10
15
20
25

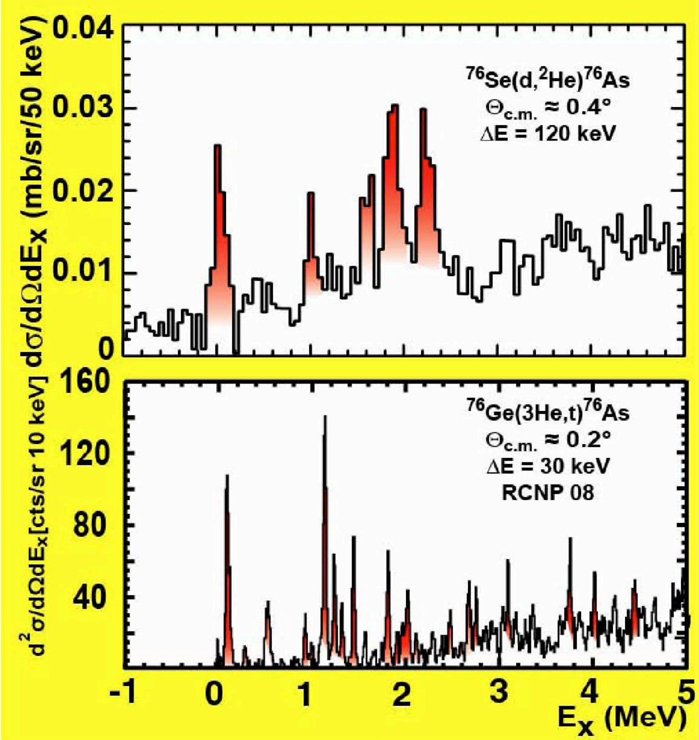
$M^{2\nu}_{GT}$ and $M^{2\nu}_{GT-cl}$ can be, in principle, experimentally determined

Cross sections of (t,^{3}He) and (d,^{2}He) reactions give B(GT$^{\pm}$) for β^+ and β^-; products of the amplitudes (B(GT)$^{1/2}$) entering the numerator of $M^{2\nu}_{GT}$

$$M^{2\nu}_{GT} = \sum_m \frac{M^{(+)}_{GT}(m)\ M^{(-)}_{GT}(m)}{Q_{\beta\beta}/2 + m_e + E_x(1^+_m) - E_0}$$



The β^- strength is dominated by the giant GT resonance. However, the β^+ strength is concentrated at low energy, little (but unknown) strength to the giant.



2νββ-matrix element

0.16 ± 0.04 MeV^{-1}

with

G$^{(2\nu)}$ = 3.4x 10^{-20} MeV^2 a^{-1}

2νββ - half-life

(1.1 ± 0.2) x 10^{21} a

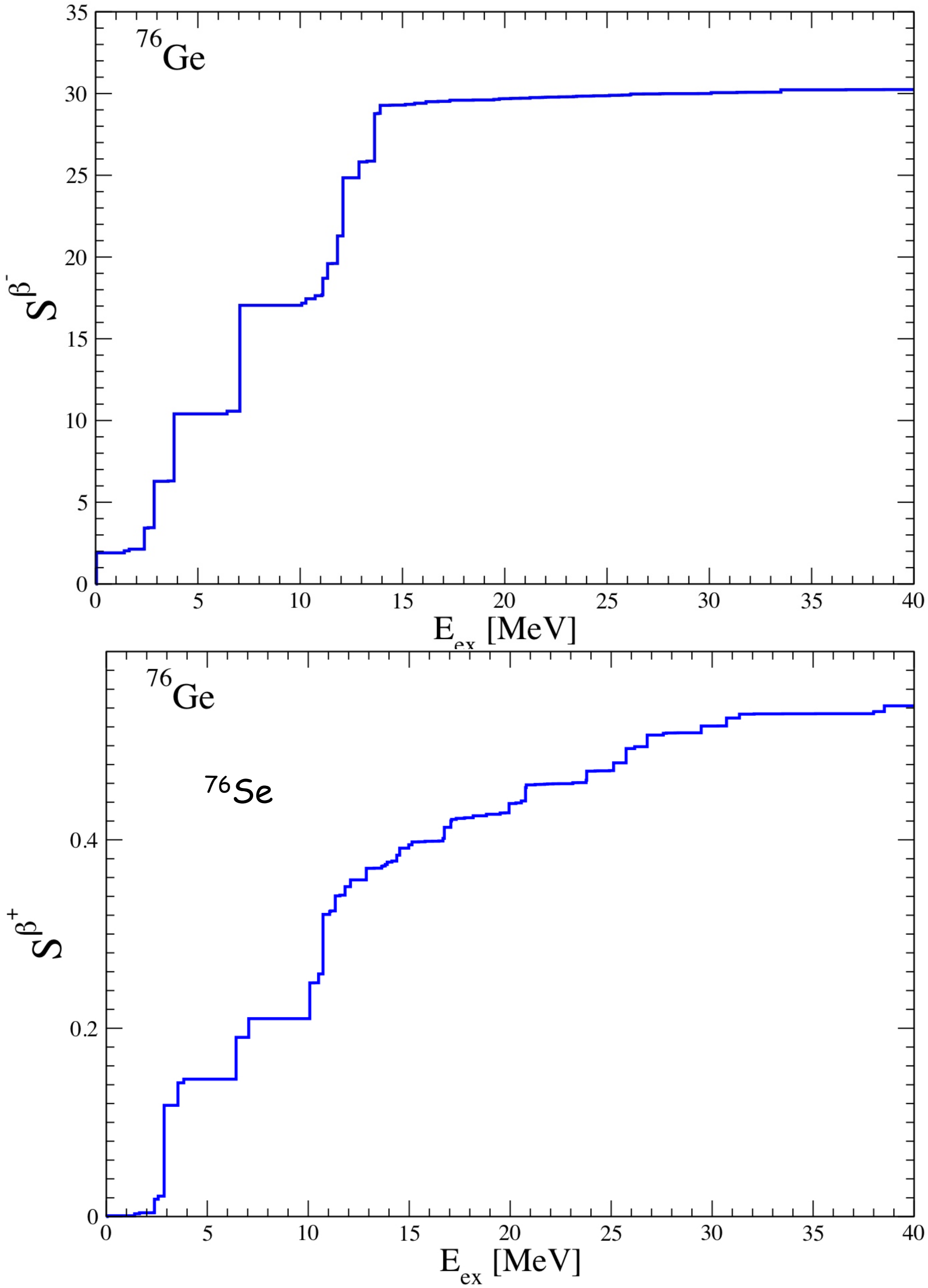
recommended. exp. value:

