Description of ^{31}Ne in Halo EFT

TECHNISCHE UNIVERSITÄT DARMSTADT

International School of Nuclear Physics 40th Course

Ettore Majorana Center, Erice, Sicily
September 16 – 24, 2018

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September 20, 2018

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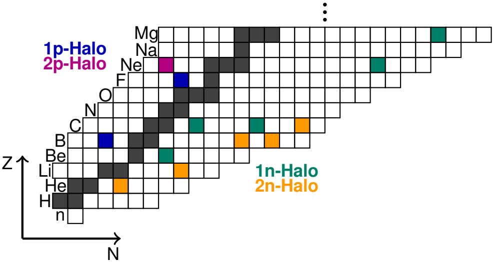
Introduction and Motivation

- Only small number of neutron halo nuclei have been identified
  - Most of them are neutron-rich light isotopes of He through C
- Until now, the heaviest 1n-halo nuclei are ^{37}Mg and $\boxed{^{31}\text{Ne}}$

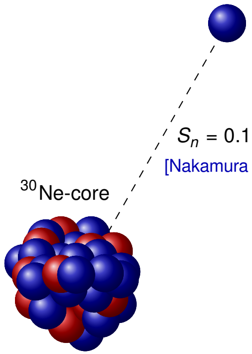


- 1n-removal reactions on C and Pb targets revealed: [Nakamura et al., 2014]
  ^{31}Ne deformed nucleus with a significant P-wave halo component

$\Rightarrow$ ^{31}Ne offers a prototype to study deformation-driven halos and understand emergent properties in heavier-near-drip-line nuclei

Introduction and Motivation

- One-neutron halo nuclei are exotic nuclear states
- Degrees of freedom: tightly bound core and a loosely bound valence neutron
- In general: valence neutron bound in a low-ℓ wave
- Quantum numbers determined: $J^P = \frac{3}{2}^-$ [Nakamura et al., 2014]



Effective Lagrangian

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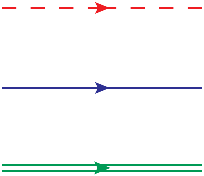
$$\mathcal{L} = c^\dagger \left[i\partial_0 + \frac{\overrightarrow{\nabla}^2}{2m_c} \right] c + n_\alpha^\dagger \left[i\partial_0 + \frac{\overrightarrow{\nabla}^2}{2m_n} \right] n_\alpha + \pi_\beta^\dagger \left[\eta \left(i\partial_0 + \frac{\overrightarrow{\nabla}^2}{2M_{nc}} \right) + \Delta \right] \pi_\beta$$

$$- g \left[\left(c \overleftrightarrow{\nabla}_i \, n_\alpha \right) \pi_\beta^\dagger \, C^{\frac{3}{2}\beta}_{(1i)(\frac{1}{2}\alpha)} + \text{H.c.} \right],$$

where $M_{nc} = m_n + m_c$ (total mass of neutron and core)

Feynman rules:



Full Dimeron Propagator

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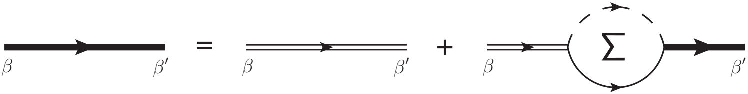
Full dimeron propagator:

Dress bare propagator with a geometric series of dimeron self-energies

$\Rightarrow$ Dyson equation:

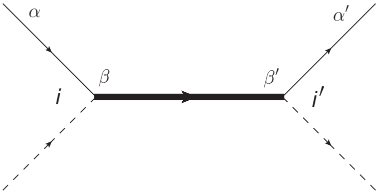


$$D_{\beta\beta'}(p_0, p) = \frac{\delta_{\beta\beta'}}{\eta\left(p_0 - p^2/(2M_{nc})\right) + \Delta - \Sigma + i\epsilon}$$

$$\Sigma = \frac{-\mu g^2 2\mu\left(p_0 - p^2/(2M_{nc})\right)}{6\pi}\left(\frac{3}{2}\Lambda^{\mathrm{PDS}} - \sqrt{-2\mu\left(p_0 - p^2/(2M_{nc})\right) - i\epsilon}\right),$$

