The Nuclear Equation of State and Neutron Stars

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The Strong Interaction: From Quarks and Gluons to Nuclei and Stars
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Neutron Star Mass and Radius Measurements

- Constraints from GR, Causality and a Neutron Star Maximum Mass
- Nuclear theoretical and experimental constraints
- Photospheric Radius Expansion Bursts
- Quiescent Low-Mass X-ray Binaries
- Pulsar Timing
- NICER and Moment of Inertia Constraints
- GW170817
Radii are highly correlated with neutron star matter pressure at $1 - 2n_s \simeq 0.16 - 0.32$ fm$^{-3}$.

Neutron star matter is nearly pure neutrons, $x \sim 0.04$.

Nuclear symmetry energy

$S(n) \equiv E_0(n) - E_{1/2}(n)$

$E_x(n) \simeq E_{1/2}(n) + S_2(n)(1-2x)^2 + \ldots$

$S(n) \simeq S_2(n) \simeq S_v + \frac{L}{3} \frac{n - n_s}{n_s} + \frac{K_{sym}}{18} \left( \frac{n - n_s}{n_s} \right)^2 R_{1.4} \ldots$

$S_v \equiv S_2(n_s) \sim 32$ MeV, $L \sim 50$ MeV; nuclear systematics.

Neutron matter energy and pressure at $n_s$:

$E_0(n_s) \simeq S_v + E_{1/2}(n_s) = S_v - B \sim 13 - 17$ MeV

$p_0(n_s) = \left( n^2 \frac{\partial E_0(n)}{\partial n} \right)_{n_s} \simeq \frac{L n_s}{3} \sim 2.1 - 3.7$ MeV fm$^{-3}$
A lower limit to the maximum mass sets a lower limit to the radius for a given mass. Similarly, a precision upper limit to $R$ sets an upper limit to the maximum mass.

$R_{1.4} > 8.15(10.9)$ km if $M_{\text{max}} \geq 2.01M_{\odot}$.

$M_{\text{max}} < 3.0(2.3) M_{\odot}$ if $R < 13$ km.

If quark matter exists in the interior, the minimum radii are substantially larger; maximum masses are considerably smaller.
Neutron matter energy should be larger than the unitary gas energy $E_{UG} = \xi_0 (3/5) E_F$

$$E_{UG} = 12.6 \left( \frac{n}{n_s} \right)^{2/3} \text{MeV}$$

The unitary gas refers to fermions interacting via a pairwise short-range s-wave interaction with an infinite scattering length and zero range. Cold atom experiments show a universal behavior with the Bertsch parameter $\xi_0 \simeq 0.37$.

$S_v \geq 28.6 \text{ MeV}; \quad L \geq 25.3 \text{ MeV}; \quad p_0(n_s) \geq 1.35 \text{ MeV fm}^{-3}; \quad R_{1.4} \geq 9.7 \text{ km}$
Theoretical and Experimental Constraints

H Chiral Lagrangian

G: Quantum Monte Carlo

neutron matter calculations from Hebeler et al. (2012)

unitary gas constraints from Tews et al. (2017)

Combined experimental constraints are compatible with unitary gas bounds.

Neutron matter calculations are compatible with both.

$10.9 \, \text{km} \leq R_{1.4} \leq 13.1 \, \text{km}$
Both the assumed minimum value of the neutron star maximum mass and the assumed matter pressure at $n_1 = 1.5 - 2n_s$ are important in restricting $M - R$ and $p - \varepsilon$ values.

$8.4 < p_1 < 20$ MeV fm$^{-3}$
Simultaneous Mass and Radius Measurements

- Measurements of flux $F_\infty = \left(\frac{R_\infty}{D}\right)^2 \sigma T_{\text{eff}}^4$ and color temperature $T_c \propto \lambda_{\text{max}}^{-1}$ yield an apparent angular size (pseudo-BB):
  \[
  R_\infty/D = \frac{(R/D)}{\sqrt{1 - 2GM/Rc^2}}
  \]

- Observational uncertainties include distance $D$, nonuniform $T$, interstellar absorption $N_H$, atmospheric composition