(1.5 ± 0.1) x 10^{21} a

Closure 2νββ-decay NME

$$M^{2\nu}_{GT-cl} = \sum_m M^{(+)}_{GT}(m)\ M^{(-)}_{GT}(m)$$

Grewe, …Frekers at al, PRC 78, 044301 (2008)



The β^- and β^+ strength function calculated in SRQRPA. Note the different scales in the two panels. In the β^- case one can Clearly see the giant GT state. Also,the strength saturates at ~15 MeV.

On the other hand, the much smaller β^+ strength, unlike the usual claims, gets also a substantial contribution from relatively high excitation energies.

Whether this high-lying β^+ strength exists or not is the crucial question.

Is there a way to test whether the sum in $M^{2\nu}$ is saturated at E_m ~5 MeV, where is experimental value of $M^{2\nu}$ is usually first reached, or whether it contains significant positive and negative contributions at ~ 10 MeV?

$$M^{2\nu} = \Sigma_m \frac{\langle f||\sigma\tau^+||m\rangle\langle m||\sigma\tau^+||i\rangle}{E_m - (M_i + M_f)/2}$$

This can be, perhaps, achieved by considering in detail the two and single electron spectra of the $2\nu\beta\beta$ decay.

Testing the convergence with respect of the intermediate nucleus 1^+ spectrum of the $2\nu\beta\beta$ matrix elements :
(see Simkovic et al, Phys. Rev. C97, 034315 (2018))

The $M^{2\nu}$ in fact depends on the electron and neutrino energies

$M_{GT}^{K,L} = \Sigma_m M_m (E_m - (E_i+E_f)/2)/[(E_m - (E_i+E_f)/2)^2 - \varepsilon_{K,L}^2]$

with the numerators $M_m = \langle f||\sigma\tau^+||m\rangle\langle m||\sigma\tau^+||i\rangle$

Here $\varepsilon_{K,L}$ depend on the final electron and neutrino energies:

$2\varepsilon_K = E_{e2} + E_{\nu 2} - E_{e1} - E_{\nu 1}$, $2\varepsilon_L = E_{e1} + E_{\nu 2} - E_{e2} - E_{\nu 1}$

The standard (quite good) approximation is to take $\varepsilon_K = \varepsilon_L = 0$.

With this approximation the rate depends separately on the phase space integral and on the nuclear matrix element.

Lets, instead, expand in $\varepsilon_{K,L}/(E_m - (E_i+E_f)/2) < 1$, and keeping just the first order term. There are now two matrix elements M_1,M_3 and two phase space integrals.

M_1 has the standard energy denominator $(E_m - (E_i+E_f)/2)$

$M_1 = \Sigma_m \langle f|| \sigma\tau^+ ||m\rangle\langle m||\sigma\tau^+ ||i\rangle/(E_m - (E_i+E_f)/2)$

while M_3 has its third power $(E_n - (E_i+E_f)/2)^3$, so it converges much faster as a function of E_n.

$M_3 = \Sigma_m \langle f|| \sigma\tau^+ ||m\rangle\langle m||\sigma\tau^+ ||i\rangle/(E_m - (E_i+E_f)/2)^3$