where $\mu \,\hat{=}$ reduced mass

Scattering Amplitude: Renormalization



$$
\left.\begin{aligned}
T_{\alpha'\alpha}(\vec{p}\,',\vec{p}) &= \frac{6\pi}{\mu}\,\frac{\frac{2}{3}\vec{p}\,'\vec{p}\,\delta_{\alpha'\alpha} + \frac{i}{3}\left(\sum_l \sigma_l\left(\vec{p}\,'\times\vec{p}\right)_l\right)_{\alpha'\alpha}}{\left(\frac{-6\pi\Delta}{\mu g^2} - \frac{3\pi\eta}{\mu^2 g^2}p^2 - \Lambda p^2 - ip^3\right)} \\
&\equiv \frac{6\pi}{\mu}\,\frac{\frac{2}{3}\vec{p}\,'\vec{p}\,\delta_{\alpha'\alpha} + \frac{i}{3}\left(\sum_l \sigma_l\left(\vec{p}\,'\times\vec{p}\right)_l\right)_{\alpha'\alpha}}{\left(\frac{-6\pi\Delta^{\mathrm{R}}}{\mu\left(g^{\mathrm{R}}\right)^2} - \frac{3\pi\eta}{\mu^2\left(g^{\mathrm{R}}\right)^2}p^2 - ip^3\right)}
\end{aligned}\right\} \Rightarrow
\boxed{\begin{aligned}
\frac{-3\pi\eta}{\mu^2 g^2} - \Lambda &= \frac{-3\pi\eta}{\mu^2\left(g^{\mathrm{R}}\right)^2} \\
\frac{\Delta}{g^2} &= \frac{\Delta^{\mathrm{R}}}{\left(g^{\mathrm{R}}\right)^2}
\end{aligned}}
$$

Matching

Now compare renormalized amplitude to effective range expansion of amplitude:

$$
\left.
\begin{aligned}
T_{\alpha'\alpha}(\vec{p}', \vec{p}) &= \frac{6\pi}{\mu} \frac{\frac{2}{3}\vec{p}'\vec{p}\delta_{\alpha'\alpha} + \frac{i}{3}\left(\sum_l \sigma_l \left(\vec{p}' \times \vec{p}\right)_l\right)_{\alpha'\alpha}}{\left(\frac{-6\pi\Delta^{\mathrm{R}}}{\mu\left(g^{\mathrm{R}}\right)^2} - \frac{3\pi\eta}{\mu^2\left(g^{\mathrm{R}}\right)^2}p^2 - ip^3\right)} \\
&= \frac{6\pi}{\mu} \frac{\frac{2}{3}\vec{p}'\vec{p}\delta_{\alpha'\alpha} + \frac{i}{3}\left(\sum_l \sigma_l \left(\vec{p}' \times \vec{p}\right)_l\right)_{\alpha'\alpha}}{\left(-\frac{1}{a_1} + \frac{r_1}{2}p^2 - ip^3\right)}
\end{aligned}
\right\} \Rightarrow
\boxed{
\begin{aligned}
a_1 &= \frac{\mu\left(g^{\mathrm{R}}\right)^2}{6\pi\Delta^{\mathrm{R}}} \\
r_1 &= -\frac{6\pi\eta}{\mu^2\left(g^{\mathrm{R}}\right)^2}
\end{aligned}
}
$$

Note:

$a_1 \mathrel{\hat{=}}$ scattering volume

$r_1 \mathrel{\hat{=}}$ P-wave effective momentum

Effective Range Parameters

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Determine effective range parameters by using measured observables:

- Demand a pole in the amplitude at $E = -S_n = -\gamma^2/(2\mu)$ ($\gamma > 0 \hat{=}$ binding momentum)

$$\Rightarrow \left(-\frac{1}{a_1} + \frac{r_1}{2} p^2 - i p^3 \right) \Bigg|_{p = i\gamma} = 0$$

$$\Rightarrow a_1 = -\frac{2}{\gamma^2 (r_1 + 2\gamma)}$$

- Assuming only Δ/g^2 to be fine-tuned (shallow P-wave state) [Hammer and Phillips, 2011]

$\Rightarrow$ P-wave effective momentum $r_1 \sim M_{\mathrm{hi}}$ (breakdown scale of the theory)