  Best chances are:
  - Isolated neutron stars with parallax (atmosphere ??)
  - Quiescent low-mass X-ray binaries (QLMXBs) in globular clusters (reliable distances, low $B$ H-atmospheres)
  - Bursting sources with peak fluxes close to Eddington limit (PREs); gravity balances radiation pressure
  \[
  F_{\text{Edd}} = \frac{cGM}{\kappa D^2} \sqrt{1 - 2GM/Rc^2}
  \]
Photospheric Radius Expansion X-Ray Bursts

\[ F_{Edd} = \frac{GMc}{\kappa D^2} \sqrt{1 - \frac{2GM}{R_{ph}c^2}} \]

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\[ A = f_c^{-4}(R_\infty/D)^2 \]


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The Nuclear Equation of State and Neutron Stars
PRE Burst Models

Ozel et al. $z_{\text{ph}} = z \quad \beta = \frac{GM}{RC^2} \quad$ Steiner et al. $z_{\text{ph}} \ll z$

\[ F_{\text{Edd}} = \frac{GMc}{\kappa D} \sqrt{1 - 2\beta} \]

\[ A = \frac{F_\infty}{\sigma T_\infty^4} = f_c^{-4} \left( \frac{R_\infty}{D} \right)^2 \]

\[ \alpha = \frac{F_{\text{Edd}} \kappa D}{\sqrt{A} F_c^2 c^3} = \beta(1 - 2\beta) \]

\[ \gamma = \frac{Af_c^4 c^3}{F_{\text{Edd}}^4 c^3} = \frac{R_\infty}{\alpha} \]

\[ \beta = \frac{1}{4} \pm \frac{1}{4} \sqrt{1 - 8\alpha} \]

\[ \alpha \leq \frac{1}{8} \quad \text{required.} \]

\[ F_{\text{Edd}} = \frac{GMc}{\kappa D} \]

\[ \alpha = \beta \sqrt{1 - 2\beta} \]

\[ \theta = \cos^{-1} \left( 1 - 54\alpha^2 \right) \]

\[ \beta = \frac{1}{6} \left[ 1 + \sqrt{3} \sin \left( \frac{\theta}{3} \right) \right] - \cos \left( \frac{\theta}{3} \right) \]

\[ \alpha \leq \sqrt{\frac{1}{27}} \approx 0.192 \quad \text{required.} \]

EXO1745-248 4U1608-522 4U1820-30 KS1731-260 SAXJ1748.9-2021

0.19 ± 0.04 0.25 ± 0.06 0.24 ± 0.04 0.20 ± 0.03 0.18 ± 0.04

observed $\alpha$ values (Ozel et al.)
PRE $M - R$ Estimates

\[ \alpha_{\text{min}} = \begin{align*} 
0.164 \pm 0.024 \\
0.153 \pm 0.039 \\
0.171 \pm 0.042 \\
0.164 \pm 0.037 \\
0.167 \pm 0.045 \\
0.198 \pm 0.047 
\end{align*} \]

Özel & Freire (2016)
QLMXBs (Guillot & Rutledge 2013)

ω Cen

Causality

common R

NGC 6397

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Reconciling sources with disparate values of $R_\infty$, with little information concerning $z$, forces a solution near the smallest $R_\infty$. Sources then have a wide range of predicted masses.
QLMXB $M - R$ Estimates

Özel & Freire (2016)
8 QLMXBs (Steiner et al. 2018)

Parameterized EOS priors

Consistent with Steiner et al. (2013)
Although simple average mass of w.d. companions is 0.23 M⊙ larger, weighted average is 0.04 M⊙ smaller.