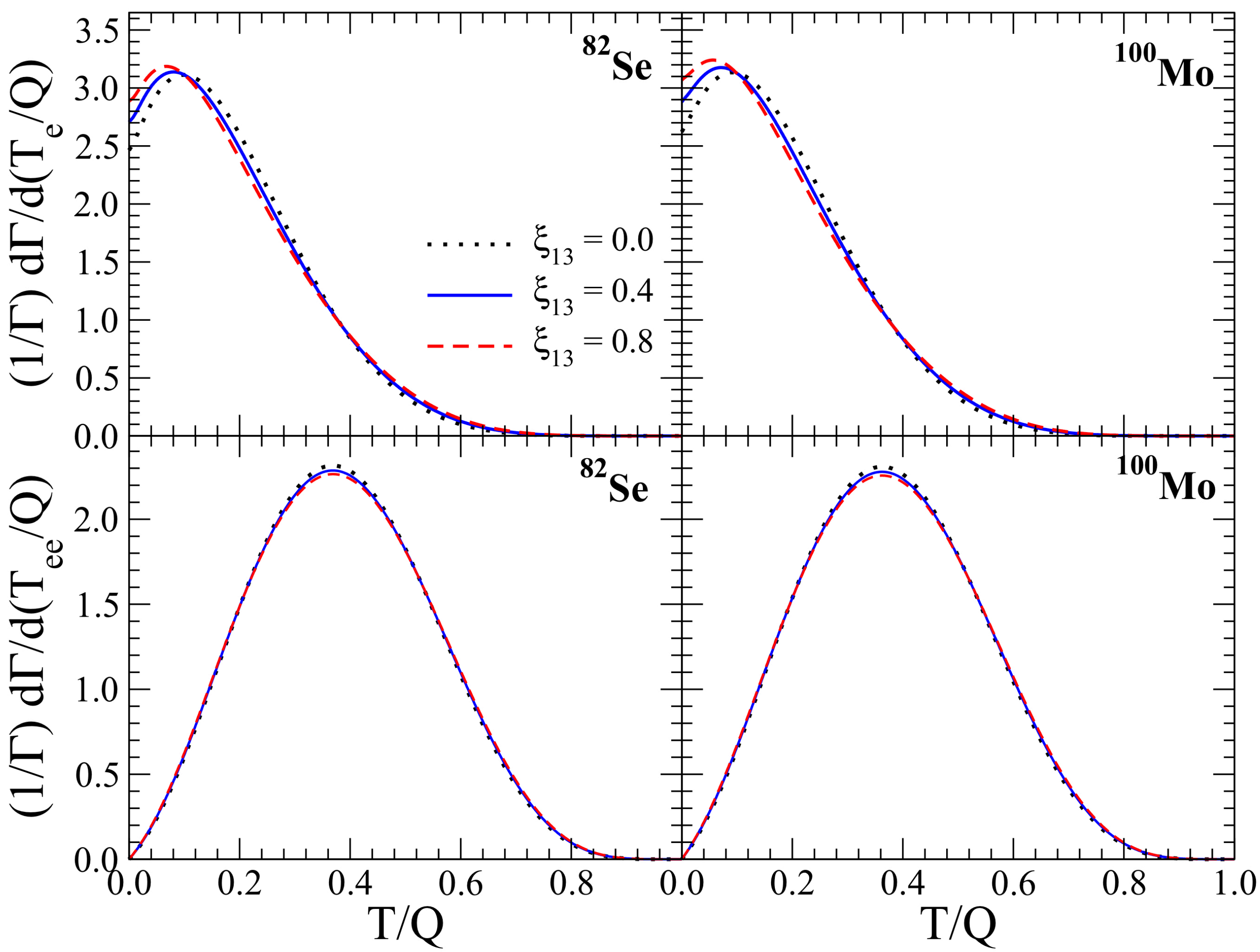
The single and full electron spectra depend (slightly) on the dimensionless ratio $\xi^{2\nu}_{31} = 4m_e^2\ M_3/M_1$. If $\xi^{2\nu}_{31}$ could be determined experimentally it would tell us how fast the sum over ``m" converges.