$\Rightarrow$ scattering volume a_1 is enhanced by $1/(M_{\mathrm{lo}}^2 M_{\mathrm{hi}})$

EM Interaction – Minimal Substitution

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Photons are included via <u>minimal substitution:</u>

$$\boxed{\partial_\mu \to D_\mu = \partial_\mu + ie\hat{q}A_\mu \quad (e > 0)}$$

- $\hat{q}$ is the charge operator acting on a c or n field
  - $\hat{q}n = 0$
  - $\hat{q}c = q_c c$ with $q_c = 10$ for Ne

- Gauge-invariant operators involving $\vec{B}$ and $\vec{E}$ might contribute to electromagnetic observables within our power counting scheme

Scalar Component of EM Current

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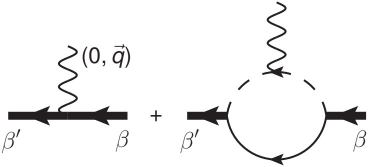
$$i\mathcal{A} = \langle \pi_{\beta'}(\vec{p}\,')|\, J_0 \,|\pi_\beta(\vec{p})\rangle =$$



$$= -i q_{\text{tot}} G_{\text{E0}}(q) \sqrt{4\pi} q^0 Y_{00}(\vec{e}_{\vec{q}}) \tilde{T}^{00}_{\beta'\beta} - i Q G_{\text{E2}}(q) \frac{1}{2}\sqrt{\frac{4\pi}{5}} q^2 \sum_M Y_{2M}(\vec{e}_{\vec{q}}) \tilde{T}^{2M}_{\beta'\beta}$$

$$\lim_{q\to 0} G_{\text{E0}}(q) \equiv 1 \quad \text{(Charge conservation given by gauge-invariance)}$$

$$\lim_{q\to 0} G_{\text{E2}}(q) \equiv 1 \quad \Rightarrow Q \text{ (this limit defines } Q\text{)}$$

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Electric Form Factors

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Monopole and Quadrupole Form Factors

$$G_{E0}(q) = \left[1 + \frac{\gamma}{|r_1|} - \frac{y^2q^2 + 2\gamma^2}{yq|r_1|} \arctan\left(\frac{yq}{2\gamma}\right)\right]$$

$$G_{E2}(q) = \frac{1}{Q}\frac{q_{\text{tot}}}{8|r_1|yq^3}\left[2\gamma yq + \left(y^2q^2 - 4\gamma^2\right) \arctan\left(\frac{yq}{2\gamma}\right)\right]$$

with $y = \frac{m_n}{M_{nc}} = \frac{1}{31}$, $\gamma \hat{=}$ binding momentum

and estimate P-wave effective momentum: $r_1 \sim M_{\text{hi}}$

- Consistency check: $\lim_{q\to 0} G_{E0}(q) \equiv 1$ ✓
- $\lim_{q\to 0} G_{E2}(q) \equiv 1 \Rightarrow \boxed{Q = \frac{y^2 q_{\text{tot}}}{3\gamma|r_1|}} \Rightarrow Q \in [0.17, 0.28]\, e\text{fm}^2$

Electric Form Factors

Charge and quadrupole radii are defined by expanding the form factors in q^2:

$$G_{E0}(q) \approx 1 - \frac{1}{6} < r_{E0}^2 > q^2 + ...$$
$$G_{E2}(q) \approx 1 - \frac{1}{6} < r_{E2}^2 > q^2 + ...$$

Now compare to the expansion of the calculated form factors:

Charge & Quadrupole Radii

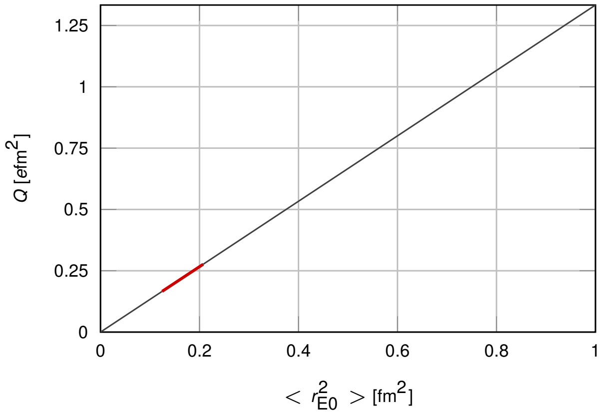
$$\Rightarrow < r_{E0}^2 > = \frac{5y^2}{2\gamma|r_1|} \qquad \Rightarrow \sqrt{< r_{E0}^2 >} \in [0.35, 0.46]\,\mathrm{fm}$$
$$\Rightarrow < r_{E2}^2 > = \frac{3y^2}{5\gamma^2} \qquad \Rightarrow \sqrt{< r_{E2}^2 >} = 0.30\,\mathrm{fm}$$

