Structure of three-body hypernuclei

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Fabian Hildenbrand Hans-Werner Hammer

Institute of Nuclear Physics
Technische Universität Darmstadt

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Bundesministerium
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Introduction

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- Hypernuclei contain at least one hyperon (H) ($S \neq 0$)
- Consider three-body hypernuclei NNH
- Restriction to Λ ($M_\Lambda = 1115.68$ MeV) particle as hyperon leads to the systems $S = -1$

$$\text{isospin}\,(I = 1) = \begin{cases} pp\Lambda \\ \frac{1}{\sqrt{2}}\,(np + pn)\,\Lambda \\ nn\Lambda \end{cases} \qquad \text{isospin}\,(I = 0) = \frac{1}{\sqrt{2}}\,(pn - np)\,\Lambda$$

- Two examples: nnΛ and hypertriton

The hypertriton

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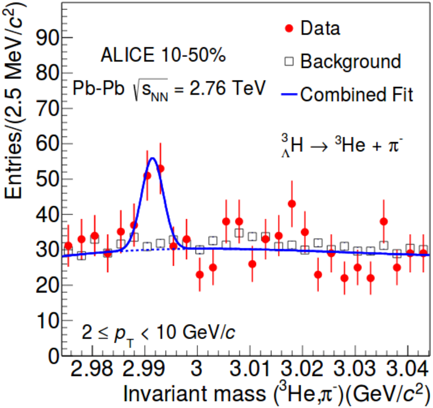


- Hypertriton is bound with a binding energy of $= 2.35 \pm 0.05$ MeV

  [M. Juric et al., Nucl. Phys. B52, 1 (1973)]

- Consists of separation energy into a deuteron ($B_d = 2.22$) MeV and a Λ $B_\Lambda = 0.13 \pm 0.05$ MeV

- Hypertriton was also produced at ALICE in Pb-Pb reaction recently

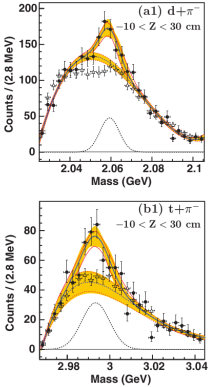
  [ALICE Collaboration Phys. Lett. B 754 (2016) 360-372]

- mean invariant mass $\mu = 2991 \pm 1 \pm 3$MeV $\Rightarrow B \approx 2$ MeV

The Λnn

- 2013 the HypHI Collaboration found evidence that the Λnn might be bound

[HypHI Collaboration C.Rappold et al. Phys.Rev.C.88(041001(R)),2013]



- d+ π^- and $t + \pi^-$final states ^{6}Li + ^{12}C reactions
- possible explanation of the observed final decay of a bound Λnn
- $^3_\Lambda n \to t + \pi^-$
  $^3_\Lambda n \to t^* + \pi^- \to d + n + \pi^-$
- mean invariant mass $\mu = 2993.7 \pm 1.3 \pm 0.6$ MeV $(t + \pi^-) \Rightarrow B \approx 1$ MeV

Theoretical framework

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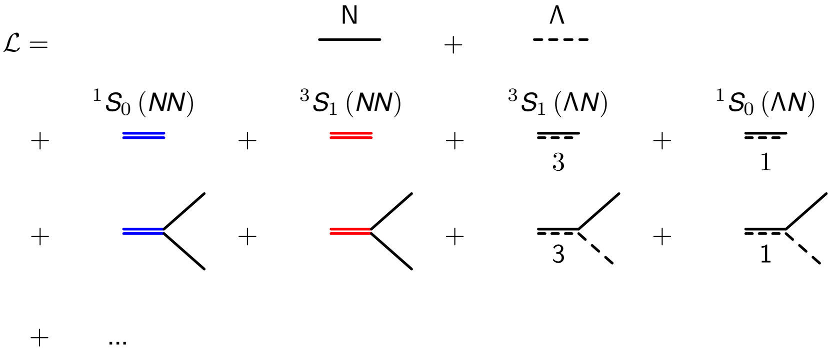
- Use pionless EFT $\rightarrow$ all interactions are contact interactions
- Exploit dibaryon formalism
- NNΛ interaction channels (only S-wave)

$$\mathrm{NN\Lambda} = \begin{cases} {}^1S_0\,(\mathrm{NN}) + \Lambda, & \Lambda\mathrm{nn} \\ {}^3S_1\,(\mathrm{NN}) + \Lambda, & \text{hypertriton} \\ {}^3S_1\,(\mathrm{N\Lambda}) + N, & \text{hypertriton and } \Lambda\mathrm{nn} \\ {}^1S_0\,(\mathrm{N\Lambda}) + N, & \text{hypertriton and } \Lambda\mathrm{nn} \end{cases}$$