The halflife $T_{1/2}$ is now, with G_0 and G_2 phase-space factors,

$1/T_{1./2}^{2\nu} = g_A^4\ (M^{2\nu})^2\ (G_0 + \xi^{2\nu}_{31}\ G_2)$

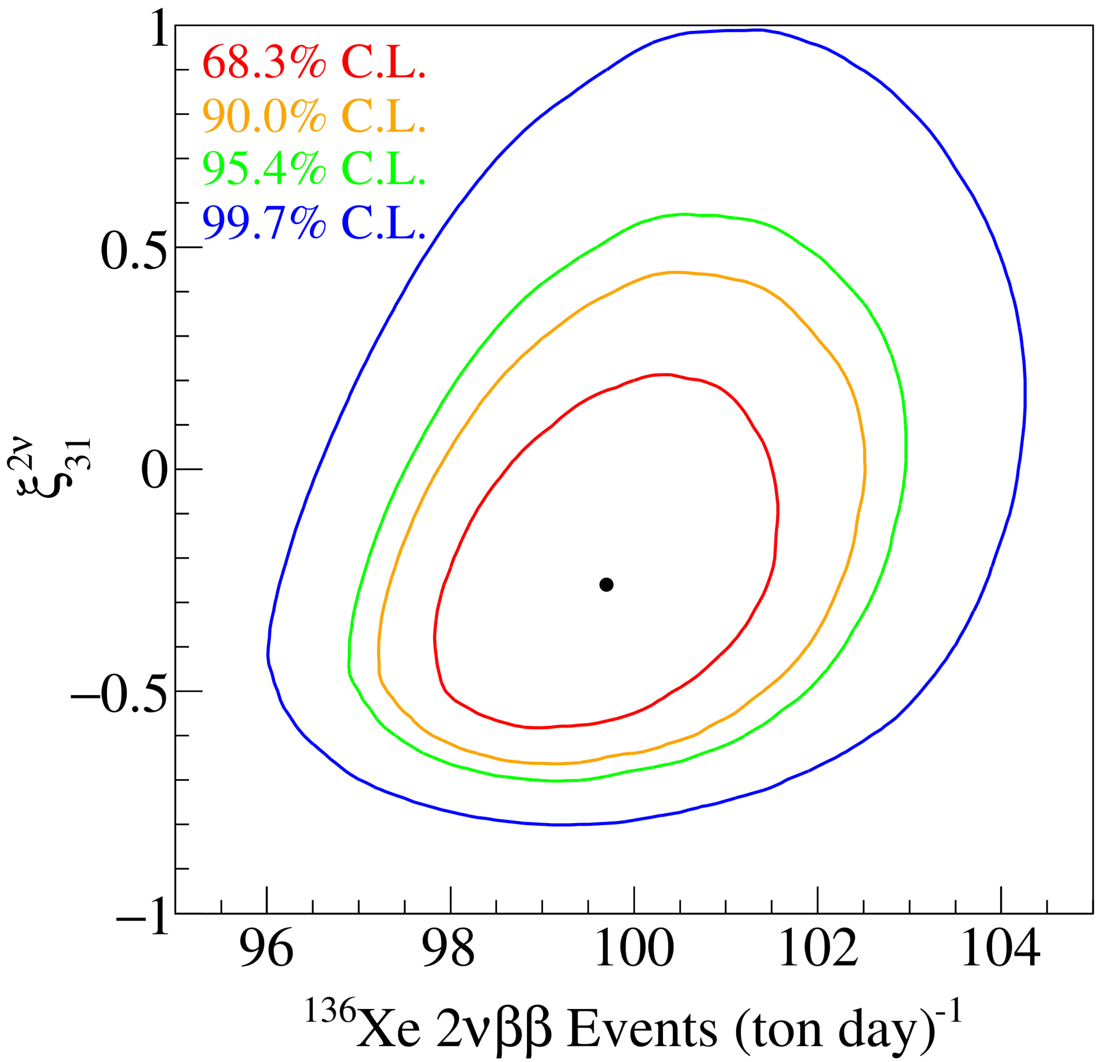
Where the second term represents a small (a few %) correction.

Illustration of the (tine) effects of $\xi^{2\nu}{}_{31}$ on the single electron (upper panels) and two electron spectra (lower panels). The shape of the effect depends only on the phase space integrals, not on the M_3 and M_1 matrix element.