Demorest et al. 2010
Fonseca et al. 2016
Antoniadis et al. 2013
Barr et al. 2016
Champion et al. 2008

vanKerkwijk 2010
Romani et al. 2012
A 1.6ms pulsar in circular 9.17h orbit with $\sim 0.03 \, M_\odot$ companion. The pulsar is eclipsed for 50-60 minutes each orbit; eclipsing object has a volume much larger than the secondary or its Roche lobe. Pulsar is ablating the companion leading to mass loss and the eclipsing plasma. The secondary may nearly fill its Roche lobe. Ablation by the pulsar leads to secondary’s eventual disappearance. The optical light curve tracks the motion of the secondary’s irradiated hot spot rather than its center of mass motion.
PSR J2215-5135

- Redback binary MSP
- $P_{\text{orb}} = 4.14$ hr
- $T_{\text{night}} = 5660^{+260}_{-380}$ K
- $T_{\text{day}} = 8080^{+470}_{-280}$ K
- $D = 2.9 \pm 0.1$ kpc
- $e = 0.144 \pm 0.002$
- Roche lobe filling factor $f = 0.95 \pm 0.01$
- $M_{\text{pulsar}} = 2.27^{+0.17}_{-0.15} M_{\odot}$
- $M_{\text{comp}} = 0.33^{+0.02}_{-0.02} M_{\odot}$

Linares et al. (2018)
Lightcurve modeling constrains the compactness $(M/R)$ and viewing geometry of a non-accreting millisecond pulsar through the depth of modulation and harmonic content of emission from rotating hot-spots, thanks to gravitational light-bending...
... while phase-resolved spectroscopy promises a direct constraint of radius $R$. 
LIGO-Virgo detected a signal consistent with a BNS merger, followed 1.7 s later by a weak sGRB.

- 16600 orbits observed over 165 s.
- $M_{\text{chirp}} = 1.1867 \pm 0.0001 \, M_\odot$
- $M_{\text{tot, max}} = 2^{6/5} M_{\text{chirp}} = 2.726 M_\odot$
- $E_{\text{rad}} > 0.025 M_\odot c^2$
- $D_L = 40 \pm 10 \, \text{Mpc}$
- $84 < \tilde{\Lambda} < 640 \, (90\%)$
- $M_{\text{ejecta}} \sim 0.06 \pm 0.02 \, M_\odot$
- Blue ejecta: $\sim 0.01 M_\odot$
- Red ejecta: $\sim 0.05 M_\odot$
- Likely r-process production

Abbott et al. (2017)
Drout et al. (2017)

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Properties of Double Neutron Star Binaries

DNS with only an upper limit to $m_p$

DNS with $\tau_{GW} = \infty$
Waveform Model Parameter Determinations

There are 13 free wave-form parameters including finite-size effects at third PN order $(\nu/c)^6$. LV17 used a 13-parameter model; De et al. (2018) used a 9-parameter model.

- Sky location (2) EM data
- Distance (1) EM data
- Inclination (1)
- Coalescence time (1)
- Coalescence phase (1)
- Polarization (1)
- Component masses (2)
- Perpendicular spins (2)
- Tidal deformabilities (2)

Extrinsic

Intrinsic
correlated with masses
Tidal deformability $\lambda$ is the ratio between the induced dipole moment $Q_{ij}$ and the external tidal field $E_{ij}$, $Q_{ij} \equiv -\lambda E_{ij}$.

$k_2$ is the dimensionless Love number. It is convenient to work with the dimensionless quantity

$$\Lambda = \frac{\lambda c^{10}}{G^4 M^5} \equiv \frac{2}{3} k_2 \left( \frac{R c^2}{GM} \right)^5 .$$

For a binary neutron star, the relevant quantity is

$$\bar{\Lambda} = \frac{16}{13} \frac{(1 + 12q) \Lambda_1 + (12 + q) q^4 \Lambda_2}{(1 + q)^5} ,$$

$$\delta \Phi_t = -\frac{117}{256} \frac{(1 + q)^4}{q^2} \left( \frac{\pi f_{GW} G M}{c^3} \right)^{5/3} \bar{\Lambda} + \cdots .$$
Deformability and the Radius