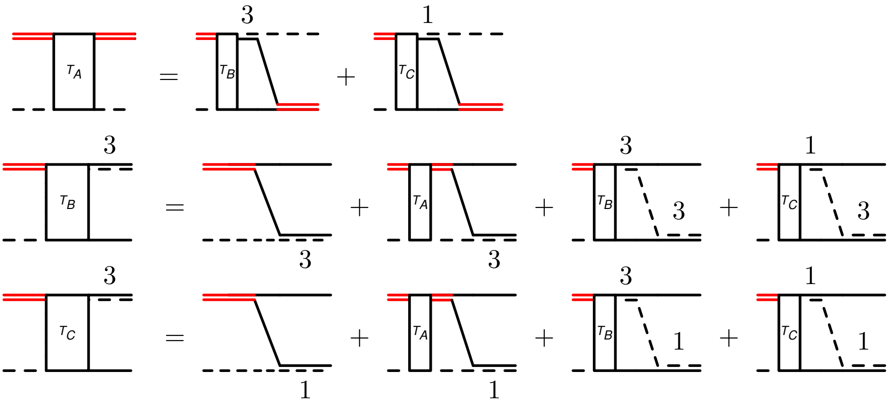
- Explicit $\Lambda \Leftrightarrow \Sigma$ conversions are not included (implicit in the 3-body force)

$$\gamma_3^\Lambda \sim 2\sqrt{MB_3^\Lambda/3} \approx 1.2\gamma_d \approx 54\ \mathrm{MeV}$$
$$\ll \sqrt{M_\Lambda\left(M_\Sigma - M_\Lambda\right)} \approx 300\ \mathrm{MeV}$$

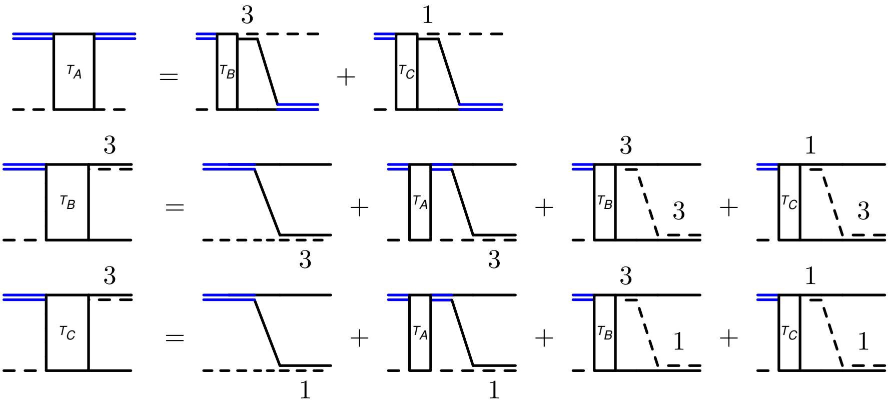
Lagrangian for the hypertriton and Λnn system



Integral equations for the hypertriton



Integral equations the Λnn system



Asymptotic Analysis of the hypertriton and Λnn system

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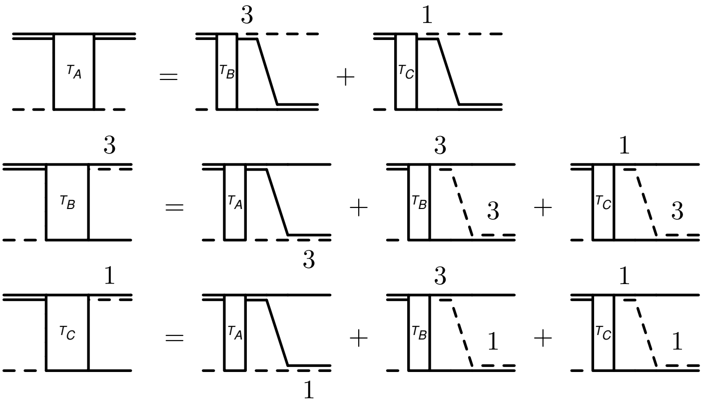
asymptotic limit
$\Lambda_c \gg p, q \gg 1/a, \gamma \sim k$
vanishing mass difference