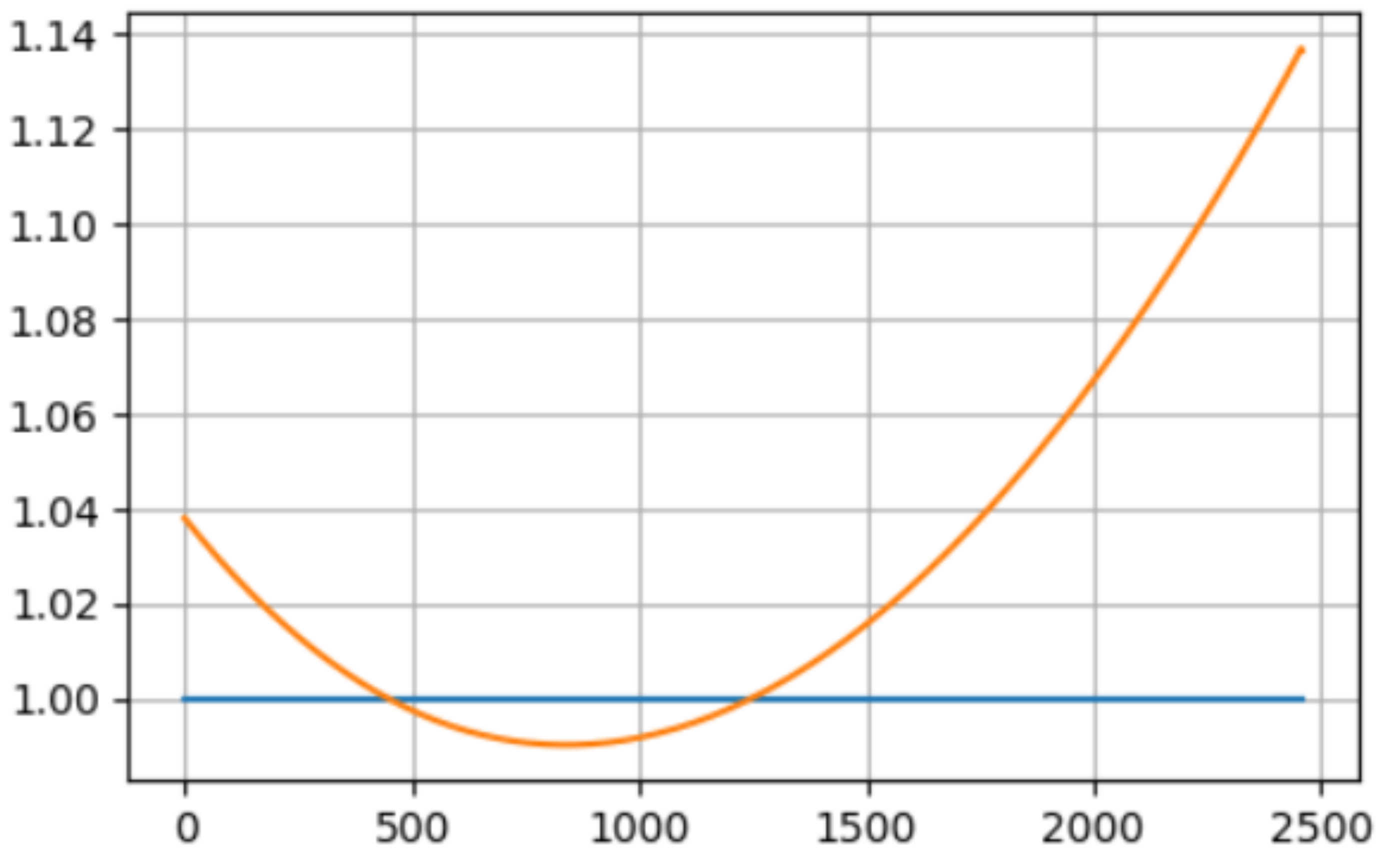


The spectrum with $\xi^{2\nu}{}_{31} > 0$ is slightly wider than the spectrum with $\xi^{2\nu}{}_{31} = 0$

$\xi^{2\nu}_{31}$ has been constrained from the $2\nu\beta\beta$ two-electron spectrum of ^{136}Xe in the KamLAND-Zen experiment (Gando et al,1901.03871, Phys.Rev.Lett. **122**, 192501 (2019).) The fit gives $\xi^{2\nu}_{31} = -0.26^{+0.31}_{-0.26}$ that agrees with both NSM and QRPA, but the error bars are a bit large to make definitive conclusions.



The effect of ξ_{31} = 0.4 on the two electron spectrum for ^{136}Xe. These curves scale with ξ_{31}. The deviation from unity become correspondingly smaller for smaller $|\xi_{31}|$. To reach sensitivity to ξ_{31} = 0.1 it would be necessary to understand the $2\nu\beta\beta$ spectrum to better than 1%.



Larger deviations appear at the low and high energy edges of the spectrum, where however, there are only few counts.

It appears that NEMO (^{100}Mo), CUORE (^{130}Te) and EXO200(^{136}Xe) are also trying to determine $\xi^{2\nu}_{31}$ from their data. The hope is that the uncertainty in $\xi^{2\nu}_{31}$ can be reduced to about +- 0.1.

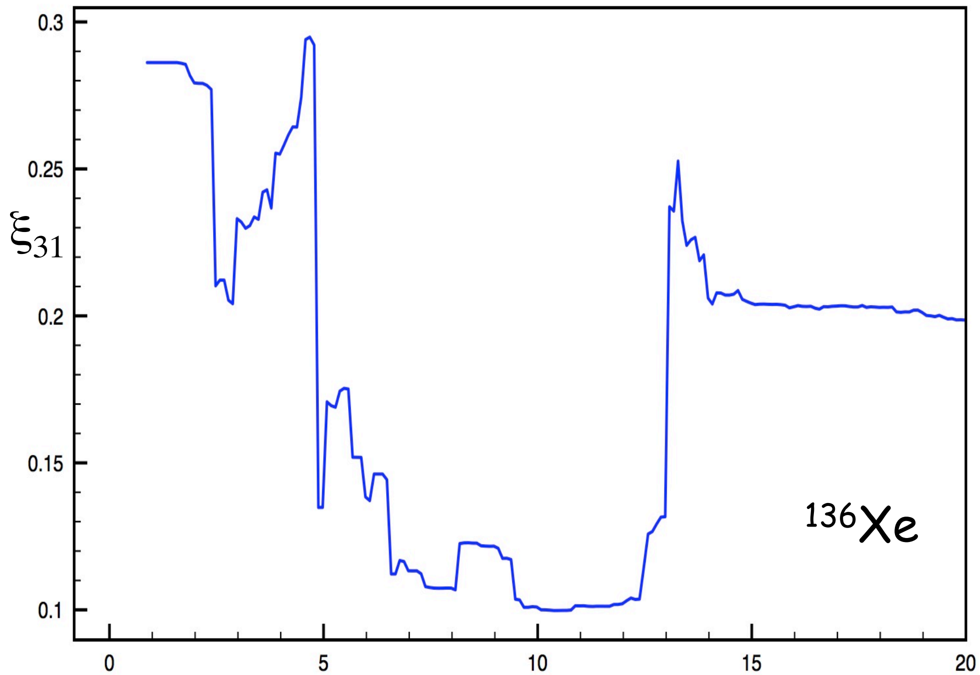
M_3 depends only on the low-lying 1^+ states., while as I argued above M_1 depends to some extend also on the higher lying states. Lets separate in M_1 the low and higher lying states, $M_1 = M_1^{low} + M_1^{high}$. Calculations, both QRPA and Shell Model, suggest that $M_1^{high} >> M_1^{low.}$

If only one (or several close states adding with the same sign) contribute, than $\xi^{2\nu}_{31} = 4m_e^2/\Delta E^2$ is positive $0 < \xi^{2\nu}_{31} < 1$ and $M_1^{high} = 0$.