\[ \Lambda = a \left( \frac{Rc^2}{GM} \right)^6 \]
\[ a = 0.0085 \pm 0.0010 \text{ for } M = 1.35 \pm 0.25 \, M_\odot \]
\[ \tilde{\Lambda} = \frac{16}{13} \left( 1 + 12q \right) \Lambda_1 + q^4 \left( 12 + q \right) \Lambda_2 \]
\[ R_1 \simeq R_2 \simeq \hat{R} \]
\[ \tilde{\Lambda} \simeq \frac{16a \left( \frac{\hat{R}c^2}{GM} \right)^6 q^{8/5} (12 - 11q + 12q^2)}{(1 + q)^{26/5}} \]
\[ \Lambda = a' \left( \frac{\hat{R}c^2}{GM} \right)^6 \]
\[ a' = 0.0035 \pm 0.0007 \text{ for } M = 1.2 \pm 0.2 \, M_\odot \]
\[ a' = 0.0039 \pm 0.0002 \text{ GW10817} \]
\[ \hat{R} = 11.5 \pm 0.3 \frac{M}{M_\odot} \left( \frac{\tilde{\Lambda}}{800} \right)^{1/6} \, \text{km} \]
\[ \hat{R} = 13.4 \pm 0.1 \left( \tilde{\Lambda}/800 \right)^{1/6} \, \text{km GW170817} \]
\[
\Lambda_1 = q^6 \Lambda_2
\]

\[
\tilde{\Lambda} = \frac{16}{13} \Lambda_1 \frac{12 - 11q + 12q^2}{q^2(1 + q)^4}
\]
Measurability of Tidal Deformability

De et al. (2018)
De18 takes advantage of the precisely-known electromagnetic source position (Soares-Santos et al., 2017).

Uses existing knowledge of the $H_0$ and the redshift of NGC 4993 to fix the distance (Cantiello et al., 2017).

Assumes both neutron stars have the same equation of state, which implies $\Lambda_1 \simeq q^6 \Lambda_2$.

Baseline model effectively has 9 instead of 13 parameters.

Explores variations of mass, spin and deformability priors.

Low-frequency cutoff taken to be 20 Hz, not 30 Hz (LV17), doubling the number of analyzed orbits.

De18 finds mild evidence ($B \sim 10$) for finite-size effects, but no evidence for spins.

Finds a 90% lower confidence bound to $\tilde{\Lambda}$.

Finds strong evidence ($B \sim 400$) for a common EOS, which argues against one star being a hybrid star.

Finds including $\Lambda - M$ correlations reduces $\tilde{\Lambda}$ by $\sim 30\%$. 

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GW170817

uniform short 20Hz

Zhou and Lattimer (2018)

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The Nuclear Equation of State and Neutron Stars
LV18 Determined $R_1 \simeq R_2$ for GW170817

$\Lambda_2 - \Lambda_1$ correlations from parameterized EOSs with $M_{\text{max}} > 1.97M_\odot$

$R_1(\text{km}) = 11.9^{+1.4}_{-1.4}$

$R_2(\text{km}) = 11.9^{+1.4}_{-1.4}$
GW170817 Summary

- It’s important to include correlations among $\Lambda_1$, $\Lambda_2$, $M_1$ and $M_2$ in a model-independent fashion.
- It’s important to include as many orbits as possible; low frequency cutoff is about 20 Hz.
- Better waveform models appear to reduce fitting uncertainties in $\tilde{\Lambda}$ (by about 15%).
- LV18 constraints on radii depend on how the EOS is modeled. Compared to using bounds on the deformability-mass correlation, their method biases results towards central values of the EOS parameter and radius ranges.
- Upper bounds to $M_{\text{max}}$ should not play a role in radius determinations, but do in the LV18 analyses.
- With a better waveform model, De et al. should obtain a 90% confidence upper limit to $\hat{R}$ of about 12.4 km.
- With unitary gas constraint, $\tilde{\Lambda} > 160$, $\hat{R} > 10.7$ km.
- Coughlin et al. (2018) claim $\tilde{\Lambda} > 197$ from EM evidence.
The Future

- A few more neutron star mergers per year should allow significant improvements to radii and maximum mass constraints. The systems are expected to be quite similar to GW170817, except they will be further away and may not have optical transients or sGRBs.
- It should be possible to establish a pulsar-timing bounds to the moment of inertia of PSR 0737-3037A, which with its known mass, will set tight radius constraints.
- NICER expects to release radius estimates with 1 km-accuracy by years’ end.
- Ongoing and planned nuclear experiments, including neutron skin measurements at J-Lab and Mainz, will offer competing information.
- Further ab-initio improvements will be important.
- Further measurements of black widow pulsar systems should give us some surprises.