→

equation becomes homogeneous
and can be decoupled
much simpler Kernel

Integral equations for the hypertriton and the Λnn system in the asymptotic limit



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$$T_i(p) = \frac{4}{\sqrt{3}\pi}\tilde{\lambda}_i \int dq \frac{1}{q} \ln\left(\frac{p^2+q^2+pq}{p^2+q^2-pq}\right) T_i(q), i \in 1,2,3$$

Efimov effect

solution: power law
$T_i(p) = p^{s_i}$

$\tilde{\lambda}_1 = 1 \Rightarrow$
complex exponent
$s_1 = 1.00624i$

one-parameter
three-body-force

different result compared to [Hammer Nucl.Phys. A 705:173 (2002)]

Asymptotic Analysis of the hypertriton and Λnn system

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→ one-parameter three-body-force

Efimov physics → universal relations and correlations

Asymptotic Analysis of the hypertriton and Λnn system-with different masses

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- Physical mass of Λ and nucleons are different
- Therefore asymptotic equations do not decouple
- Since the result of a power law should be reproduced for the case of $y = 0$ we choose as an ansatz

$$T_j\left(p\right) = \alpha_j p^{s_1} + \beta_j p^{s_2} + \gamma_j p^{s_3}, j \in \{A, B, C\}$$

- Since kernel is more complex now $\Rightarrow$ integrate term by term

Scaling factor as a function of the mass ratio

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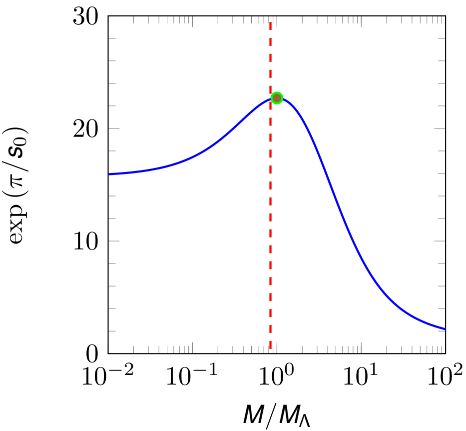

- Ansatz reproduces the $M = M_\Lambda$ result
- $s_0 = 1.00760(M/M_\Lambda = 0.84)$
- Results are consistent with Braaten and Hammer
  [Braaten, Hammer Phys. Rept. 428, 259–390 (2006)]
- Used chiral EFT prediction for Λ-N scattering lengths ($a_3 = -1.45 - 1.70$ fm, $a_1 = -2.90 - 2.91$ fm)
  [Haidenbauer et al Nucl. Phys. A 915 (2013) 24-58]
- considered to be large since ΛN range is given by the 2π exchange $\sim \frac{1}{2m_\pi} \approx 0.7$fm

Introduction of the 3-body force

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introduce three-body force term $H(\Lambda)$ in T_1
includes $\Lambda \Leftrightarrow \Sigma$ conversions

$$T_1(p) = \frac{4}{\sqrt{3}\pi} \int^{\Lambda} dq \frac{1}{q} \left[\ln\left(\frac{p^2+q^2+pq}{p^2+q^2-pq} \right) + 2\frac{H(\Lambda)}{\Lambda^2} pq \right] T_1(q)$$

transform back to original amplitudes T_A, T_B, T_C

theoretical expected form
$$H(\Lambda) = -\frac{\sin\left(s \log\left(\frac{\Lambda}{\Lambda_*}\right) - \arctan\left(\frac{1}{s}\right)\right)}{\sin\left(s \log\left(\frac{\Lambda}{\Lambda_*}\right) + \arctan\left(\frac{1}{s}\right)\right)}$$