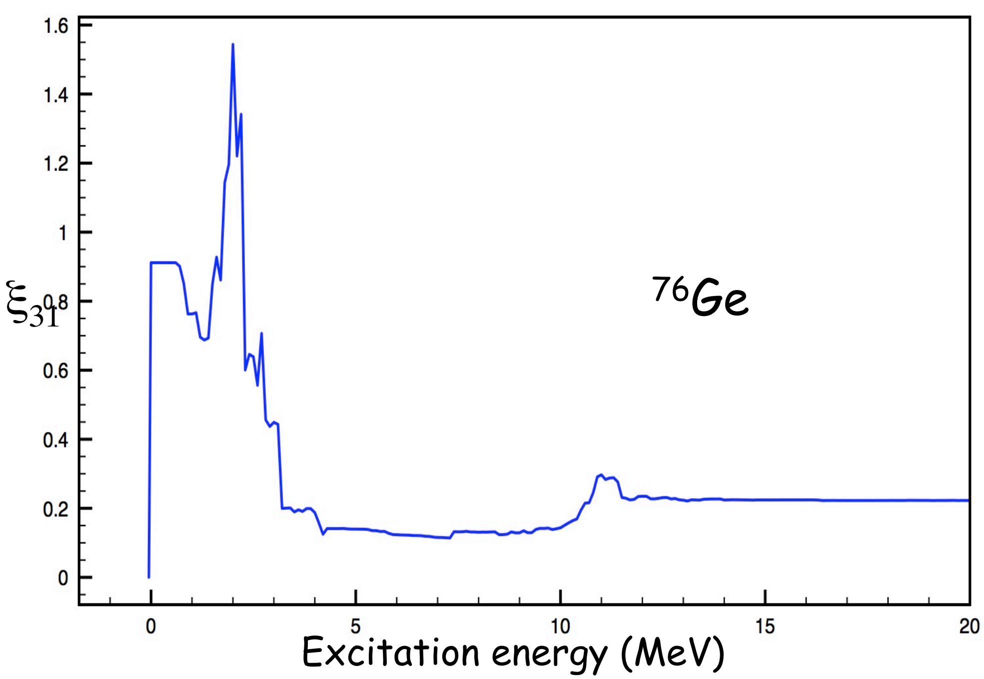
Generally, $\xi^{2\nu}_{31} \sim 4m_e^2/\Delta E^2 (1 - M_1^{high}/M_1^{low})$ which is positive and less than unity.

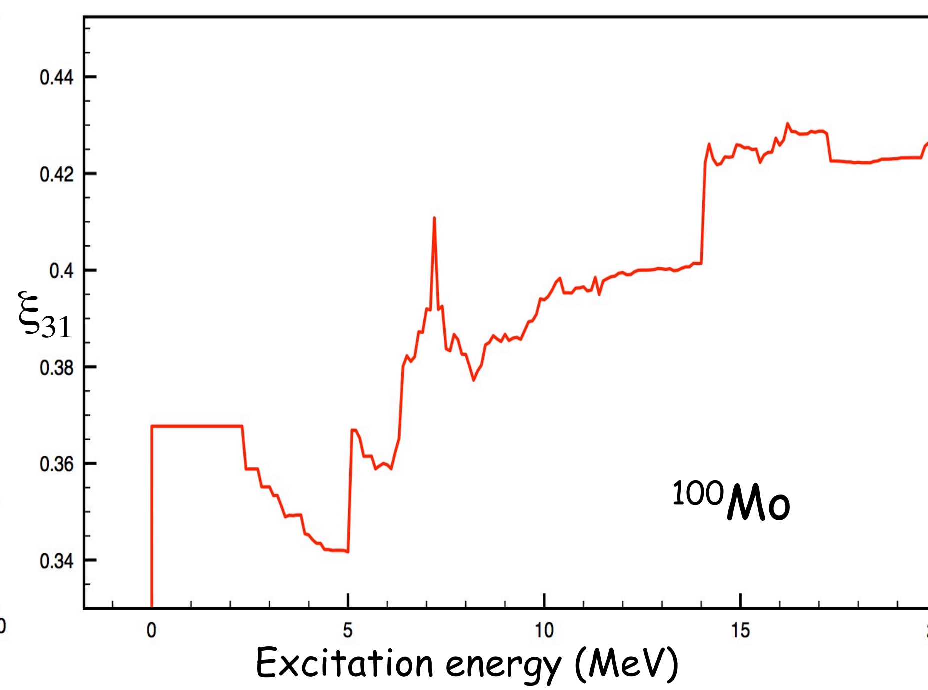
Conclusions:

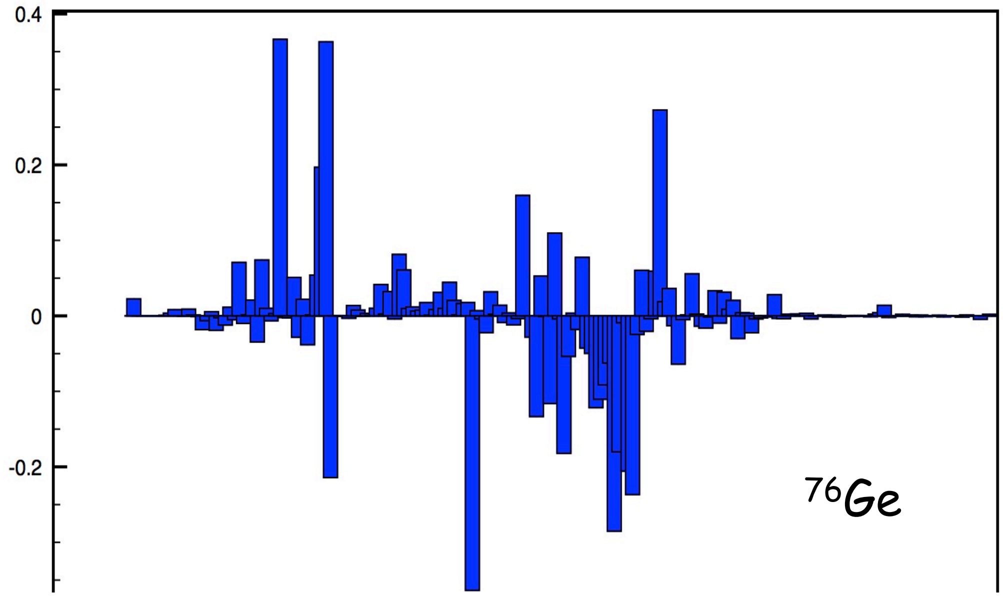
1) Determination of the magnitude of $M^{2\nu}_{cl}$ is important, but challenging
2) The issue is whether the virtual intermediate states at 5-10 MeV contribute (or not) to the $M^{2\nu}_{cl}$ and $M^{2\nu}$. The QRPA and Nuclear Shell Model disagree on that feature of the $M^{2\nu}_{cl}$.
3) It is suggested that a detailed determination of the shape of two and single electron spectra of $2\nu\beta\beta$ decay, interesting by itself, might help in resolving the problem.



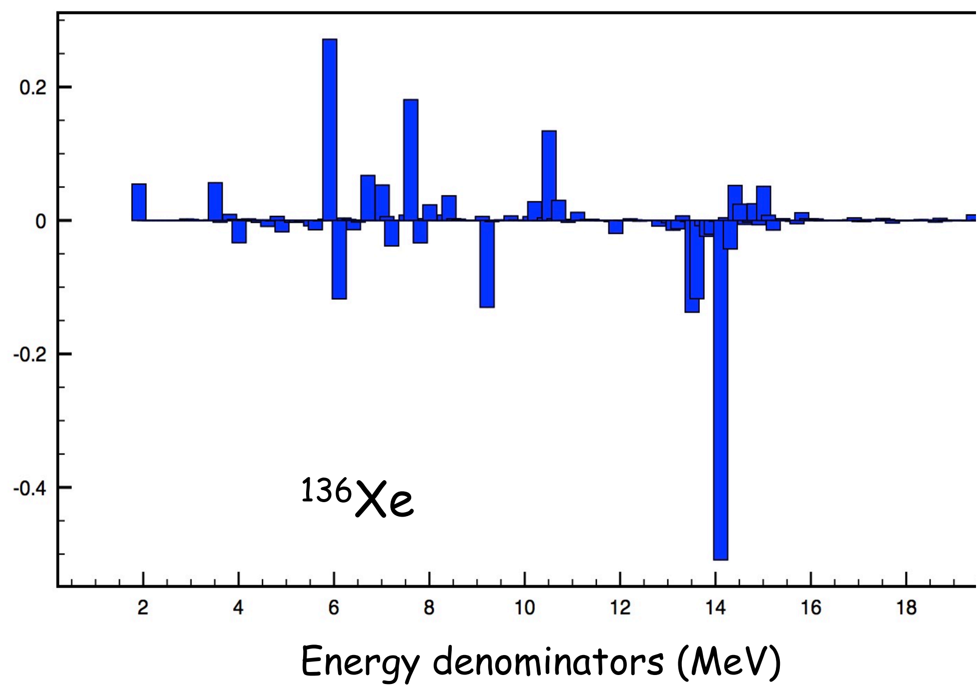
Calculated ξ_{31} values (in QRPA) as a function of the excitation energy. Clearly, the asymptotic values and the values at cut-off energy at, say 5 MeV, differ by more than 10%. Thus, 10% exp. determination of ξ_{31} would be able to decide who is right.