Correlations

$Q \longleftrightarrow < r^2_{\mathrm{E0}} >$:

- $< r^2_{\mathrm{E0}} > = \frac{5y^2}{2\gamma|r_1|}$
- $Q = \frac{y^2 q_{\mathrm{tot}}}{3\gamma|r_1|}$

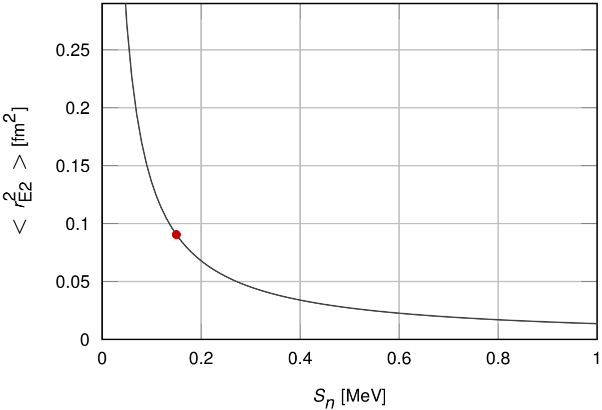
$$\Rightarrow \boxed{Q = \frac{2}{15} q_c e < r^2_{\mathrm{E0}} >}$$



Correlations

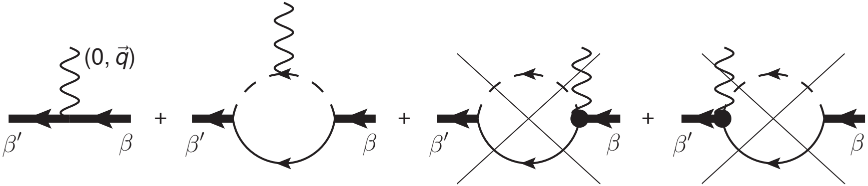
$< r_{\mathrm{E2}}^2 > \longleftrightarrow S_n$:

$$\boxed{< r_{\mathrm{E2}}^2 >= \frac{3y^2}{10\mu}\frac{1}{S_n}}$$

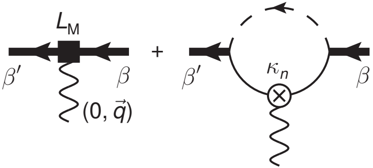


Vector Component of EM Current

Contributions to $\langle \pi_{\beta'}(\vec{p}')| \, J_k \, |\pi_\beta(\vec{p})\rangle$ due to minimal substitution:



Contributions to $\langle \pi_{\beta'}(\vec{p}')| \, J_k \, |\pi_\beta(\vec{p})\rangle$ due to magnetic moment coupling:



Vector Component of EM Current

$$\langle \pi_{\beta'}(\vec{p}')|\, J_k \,|\pi_\beta(\vec{p})\rangle$$

$$= \left(i q_{\text{tot}} G_{\text{E0}}(q) \sqrt{4\pi} q^0 Y_{00}(\vec{e}_{\vec{q}})\, \tilde{T}^{00}_{\beta'\beta} + i Q G_{\text{E2}}(q) \frac{1}{2} \sqrt{\frac{4\pi}{5}} q^2 \sum_M Y_{2M}(\vec{e}_{\vec{q}})\, \tilde{T}^{2M}_{\beta'\beta} \right) \frac{(\vec{p}' + \vec{p})^*_k}{2M_{nc}}$$

$$+ i\mu_{\text{M}} G_{\text{M1}}(q) \sqrt{\frac{4\pi}{3}} q^1 \sum_M \sqrt{2}\; C^{1M+k}_{(1k)(1M)} Y^*_{1M+k}(\vec{e}_{\vec{q}}) \left[\tilde{T}^{1M}_{\beta'\beta} \right]^\dagger$$

$$+ i o_{\text{M}} G_{\text{M3}}(q) \frac{1}{2} \sqrt{\frac{4\pi}{7}} q^3 \sum_M \sqrt{2}\; C^{3M+k}_{(1k)(3M)} Y^*_{3M+k}(\vec{e}_{\vec{q}}) \left[\tilde{T}^{3M}_{\beta'\beta} \right]^\dagger$$

Note:

For $J = 1/2$ the electric quadrupole and the magnetic octupole moment is not observable!