compare

solve numerical
use B as input
to fix H

H(Λ) for the hypertriton

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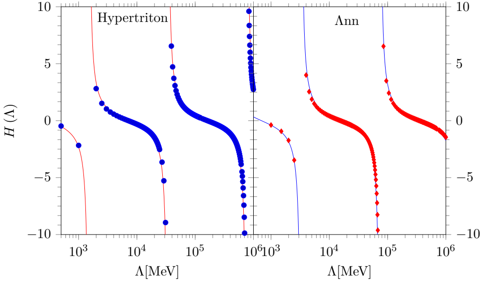
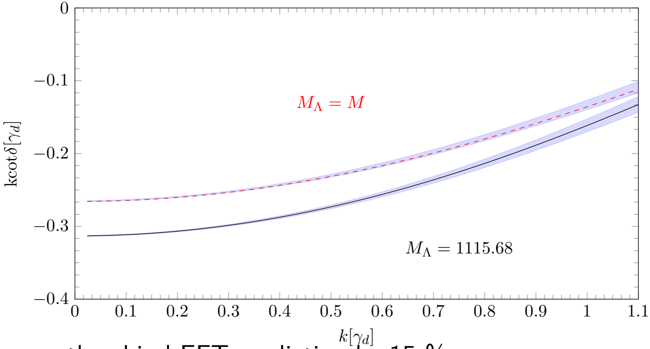


- ($a_3 = -1.64$ fm, $a_1 = -2.91$ fm)
- 3-body force shows the expected limit cycle behavior
- It is not possible to fix the Λnn 3-body force with the hypertriton one due to different isospin channels ($I = 1$ and $I = 0$)
- extract the three-body parameter $\Lambda_* \approx (0.1395 + 0.002)\gamma_d$ (hypertriton) $\Lambda_* \approx 0.3053\gamma_d$ (Λnn)
- for all further calculation absorbed into the cutoff

Scattering phase shift for deuteron Λ scattering



- Variation on the chiral EFT prediction by 15 %
- Scattering phase shift is insensitive towards exact values of the ΛN scattering lengths

Universal relation for the hypertriton

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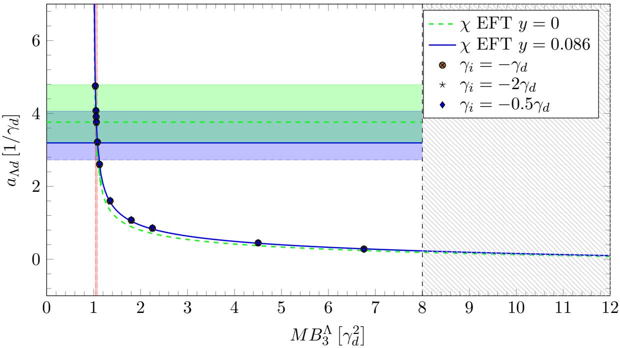
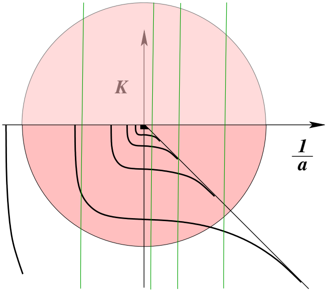


- Phillips line for the hypertriton
- Independent of the ΛN pole-position
- Differs for unphysical regions, defined by the pion mass

Λnn theory exspectations

- Λnn is always bound in this theory by construction system shows the effimov effect
- BUT! Λnn may not be within range of applicability