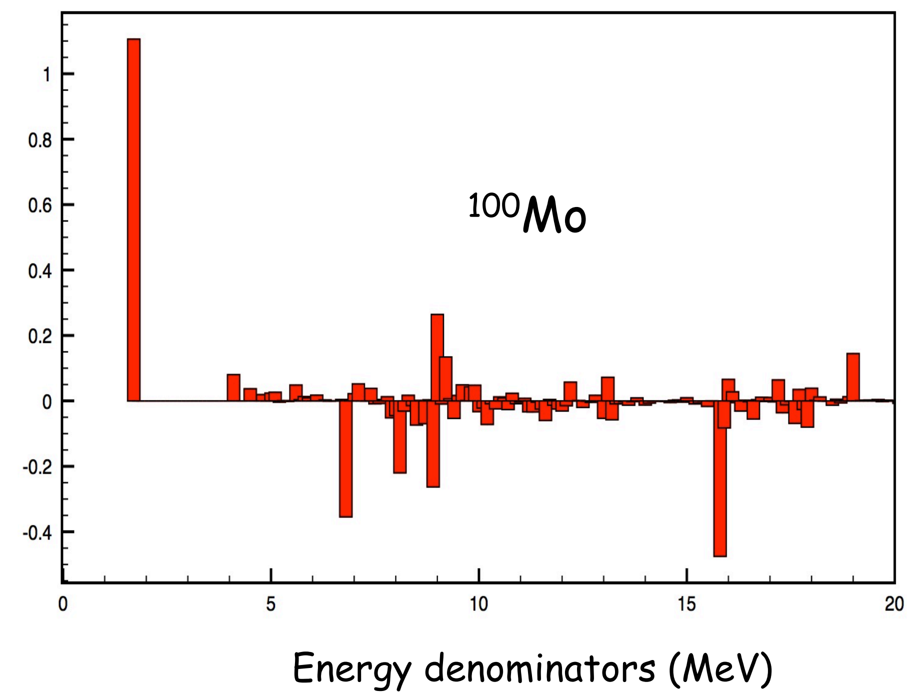




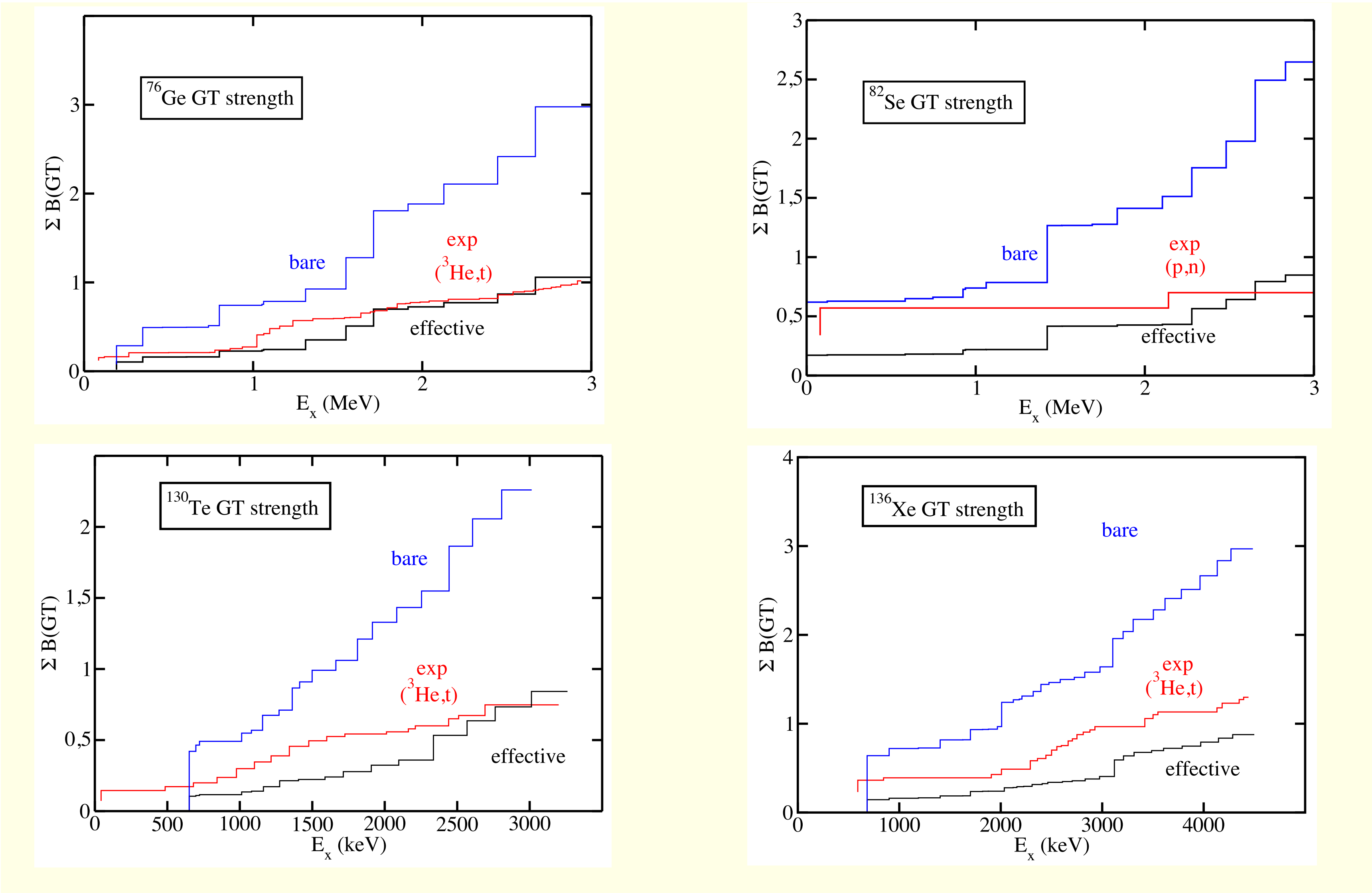


Distribution of the numerators,
<f|| $\sigma\,\tau^+$||n><n|| $\sigma\,\tau^+$ ||i>
evaluated in QRPA, for the indicated $2\nu\beta\beta$ decays.
In ^{100}Mo the ground 1^+ state of ^{100}Tc dominates, but there are significant positive and negative contributions at 5-15 MeV. For ^{136}Xe and ^{76}Ge there are significant, although mutually cancelling, contributions at ~ 10 MeV, at energy of the giant GT state.





Calculated and measured GT strength, Corragio et al, talk at INT-2018



However, the total GT strength (using the Ikeda sum rule and neglecting the β^+) is ^{76}Ge (36), ^{82}Se (42), ^{130}Te (78), ^{136}Xe (84). Thus, only a tiny fraction of the total GT strength is displayed.