Magnetic Form Factors

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Octupole Form Factor

$$G_{M3}(q) = \frac{3\gamma}{2(1-y)^3 q^3}\left[2\gamma(1-y)q + \left((1-y)^2 q^2 - 4\gamma^2\right)\arctan\left(\frac{(1-y)q}{2\gamma}\right)\right]$$
$$= G_{E2}[y \to (1-y)]$$

Octupole Moment and Radius

$$o_M = \frac{(1-y)^2 \kappa_n \mu_N}{10\sqrt{6}\gamma|r_1|} \qquad \Rightarrow o_M \in [-23, -14]\, \mu_N \mathrm{fm}^2$$

$$< r_{M3}^2 > = \frac{3(1-y)^2}{10\mu}\frac{1}{S_n} \qquad \Rightarrow \sqrt{< r_{M3}^2 >} = 8.95 \text{ fm}$$

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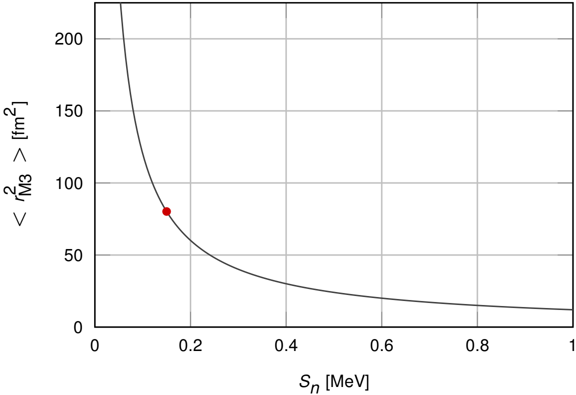
Correlations

$< r^2_{\mathrm{M3}} > \longleftrightarrow S_n$:

$$\boxed{< r^2_{\mathrm{M3}} >= \frac{3(1-y)^2}{10\mu}\frac{1}{S_n}}$$



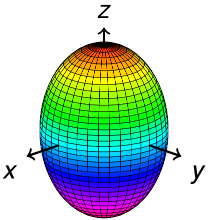
Nuclear Deformation

Consider a quadrupolar deformed nucleus with sharp edge at radius:

$$\boxed{R_{\mathrm{def}} = R_0\left[1 + \beta_2 Y_{20}\right]/N}$$

where $R_0 \hat{=}$ equilibrium radius,

$N \hat{=}$ volume normalization constant



$\Rightarrow$ Quadrupole moment:

$$Q(3/2) = \frac{1}{5}Q_0 = \frac{1}{5}\sqrt{\frac{16\pi}{5}}\frac{3}{4\pi}Ze\,R_0^2\,\beta_2$$

$$= \sqrt{\frac{1}{5\pi}}Ze\,\beta_2 < r_{E0}^2 >$$

$\xrightarrow{\text{Compare to EFT result}}$ $\boxed{\beta_2 = 0.53}$

Summary and Outlook

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Summary

- Effective Lagrangian
  - Scattering Amplitude
  - Renormalization
- Electric and Magnetic Properties
- Correlations
- Nuclear Deformation

Outlook

- Matter Radii
- Coulomb Breakup
- Improvement of Power Counting

$\Rightarrow$ Incorporation of Additional Field(s)

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Thank you for your attention!

References

Hammer, H. W. and Phillips, D. R. (2011).
Electric properties of the Beryllium-11 system in Halo EFT.
Nucl. Phys., A865:17–42.

Nakamura, T. et al. (2014).
Deformation-Driven *P*-Wave Halos at the Drip Line: Ne^{31}.
Phys. Rev. Lett., 112(14):142501.

Shamsuzzoha Basunia, M. (2010).
Nuclear Data Sheets for A = 30.
Nucl. Data Sheets, 111:2331–2424.