Universal relations for the Λnn system

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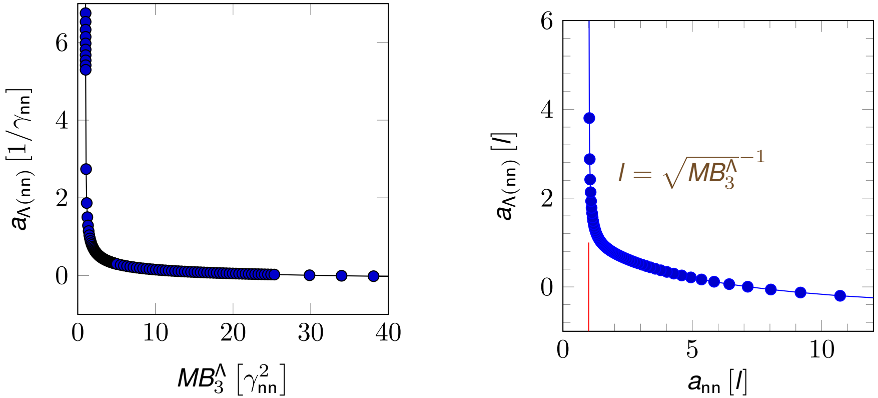
Use hypothetical positive scattering lengths for the n-n scattering lengths

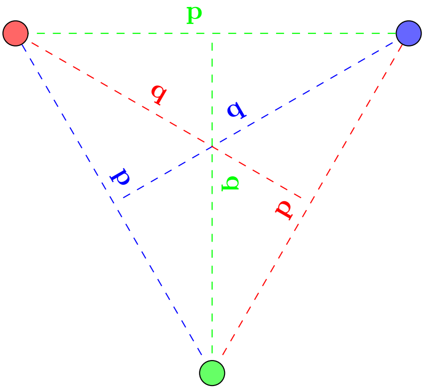


Wavefunction for the hypertriton

- 3 particle system ⇒ Introduce Jacobi-koordinates in momentum space presecribtion
- Use the same prescription used for 2-n Halo Systems.

[Canham, Hammer Eur.Phys.J.A37:367-380,2008]

[Hammer, Ji, Phillips J. Phys. G44, 103002,2017]



boundststate equation for the hypertriton

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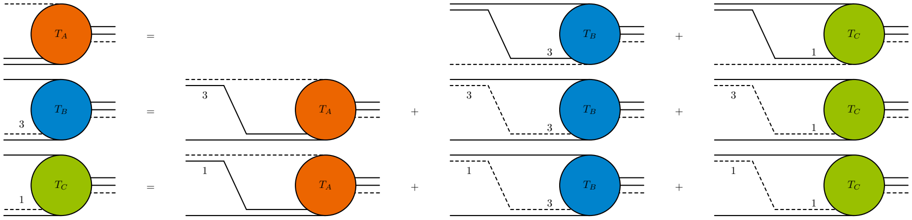
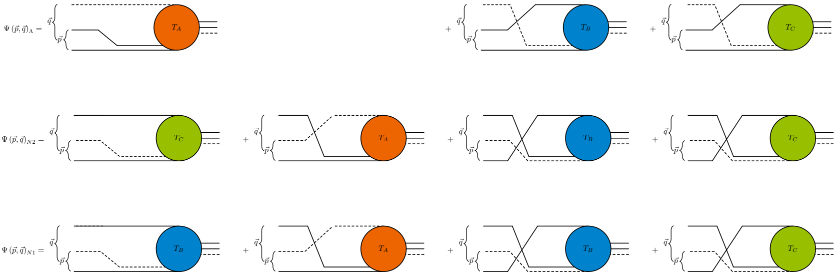


wavefunctions for the hypertriton



- Calculate Formfactors out of the wavefunctions for different observer particles
- Extract Observables like the matter nucleus (Expectation ≈ 10 fm)

Conclusion and Outlook

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- Presented an EFT approach to strangeness $S = -1$ three-body hypernuclei
- Showed you universal relations for the Λnn system and the hypertriton
- Study Λnn system dependence on input parameters
- Include explicit $\Lambda \Leftrightarrow \Sigma$ Conversions to check estimate

$$\gamma_3^{\Lambda} \sim 2\sqrt{MB_3^{\Lambda}/3} \approx 1.2\gamma_d \approx 54 \text{ MeV}$$
$$\ll \sqrt{M_{\Lambda}\left(M_{\Sigma} - M_{\Lambda}\right)} \approx 300 \text{ MeV}$$

Conclusion and Outlook

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- Calculate wave-function of the hypertriton
- Use this to calculate observable like matter-radius
- Test Sensitivity on $B_{\mathrm{np}\Lambda} - B_{\mathrm{d}}$